

L.A.P.P seminar, April 9th 2021

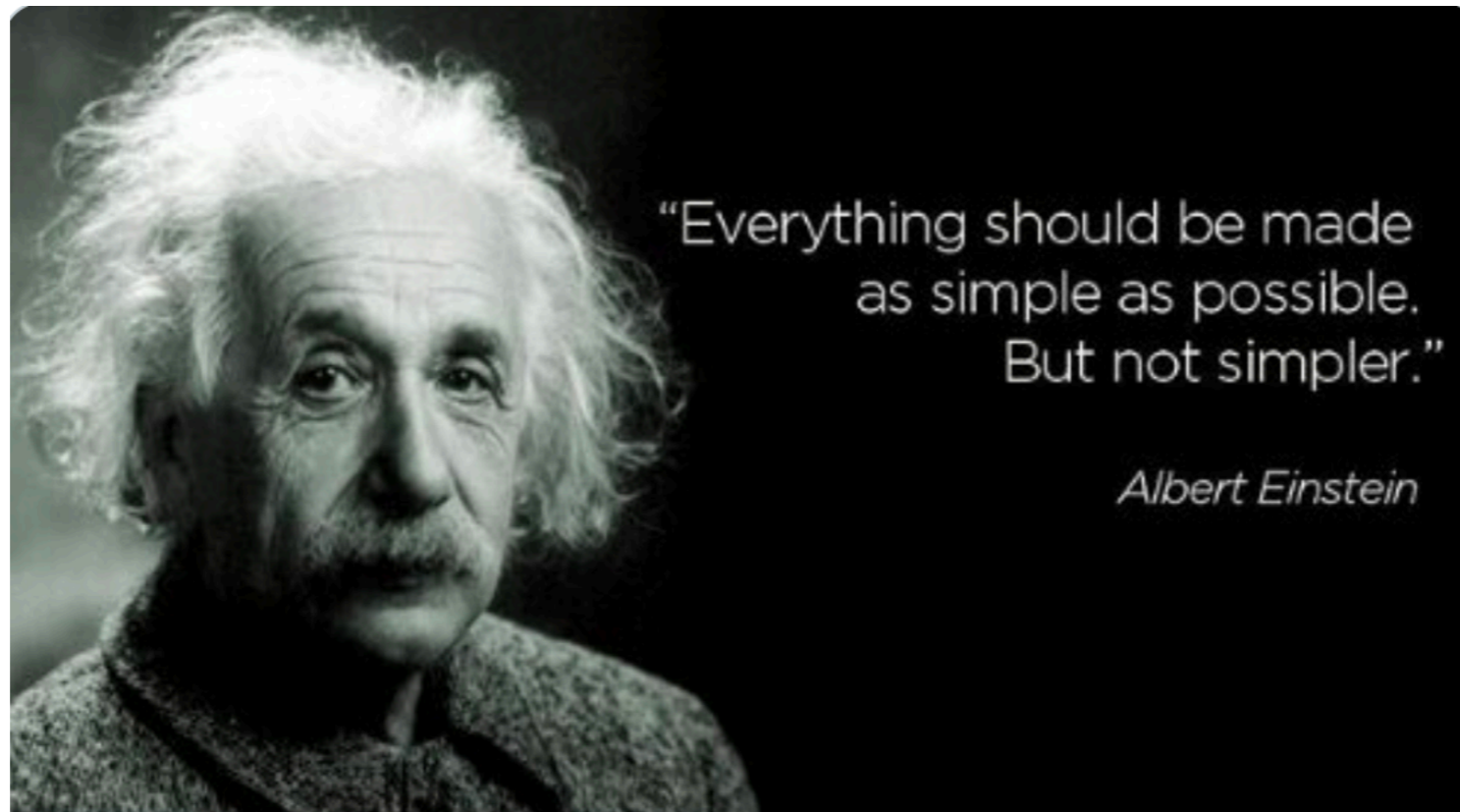
SMEFT interpretations of ATLAS measurements in the EW and Higgs sector.

Ana Cueto (CERN)



Effective field theory

- ▶ Effective field theory is not another model
- ▶ It is a very powerful tool used in different fields of physics
- ▶ Effective field theories allows to test rigorously a theory adapting it to your testing conditions (e.g. low energy)

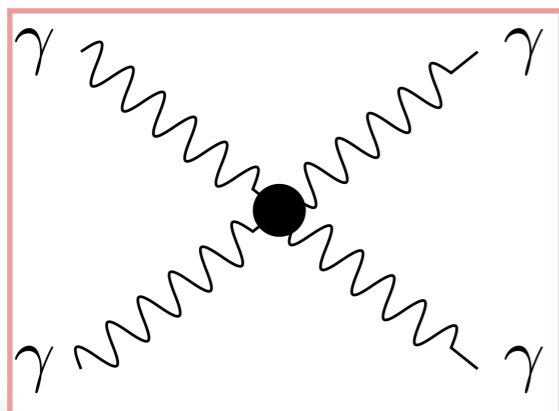


A simple example: Light-by-light at low energies (Euler-Heisenberg Lagrangian)

- ▶ Experiment: two photon beams with $E_\gamma \ll 2m_e$
- ▶ Effective Lagrangian only with photons preserving: gauge and Lorentz invariance, charge conjugation, parity constraints
 - * Expand in $1/m_e$

$$\mathcal{L}_{\text{eff}} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{a}{m_e^4}(F^{\mu\nu}F_{\mu\nu})^2 + \frac{b}{m_e^4}F^{\mu\nu}F_{\nu\sigma}F^{\sigma\rho}F_{\rho\mu} + \mathcal{O}\left(\frac{F^6}{m_e^8}\right) \quad [F] = 2$$

- ▶ At low-energies, QED dynamics determined by two terms
 - * In this case we know the high-energy theory, but a and b could be determined from the experiment

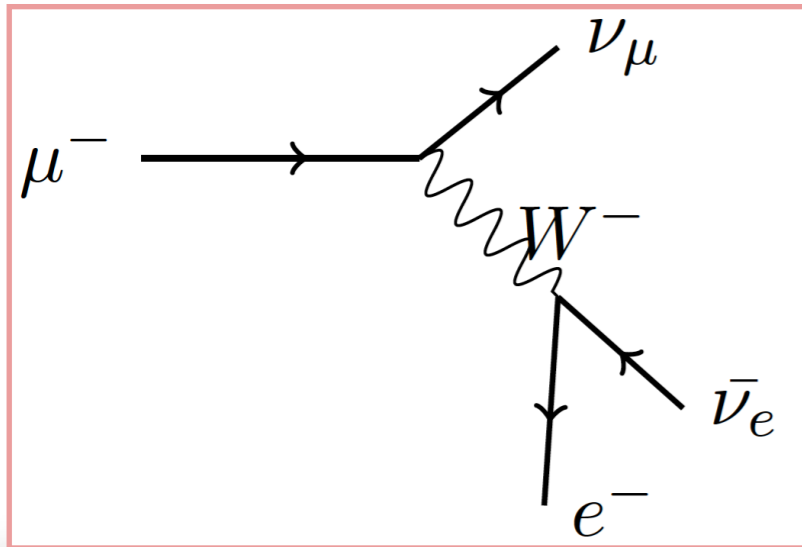


$$A_{2\gamma} \sim \frac{\alpha^2 E^4}{m_e^4} \longrightarrow \sigma_{2\gamma} \sim \left(\frac{\alpha^2 E^4}{m_e^4}\right)^2 \frac{1}{E^2} = \frac{\alpha^4 E^6}{m_e^8}$$

each photon carries a factor e and each gradient produces a power of energy

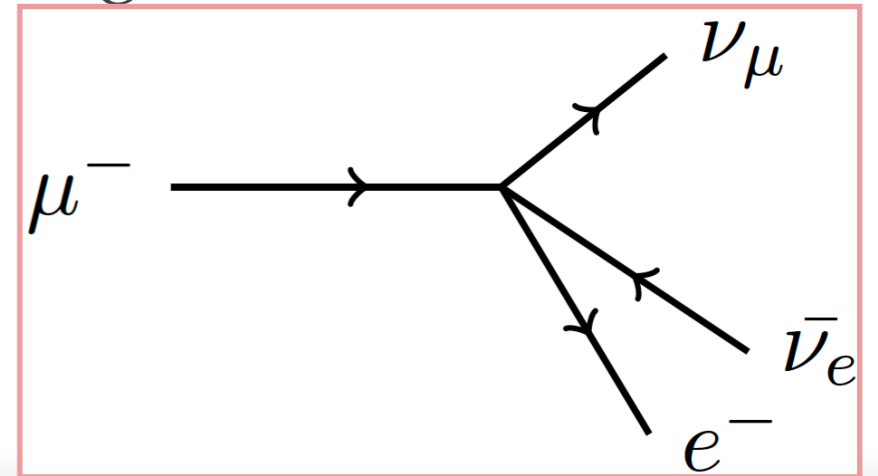
Another example: Fermi theory

- ▶ In SM, muons decay to electrons and neutrinos mediated by a W boson



$$\frac{-g_{\mu\nu} + q_\mu q_\nu / M_W^2}{q^2 - M_W^2} \xrightarrow{q^2 \ll M_W^2} \frac{g_{\mu\nu}}{M_W^2}$$

- ▶ Momentum transfer carried by the W very small compared to M_W . Contact interaction at low energies

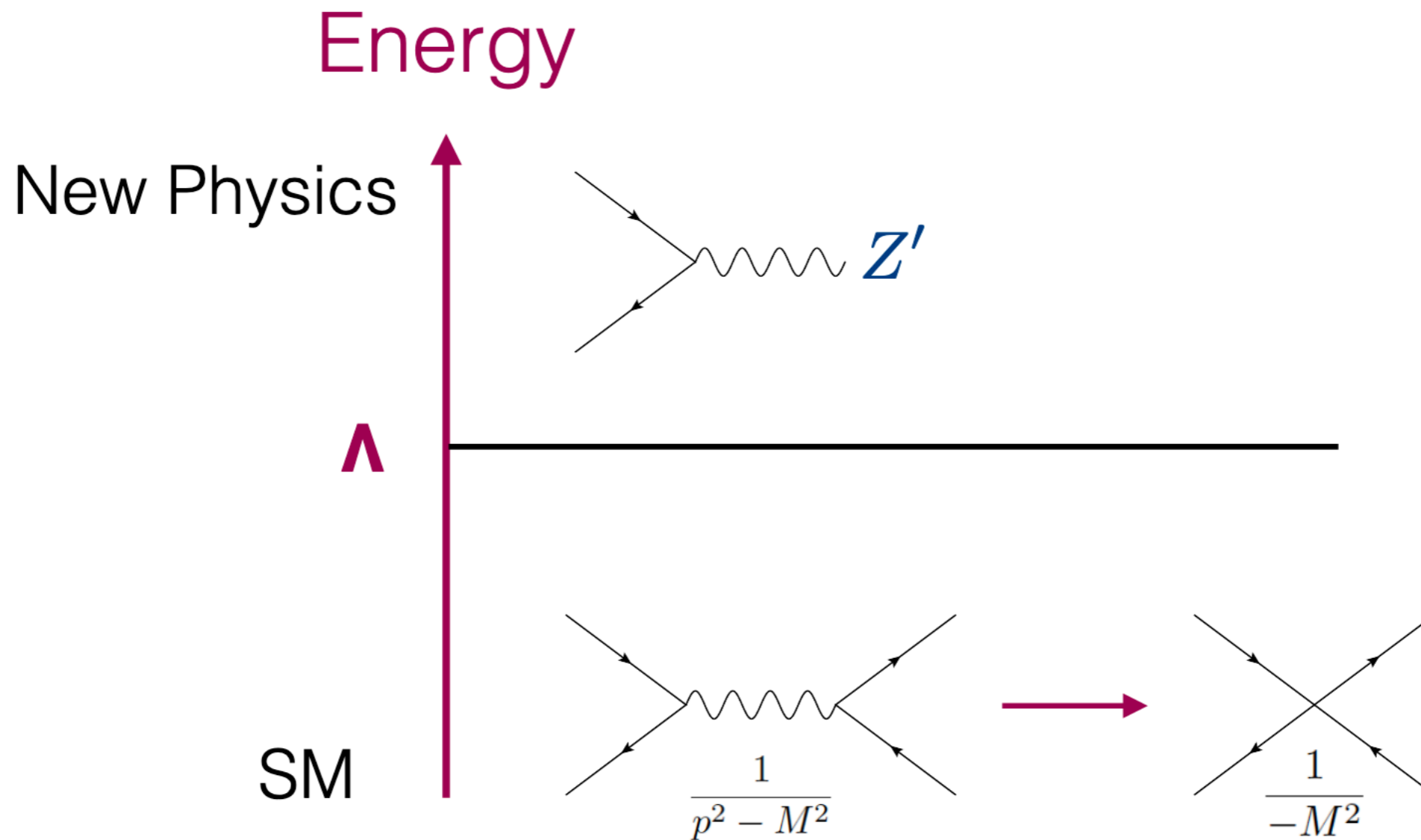


$$\mathcal{L}_f = -\frac{g}{2\sqrt{2}} \{W_\mu^\dagger J^\mu + h.c.\}$$

$$\mathcal{L}_{eff} = -\frac{G_F}{\sqrt{2}} \{J_\mu^\dagger J^\mu\}, \quad G_F = \frac{g^2}{8M_W^2}$$

- ▶ Effective Lagrangian predicts tau BR of 17.79% **Experimentally measured at (17.83±0.04)%**
- ▶ Cross sections grows with energy. Is it wrong? No, it is only valid in a given regime

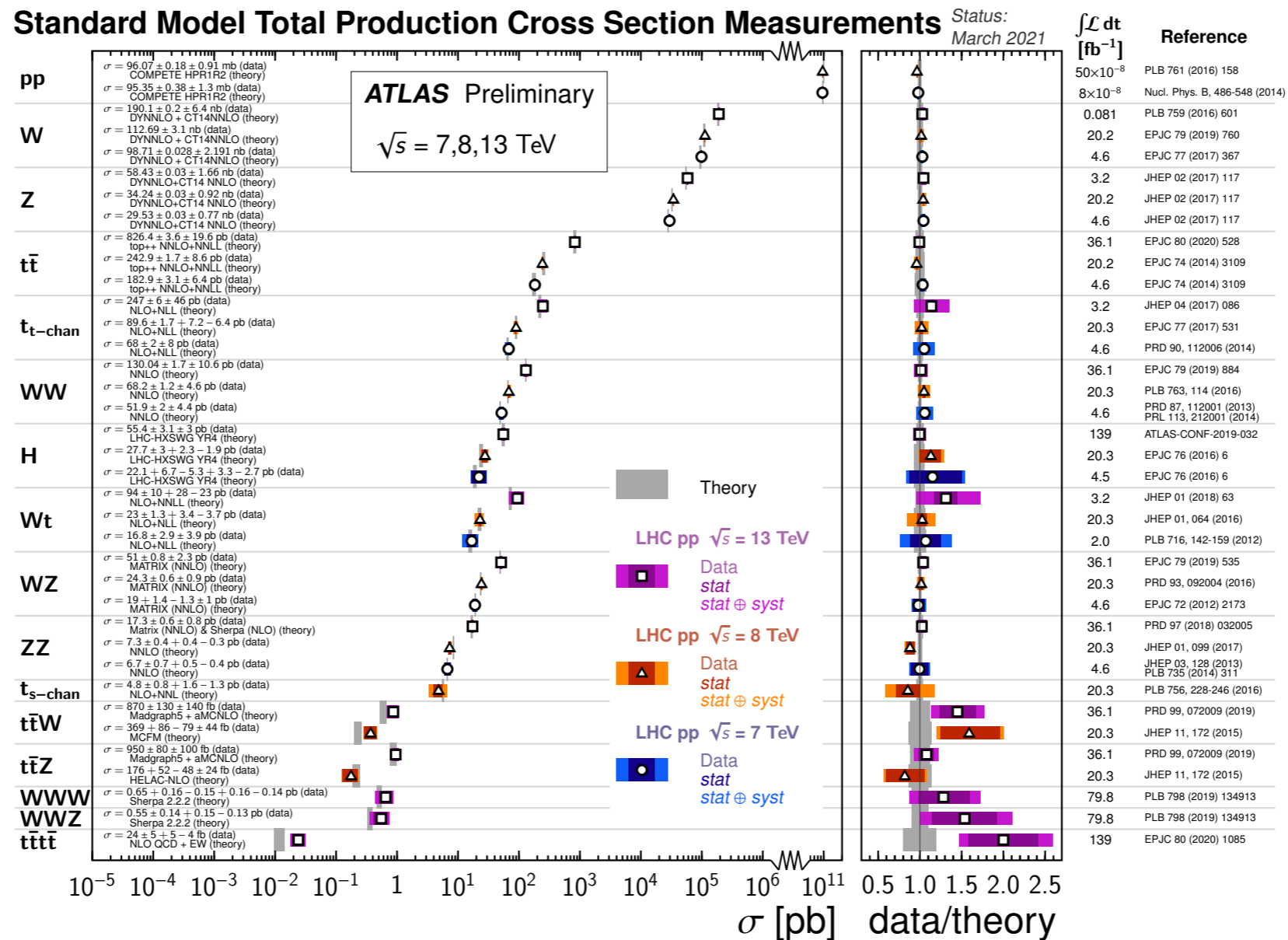
General idea in Standard Model EFT



$$\frac{1}{p^2 - M^2} = \frac{1}{-M^2} \left[1 + \left(\frac{p^2}{M^2} \right) + \left(\frac{p^2}{M^2} \right)^2 + \dots \right]$$

Why SMEFT at the LHC?

- Large success of the SM so far at the LHC and no clear evidence of BSM physics from direct searches



- SMEFT allows a systematic interpretation of large datasets with the only assumption that new physics is happening at larger scales

Standard Model EFT

- ▶ We assume that the SM is just an EFT. Opposite to the previous cases we do not know the high-energy theory
- ▶ Take an energy cut-off $\Lambda \gg \text{vev}$ and write down the most general Lagrangian preserving SM symmetries and particle content

$$\mathcal{L}_{SMEFT} = \mathcal{L}_{SM} + \sum_i \frac{c_i^{d=6}}{\Lambda^2} \mathcal{O}^{d=6} + \sum_i \frac{c_i^{d=8}}{\Lambda^4} \mathcal{O}^{d=8} + \dots$$

c_i are the so-called Wilson coefficients

- ▶ Only c_i / Λ^{d-4} is measurable
- ▶ Constrain EFT coefficients \rightarrow constrain large classes of UV theories
- ▶ SMEFT is a complete QFT compatible with NLO calculations, in contradistinction to kappa framework or anomalous couplings interpretations

Bases

- ▶ At each dimension, several bases can be worked out

Basis: Complete set of not-redundant operators. Takes into account:

- * Group identities (Fierz)
- * Equations of motion
- * Integration by parts

- ▶ Some examples for **dimension 6**:

Basis	Underlying gauge symmetry	Fields used in the Lagrangian
Warsaw, SILH	$SU(3)_C \times SU(2)_L \times U(1)_Y$	Gauge-eigenstates
BSM primaries, Higgs	$SU(3)_C \times SU(2)_L \times U(1)_Y$	Mass-eigenstates
Higgs/BSM characterisation	$SU(3)_C \times U(1)_{EM}$	Mass-eigenstates

From Eur. Phys. J. C (205) 75:583

- ▶ Full RGE for **Warsaw** basis (being standardised in experiments but translation is always possible)
- ▶ Number of operators depends on flavour assumptions
 - * 2499 in $d=6$ for $N_f=3$; 76 for $N_f=1$

Some operators in the Warsaw basis

Z,W couplings

$$\begin{aligned}
 Q_{HI}^{(1)} &= (iH^\dagger \overleftrightarrow{D}_\mu H)(\bar{l}\gamma^\mu l) \\
 Q_{He} &= (iH^\dagger \overleftrightarrow{D}_\mu H)(\bar{e}\gamma^\mu e) \\
 Q_{Hq}^{(1)} &= (iH^\dagger \overleftrightarrow{D}_\mu H)(\bar{q}\gamma^\mu q) \\
 Q_{Hq}^{(3)} &= (iH^\dagger \overleftrightarrow{D}_\mu^i H)(\bar{q}\sigma^i\gamma^\mu q) \\
 Q_{Hu} &= (iH^\dagger \overleftrightarrow{D}_\mu H)(\bar{u}\gamma^\mu u) \\
 Q_{Hd} &= (iH^\dagger \overleftrightarrow{D}_\mu H)(\bar{d}\gamma^\mu d)
 \end{aligned}$$

$$\begin{aligned}
 Q_{HD} &= (D_\mu H^\dagger H)(H^\dagger D^\mu H) \\
 Q_{HWB} &= (H^\dagger \sigma^i H)W_{\mu\nu}^i B^{\mu\nu} \\
 Q_{HI}^{(3)} &= (iH^\dagger \overleftrightarrow{D}_\mu^i H)(\bar{l}\sigma^i\gamma^\mu l) \\
 Q'_{ll} &= (\bar{l}_p\gamma^\mu l_r)(\bar{l}_r\gamma^\mu l_p)
 \end{aligned}$$

input quantities

$$Q_W = \varepsilon_{ijk} W_\mu^{i\nu} W_\nu^{j\rho} W_\rho^{k\mu}$$

TGC

Bhabha scattering

$$\begin{aligned}
 Q_{ee} &= (\bar{e}\gamma^\mu e)(\bar{e}\gamma^\mu e) \\
 Q_{le} &= (\bar{l}\gamma^\mu l)(\bar{e}\gamma^\mu e) \\
 Q_{ll} &= (\bar{l}_p\gamma^\mu l_p)(\bar{l}_r\gamma^\mu l_r)
 \end{aligned}$$

$$\begin{aligned}
 Q_{Hbox} &= (H^\dagger H) \square (H^\dagger H) \\
 Q_{HG} &= (H^\dagger H) G_{\mu\nu}^a G^{a\mu\nu} \\
 Q_{HB} &= (H^\dagger H) B_{\mu\nu} B^{\mu\nu} \\
 Q_{HW} &= (H^\dagger H) W_{\mu\nu}^i W^{i\mu\nu} \\
 Q_{uH} &= (H^\dagger H)(\bar{q}\tilde{H}u) \\
 Q_{dH} &= (H^\dagger H)(\bar{q}Hd) \\
 Q_{eH} &= (H^\dagger H)(\bar{q}He) \\
 Q_G &= \varepsilon_{abc} G_\mu^{a\nu} G_\nu^{b\rho} G_\rho^{c\mu} \\
 Q_{uG} &= (\bar{q}\sigma^{\mu\nu} T^a \tilde{H}u) G_{\mu\nu}^a
 \end{aligned}$$

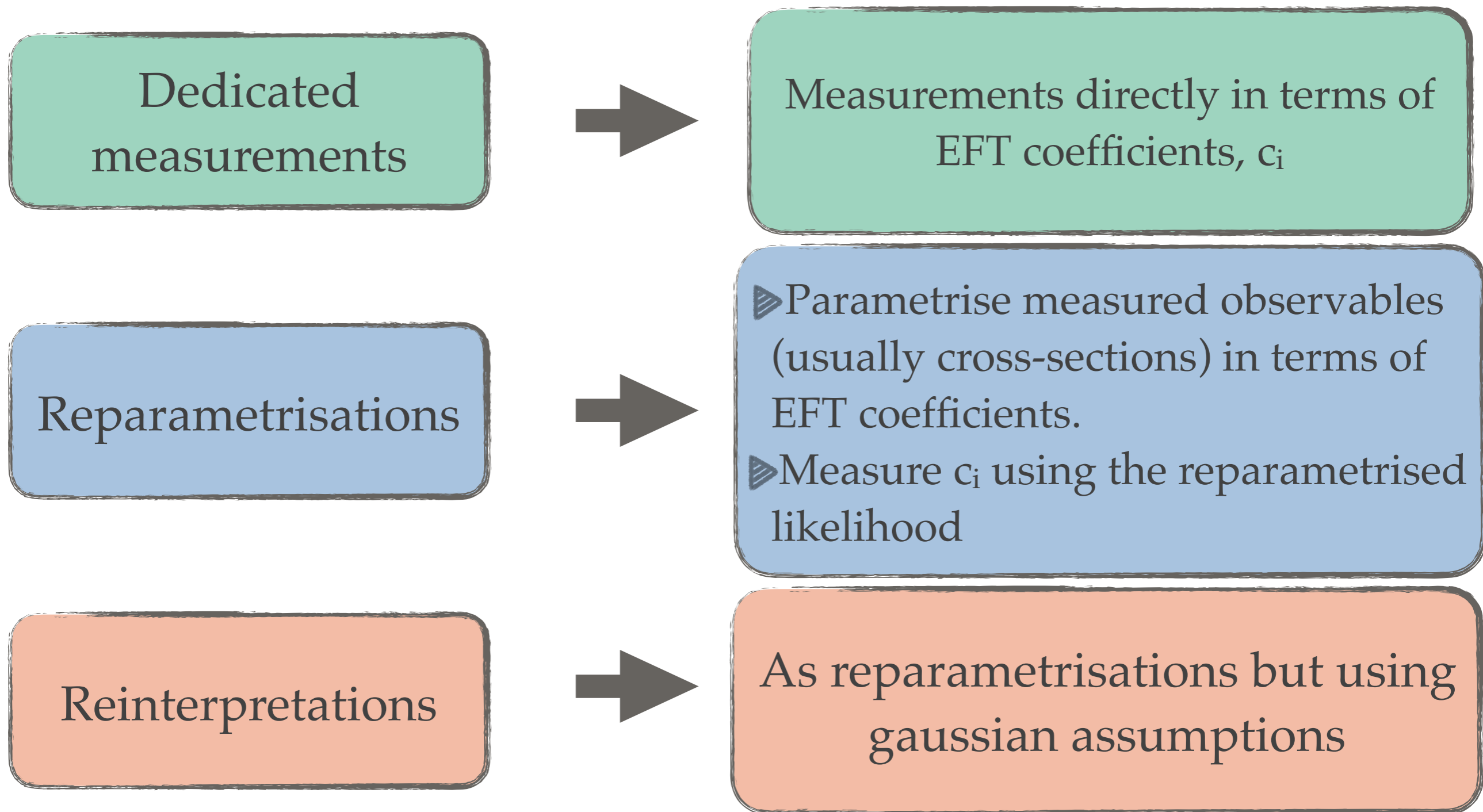
H processes

Common input schemes:

→ (mW,mZ,GF)

→ (α, mZ, GF)

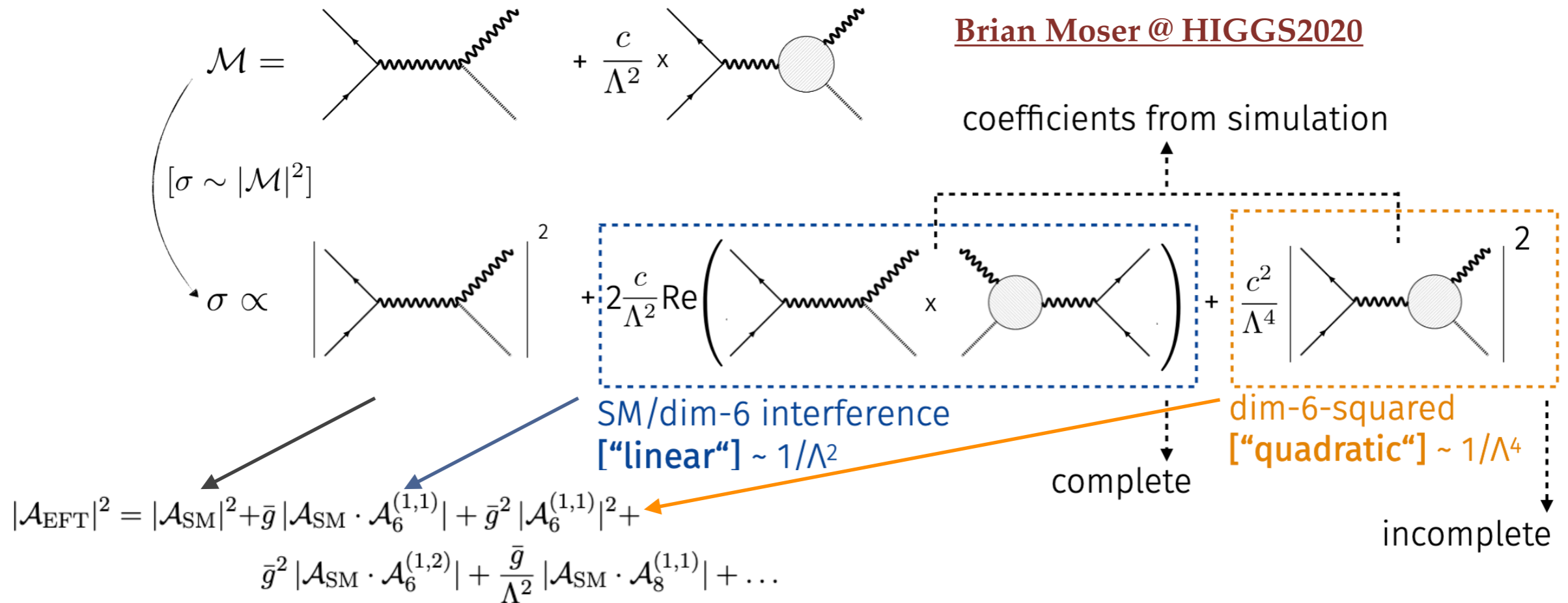
Implementation of EFT analyses



More assumptions means also less sensitivity but also easier implementation

From EFT to bins parametrisation

Brian Moser @ HIGGS2020



Common workflow:

- * Simulate events in any generator able to read models from UFO files
- * Analyse you events at particle level
- * Take the ratios to SM of the linear and quadratic terms (if included)
- * Assume same unfolding efficiencies as SM

Tools

► Main tools used in Run-2 analyses

SMEFTsim

- LO tool with effective vertices for $gg(g)H$, Hyy and $HZ\gamma$
- Truncation of the lagrangian at $1/\Lambda^2$
- Two different input parameter schemes
- Several flavour assumptions

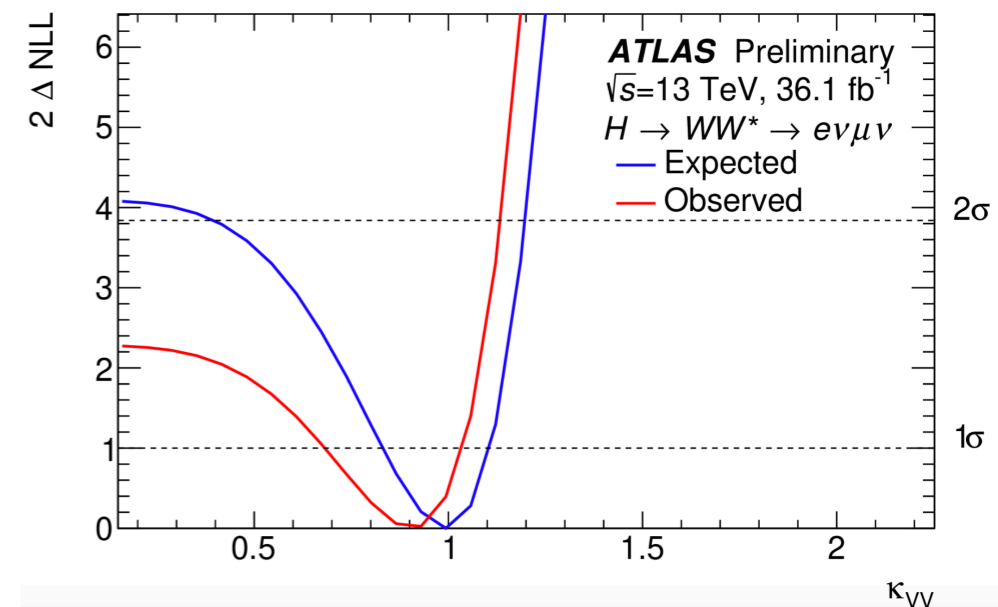
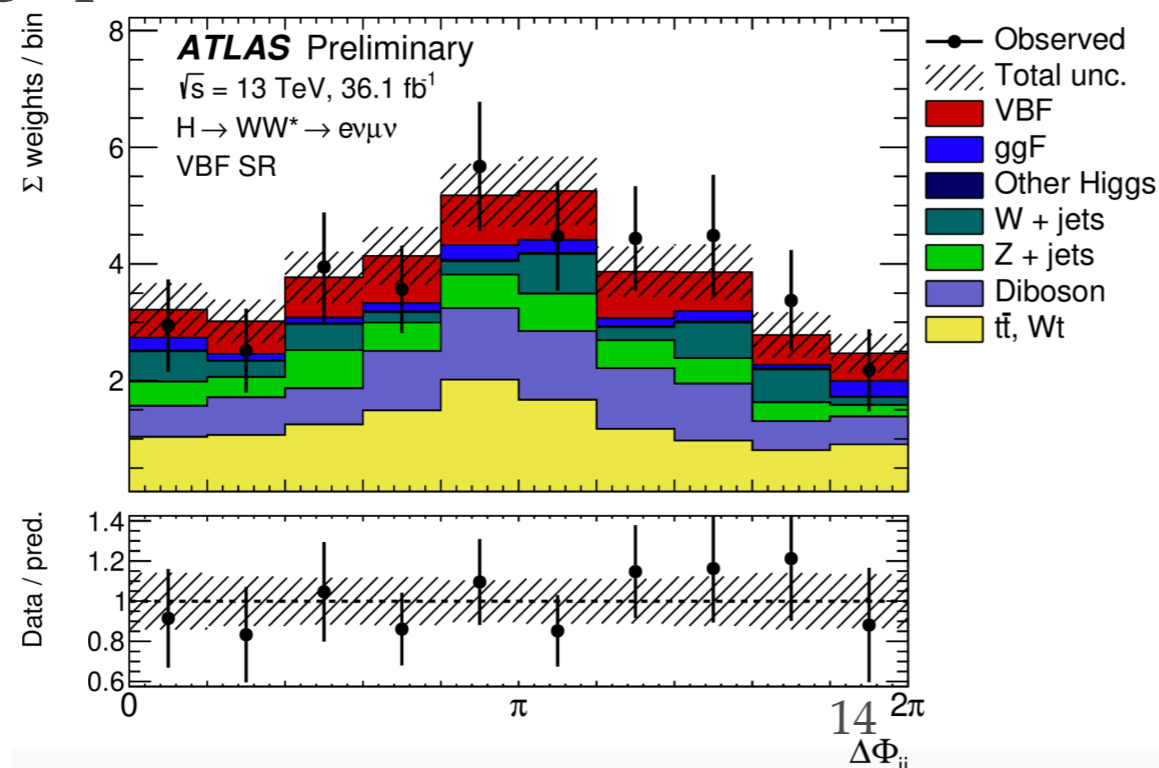
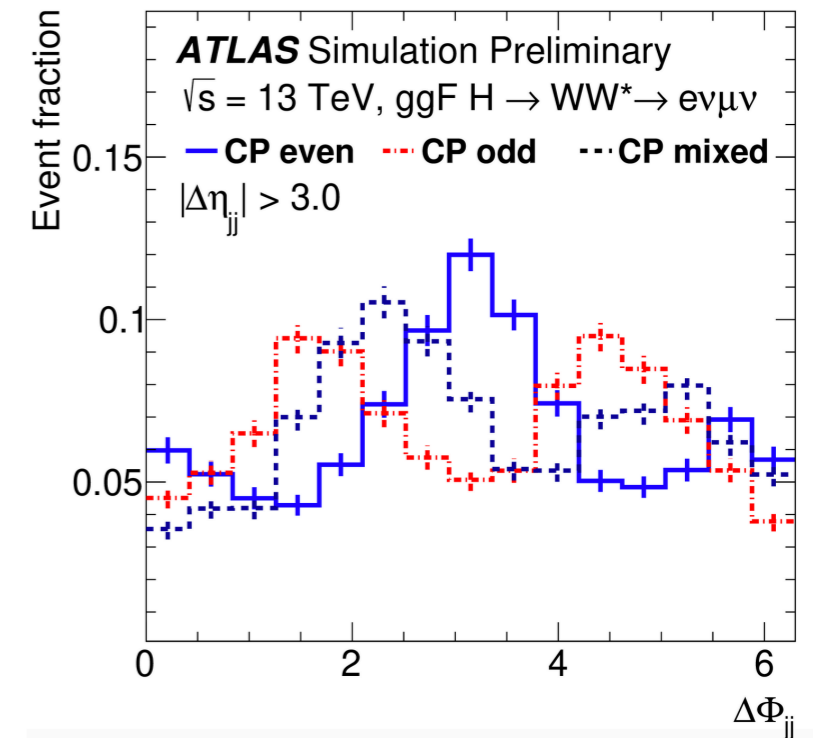
SMEFT@NLO

- Compatible with NLO QCD calculations
- $m\bar{W}$ scheme
- Exact $U(2)_q \times U(2)_d \times U(3)_d \times (U(1)_l \times U(1)_e)^3$ flavour symmetry forced. 5FS by default.
- Following LHC Top WG standards

Higgs interpretations

Dedicated analyses

- ▶ Measuring CP properties of HVV, Hff, Hgg vertices
- ▶ Example from a $H \rightarrow WW^* + jj$ analyses measuring the CP properties of the Higgs couplings in the ggH+jj and VBF from signed $\Delta\phi_{jj}$
 - * Other ATLAS Higgs CP analyses: ttH(yy), ttH(bb), H->4l, H->tautau
- ▶ Estimate c_i -dependence of reco-level observables
 - * Using morphing: interpolate between yields from multiple samples produced for various couplings points



Simplified template cross sections

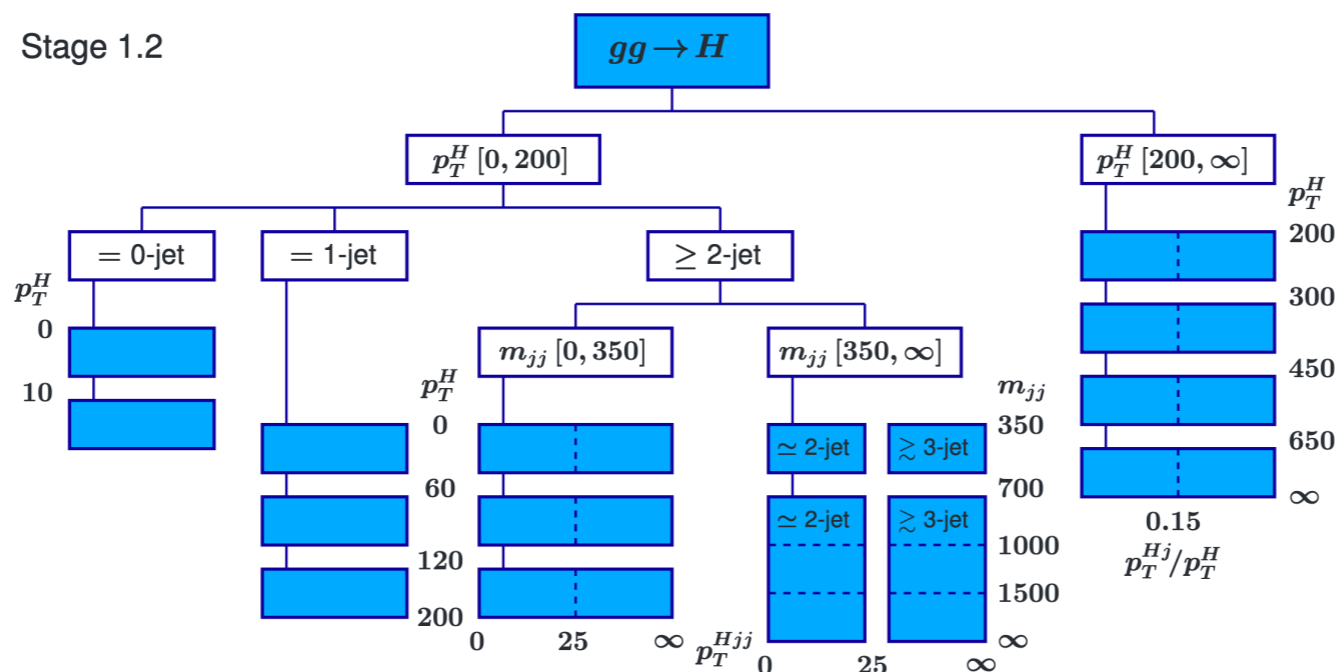
► STXS measurements broadly used in ATLAS and CMS to probe Higgs boson couplings. Designed for:

- Maximizing experimental sensitivity
- Isolation of possible BSM effects
- Not fully fiducial

- Minimizing the theoretical uncertainties
- Suitable for global combinations
- No Higgs decay information

► Several “Stages” depending on the kinematic information exploited in the different production modes. Currently using Stage 1.2

* Stage 0 corresponds to production mode measurements



ggH bins in Stage 1.2

Parametrisation in Higgs analyses

- Parametrise $\sigma_{\text{STXS}}(c)$ in each STXS region and BRs of the considered decay channels

$$(\sigma \times B)^{i,H \rightarrow X} = (\sigma \times B)_{\text{SM},(\text{N}(\text{N}))\text{NLO}}^{i,H \rightarrow X} \left(1 + \frac{\sigma_{\text{int},(\text{N})\text{LO}}^i}{\sigma_{\text{SM},(\text{N})\text{LO}}^i} + \frac{\sigma_{\text{BSM},(\text{N})\text{LO}}^i}{\sigma_{\text{SM},(\text{N})\text{LO}}^i} \right) \left(\frac{1 + \frac{\Gamma_{\text{int}}^{H \rightarrow X}}{\Gamma_{\text{SM}}^{H \rightarrow X}} + \frac{\Gamma_{\text{BSM}}^{H \rightarrow X}}{\Gamma_{\text{SM}}^{H \rightarrow X}}}{1 + \frac{\Gamma_{\text{int}}^H}{\Gamma_{\text{SM}}^H} + \frac{\Gamma_{\text{BSM}}^H}{\Gamma_{\text{SM}}^H}} \right)$$

- A_i and B_{ij} obtained from SMEFTsim except for:

- * ggH, ggZH and H->gg loop processes done with SMEFT@NLO
- * H->yy taken from its analytic form in NLO QED at interference level (not yet available in any MC tool)

$$\frac{\sigma_{\text{int}}^i}{\sigma_{\text{SM}}^i} = \sum_j A_j^{\sigma_i} c_j$$

$$\frac{\sigma_{\text{BSM}}^i}{\sigma_{\text{SM}}^i} = \sum_{jk} B_{jk}^{\sigma_i} c_j c_k$$

$$\frac{\Gamma_{\text{int}}^{H \rightarrow X}}{\Gamma_{\text{SM}}^{H \rightarrow X}} = \sum_j A_j^{\Gamma^{H \rightarrow X}} c_j$$

$$\frac{\Gamma_{\text{BSM}}^{H \rightarrow X}}{\Gamma_{\text{SM}}^{H \rightarrow X}} = \sum_{jk} B_{jk}^{\Gamma^{H \rightarrow X}} c_j c_k$$

$$\frac{\Gamma_{\text{int}}^H}{\Gamma_{\text{SM}}^H} = \sum_j A_j^{\Gamma^H} c_j$$

$$\frac{\Gamma_{\text{BSM}}^H}{\Gamma_{\text{SM}}^H} = \sum_{jk} B_{jk}^{\Gamma^H} c_j c_k,$$

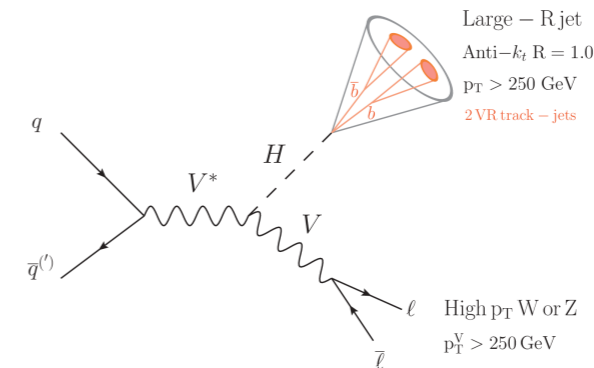
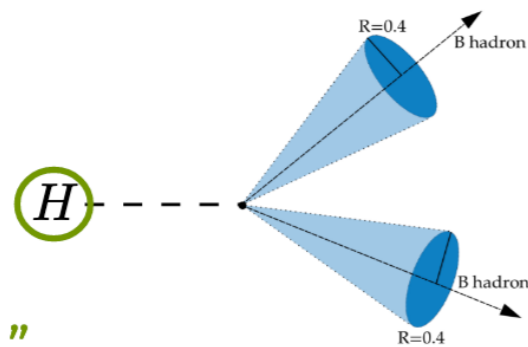
- **Linear model:** keep Λ^{-2} dependence by Taylor expanding the BR
- **Linear+quadratic model:** Full BR ratio, not defined dependence on Λ

VH, H → bb

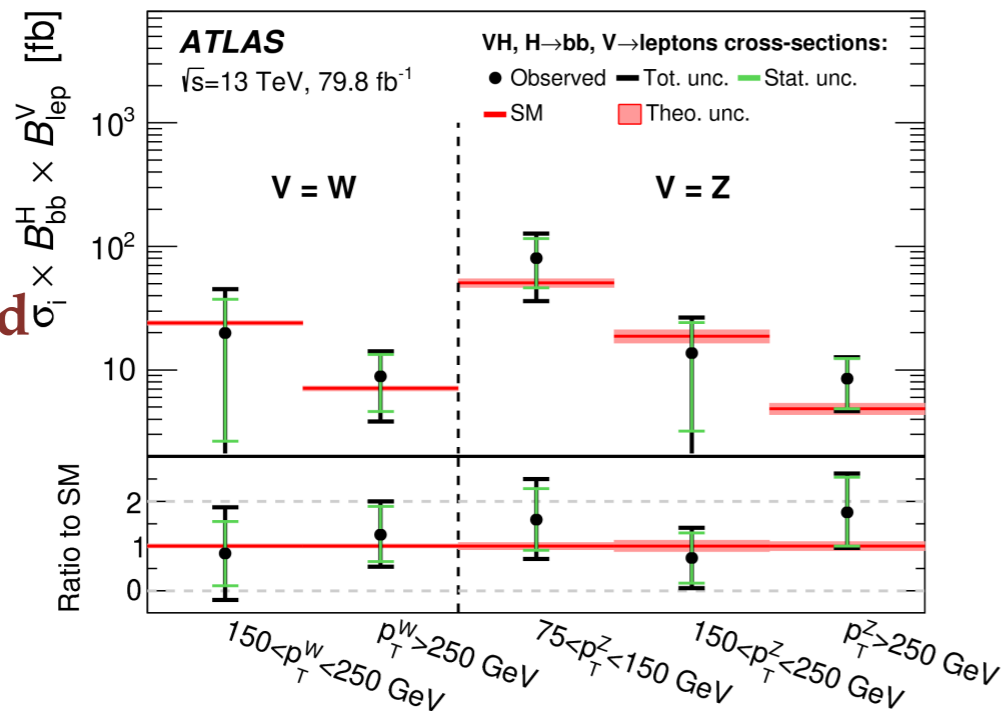
arXiv:2007.02873

arXiv:2008.02508

- ❖ Two analyses (resolved and boosted) using the same strategy for EFT interpretation
 - ▶ Warsaw basis as implemented in SMEFTsim in the Mw scheme.

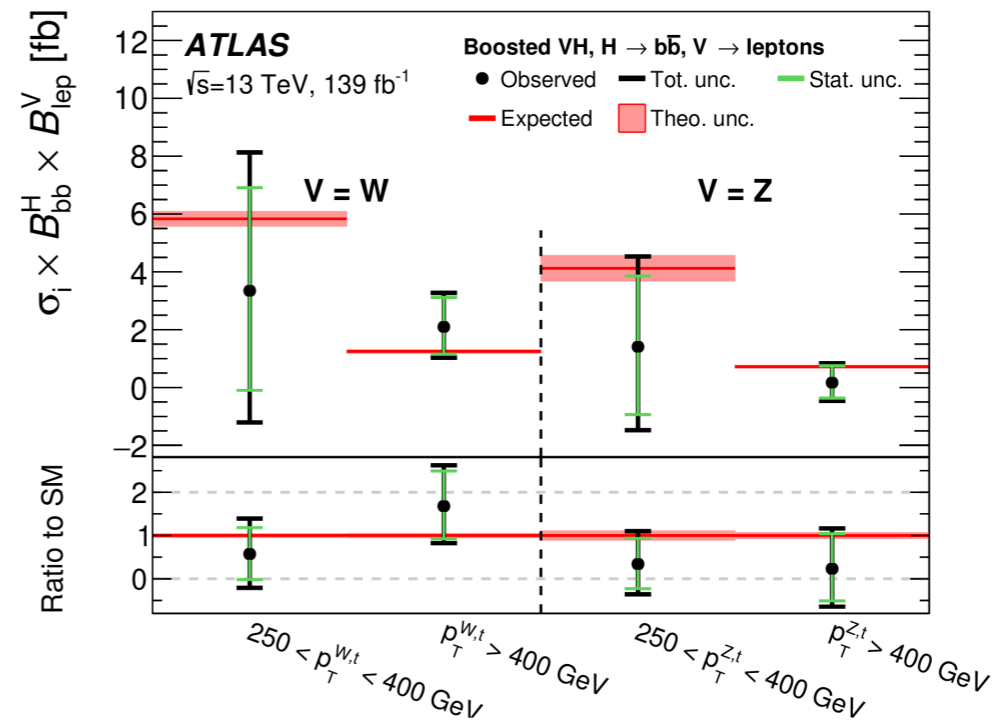


Resolved



Inclusive bin for $p_T^V > 250$ GeV

Boosted



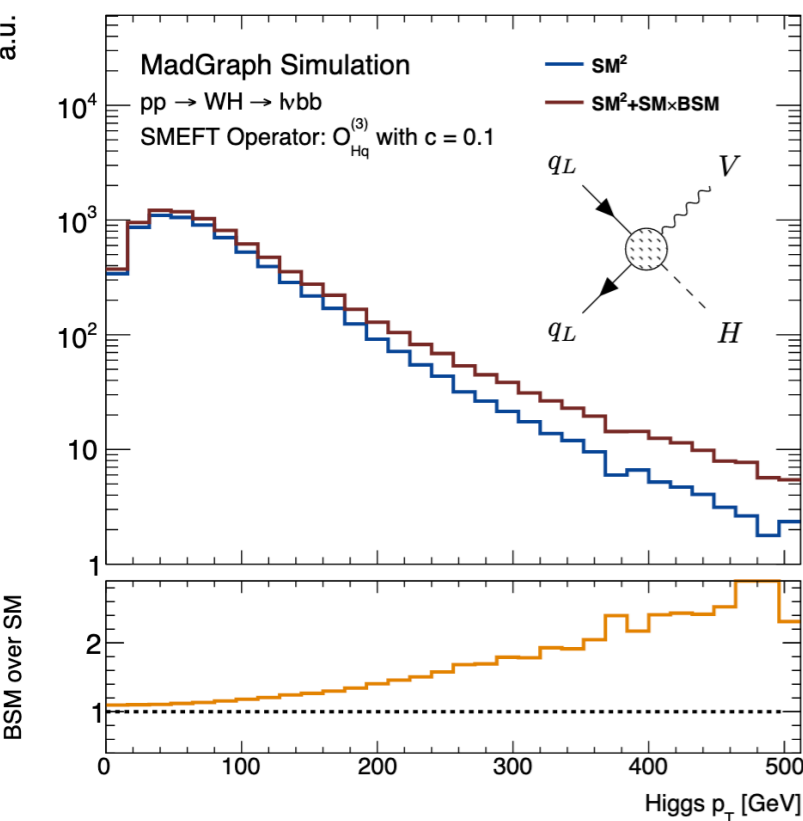
Additional splitting at high p_T^V

VH, H → bb

[arXiv:2007.02873](https://arxiv.org/abs/2007.02873)

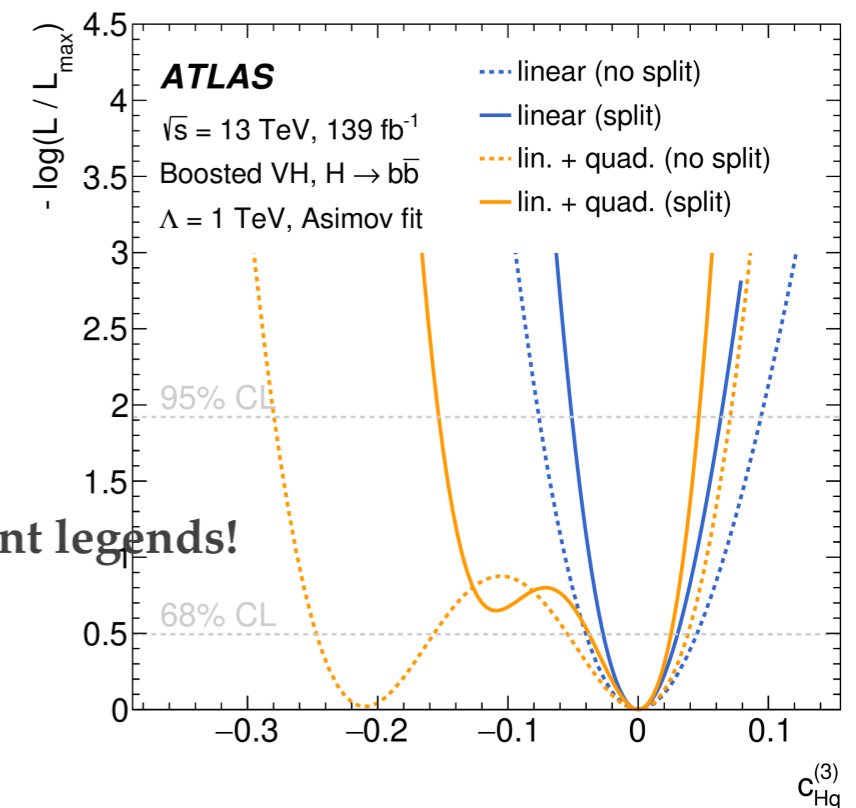
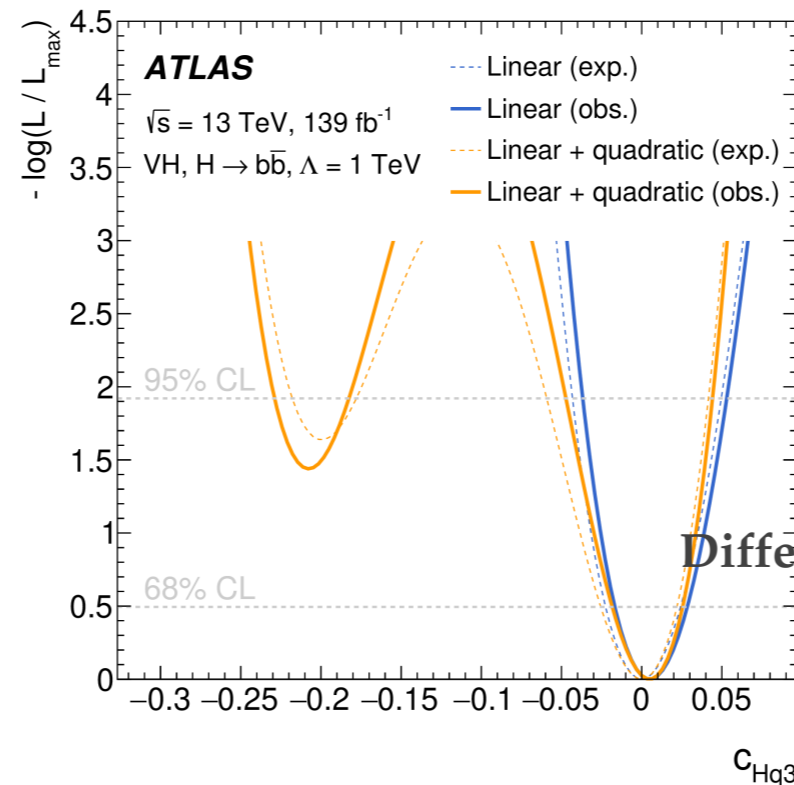
[arXiv:2008.02508](https://arxiv.org/abs/2008.02508)

- ❖ How much improvement can be achieved with higher granularity at high p_T^V ?
 - ▶ 1-D likelihood scans (all other Wilson coefficients set to 0) to c_{Hq3} which shows an energy growth
 - ▶ Boosted analysis less precise but achieves competitive constraints thanks to higher reach in p_T^H



Impact of c_{Hq3} in p_T^H

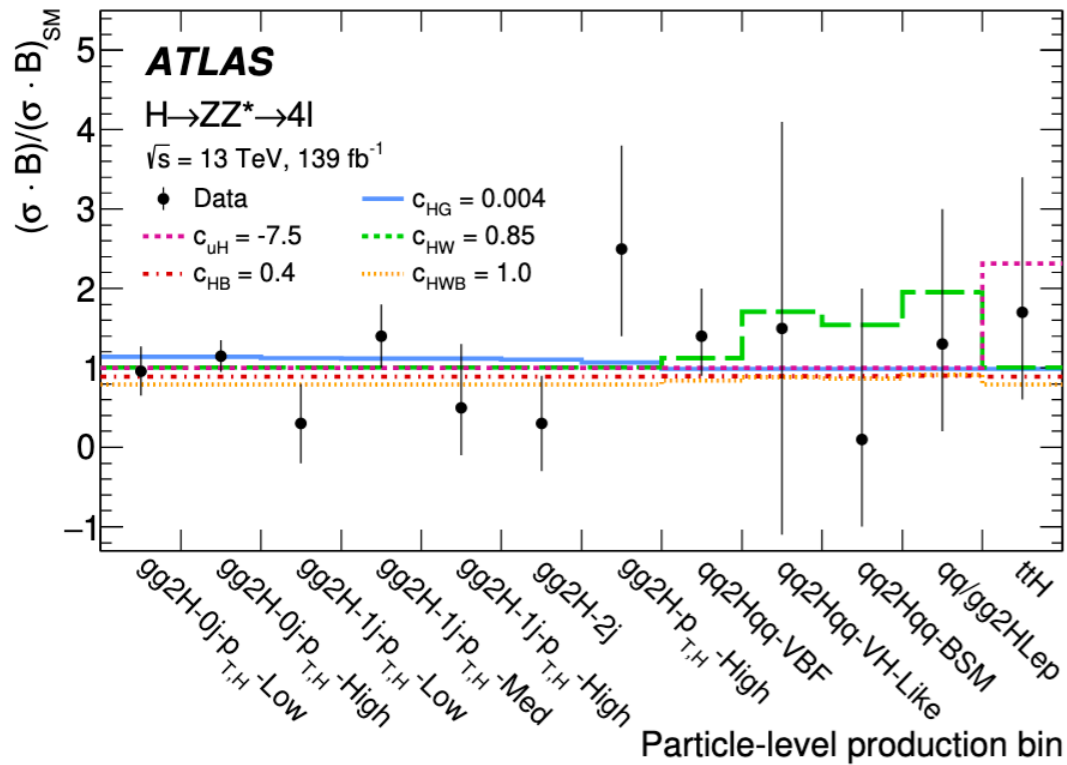
$$p_T^V \sim p_T^H$$



Split at 400 GeV can improve limits by a factor of ~two

H → 4l

arXiv:2004.03447



- ❖ Interpretation of STXS measurements in the Warsaw basis with Mw scheme
- ❖ Main operators affecting the measurement are selected
- ❖ Linear+quadratic terms included in the parametrisation as well as CP-even and CP-odd operators

- ❖ CP-odd operators only appear in the quadratic terms
- ❖ For several operators, quadratic terms are relevant

CP-even			CP-odd			Impact on	
Operator	Structure	Coeff.	Operator	Structure	Coeff.	production	decay
O_{uH}	$HH^\dagger \bar{q}_p u_r \tilde{H}$	c_{uH}	$O_{\tilde{u}H}$	$HH^\dagger \bar{q}_p u_r \tilde{H}$	$c_{\tilde{u}H}$	ttH	-
O_{HG}	$HH^\dagger G_{\mu\nu}^A G^{\mu\nu A}$	c_{HG}	$O_{H\tilde{G}}$	$HH^\dagger \tilde{G}_{\mu\nu}^A G^{\mu\nu A}$	$c_{H\tilde{G}}$	ggF	Yes
O_{HW}	$HH^\dagger W_{\mu\nu}^l W^{\mu\nu l}$	c_{HW}	$O_{H\tilde{W}}$	$HH^\dagger \tilde{W}_{\mu\nu}^l W^{\mu\nu l}$	$c_{H\tilde{W}}$	VBF, VH	Yes
O_{HB}	$HH^\dagger B_{\mu\nu} B^{\mu\nu}$	c_{HB}	$O_{H\tilde{B}}$	$HH^\dagger \tilde{B}_{\mu\nu} B^{\mu\nu}$	$c_{H\tilde{B}}$	VBF, VH	Yes
O_{HWB}	$HH^\dagger \tau^l W_{\mu\nu}^l B^{\mu\nu}$	c_{HWB}	$O_{H\tilde{W}B}$	$HH^\dagger \tau^l \tilde{W}_{\mu\nu}^l B^{\mu\nu}$	$c_{H\tilde{W}B}$	VBF, VH	Yes

H → 4l (ATLAS)

[arXiv:2004.03447](https://arxiv.org/abs/2004.03447)

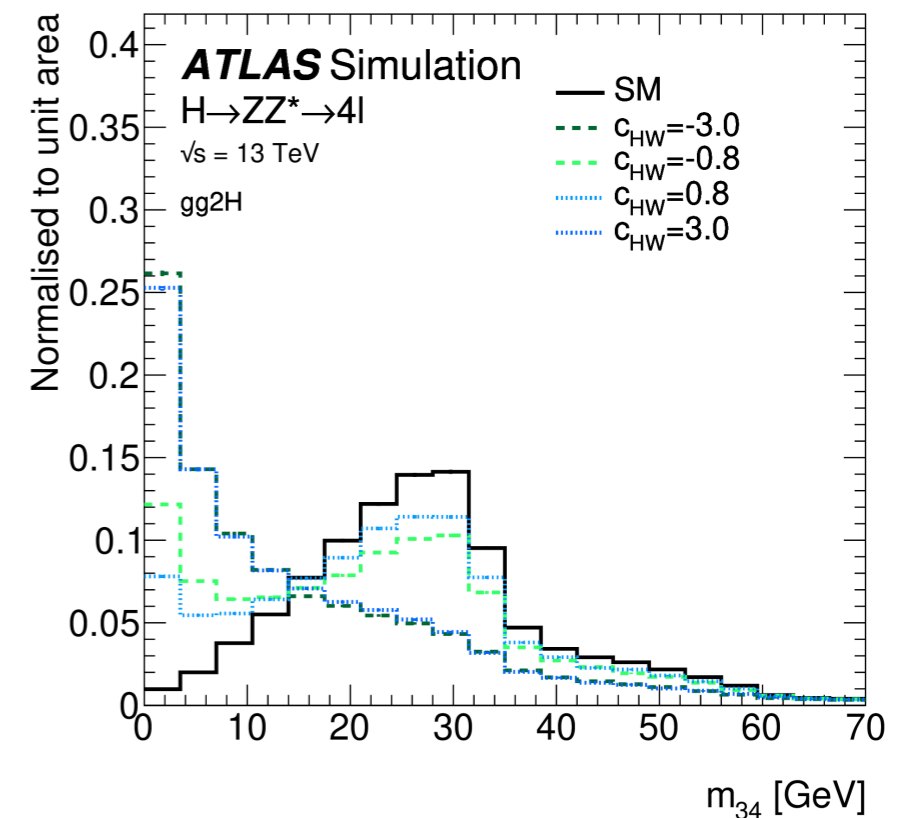
- ❖ Reconstruction-level requirements on m_{12} and m_{34} to target $H \rightarrow ZZ^*$
- ❖ EFT does not have the same acceptance as SM and needs to be corrected in the parametrization
- ❖ Other effects: differences in efficiencies or classification in reco bins used in the analysis found to be negligible

Strategy:

- * Mimic reco selection at particle level
- * Fit a 3-D Lorentzian function for c_{HW} , c_{HB} , c_{HWB} for the acceptance correction relative to SM (or their CP-odd analogous assuming the CP-even ones vanish)

$$\frac{A(\vec{c})}{A_{SM}} = \alpha_0 + (\alpha_1)^2 \cdot \left[\alpha_2 + \sum_i \delta_i \cdot (c_i + \beta_i)^2 + \sum_{\substack{ij \\ i \neq j}} \delta_{(i,j)} \cdot c_i c_j + \sum_{\substack{ij \\ i \neq j \neq k}} \delta_{(i,j,k)} \cdot c_i c_j c_k \right]^{-1}$$

i, j, k run over c_{HW} , c_{HB} , c_{HWB}
 α , β and δ parameters are free in the fit

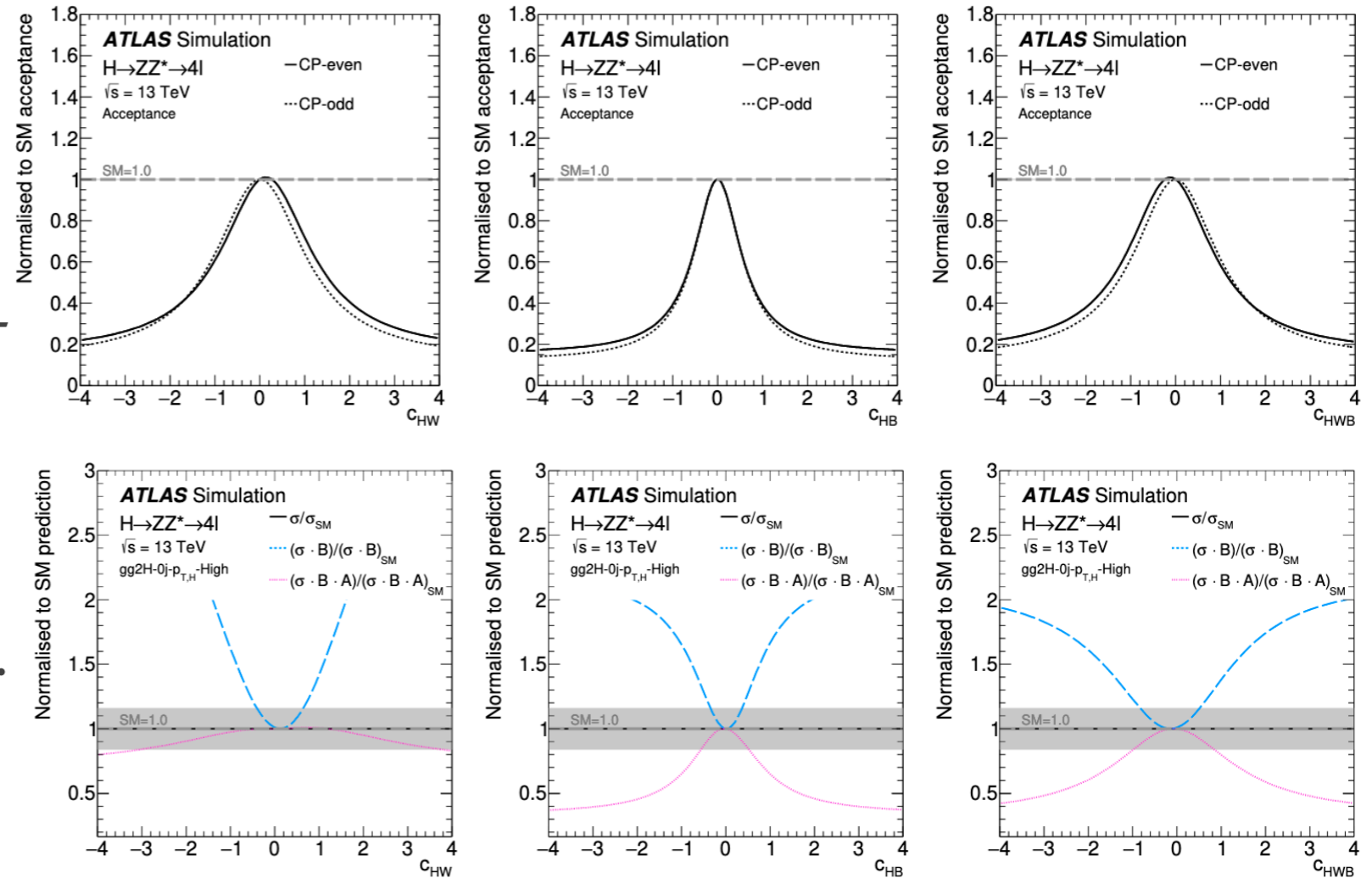


$m_{34} > 12 \text{ GeV}$ at reco level

H \rightarrow 4l

arXiv:2004.03447

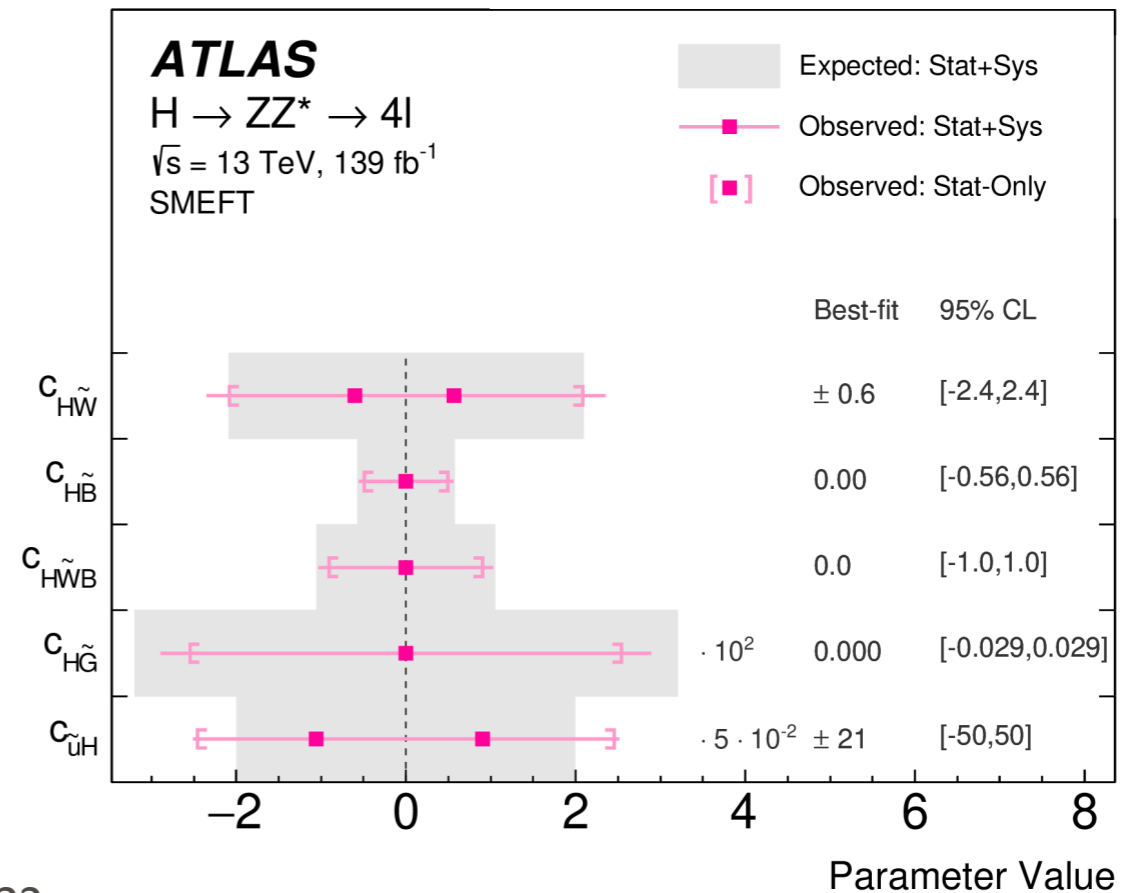
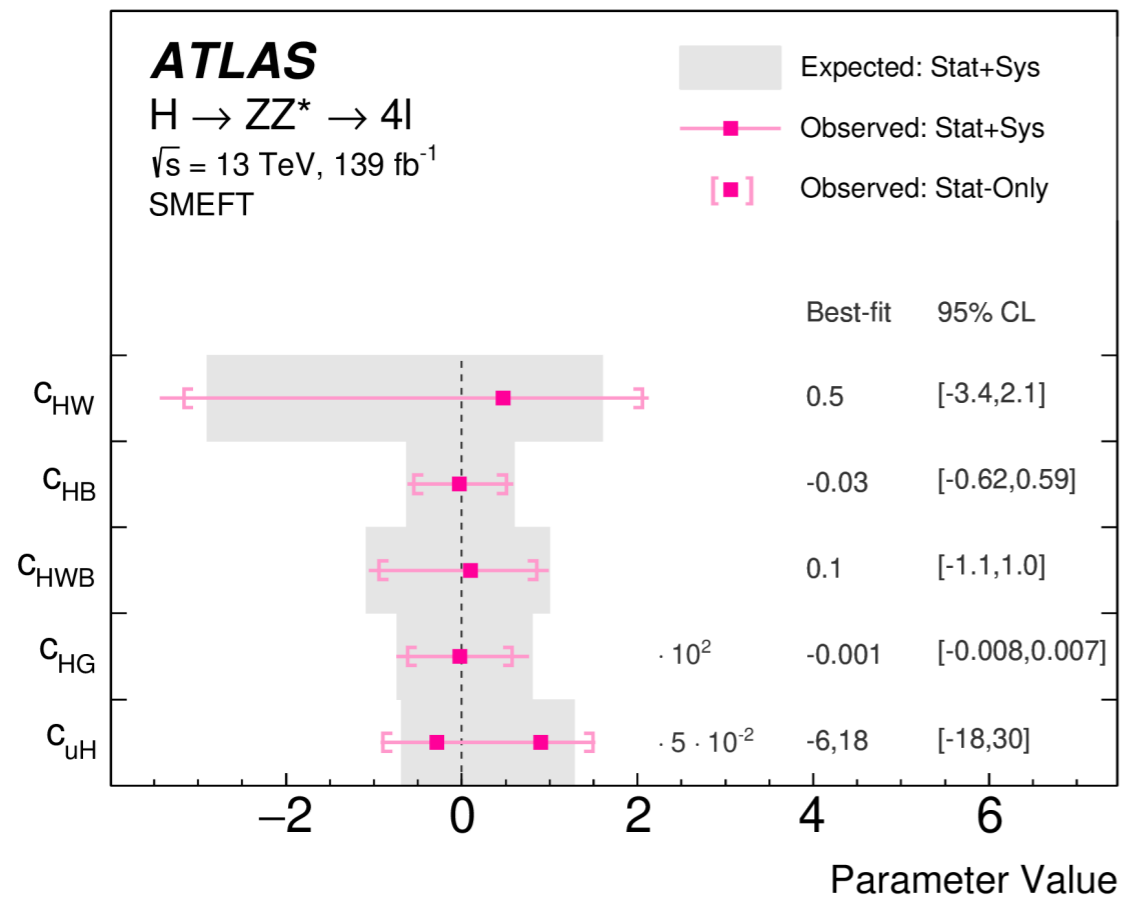
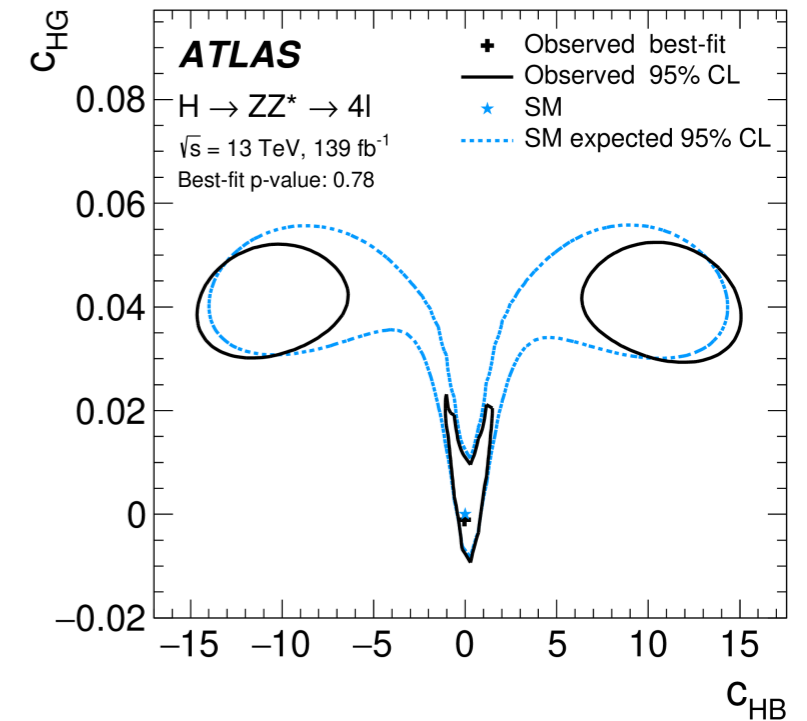
- Acceptance ratios varying one parameter at a time.
- Similar for CP-even and CP-odd operators (mostly from quadratic terms)
- Common acceptance parametrisation for all prod. modes
- Effects on the expected yields normalised to SM



Blue line: w.o. acceptance
Pink line: with acceptance

H → 4l

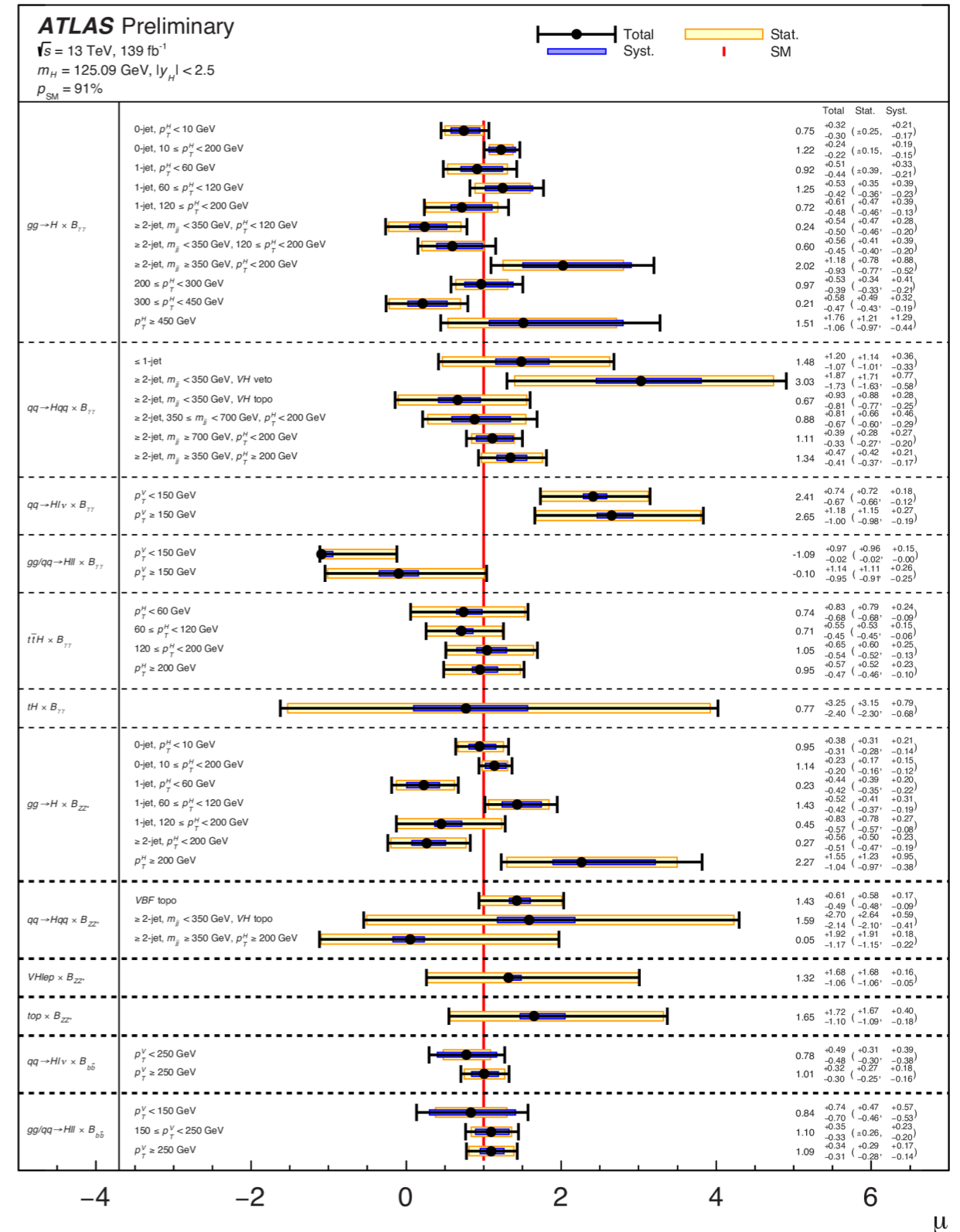
- ❖ Limits from 1-D fits (all others set to SM), correlations studied through 2D scans
 - ▶ Not trivial correlations between most of the parameter pairs



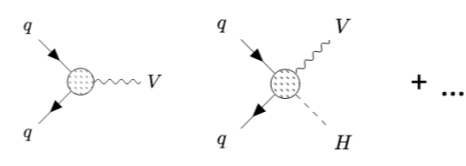
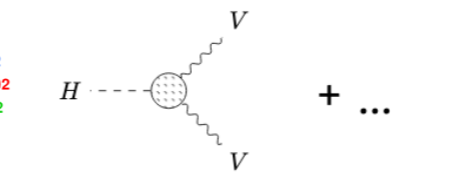
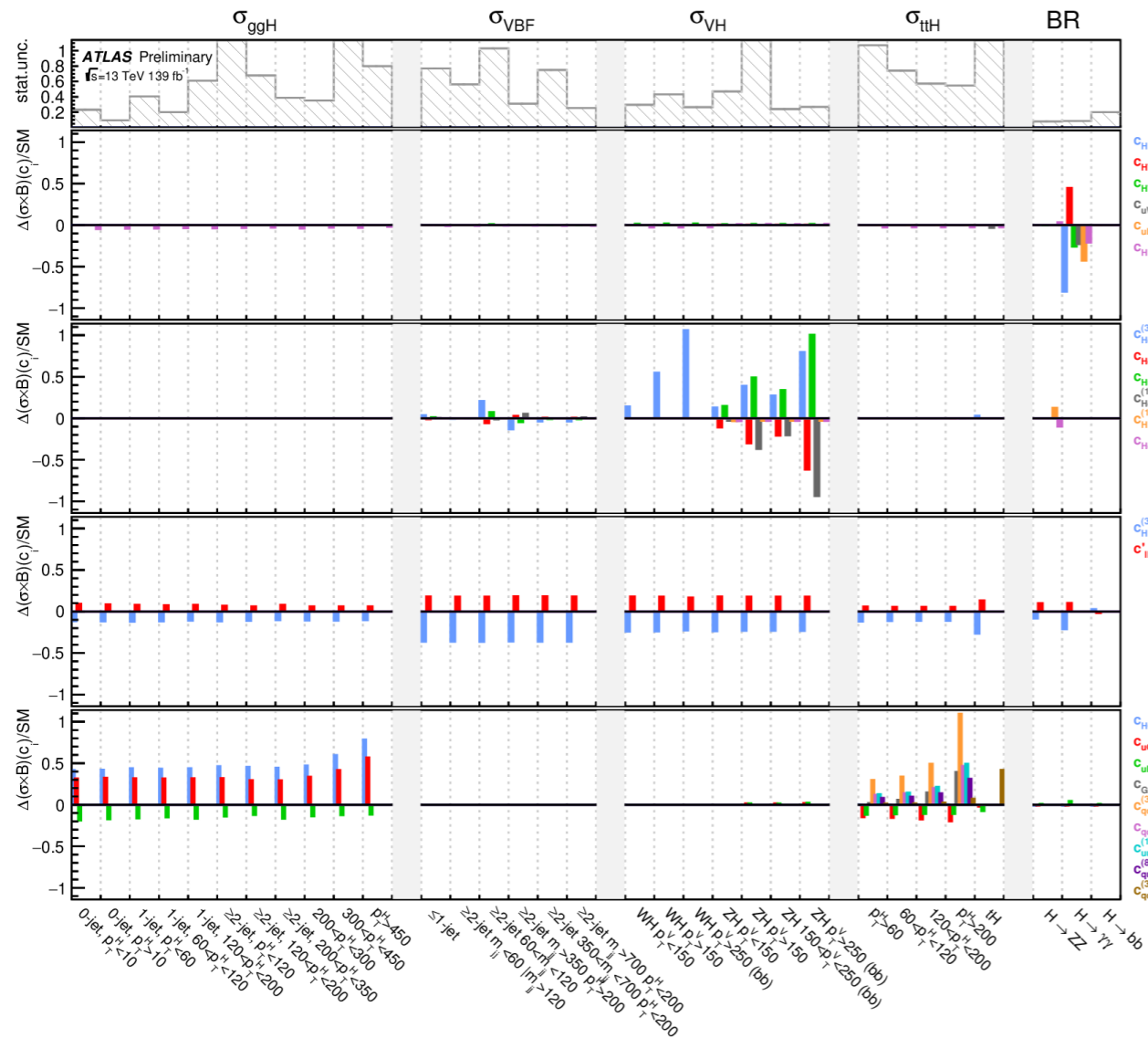
Higgs combination

ATLAS-CONF-2020-053

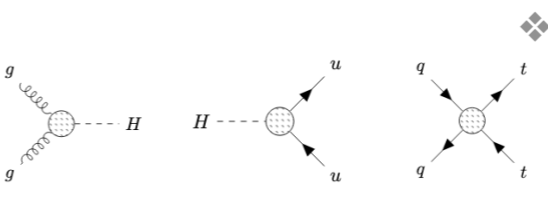
- ❖ Stage 1.2 STXS combination of $H \rightarrow \gamma\gamma$, $VH(H \rightarrow bb)$ and $H \rightarrow ZZ^* \rightarrow 4l$ for full Run 2
- ❖ Based on $\sigma_{STXS_i} \times BR_{H \rightarrow X}$ signal strength measurement
 - ▶ Warsaw basis in Mw scheme
 - ▶ Lowest order of each production mode or decay channel: NLO QCD for ggH and $ggZH$ from SMEFT@NLO, NLO EW for $H \rightarrow \gamma\gamma$, LO for the rest from SMEFTsim
 - ▶ Only CP-even operators (no linear contribution from CP-odd ones and not available in SMEFT@NLO)
 - ▶ Include the acceptance effects in $H \rightarrow ZZ^* \rightarrow 4l$ for cHW, cHB and cHWB



Higgs combination



Shifts to G_f



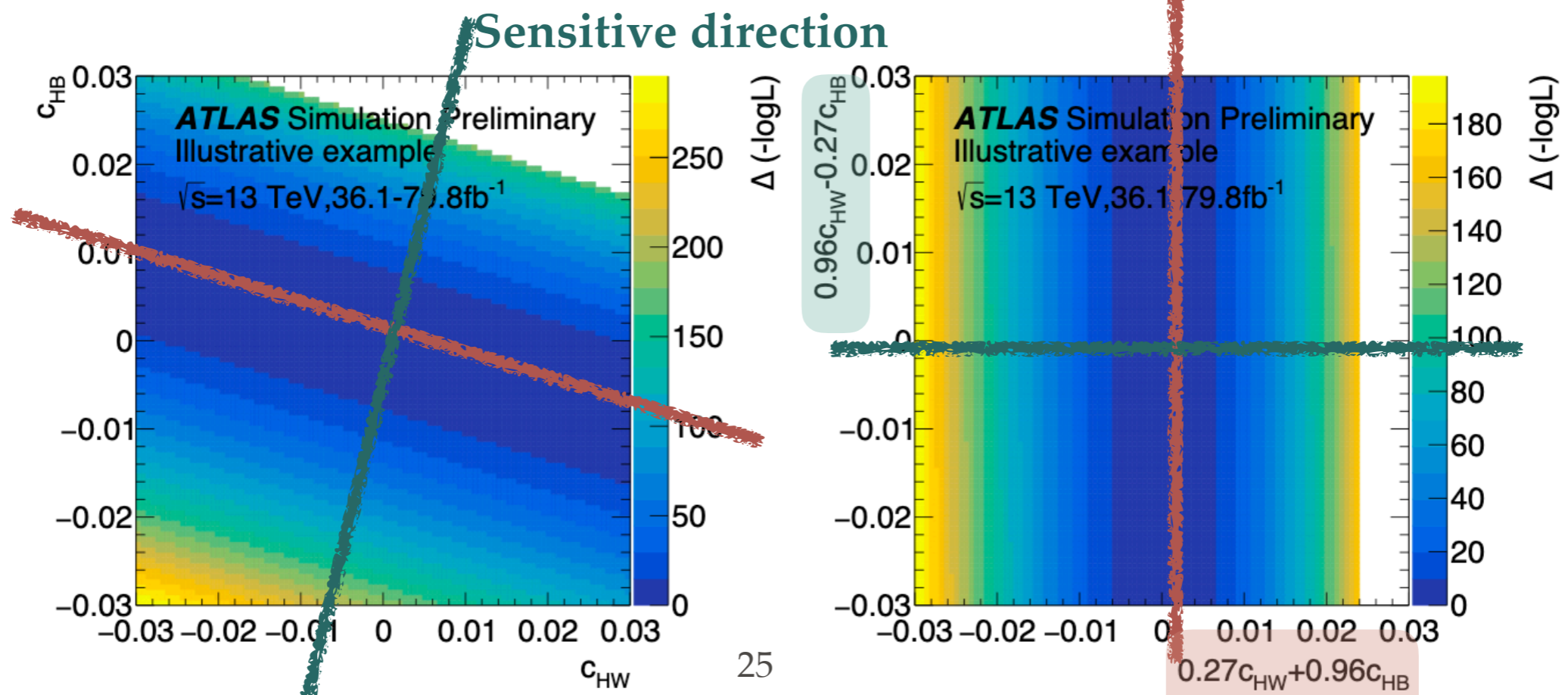
❖ Retain all operators that modify the production modes or BRs

❖ They can modify the couplings, introduce new diagrams, enter through field redefinitions or shifts to input quantities

❖ Simultaneous fit to all relevant single coefficients not possible due to degeneracies

Higgs combination

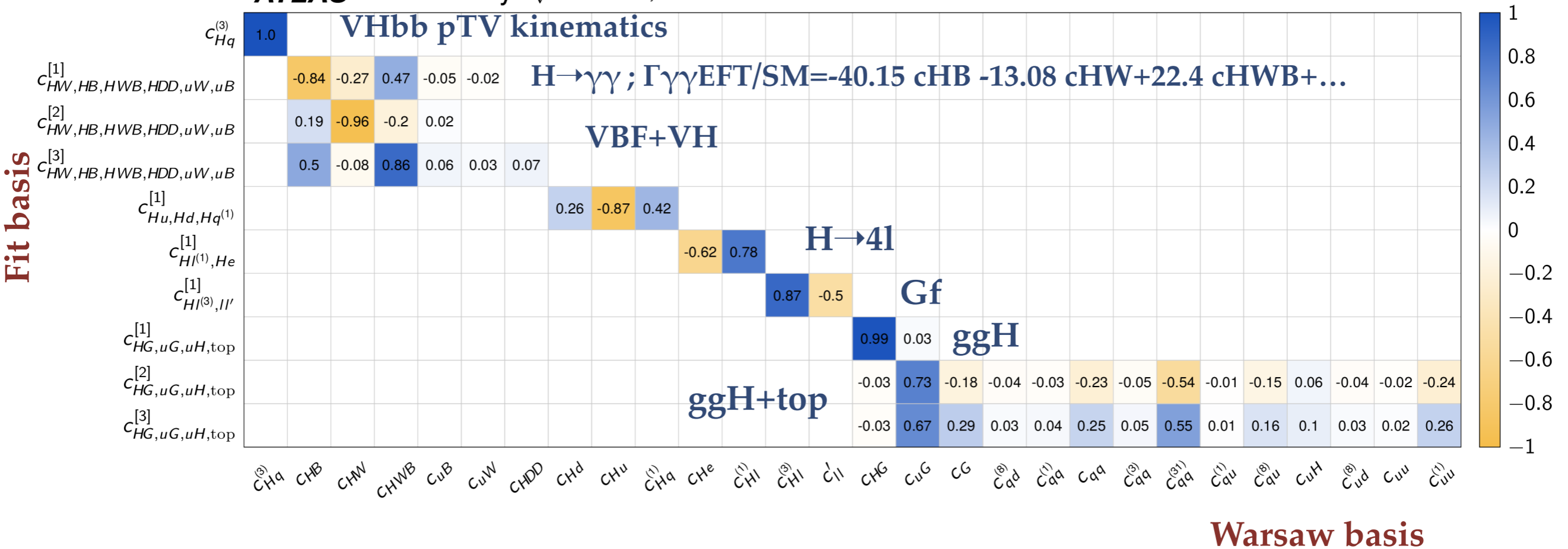
- ❖ No straightforward EFT dependence of observables
 - ▶ Combination of multiple of them in each STXS bin or decay width
 - ▶ Some are constant throughout the bins (field redefinitions, shifts to input quantities, ...), while others show a momentum dependence.
 - ▶ EFT parameters highly correlated
- ❖ Keep all operators but remove **flat directions** from the fit
 - ▶ No model dependence since there is no sensitivity to them.



Higgs combination

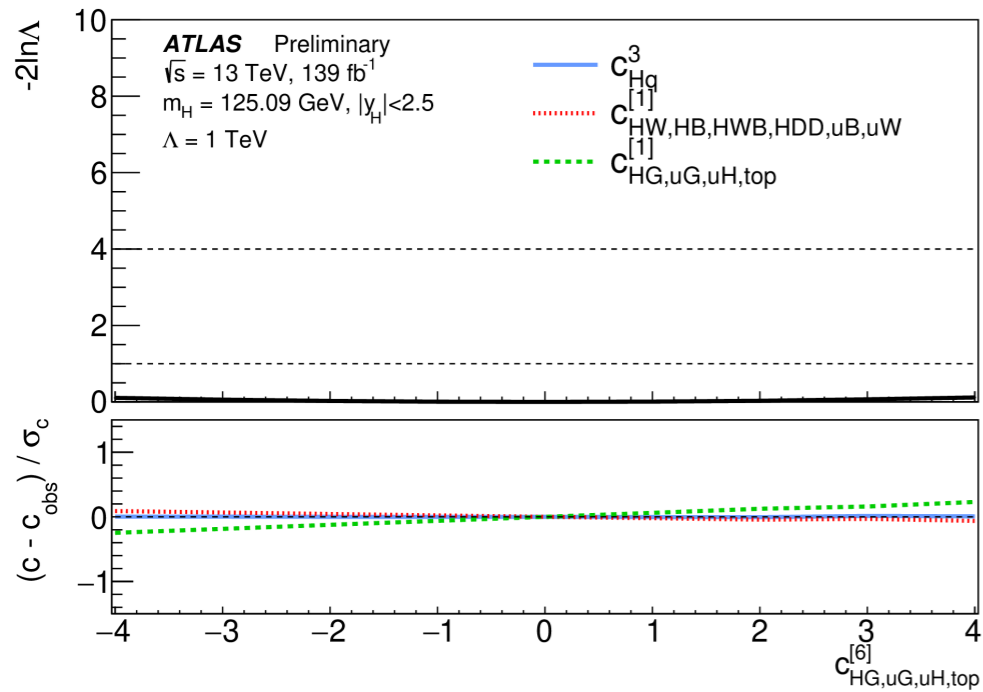
- To reduce the dimensionality of the fit a PCA is performed using the covariance matrix of the STXS measurement and propagating the EFT parametrisation
 - Second PCA on sub-covariance matrices grouping operators affecting the same prod. mode or decay rates. Identify sensitive directions and neglect blind directions
- $$\{c_i\} = \{c_{Hq}^{(3)}\} \times \{c_{HG}, c_{uG}, c_{uH}, c_{qq}^{(1)}, c_{qq}^{(3)}, c_{qq}^{(31)}, c_{uu}, c_{uu}^{(1)}, c_{ud}^{(8)}, c_{qu}^{(1)}, c_{qu}^{(8)}, c_{qd}^{(8)}, c_G\} \times \{c_{HW}, c_{HB}, c_{HWB}, c_{HDD}, c_{uW}, c_{uB}, \} \times \{c_{Hl}^{(1)}, c_{He}\} \times \{c_{Hl}^{(3)}, c'_{ll}\} \times \{c_{Hu}, c_{Hd}, c_{Hq}^{(1)}\}$$

ATLAS Preliminary $\sqrt{s} = 13 \text{ TeV}, 139 \text{ fb}^{-1}$



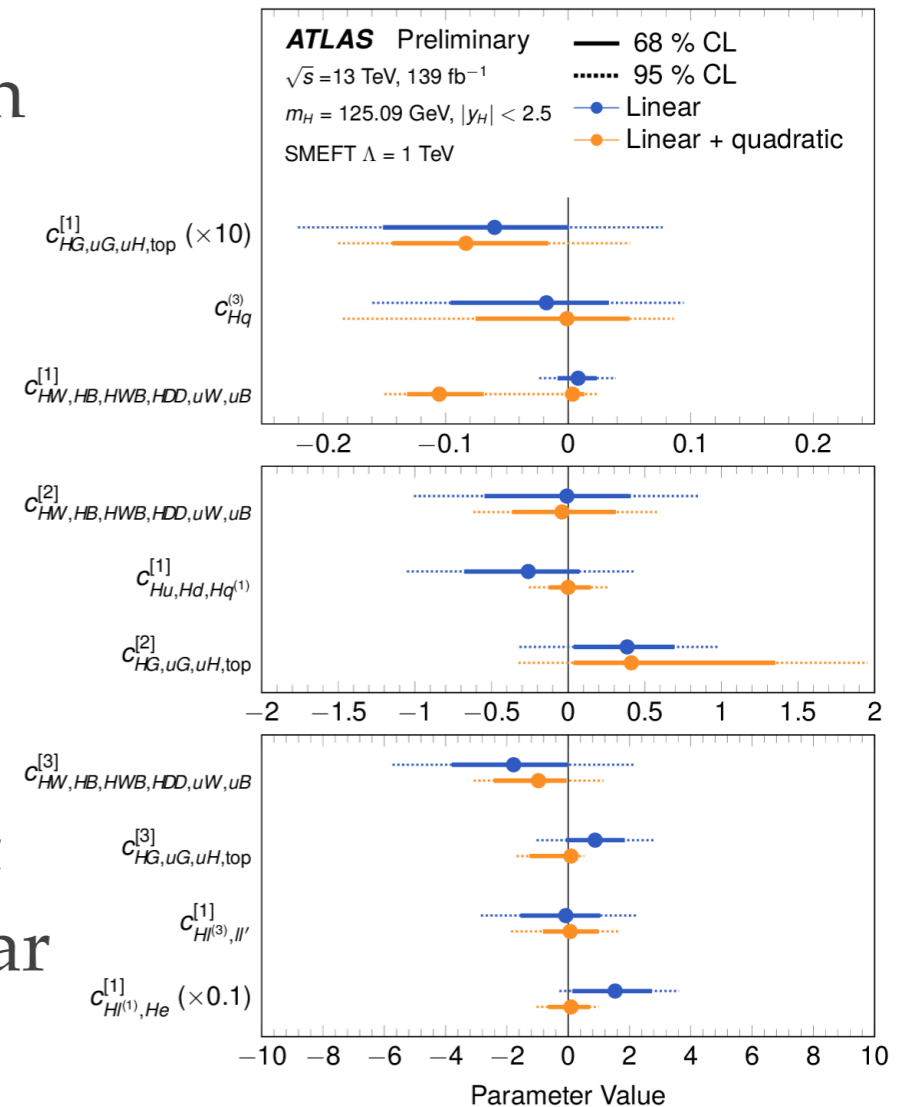
Higgs combination

ATLAS-CONF-2020-053

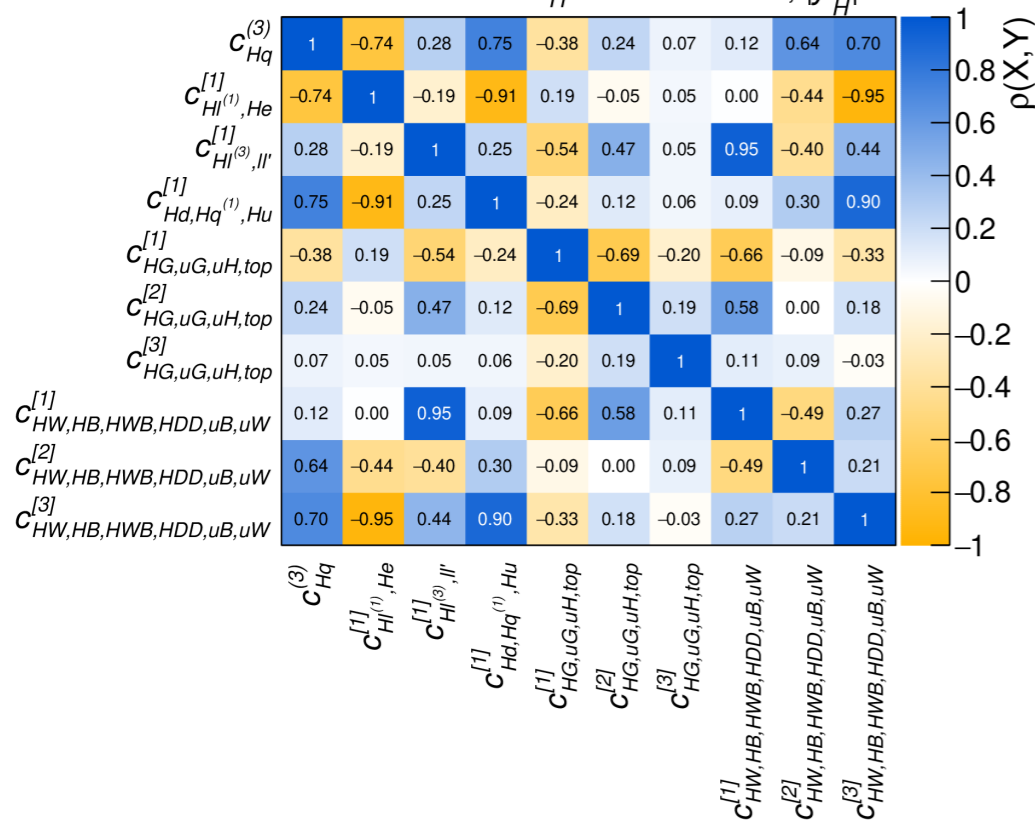


❖ Insensitivity to neglected direction and negligible impact on fitted POIs checked

❖ No reduction of “experimental” correlations (exact EVs not fitted) but checked to be linear



ATLAS Preliminary $\sqrt{s} = 13 \text{ TeV}, 139 \text{ fb}^{-1}$
 $m_H = 125.09 \text{ GeV}, |y_H| < 2.5$



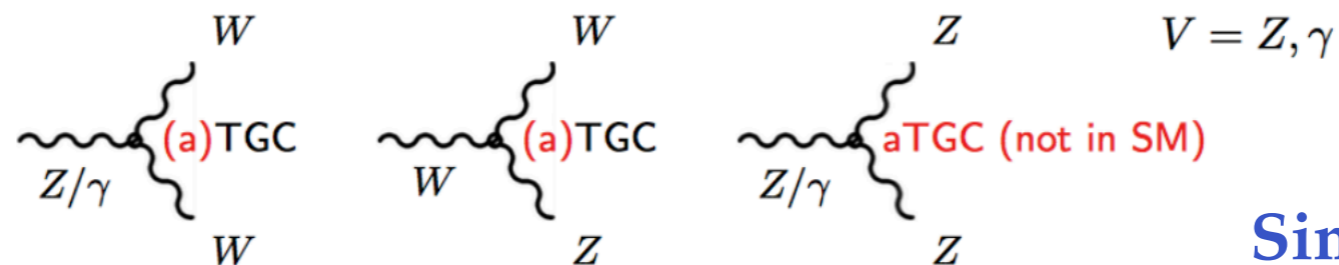
❖ Very good sensitivity to the 10 fitted POI

❖ Quadratic terms relevant when constrained from low stat bins.

EW interpretations

From aTGCs to EFT

- ▶ Anomalous couplings typically used to look for SM deviations in gauge boson couplings



Similarly for aQGCs, but with d=8 operators

- ▶ Extend the Lagrangian only with the needed terms

$$-ig_{WWV}[g_1^V(W_{\mu\nu}^+ W^{-\mu} V^\nu - W_{\mu\nu}^- W^{+\mu} V^\nu) + \kappa_V W_\mu^+ W_\nu^- V^{\mu\nu}] - i\frac{\lambda_V}{m_W^2} V^{\mu\nu} W_\nu^{+\rho} W_{\rho\mu}^-$$

- ▶ Not necessarily gauge invariance

- ▶ Possible translation to EFT:

$$g_1^Z = 1 + c_W \frac{m_Z^2}{\Lambda^2}$$

$$\kappa_\gamma = 1 + (c_W + c_B) \frac{m_W^2}{2\Lambda^2} \longrightarrow$$

...

$$\mathcal{O}_B = (D_\mu H^\dagger) B^{\mu\nu} D_\nu H$$

$$\mathcal{O}_W = (D_\mu H)^\dagger W^{\mu\nu} D_\nu H$$

$$\mathcal{O}_{WWW} = \text{Tr}[W_{\mu\nu} W_\rho^\nu W^{\rho\mu}]$$

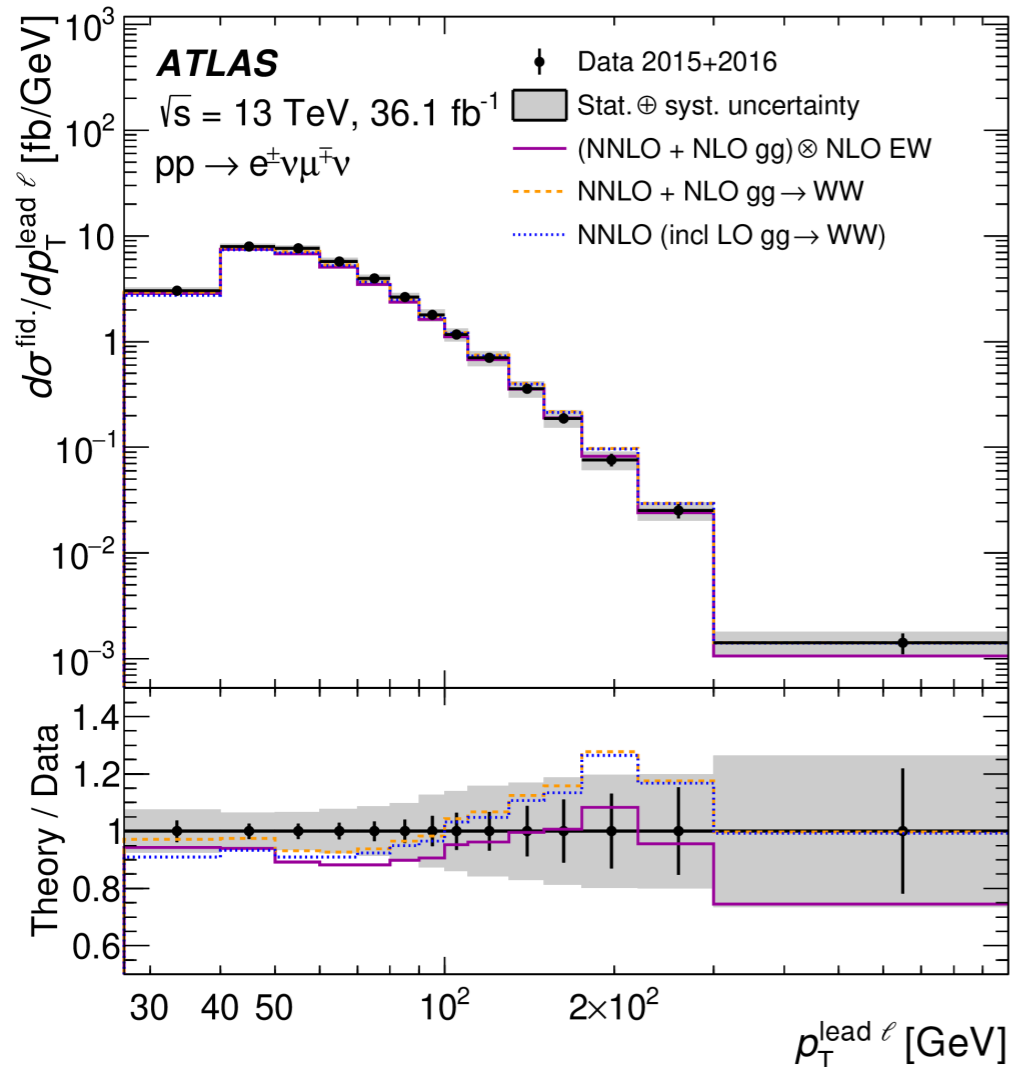
HISZ basis commonly

used in Run 1

$$\mathcal{O}_{\tilde{W}} = (D_\mu H)^\dagger \tilde{W}^{\mu\nu} D_\nu H$$

$$\mathcal{O}_{W\tilde{W}} = \text{Tr}[W_{\mu\nu} W_\rho^\nu \tilde{W}^{\rho\mu}]$$

- ▶ In EFT many other operators affect vector-boson measurements, typically not considered since they were well constrained at LEP (this is basis dependent)



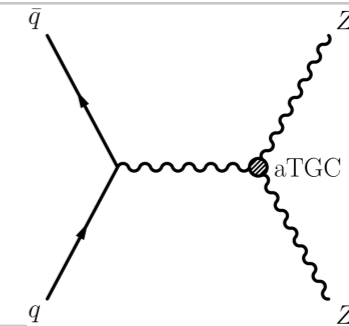
Operator	95% CL (linear and quadratic terms)	95% CL (linear terms only)
c_{WWW}/Λ^2	$[-3.4 \text{ TeV}^{-2}, 3.3 \text{ TeV}^{-2}]$	$[-179 \text{ TeV}^{-2}, -17 \text{ TeV}^{-2}]$
c_W/Λ^2	$[-7.4 \text{ TeV}^{-2}, 4.1 \text{ TeV}^{-2}]$	$[-13.1 \text{ TeV}^{-2}, 7.1 \text{ TeV}^{-2}]$
c_B/Λ^2	$[-21 \text{ TeV}^{-2}, 18 \text{ TeV}^{-2}]$	$[-104 \text{ TeV}^{-2}, 101 \text{ TeV}^{-2}]$

- ❖ $WW \rightarrow e\nu\mu\nu$. More background than WZ, need to suppress $t\bar{t}$ with jet veto
- ❖ Limits from unfolded leading p_T^1 differential cross section
 - BSM terms behave as SM in the unfolding
- ❖ Large EW corrections in the p_T^1 tail
- ❖ Less sensitive to O_W, O_{WWW} than WZ
- ❖ Studied relevance of quadratic terms
 - Relevant especially for O_{WWW}

Better limits from CMS from the inclusion of WW+1jet

Parameter	Observed 95% CL [TeV^{-2}]	Expected 95% CL [TeV^{-2}]
c_{WWW}/Λ^2	$[-3.4, 3.3]$	$[-3.0, 3.0]$
c_W/Λ^2	$[-7.4, 4.1]$	$[-6.4, 5.1]$
c_B/Λ^2	$[-21, 18]$	$[-18, 17]$
$c_{\tilde{W}WW}/\Lambda^2$	$[-1.6, 1.6]$	$[-1.5, 1.5]$
$c_{\tilde{W}}/\Lambda^2$	$[-76, 76]$	$[-91, 91]$

ZZ → 2l2ν

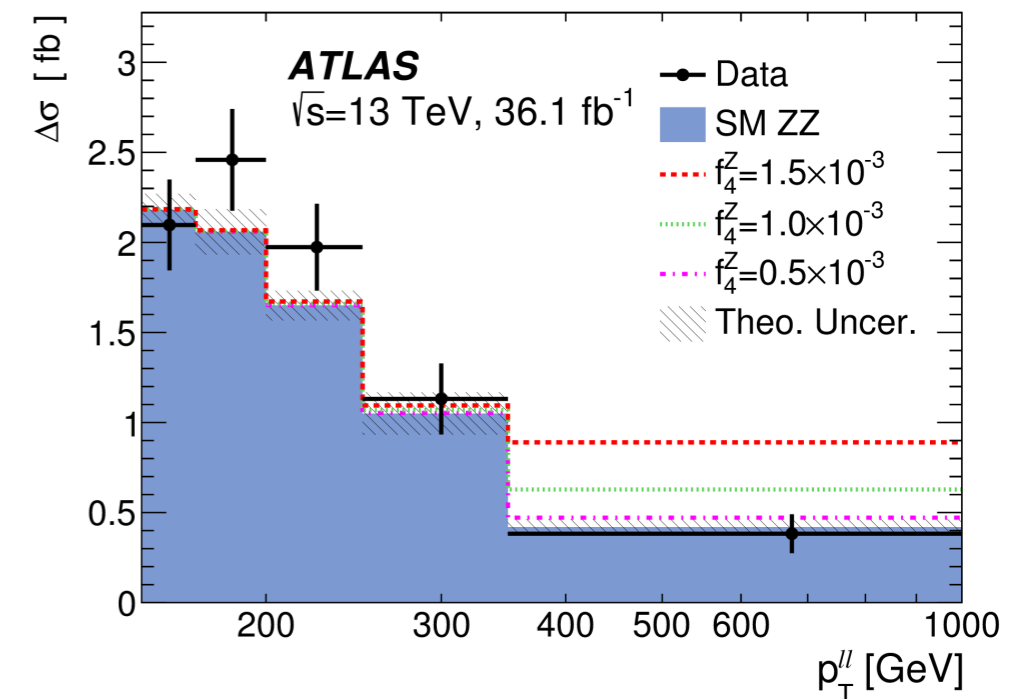


- ❖ ZZ → 2l2ν. Larger branching fraction than 4l
 - Also larger backgrounds
- ❖ nTGC limits from unfolded p_{T}^{ll} (>150 GeV) distribution
- ❖ Sensitivity range found to be within unitarity bounds, no form factors applied.
- ❖ Sensitivity limited by statistical uncertainty in data (40%)
- ❖ Vertex-approach for interpretation

$$g_{ZZV}\Gamma_{ZZV}^{\alpha\beta\mu} = e \frac{P^2 - M_V^2}{M_Z^2} \left[i f_4^V (P^\alpha g^{\mu\beta} + P^\beta g^{\mu\alpha}) + i f_5^V \epsilon^{\mu\alpha\beta\rho} (q_1 - q_2)_\rho \right],$$

	f_4^Y	f_4^Z	f_5^Y	f_5^Z
Expected [$\times 10^{-3}$]	[-1.3, 1.3]	[-1.1, 1.1]	[-1.3, 1.3]	[-1.1, 1.1]
Observed [$\times 10^{-3}$]	[-1.2, 1.2]	[-1.0, 1.0]	[-1.2, 1.2]	[-1.0, 1.0]

1-dimensional 95% CL



Older analyses constraining nTGC:

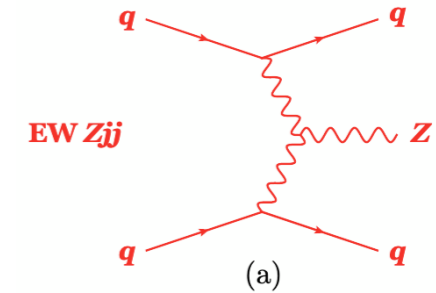
$Z\gamma\gamma$ constrained by [ATLAS \$Z\(\nu\nu\)\gamma\$ analysis](#)

ZZZ and $ZZ\gamma$ in [ATLAS \$ZZ \rightarrow 4l\$](#)

analysis. But better constraints from [CMS](#) with full Run-2 dataset.

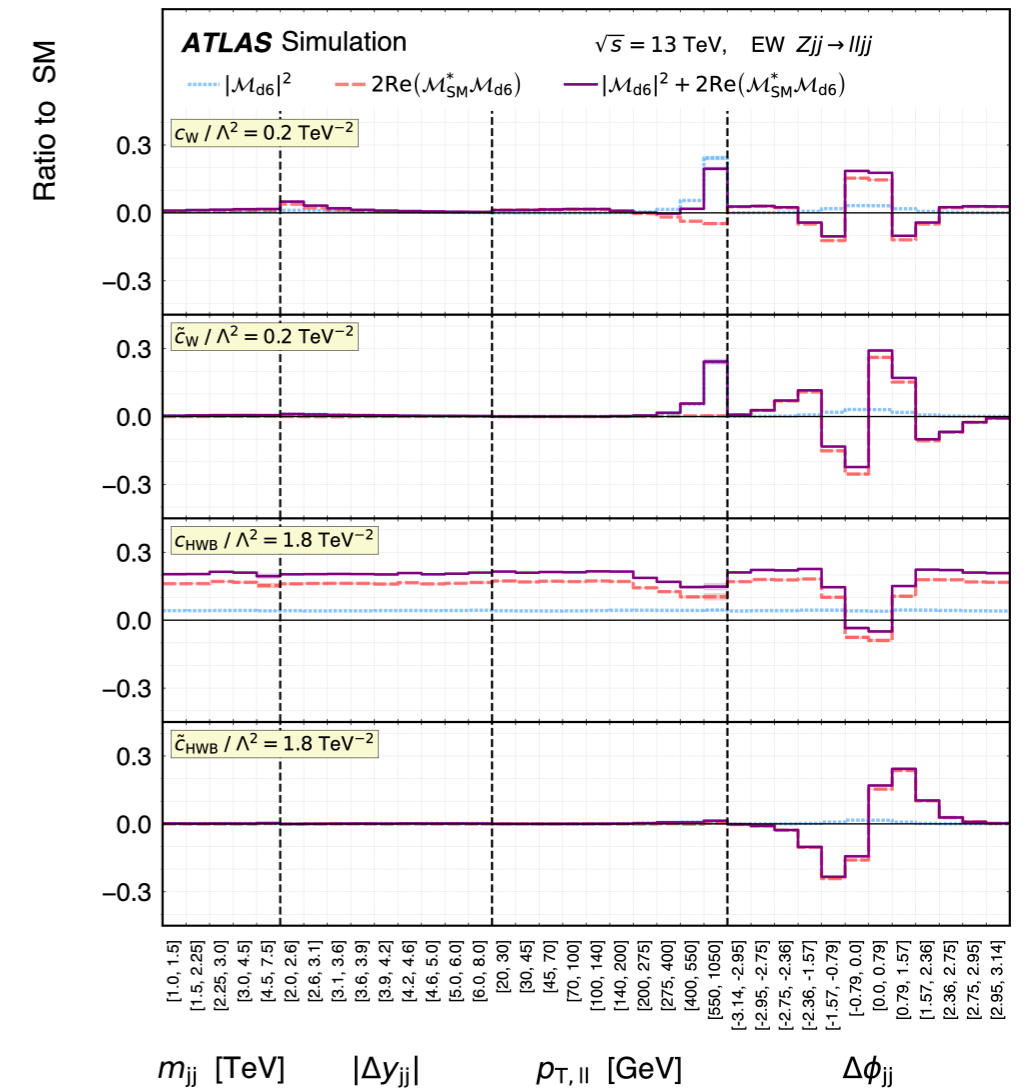
Examples of CMS analyses constraining aQGC in backup

EW Zjj



- ❖ Differential cross sections for EW Zjj production (Z to ee or $\mu\mu$) for the first time. Full Run 2 analysis
- ❖ Using Warsaw basis as implemented in SMEFTsim package
- ❖ Also exploits parity odd observables, $\Delta\phi_{jj}$, for the constraint of CP-even and CP-odd operators
- ❖ Checked importance of quadratic terms
 - Constraints mainly from interference (test of EFT convergence), no unitarity violation issues.

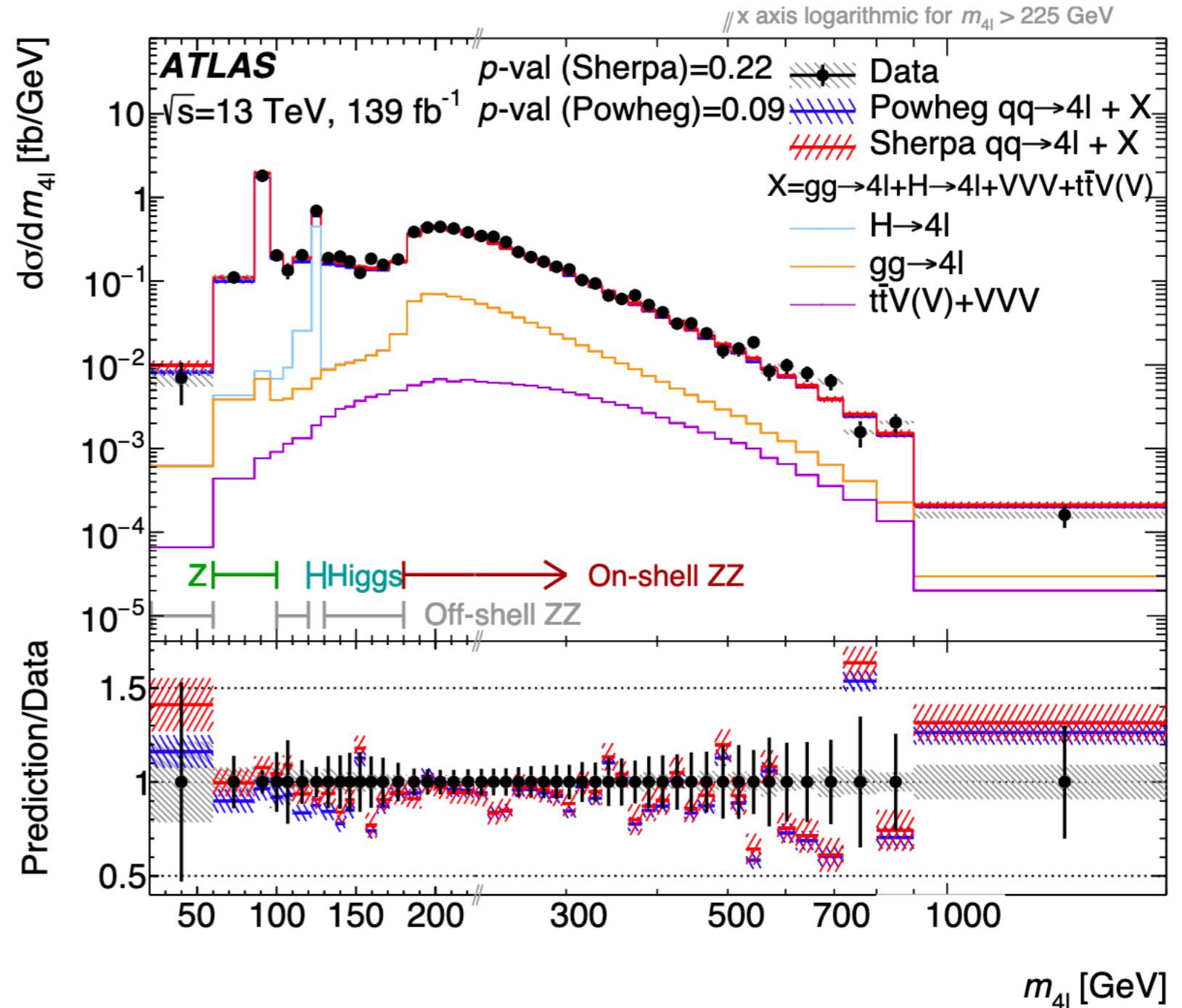
Wilson coefficient	Includes $ \mathcal{M}_{d6} ^2$	95% confidence interval [TeV^{-2}]		p -value (SM)
		Expected	Observed	
c_W/Λ^2	no	[-0.30, 0.30]	[-0.19, 0.41]	45.9%
	yes	[-0.31, 0.29]	[-0.19, 0.41]	43.2%
\tilde{c}_W/Λ^2	no	[-0.12, 0.12]	[-0.11, 0.14]	82.0%
	yes	[-0.12, 0.12]	[-0.11, 0.14]	81.8%
c_{HWB}/Λ^2	no	[-2.45, 2.45]	[-3.78, 1.13]	29.0%
	yes	[-3.11, 2.10]	[-6.31, 1.01]	25.0%
$\tilde{c}_{HWB}/\Lambda^2$	no	[-1.06, 1.06]	[0.23, 2.34]	1.7%
	yes	[-1.06, 1.06]	[0.23, 2.35]	1.6%



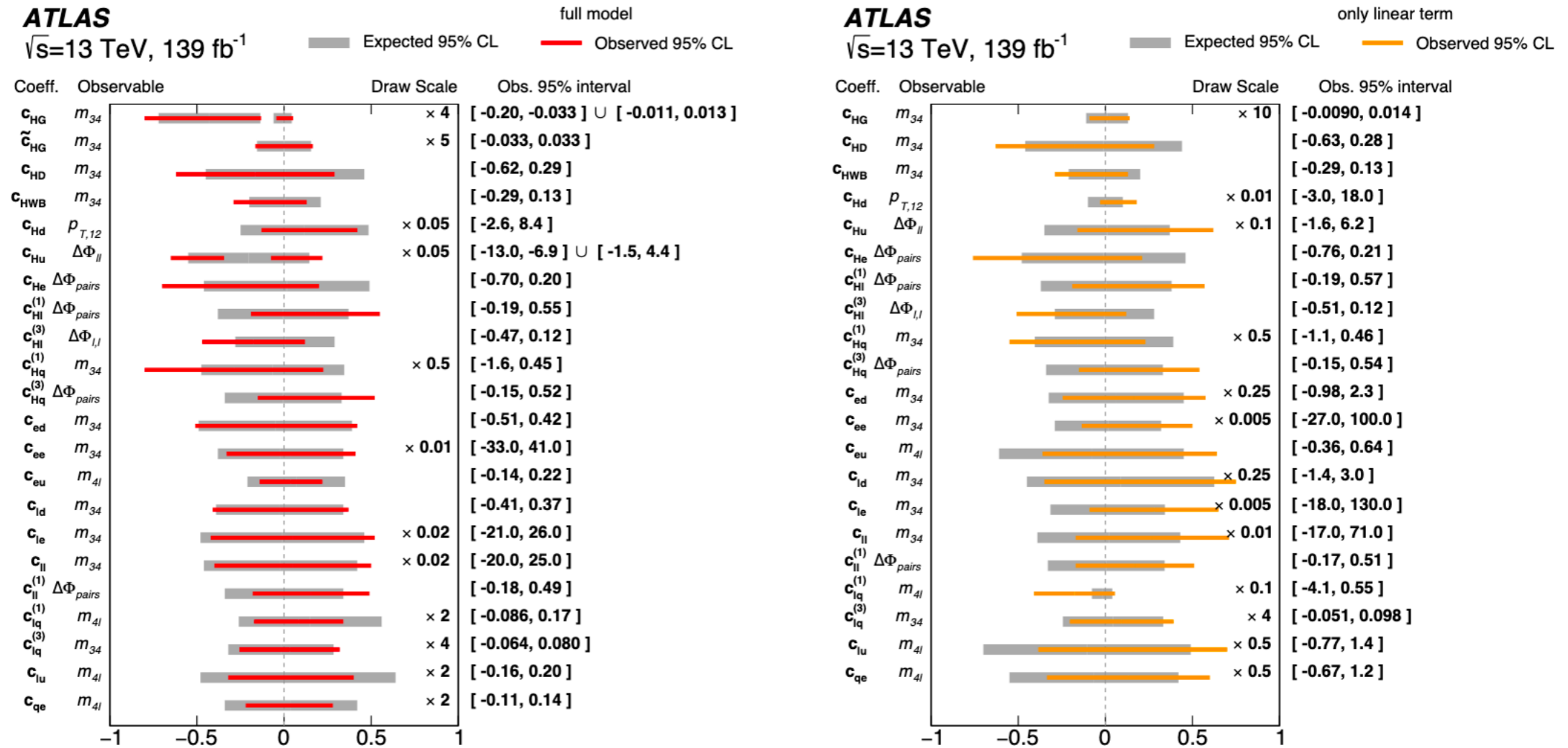
$$\Delta\phi_{jj} = y_f - y_b \text{ with } y_f > y_b$$

4l diff. Xs

- ❖ Using Warsaw basis as implemented in SMEFTsim package
- ❖ Considering all operators changing the cross section of the 4l processes
- ❖ One operator at a time
- ❖ Linear and linear+quadratic fits
 - tcHG only from quadratic terms



4l diff. Xs

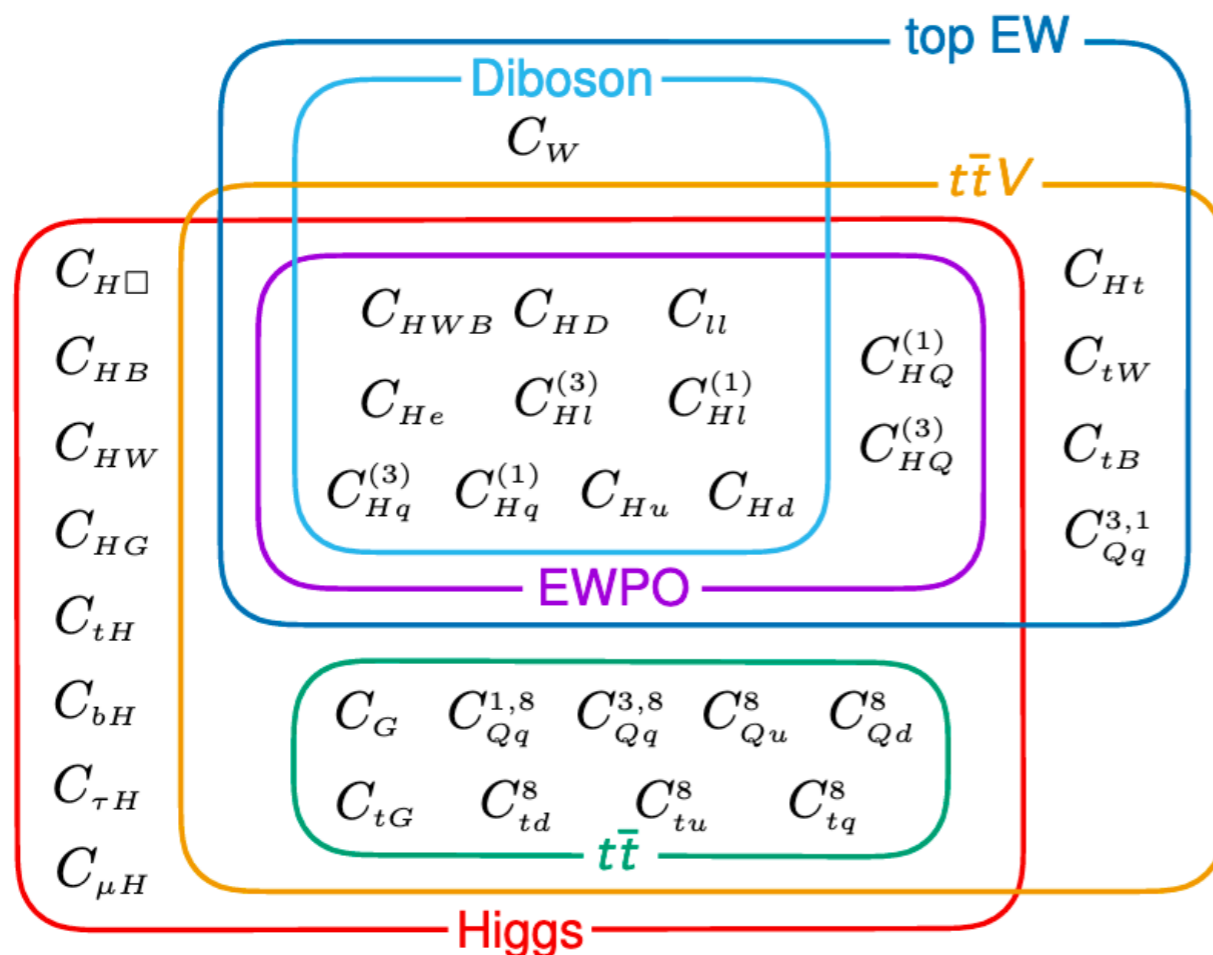


- ❖ Several four-fermion ^(a) contact interaction operators receive significant contribution ^(b) from the quadratic terms
- ❖ Less (more) stringent limits in c_{HG} (c_{HWB}) than the $H \rightarrow 4l$ analysis
- ❖ Parameters affecting $Z \rightarrow ll$ vertices better constrained by LEP

Towards global fits from the
experimental community

Global fits

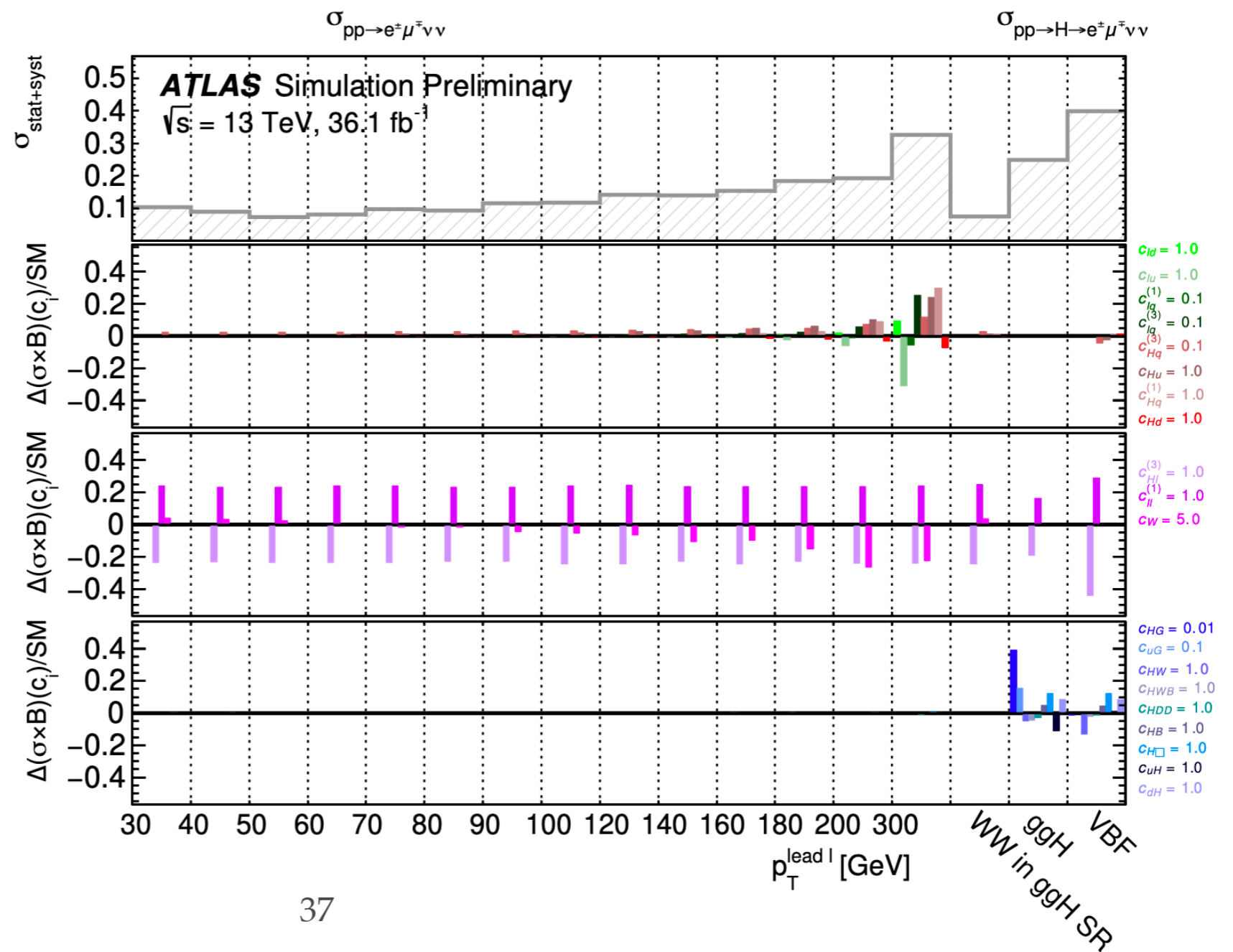
- ❖ Global fits of EW+Higgs+Top data so far only attempted by the theory community
 - ▶ Several assumptions made since full likelihoods are not published
- ❖ Overlap and complementarity from different datasets



- ❖ Tools usually made public. Examples:
 - ▶ FitMaker
 - ▶ SMEFiT using part of the NNPDF code
- ❖ Overview of tools available up to 2019 here.

$H \rightarrow WW^* + WW$ combination

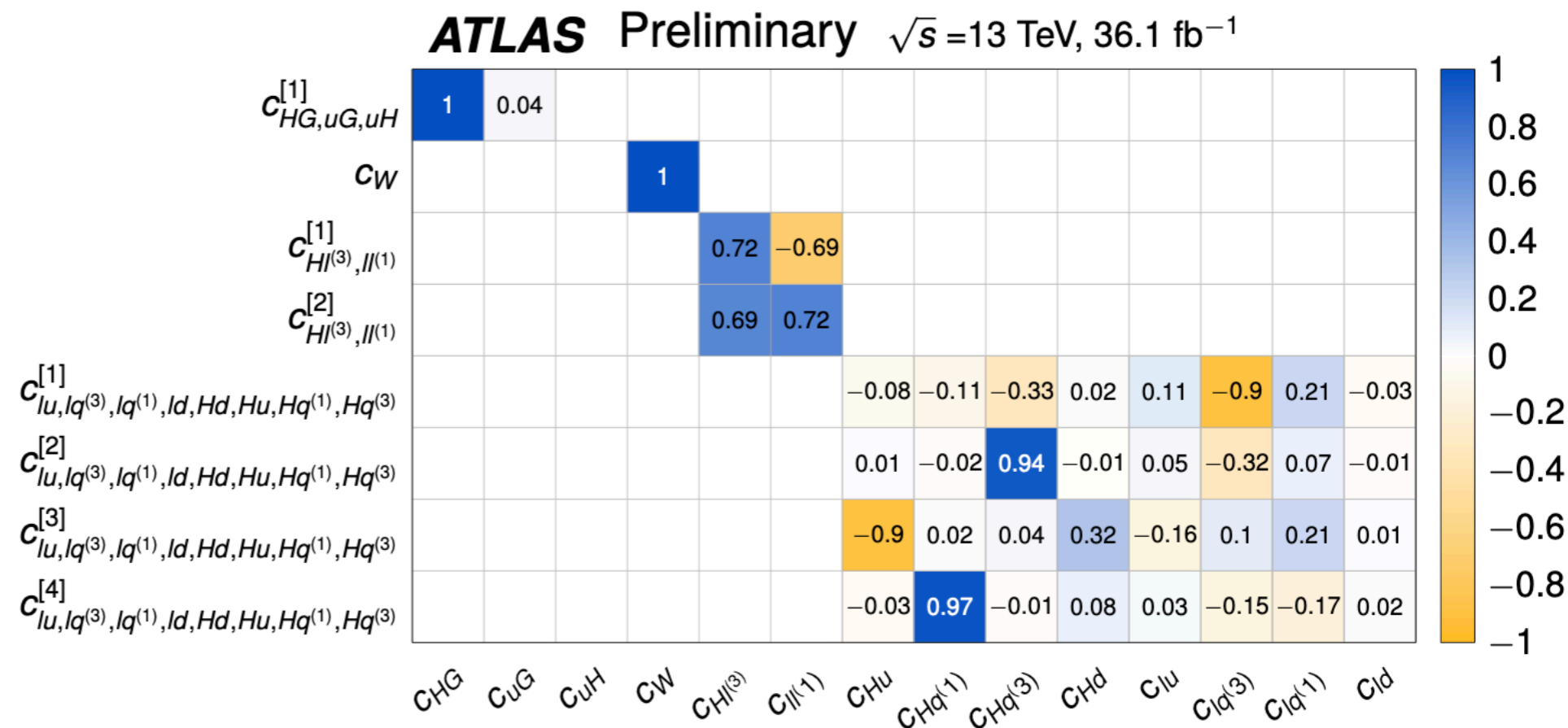
- ❖ Combination of $H \rightarrow WW^*$ production mode signal strengths in the VBF and ggH channel with differential WW cross sections.
- ❖ Measured in orthogonal regions and uncertainties correlated appropriately
- ❖ Using SMEFTsim in MW scheme and only CP-even operators
- ❖ Higgs predictions in the NWA
- ❖ Reduction of sensitivity from the removal of overlapping regions



$H \rightarrow WW^* + WW$ combination

- ❖ Differences in acceptance between EFT and SM are taken into account:
 - ▶ in the $H \rightarrow WW^*$ BR as multiplicative factors affecting mainly to c_{HW} (+10% wrt. SM) and c_{Hl3} (-1.8% wrt. SM)
 - ▶ In the WW background of the $H \rightarrow WW$ signal region
 - ▶ Neglected elsewhere

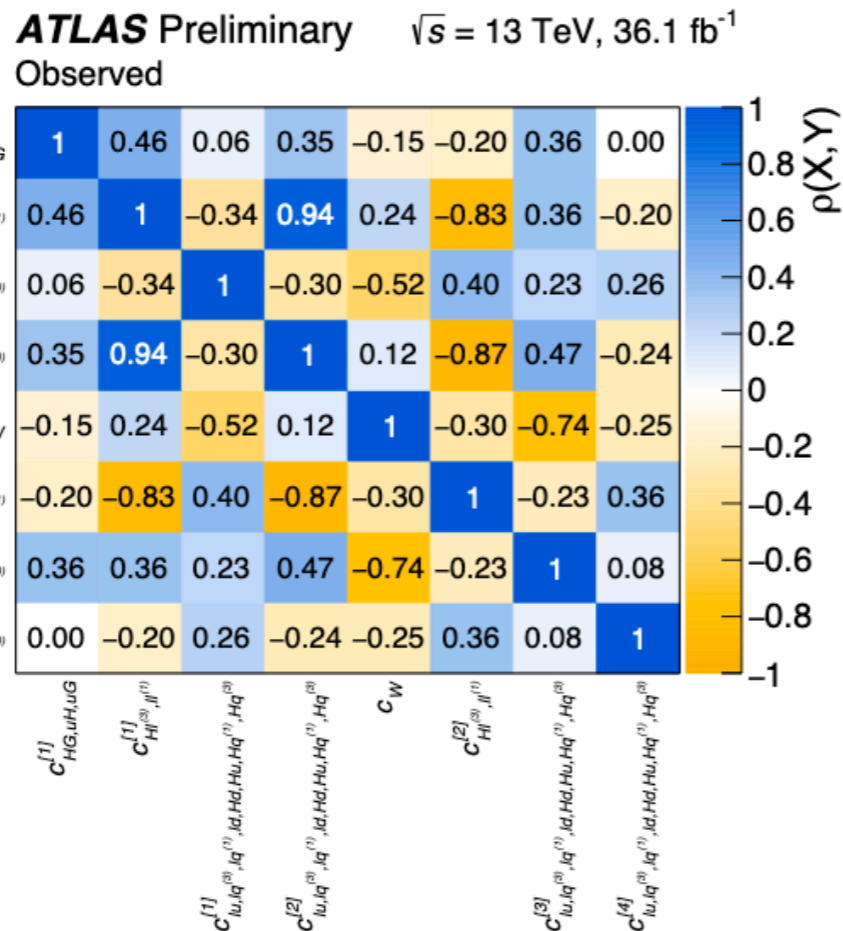
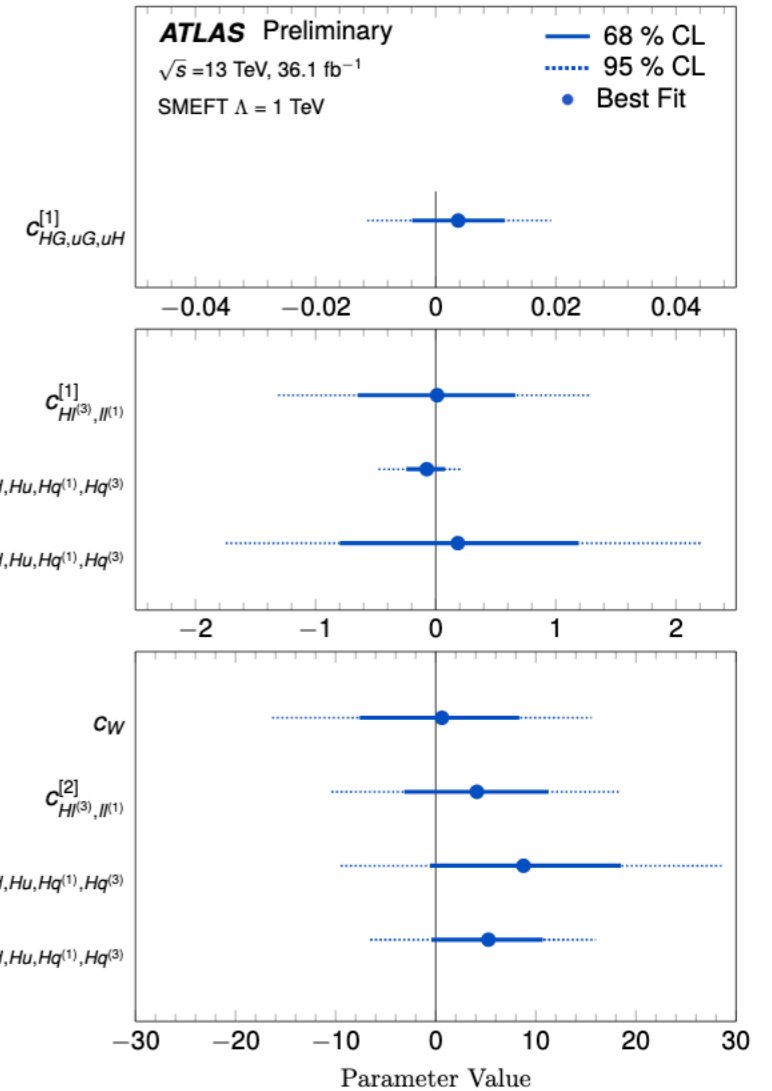
- ❖ Similar PCA analysis as for the STXS combination



H → WW* + WW combination

- Results shown only for the linear case
 - cW expected to have large contribution from quadratic terms (interference largely suppressed)

- No significant deviations from the SM found



EFT harmonisation

- ▶ EFT interpretations used to be done following different conventions / bases
- ▶ Significant steps given in the past years:
 - * Agreement on a common basis: dimension-6 Warsaw basis. No dim-8 needed for aQGC
 - * LO (SMEFTsim) and NLO (SMEFT@NLO) complete tools in this basis
 - * Several flavour structures in the former. Using $U(3)_5$ for Higgs or EW measurements but $U(2)_{q,u,d}^3 U(3)_{l,e}^2$ well suited for top measurements
 - * Two sets of input parameters in the former, (m_W, m_Z, G_F) usually preferred for LHC, but LEP constraints derived for (α, m_Z, G_F) . How to combine?
 - * Linear model better defined (complete), but when possible provide also linear+quadratic

Open questions: EFT validity

Presence of BSM light states: EFT (by construction) will not be able to reproduce the data

Sensitivity to only very large values of c_i/Λ : analysis cannot exclude $M > E$, unitarity issues



Clipping approach

- Use the EFT prediction only up to a clipping energy $\sqrt{s} = E_{\text{clip}}$ and set any contribution from this theory to 0 beyond this energy
- The clipping is done at parton level
- The SM predictions as well as the data remain untouched
- Derive limits for various E_{clip}
- Considering to use: Last data point can be use as reference point to start clipping scan

Joany Manjarres

Open questions: EFT uncertainties

EFT Truncation: additional insertions,
higher dimension terms

Interplay with QCD/EW corrections?

Unknown SMEFT effects on α_s running, PDFs,
hadronisation etc...

To address these and many other open questions with the theory and experiment community: LHC EFT WG

Conclusions

- ▶ SMEFT is a very powerful tool for data interpretation
- ▶ SMEFT interpretations are becoming quite common in LHC measurements
- ▶ ATLAS is taking efforts to improve these measurements and to adopt the latest tools provided by the theory community
- ▶ Target goal: Global EFT fits (LEP+Higgs+EW+Top+ B-physics+...)
- ▶ Step-by-step process: several open questions and still a lot to learn!

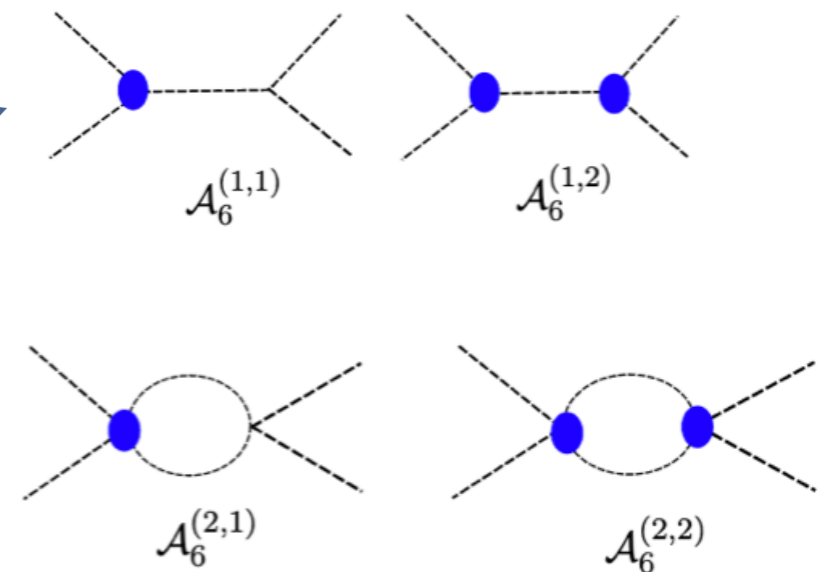
Sorry for the uncovered aspects or analyses: EFT is very broad!

Thanks!

Back up

Higher orders in SMEFT and other concepts

- ▶ Apart from adding additional dimensions, one can add higher orders by adding:
 - * More insertions (needs higher dimension counter-terms)
 - * More loops
 - ◆ Usually LO simulations are considered although NLO is possible (unlike κ -framework) and available
- ▶ No clear recommendations on uncertainties for EFT predictions
- ▶ In differential measurements, effect of operators usually growing with $(E/\Lambda)^{d-4}$
 - * Measure in tails of distributions
- ▶ Growth of amplitude with energy can lead to unitarity violation
 - * EFT no longer valid

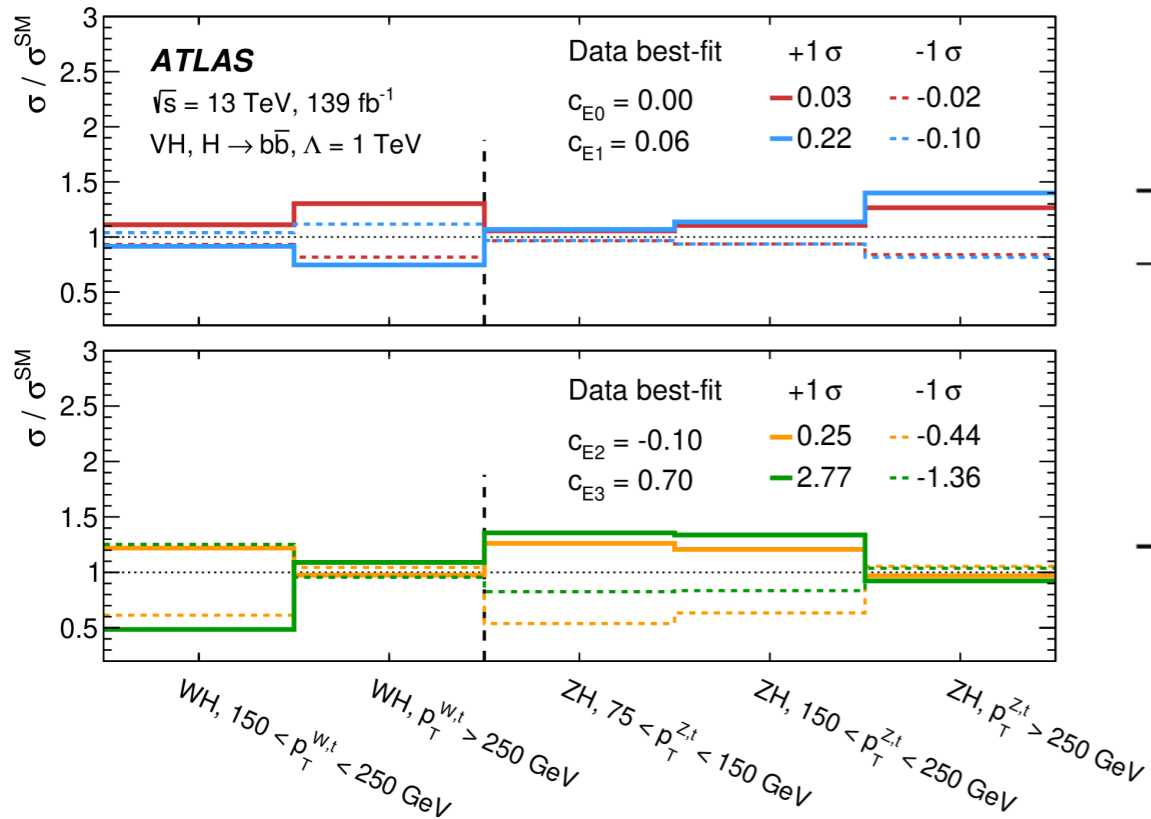


VH, H → bb

[arXiv:2007.02873](https://arxiv.org/abs/2007.02873)

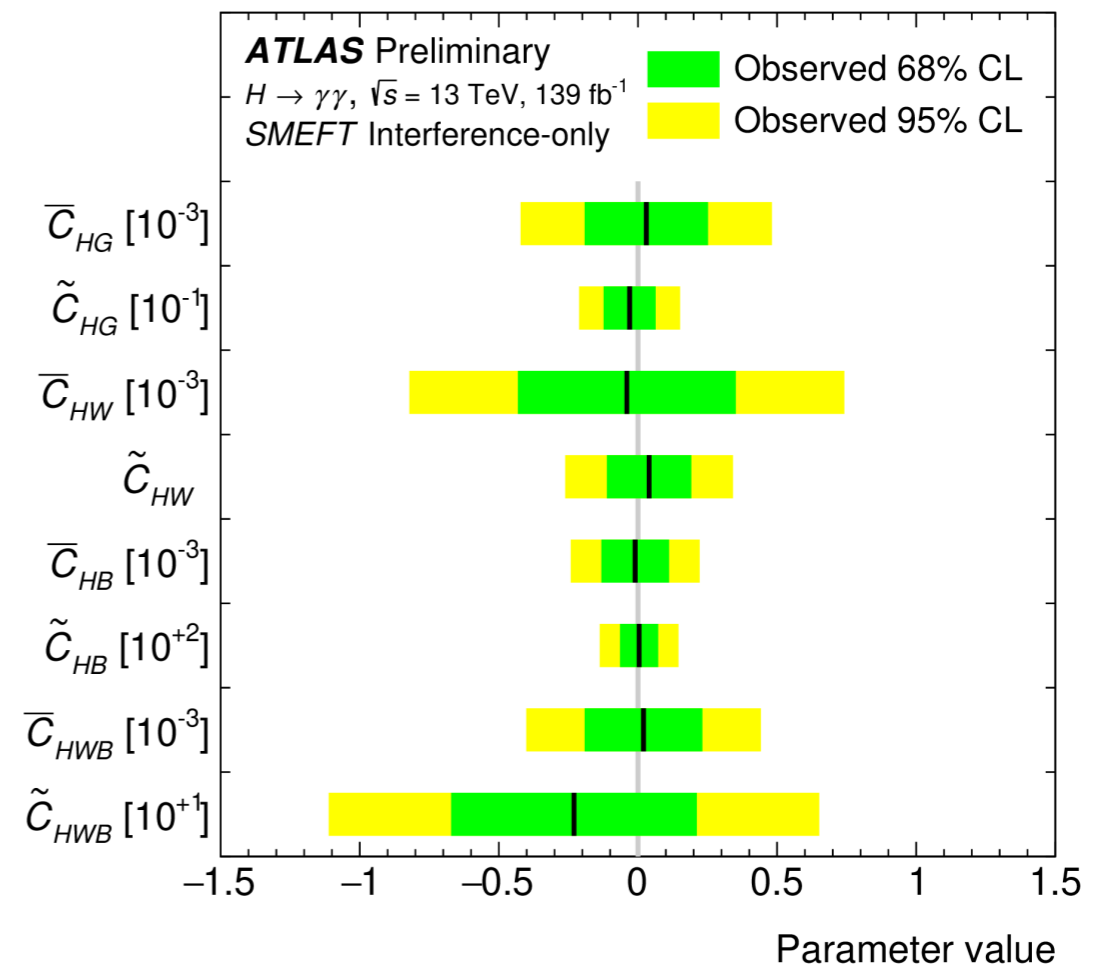
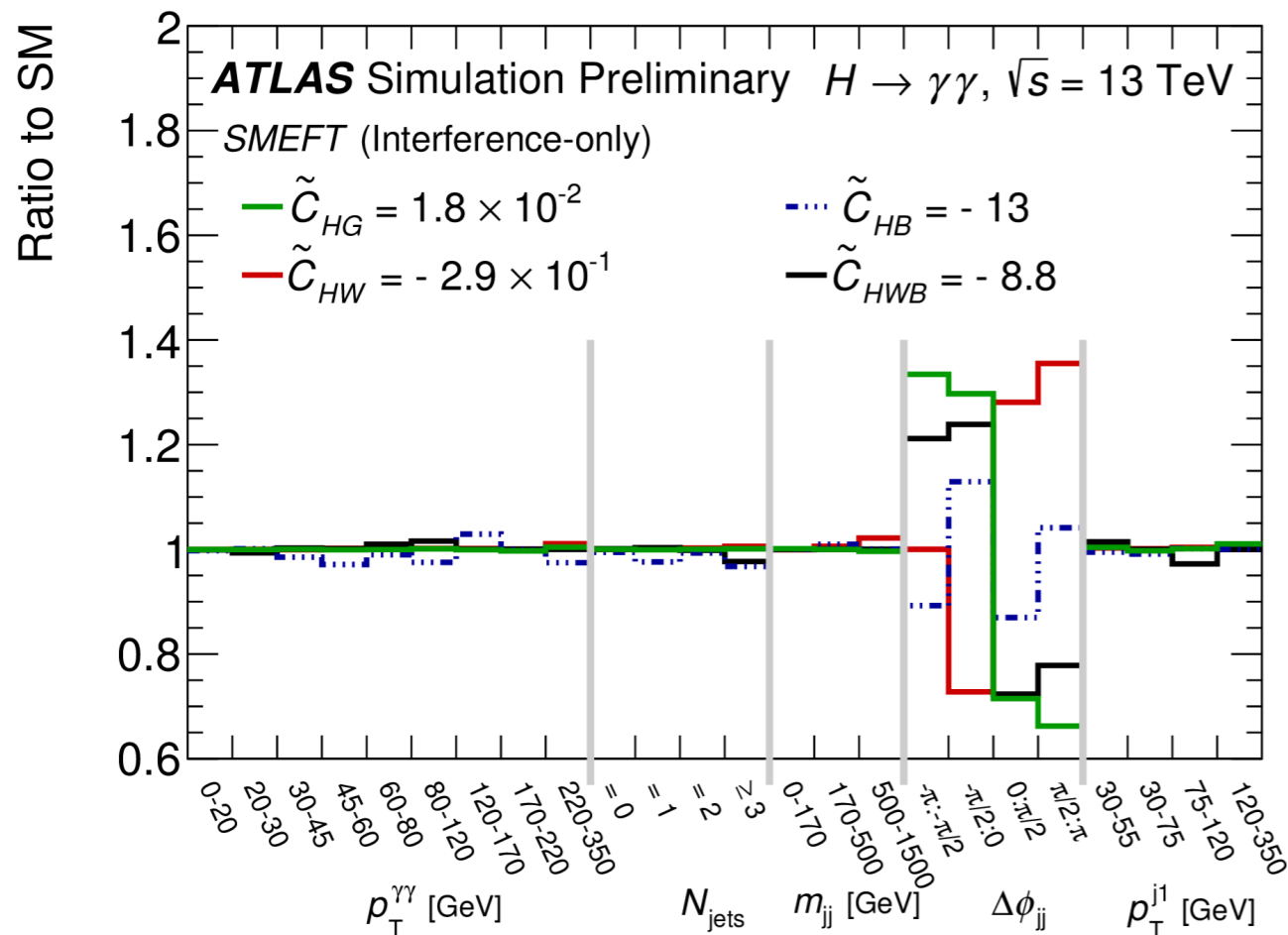
[arXiv:2008.02508](https://arxiv.org/abs/2008.02508)

- ❖ Several operators affecting VH, H → bb in the Warsaw basis
 - ▶ Not possible to constrain all with 4/5 measured STXS bins
 - ▶ Do a principal component analysis (PCA). Methodology from [ATL-PHYS-PUB-2019-042](https://arxiv.org/abs/1904.042)
 - ▶ Fit simultaneously the 5 sensitive directions



Wilson coefficient	Eigenvalue	Eigenvector
c_{E0}	2000	$0.98 \cdot c_{Hq3}$
c_{E1}	38	$0.85 \cdot c_{Hu} - 0.39 \cdot c_{Hq1} - 0.27 \cdot c_{Hd}$
c_{E2}	8.3	$0.70 \cdot \Delta\text{BR}/\text{BR}_{\text{SM}} + 0.62 \cdot c_{HW}$
c_{E3}	0.2	$0.74 \cdot c_{HWB} + 0.53 \cdot c_{Hq1} - 0.32 \cdot c_{HW}$
c_{E4}	$6.4 \cdot 10^{-3}$	$0.65 \cdot c_{HW} - 0.60 \cdot \Delta\text{BR}/\text{BR}_{\text{SM}} + 0.35 \cdot c_{Hq1}$

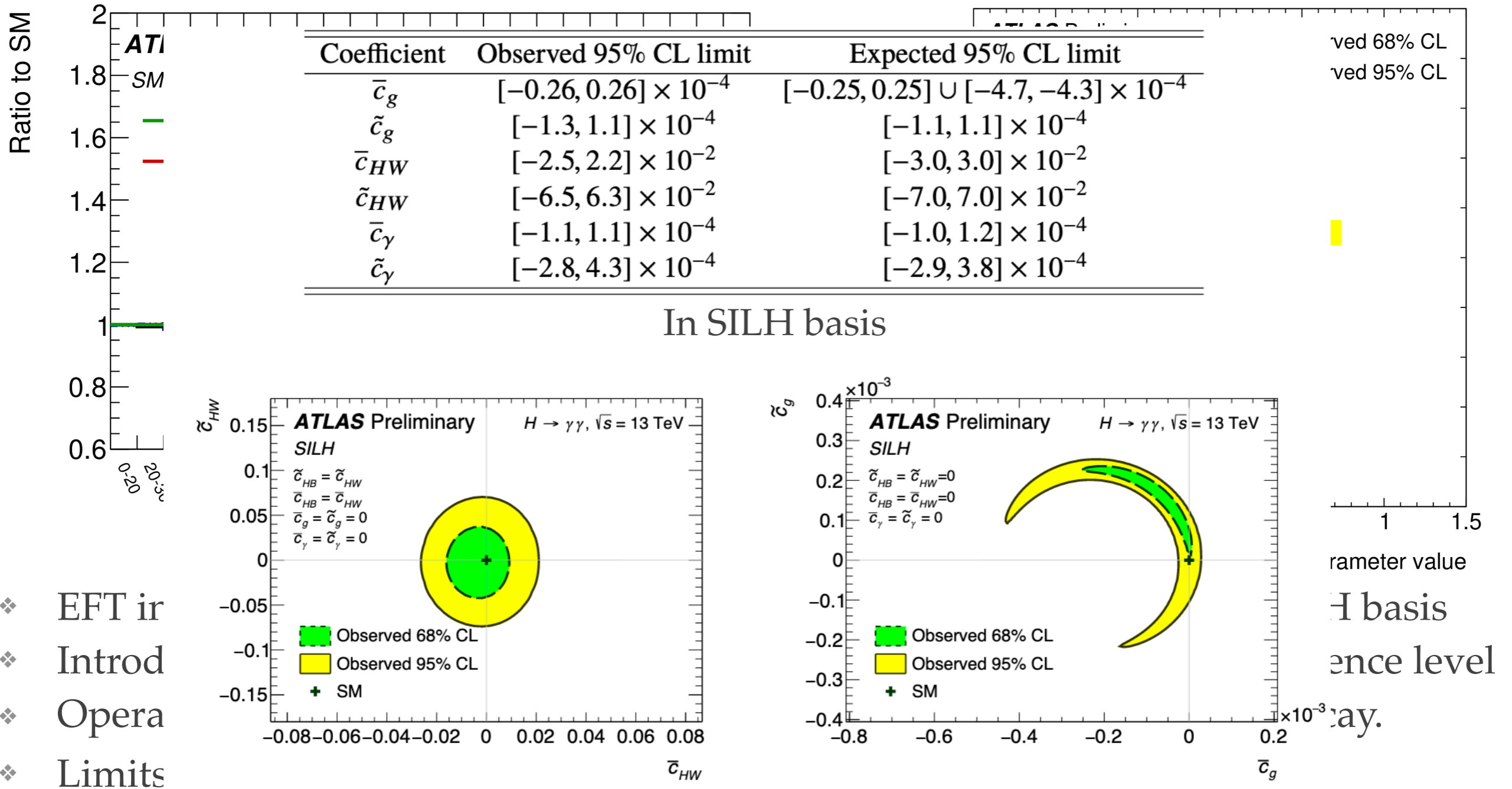
ATLAS: $H \rightarrow \gamma\gamma$



- ❖ EFT interpretation from differential cross sections using Warsaw and SILH bases
- ❖ Introduced CP-odd observables to constrain CP-odd operators at interference level
- ❖ Operators studied are the ones modifying mainly ggH and the $H \rightarrow \gamma\gamma$ decay.
- ❖ Limits from 1-D fits

$$\mathcal{L} = \frac{1}{\sqrt{(2\pi)^k |C|}} \exp\left(-\frac{1}{2} (\vec{\sigma}_{\text{data}} - \vec{\sigma}_{\text{pred}})^T C^{-1} (\vec{\sigma}_{\text{data}} - \vec{\sigma}_{\text{pred}})\right),$$

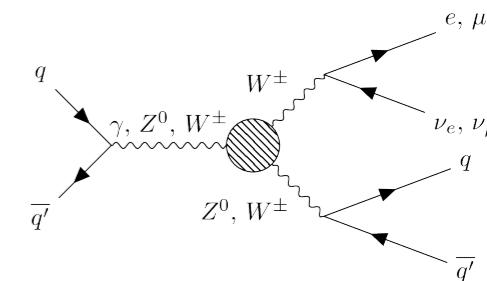
ATLAS: $H \rightarrow \gamma\gamma$



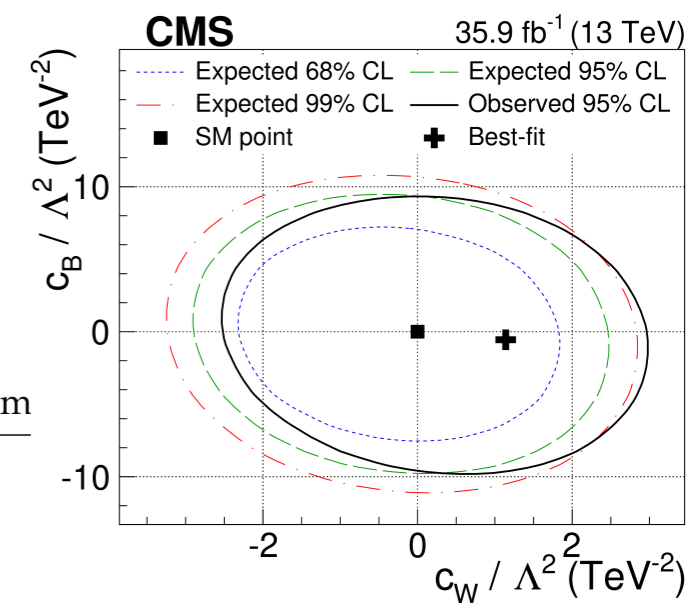
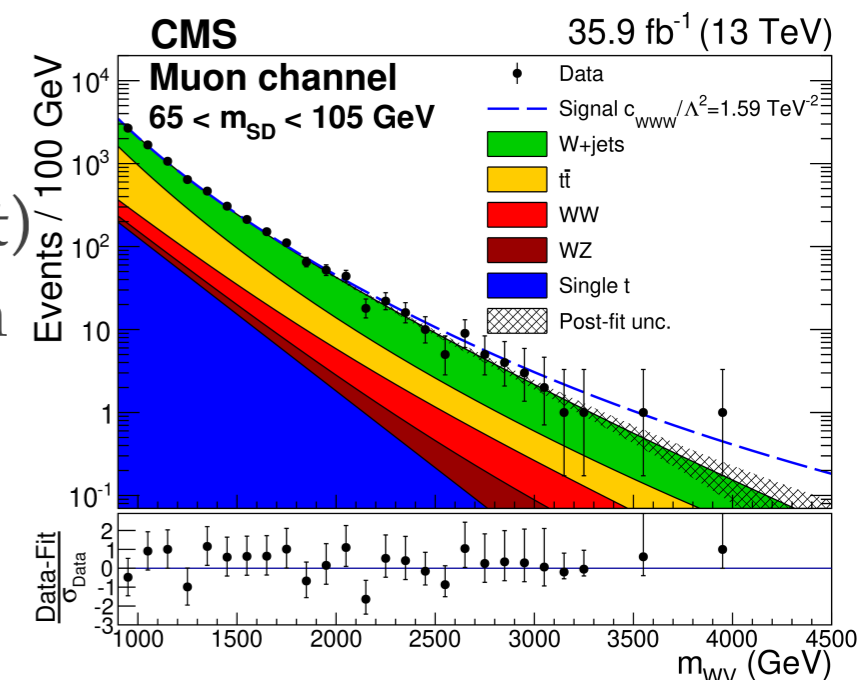
- ❖ EFT in
- ❖ Introduct
- ❖ Opera
- ❖ Limits

$$\chi^2 = \frac{1}{\sqrt{(2\pi)^k |C|}} \exp \left(-\frac{1}{2} (\mu_{\text{data}} - \mu_{\text{pred}})^T C^{-1} (\mu_{\text{data}} - \mu_{\text{pred}}) \right)$$

CMS: WW and WZ

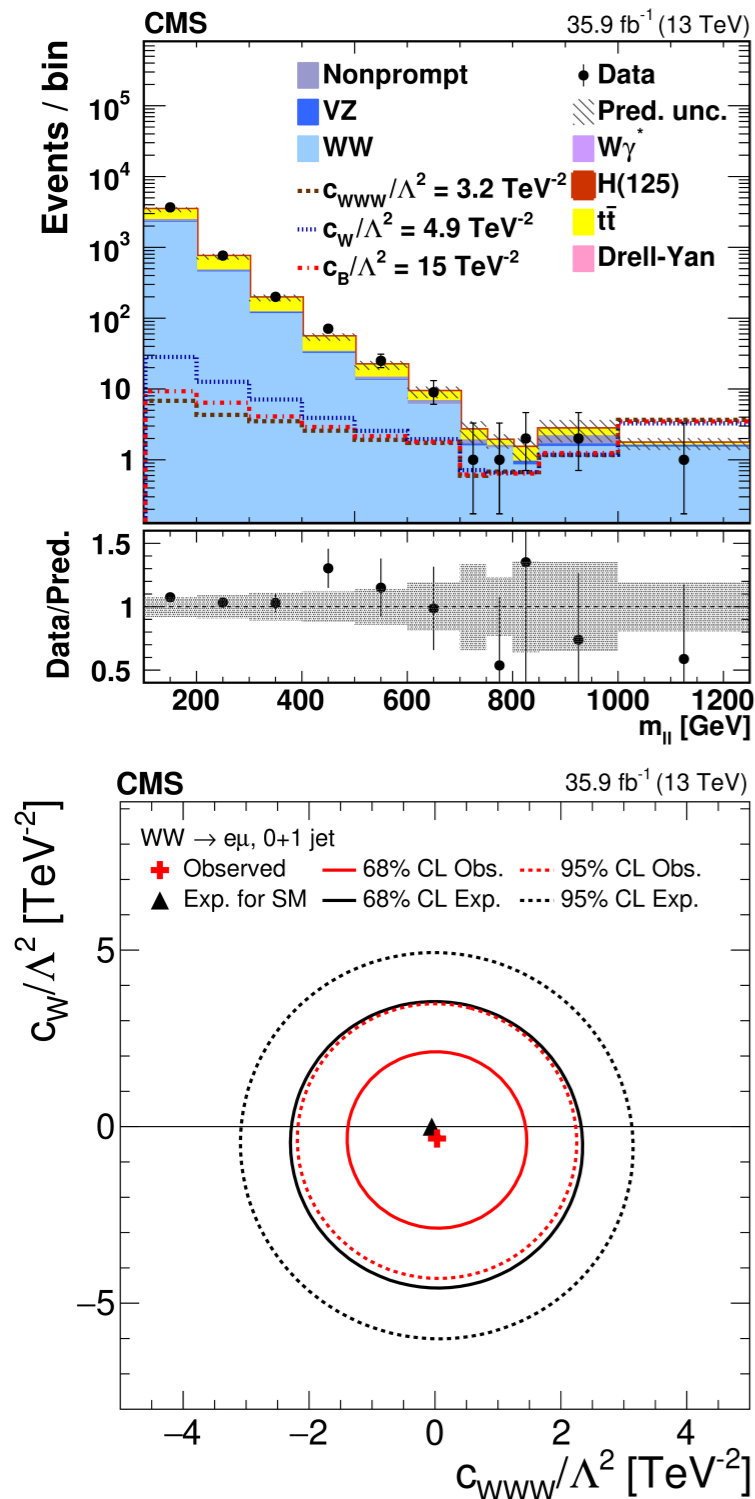


- ❖ Dedicated measurement for constraining anomalous WW_γ and WWZ couplings
- ❖ W decaying leptonically and Z or W hadronically (fat jet)
 - Semi-leptonic channels offer a good balance between purity and efficiency
- ❖ Limits from 2D unbinned LH fits to (m_{SD}, m_{WV})
- ❖ c_{WW} and c_W similar impact in WW and WZ, c_B much greater in WW region.
 - Little separation power between c_{WW} and c_W
- ❖ Improvement wrt. 8 TeV results



Parametrization	aTGC	Expected limit	Observed limit	Observed best-fit	8 TeV observed lim
EFT	c_{WWW} / Λ^2 (TeV^{-2})	[-1.44, 1.47]	[-1.58, 1.59]	-0.26	[-2.7, 2.7]
	c_W / Λ^2 (TeV^{-2})	[-2.45, 2.08]	[-2.00, 2.65]	1.21	[-2.0, 5.7]
	c_B / Λ^2 (TeV^{-2})	[-8.38, 8.06]	[-8.78, 8.54]	1.07	[-14, 17]
LEP	λ_Z	[-0.0060, 0.0061]	[-0.0065, 0.0066]	-0.0010	[-0.011, 0.011]
	Δg_1^Z	[-0.0070, 0.0061]	[-0.0061, 0.0074]	0.0027	[-0.009, 0.024]
	$\Delta \kappa_Z$	[-0.0074, 0.0078]	[-0.0079, 0.0082]	-0.0010	[-0.018, 0.013]

CMS: WW



- ❖ Two methodologies (sequential cuts and random forests) studied for background estimation.
- ❖ WW → l⁺νl⁻ν with 0 or 1-jet
- ❖ Limits from m_{eμ} templates (not sensitive to higher-order QCD effects or jet energy scale). BSM terms behave as SM in the unfolding
- ❖ Only different flavour event sample
 - Same flavour has larger contamination from DY
 - m_{eμ} > 100 GeV to reduce Higgs contribution
- ❖ Almost a factor 2 better more stringent than ATLAS
 - Due to the usage of 1-jet measurement

WW+1jet measurement in ATLAS,
but no interpretation

Coefficients (TeV ⁻²)	68% confidence interval		95% confidence interval	
	expected	observed	expected	observed
c_{WWW}/Λ^2	[-1.8, 1.8]	[-0.93, 0.99]	[-2.7, 2.7]	[-1.8, 1.8]
c_W/Λ^2	[-3.7, 2.7]	[-2.0, 1.3]	[-5.3, 4.2]	[-3.6, 2.8]
c_B/Λ^2	[-9.4, 8.4]	[-5.1, 4.3]	[-14, 13]	[-9.4, 8.5]

Beyond dim-6: nTGC and aQGC

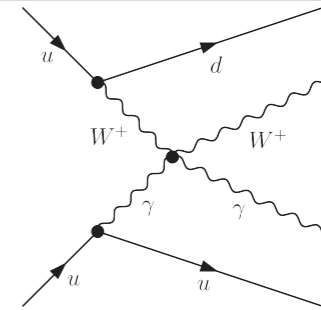
- ❖ No neutral gauge couplings in SM or from dimension-6 operators at tree-level
- ❖ They first appear at dimension 8

$$\begin{aligned}\mathcal{O}_{B\tilde{W}} &= iH^\dagger \tilde{B}_{\mu\nu} W^{\mu\rho} \{D_\rho, D^\nu\} H & \mathcal{O}_{WW} &= iH^\dagger W_{\mu\nu} W^{\mu\rho} \{D_\rho, D^\nu\} H \\ \mathcal{O}_{BW} &= iH^\dagger B_{\mu\nu} W^{\mu\rho} \{D_\rho, D^\nu\} H & \mathcal{O}_{BB} &= iH^\dagger B_{\mu\nu} B^{\mu\rho} \{D_\rho, D^\nu\} H\end{aligned}$$

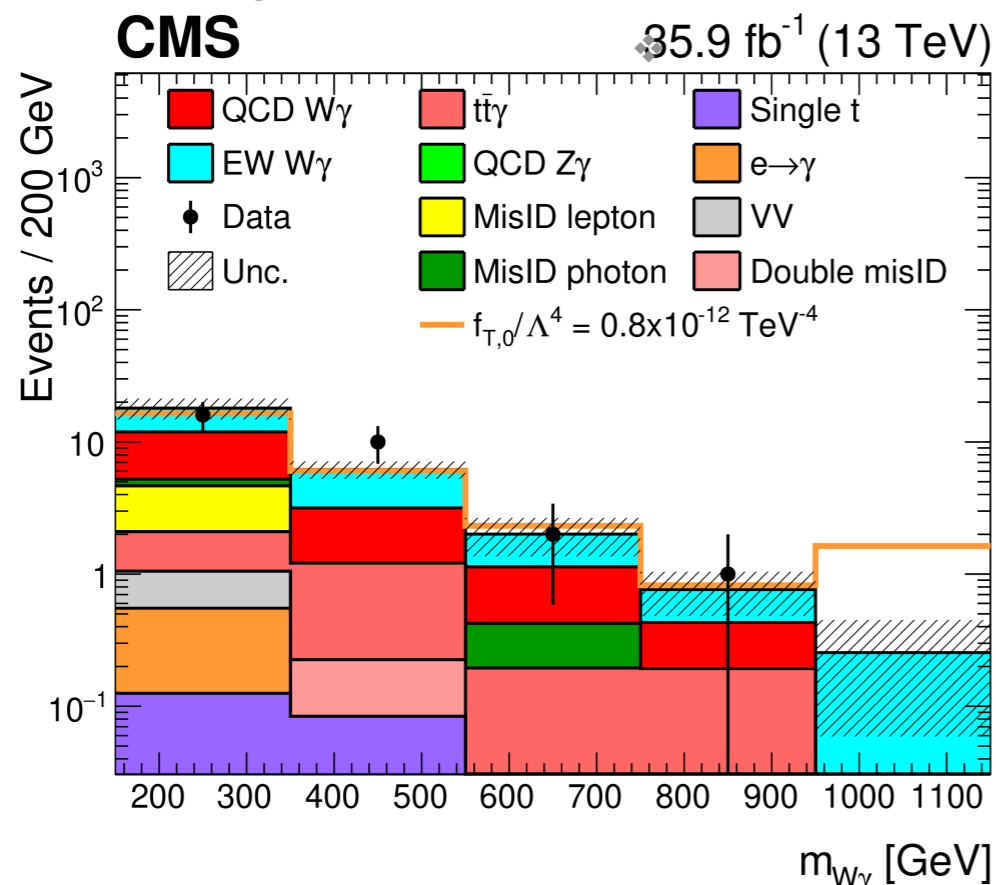
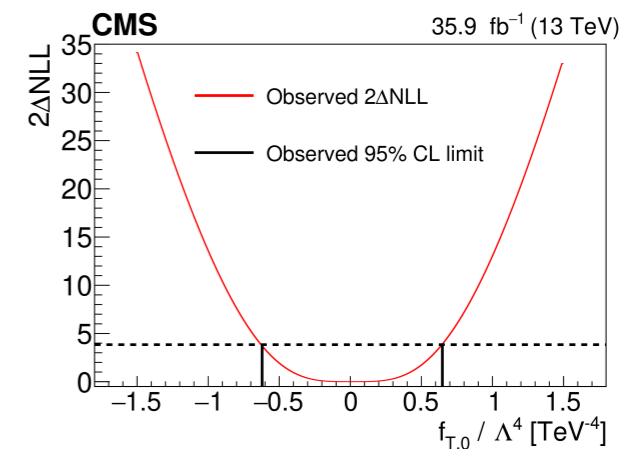
- ❖ Operators with quartic vertices appear at dimension 8
- ❖ Assume processes probing aQGC have negligible contribution from dimension-6 operators (constrained by other measurements)
- ❖ Lagrangian terms:

$$\mathcal{L}_{S,0-1} \propto (D_\mu \Phi)^4, \quad \mathcal{L}_{M,0-7} \propto (F^{\mu\nu})^2 (D_\mu \Phi)^2, \quad \mathcal{L}_{T,0-9} \propto (F^{\mu\nu})^4$$

CMS: $W\gamma$ VBS

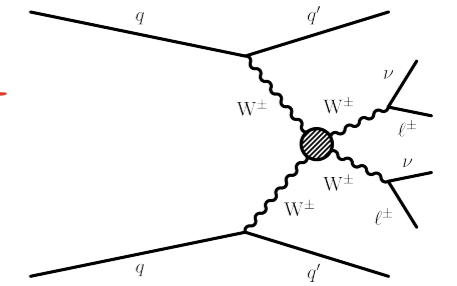


- ❖ W decaying in the leptonic (e or μ) channel
- ❖ $p_{T\gamma} > 25$ GeV, $m_{jj} > 500$ GeV, $|\Delta\eta_{jj}| > 2.5$
 - EW extraction from 2-D template fits to $(m_{jj}, m_{l\gamma})$
- ❖ aQGC limits from fits to $m_{\gamma W}$ distribution
- ❖ Using Eboli basis
- ❖ Limits set from profile likelihood test statistic
- ❖ Most stringent limits on $f_{M,2-5}$ and $f_{T,6-7}$

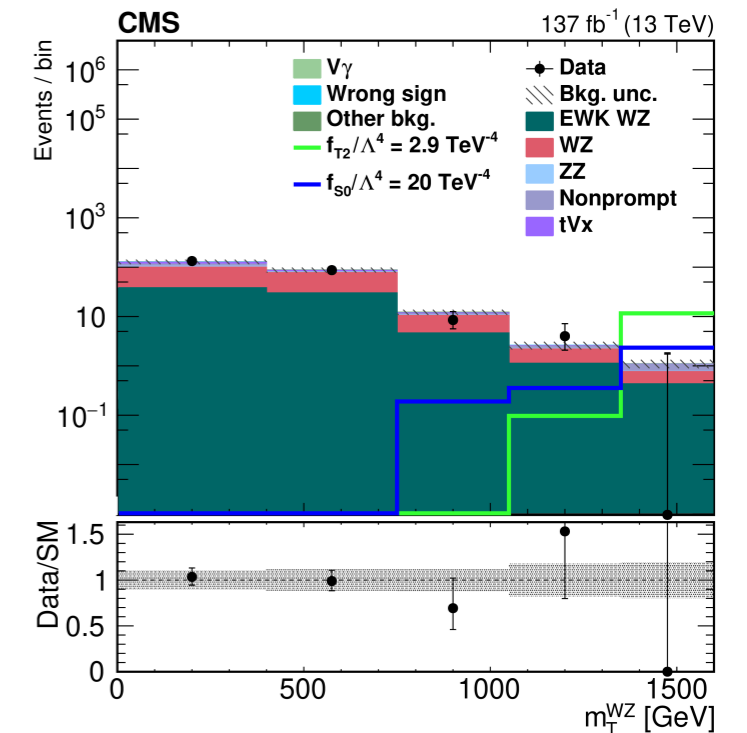


Parameters	Exp. limit	Obs. limit	U_{bound}
$f_{M,0}/\Lambda^4$	$[-8.1, 8.0]$	$[-7.7, 7.6]$	1.0
$f_{M,1}/\Lambda^4$	$[-12, 12]$	$[-11, 11]$	1.2
$f_{M,2}/\Lambda^4$	$[-2.8, 2.8]$	$[-2.7, 2.7]$	1.3
$f_{M,3}/\Lambda^4$	$[-4.4, 4.4]$	$[-4.0, 4.1]$	1.5
$f_{M,4}/\Lambda^4$	$[-5.0, 5.0]$	$[-4.7, 4.7]$	1.5
$f_{M,5}/\Lambda^4$	$[-8.3, 8.3]$	$[-7.9, 7.7]$	1.8
$f_{M,6}/\Lambda^4$	$[-16, 16]$	$[-15, 15]$	1.0
$f_{M,7}/\Lambda^4$	$[-21, 20]$	$[-19, 19]$	1.3
$f_{M,0}/\Lambda^4$	$[-0.6, 0.6]$	$[-0.6, 0.6]$	1.4
$f_{M,1}/\Lambda^4$	$[-0.4, 0.4]$	$[-0.3, 0.4]$	1.5
$f_{M,2}/\Lambda^4$	$[-1.0, 1.2]$	$[-1.0, 1.2]$	1.5
$f_{M,5}/\Lambda^4$	$[-0.5, 0.5]$	$[-0.4, 0.4]$	1.8
$f_{M,6}/\Lambda^4$	$[-0.4, 0.4]$	$[-0.3, 0.4]$	1.7
$f_{M,7}/\Lambda^4$	$[-0.9, 0.9]$	$[-0.8, 0.9]$	1.8

CMS: WZ and ss WW



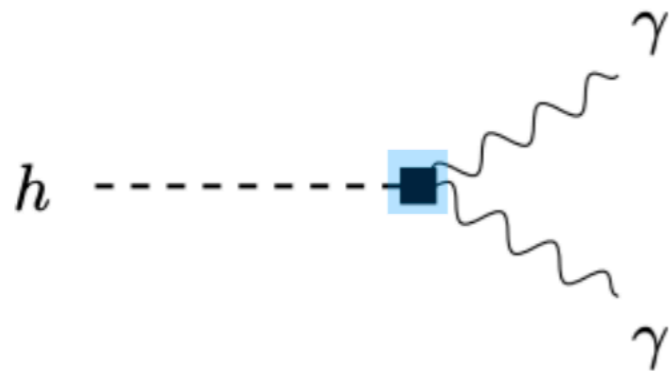
- ❖ $W^\pm Z \rightarrow l^\pm \nu l'^\pm l'^\mp$ and $WW \rightarrow l^\pm \nu l'^\pm \nu$
 - ssWW cleanest channel in terms of EW signal to QCD bkg. ratio
- ❖ EW WZ signal separated from WZ QCD process using a BDT approach
- ❖ aQGC limits from fits to the transverse mass of the diboson system distribution
 - Eboli basis. Cutting the EFT integration at the unitarity limit
- ❖ Improvement over other leptonic measurements of WZ and WW
 - But less restrictive than semileptonic final states



Including unitarization

	Observed ($W^\pm W^\pm$) (TeV^{-4})	Expected ($W^\pm W^\pm$) (TeV^{-4})	Observed (WZ) (TeV^{-4})	Expected (WZ) (TeV^{-4})	Observed (TeV^{-4})	Expected (TeV^{-4})
f_{T0}/Λ^4	[-1.5, 2.3]	[-2.1, 2.7]	[-1.6, 1.9]	[-2.0, 2.2]	[-1.1, 1.6]	[-1.6, 2.0]
f_{T1}/Λ^4	[-0.81, 1.2]	[-0.98, 1.4]	[-1.3, 1.5]	[-1.6, 1.8]	[-0.69, 0.97]	[-0.94, 1.3]
f_{T2}/Λ^4	[-2.1, 4.4]	[-2.7, 5.3]	[-2.7, 3.4]	[-4.4, 5.5]	[-1.6, 3.1]	[-2.3, 3.8]
f_{M0}/Λ^4	[-13, 16]	[-19, 18]	[-16, 16]	[-19, 19]	[-11, 12]	[-15, 15]
f_{M1}/Λ^4	[-20, 19]	[-22, 25]	[-19, 20]	[-23, 24]	[-15, 14]	[-18, 20]
f_{M6}/Λ^4	[-27, 32]	[-37, 37]	[-34, 33]	[-39, 39]	[-22, 25]	[-31, 30]
f_{M7}/Λ^4	[-22, 24]	[-27, 25]	[-22, 22]	[-28, 28]	[-16, 18]	[-22, 21]
f_{S0}/Λ^4	[-35, 36]	[-31, 31]	[-83, 85]	[-88, 91]	[-34, 35]	[-31, 31]
f_{S1}/Λ^4	[-100, 120]	[-100, 110]	[-110, 110]	[-120, 130]	[-86, 99]	[-91, 97]

We already know quadratics est. fails for loop processes



- How much does dim 8 effect things? A lot.

Hays, Helset, Martin Trott: 2007.00565

- A process dependent error estimate seems to be required.

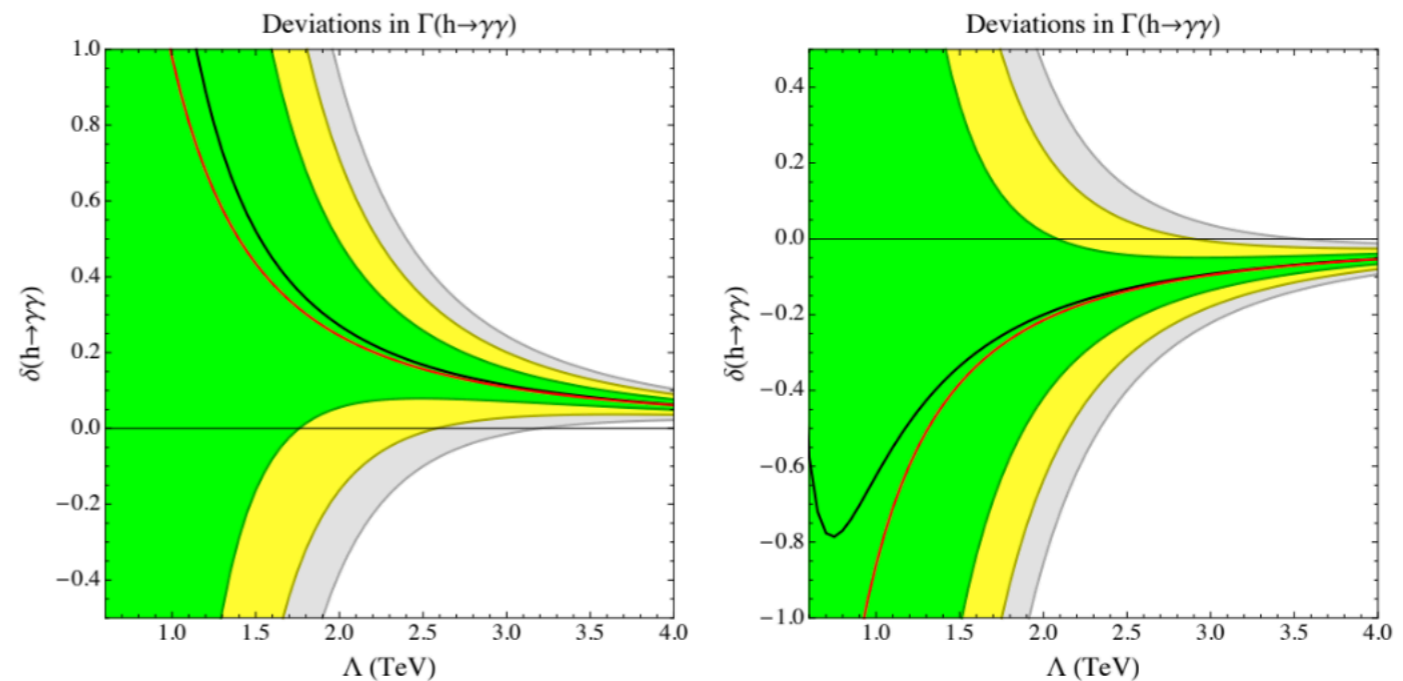


Figure 1. The deviations in $h \rightarrow \gamma\gamma$ from the $\mathcal{O}(v^2/\Lambda^2)$ (red line) and partial-square (black line) results, and the full $\mathcal{O}(v^4/\Lambda^4)$ results (green $\pm 1\sigma_\delta$, yellow $\pm 2\sigma_\delta$, and grey $\pm 3\sigma_\delta$ regions). In the left panel the coefficients determining the $\mathcal{O}(v^2/\Lambda^2)$ and partial-square results are $C_{HB}^{(6)} = -0.01$, $C_{HW}^{(6)} = 0.004$, $C_{HWB}^{(6)} = 0.007$, $C_{HD}^{(6)} = -0.74$, and $\delta G_F^{(6)} = -1.6$. In the right panel they are $C_{HB}^{(6)} = 0.007$, $C_{HW}^{(6)} = 0.007$, $C_{HWB}^{(6)} = -0.015$, $C_{HD}^{(6)} = 0.50$, and $\delta G_F^{(6)} = 1.26$.

- Quadratics not working well here. They are the black line. If we do real error variation then we will pick this up. Its the same story for ggh (in preparation)

Vertex function approach

$$\begin{aligned}\Gamma_V^{\alpha\beta\mu} = & f_1^V (q - \bar{q})^\mu g^{\alpha\beta} - \frac{f_2^V}{M_W^2} (q - \bar{q})^\mu P^\alpha P^\beta + f_3^V (P^\alpha g^{\mu\beta} - P^\beta g^{\mu\alpha}) \\ & + i f_4^V (P^\alpha g^{\mu\beta} + P^\beta g^{\mu\alpha}) + i f_5^V \epsilon^{\mu\alpha\beta\rho} (q - \bar{q})_\rho \\ & - f_6^V \epsilon^{\mu\alpha\beta\rho} P_\rho - \frac{f_7^V}{m_W^2} (q - \bar{q})^\mu \epsilon^{\alpha\beta\rho\sigma} P_\rho (q - \bar{q})_\sigma\end{aligned}$$

- ❖ Momentum-space analogue of the Lagrangian approach
- ❖ P, q, \bar{q} are the four-momenta of V, W_-, W_+ , respectively.

Eboli basis

a. *Operators containing just $D_\mu\Phi$*

The two independent operators in this class are

$$\mathcal{L}_{S,0} = \left[(D_\mu\Phi)^\dagger D_\nu\Phi \right] \times \left[(D^\mu\Phi)^\dagger D^\nu\Phi \right] \quad (\text{A5})$$

$$\mathcal{L}_{S,1} = \left[(D_\mu\Phi)^\dagger D^\mu\Phi \right] \times \left[(D_\nu\Phi)^\dagger D^\nu\Phi \right] \quad (\text{A6})$$

b. *Operators containing $D_\mu\Phi$ and field strength*

The operators in this class are:

$$\mathcal{L}_{M,0} = \text{Tr} \left[\hat{W}_{\mu\nu} \hat{W}^{\mu\nu} \right] \times \left[(D_\beta\Phi)^\dagger D^\beta\Phi \right] \quad (\text{A7})$$

$$\mathcal{L}_{M,1} = \text{Tr} \left[\hat{W}_{\mu\nu} \hat{W}^{\nu\beta} \right] \times \left[(D_\beta\Phi)^\dagger D^\mu\Phi \right] \quad (\text{A8})$$

$$\mathcal{L}_{M,2} = [B_{\mu\nu} B^{\mu\nu}] \times \left[(D_\beta\Phi)^\dagger D^\beta\Phi \right] \quad (\text{A9})$$

$$\mathcal{L}_{M,3} = [B_{\mu\nu} B^{\nu\beta}] \times \left[(D_\beta\Phi)^\dagger D^\mu\Phi \right] \quad (\text{A10})$$

$$\mathcal{L}_{M,4} = \left[(D_\mu\Phi)^\dagger \hat{W}_{\beta\nu} D^\mu\Phi \right] \times B^{\beta\nu} \quad (\text{A11})$$

$$\mathcal{L}_{M,5} = \left[(D_\mu\Phi)^\dagger \hat{W}_{\beta\nu} D^\nu\Phi \right] \times B^{\beta\mu} \quad (\text{A12})$$

$$\mathcal{L}_{M,6} = \left[(D_\mu\Phi)^\dagger \hat{W}_{\beta\nu} \hat{W}^{\beta\nu} D^\mu\Phi \right] \quad (\text{A13})$$

$$\mathcal{L}_{M,7} = \left[(D_\mu\Phi)^\dagger \hat{W}_{\beta\nu} \hat{W}^{\beta\mu} D^\nu\Phi \right] \quad (\text{A14})$$

Eboli basis

$$\mathcal{L}_{T,0} = \text{Tr} [\hat{W}_{\mu\nu} \hat{W}^{\mu\nu}] \times \text{Tr} [\hat{W}_{\alpha\beta} \hat{W}^{\alpha\beta}] \quad (\text{A15})$$

$$\mathcal{L}_{T,1} = \text{Tr} [\hat{W}_{\alpha\nu} \hat{W}^{\mu\beta}] \times \text{Tr} [\hat{W}_{\mu\beta} \hat{W}^{\alpha\nu}] \quad (\text{A16})$$

$$\mathcal{L}_{T,2} = \text{Tr} [\hat{W}_{\alpha\mu} \hat{W}^{\mu\beta}] \times \text{Tr} [\hat{W}_{\beta\nu} \hat{W}^{\nu\alpha}] \quad (\text{A17})$$

$$\mathcal{L}_{T,3} = \text{Tr} [\hat{W}_{\alpha\mu} \hat{W}^{\mu\beta} \hat{W}^{\nu\alpha}] \times B_{\beta\nu} \quad (\text{A18})$$

$$\mathcal{L}_{T,4} = \text{Tr} [\hat{W}_{\alpha\mu} \hat{W}^{\alpha\mu} \hat{W}^{\beta\nu}] \times B_{\beta\nu} \quad (\text{A19})$$

$$\mathcal{L}_{T,5} = \text{Tr} [\hat{W}_{\mu\nu} \hat{W}^{\mu\nu}] \times B_{\alpha\beta} B^{\alpha\beta} \quad (\text{A20})$$

$$\mathcal{L}_{T,6} = \text{Tr} [\hat{W}_{\alpha\nu} \hat{W}^{\mu\beta}] \times B_{\mu\beta} B^{\alpha\nu} \quad (\text{A21})$$

$$\mathcal{L}_{T,7} = \text{Tr} [\hat{W}_{\alpha\mu} \hat{W}^{\mu\beta}] \times B_{\beta\nu} B^{\nu\alpha} \quad (\text{A22})$$

$$\mathcal{L}_{T,8} = B_{\mu\nu} B^{\mu\nu} B_{\alpha\beta} B^{\alpha\beta} \quad (\text{A23})$$

$$\mathcal{L}_{T,9} = B_{\alpha\mu} B^{\mu\beta} B_{\beta\nu} B^{\nu\alpha} \quad (\text{A24})$$