

Simulating gas-based detectors

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Operating principles:

- ▶ The **charged particle** or photon we want to observe passes through the gas and **ionises** molecules;
- ▶ the **electric field** in the gas volume **transports** the ionisation electrons and ions;
- ▶ electrons **multiply** in high field areas;
- ▶ moving electrons and ions **induce currents** in electrodes.

Ionisation

[Four Curies: Pierre, Marie, Irène and
Pierre's father, around 1904 at the BIPM]



1896: Ionisation by radiation

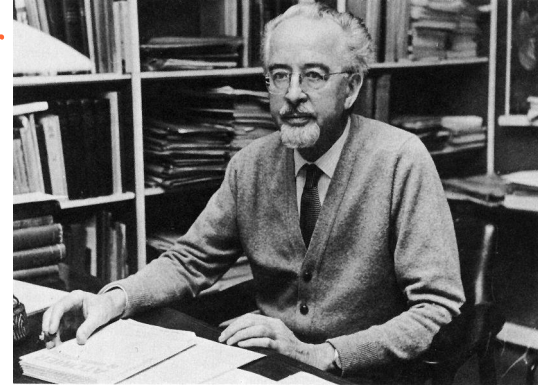
- ▶ Early in the study of radioactivity, ionisation by radiation was recognised:

” Becquerel discovered in 1896 the special radiating properties of uranium and its compounds. Uranium emits very weak rays which leave an impression on photographic plates. These rays pass through black paper and metals; **they make air electrically conductive.** “

[Pierre Curie, Nobel Lecture, June 6th 1905]

“A sphere of charged uranium, which discharges spontaneously in the air under the influence of its own radiation, retains its charge in an absolute vacuum. The exchanges of electrical charges that take place between charged bodies under the influence of the new rays, are the **result of a special conductivity imparted to the surrounding gases**, a conductivity that persists for several moments after the radiation has ceased to act.”

[Antoine Henri Becquerel, Nobel Lecture, December 11th 1903]



1930-1933: EM energy loss

- ▶ 1930 - Hans Bethe, non-relativistic quantum calculation:

The loss in kinetic energy per centimeter path is

$$-\frac{dT}{dx} = N E = \frac{4\pi e^4 z^2 N}{m v^2} \ln \frac{(2) m v^2}{c R h} .$$

Z: "hidden in N"

(2): only for electrons

R: Rydberg constant

- ▶ 1931 - Christian Møller solves relativistic e^- scattering.
- ▶ 1932 - Hans Bethe, relativistic quantum calculation:

Ein Teilchen der Ladung ez möge sich mit der Geschwindigkeit v durch eine Substanz hindurchbewegen, welche in der Volumeneinheit N Atome der Ordnungszahl Z enthält. Dann verliert das Teilchen pro Zentimeter Weg die Energie

$$-\frac{dT}{dx} = \frac{2\pi e^4 N Z z^2}{m v^2} \left(\lg \frac{2 m v^2 W}{\bar{E}^2 \left(1 - \frac{v^2}{c^2}\right)} - \frac{v^2}{c^2} \right),$$

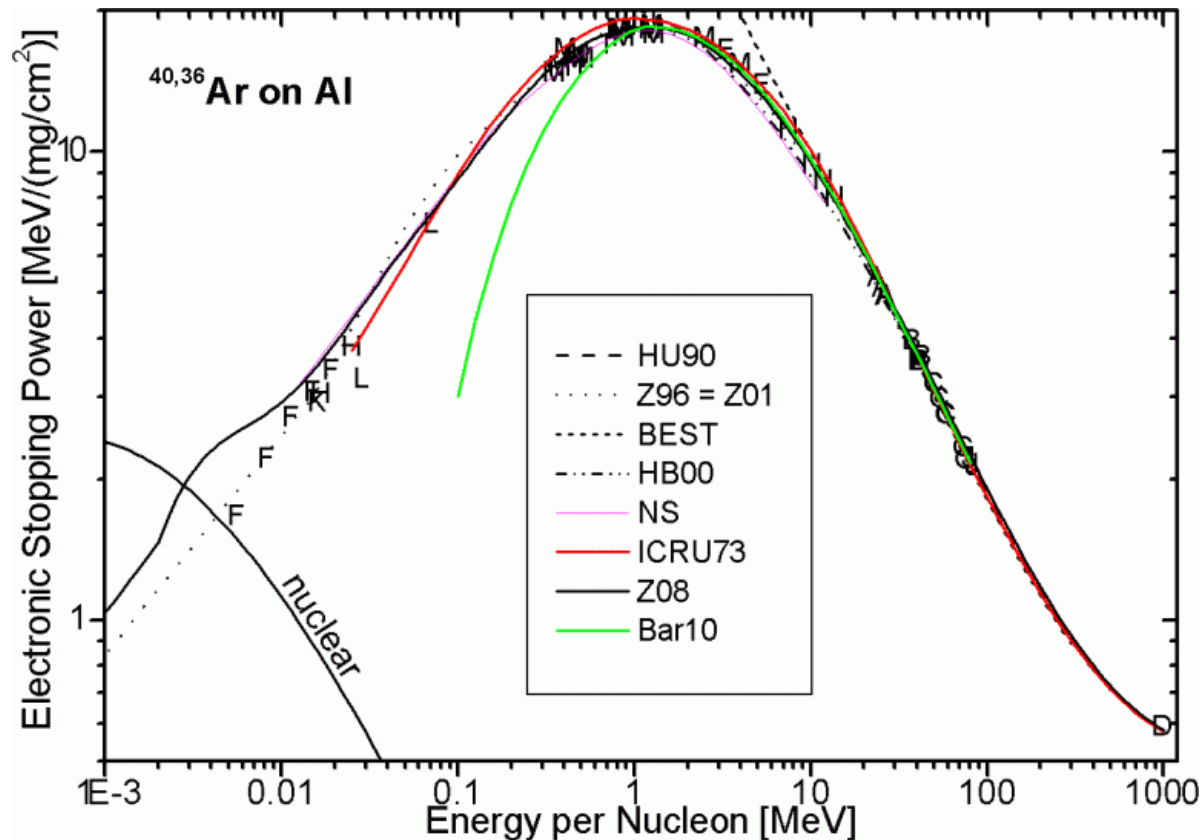
falls wir nur den Energieverlust durch solche Stöße ins Auge fassen, bei denen im einzelnen höchstens die Energie W auf das Atom übertragen wird³⁾.

\bar{E} : average atomic
ionisation energy

W : largest energy
transfer per collision

Electromagnetic losses at low energy

- ▶ Nuclear effects abound at low energy.
- ▶ Numerous models: SRIM, MSTAR, CasP, PASS ...



- ▶ Ref: Helmut Paul, <http://www.exphys.uni-linz.ac.at/Stopping/>



1944: Energy loss fluctuations

- ▶ Given a single-collision energy loss distribution $w(\epsilon)$, the distribution $f(s)$ of the energy loss s after many collisions is *schematically* given by the Laplace transform:

$$L f(x, s) = \bar{f}(x, p) = e^{-x \int_0^{\infty} (1 - e^{-p\epsilon}) w(\epsilon) d\epsilon}$$

x : direction of travel

$s \leftrightarrow p$: total energy loss

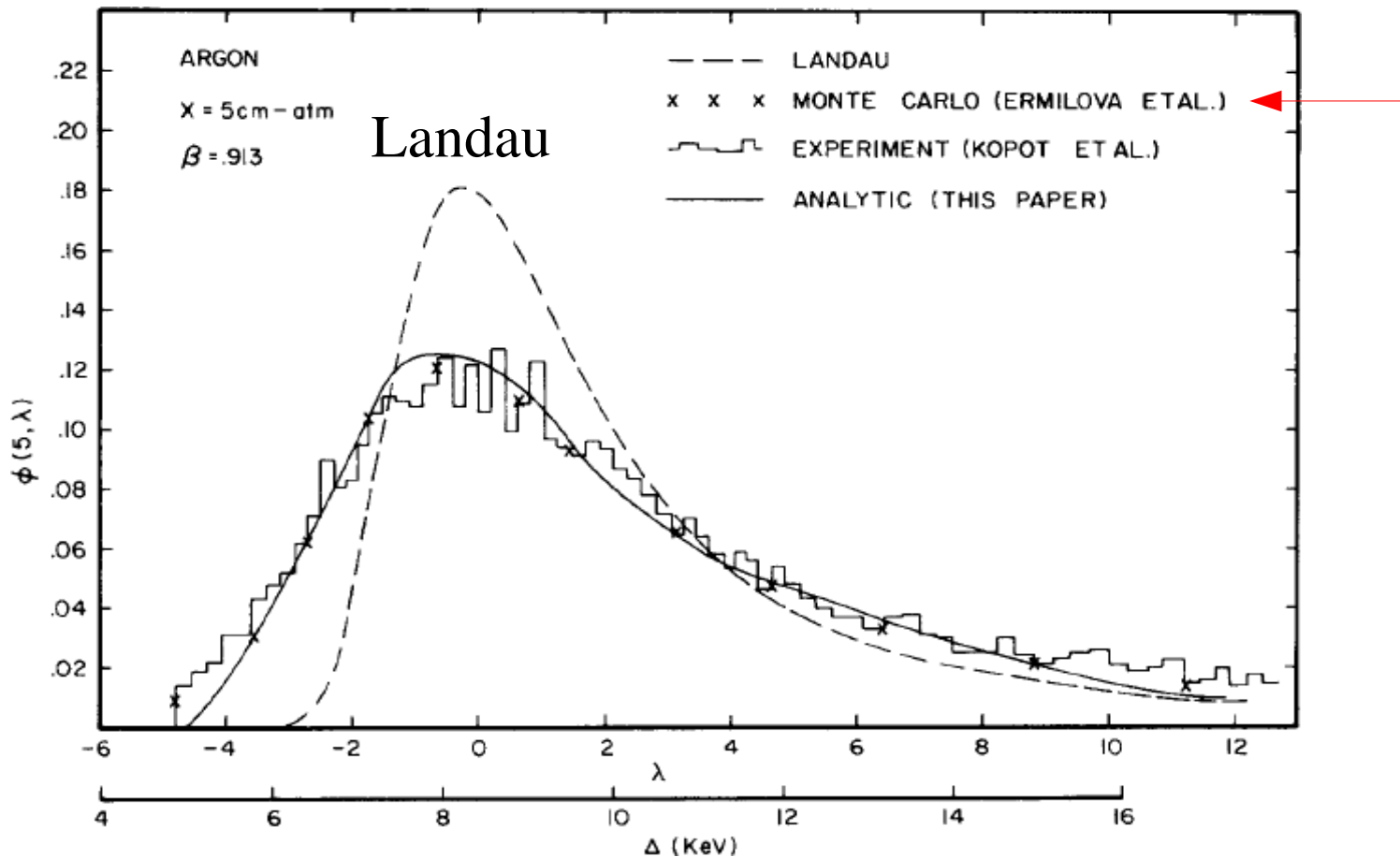
- ▶ Landau showed (1944), assuming in particular:

- ▶ **thick layers**: numerous small energy losses;
- ▶ Rutherford-inspired energy loss distribution $w(\epsilon) \sim 1/\epsilon^2$;
- ▶ neglect of the atomic structure:

$$L f(s) \approx s^s$$

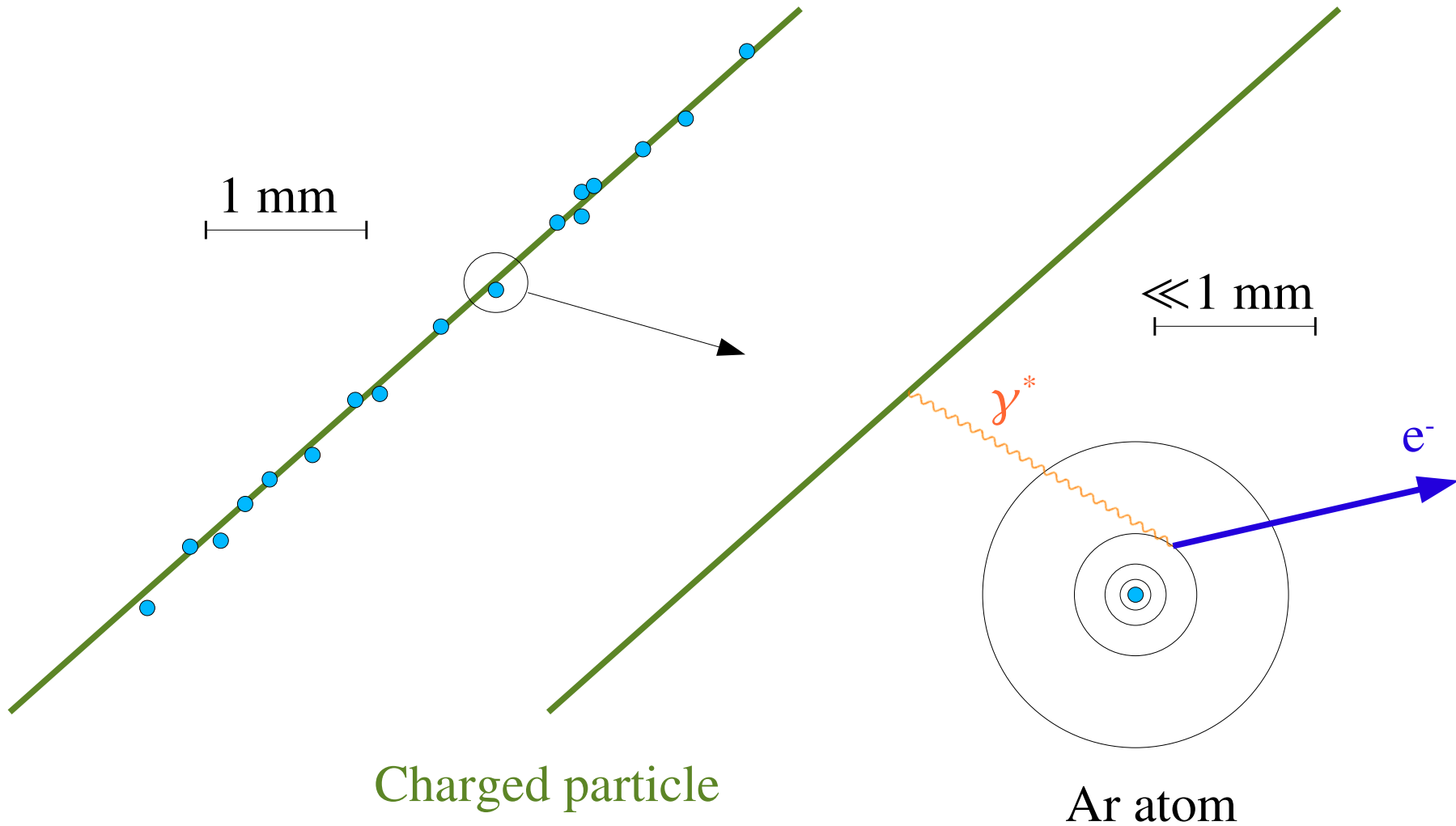
Landau: too narrow for thin layers

- ▶ 2 GeV protons on an (only !) 5 cm thick Ar gas layer:



[Diagram: Richard Talman, NIM A 159 (1979) 189-211]

Virtual photon exchange model



PAI model



Wade Allison



John Cobb

► Key ingredient: photo-absorption cross section $\sigma_y(E)$

$$\frac{\beta^2 \pi}{\alpha} \frac{d\sigma}{dE} = \frac{\sigma_y(E)}{E} \log \left(\frac{1}{\sqrt{(1-\beta^2 \epsilon_1)^2 + \beta^4 \epsilon_2^2}} \right) +$$

Cross section to transfer energy E

$$\frac{1}{N \bar{h} c} \left(\beta^2 - \frac{\epsilon_1}{|\epsilon|^2} \right) \theta +$$

$$\frac{\sigma_y(E)}{E} \log \left(\frac{2 m_e c^2 \beta^2}{E} \right) +$$

$$\frac{1}{E^2} \int_0^E \sigma_y(E_1) dE_1$$

Relativistic rise

Čerenkov radiation

Resonance region

Rutherford scattering

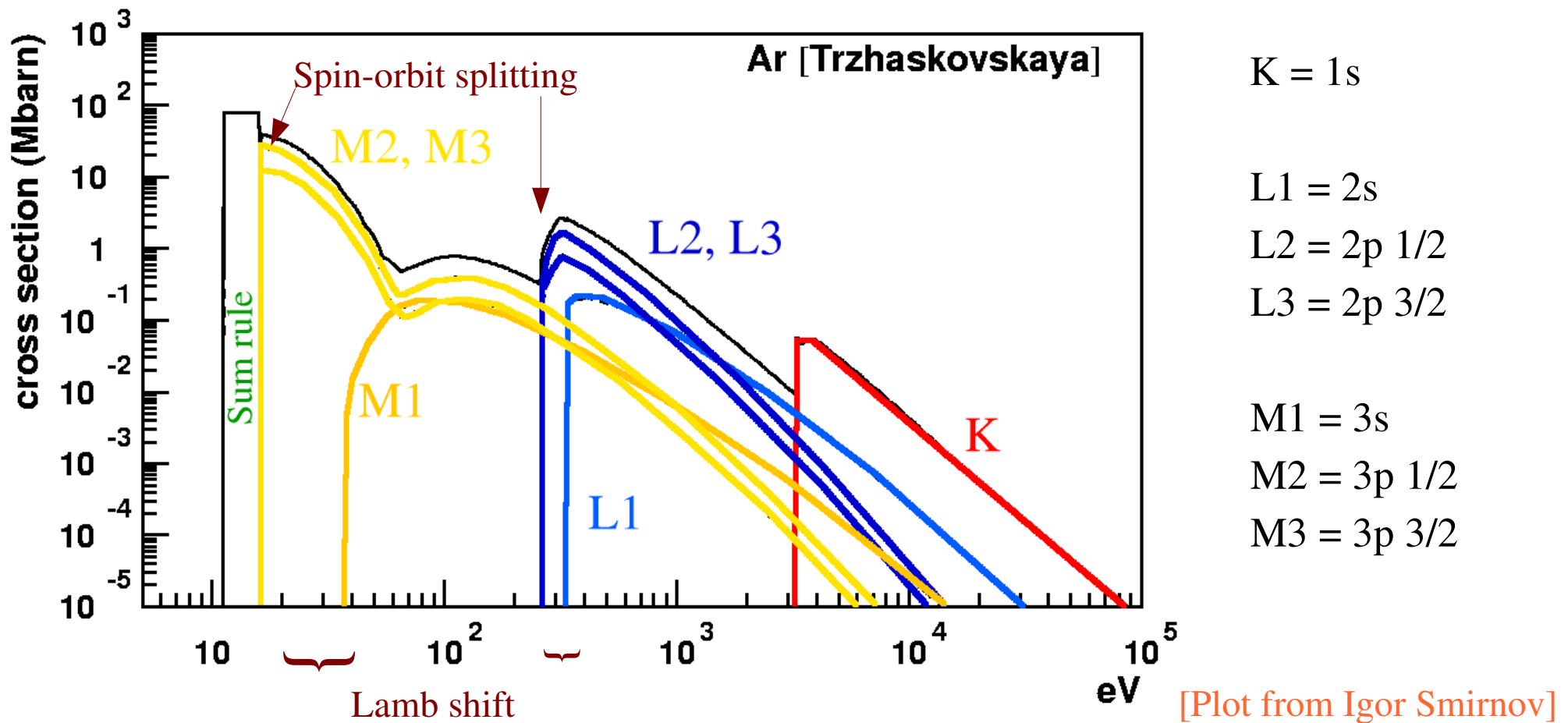
With:
$$\epsilon_2(E) = \frac{N_e \bar{h} c}{E Z} \sigma_y(E)$$

$$\epsilon_1(E) = 1 + \frac{2}{\pi} \text{P} \int_0^\infty \frac{x \epsilon_2(x)}{x^2 - E^2} dx$$

$$\theta = \arg(1 - \epsilon_1 \beta^2 + i \epsilon_2 \beta^2) = \frac{\pi}{2} - \arctan \frac{1 - \epsilon_1 \beta^2}{\epsilon_2 \beta^2}$$

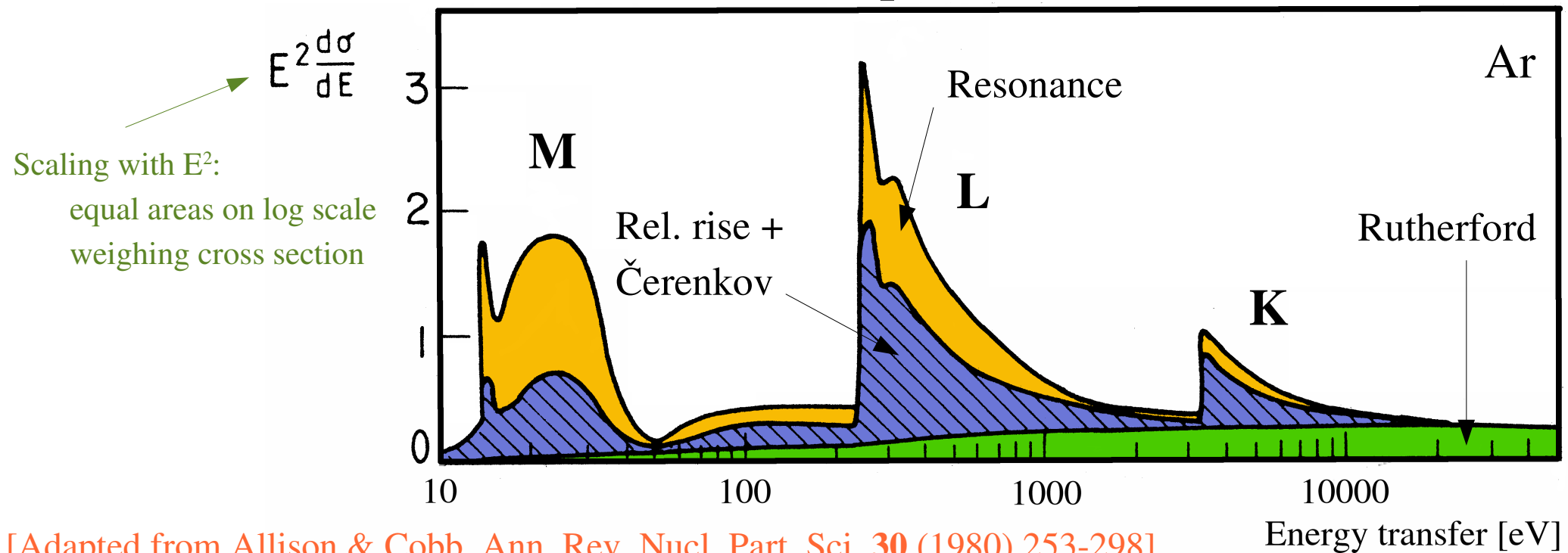
Photo-absorption in argon

- ▶ Argon has 3 shells, hence 3 groups of lines:



Importance of the PAI model terms

- ▶ All electron orbitals (shells) participate:
 - ▶ outer shells: frequent interactions, few electrons;
 - ▶ inner shells: few interactions, many electrons.
- ▶ All terms in the formula are important.

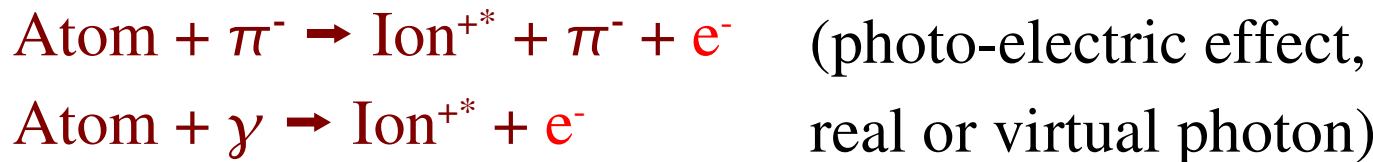


Heed – computer model



Igor Smirnov

- ▶ PAI model or absorption of real photons:



- ▶ Decay of excited states:



- ▶ Treatment of:

- ▶ secondary photons, returning to the PAI model,

- ▶ ionising photo-electrons and Auger-electrons, collectively known as δ -electrons:



Degrad – Steve Biagi

- ▶ Auger cascade model for **electron thermalisation** in gas mixtures produced by photons or particles in electric and magnetic fields.
- ▶ Applications:
 - ▶ **Work function** and **Fano factor** (including shape)
 - ▶ Cloud size after **β -decay**
 - ▶ **Cluster size** & density distribution for charged particle tracks
- ▶ Degrad shares its electron impact cross sections with Magboltz and is a successor to MIP.

Fields

- ▶ “Classic” devices – metal wires and planes – 2d:
 - ▶ elegant, closed expressions.
- ▶ MPGDs – dielectric media – 3d:
 - ▶ finite elements;
 - ▶ boundary elements.

MWPC

- ▶ First gaseous tracking device
- ▶ 1968: Georges Charpak

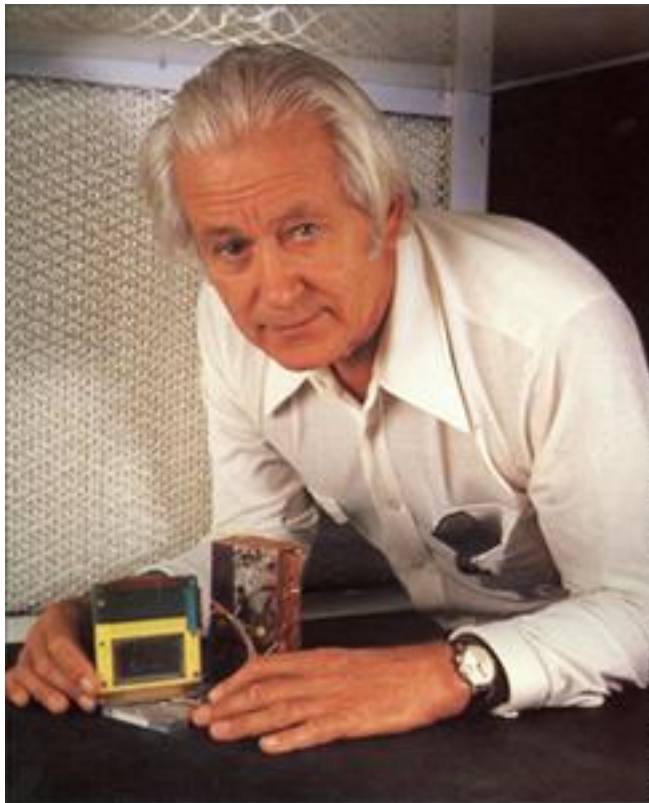
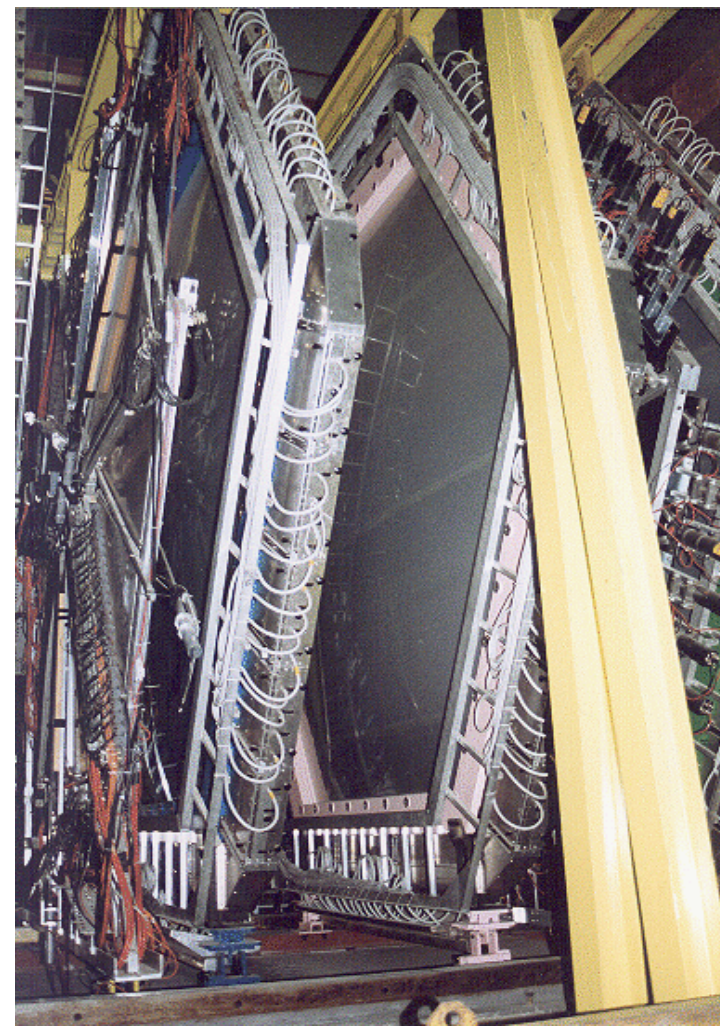


Photo: D. Parker, Science Photo Lab, UK

Georges Charpak



One of the NA60 muon chambers



TPC

- ▶ Typically very large
- ▶ Almost empty inside
- ▶ Excellent for dealing with large numbers of tracks
- ▶ 1976: David Nygren (for PEP4)



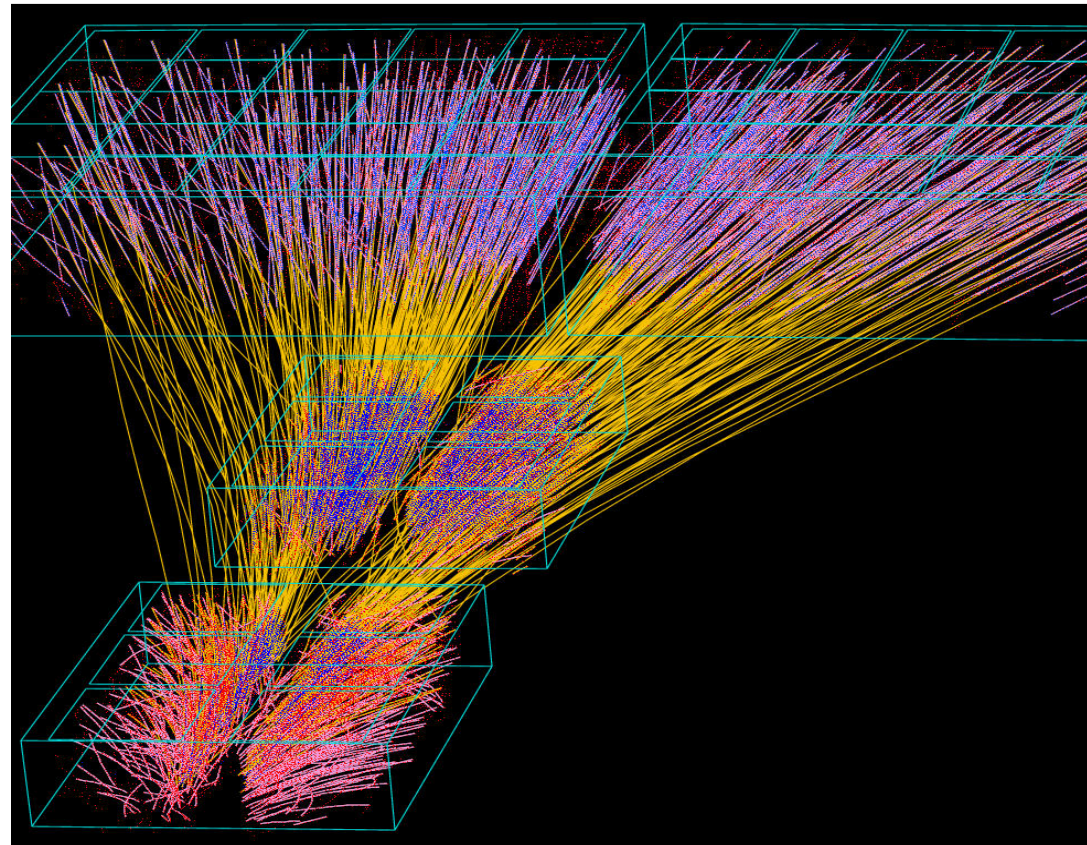
David Nygren



Alice



Star



NA49

1749: Cauchy-Riemann equations



Jean le Rond d'Alembert
(Nov 16th 1717 – Oct 29th 1783)

► Core of the complex potential method.

► Existence of a derivative of a complex analytic function $f = u + i v$

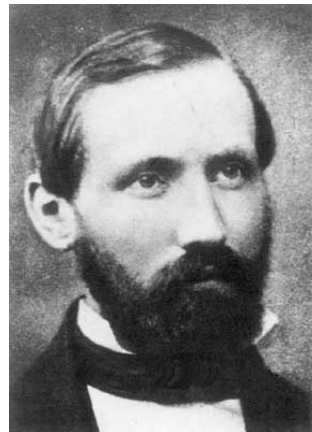
$$\begin{aligned} f'(z) &= \frac{\partial f}{\partial x} = \frac{\partial u}{\partial x} + i \frac{\partial v}{\partial x} & \frac{\partial u}{\partial x} &= \frac{\partial v}{\partial y} \\ &= \frac{\partial f}{\partial i y} = -i \frac{\partial u}{\partial y} + \frac{\partial v}{\partial y} & \frac{\partial v}{\partial x} &= -\frac{\partial u}{\partial y} \end{aligned}$$

► implies that $\text{Re}(f)$ is harmonic:

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 v}{\partial x \partial y} = \frac{\partial^2 v}{\partial y \partial x} = \frac{-\partial^2 u}{\partial y \partial y} \quad \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$



Augustin Louis Cauchy
(Aug 21st 1789 – May 23rd 1857)

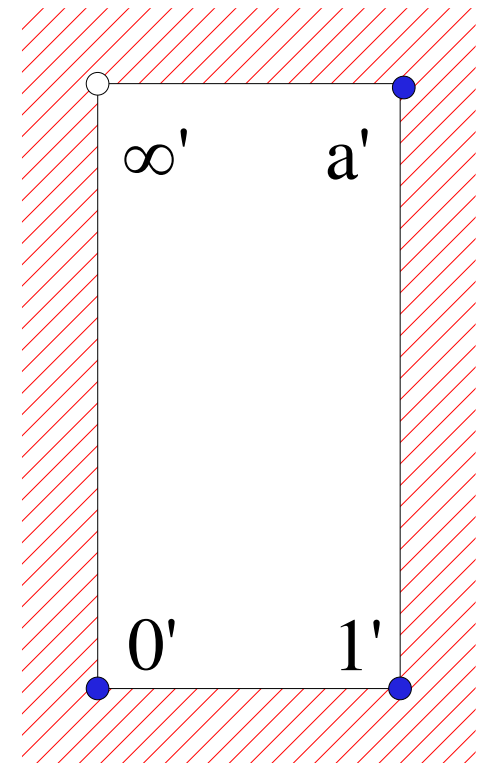
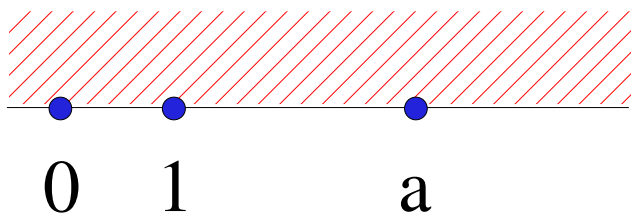


Georg Friedrich Bernhard Riemann
(Sep 17th 1826 – Jul 20th 1866)

Conformal mappings – an example

- ▶ Schwartz-Christoffel transformation of a half-plane to the external part of a rectangle:

$$z \rightarrow \int_0^z \frac{d\xi}{\sqrt{\xi(\xi-1)(\xi-a)}}$$
$$= \frac{2}{\sqrt{a}} \operatorname{sn}^{-1}\left(\sqrt{z}, \frac{1}{\sqrt{a}}\right)$$

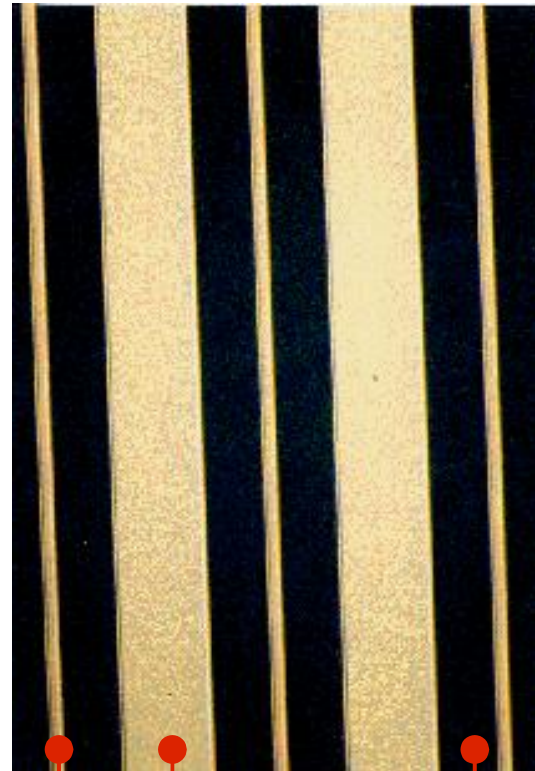


MPGD

- ▶ **M**icro-**P**attern **G**as-based **D**etectors
 - ▶ have small structural elements
 - ▶ use 3d electrodes to generate electric fields.

MSGC: an early MPGD

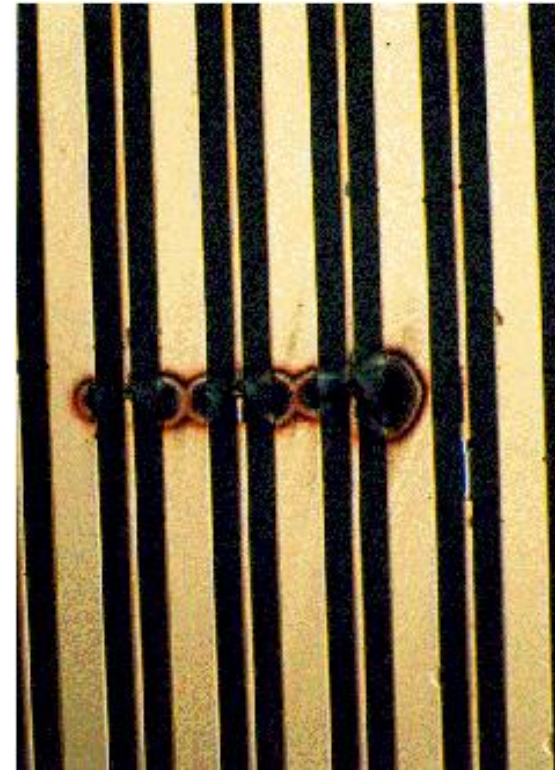
- ▶ Built using solid-state techniques;
 - ▶ good resolution;
 - ▶ poor resistance to high rates.
-
- ▶ 1988: Anton Oed



Anode

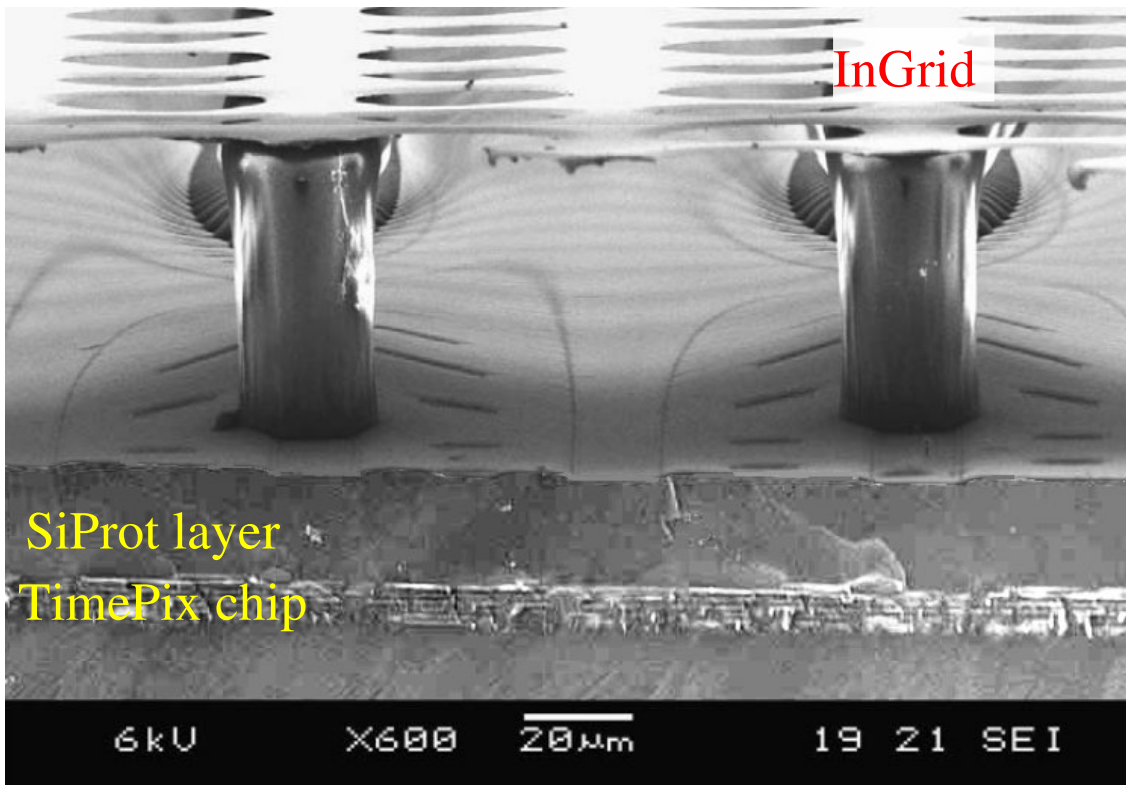
Cathode

Substrate

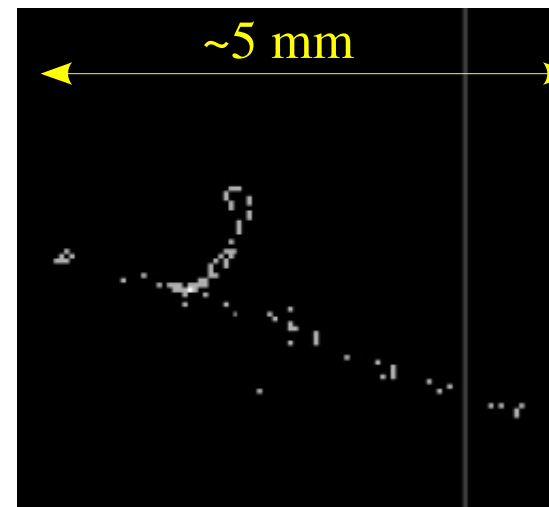


Gossip

- ▶ The “electronic bubble chamber”.



Harry van der Graaf (r)

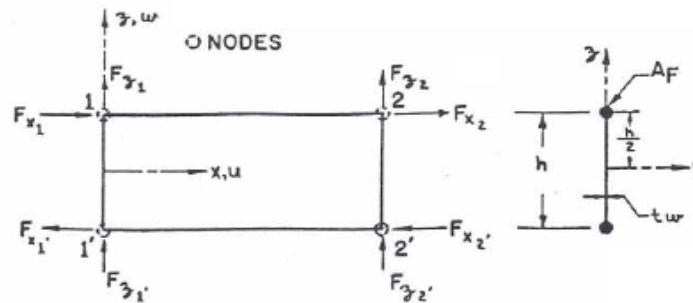


δ -electrons made visible in He/ iC_4H_{10} ,
using a modified MediPix, ~2004.

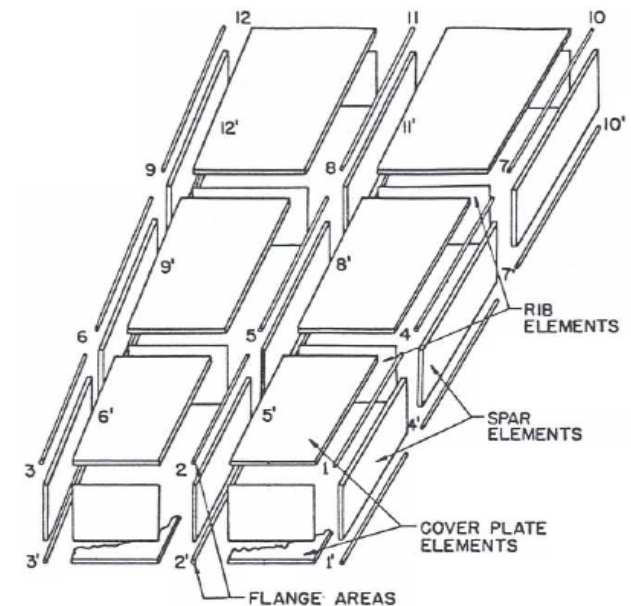


Aircraft wings – finite elements

- ▶ “*Stiffness and Deflection Analysis of Complex Structures*”, a study in the use of the finite element technique (then called “direct stiffness method”) for aircraft wing design.



$$[K] = \frac{6EI}{Lh^2(1+4n)} \begin{bmatrix} (4/3)(1+n) & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -(h/L) & 0 & h^2/L^2 & 0 & 0 & 0 \\ (2/3)(1-2n) & 0 & -(h/L) & (4/3)(1+n) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ h/L & 0 & -(h^2/L^2) & h/L & 0 & h^2/L^2 \end{bmatrix}$$

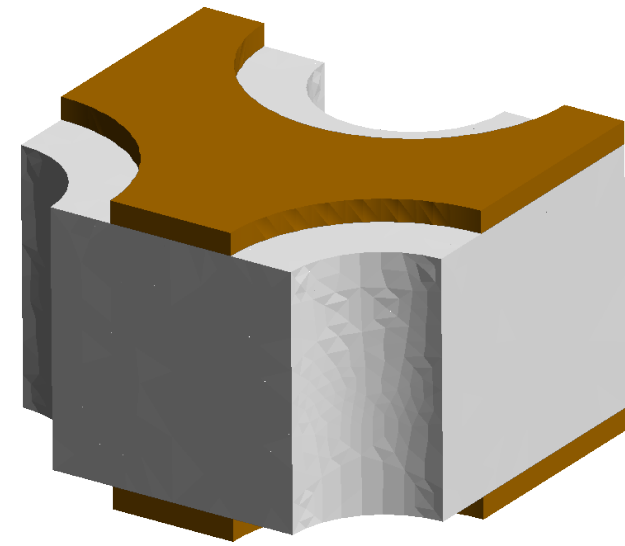


[M.J. Turner, R.W. Clough, H.C. Martin and L.J. Topp, *Stiffness and Deflection Analysis of Complex Structures*, J. Aero. Sc. **23** (1956), 805-824. MJT & LJT with Boeing.]

FEM – the price to pay

- ▶ Finite element programs are flexible but they focus on the wrong thing: they solve V well, but we do not really need it:
 - ▶ quadratic shape functions do a fair job at approximating $V \approx \log(r)$ potentials;
 - ▶ potentials are continuous;
 - ▶ potentials and fields are **not Maxwell compliant**.
- ▶ E is what we use to transport charges, but:
 - ▶ gradients of quadratic shape functions are linear and **not suitable to approximate $E \approx 1/r$** , left alone $E \approx 1/r^2$ fields;
 - ▶ electric fields are **discontinuous** at element boundaries;
 - ▶ a local accuracy of $\sim 50\%$ in high-field areas is not unusual.

Open source finite elements



- ▶ Finite element programs are widely used in engineering and nearly always commercial.
- ▶ A rare exception is Gmsh/Elmer. Josh Renner (Berkeley) has written a Garfield++ interface.

C. Geuzaine and J.-F. Remacle, *Gmsh: a three-dimensional finite element mesh generator with built-in pre- and post-processing facilities*, <http://geuz.org/gmsh/>.

CSC – IT Center for Science, *Elmer: Open Source Finite Element Software for Multiphysical Problems*, <http://www.csc.fi/english/pages/elmer>.

Boundary element methods

- ▶ Contrary to the finite element method, the elements are on the boundaries, not inside the problem domain. Charges are computed for the boundary elements.
- ▶ The field in the problem domain is calculated as the sum of **Maxwell-compliant field functions**, each extending over the entire problem domain. There are **no discontinuities**.
- ▶ But ... the method poses substantial numerical challenges: large non-sparse matrices and inherent singularities. The technique is time consuming.



neBEM – the price to pay

$$\Phi = \frac{1}{2} ((z_M Y^2 - XG)(LP_1 + LM_1 - LP_2 - LM_2) + i|Y|(z_M X + G)(LP_1 - LM_1 - LP_2 + LM_2) - S_1 X (\tanh^{-1}(\frac{R_1 + iI_1}{D_{11}|Z|}) + \tanh^{-1}(\frac{R_1 - iI_1}{D_{11}|Z|}) - \tanh^{-1}(\frac{R_1 + iI_2}{D_{21}|Z|}) - \tanh^{-1}(\frac{R_1 - iI_2}{D_{21}|Z|})) + i|Y| (\tanh^{-1}(\frac{R_1 + iI_1}{D_{11}|Z|}) - \tanh^{-1}(\frac{R_1 - iI_1}{D_{11}|Z|}) - \tanh^{-1}(\frac{R_1 + iI_2}{D_{21}|Z|}) + \tanh^{-1}(\frac{R_1 - iI_2}{D_{21}|Z|})) + \frac{2G}{\sqrt{1+z_M^2}} \log(\frac{\sqrt{1+z_M^2}D_{12} - E_1}{\sqrt{1+z_M^2}D_{21} - E_2}) + 2Z \log(\frac{D_{21} - X + 1}{D_{11} - X})) + C$$



- ▶ For computing the field at any point, neBEM sums the fields due to each element on that point.

$$D_{11} = \sqrt{(X - x_1)^2 + Y^2 + (Z - z_1)^2}; D_{12} = \sqrt{(X - x_1)^2 + Y^2 + (Z - z_2)^2}$$

$$D_{21} = \sqrt{(X - x_2)^2 + Y^2 + (Z - z_1)^2}; I_1 = (X - x_1) |Y|; I_2 = (X - x_2) |Y|$$

$$S_1 = \text{sign}(z_1 - Z); R_1 = Y^2 + (Z - z_1)^2$$

- ▶ Evaluating the Green's functions, especially the one for triangular elements, is costly.

$$LP_1 = \frac{1}{G - iz_M |Y|} \log\left(\frac{(H_1 + GD_{12}) + i|Y|(E_1 - iz_M D_{12})}{-X + i|Y|}\right)$$

- ▶ For a modest doubly-periodic 1000-element model there would be $\sim 10^8$ function evaluations. For an avalanche study we would love to have 1000. We then need to compute the field at $\sim 10^7$ points.

$$LM_1 = \frac{1}{G + iz_M |Y|} \log\left(\frac{(H_1 + GD_{12}) + i|Y|(E_1 - iz_M D_{12})}{-X - i|Y|}\right)$$

$$LP_2 = \frac{1}{G - iz_M |Y|} \log\left(\frac{(H_2 + GD_{21}) + i|Y|(E_2 - iz_M D_{21})}{1 - X + i|Y|}\right)$$

$$LM_2 = \frac{1}{G + iz_M |Y|} \log\left(\frac{(H_2 + GD_{21}) + i|Y|(E_2 - iz_M D_{21})}{1 - X - i|Y|}\right)$$

and C denotes a constant of integration.

Electron transport

- ▶ Interaction of electrons up to a few eV with gas.

LXcat

- ▶ Art Phelps,
- ▶ Leanne Pitchford – Toulouse,
- ▶ Klaus Bartschat – Iowa,
- ▶ Oleg Zatsarinny – Iowa,
- ▶ Michael Allan – Fribourg,
- ▶ Steve Biagi

Leanne Pitchford



Michael Allan



Klaus Bartschat



LXcat (pronounced *elecscat*) is an open-access website for collecting, displaying, and downloading ELECTron SCATtering cross sections and swarm parameters (mobility, diffusion coefficient, reaction rates etc.) required for modeling low temperature plasmas. [...]"

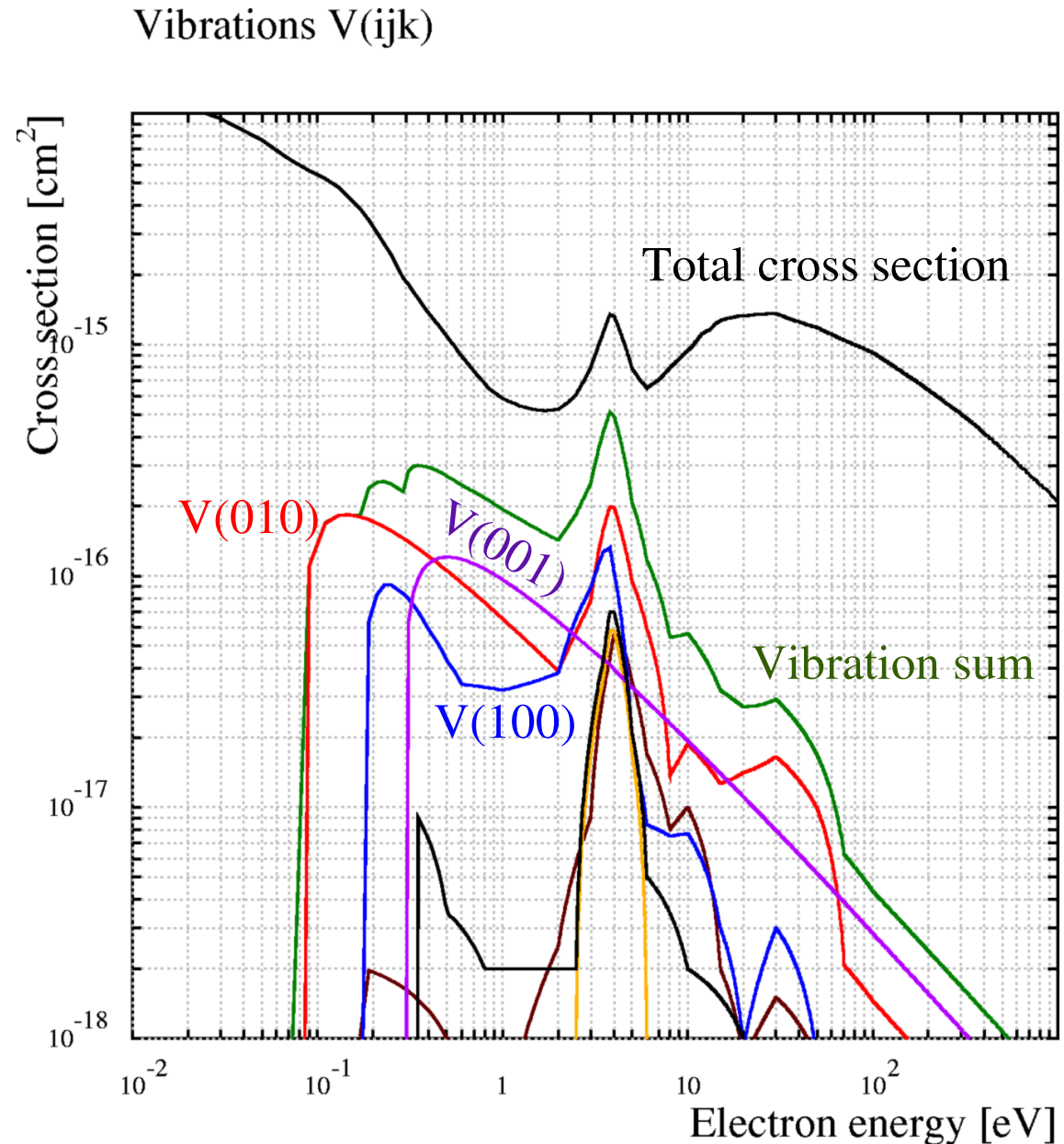
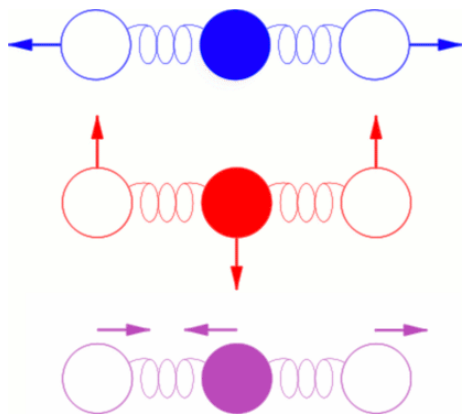
<http://www.lxcat.laplace.univ-tlse.fr/>

Magboltz

- ▶ A large number of cross sections for 60 molecules...
 - ▶ Noble gases (He, Ne, Ar, Kr, Xe):
 - ▶ elastic scattering,
 - ▶ 44 excited states (Ar) and
 - ▶ 7 ionisations (Ar);
 - ▶ numerous organic gases, additives, *e.g.* CO₂:
 - ▶ elastic scattering,
 - ▶ 44 inelastic cross sections (5 vibrations and 30 rotations + super-elastic and 9 polyads),
 - ▶ attachment,
 - ▶ 6 excited states and
 - ▶ 3 ionisations.

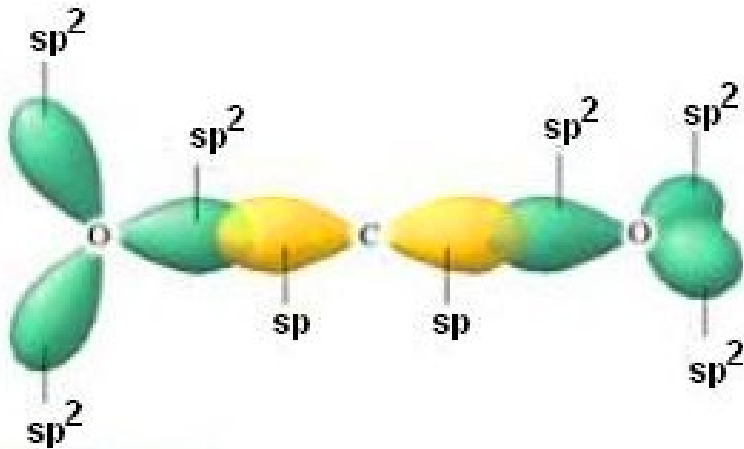
CO₂ – vibration modes

- ▶ CO₂ is linear:
- ▶ O – C – O
- ▶ Vibration modes are numbered $V(ijk)$
 - ▶ i : symmetric,
 - ▶ j : bending,
 - ▶ k : anti-symmetric.

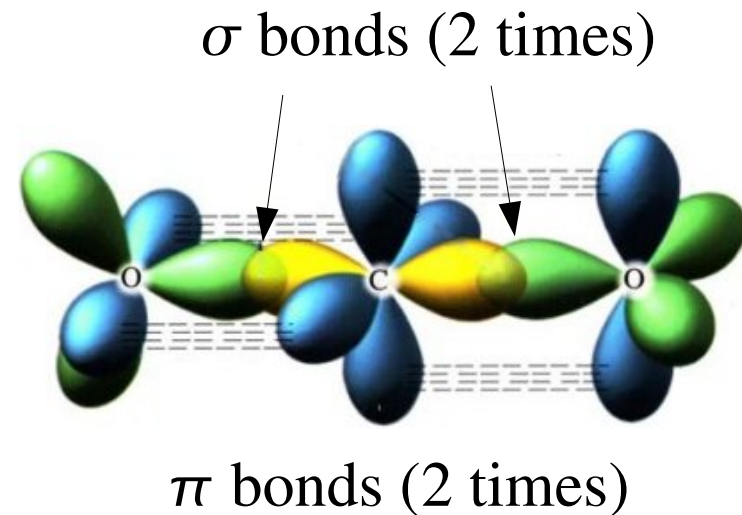


Attachment in CO₂

- ▶ CO₂ is a linear molecule:

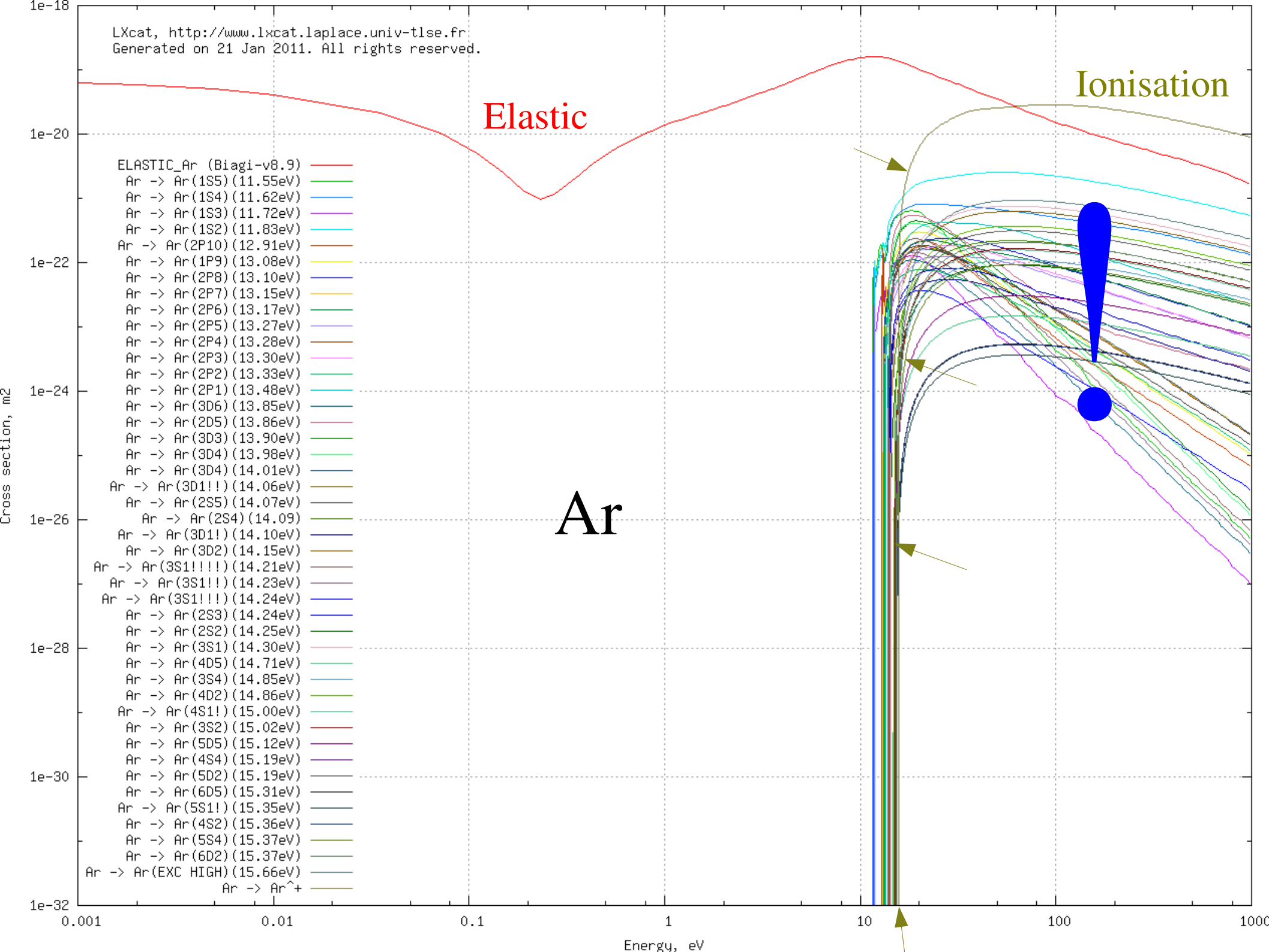


hybrid orbitals only,
p-orbitals not shown



- ▶ CO₂ with an extra e⁻: instable ($\tau \ll 1$ ps, $\epsilon_{\text{VEA}} \approx -3.8$ eV).
Low energy e⁻ collisions produce O⁻, not CO₂⁻.
- ▶ With an e⁻ added, a bent structure (134°) is favoured.
Long lifetime ($\tau \approx 90$ μs) but still *negative* electron affinity ($\epsilon_{\text{AEA}} \approx -0.6$ eV). metastable.
- ▶ Attachment works in [CO₂]_n clusters: vibration and rotation modes absorb excess energy.

- ▶ [Source: presumably SS Zumdahl, Chemistry (1983) DC Heath and Company.]



Sir John Sealy Edward Townsend
(1868-1957)



Electron multiplication (1901)

- ▶ John Townsend “*The conductivity produced in gases by the motion of negatively charged ions*”:

Negative Ionen erzeugen bei ihrer Wanderung durch Berührung neue Ionen. Die Anzahl der negativen Ionen in einer Entfernung x vom Punkte der Entstehung ist deshalb durch die Exponentialfunktion in der Form $N = N_0 e^{\alpha x}$ gegeben. Diese Formel wird durch Versuche bestätigt. Dabei ist α eine Größe, die außer von der Temperatur noch von der Größe der die Ionen bewegenden Kraft X und dem Druck p abhängt, und zwar ergibt sich diese Abhängigkeit in der Form $\alpha = pf(X/p)$.

[JS Townsend, “*The conductivity produced in gases by the motion of negatively charged ions*”, *Phil. Mag.* **6-1** (1901) 198-227. Access to *Phil. Mag* is restricted. The above German-language abstract is available at <http://jfm.sub.uni-goettingen.de/>.]

“Yule - Furry” model (1924 - 1937)

- ▶ The exponential avalanche size distribution was found by WH Furry, independently by RA Wijsman and others.

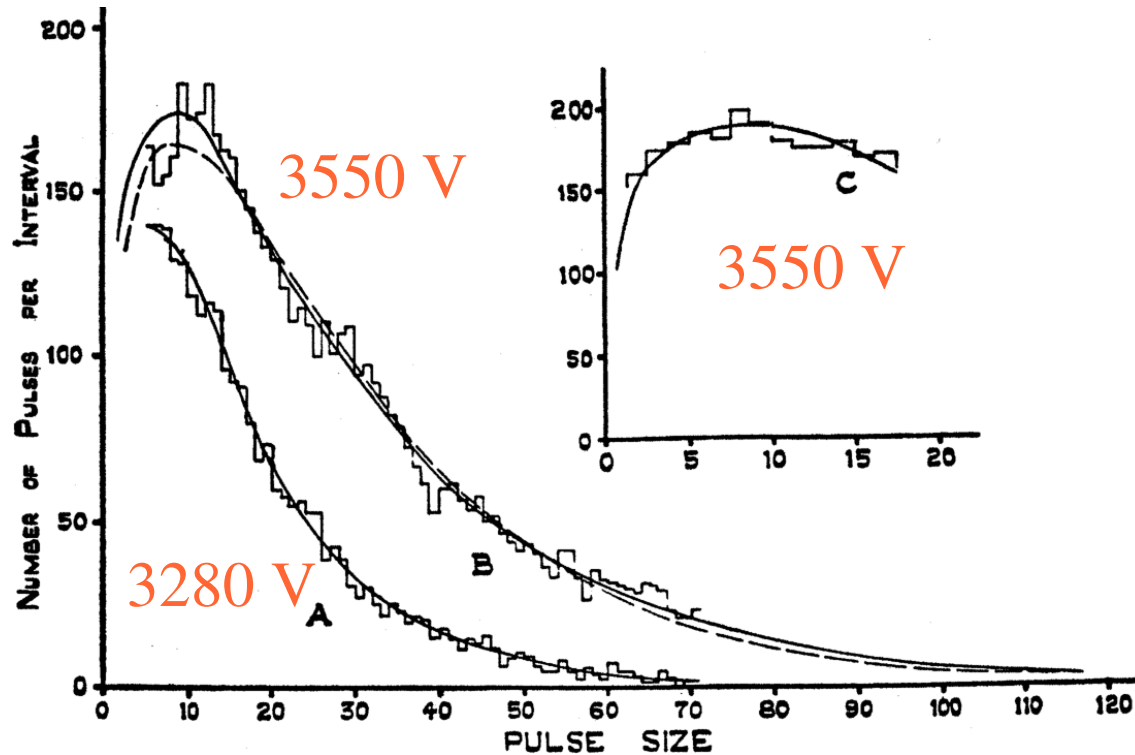
$$p(n) = \frac{1}{\bar{p}} \left(1 - \frac{1}{\bar{p}}\right)^{n-1} \approx \frac{e^{-n/\bar{p}}}{\bar{p}} \quad \text{when } \bar{p} \gg 1$$

$$\text{mean: } \bar{p}, \quad \text{width: } \bar{p} \sqrt{1 - \frac{1}{\bar{p}}}$$

- ▶ Assumptions:
 - ▶ probability to ionise over a distance δ is $\delta\alpha$;
 - ▶ Townsend coefficient α is constant;
 - ▶ further development is independent of history.

SC Curran (1949)

- ▶ SC Curran *et al.* measure the pulse height distribution in a cylindrical counter ($d = 150 \mu\text{m}$ wire, Ar 50 % CH₄ 50 %, $p = 670$ mbar) at $G \sim 10^4$ - 10^5 :



Fit curve:

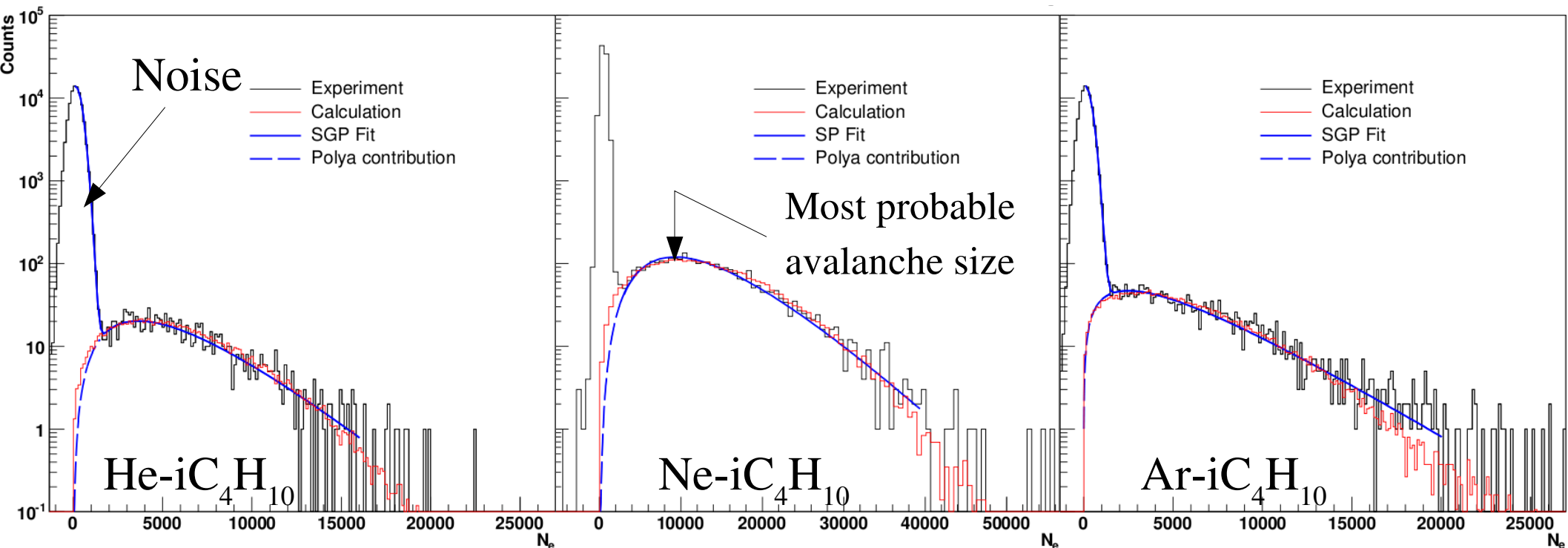
$$p(n) = \sqrt{n} e^{-n}$$
$$\left(\frac{\sigma}{\mu}\right)^2 \approx \frac{2}{3}$$

Heinz Raether's group (Hamburg)

- ▶ Stress that electrons have to **travel a minimum distance** before their energy exceeds the ionisation potential U .
- ▶ The parameter $\kappa = E/\alpha U$ is found to be an indicator of the extent to which the Furry distribution applies.
 - ▶ Lothar Frommhold (1956):
 - ▶ $\kappa = 12-110$: exponential
 - ▶ Hans Schlumbohm (1958):
 - ▶ $\kappa > 23$: exponential
 - ▶ $23 > \kappa > 10$: levels off towards small sizes
 - ▶ $10 > \kappa$: a maximum appears
- ▶ Werner Legler's model (1961) for any κ .

Avalanche size fluctuations

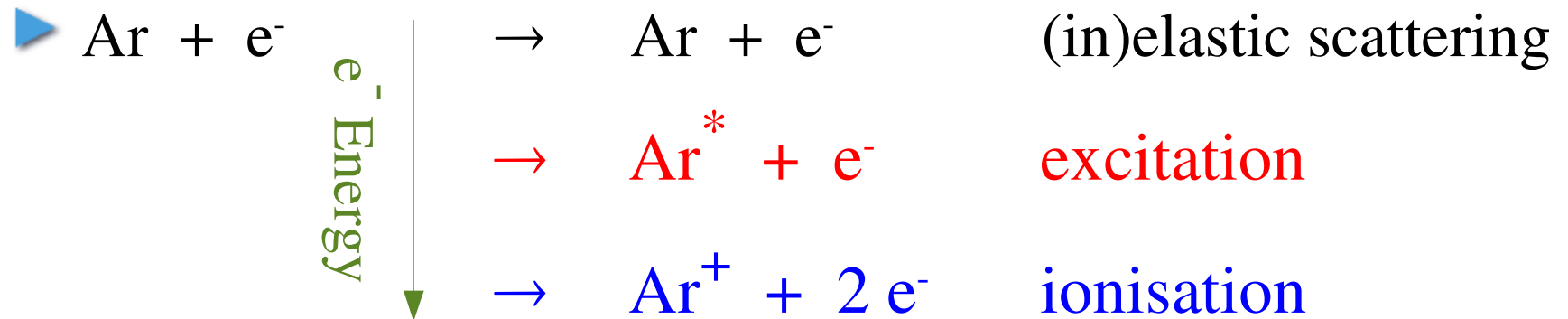
- ▶ Distribution of the size of avalanches triggered by a single electron: measurements and microscopic simulation.



- ▶ [Measurements: Thomas Zerguerras *et al.*, calculations Fabian Kuger]

Ionisation: direct & through excitation

▶ Electron impact on atoms:



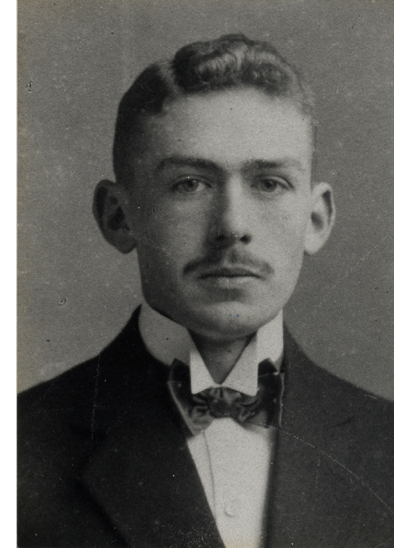
▶ Interaction of excited noble gas and quencher:



provided Ar^* excitation energy $>$ CO_2^+ ionisation energy;

▶ quenching rate constants are comparable to hard-sphere scattering, i.e. faster than radiative decay of (esp. higher) excited states.

Frans Michel Penning,
photo taken around 1921
(1894-1953)



1928: Ar-Ne-Hg Penning effects

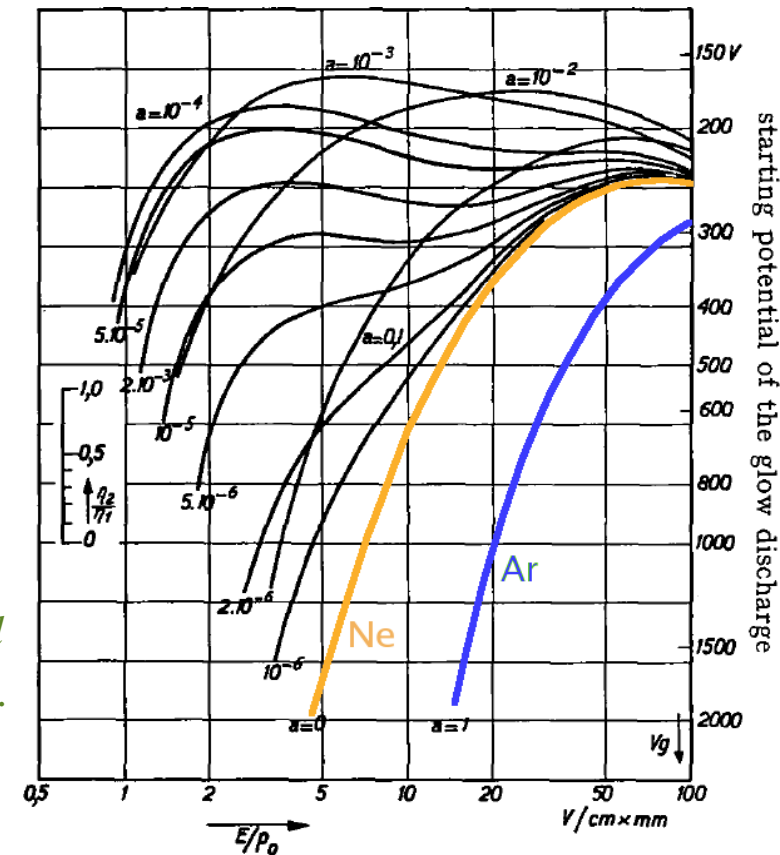
▶ Frans Michel Penning worked from 1924 on gas discharges at the Philips Natuurkundig Laboratorium.

Der Einfluß sehr geringer Beimischung von Hg und Ar auf die Zündspannung V_z des Neons wurde quantitativ bestimmt. Die Erklärung wurde gefunden in der Ionisierung der Fremdatome durch die metastabilen Atome des Neons. Die notwendige Bedingung für diesen Vorgang: $V_i < V_{met.}$ wurde geprüft bei Ne, Ar und He als Hauptgasen mit verschiedenen anderen Gasen als Beimischung und stets bestätigt gefunden. Andererseits erniedrigten immer Beimischungen, wobei $V_i < V_{met.}$ die Zündspannung; nur NO in Ar machte eine Ausnahme von dieser Regel, was aber auf Grund des Termschemas von NO nicht zu verwundern braucht.

F. M. Penning, *Über den Einfluß sehr geringer Beimischungen auf die Zündspannung der Edelgase*, *Z. Phys.* **46** (1928) 334-348.

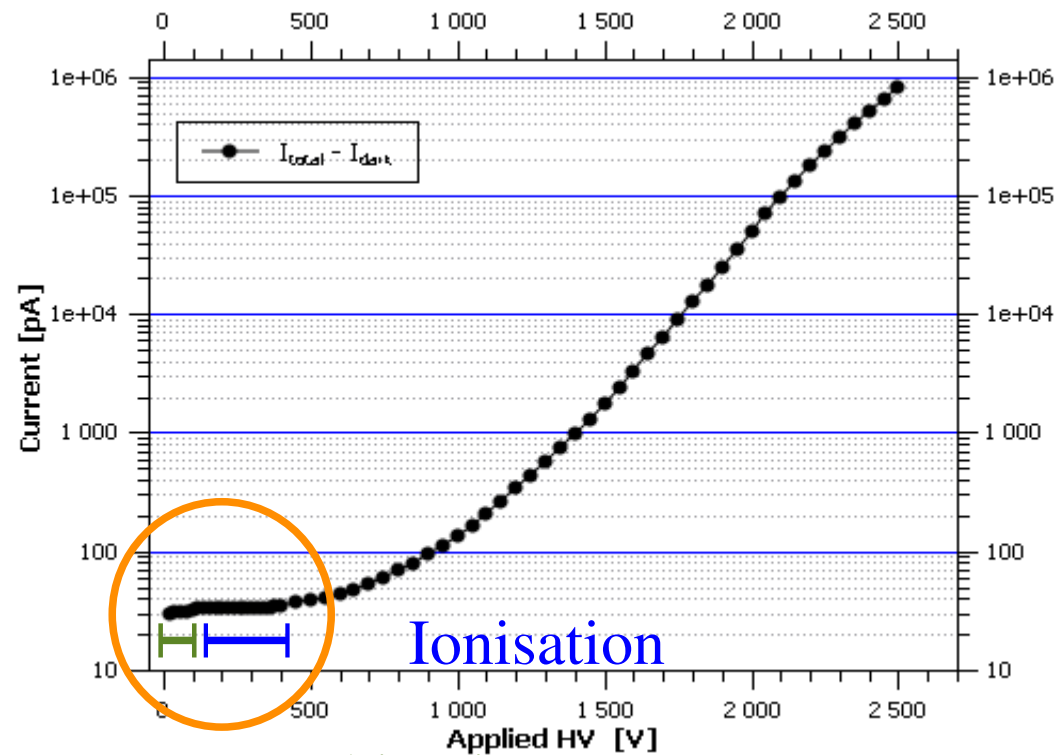
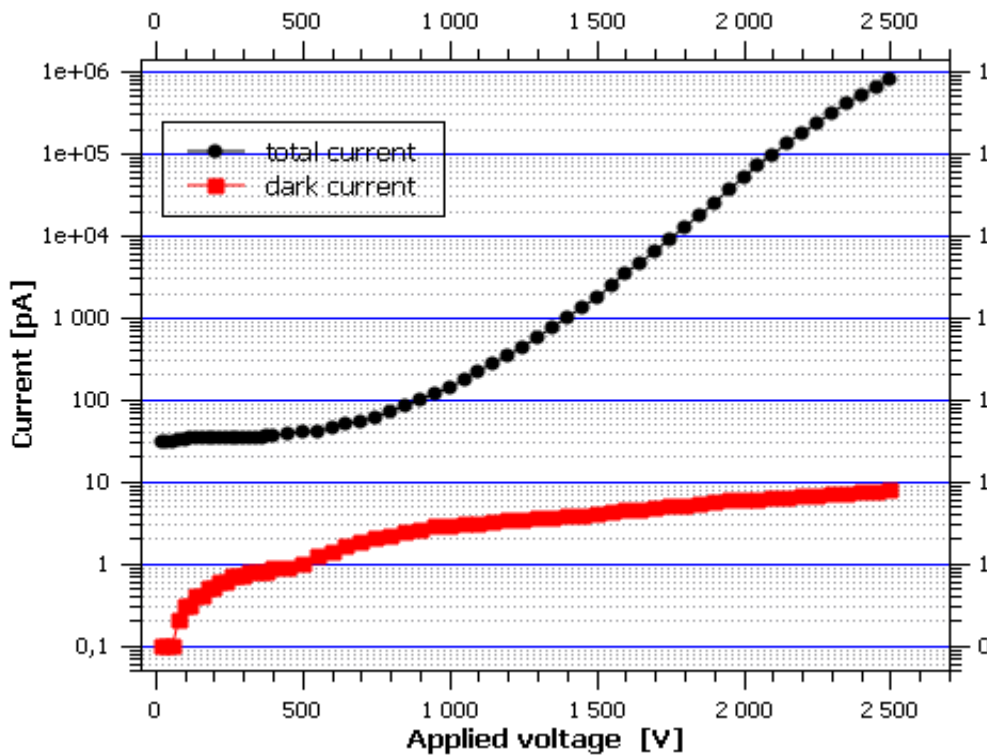
F.M. Penning, *The starting potential of the glow discharge in neon argon mixtures between large parallel plates: II. Discussion of the ionisation and excitation by electrons and metastable atoms*, *Physica* **1** (1934) 1028-1044.

F.M. Penning, *Electrische gasontladingen*, Philips Technische Bibliotheek (posthumous, 1955). Translated in various languages.



Example of gain measurements

- ▶ Ar-CO₂-N₂ [91.1-6.4- 2.5] at $p = 0.2$ MPa;
- ▶ dark current measurement and subtraction:

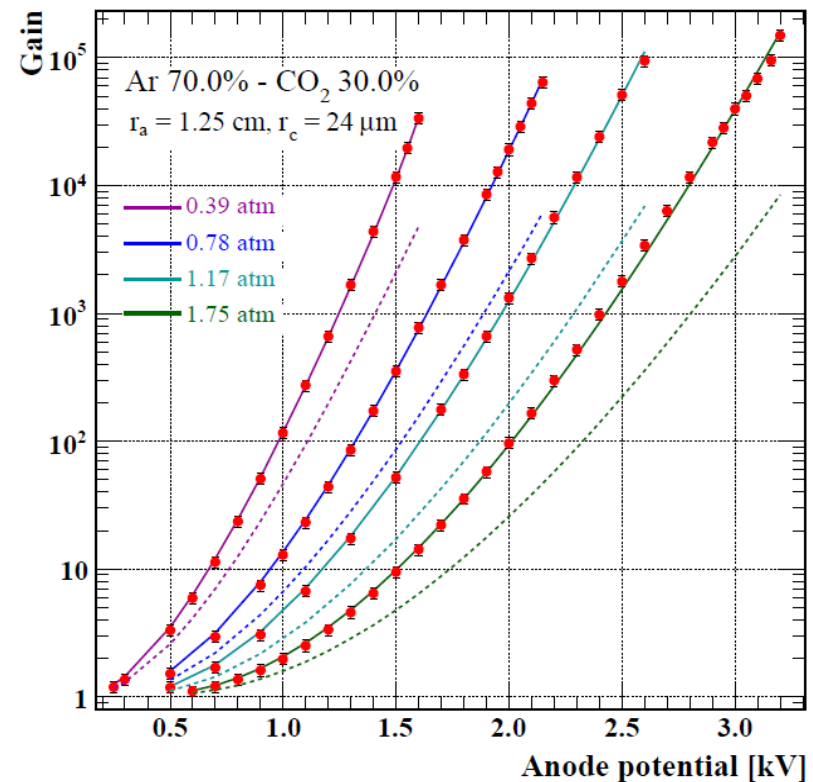
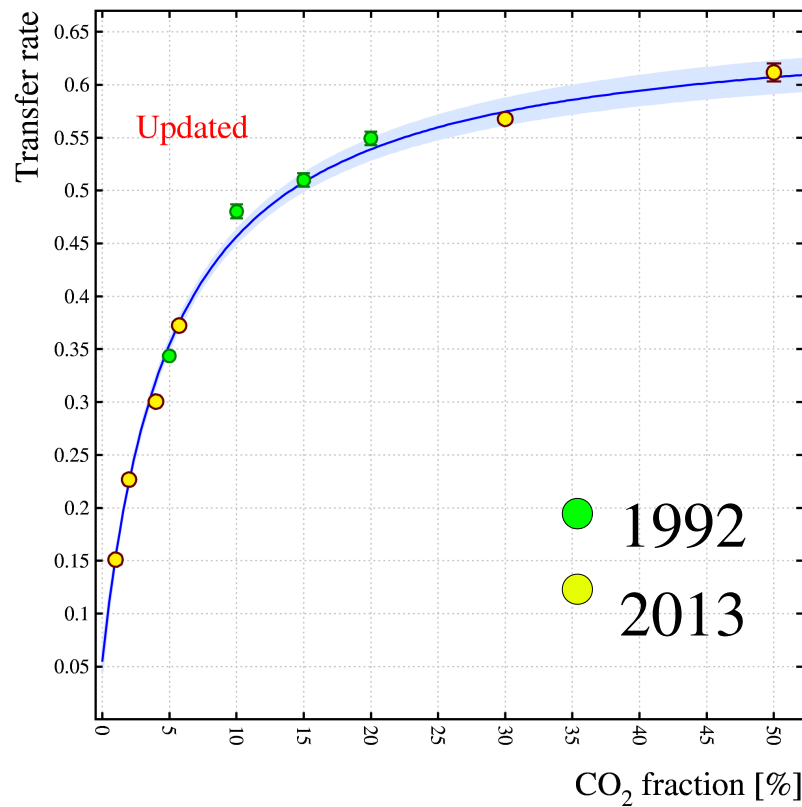


Recombination

Ionisation

Ar-CO₂

- ▶ Reproduce the gain curves by:
 - ▶ ionisation rates + r_p excitation rates.
 - ▶ r_p is called the Penning or transfer rate.
- ▶ Measurements: Tadeusz Kowalski *et al.* AGH Kraków.
Fits: Özkan Şahin, Uludağ.

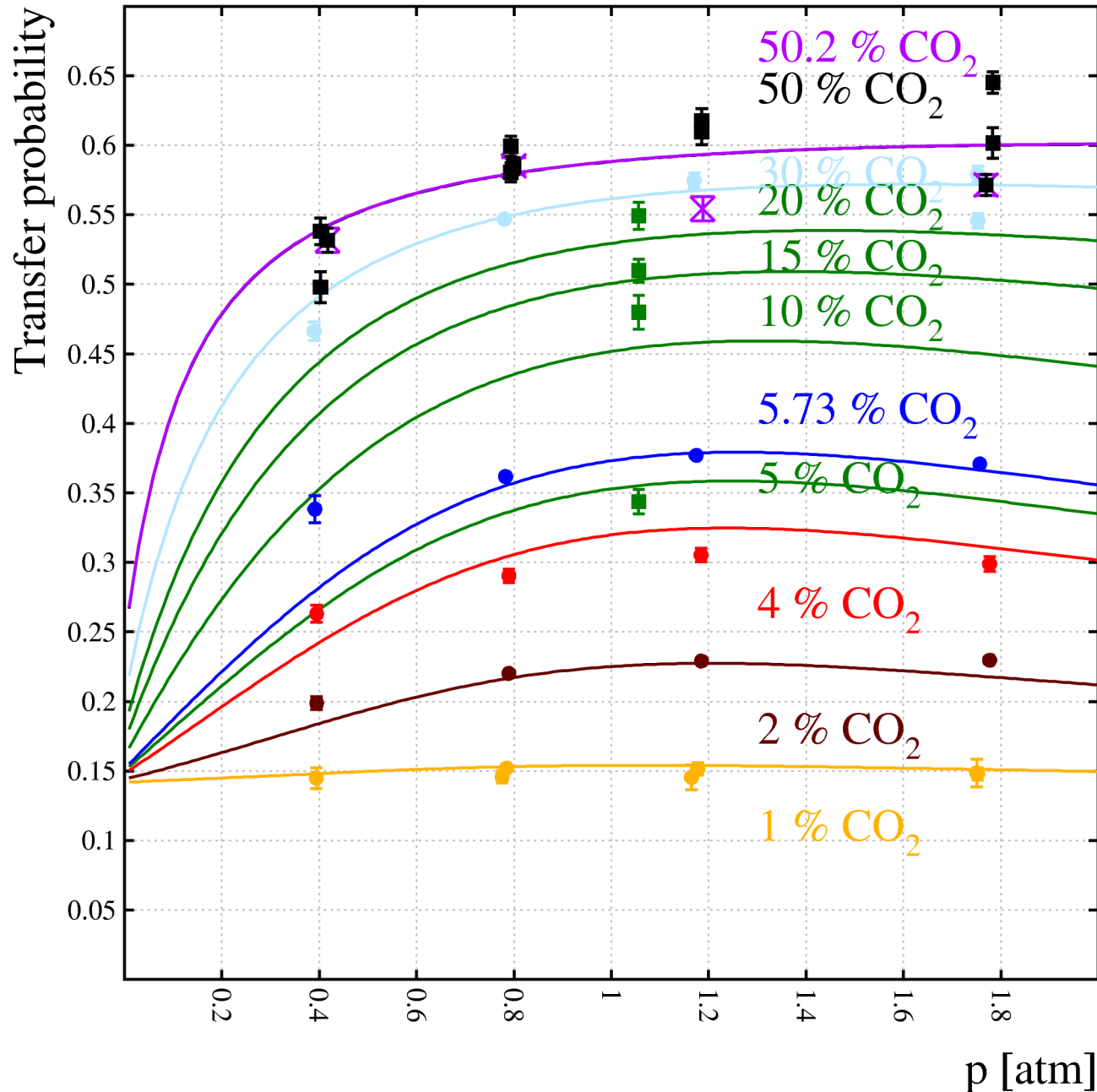


Pressure

► Transfer rates for Ar-CO₂, from experimental gain curves

► Hint of 3-body interactions.

[Tadeusz Kowalski and Özkan Şahin]

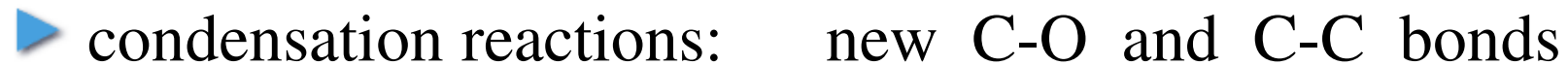


Ion transport

- ▶ If electrons seemed complicated ...

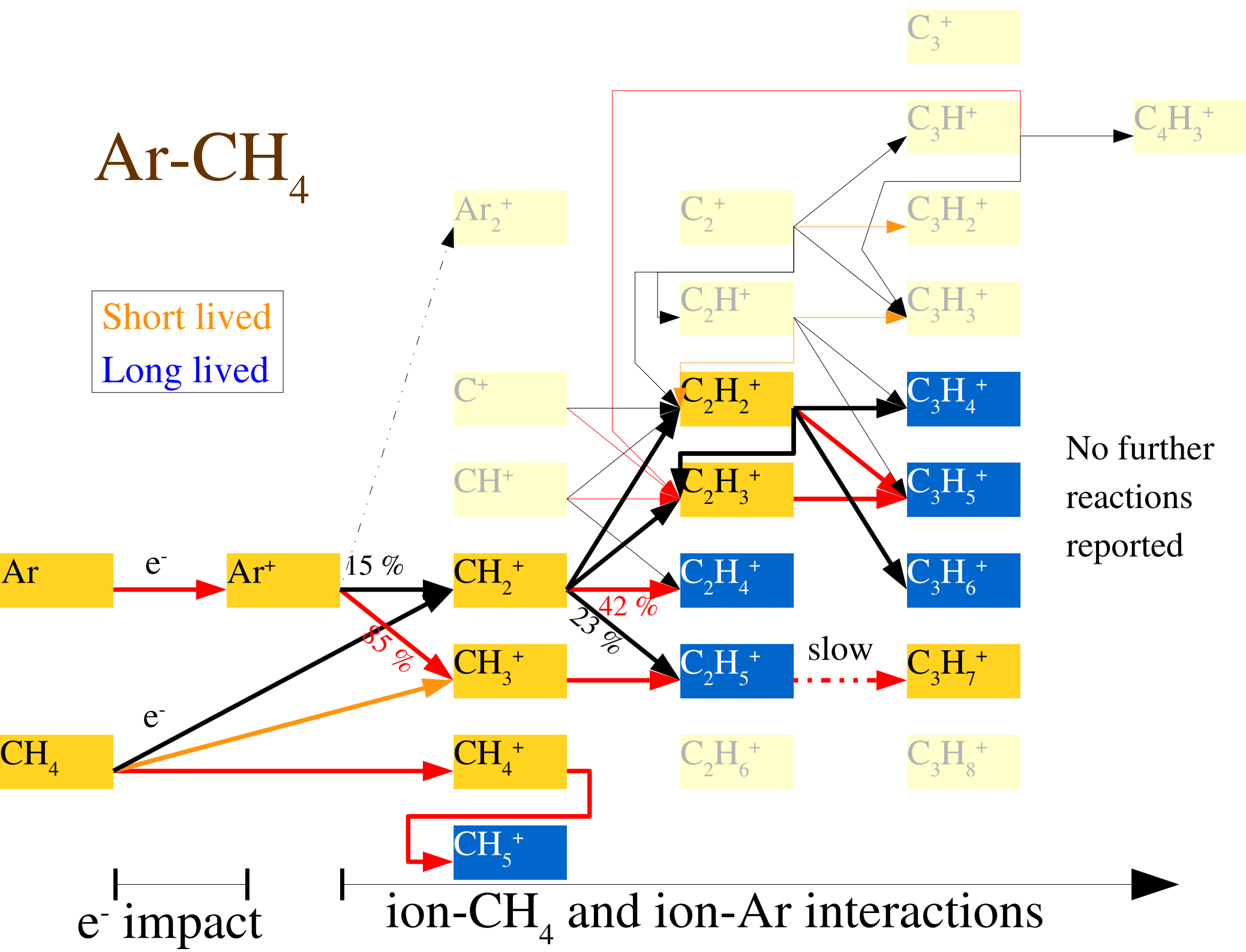
Ion chemistry – rate constants

- ▶ Ions react with the gas in which they move:



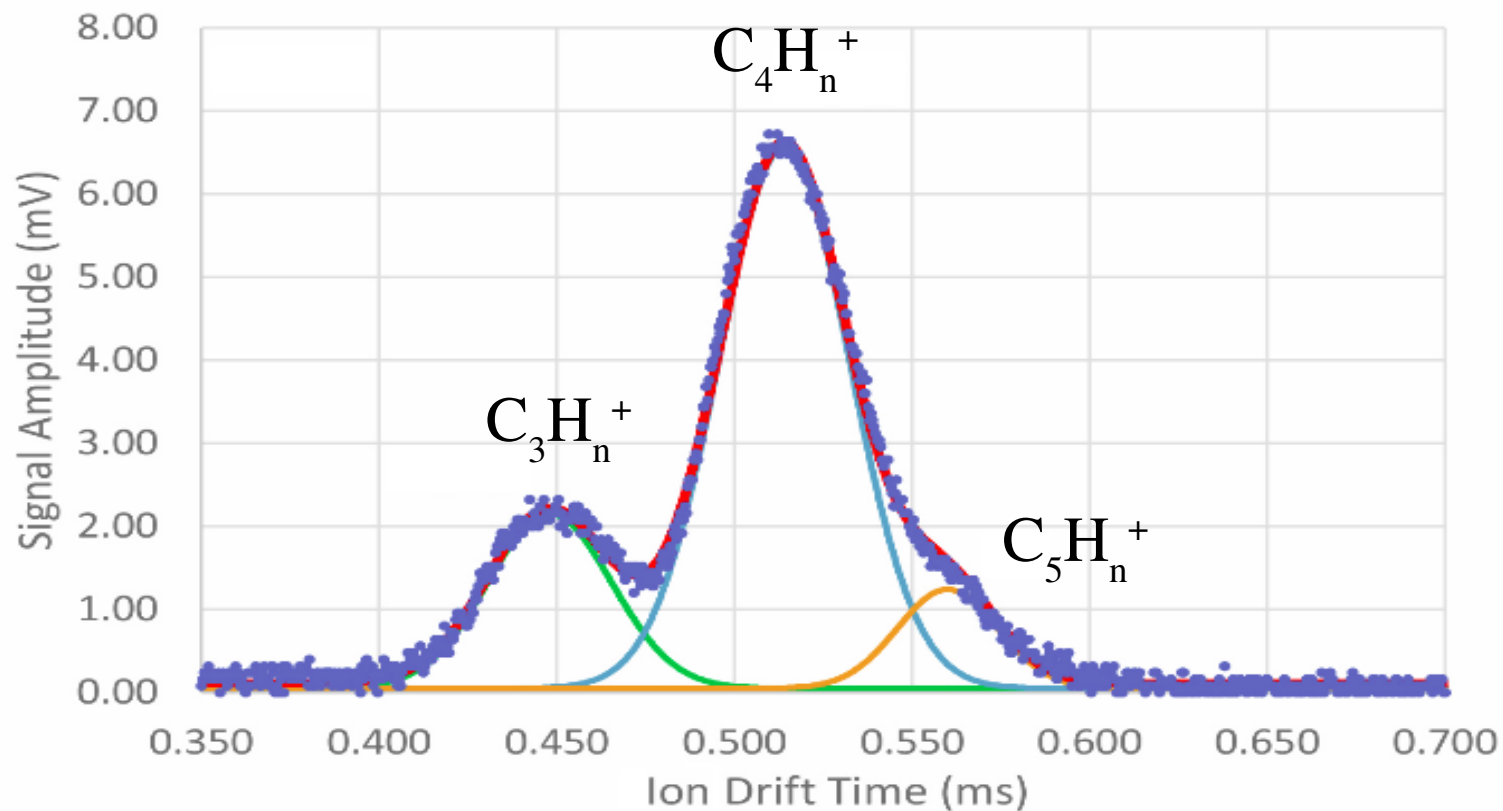
- ▶ ...

- ▶ Rate constants range from 10^{-9} to 10^{-14} cm³/s. At atmospheric pressure, in a pure gas, this corresponds to characteristic times of 40 ps to 4 μs.



Ion transport

- ▶ Ar 90 % - C₂H₆ 10 %, at low pressure

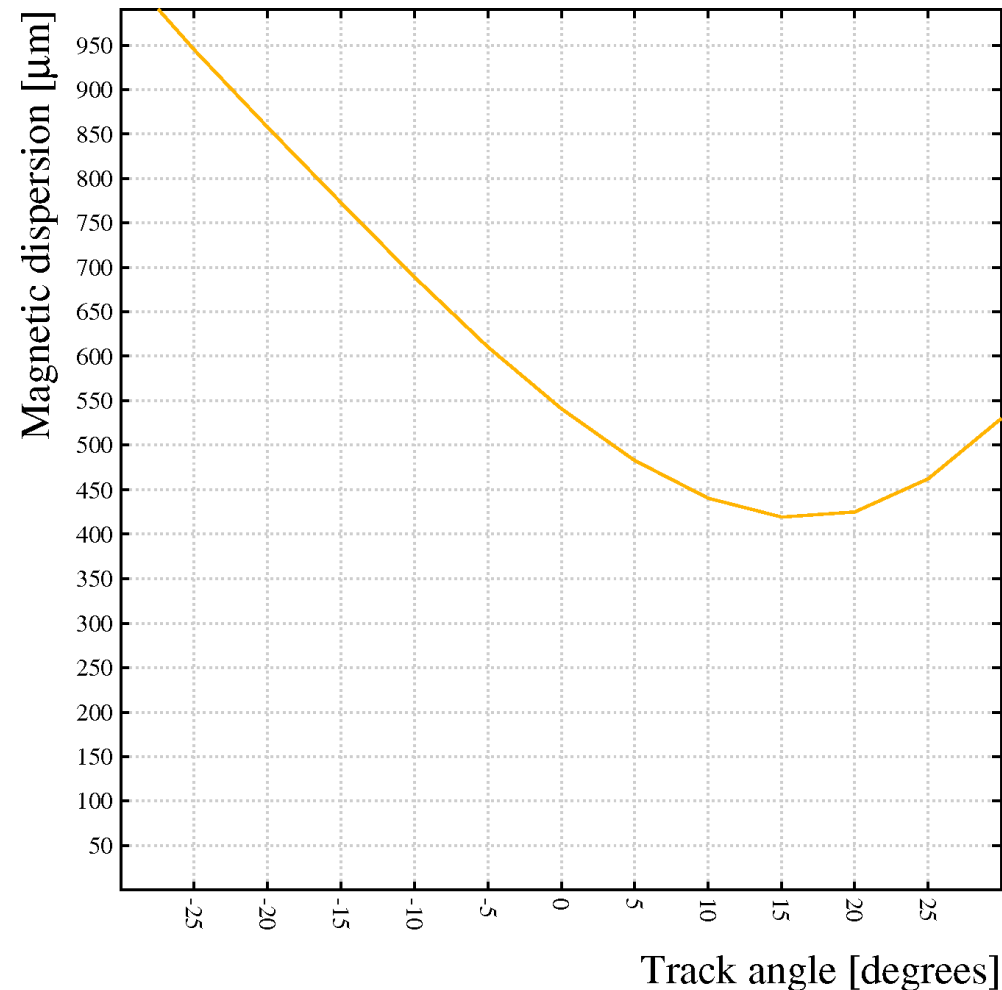
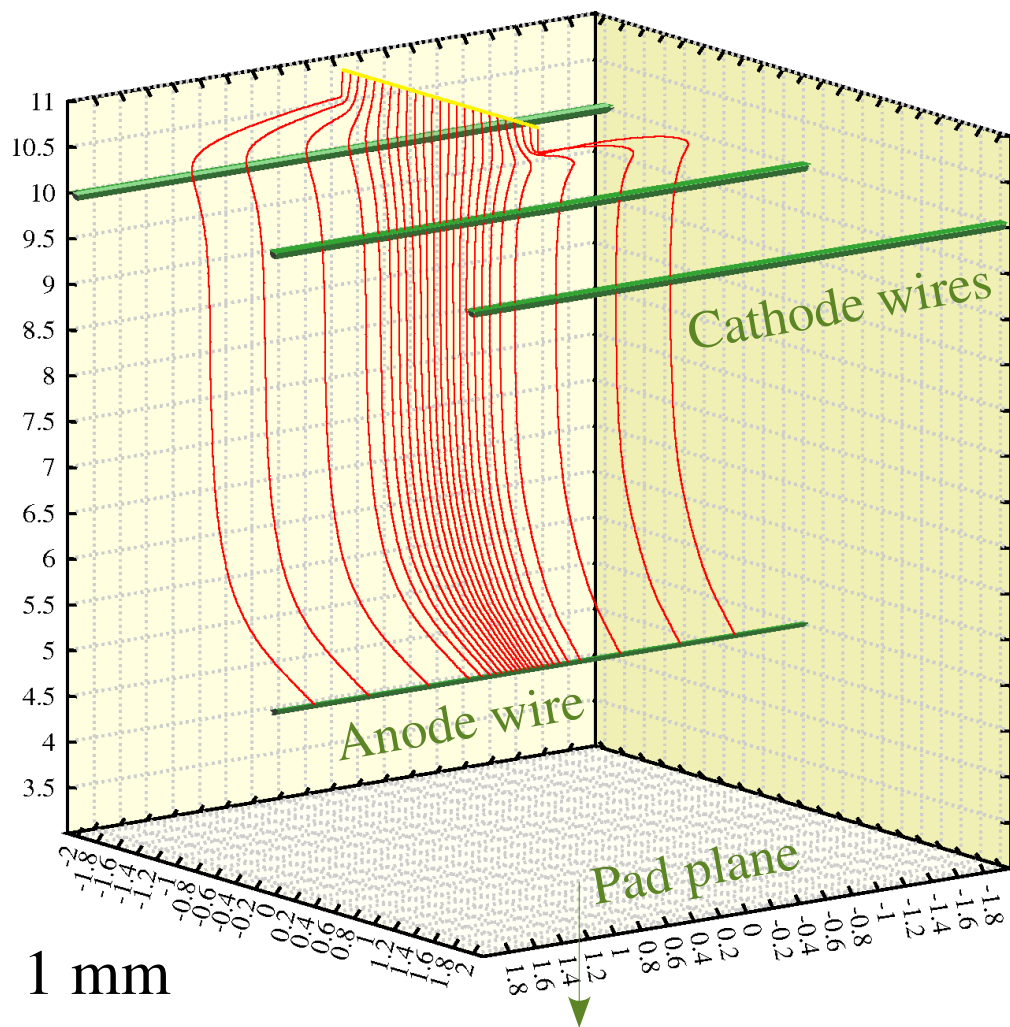


[André Cortez et al. 10.1088/1748-0221/8/12/P12012]

Putting it together

Classic: e^- and ion^+ trajectories

- ▶ Example (Harp): $E \times B$ effect in an enlarged \aleph read-out cell.



Scale

- ▶ Recall:

- ▶ Mean free path of e^- in argon: 2-5 μm ,
- ▶ diffusion: $\sim 80 \mu\text{m}$ for 1 mm.

- ▶ Compare with:

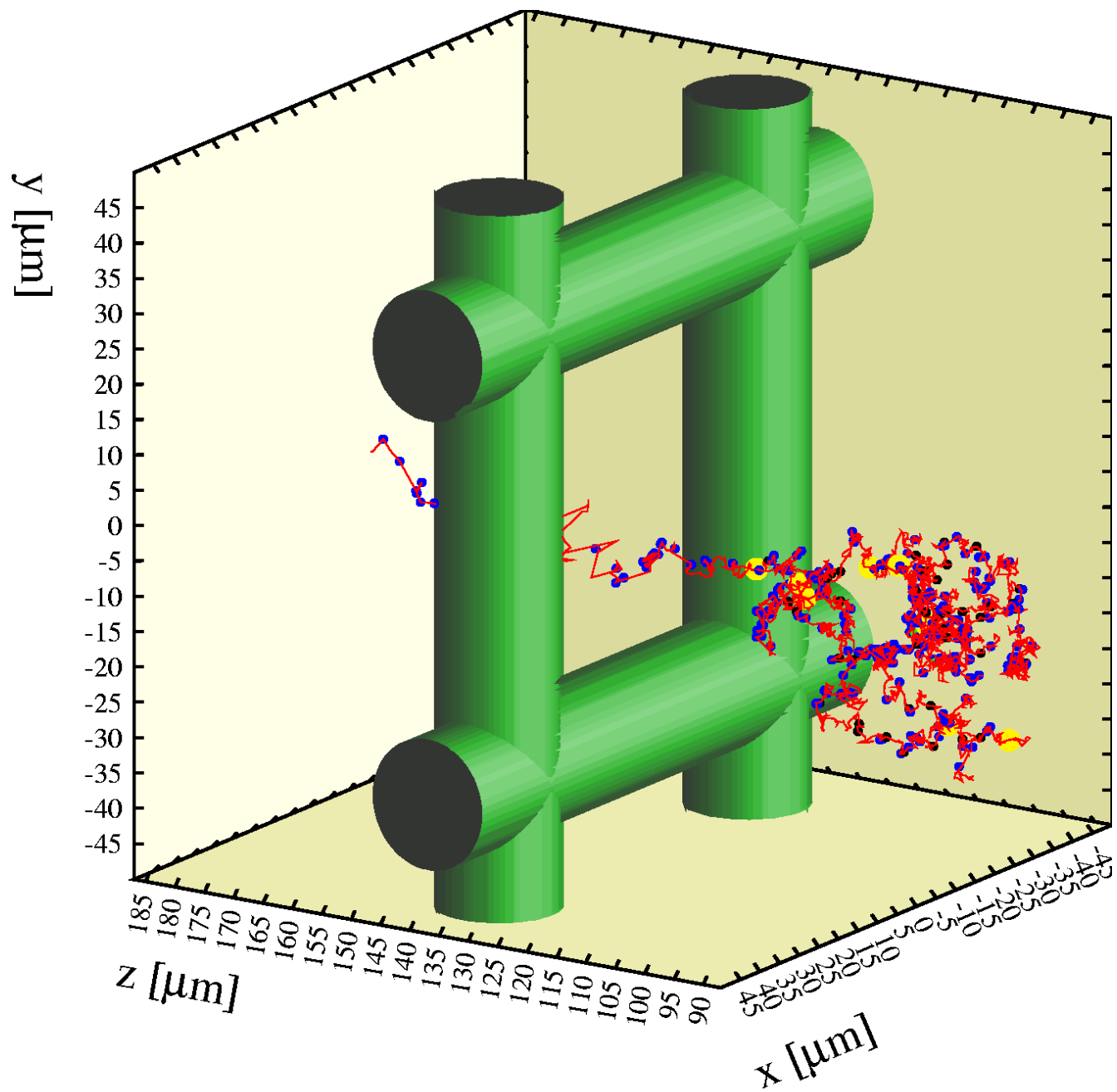
- ▶ Micromegas mesh pitch: 63.5 μm
- ▶ GEM polyimide thickness: 50 μm
- ▶ Micromegas wire thickness: 18 μm
- ▶ GEM conductor thickness: 5 μm

- ▶ Hence:

- ▶ mean free path approaches small structural elements;
- ▶ diffusion is not likely to be Gaussian.

Microscopic Micromegas

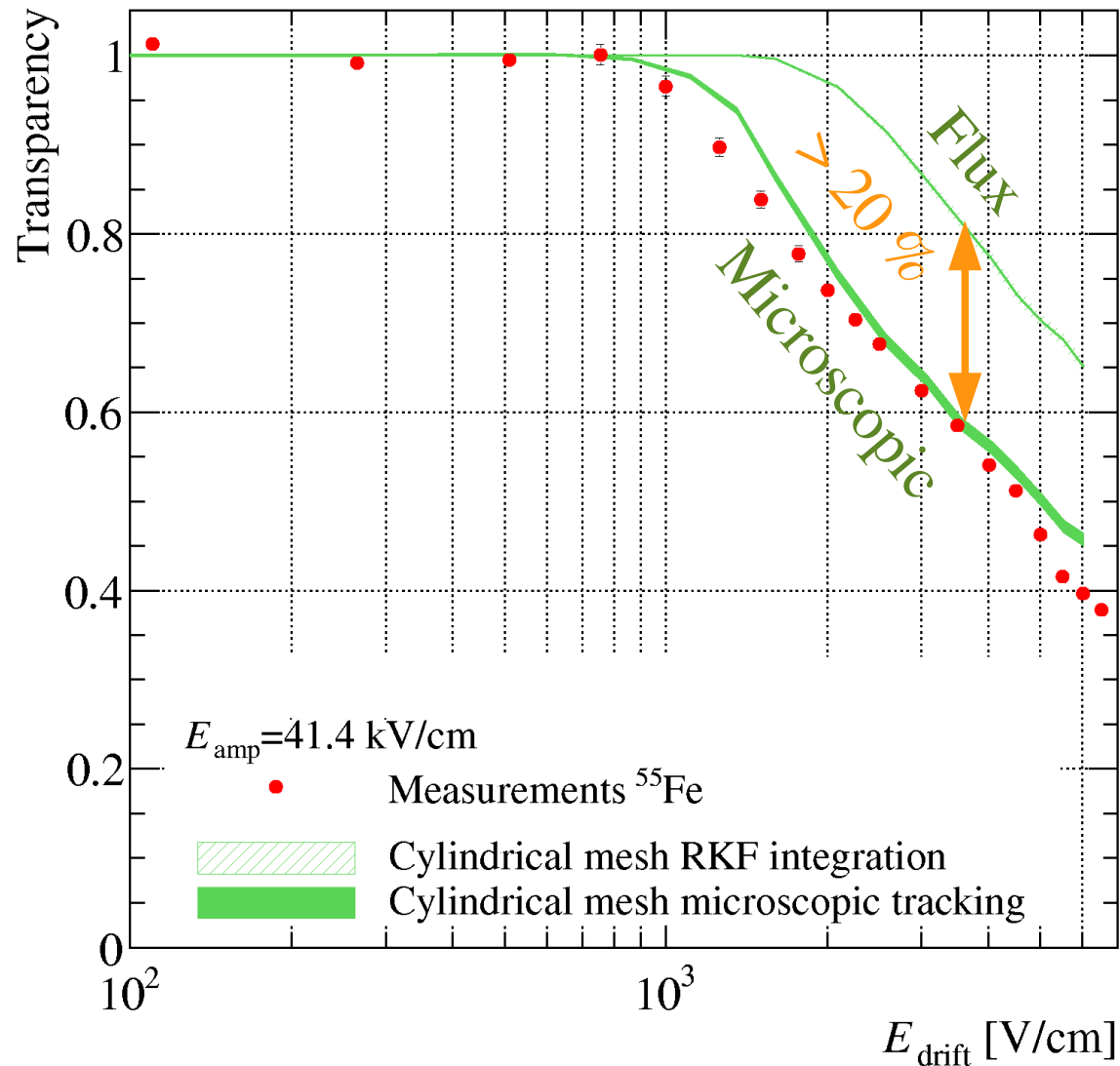
- ▶ Legend:
- ▶ — electron
 - ▶ ○ inelastic
 - ▶ ○ excitation
 - ▶ ○ ionisation



Flux vs microscopic ?

- ▶ A diffusion-free flux argument does not reproduce the data ...
- ▶ but the microscopic approach works.

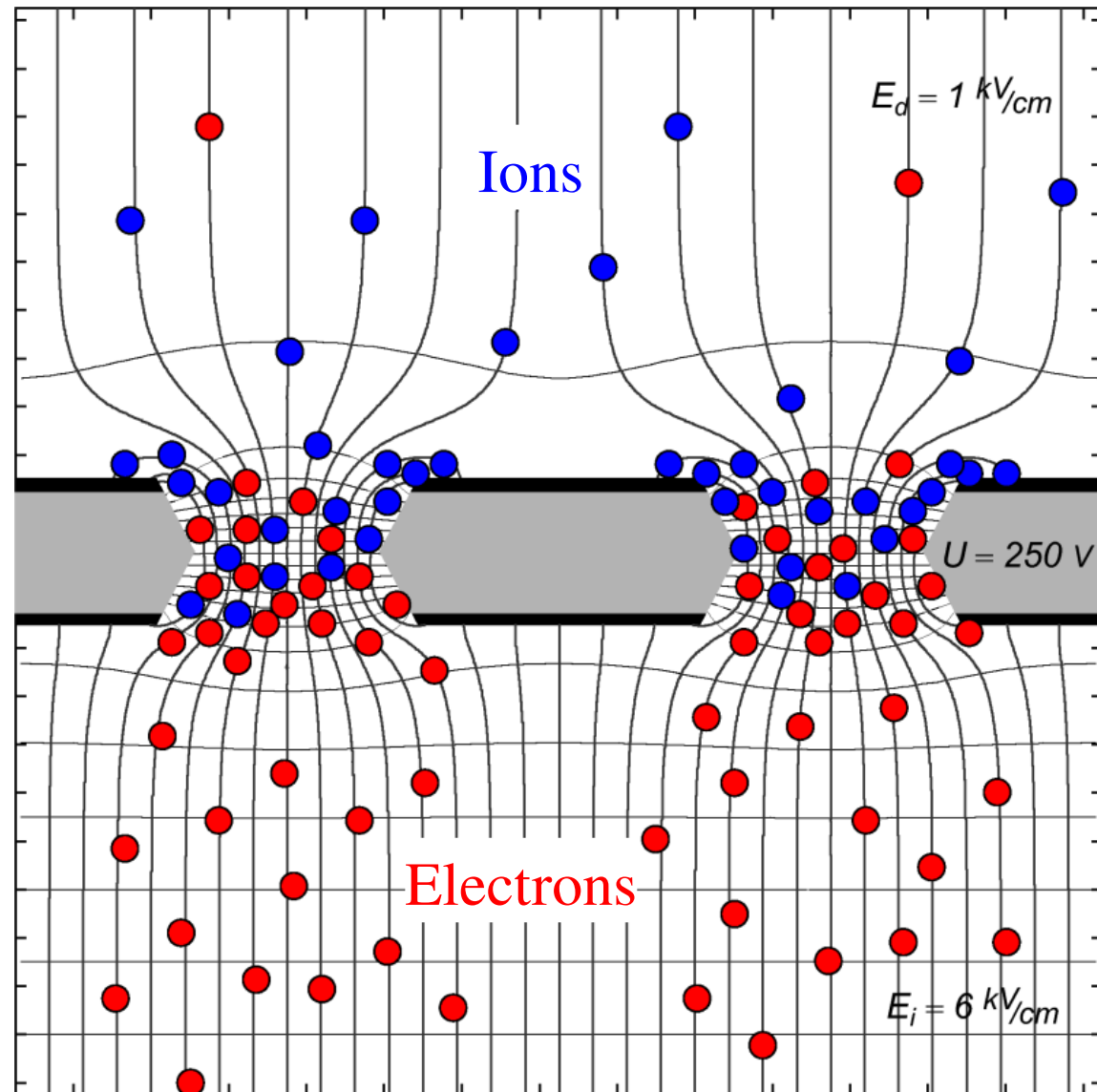
Field calculations: finite elements.



GEM, textbook

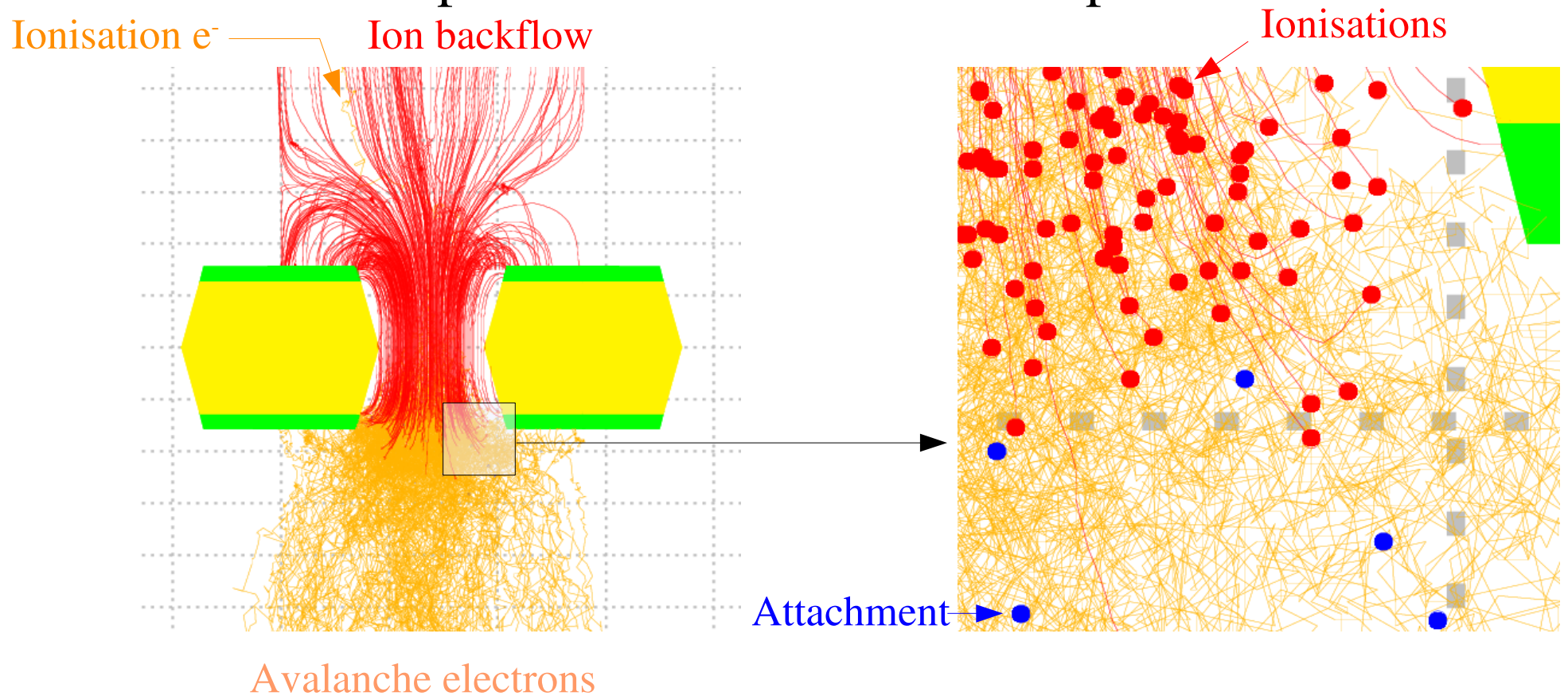
- ▶ e^- & ion^+ follow the “field lines”

[DESY FLC/TPC, based on a CERN GDD drawing]



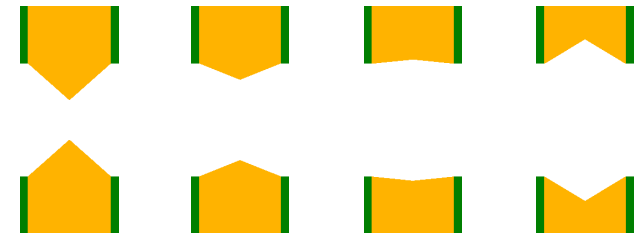
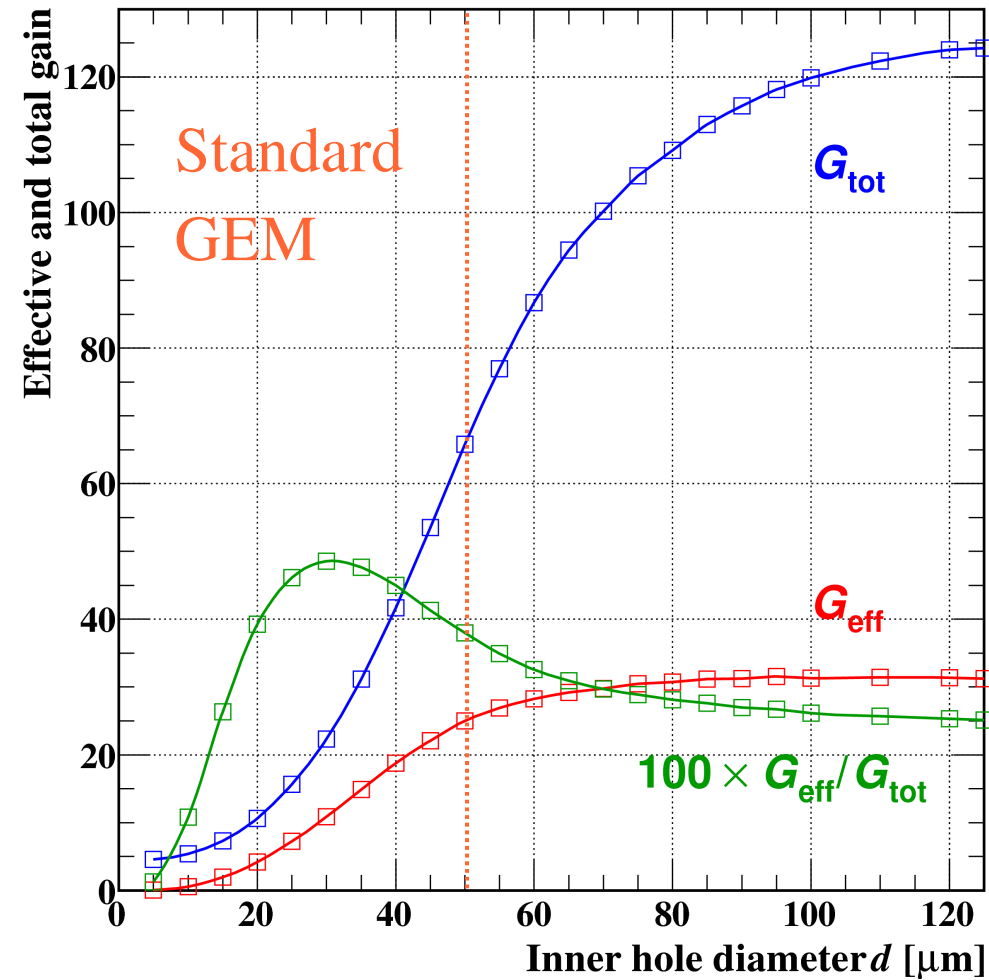
GEM, microscopic view

- ▶ Micropattern devices have characteristic dimensions that are comparable with the mean free path.



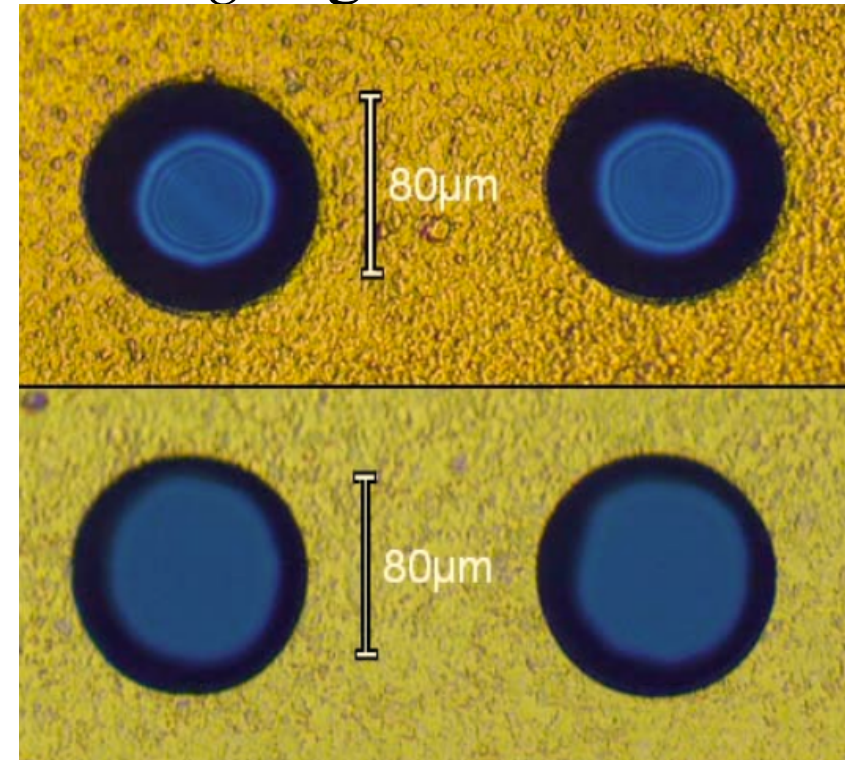
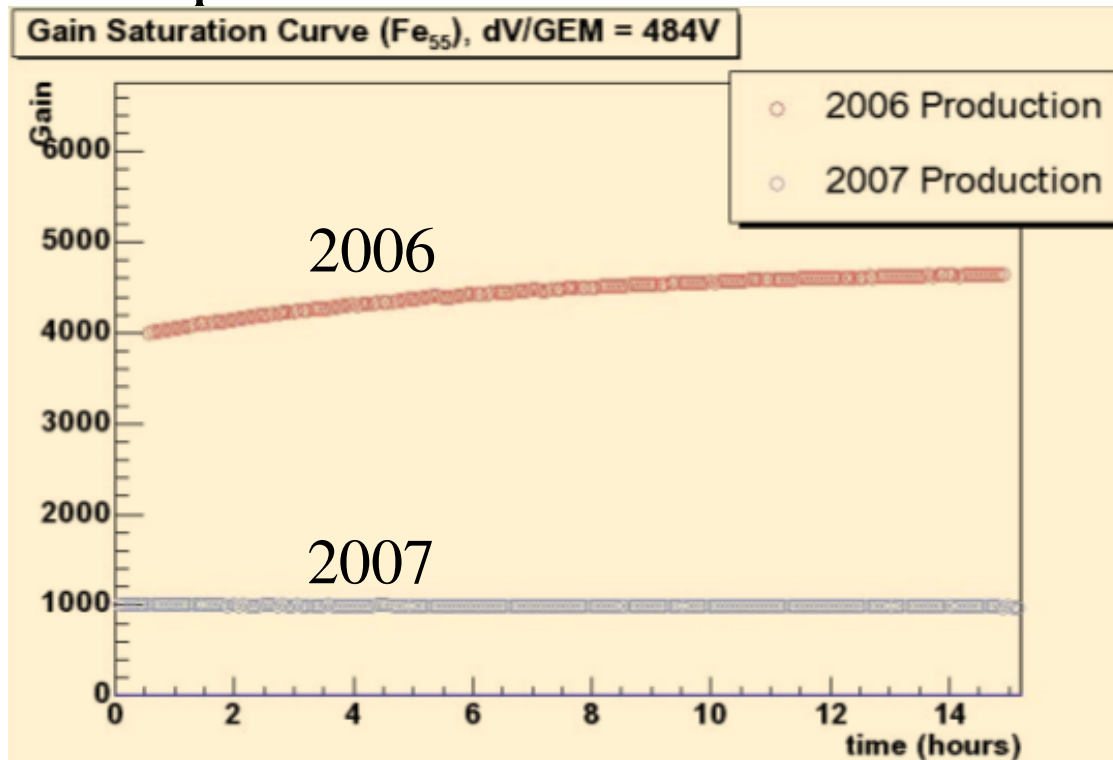
Gain calculations in a pristine GEM

- ▶ Calculations predict that G_{tot} and G_{eff} rise with increasing inner hole diameter.
- ▶ G_{eff} rises mainly because the losses of incoming electrons diminish;
- ▶ G_{tot} rises because the exit electrode becomes more accessible.



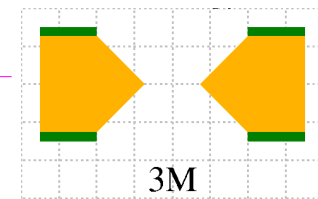
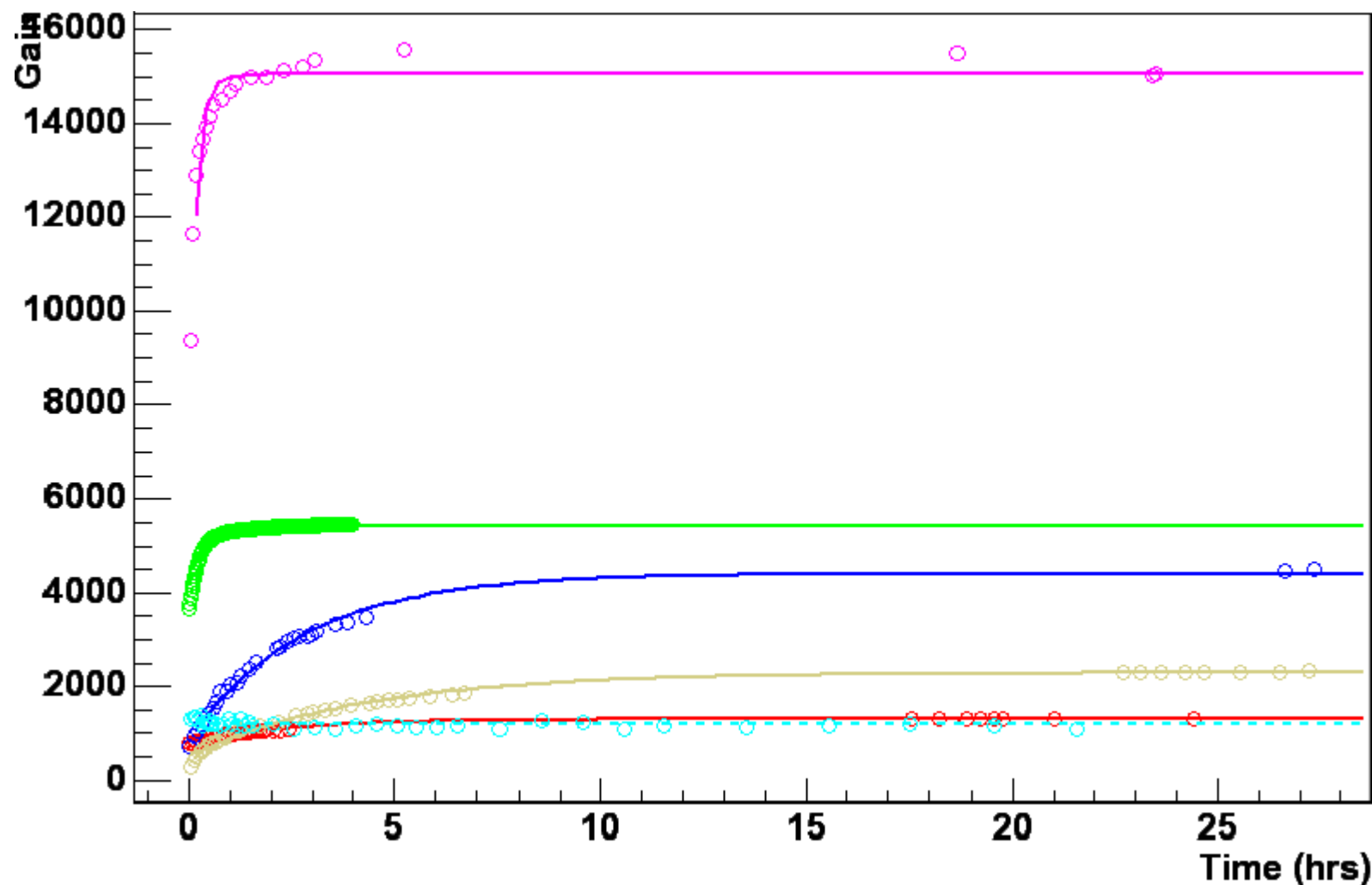
HBD data

- ▶ Measurements for 2 triple GEMs with different hole shape shows that smaller holes lead to *larger* gain !



- ▶ [W. Anderson *et al.* 10.1109/NSSMIC.2007.4437147]

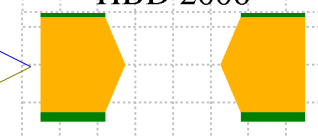
GEMs of various manufacturers



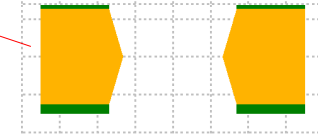
3M



HBD 2006



Tech-Etch



CERN

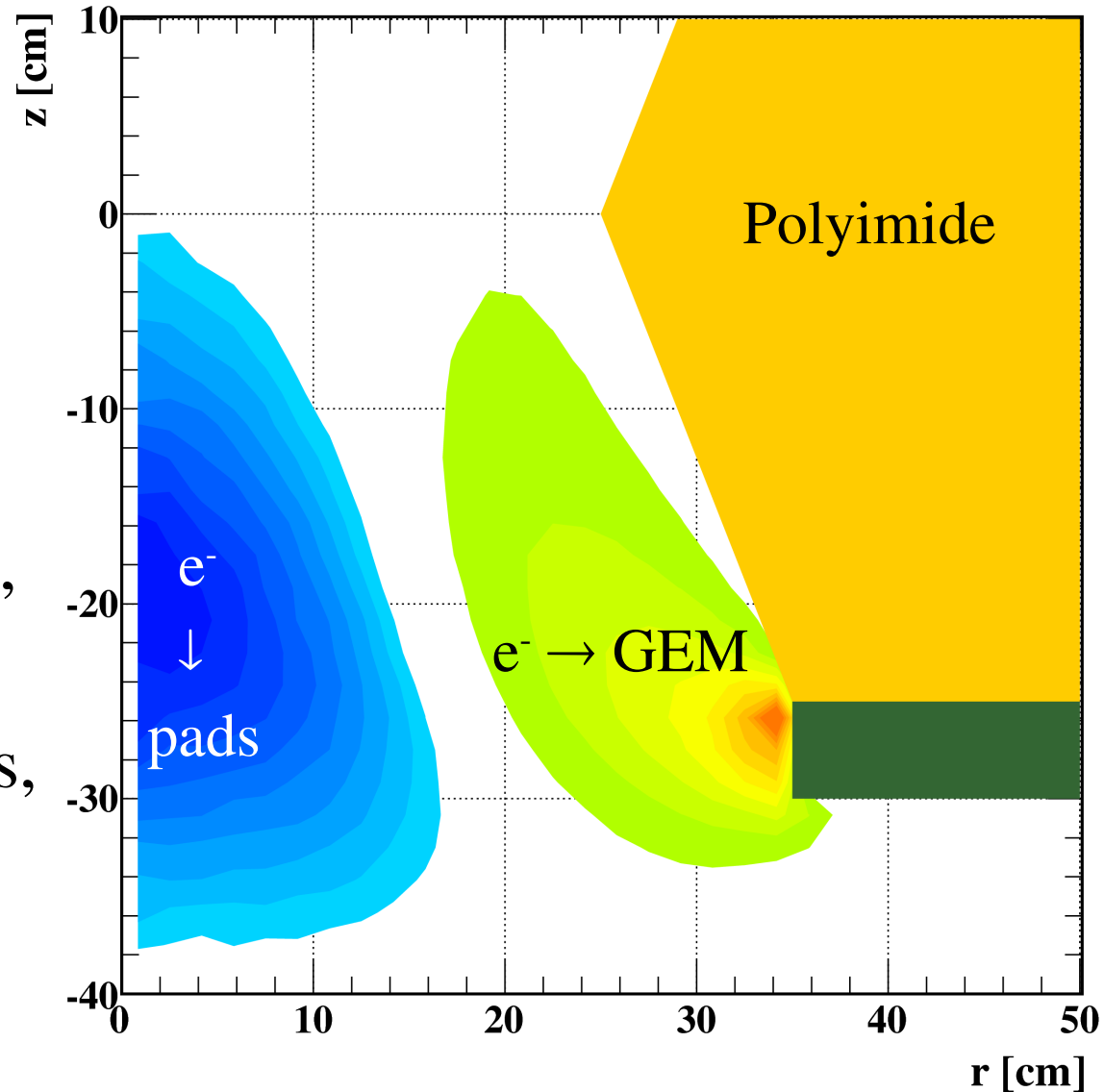


Scienergy

► [B. Azmoun *et al.* 10.1109/NSSMIC.2006.353830]

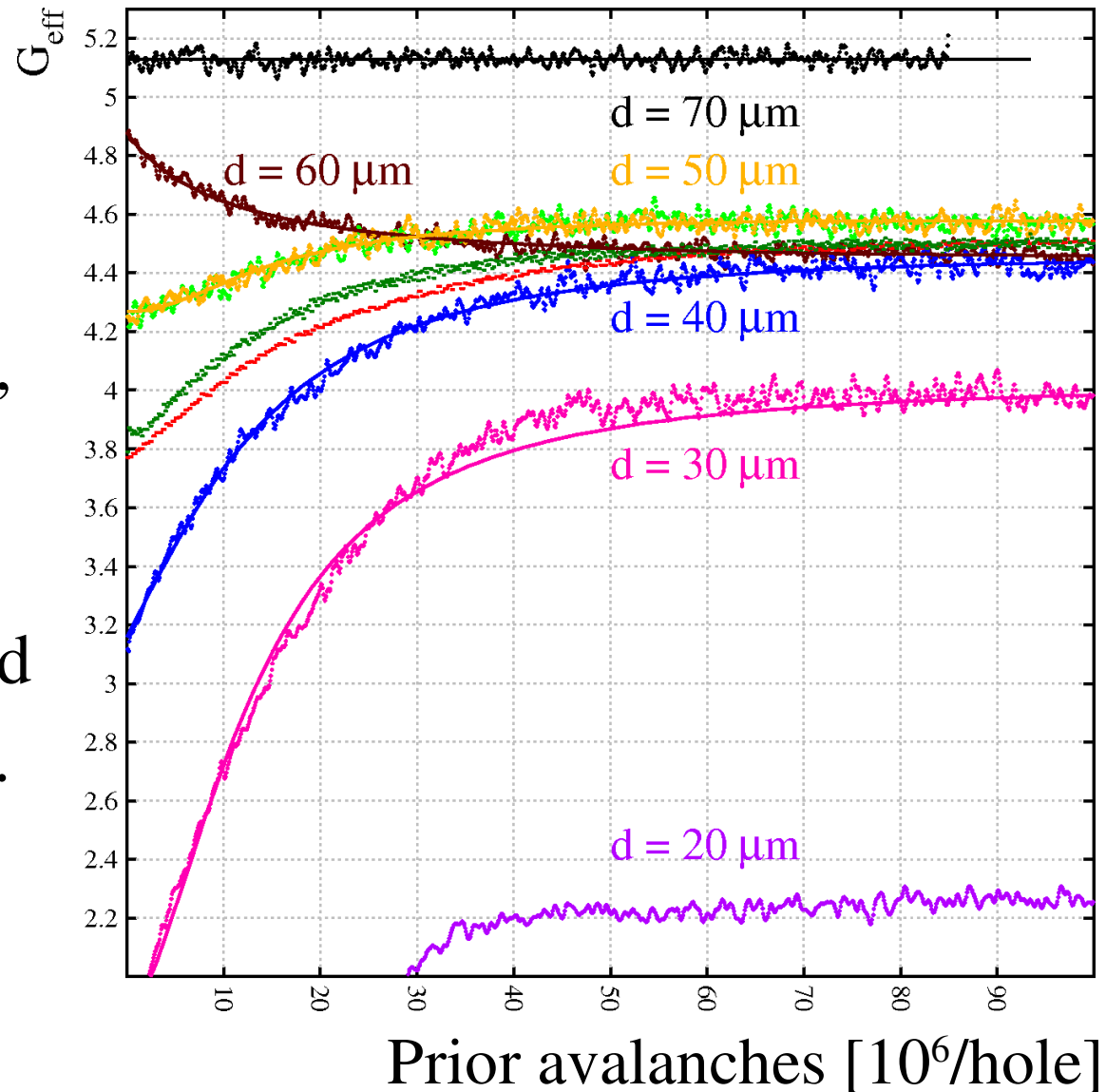
Avalanche regions

- ▶ Vicinity GEM anode:
large multiplication,
 $e^- \rightarrow$ GEM anode,
polyimide,
 $ion^+ \rightarrow$ polyimide,
GEM cathode,
drift region.
- ▶ Hole centre: lower fields,
less multiplication,
 $e^- \rightarrow$ pad plane,
 $ion^+ \rightarrow$ drift region.

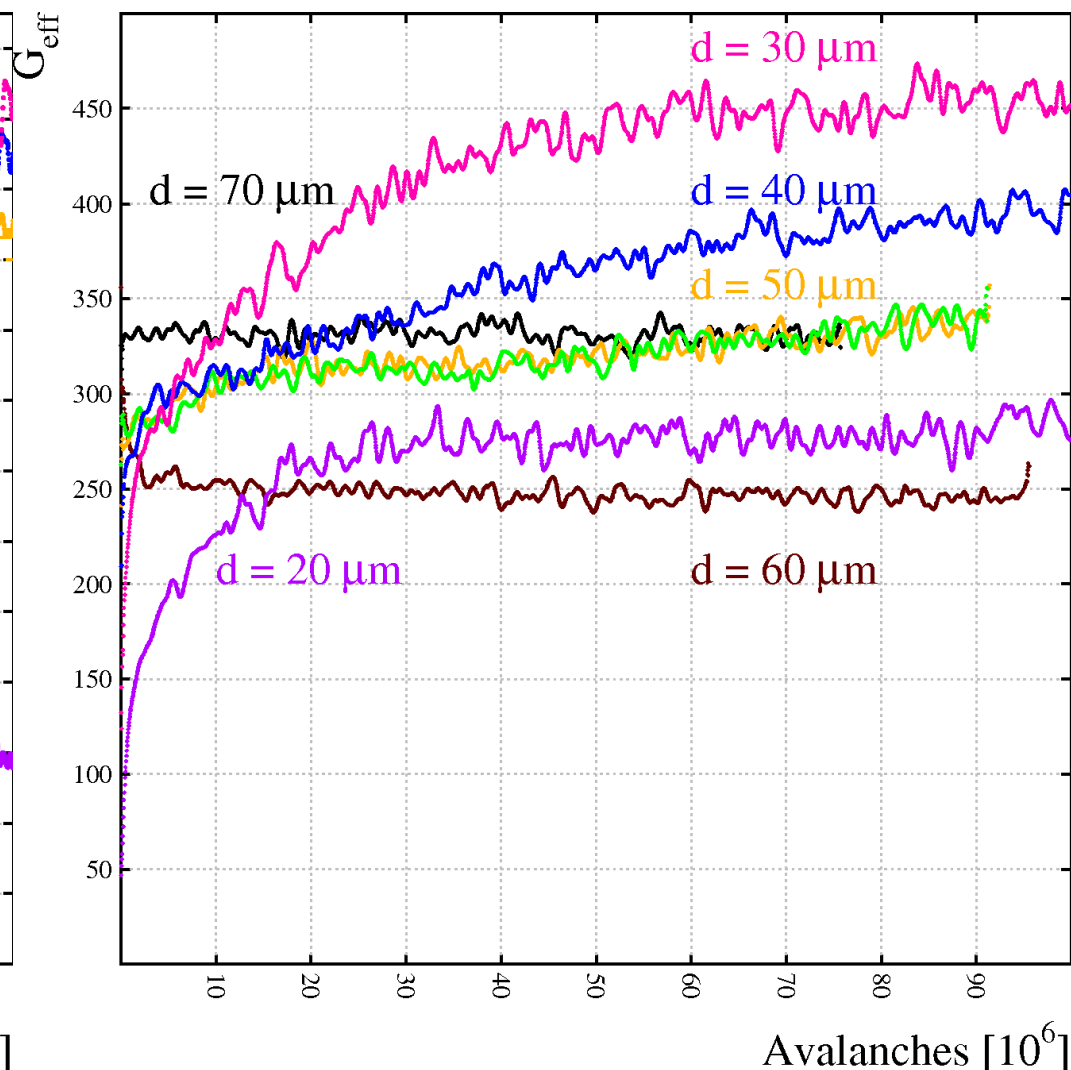
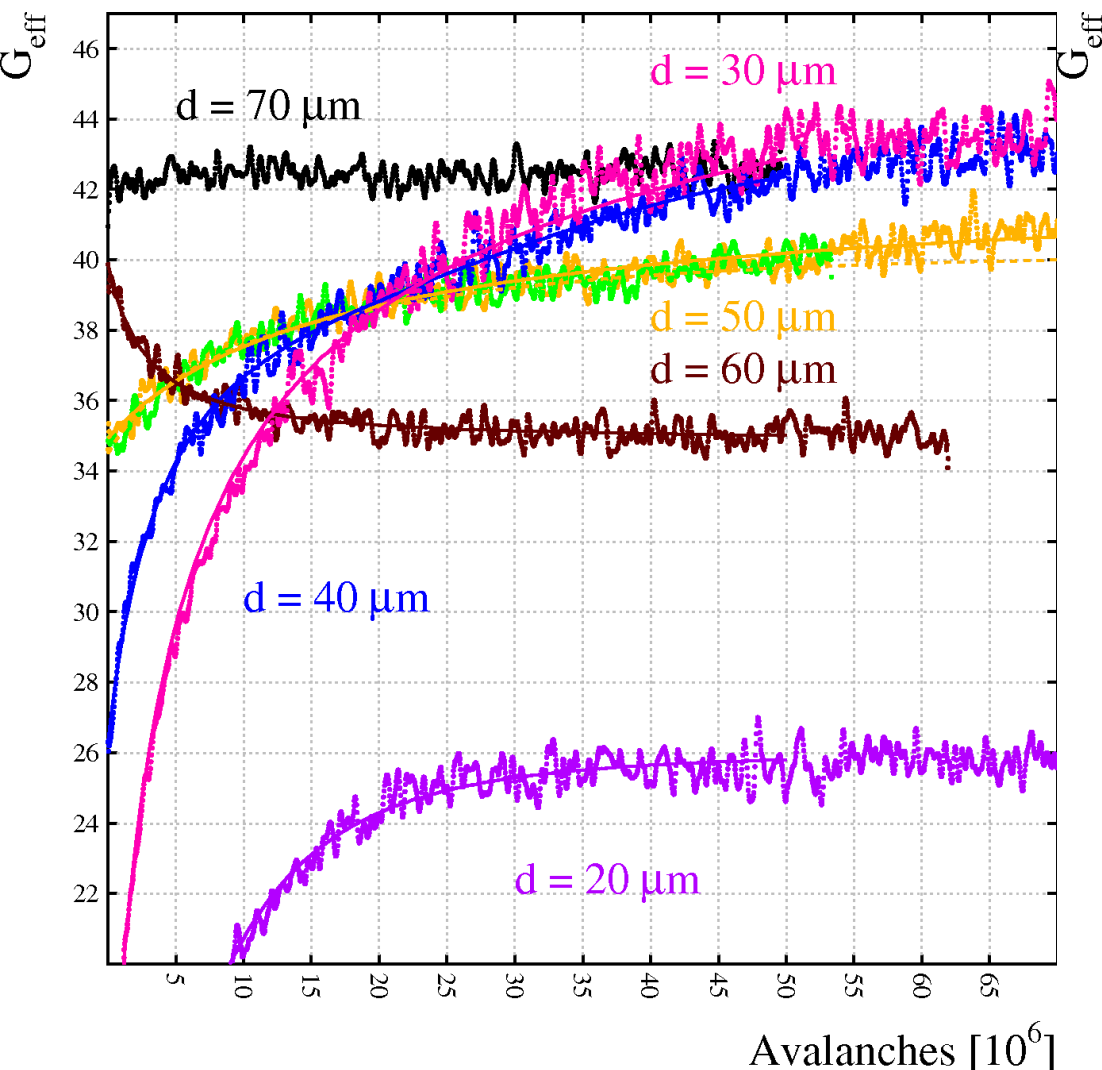


Effect of surface charge ($V_{\text{GEM}} = 300 \text{ V}$)

- ▶ In a clean GEM, small holes give lower gain.
- ▶ As charge accumulates, the gain curves cross.
- ▶ Effect more pronounced at higher GEM voltage.



$V_{\text{GEM}} = 400 \text{ V}$ and $V_{\text{GEM}} = 500 \text{ V}$



Trends

- ▶ Wires have fallen out of favour:
 - ▶ finite element and boundary elements.
- ▶ Electron transport:
 - ▶ integrated transport and fields for small structures;
 - ▶ role of excited noble gas atoms, transfer measurements;
 - ▶ charges in dielectrics, surface charge, space charge.
- ▶ Ion transport:
 - ▶ understanding ion chemistry;
 - ▶ measurement of ion transport.
- ▶ Current activities
 - ▶ semi-conductive layers.

Backup slides

De-excitation



Ralph de Laer Kronig
(1904-1995)



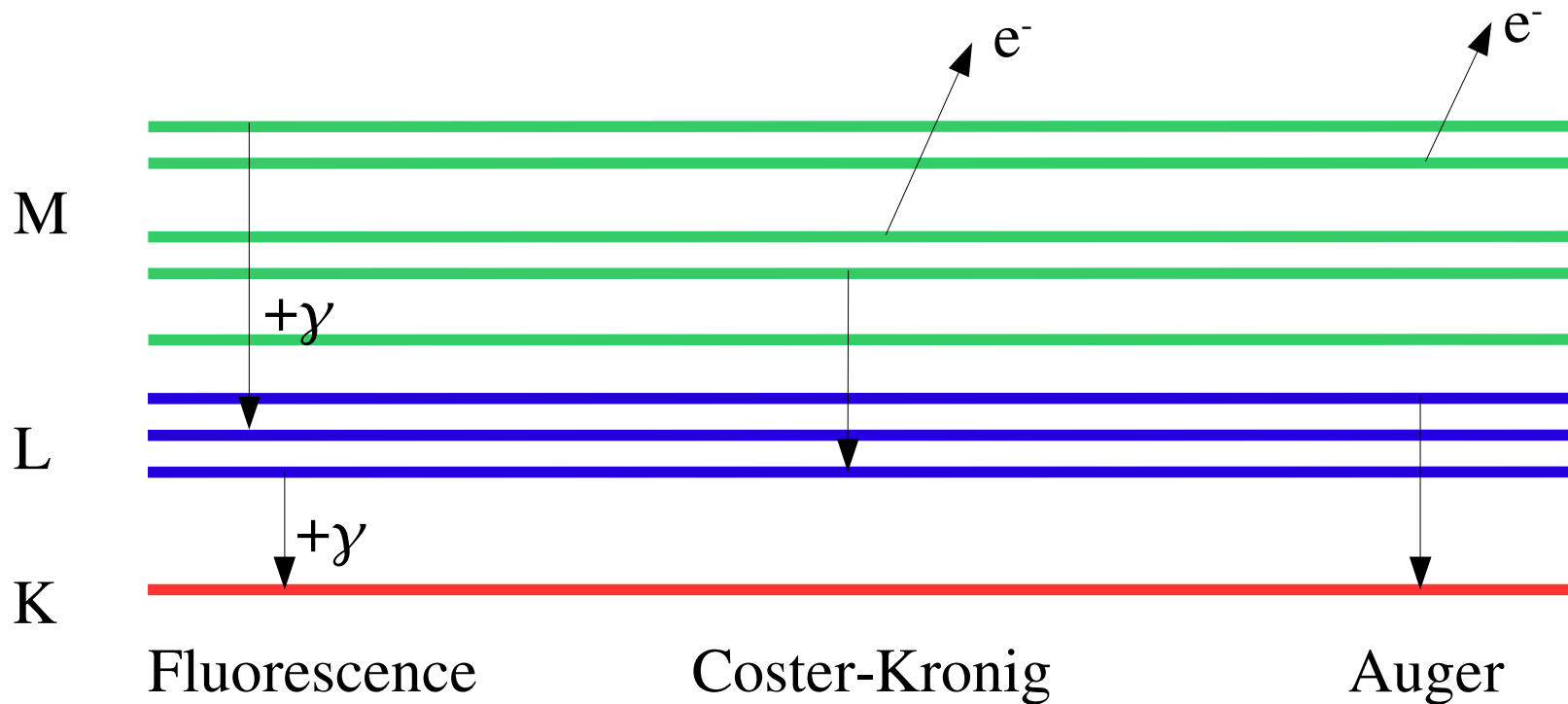
Dirk Coster
(1889-1950)



Lise Meitner
(1878-1968)



Pierre Victor Auger
(1899-1993)



References:

D. Coster and R. de L. Kronig, *Physica* **2** (1935) 13-24.

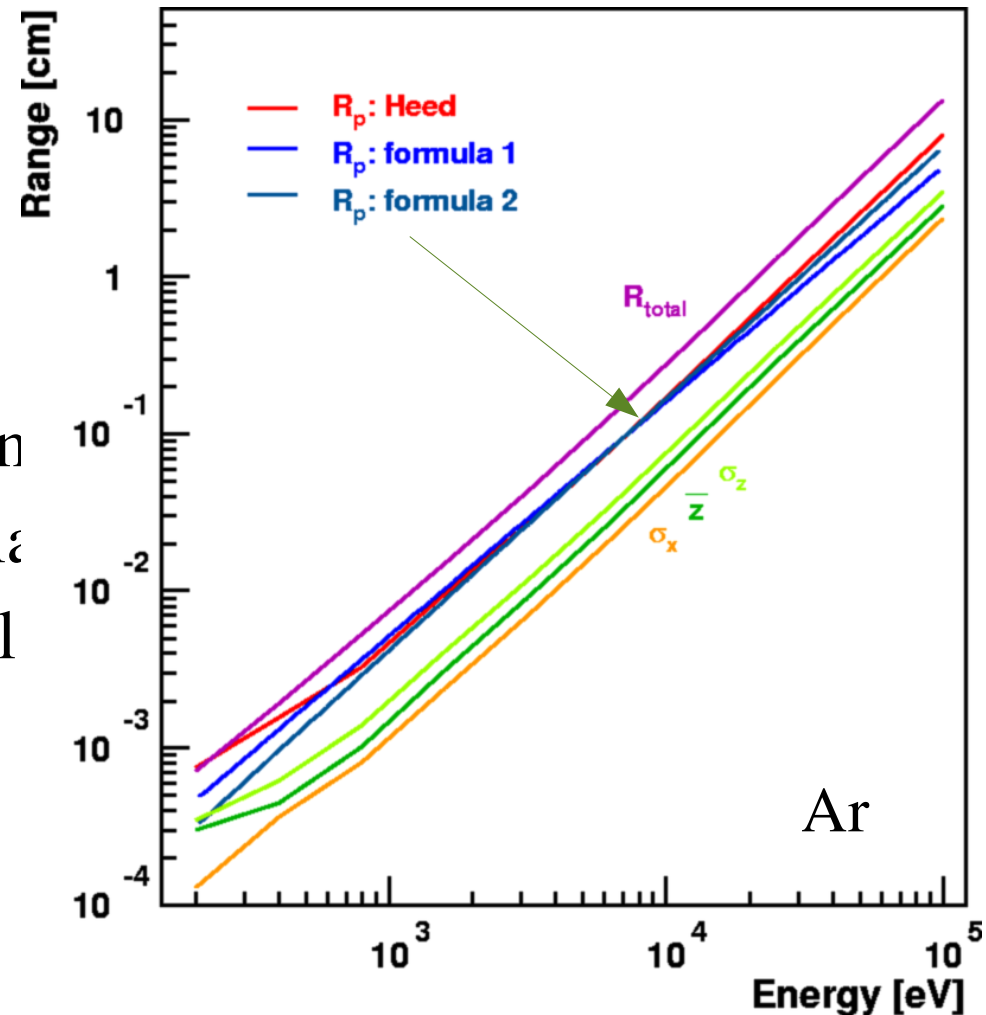
Lise Meitner, *Über die β -Strahl-Spektren und ihren Zusammenhang mit der γ -Strahlung*, *Z. Phys.* **11** (1922) 35-54.

L. Meitner, *Das β -Strahlenspektrum von UX_1 und seine Deutung*, *Z. Phys.* **17** (1923) 54-66.

P. Auger, *J. Phys. Radium* **6** (1925) 205.

Range of photo- and Auger-electrons

- ▶ Electrons scatter in a gas.
- ▶ Measures of the range:
 - ▶ R_{total} : total path length
 - ▶ R_p : practical range
 - ▶ \bar{z} : cog in direction of initial n
 - ▶ σ_x : RMS in direction of initial n
 - ▶ σ_z : RMS transverse to initial n



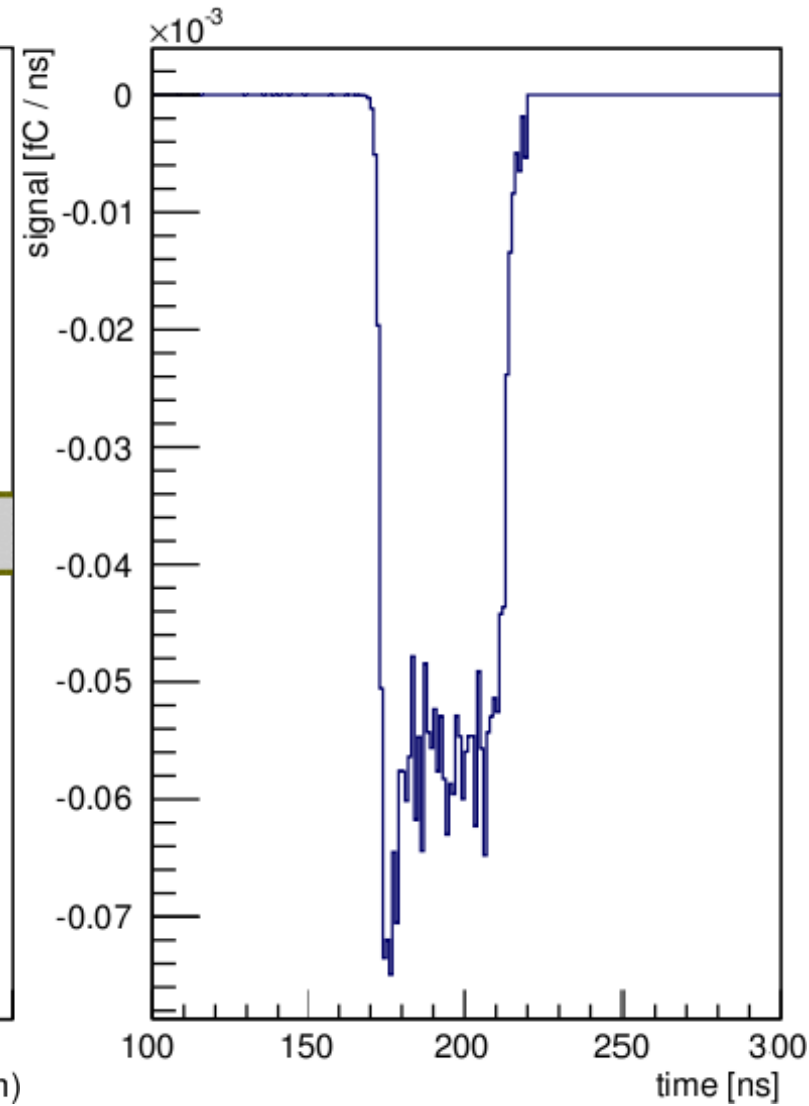
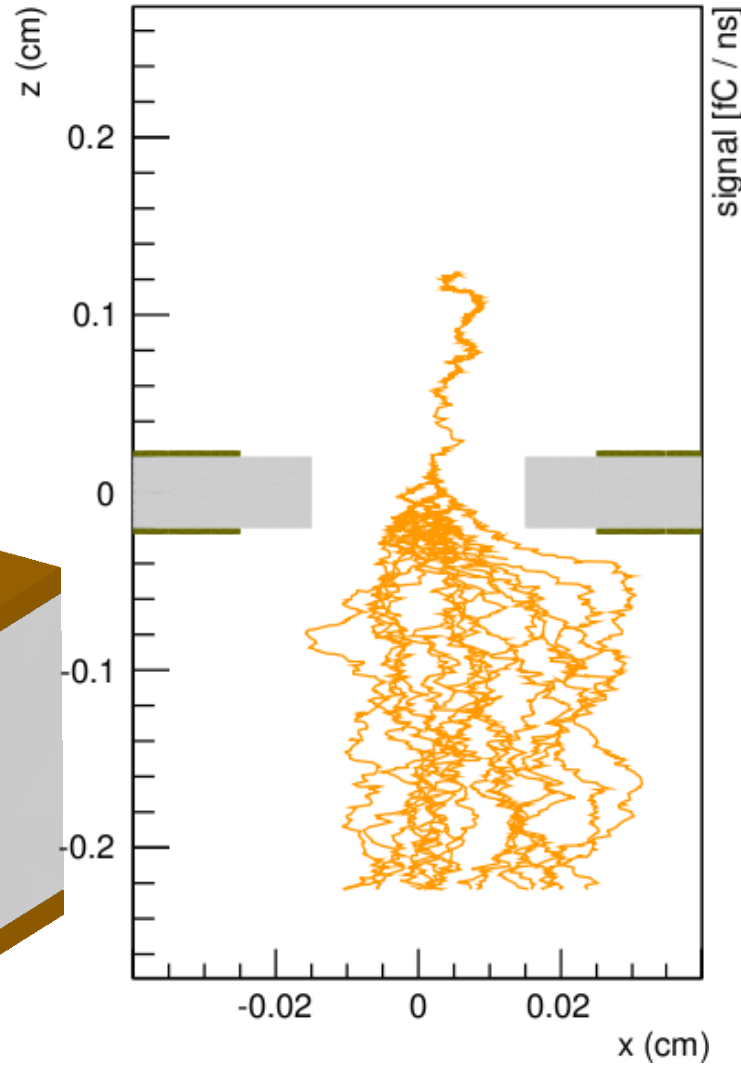
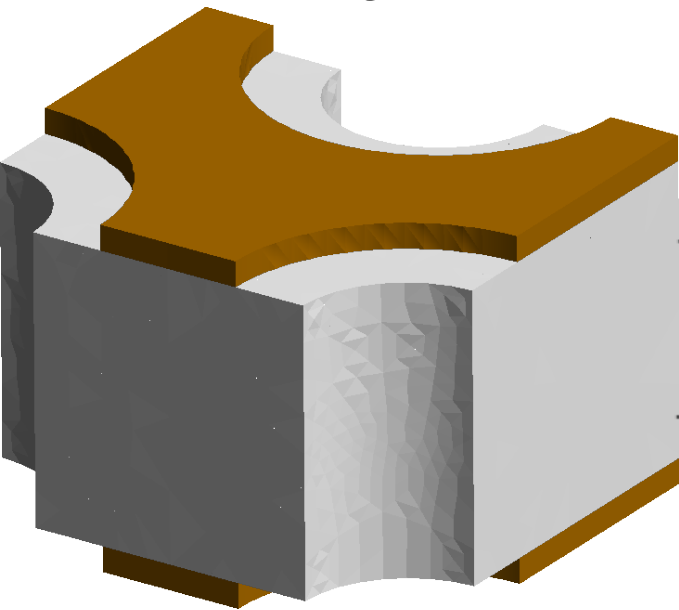
Practical range: distance at which the tangent through the inflection point of the descending portion of the depth- absorbed dose curve meets the extrapolation of the Bremsstrahlung background (ICRU report 35, 1984)

Field calculation techniques

- ▶ Closed expressions:
 - ▶ almost all 2d structures of wires, planes + periodicities;
 - ▶ dielectrics and space/surface charge are laborious;
 - ▶ fast and precise, if applicable – not suitable for MPGDs.
- ▶ **Finite elements:**
 - ▶ 2d and 3d structures, with or without dielectrics;
 - ▶ several major intrinsic shortcomings.
- ▶ Integral equations or **Boundary element methods:**
 - ▶ equally comprehensive without the intrinsic flaws;
 - ▶ technically challenging and emerging.
- ▶ Finite differences:
 - ▶ used for iterative, time-dependent calculations.

Gmsh/Elmer example

- ▶ Thick GEM
- ▶ unit cell,
- ▶ avalanche,
- ▶ signal.



The Degrad model

- ▶ Atomic/molecular cascade:

- ▶ Auger and Coster-Kronig decay;
- ▶ fluorescence;
- ▶ outer shell electron shake off.

Xe: innermost 17 shells

- ▶ Photon absorption:

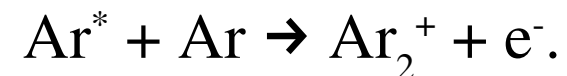
- ▶ photoelectric effect;
- ▶ Compton scattering.

- ▶ Electron scattering:

- ▶ rotational, vibrational, excitation and ionisation scattering.

- ▶ Atomic de-excitation:

- ▶ Penning;
- ▶ Hornbeck-Molnar.

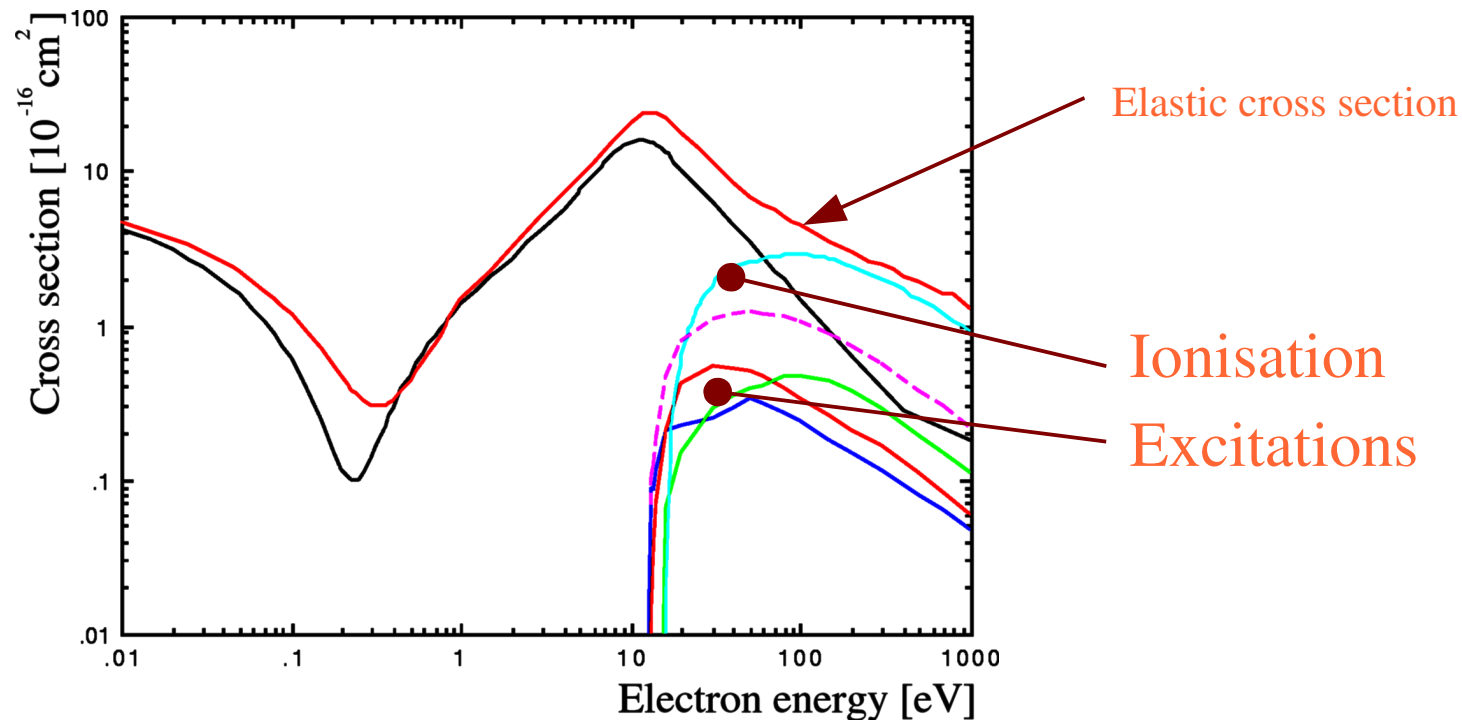


- ▶ Not included:

- ▶ Bremsstrahlung, pair production.

Cross section of argon

- ▶ Cross section in a hard-sphere model:
 - ▶ Radius: ~ 70 pm (<http://www.webelements.com>)
 - ▶ Surface: $\sigma = \pi (70 \cdot 10^{-10} \text{ cm})^2 \approx 1.5 \cdot 10^{-16} \text{ cm}^2$
- ▶ Simplified cross sections used by Magboltz:



Mean free path in argon

▶ Given:

▶ Cross section of 1 atom: $\sigma \approx 1.5 \cdot 10^{-16} \text{ cm}^2$

▶ Atoms per volume: $\mathcal{L} \approx 2.5 \cdot 10^{19} \text{ atoms/cm}^3$

▶ Mean free path for an electron ?

▶ An electron hits all atoms of which the centre is less than a cross section radius from its path

▶ Over a distance L , the electron hits $\mathcal{L} \sigma L$ atoms

▶ Hence, the mean free path is $\lambda_e = 1/(\mathcal{L} \sigma) \approx 2.7 \mu\text{m}$

▶ Much larger than the distance between atoms, 3.5 nm and typical gas molecule diameters, 140-600 pm.

Drift velocity in electric fields

► Imagine that an electron stops every time it collides with a gas molecule and then continues along E .

► To cover a distance λ_e , it will need a time t :

$$\frac{1}{2} \frac{q E}{m_e} t^2 = \lambda_e, \quad \text{i.e.} \quad t = \sqrt{\frac{2 \lambda_e m_e}{q E}}, \quad \text{i.e.} \quad \bar{v} = \frac{\lambda_e}{t} = \sqrt{\frac{\lambda_e q E}{2 m_e}}$$

► For example:

$$\bar{v} \approx 13 \text{ cm}/\mu\text{s} \quad \text{for } E = 1 \text{ kV/cm}$$

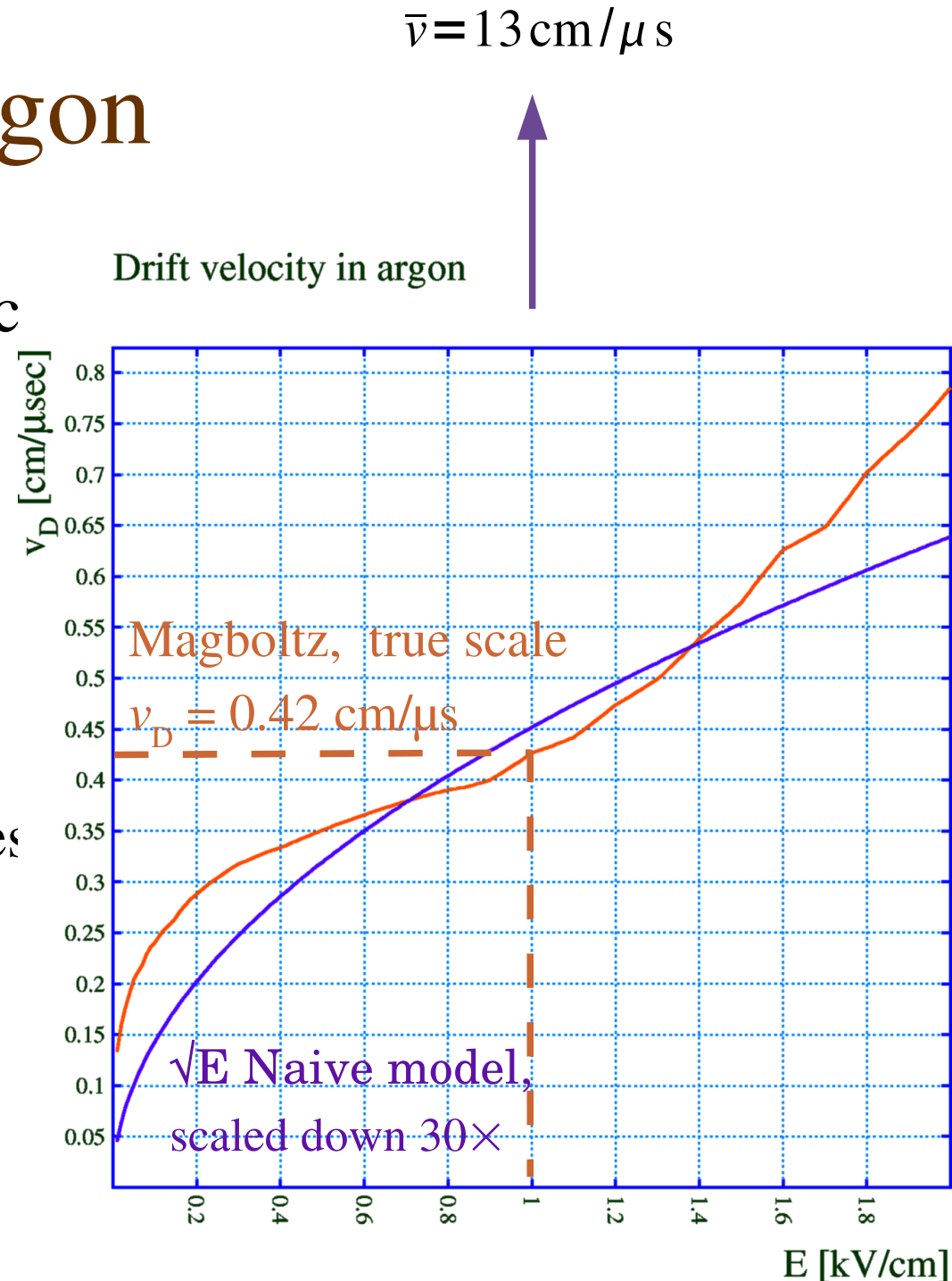
Drift velocity in argon

▶ Compare with a Magboltz c

▶ E dependence is OK;

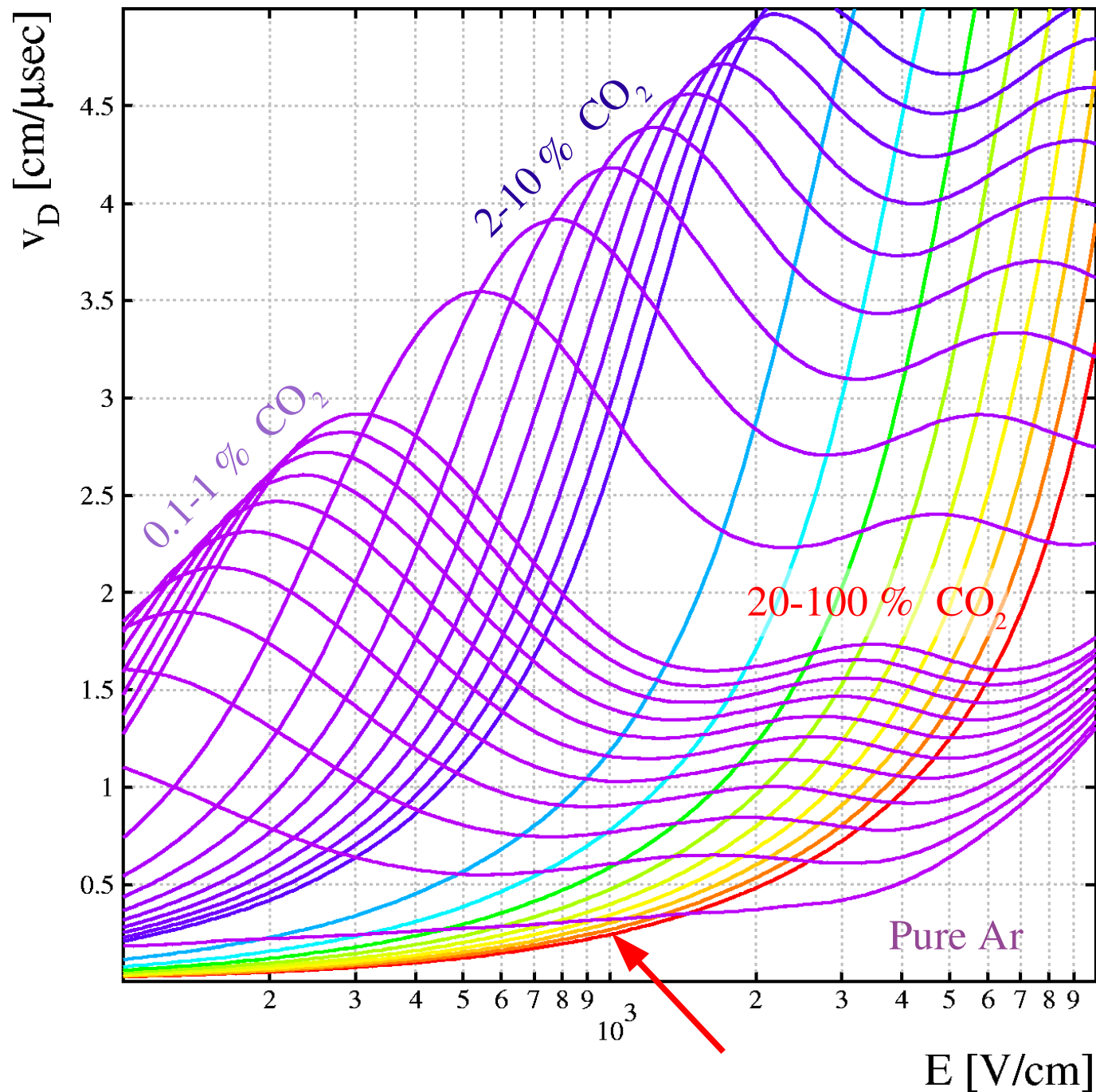
BUT

▶ the velocity is *vastly* overes

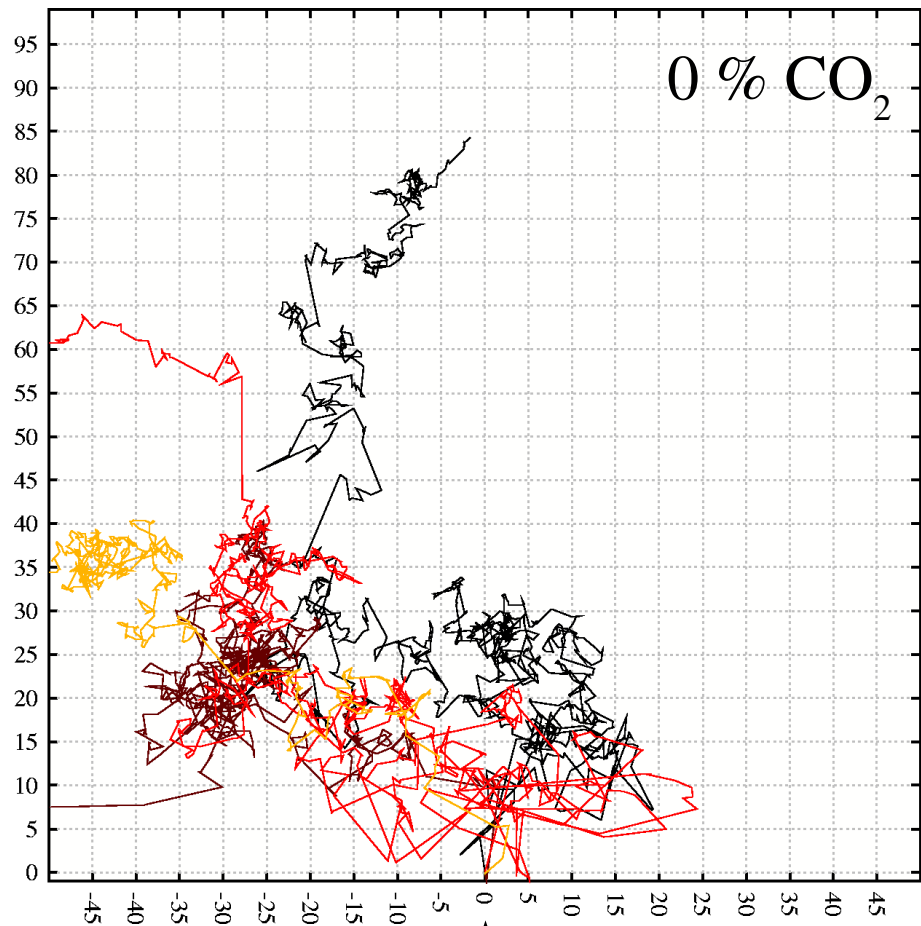


Adding CO₂

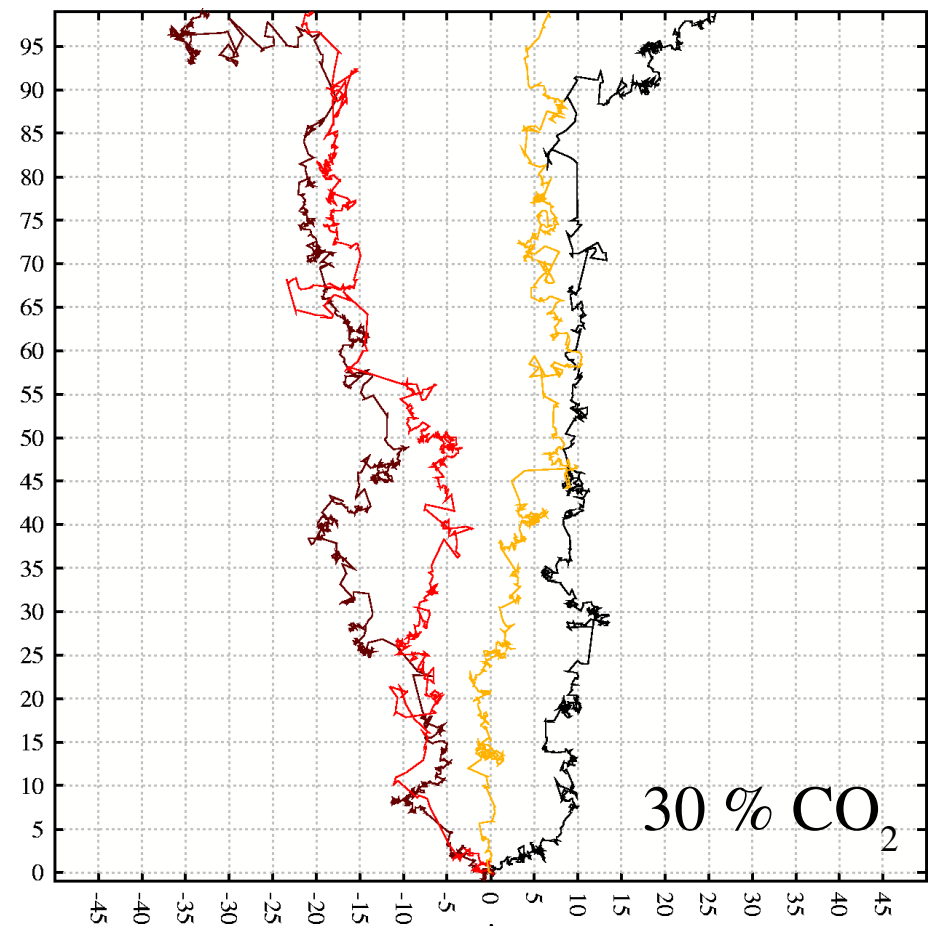
- ▶ CO₂ makes the gas faster, dramatically.
- ▶ Calculated by Magboltz for Ar/CO₂ at 3 bar.



Electrons in Ar/CO₂ at $E=1$ kV/cm



Starting point



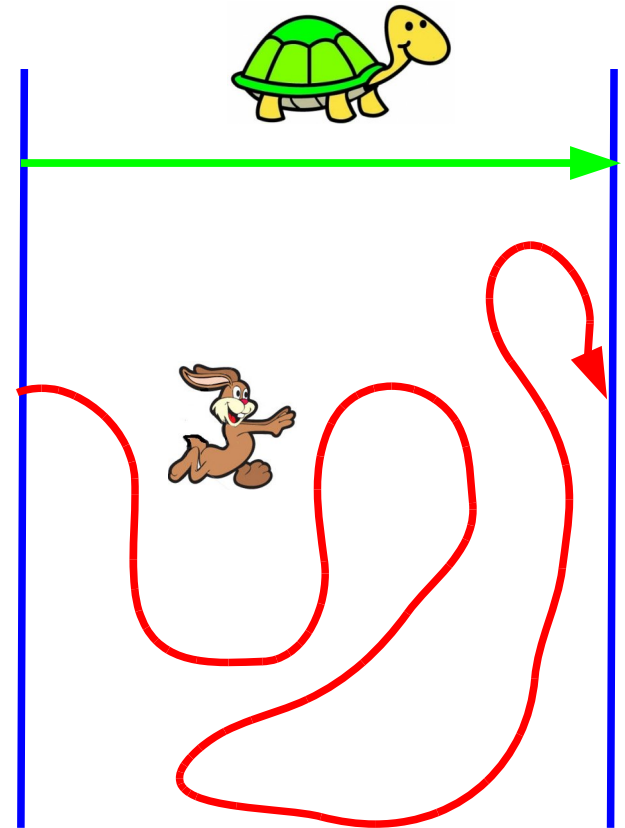
Starting point

Drift velocity vs Mean velocity

- ▶ Drift velocity: distance effectively travelled divided by time needed.
- ▶ Imagine they take equal time:

$$v_D = \bar{v}$$

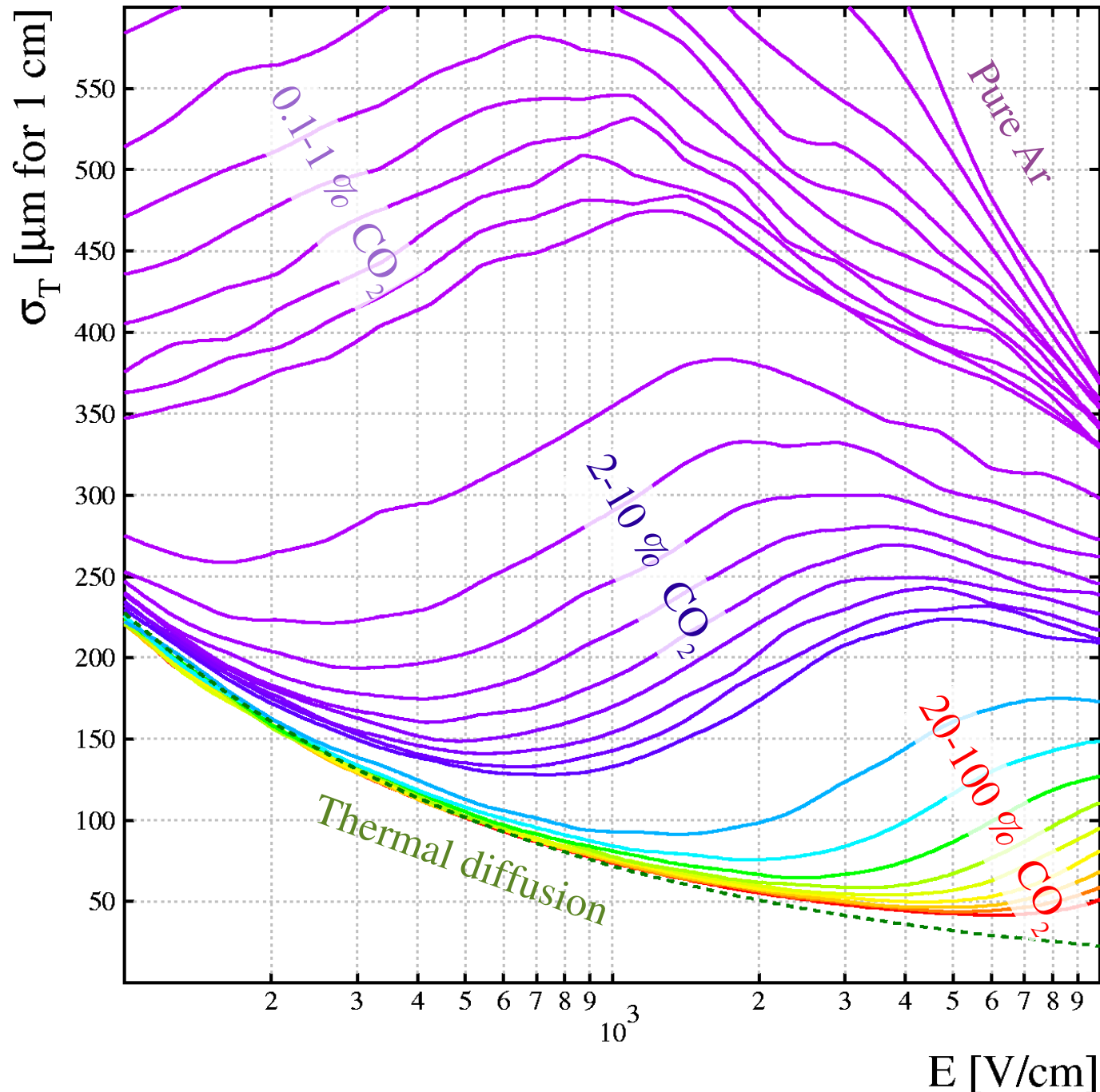
$$v_D \ll \bar{v}$$



Adding CO₂

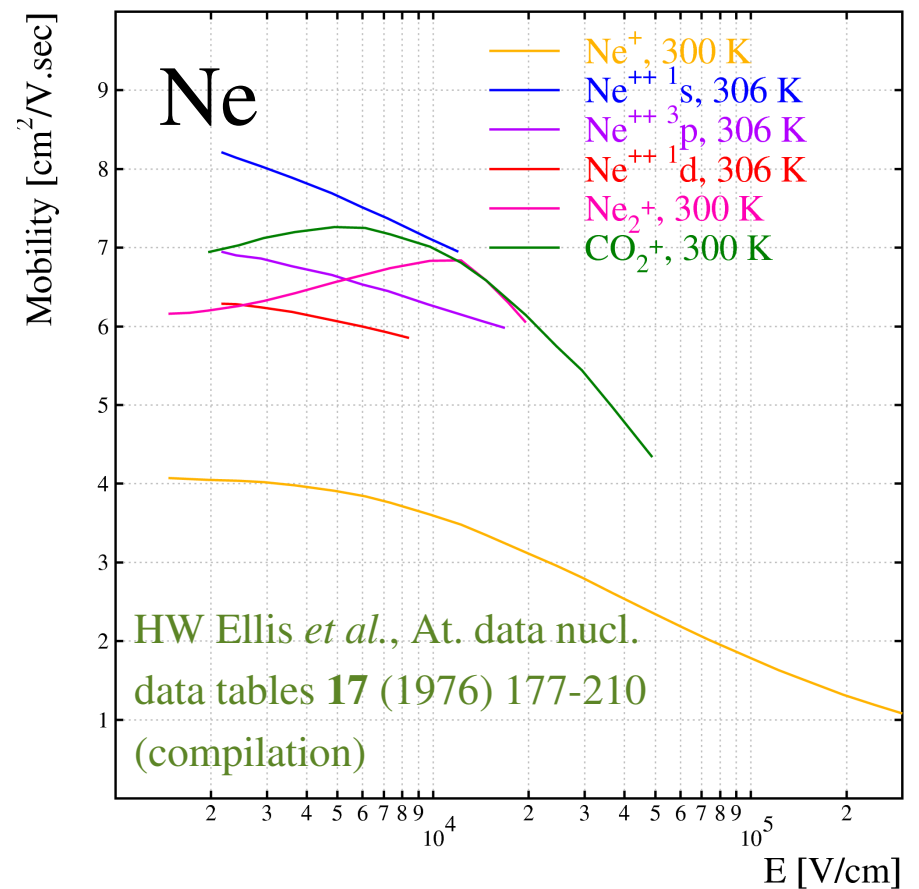
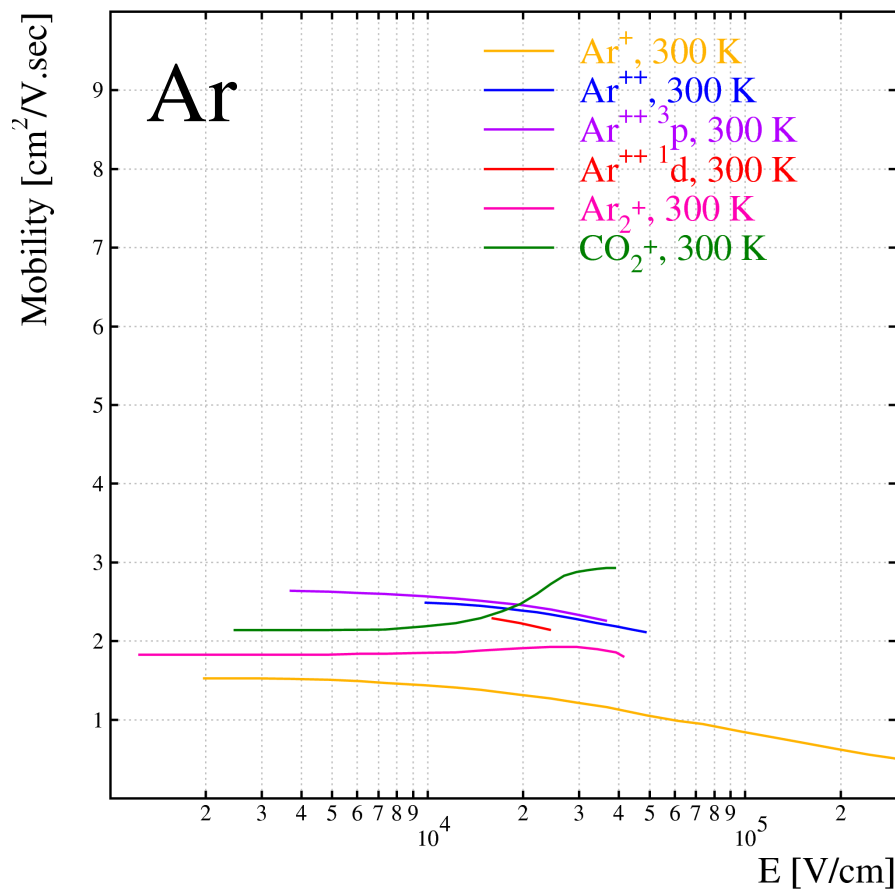
▶ Transverse diffusion is much reduced by CO₂.

▶ Calculated by Magboltz for Ar/CO₂ at 3 bar.



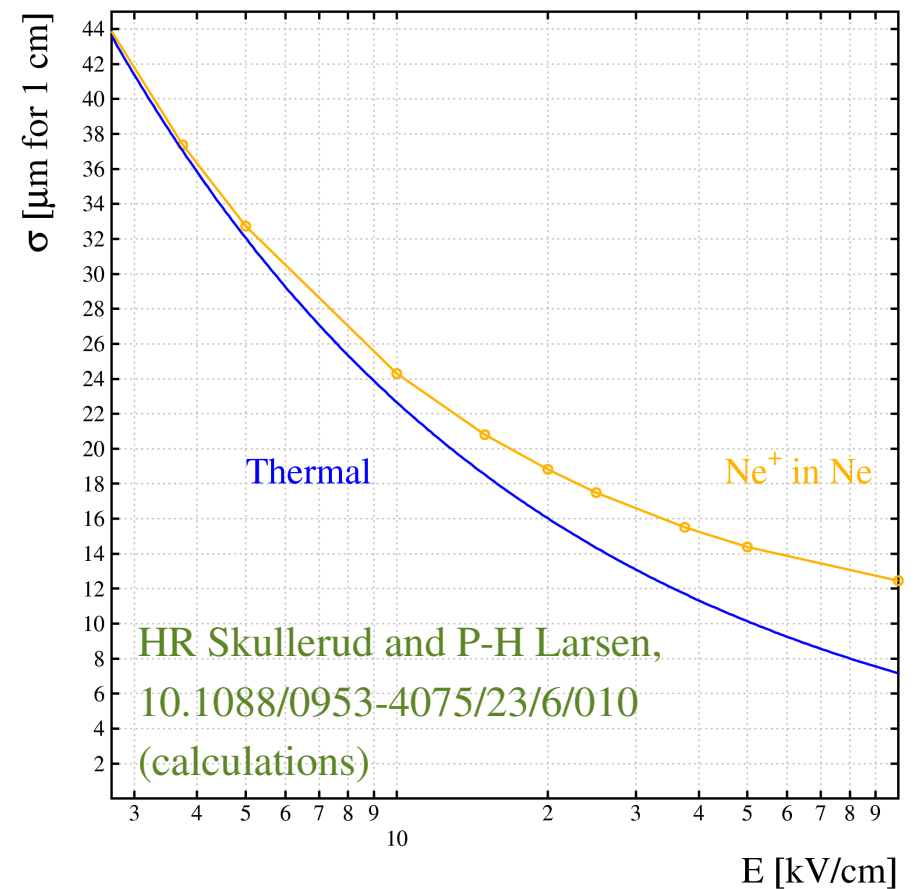
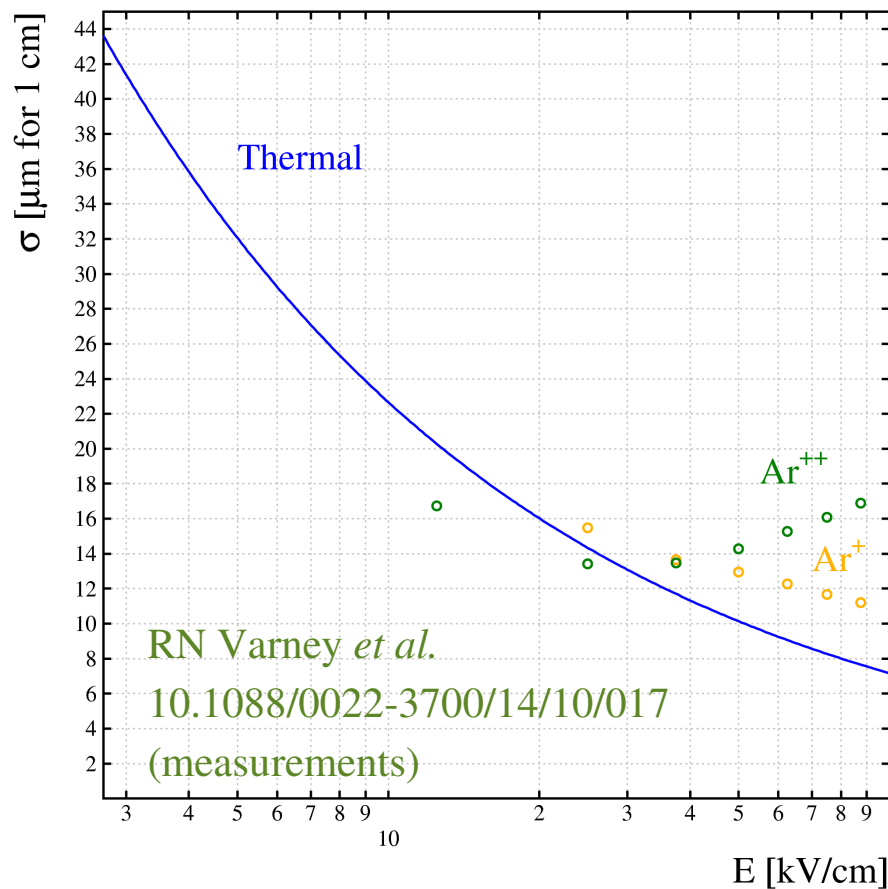
Ar⁺ and Ne⁺ mobility

- ▶ Avalanches take a few ns: <http://cern.ch/garfieldpp/examples/gemgain>
- ▶ Ion velocity at 3 kV/cm: Ar: ~20 μs/mm, Ne: ~8 μs/mm



Diffusion of Ar^+ in Ar and Ne^+ in Ne

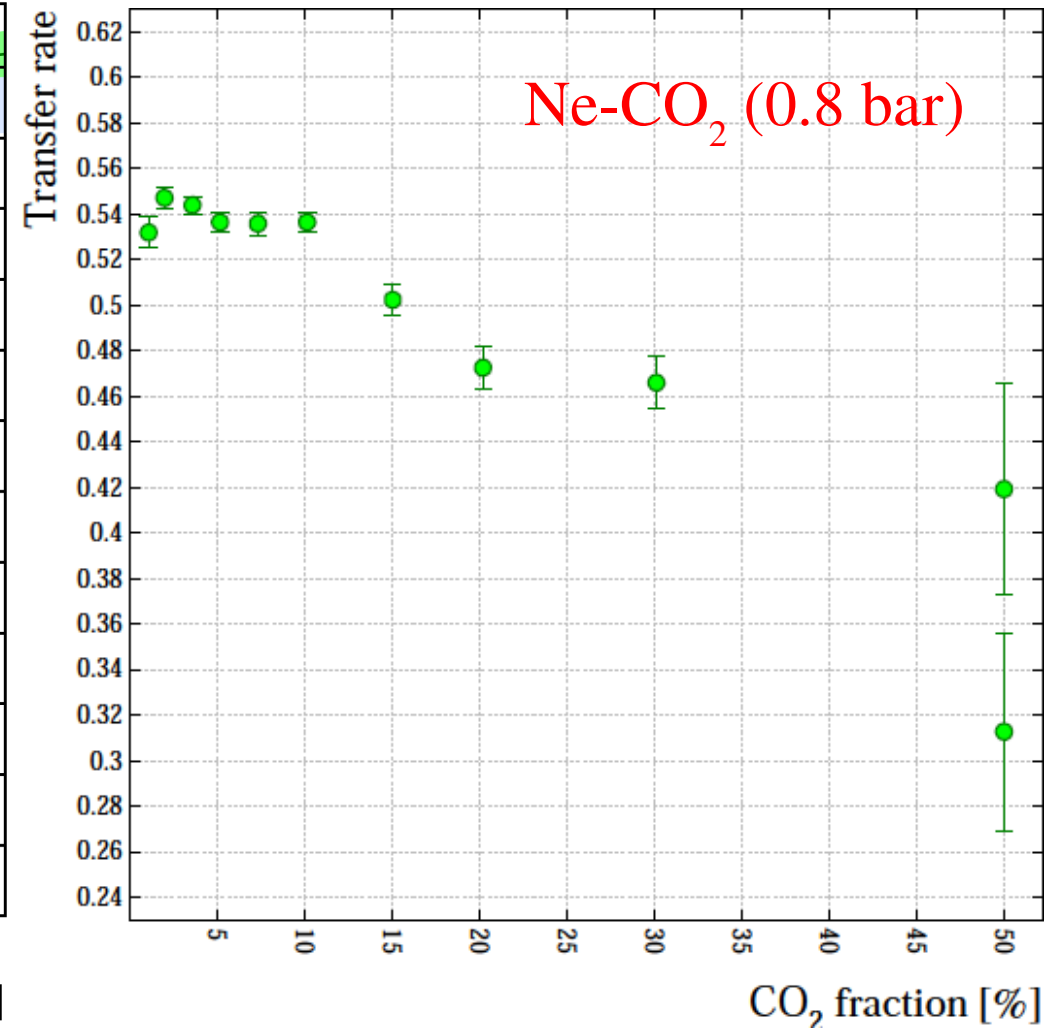
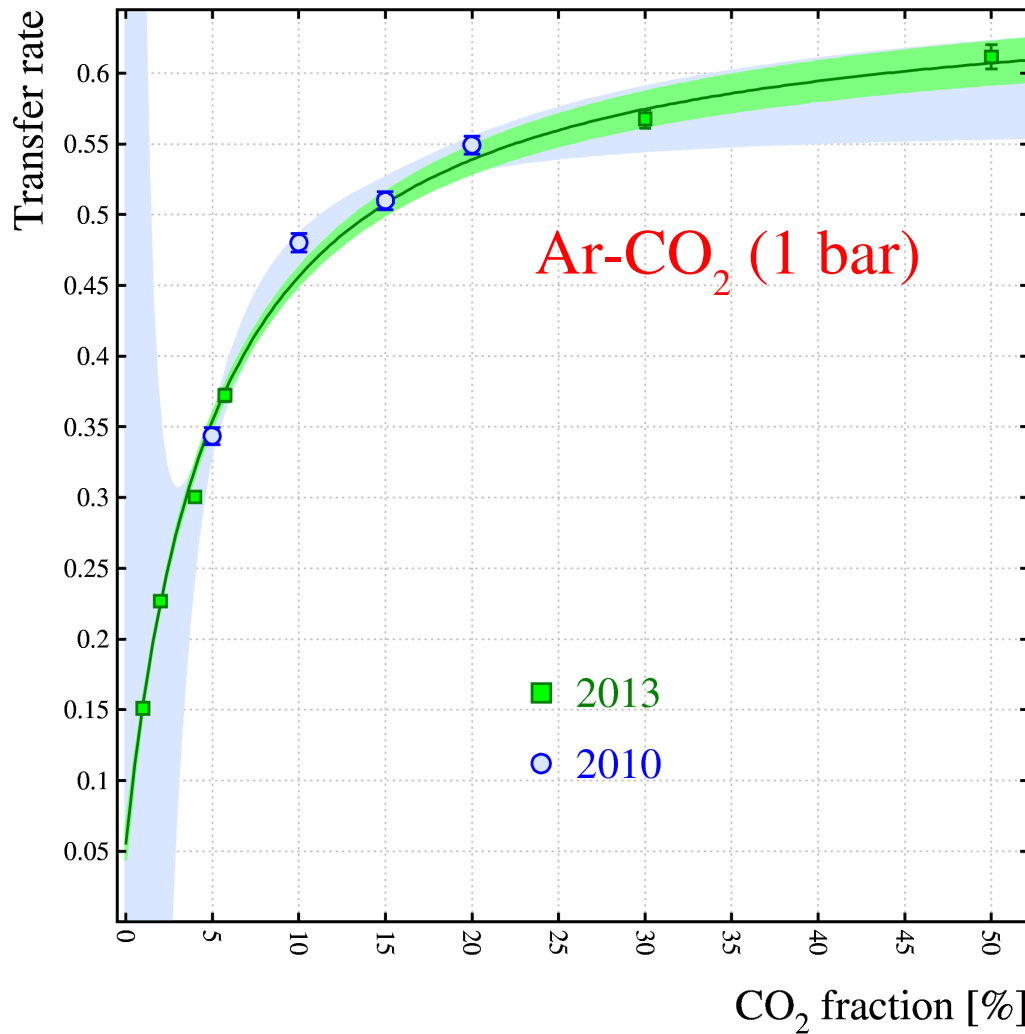
- ▶ Little experimental data, in particular at low fields.
extrapolated from higher fields: $\sim 10 \mu\text{m}$ for 1 mm



PhD students: solve this puzzle and win
a fondue during your next visit to CERN.

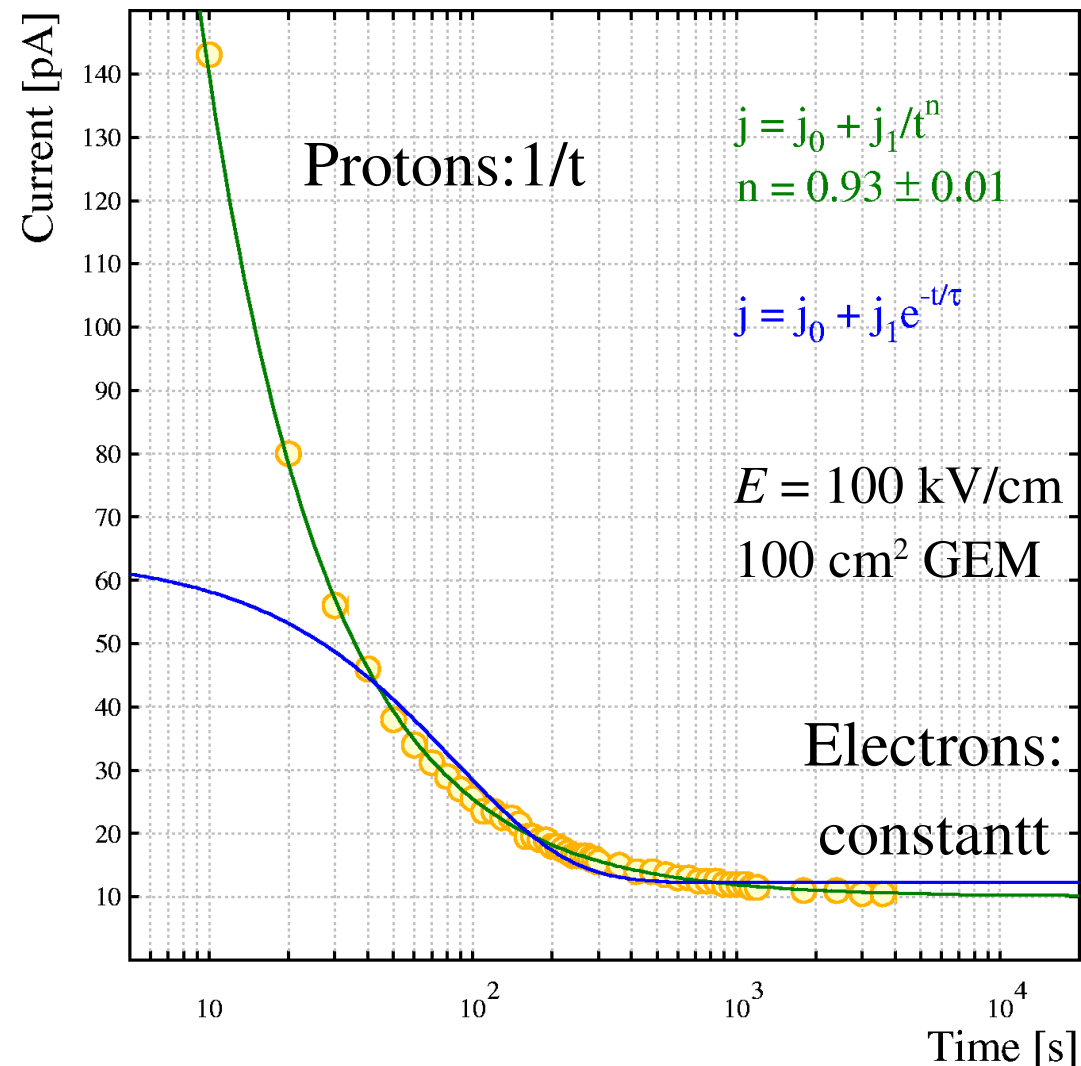
Why does the rate fall in Ne-CO₂ ?

► Ar-CO₂ is best studied mixture



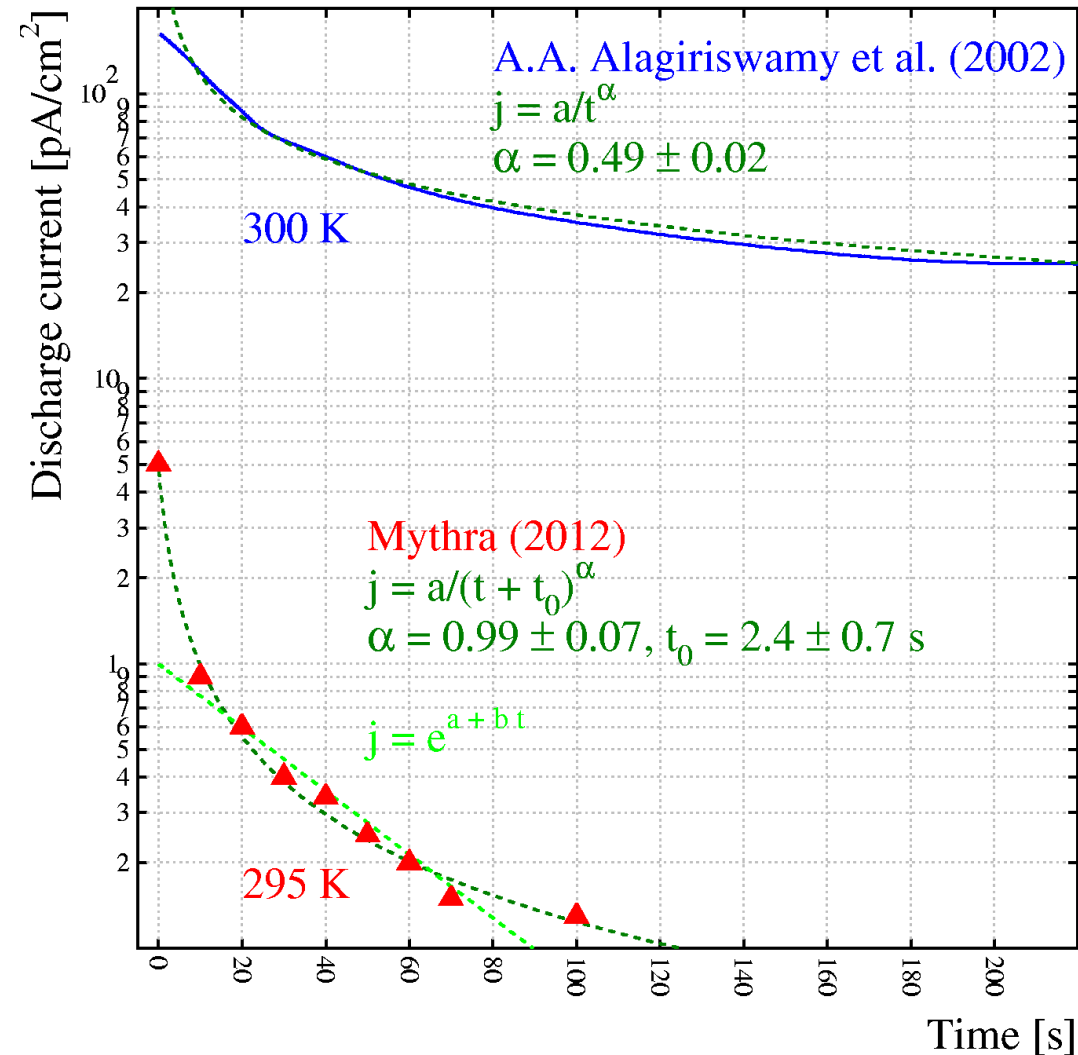
Charging-up current

- ▶ When applying voltage across a new GEM, a current flows:
 - ▶ *not* constant (i.e. not a resistor)
 - ▶ decay is *not* exponential (i.e. not a capacitor);
 - ▶ decay is *not* linear (i.e. not evacuation);
 - ▶ but a *power law*.



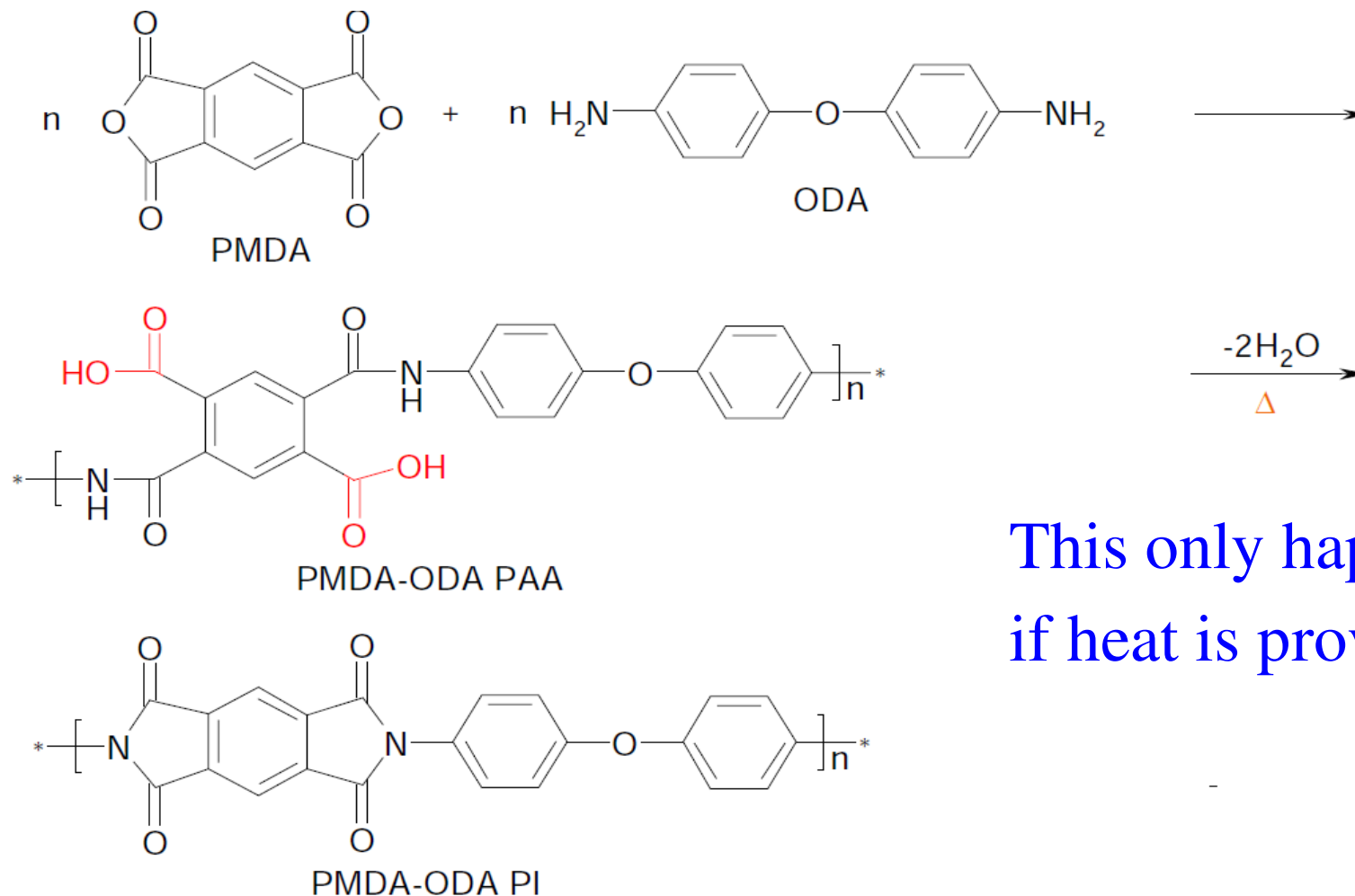
Discharge current

- ▶ The initial charge carriers stay in the polyimide, as can be seen by switching off the HV.
- ▶ The discharge current has reverse polarity and obeys a Kohlrausch law.



Protons: polyamic acid (PAA)

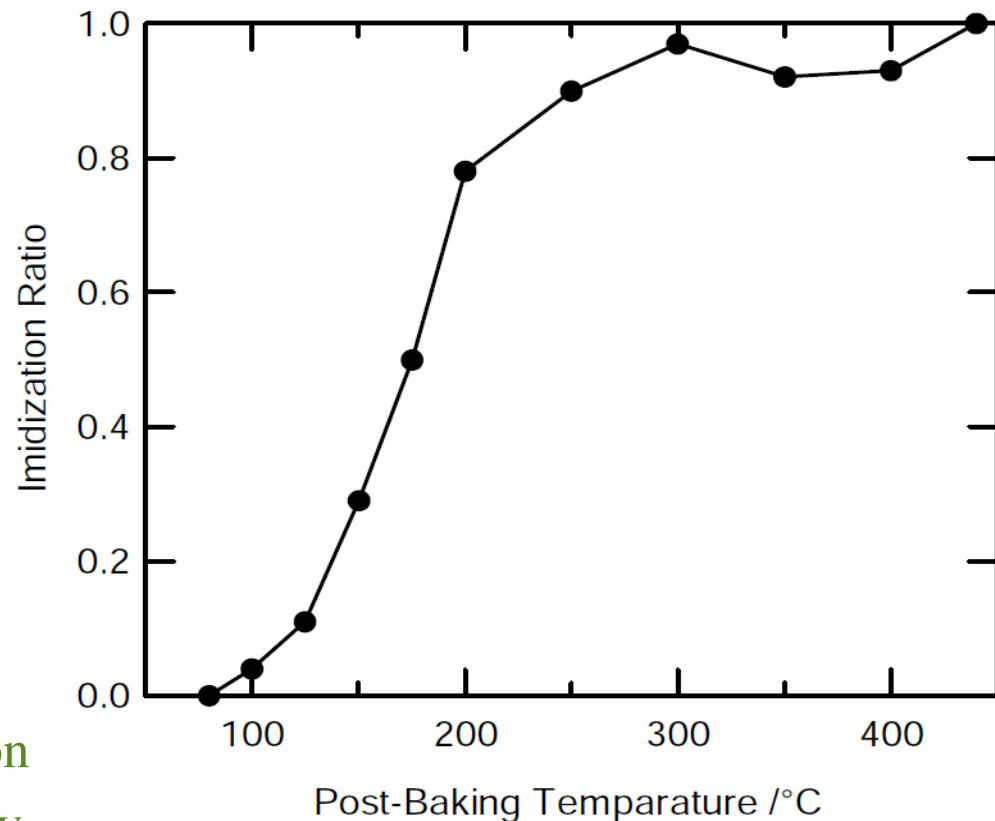
► Note the intermediate **acid**, i.e. an **H⁺** donor:



This only happens
if heat is provided.

PAA \rightarrow PI vs baking temperature

- ▶ The quantity of remaining PAA depends on the baking temperature.
- ▶ The proton density therefore also varies.
- ▶ [H. Oji *et al.*, *Memoirs of the Synchrotron Radiation Center, Ritsumeikan University, Kyoto, Japan* **8** (2006) 187-188.]



Rudolf Hermann Arndt Kohlrausch
(November 6th 1809, Göttingen -
March 8th 1858, Erlangen)



Kohlrausch relaxation

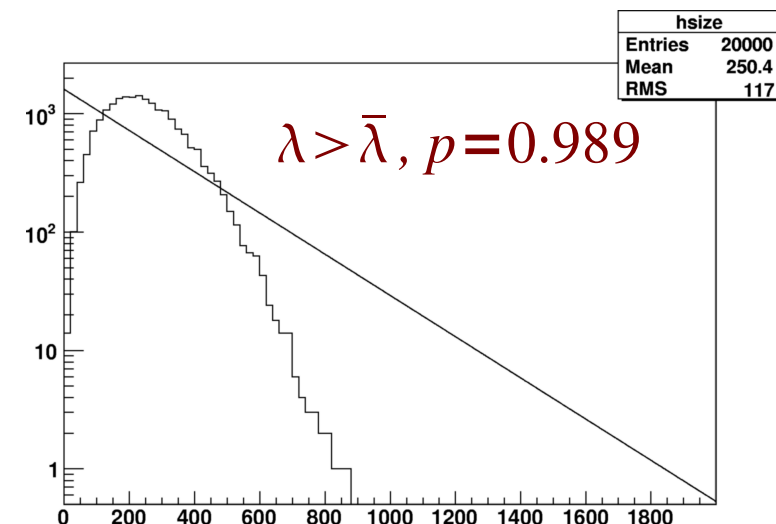
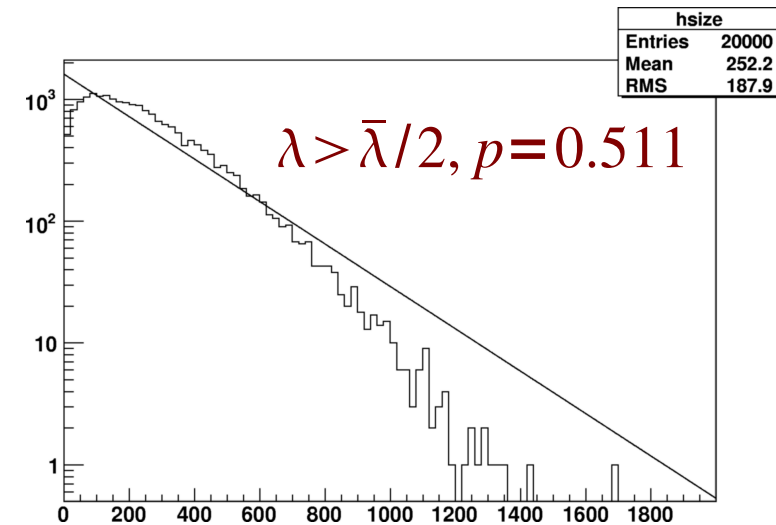
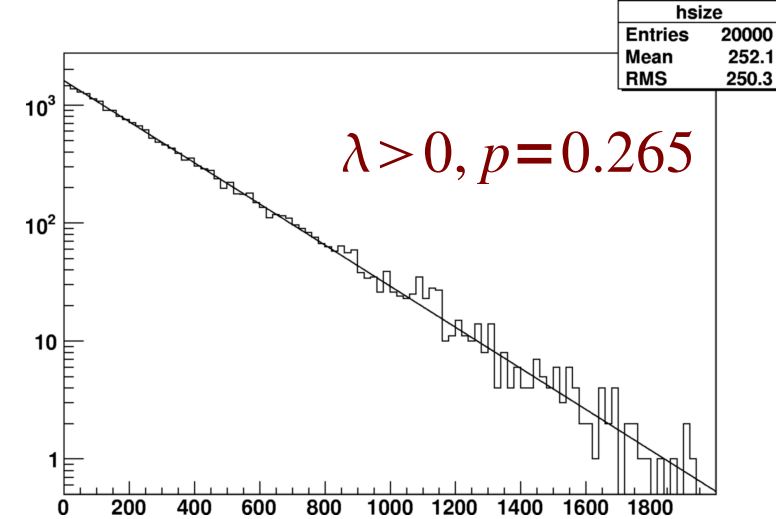
- ▶ This time dependence is known since 1854 at least. Also known as Curie-von Schweidler behaviour.
- ▶ Numerous models have been proposed
 - ▶ H. Kliem, *Kohlrausch relaxations: new aspects about the everlasting story*, doi: [10.1109/TDEI.2005.1511096](https://doi.org/10.1109/TDEI.2005.1511096).
- ▶ One of the simplest models specifically assumes ions (e.g. protons, not electrons) as charge carriers and has thin insulating barriers between dielectric medium and electrodes.

Charges in GEMs

- ▶ In GEMs, active gas comes in contact with dielectrics; in breach of a fundamental law of gas-based devices. This results in charge accumulations on the plastic which distort the field.
- ▶ Space charge affects ion back flow.
- ▶ Polyimide can contain mobile charges. These migrate through the plastic and modify the field.

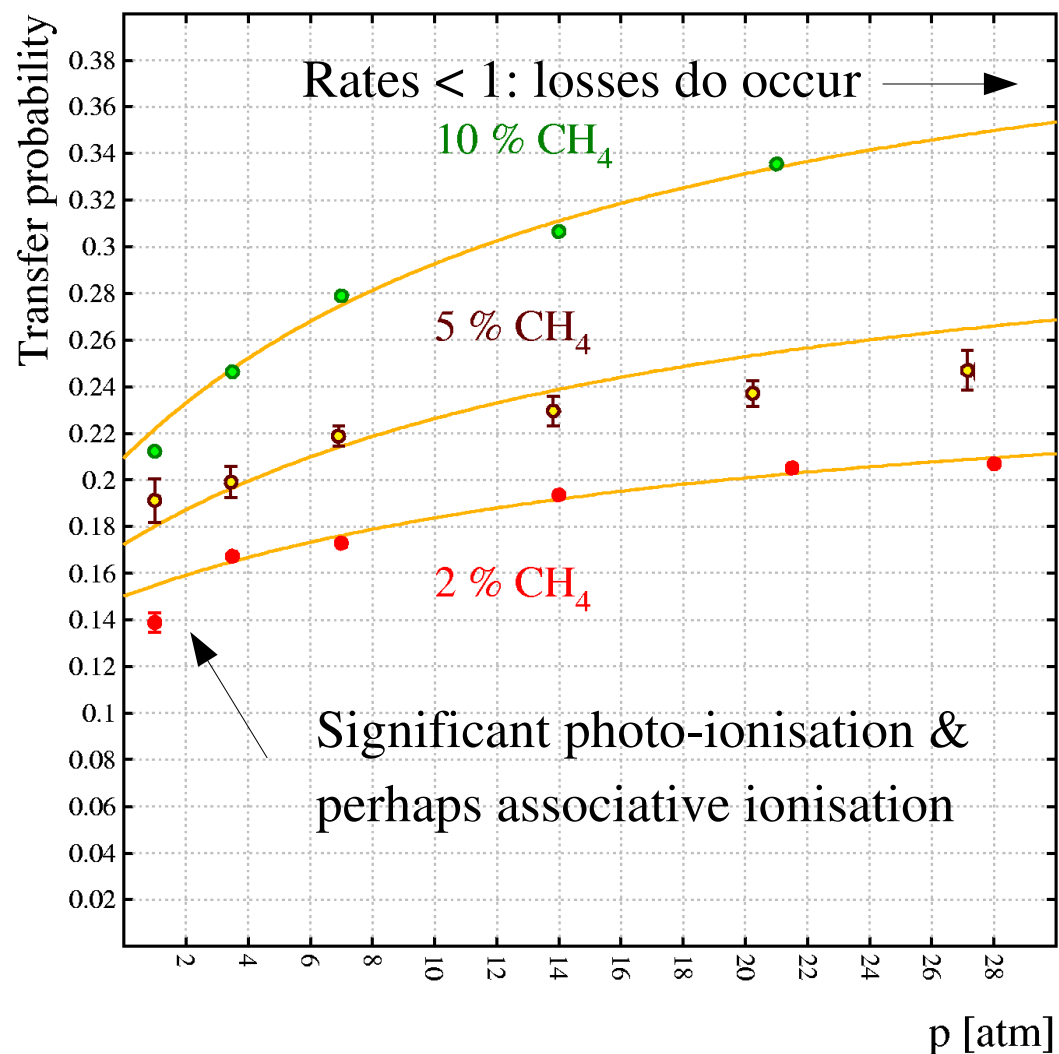
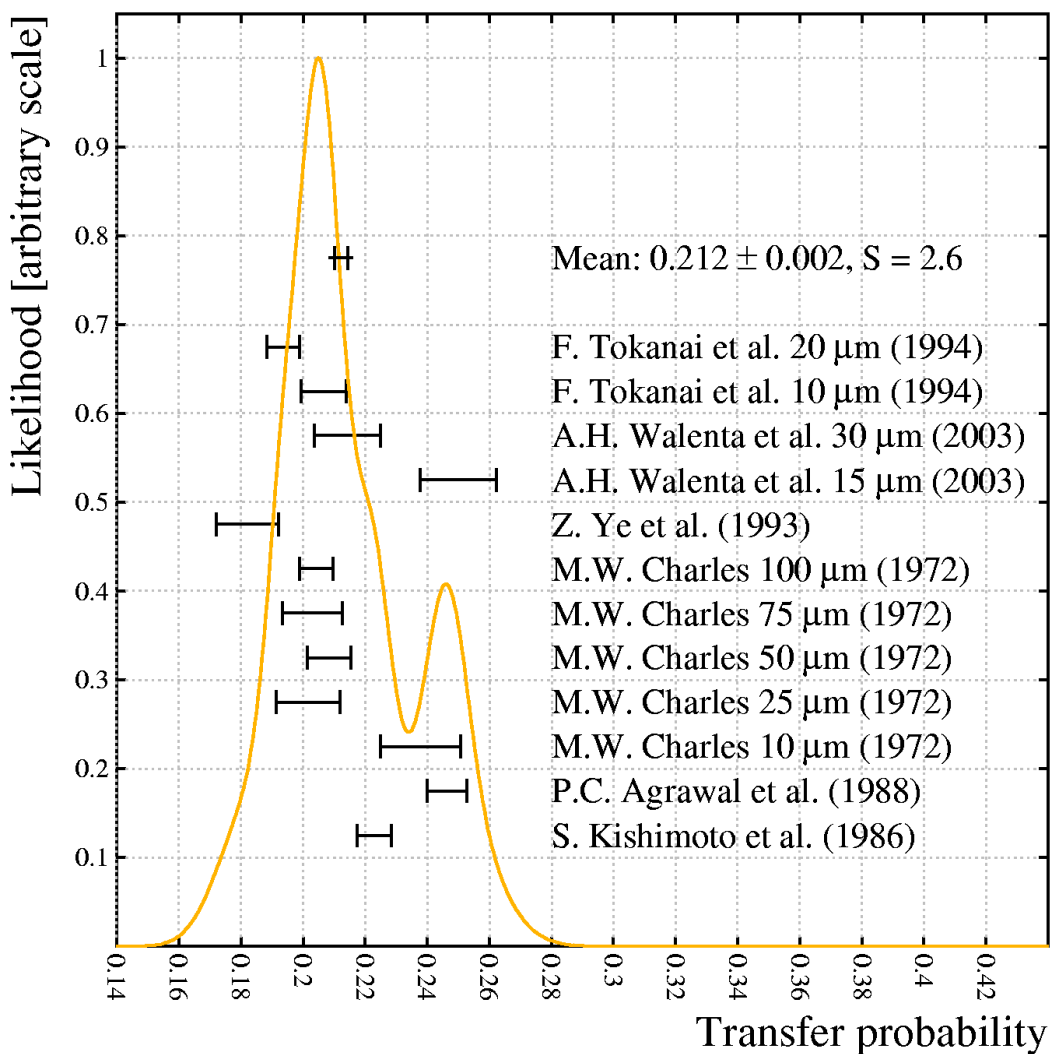
Minimum step length

- ▶ Exponential not affected by:
 - ▶ steps with an exponentially distributed length, nor
 - ▶ ionisation probability proportional to the length of the step,
- ▶ But ... a minimum step length for ionisation leads to radical changes.
- ▶ Illustrated here for a drift path with only 20 steps, $d = 20\bar{\lambda}$.

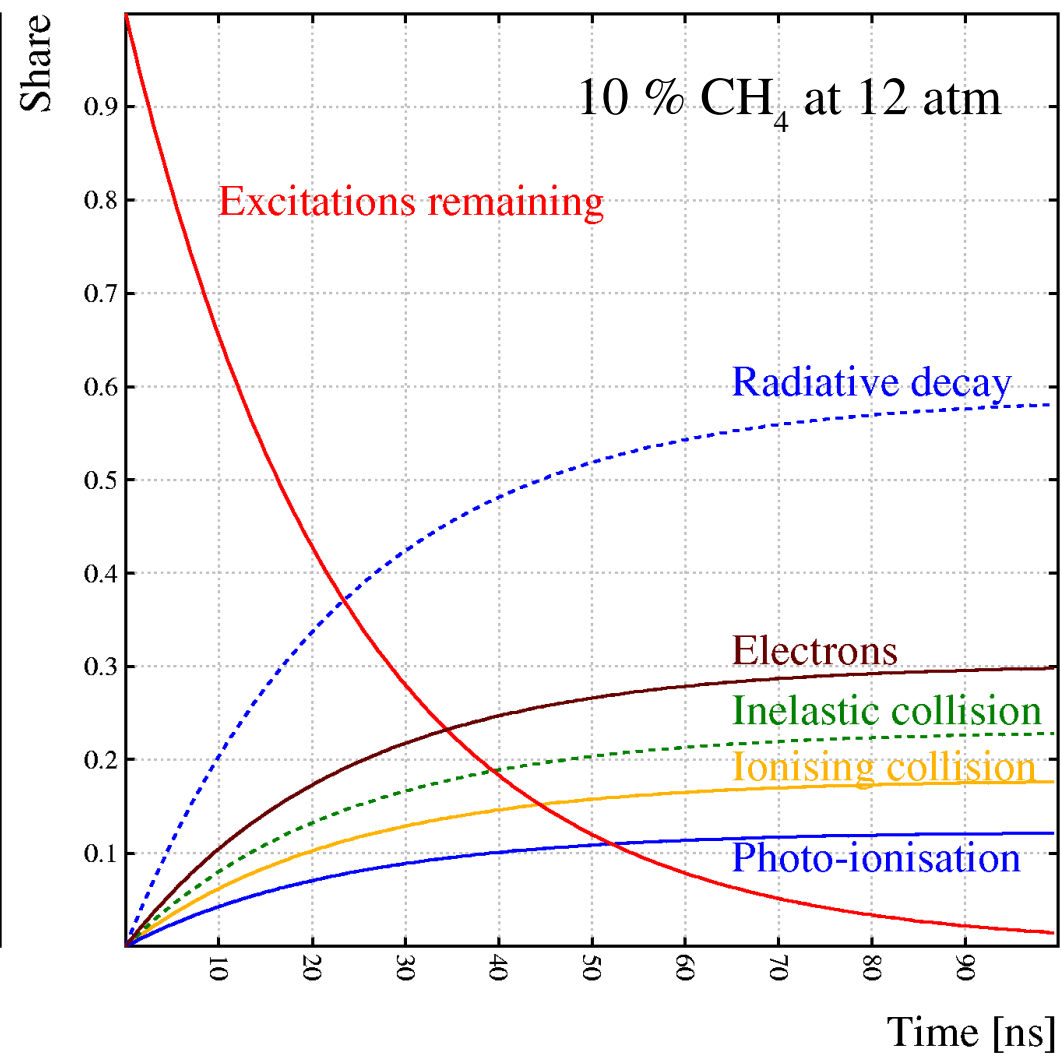
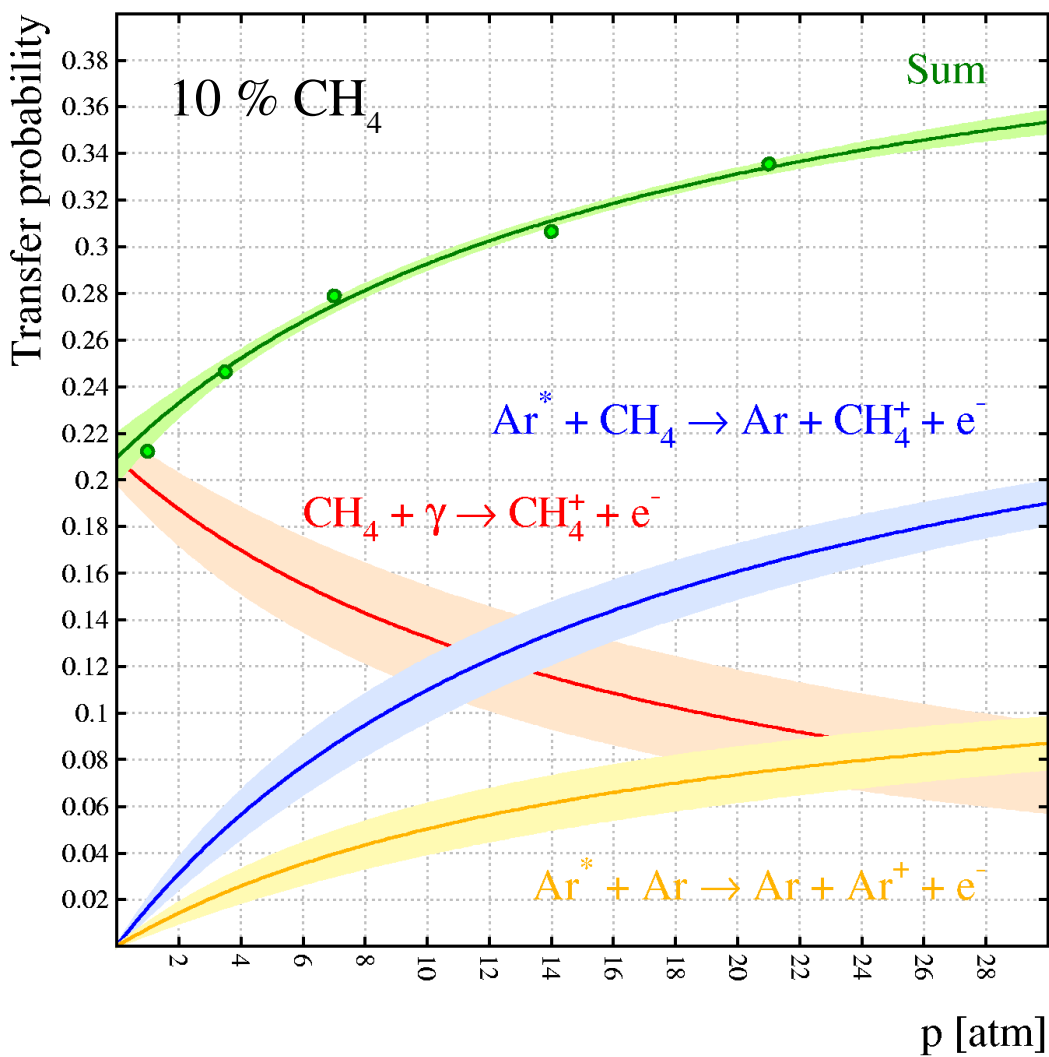


Ar-CH₄

► Large numbers of gain curves are available.



Ar-CH₄: processes and timing



Towards a faster neBEM

- ▶ Optimisation has made the program 75 % faster.
- ▶ Introduction of a “fast volume” where the field is
 - ▶ pre-computed on a mesh;
 - ▶ interpolated (finite element style).
- ▶ Outside, the field is computed as is done now.



Drift field (V/cm)	Transparency Exact computation	Transparency Fast algorithm	Gain (RKF) Exact Computation	Gain (RKF) Fast algorithm
200	99.835	99.864	250.133	250.139
400	99.669	99.471		
750	96.364	95.774		
1500	70.248	71.214		
2000	59.669	58.791		
	Time taken for 500 electrons ~150 hours	Time taken for 20,000 electrons ~ 25 hours	Time ~ 40 minutes	Time ~ less than a minute

Level diagram argon and admixtures

