What's the fuss about fuzzy dark matter?

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Basic definition

Fuzzy dark matter involves a very light (pseudo)scalar field, with associated quantum phenomena at astrophysically large scales

I will cover the two aspects which you may not be familiar with:

I.

Scalar fields?!?

Where are the DM 'particles'? How do you produce the DM?

(Generic for light (pseudo)scalar candidates, also axion to some extent)

II.

What's the peculiar phenomenology related to the fuzziness?

Part I.

The cosmology of these scalars, in a nutshell

Key notions and difference with respect to WIMPs

- **The DM behaviour is obtained as the "classical field" limit of the new dof**
- **The implementation typically requires light mass terms and BSM physics** (e.g. new symmetry breaking) **at very-high energies,** typically no link with EW scale/collider ones

What I want to show you:

- **The conditions under which a scalar field in the early universe behaves as DM**
- **The conditions needed to match the DM abundance**

For that, one needs some notions on scalar fields in the early universe!

Eq. of motion and stress-energy of a scalar field *X*

In flat FLRW: $$ds^2 = dt^2 - a(t)^2 d\mathbf{x}^2$$

For simplicity, consider
free massive particle potential $$V = M_X^2 X^2/2$$

$$a^3[\ddot{X} + 3H\,\dot{X} + M_X^2 X] = 0$$

A scalar field *X* is also associated to a stress-energy tensor.
FRLW symmetries require it to be of the "perfect fluid" form

$$T_{\alpha\beta}(X) = (\rho + P)u_\mu u_\nu - P g_{\mu\nu}$$

One can prove that:

$$\rho = \frac{1}{2}\dot{X}^2 + V(X) \qquad P = \frac{1}{2}\dot{X}^2 - V(X)$$

Early time solution

$$a^3[\ddot{X} + 3H\,\dot{X} + M_X^2 X] = 0$$

if mass term negligible wrt expansion rate (i.e. at sufficiently high temperatures)

$$H^2 \gg M_X^2$$

by setting $\dot{X} = W$ the equation reduces approximately to

$$\dot{W} + 3H\,W \simeq 0$$

whose solution is a constant (plus a transient)

$$X(t) = X_1 + W_1 \int_{t_1}^{t} \left(\frac{a_1}{a}\right)^3 dt$$

X "gets frozen" due to the high expansion rate, acting like friction (overdamping)

Late time solution

$$a^3[\ddot{X} + 3H\,\dot{X} + M_X^2 X] = 0$$

If mass term large wrt expansion rate
(i.e. at sufficiently low temperatures)

$$H^2 \ll M_X^2$$

The field oscillates fast, on the top of which "slow" evolution driven by *H*

In fact, consider the energy density

$$\rho = \frac{1}{2}(\dot{X}^2 + M_X^2 X^2)$$

From Fried. Eq., averaging over times much longer than M_X^{-1} but shorter than H^{-1}

$$\langle \dot{\rho} \rangle = -3\,H\,\langle \dot{X}^2 \rangle$$

and using virial theorem

$$\langle \dot{X}^2 \rangle = 2\langle K \rangle = \langle K \rangle + \langle V \rangle$$

valid for harmonic potential

$$\langle \dot{\rho} \rangle = -3\,H\,\langle \rho \rangle \Rightarrow \langle \rho \rangle = \langle \rho \rangle_1 \left(\frac{a_1}{a} \right)^3$$

The **field average energy** density evolves as the one for **cold dark matter**!

DM from 'misalignment'

$$\rho_0 = M_X\, n_X^* \left(\frac{a_*}{a}\right)^3 \simeq M_X \frac{\rho_*}{M_X} \left(\frac{a_*}{a}\right)^3 \simeq M_X^2 A_*^2 \left(\frac{a_*}{a_0}\right)^3$$

↓

$$\rho_0 \simeq M_X^2 A_*^2 \frac{g_S(T_0) T_0^3}{g_S(T_*) T_*^3}$$

↓

where T* is given roughly by the condition **3H(T*)=M_X**, which clearly yields (in the radiation era) **T*~(M_{Pl} M_X)$^{1/2}$**. The scaling is thus

$$\rho_0 \propto M_X^{1/2} A_*^2 \,,$$

$$\rho_0 \sim 10^{-5} \mathrm{GeV\,cm^{-3}} \sqrt{\frac{M_X}{\mathrm{eV}}} \left(\frac{A_*}{10^{12}\,\mathrm{GeV}}\right)^2 , \Leftrightarrow \Omega_X h^2 \sim 0.1 \sqrt{\frac{M_X}{100\,\mathrm{meV}}} \left(\frac{A_*}{10^{12}\,\mathrm{GeV}}\right)^2$$

Note: light particles + large values for the initial field displacement needed

Morally ok, but scaling different for the axion case (mass "time dependent" etc.…)

Part II.

Fuzzy DM

As above, for light bosons ($m \sim 10^{-22}$ eV) hence with kpc-sized De Broglie wavelength

Introduced by *W. Hu, R. Barkana and A. Gruzinov, PRL 85, 1158 (2000) [astro-ph/0003365]*

Revived by *L. Hui, J.P. Ostriker, S. Tremaine and E. Witten, PRD 95, 043541 (2017) [1610.08297]*

Main historical motivation:

Halo cutoff at low masses and profile flattening due to "uncertainty principle"

"Quantum nature" smooths small-scale structures (solitonic cores) possibly solving "small scale problems" (like cusped halos predicted in pure CDM vs cored halos inferred)

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Existing virialized structures require

$$\frac{\lambda_{DB}}{2\pi} \lesssim \frac{G M_{\rm halo}}{v_{\rm vir}^2}$$

Or better, radius containing 1/2 mass of a spherically symmetric, time-independent, self-gravitating system of FDM satisfies

$$r_{1/2}\, M_{\rm halo} \geq 3.925 \frac{\hbar^2}{G\, m^2} \simeq 0.3\,{\rm kpc} \frac{10^9\, M_\odot}{M} \left(\frac{10^{-22}\,{\rm eV}}{m}\right)^2$$

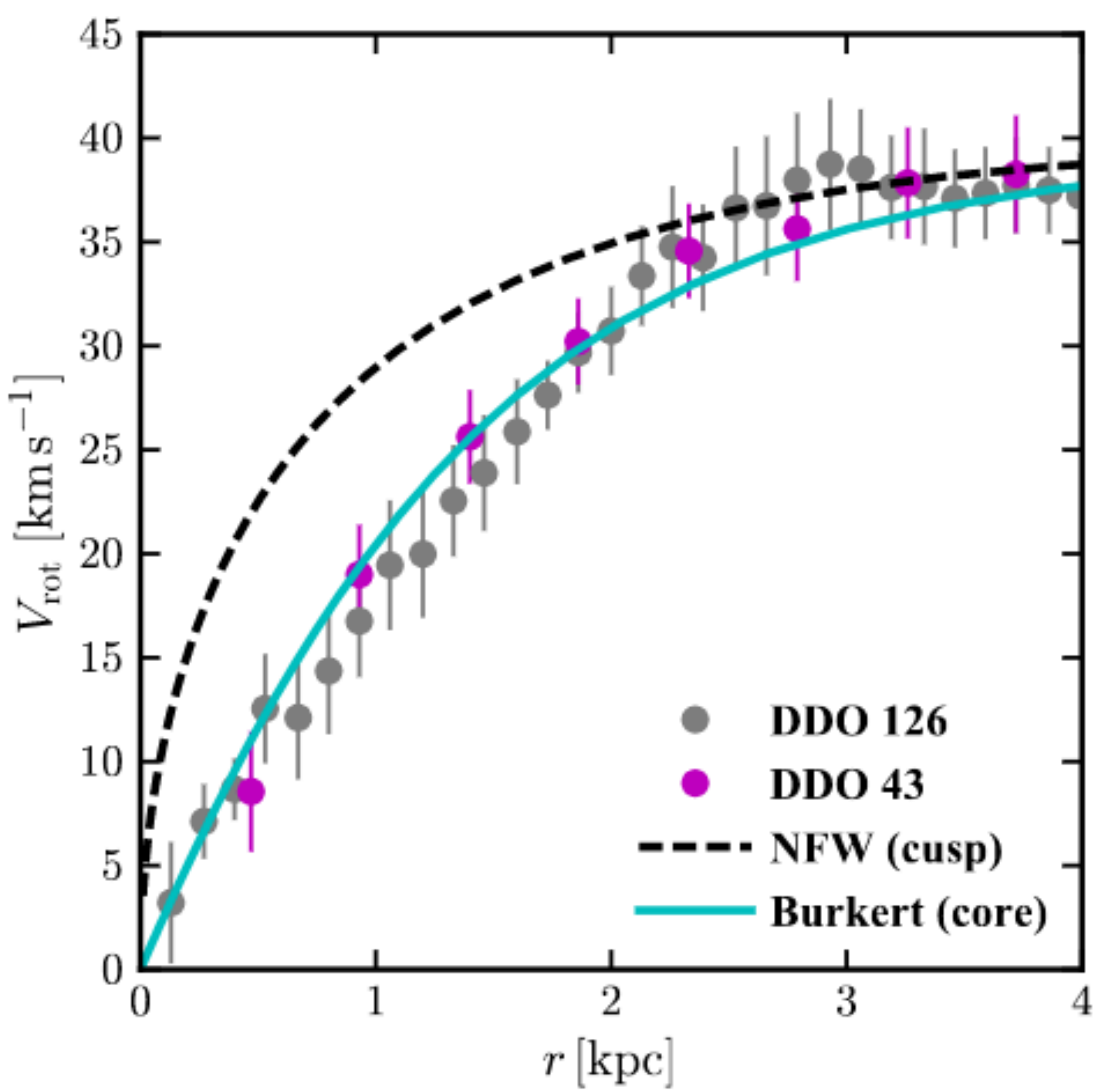
Possible motivation: Cusp/core problem ?

B. Moore, "Evidence against dissipationless dark matter from observations of galaxy haloes," Nature 370, 629 (1994)

R.A. Flores & J.R. Primack, "Observational and theoretical constraints on singular dark matter halos," ApJ. 427, L1 (1994)

Central regions of DM dominated galaxies as inferred from rotation curves tend to be both ***less dense*** (in normalization) ***and less cuspy*** (in inferred density profile slope) ***than predicted*** for CDM-only halos.

The issue is most prevalent for dwarf and low surface brightness galaxies



Plot from and more details in

J. S. Bullock and M. Boylan-Kolchin, "Small-Scale Challenges to the ΛCDM Paradigm," Ann. Rev. Astron. Astrophys. 55, 343 (2017)[1707.04256]

I. Unclear to which extent this is a problem (baryonic effects neglected!)

II. That FDM provides good quantitative agreement with the data is challenged in

A. Burkert, ApJ 904, no.2, 161 (2020) [2006.11111]

More formal derivation of "quantum pressure" term

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2\,m}\nabla^2\psi + m\Phi\psi$$

"semiclassical"
Schrödinger-Poisson eq.

$$\nabla^2\Phi = 4\pi G\, m|\psi|^2$$

Madelung transform

$$\psi(\vec{r},t) = |\psi|(\vec{r},t)e^{iS(\vec{r},t)} = \sqrt{\frac{\rho(\vec{r},t)}{m}}e^{iS(\vec{r},t)}$$

$$\rho(\vec{r},t) = m|\psi|^2 \qquad \vec{j}(\vec{r},t) = \frac{\hbar}{2im}\left(\psi^*\vec{\nabla}\psi - \psi\vec{\nabla}\psi^*\right) = n\frac{\hbar}{m}\vec{\nabla}S$$

Introduce a density and current, related to modulus and phase gradient

Or, in terms of velocity: $\vec{j} = n\vec{v} \qquad \vec{v} = \frac{\hbar}{m}\vec{\nabla}S$

The Schr. Eq. is equivalent to continuity + 'modified' Euler equation

$$\frac{\partial\rho}{\partial t} + \vec{\nabla}\cdot(\rho\vec{v}) = 0 \qquad \rho\frac{\partial\vec{v}}{\partial t} + \rho(\vec{v}\cdot\vec{\nabla})\vec{v} = -\rho\vec{\nabla}Q - \rho\vec{\nabla}\Phi$$

Euler equation contains a 'quantum pressure' due to "Bohm potential" *Q*

$$Q = -\frac{\hbar^2}{2m^2}\frac{\Delta\sqrt{\rho}}{\sqrt{\rho}}$$

Some phenomenological consequences

More and more refined studies appearing

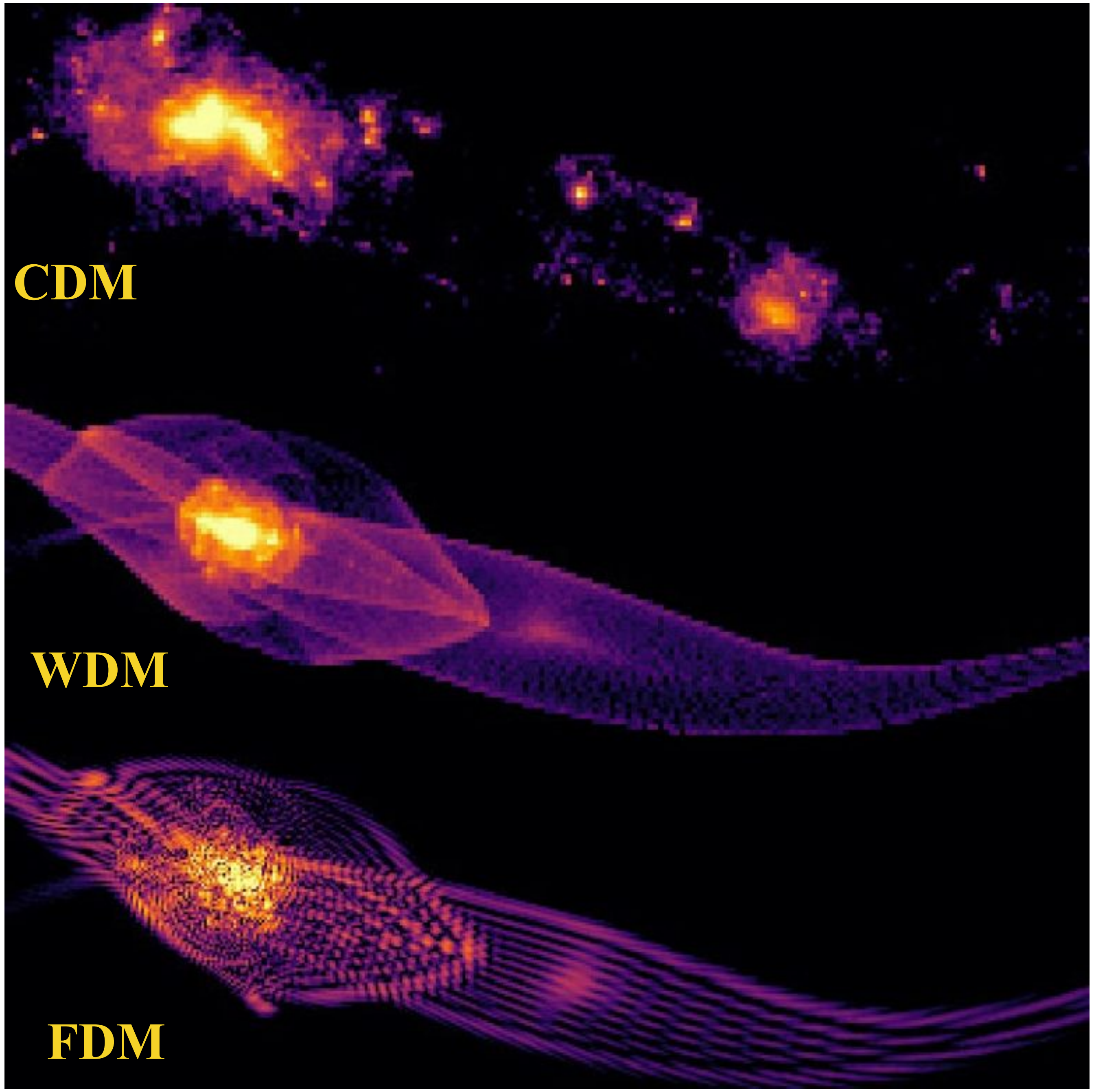
e.g. simulations including baryonic effects *J. Chan et al. MNRAS (2018) 478, 2686*

Comparative studies CDM/WDM/FDM *P.Mocz et al., PRL 123, 141301 (2019)*

Mixed CDM+FDM *B. Schwabe et al., PRD 102 083518 (2020)*

Qualitatively like warm DM: smoother features, suppresses small-scale halos (but due to quantum nature, not dispersion velocity!)

Further "quantum" effects appear, like interference patterns, time-dependent potential… which are in principle a smoking gun

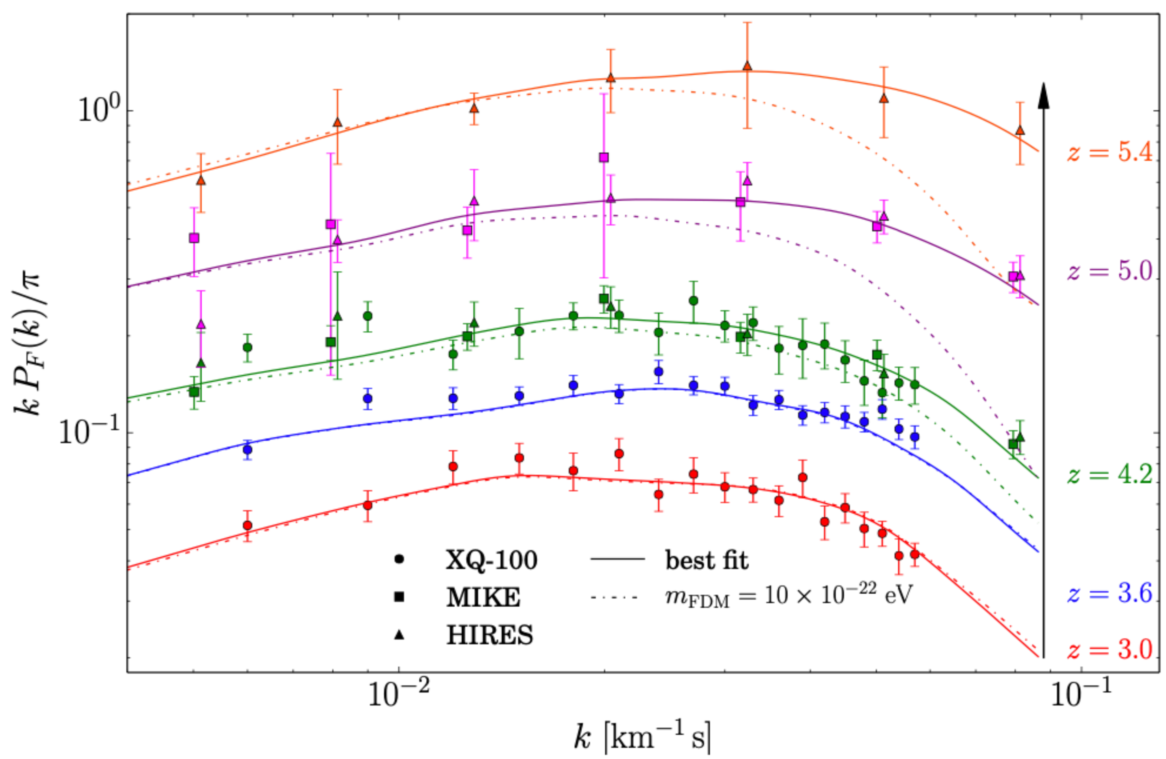


Constraints

Range interesting for addressing "small scale problems" severely constrained by

Ly-α forest

V. Irsic, M.Viel, PRL 119, no.3, 031302 (2017) [1703.04683]



Survival of the old star cluster in Eridanus-II dwarf galaxy under time-dependent fluctuations in the density on time scales which are shorter than the gravitational timescale

D. J. E. Marsh and J. C. Niemeyer, PRL 051103 (2019) [1810.08543]