GPDs & spatial structure of hadrons

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Outline

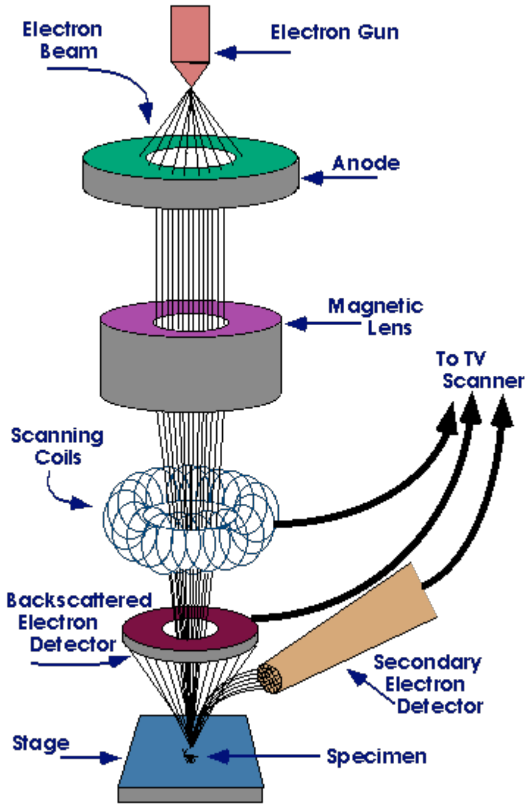
➡ lectures 1 & 2:

- What do we know about the spatial structure of hadrons from elastic form factors ?
- How do we define an imaging of hadrons in QFT ?
- What do we know about quark charge densities in hadrons ?

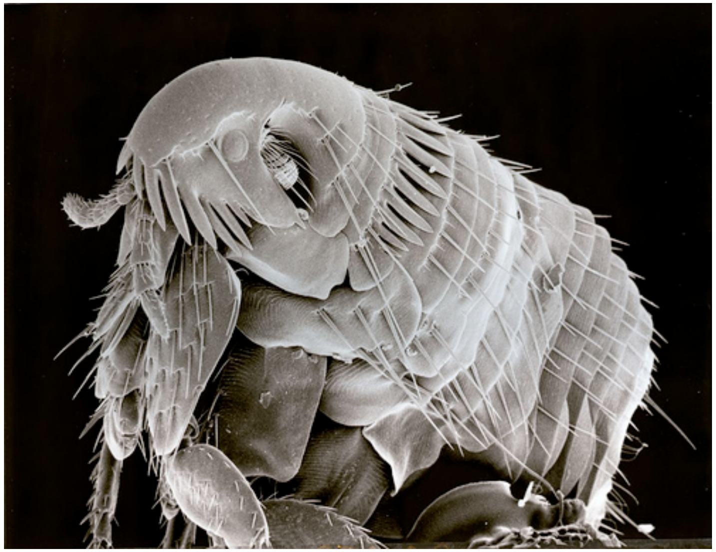
➡ lectures 3 & 4:

- What are Generalized Parton Distributions (GPDs) and what is the physics contained in them ?
- How can GPDs shed a new light on nucleon spin ?
- How can we access GPDs in experiment ?

spatial distributions and form factors

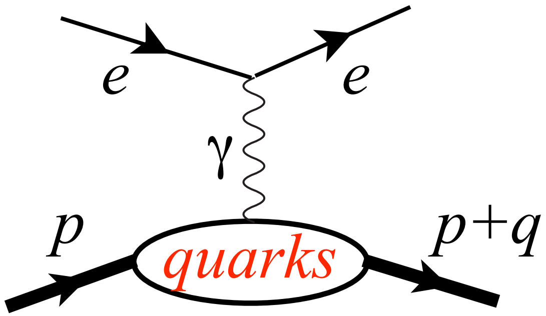


Electron microscopy



when the target is static ($m_{constituent}$, m_{target} >> Q)

-> the 3dim Fourier transform of the form factors gives the spatial distribution of electric charge and magnetization

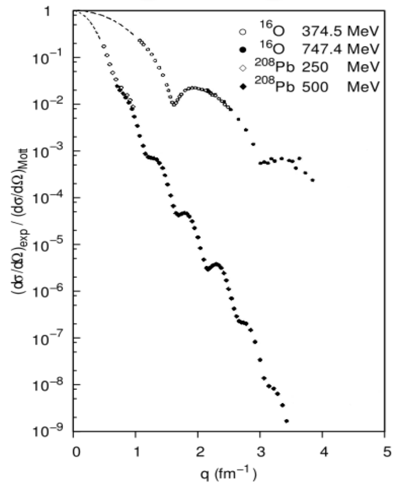


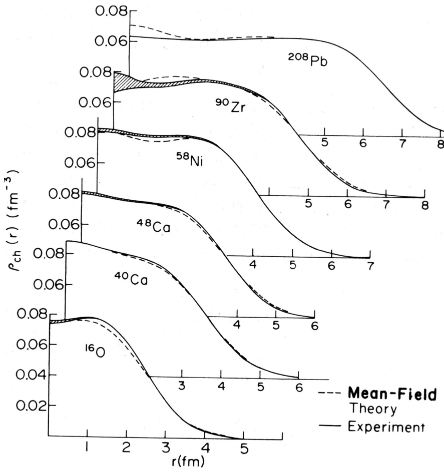
size and shape of non-relativistic many-body systems

Sizes of nuclei

as revealed through

elastic electron scattering

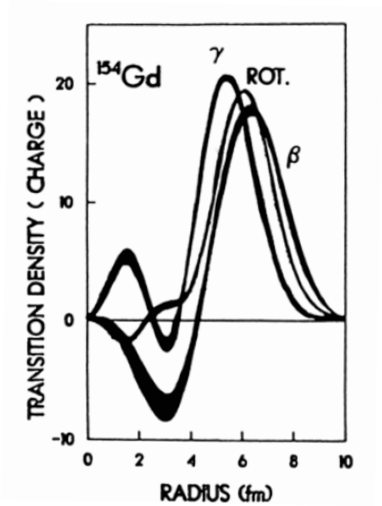


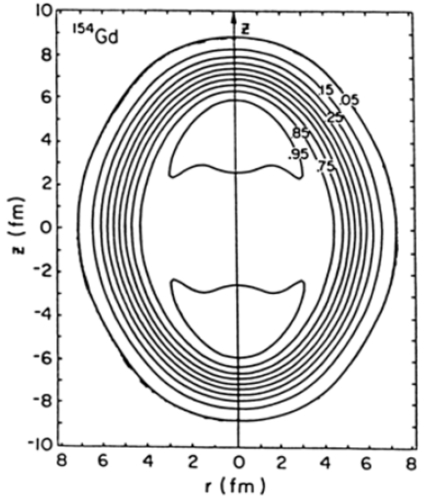


Shapes of deformed nuclei

as revealed through

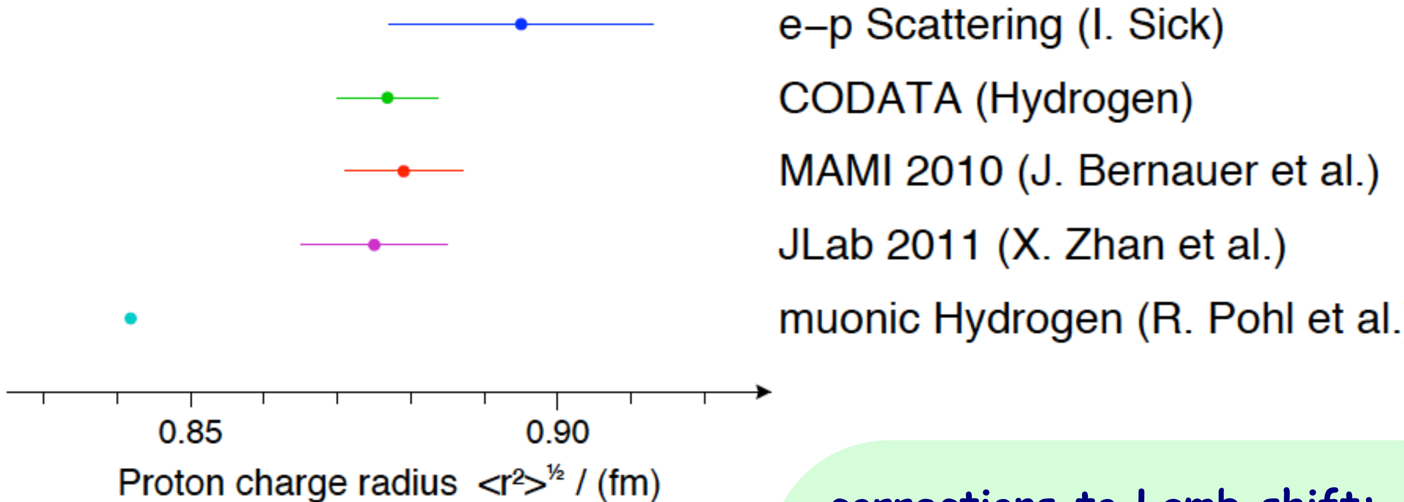
inelastic electron scattering





perspective on "Shape of Hadrons" : Alexandrou, Papanicolas, Vdh (2011)

size of proton : electric charge radius



μH data :
Pohl et al.

RE = 0.8418 ± 0.0007 fm

7.7σ difference !?

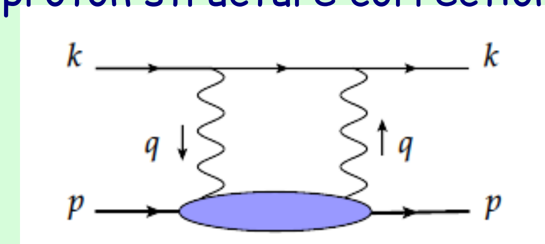
ep-data :
CODATA,
Bernauer et al.,
Zhan et al.

RE = 0.8772 ± 0.0046 fm

see also Hill, Paz
(z-exp) : RE = 0.871 ± 0.009 ± 0.002 ± 0.002 fm

corrections to Lamb shift:
300 μeV below expectation

proton structure corrections:

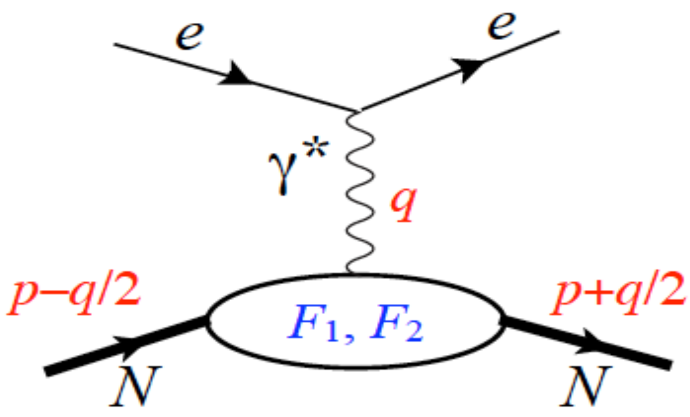


ΔE = - 35.1 μeV — Pachucki (1999)

ΔE = - 34.5 μeV — Martynenko (2006)

ΔE = (- 36.9 ± 2.4) μeV — Carlson, Vdh (2011)

spin-1/2 electromagnetic form factors



- Elastic e p -> e p scattering is like an electron microscope to investigate nucleon structure

- In 1-photon exchange approximation : nucleon structure parameterized by 2 form factors

$$A^{\mu}_{\lambda\lambda'} = \langle p + \tfrac{1}{2}q, \lambda' | J^{\mu}(0) | p - \tfrac{1}{2}q, \lambda \rangle$$

$$= \bar{u}(p + \tfrac{1}{2}q, \lambda') \left[F_1(Q^2)\gamma^{\mu} + F_2(Q^2) \frac{i}{2m} \sigma^{\mu\nu} q_{\nu} \right] u(p - \tfrac{1}{2}q, \lambda)$$

Dirac **Pauli**

F_1 helicity conserving , F_2 helicity flip form factors

- Alternatively, the Sachs form factors

$G_E(Q^2) = F_1(Q^2) - \tau\, F_2(Q^2)$, $G_M(Q^2) = F_1(Q^2) + F_2(Q^2)$, with $\tau = Q^2/4\, m^2$

Traditionally : it is assumed that in the Breit frame, and for non-relativistic systems with m >> Q, G_E and G_M are 3-dim Fourier transforms of charge- and current distributions.

measurement of nucleon Form Factors : Rosenbluth separation method

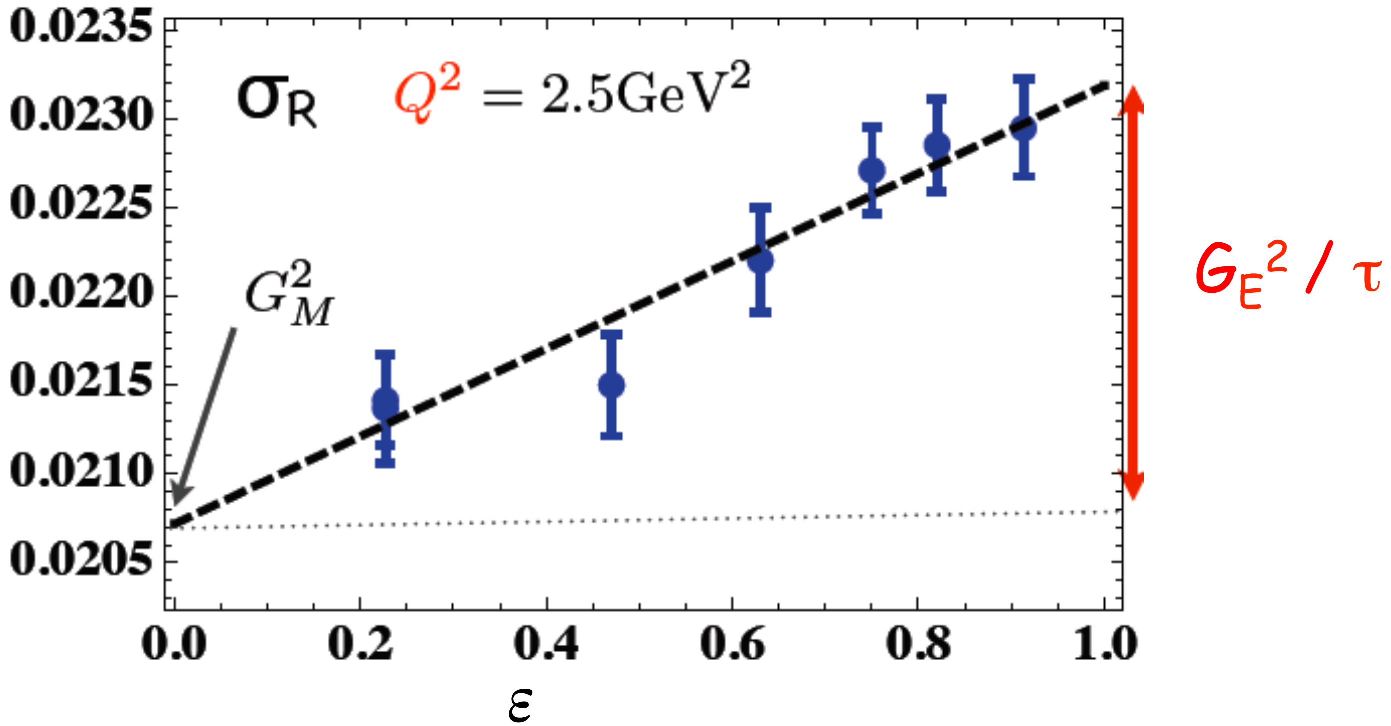
One-photon exchange elastic electron-nucleon cross section

$$\sigma_R = G_M^2 + \frac{\varepsilon}{\tau} G_E^2$$

SLAC :

Andivahis et al. (1994)

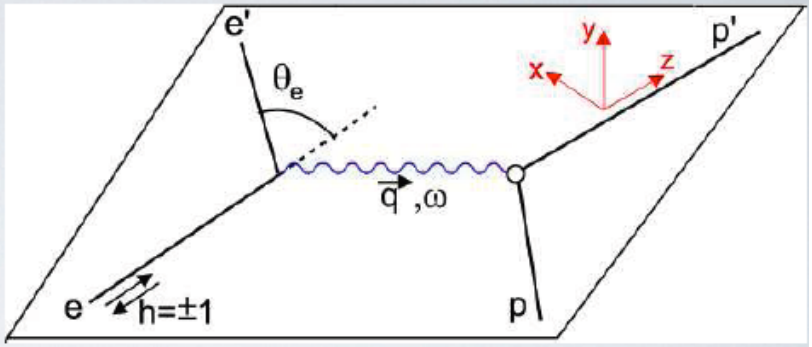
$$\tau \equiv \frac{Q^2}{4M^2} \qquad \frac{1}{\varepsilon} \equiv 1 + 2(1+\tau)\tan^2\frac{\theta}{2}$$



polarization transfer method

$$\vec{e} + p \rightarrow e + \vec{p}$$

Akhiezer, Rekalo (1974)



$$d\sigma_{pol} = d\sigma_{unpol}(1 + h\,S_x\,P_t\; + \;h\,S_z P_l)$$

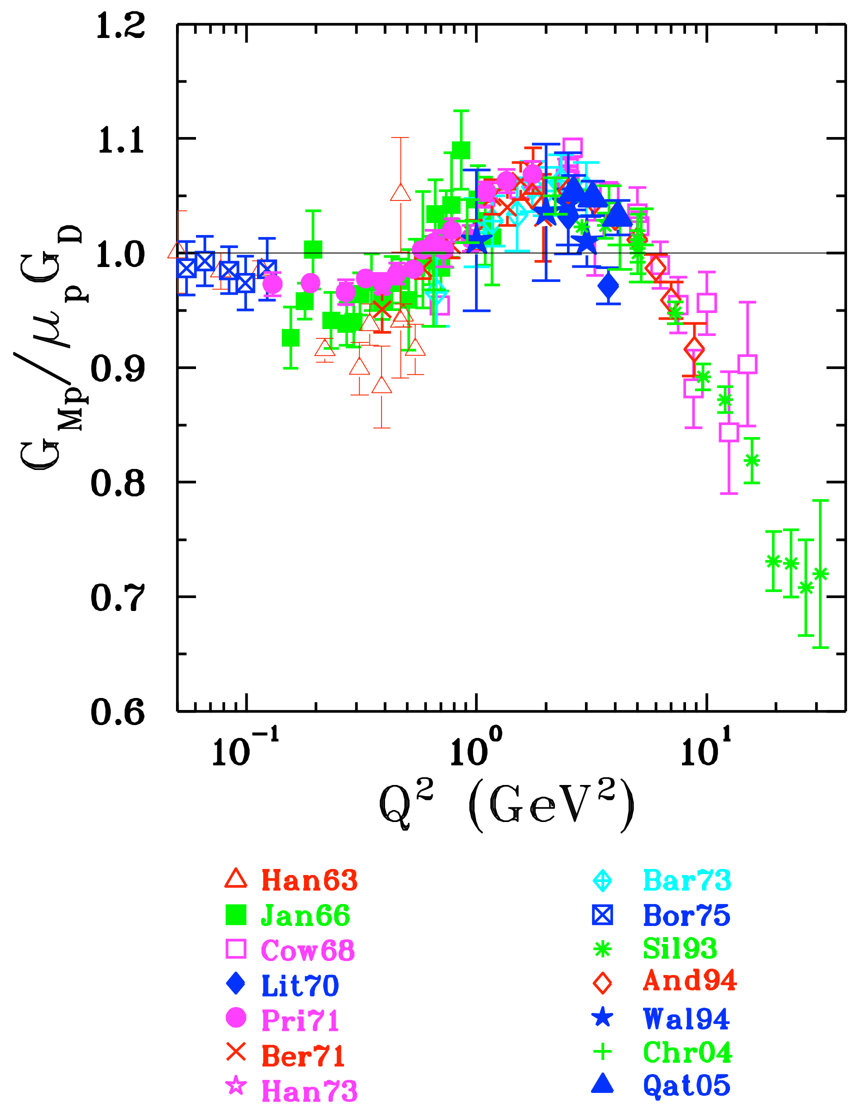
in one-photon exchange approximation :

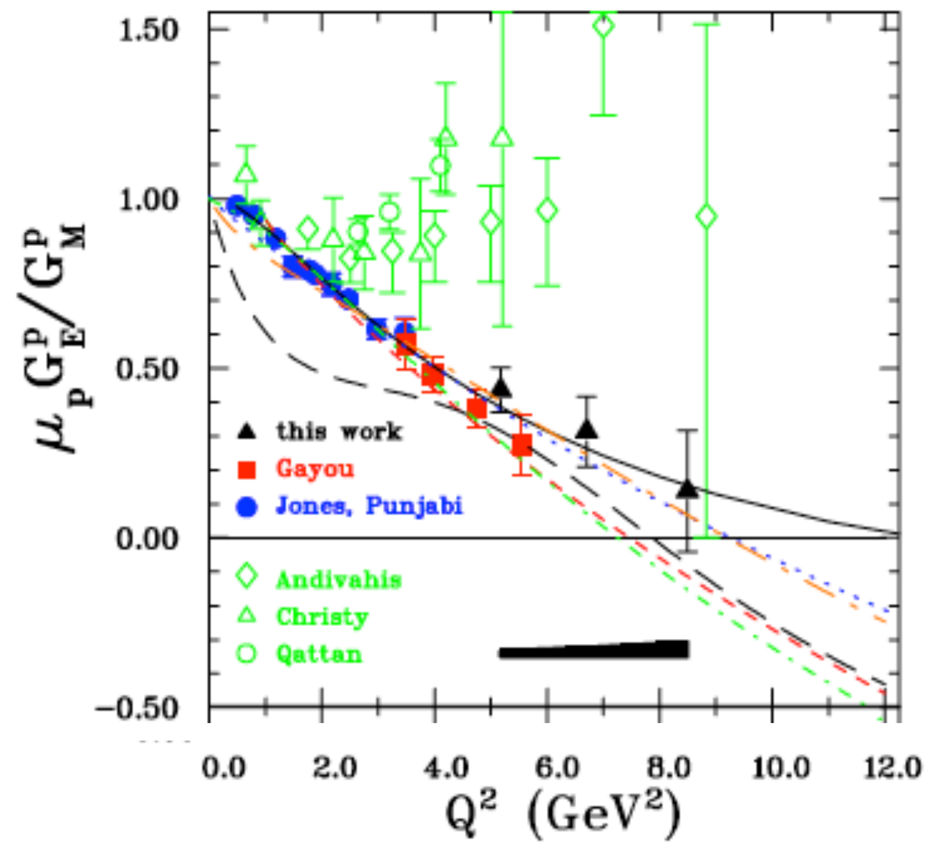
$$P_t = -\sqrt{\frac{2\varepsilon(1-\varepsilon)}{\tau}}\,\frac{G_E G_M}{\tau\,\sigma_R}$$

$$P_l = \sqrt{1-\varepsilon^2}\;\frac{G_M^2}{\tau\,\sigma_R}$$

$$\frac{P_t}{P_l} = -\sqrt{\frac{2\varepsilon}{\tau(1+\varepsilon)}}\,\frac{G_E}{G_M}$$

proton e.m. form factor : status



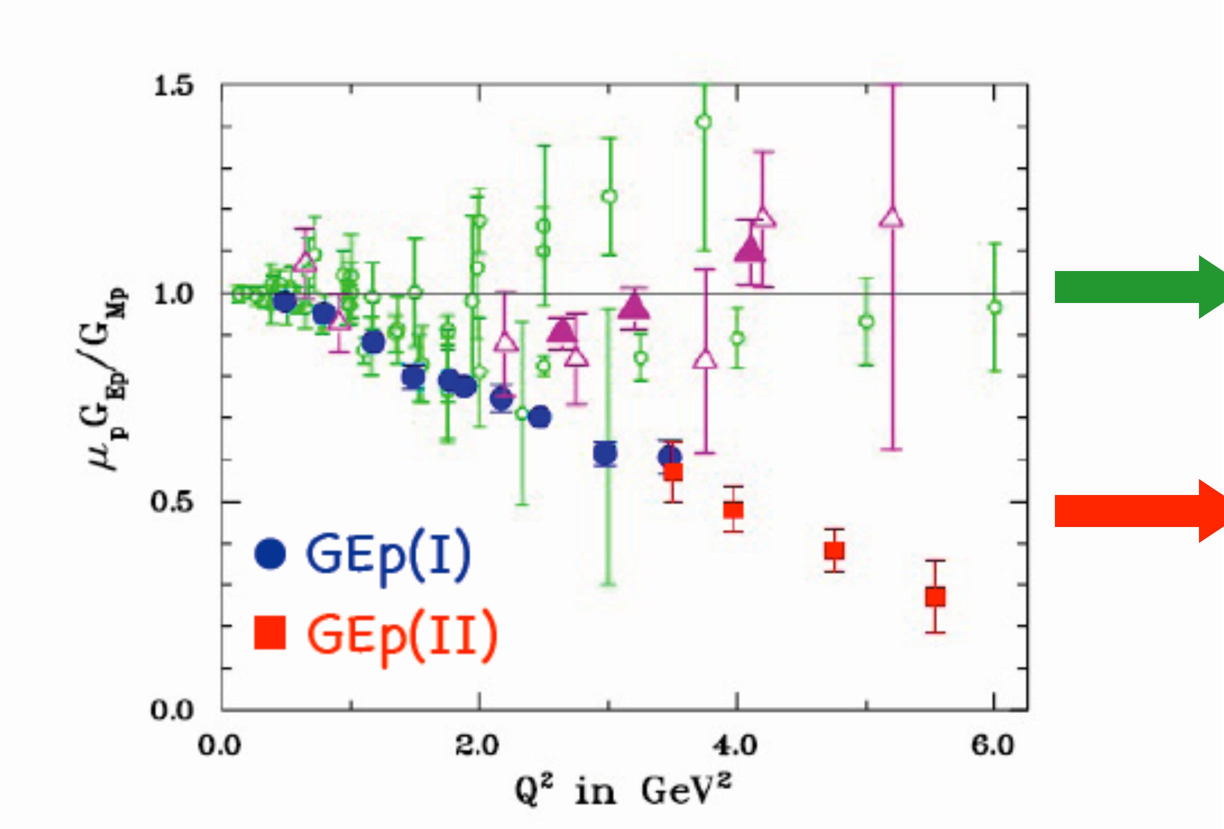


green : Rosenbluth data (SLAC, JLab)

other : recoil pol. data

(JLab/Hall A,C)

Rosenbluth vs polarization transfer measurements of G_E/G_M of proton



SLAC,

Jlab (Hall A, Hall C)

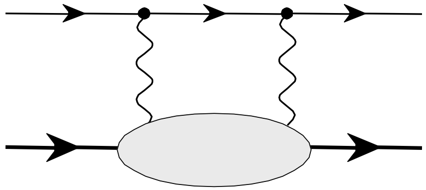
Rosenbluth data

Jlab/Hall A

Polarization data

Jones et al. (2000)

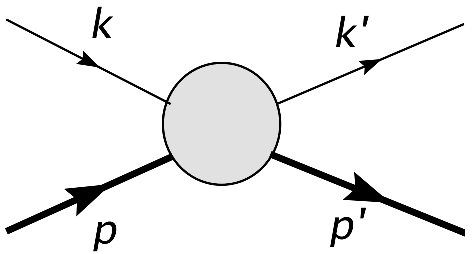
Gayou et al. (2002)



Two methods, two different results !

2γ exchange proposed as explanation

elastic eN scattering beyond 1γ-exchange approximation



Kinematical invariants :

$$P \equiv \frac{p+p'}{2}, \qquad K \equiv \frac{k+k'}{2}$$
$$Q^2 = -(p-p')^2$$
$$\nu = K \cdot P = (s-u)/4$$

for $m_e = 0$

$$T^{non-flip}_{h'\lambda'_N, h\lambda_N} = \frac{e^2}{Q^2} \bar{u}(k',h')\gamma_\mu u(k,h) \times \bar{u}(p',\lambda'_N)\left(\tilde{G}_M\,\gamma^\mu - \tilde{F}_2 \frac{P^\mu}{M} + \tilde{F}_3 \frac{\gamma.K P^\mu}{M^2}\right) u(p,\lambda_N)$$

$$\tilde{G}_M(\nu,Q^2) = G_M(Q^2) + \delta\tilde{G}_M$$
$$\tilde{F}_2(\nu,Q^2) = F_2(Q^2) + \delta\tilde{F}_2$$
$$\tilde{F}_3(\nu,Q^2) = 0 + \delta\tilde{F}_3$$

equivalently

$$\tilde{G}_E \equiv \tilde{G}_M - (1+\tau)\,\tilde{F}_2$$
$$\tilde{G}_E(\nu,Q^2) = G_E(Q^2) + \delta\tilde{G}_E$$

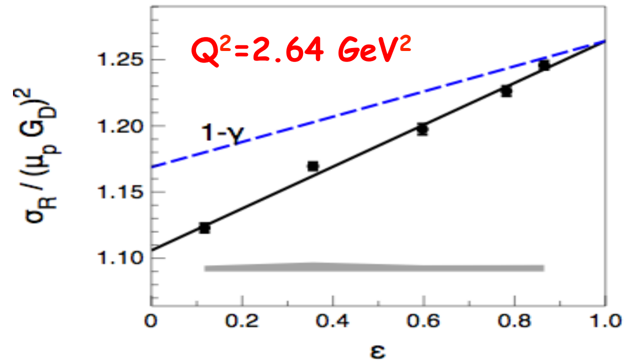
for real part → 3 independent observables

$$Y^M_{2\gamma}(\nu,Q^2) \equiv \mathcal{R}\left(\frac{\delta\tilde{G}_M}{G_M}\right)$$
$$Y^E_{2\gamma}(\nu,Q^2) \equiv \mathcal{R}\left(\frac{\delta\tilde{G}_E}{G_M}\right)$$
$$Y^3_{2\gamma}(\nu,Q^2) \equiv \frac{\nu}{M^2}\mathcal{R}\left(\frac{\tilde{F}_3}{G_M}\right)$$

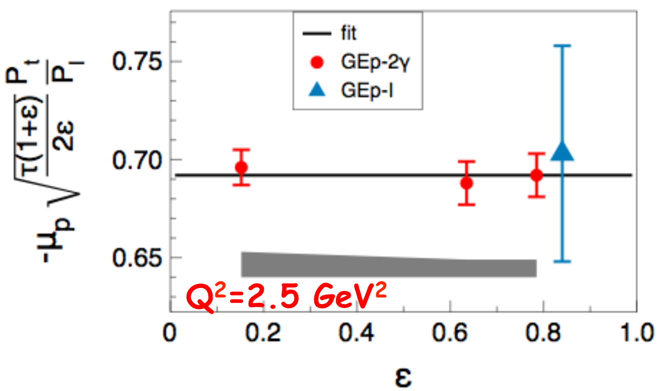
Guichon, Vdh (2003)

observables including two-photon exchange

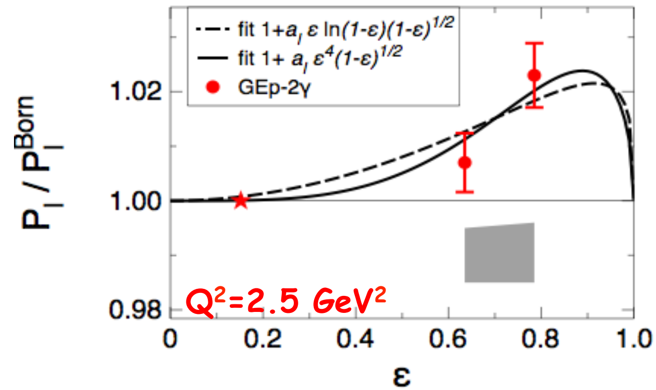
$$\frac{\sigma_R}{G_M^2} = 1 + \frac{\varepsilon}{\tau}\frac{G_E^2}{G_M^2} + 2Y_{2\gamma}^M + 2\varepsilon\frac{G_E}{\tau G_M}Y_{2\gamma}^E + 2\varepsilon\left(1+\frac{G_E}{\tau G_M}\right)Y_{2\gamma}^3 + \mathcal{O}(e^4)$$



$$-\sqrt{\frac{\tau(1+\varepsilon)}{2\varepsilon}}\frac{P_t}{P_l} = \frac{G_E}{G_M} + Y_{2\gamma}^E - \frac{G_E}{G_M}Y_{2\gamma}^M + \left(1-\frac{2\varepsilon}{1+\varepsilon}\frac{G_E}{G_M}\right)Y_{2\gamma}^3 + \mathcal{O}(e^4)$$



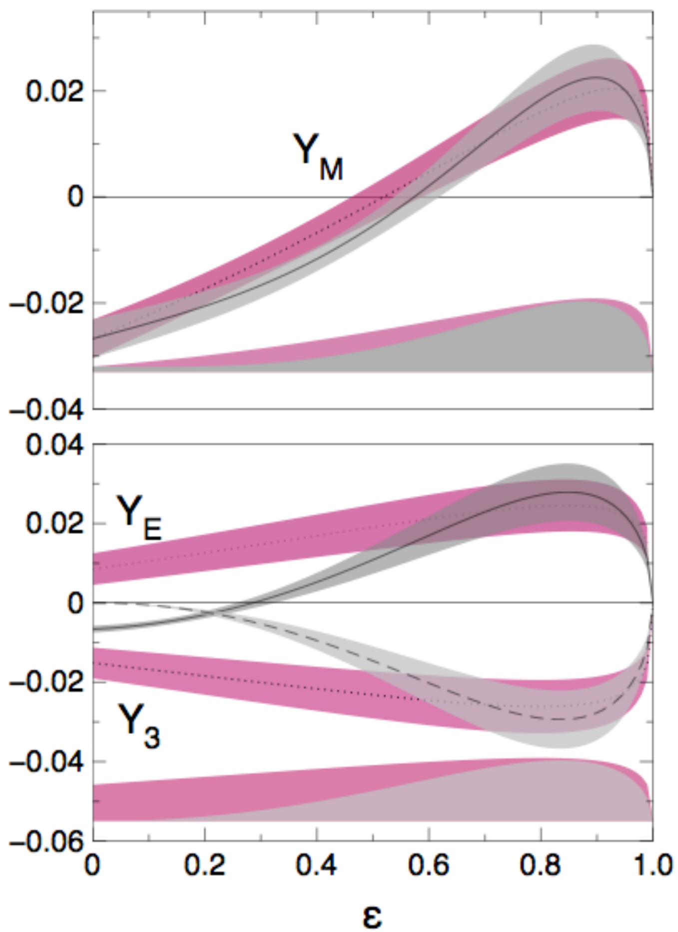
$$\frac{P_l}{P_l^{Born}} = 1 - 2\varepsilon\left(1+\frac{\varepsilon}{\tau}\frac{G_E^2}{G_M^2}\right)^{-1}\left\{\left[\frac{\varepsilon}{1+\varepsilon}\left(1-\frac{G_E^2}{\tau G_M^2}\right)+\frac{G_E}{\tau G_M}\right]Y_{2\gamma}^3 + \frac{G_E}{\tau G_M}\left[Y_{2\gamma}^E - \frac{G_E}{G_M}Y_{2\gamma}^M\right]\right\} + \mathcal{O}(e^4)$$



Jlab data : Meziane et al. (2011)

Extraction of two-photon amplitudes

Q^2=2.64 GeV^2



Guttmann, Kivel, Vdh (2011)

Extracted

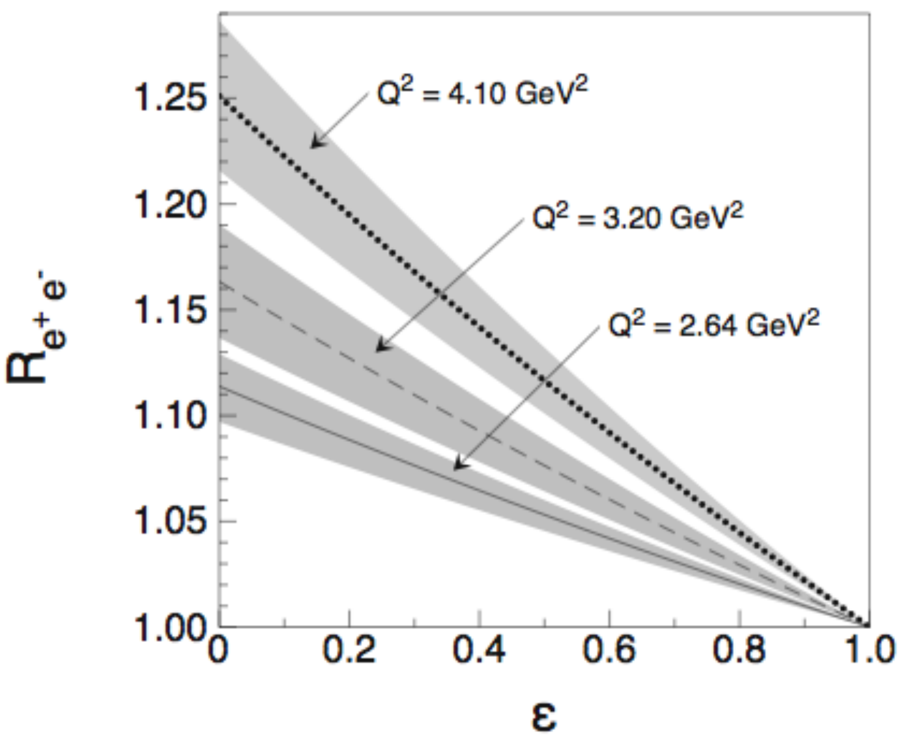
2γ -amplitudes

are in the (expected)

2 -3 % range

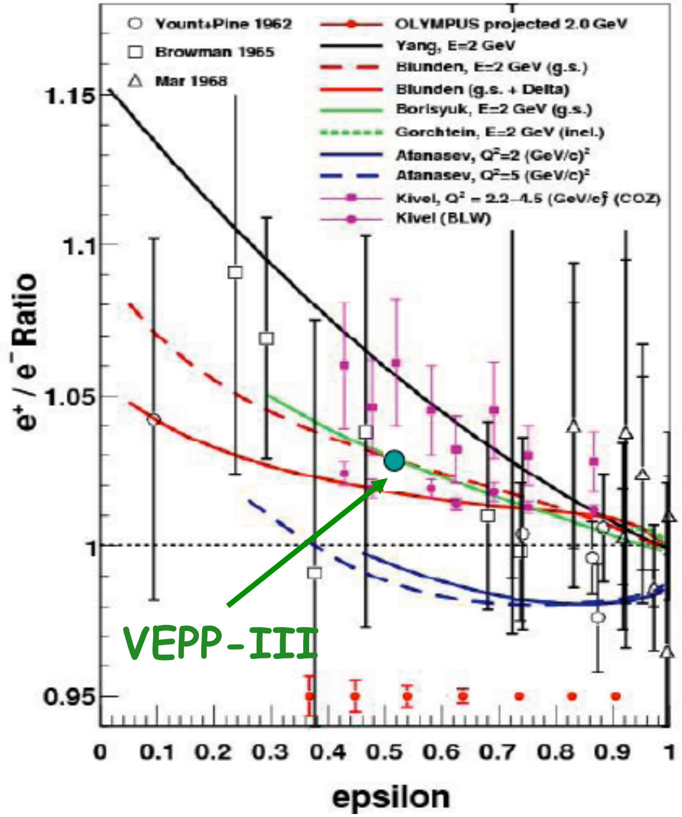
experimental test : e^+/e^- ratio

prediction



first (prelim) results from

VEPP-III (Q^2=1.43 GeV^2)

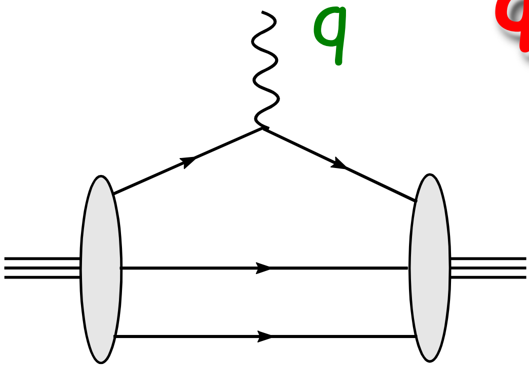


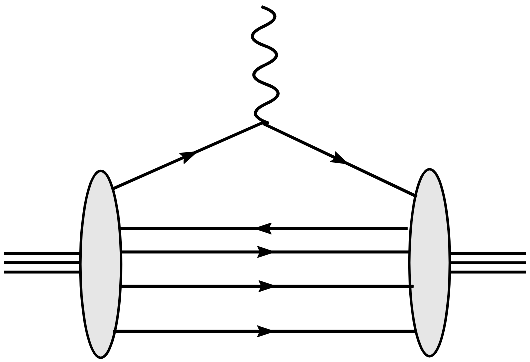
experiments underway :

VEPP-III, CLAS, Olympus@DESY

Spin-1/2 transverse densities

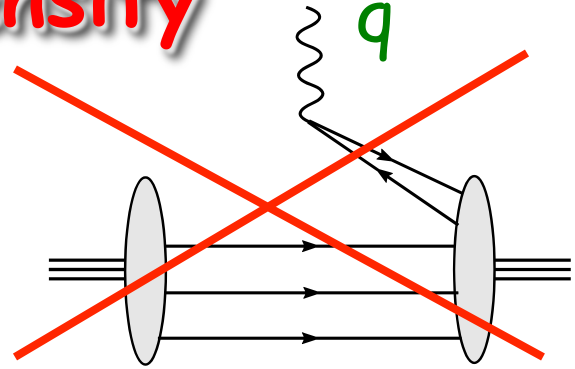
interpretation of Form Factor as quark density





overlap of wave function Fock components with same number of quarks

interpretation as probability/ charge density



overlap of wave function Fock components with different number of constituents

NO probability/charge density interpretation

absent in a LIGHT-FRONT frame !

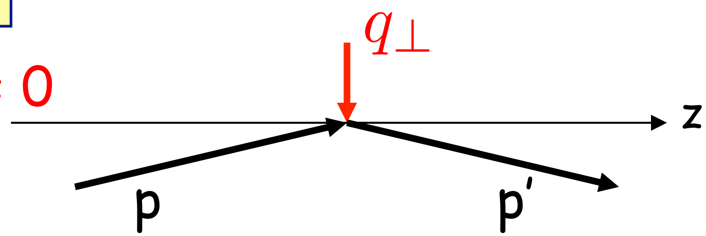
$$q^{+} = q^{0} + q^{3} = 0$$

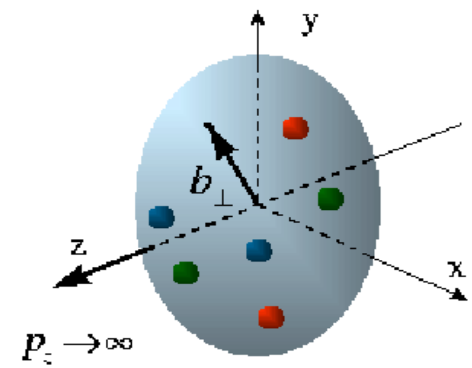
quark transverse charge densities in nucleon

light-front

- $q^+ = q^0 + q^3 = 0$

  $Q^2 \equiv \vec{q}_\perp^{\,2}$

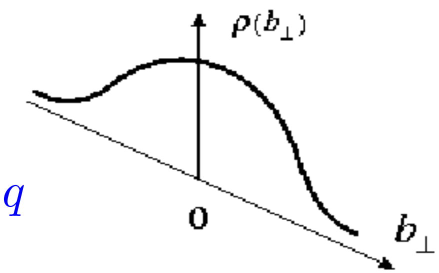




photon only couples to forward moving quarks

- quark charge density operator

$$J^+ \equiv J^0 + J^3 = \bar{q}\gamma^+ q = 2\, q_+^\dagger\, q_+, \quad \text{with} \quad q_+ \equiv \frac{1}{4}\gamma^-\gamma^+ q$$



☆ longitudinally polarized nucleon

$$\rho_0^N(\vec{b}) \equiv \int \frac{d^2\vec{q}_\perp}{(2\pi)^2}\, e^{-i\,\vec{q}_\perp\cdot\vec{b}}\, \frac{1}{2P^+}\, \langle P^+, \frac{\vec{q}_\perp}{2}, \lambda \,|\, J^+(0)\,|\, P^+, -\frac{\vec{q}_\perp}{2}, \lambda\rangle$$

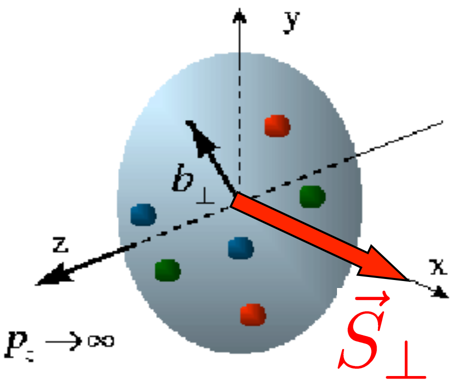
$$= \int_0^\infty \frac{dQ}{2\pi} Q\, J_0(b\,Q) F_1(Q^2)$$

Soper (1997)
Burkardt (2000)

Miller (2007)

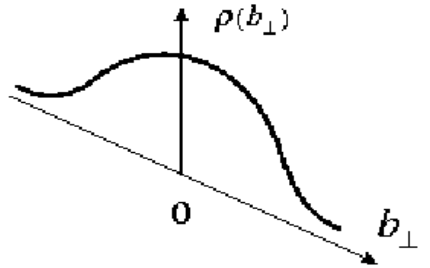
quark transverse charge densities in nucleon (II)

☆ transversely polarized nucleon



transverse spin $\vec{S}_\perp = \cos\phi_S\,\hat{e}_x + \sin\phi_S\,\hat{e}_y$

e.g. along x-axis : $\phi_S = 0$



$$\vec{b} = b\,(\cos\phi_b\,\hat{e}_x + \sin\phi_b\,\hat{e}_y)$$

$$\rho_T^N(\vec{b}) \equiv \int \frac{d^2\vec{q}_\perp}{(2\pi)^2}\, e^{-i\,\vec{q}_\perp\cdot\vec{b}}\, \frac{1}{2P^+}\, \langle P^+, \frac{\vec{q}_\perp}{2}, s_\perp = +\frac{1}{2}\,|\,J^+(0)\,|\,P^+, -\frac{\vec{q}_\perp}{2}, s_\perp = +\frac{1}{2}\rangle$$

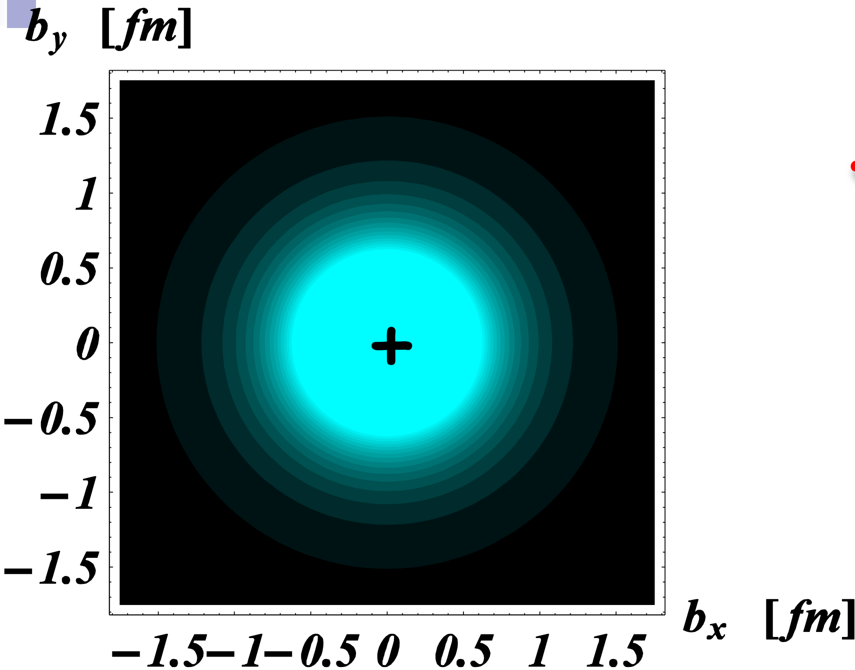
$$= \rho_0^N(b) + \sin(\phi_b - \phi_S)\int_0^\infty \frac{dQ}{2\pi}\,\frac{Q^2}{2M_N}\,J_1(b\,Q)F_2(Q^2)$$

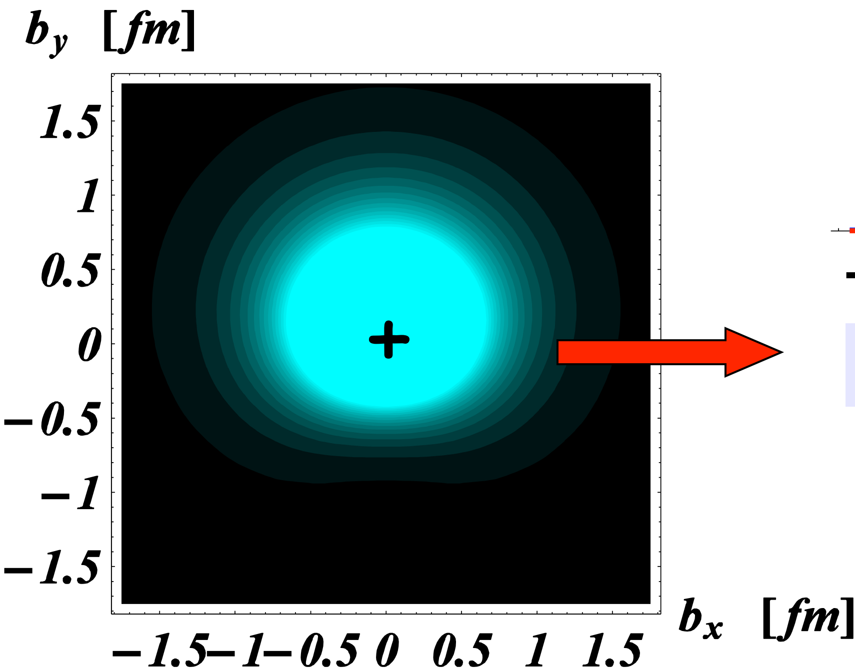
↓

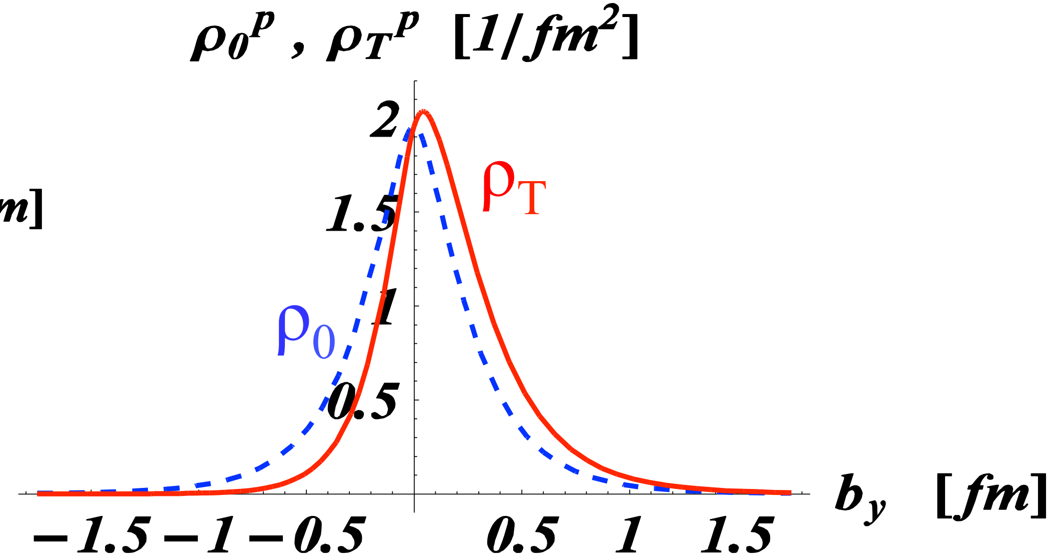
dipole field pattern

Carlson, Vdh (2007)

empirical quark transverse densities in proton







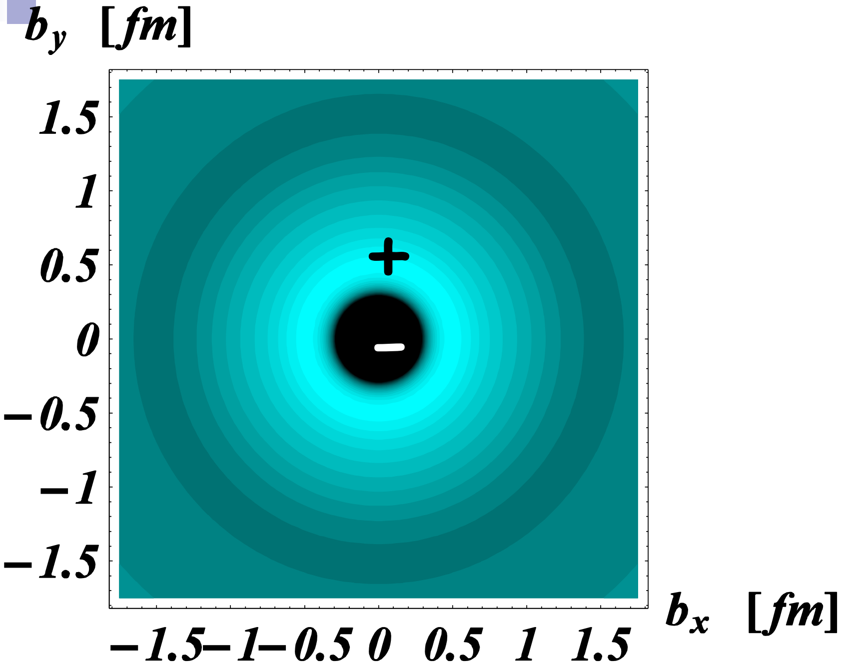
induced EDM : $d_y = F_{2p}(0) \,.\, e / (2 M_N)$

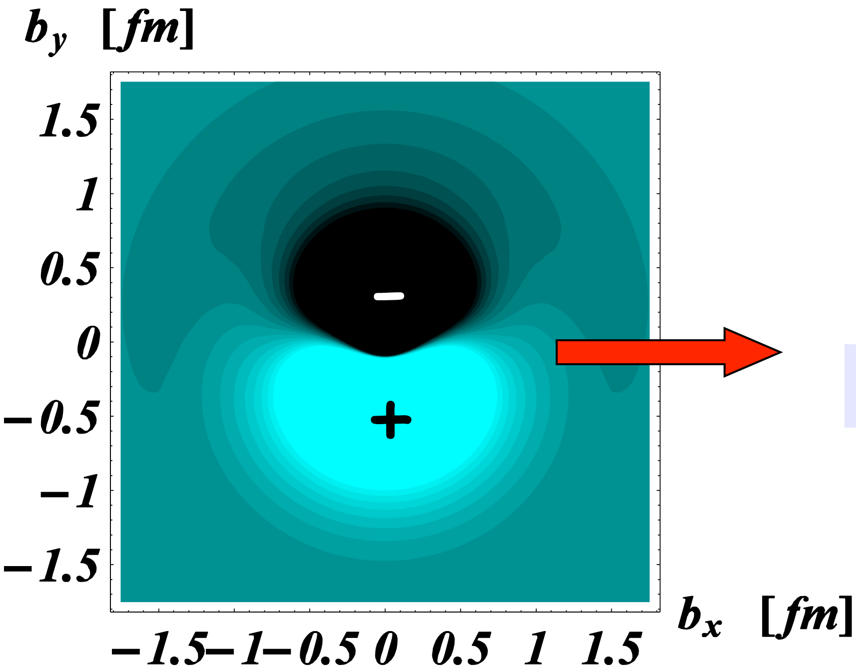
Burkardt (2000)

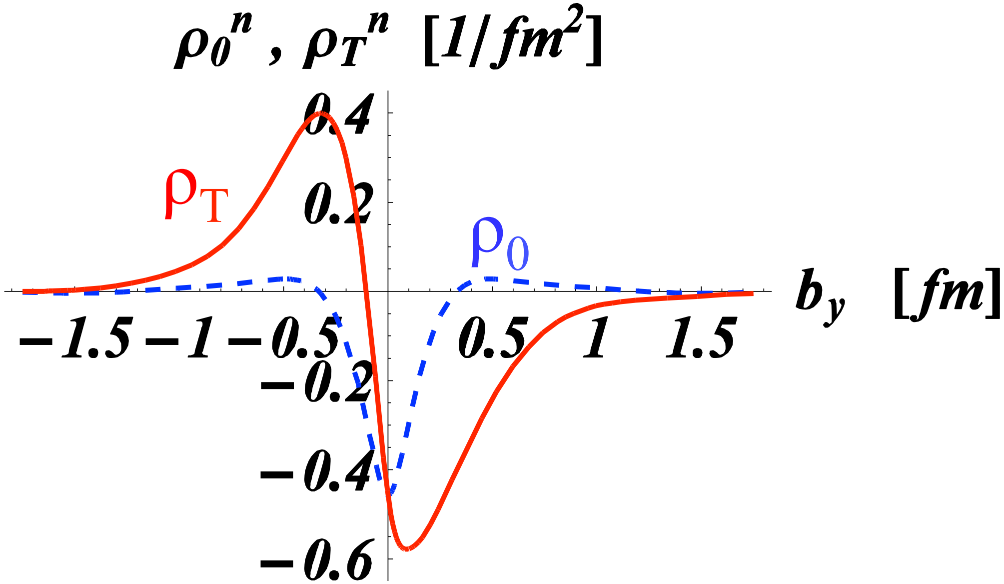
data : Arrington, Melnitchouk, Tjon (2007)

densities : Miller (2007); Carlson, Vdh (2007)

empirical quark transverse densities in neutron







induced EDM : $d_y = F_{2n}(0) \cdot e / (2 M_N)$

data: Bradford, Bodek, Budd, Arrington (2006)

densities: Miller (2007); Carlson, Vdh (2007)

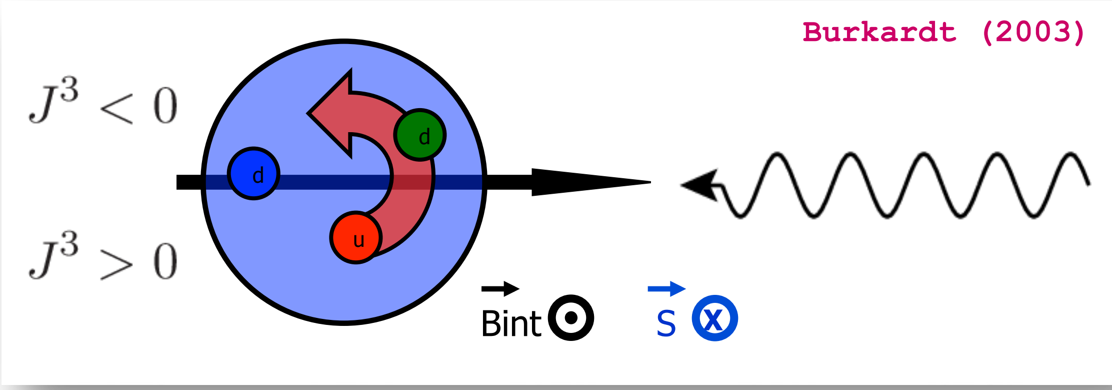
current of classical spinning particle

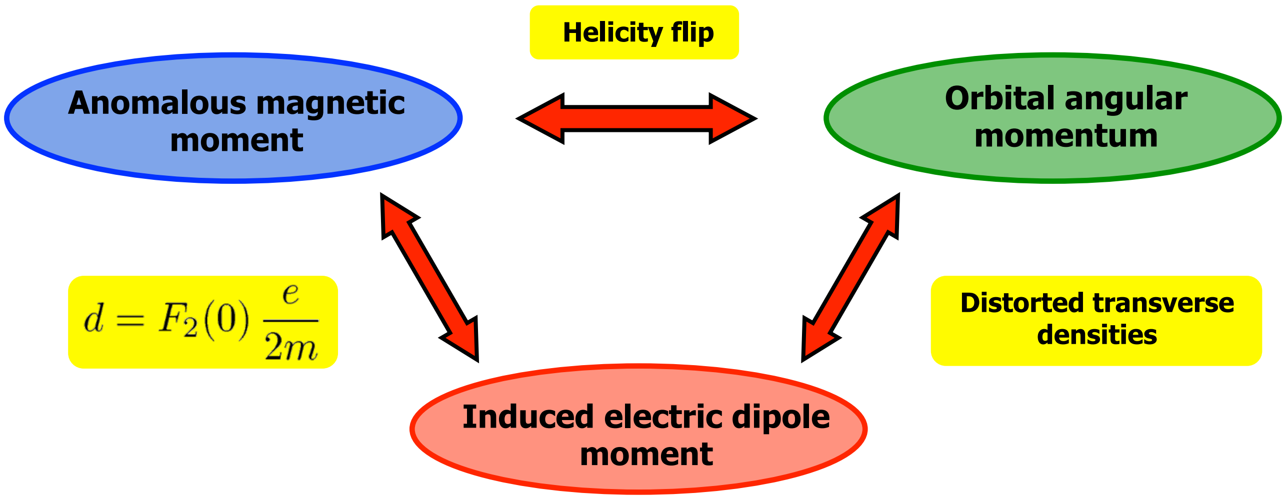
transversely polarized nucleon

$$J^+ = \frac{1}{\sqrt{2}}(J^0 + J^3)$$

induced EDM

$d_y = F_2(0) . e / (2 M)$





Spin-1 transverse densities

deuteron e.m. FFs

$$\langle p_2, \lambda_2 | J^\mu | p_1, \lambda_1 \rangle = -(\varepsilon_2^* \cdot \varepsilon_1) 2P^\mu G_1(Q^2) - \left(\varepsilon_1^\mu \varepsilon_2^* \cdot q - \varepsilon_2^{\mu *} \varepsilon_1 \cdot q\right) G_M(Q^2) + q \cdot \varepsilon_1 \, q \cdot \varepsilon_2^* \frac{P^\mu}{M_d^2} G_3(Q^2)$$

mass M_d $\quad P \equiv (p_1 + p_2)/2$

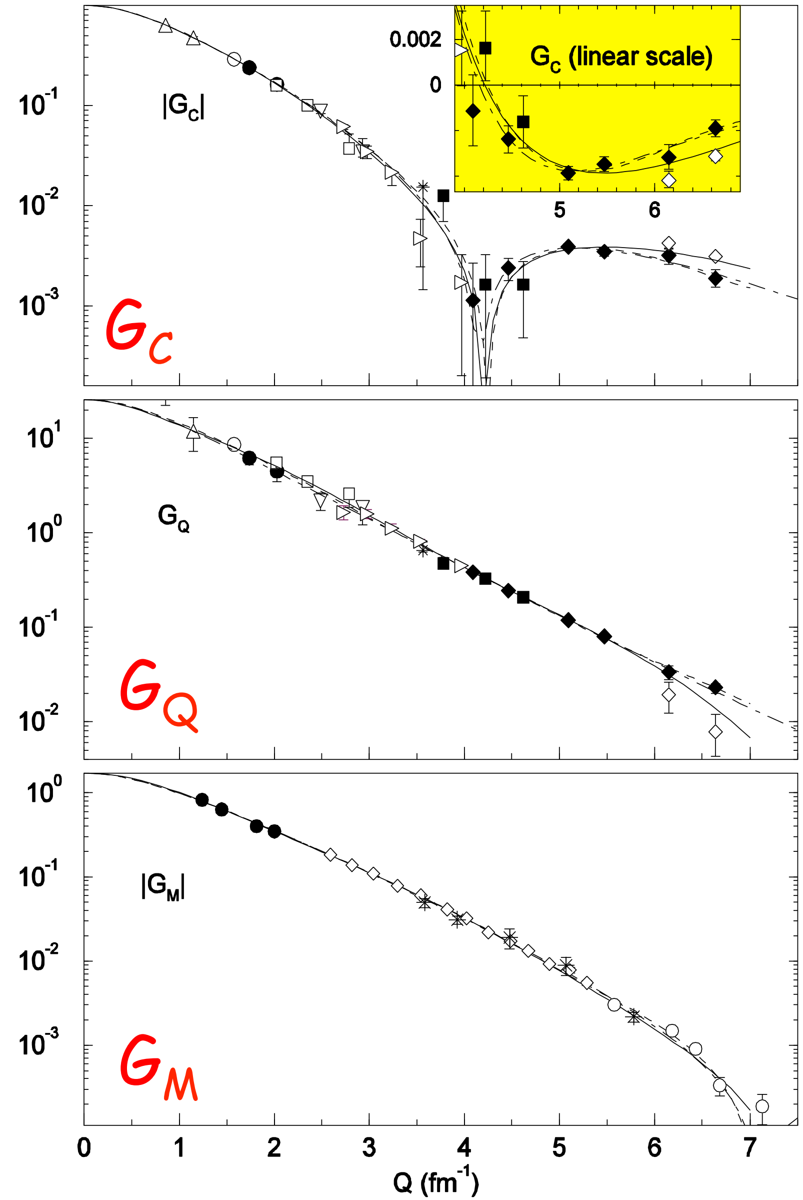
spin-1 polarization vectors → $\varepsilon_1^\mu \,,\ \varepsilon_2^\mu$

$$G_C = G_1 + \frac{2}{3}\eta\, G_Q$$
$$G_Q = G_1 + (1+\eta)\, G_3 - G_M$$
$$\eta \equiv Q^2/(4M_d^2)$$

G_C : **charge monopole**

G_Q : **charge quadrupole**

G_M : **magnetic dipole**



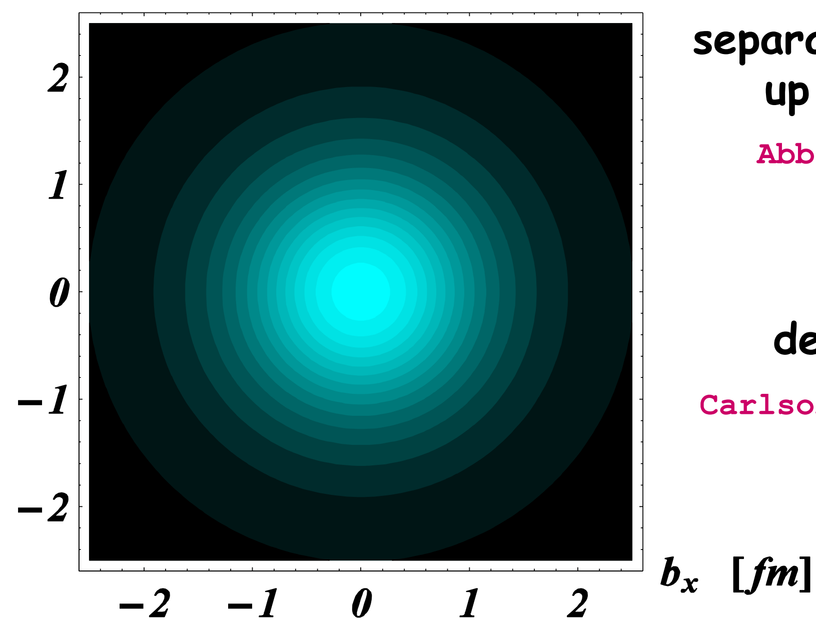
Abbott et al.(2000)

empirical quark transverse densities in deuteron

$$\rho^d_\lambda(b) \equiv \int \frac{d^2\vec{q}_\perp}{(2\pi)^2} e^{-i\,\vec{q}_\perp\cdot\vec{b}} \frac{1}{2P^+} \langle P^+, \frac{\vec{q}_\perp}{2}, \lambda | J^+ | P^+, \frac{-\vec{q}_\perp}{2}, \lambda \rangle$$
$$= \int_0^\infty \frac{dQ}{2\pi} Q\, J_0(b\,Q)\, G^+_{\lambda\lambda}(Q^2)$$

$$G^+_{11} = \frac{1}{1+\eta}\left\{G_C + \eta\, G_M + \frac{\eta}{3} G_Q\right\}$$
$$G^+_{00} = \frac{1}{1+\eta}\left\{(1-\eta)\,G_C + 2\eta\, G_M - \frac{2\eta}{3}(1+2\eta)G_Q\right\}$$

$\lambda = \pm 1$



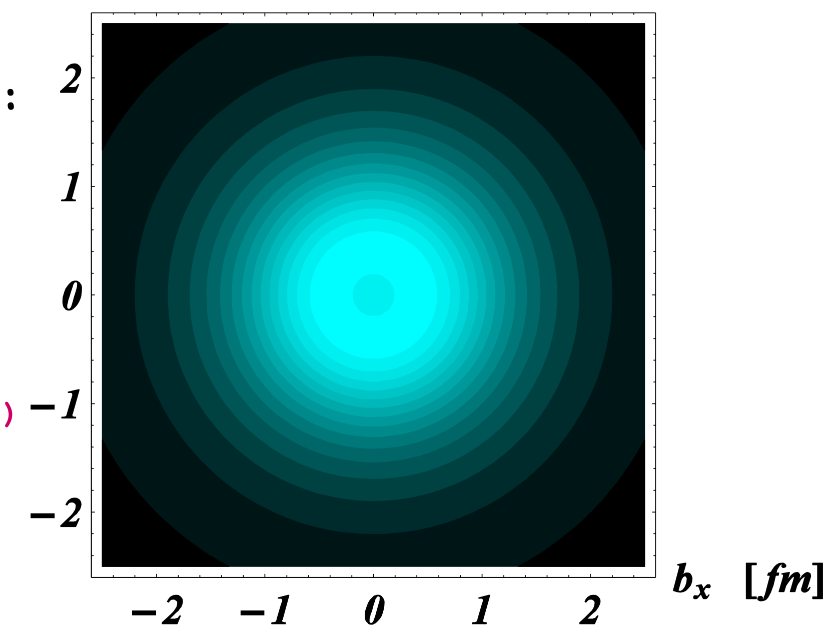
separated data up to 2 GeV^2 :

Abbott et al. (2000)

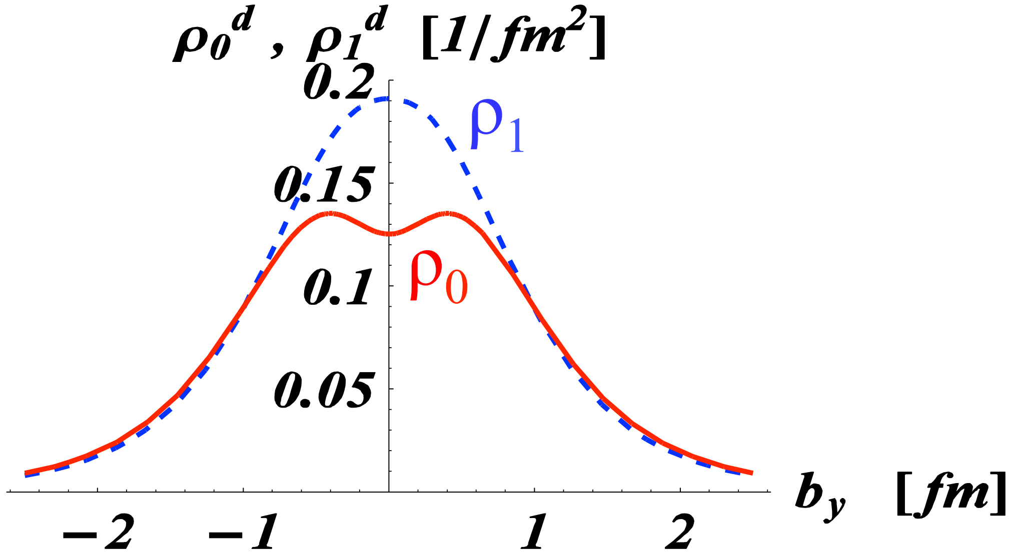
densities :

Carlson, Vdh (2008)

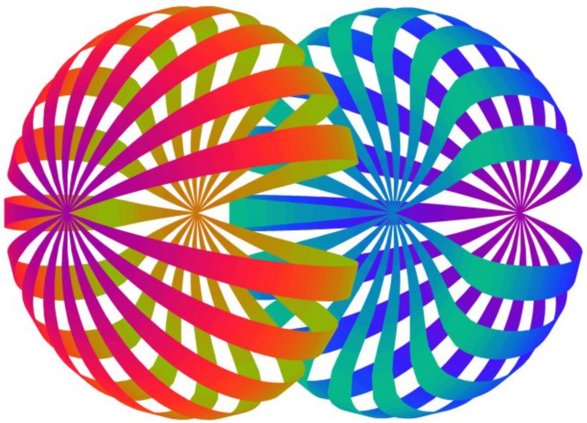
$\lambda = 0$



longitudinally polarized deuteron



$\lambda = \pm 1$



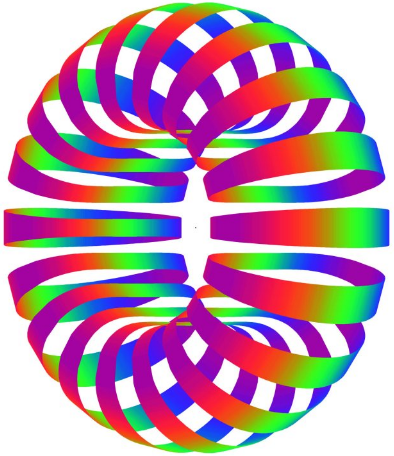
deuteron equidensity surfaces

(ρ_d = 0.24 fm^{-3})

from Argonne v_{18} :

Forest et al. (1996)

$\lambda = 0$



transversely polarized deuteron

$$Q^d_{s_\perp} \equiv e \int d^2\vec{b}\, (b_x^2 - b_y^2)\, \rho^d_{T\, s_\perp}(\vec{b})$$

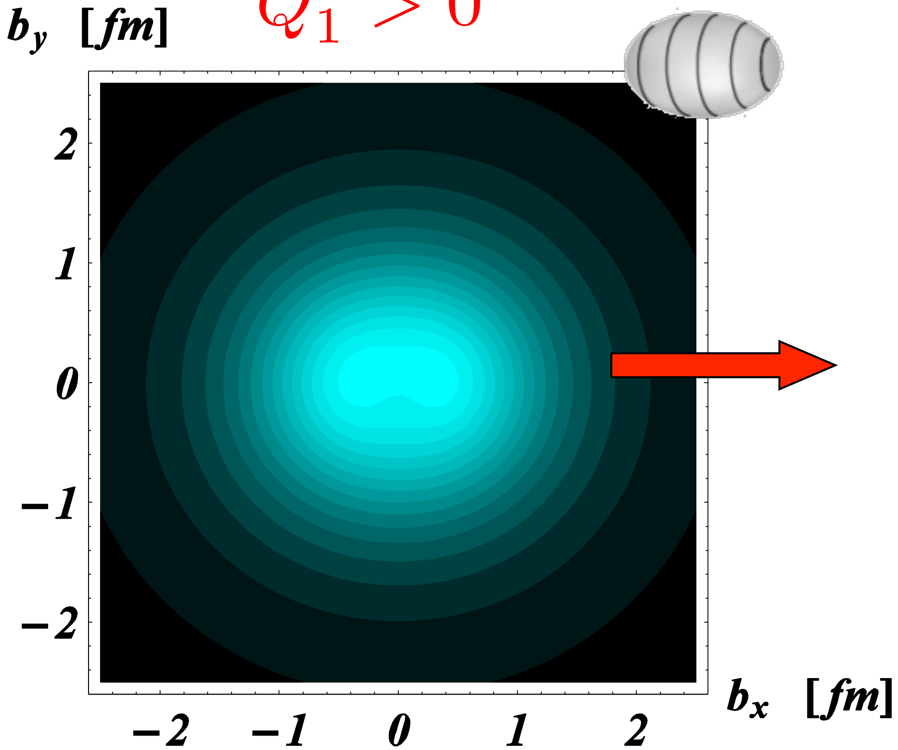
$$Q^d_1 = -\frac{1}{2} Q^d_0 = \frac{1}{2} \left\{ [G_M(0) - 2] + [G_Q(0) + 1] \right\} \left(\frac{e}{M_d^2} \right)$$

experiment :

$$G_M(0) = 1.71$$
$$G_Q(0) = 25.84(3)$$

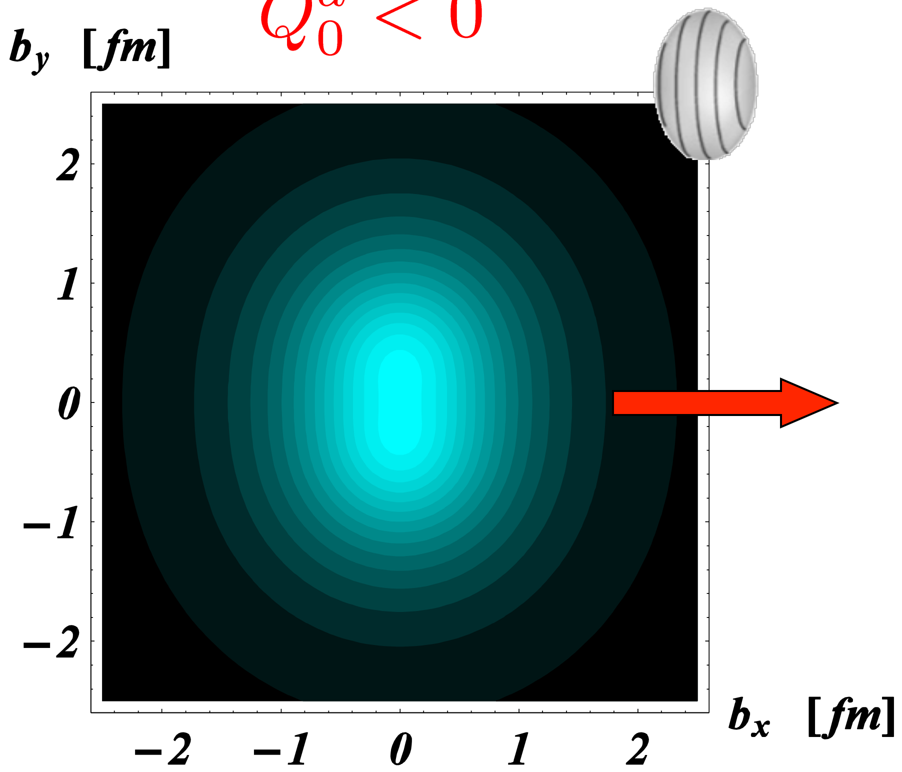
$s_\perp = +1$

$Q^d_1 > 0$



$s_\perp = 0$

$Q^d_0 < 0$



e.m. moments of W bosons

for spin-1 point particle

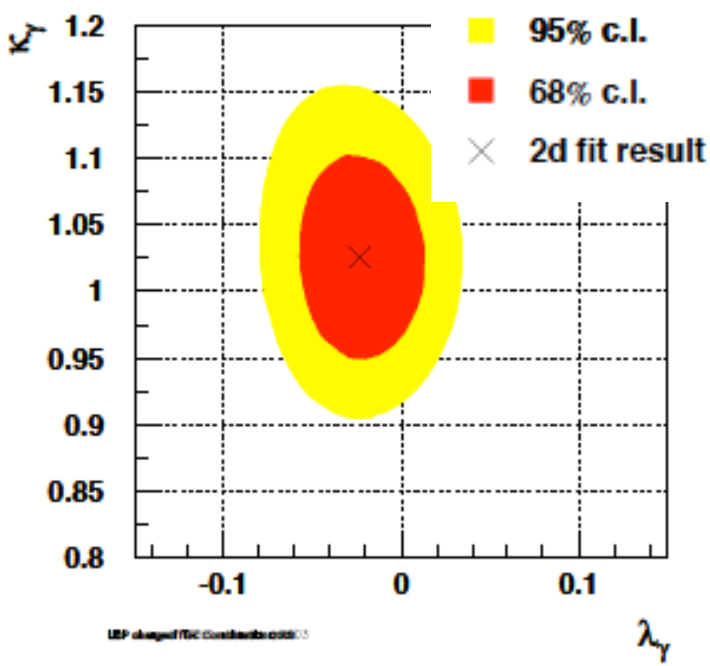
$G_M(0) = 2$ and $G_Q(0) = -1$

$$\mu = \frac{e}{2M_W}\{2 + (\kappa - 1) + \lambda\}$$

$$Q = -\frac{e}{M_W^2}\{1 + (\kappa - 1) - \lambda\}$$

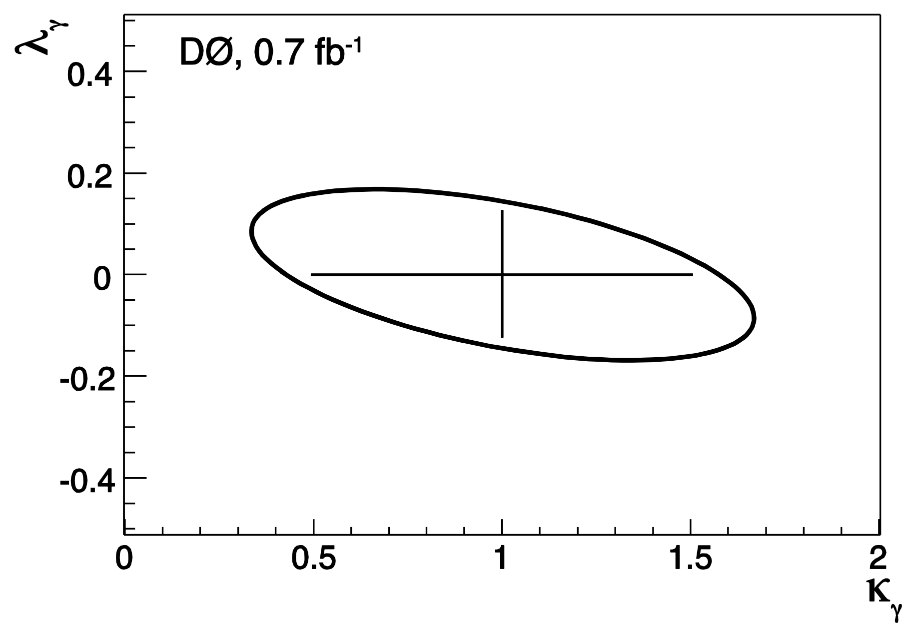
LEP Electroweak working group

hep-ex/0612034



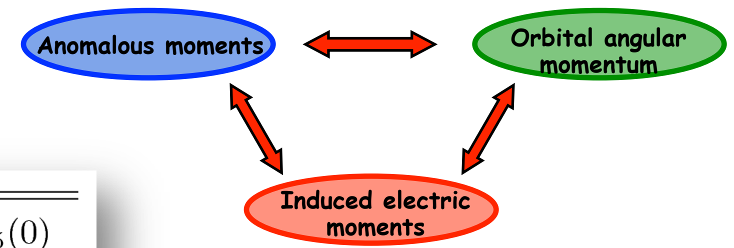
D0 Collaboration

PRL100, 241805 (2008)



particle with arbitrary spin: natural values for e.m. moments

Spin j : $2j+1$ multipoles



j	$G_{E0}(0)$ (e)	$G_{M1}(0)$ $(e/2M)$	$G_{E2}(0)$ (e/M^2)	$G_{M3}(0)$ $(e/2M^3)$	$G_{E4}(0)$ (e/M^4)	$G_{M5}(0)$ $(e/2M^5)$
0	1					
1/2	1	1				
1	1	2	-1			
3/2	1	3	-3	-1		
2	1	4	-6	-4	1	
$\vdots$						
j	C^0_{2j}	C^1_{2j}	$-C^2_{2j}$	$-C^3_{2j}$	C^4_{2j}	C^5_{2j}

Standard Model

Supergravity

Charge normalization

Universal g=2 factor

$$G_{M1}(0) = 2j$$

Structureless particle

↓

No OAM

↓

No distortions

↓

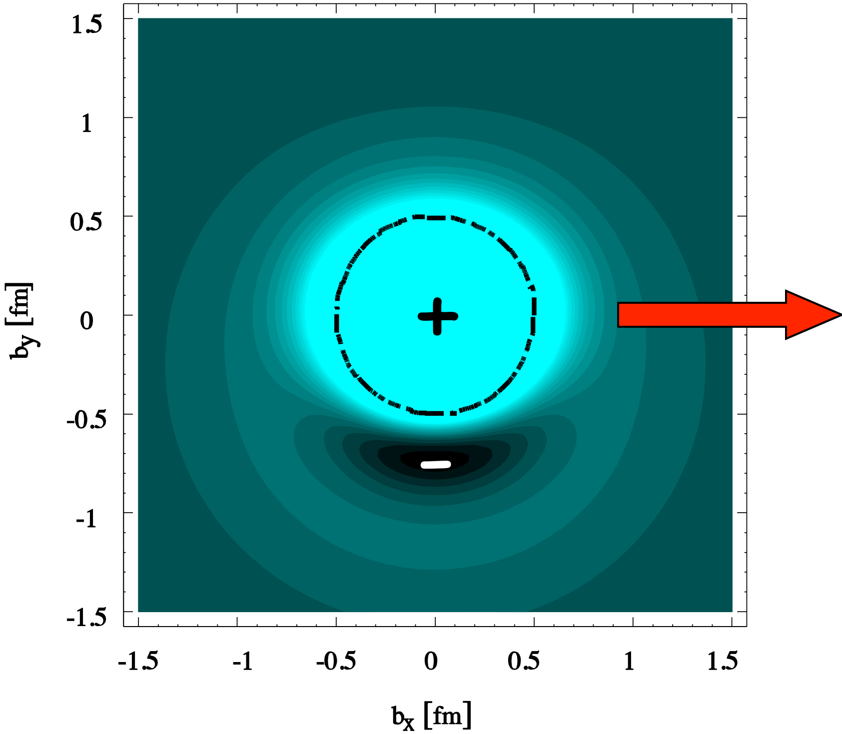
No anomalous moments

↓

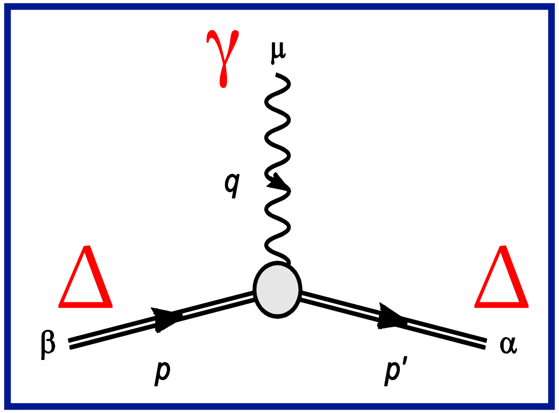
Natural EM moments

Lorcé (2008)

Spin-3/2 transverse densities



$\gamma^* \Delta\, \Delta$ vertex



$$\langle \Delta(p',\lambda') \,|\, J^\mu(0) \,|\, \Delta(p,\lambda) \rangle$$
$$= -\bar{u}_\alpha(p',\lambda') \left\{ \left[F_1^*(Q^2) g^{\alpha\beta} + F_3^*(Q^2) \frac{q^\alpha q^\beta}{(2M_\Delta)^2} \right] \gamma^\mu + \left[F_2^*(Q^2) g^{\alpha\beta} + F_4^*(Q^2) \frac{q^\alpha q^\beta}{(2M_\Delta)^2} \right] \frac{i\sigma^{\mu\nu} q_\nu}{2M_\Delta} \right\} u_\beta(p,\lambda)$$

mass M_Δ

multipole transitions

$$G_{E0} = (F_1^* - \tau F_2^*) + \frac{2}{3}\tau G_{E2}$$
$$G_{E2} = (F_1^* - \tau F_2^*) - \frac{1}{2}(1+\tau)\,(F_3^* - \tau F_4^*)$$
$$G_{M1} = (F_1^* + F_2^*) + \frac{4}{5}\tau G_{M3}$$
$$G_{M3} = (F_1^* + F_2^*) - \frac{1}{2}(1+\tau)\,(F_3^* + F_4^*)$$

$$\tau \equiv Q^2/(4M_\Delta^2)$$

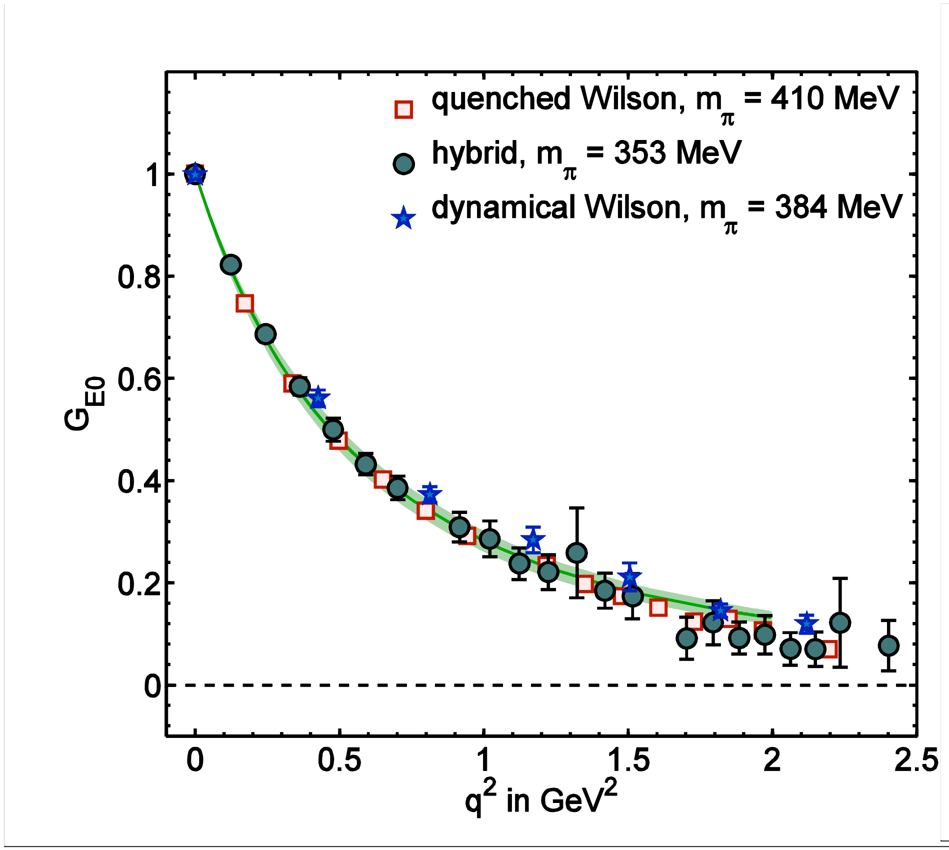
electric charge $e_\Delta = G_{E0}(0)$

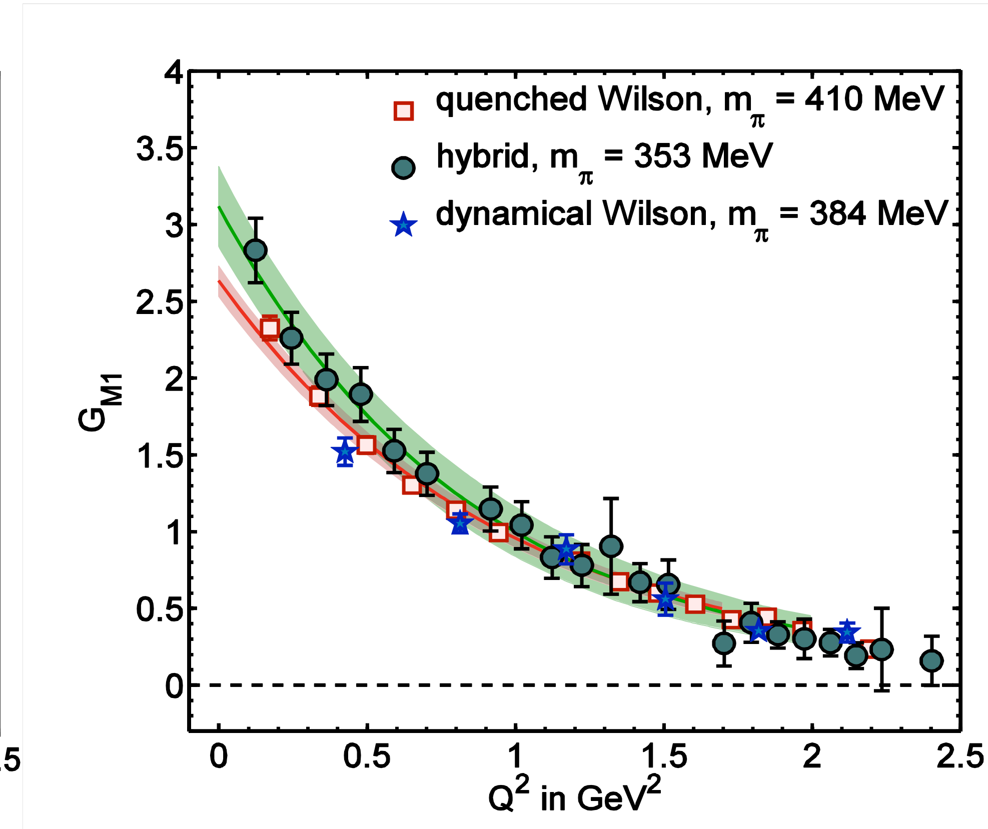
charge quadrupole $Q_\Delta = \frac{e}{M_\Delta^2} G_{E2}(0)$

magnetic dipole $\mu_\Delta = \frac{e}{2M_\Delta} G_{M1}(0)$

magnetic octupole $O_\Delta = \frac{e}{2M_\Delta^3} G_{M3}(0)$

e.m. Δ to Δ transition : lattice QCD

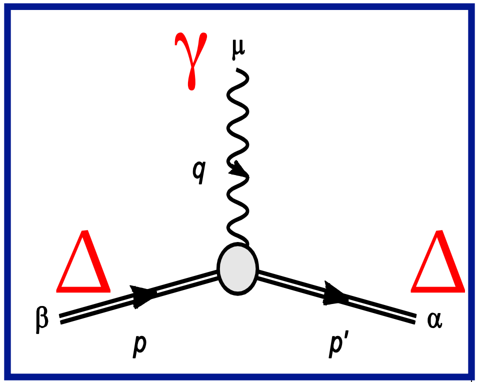




lattice analysis :

Alexandrou, Korzec, Koutsou, Leontiou, Lorcé, Negele, Pascalutsa, Tsapalis, Vdh (2008)

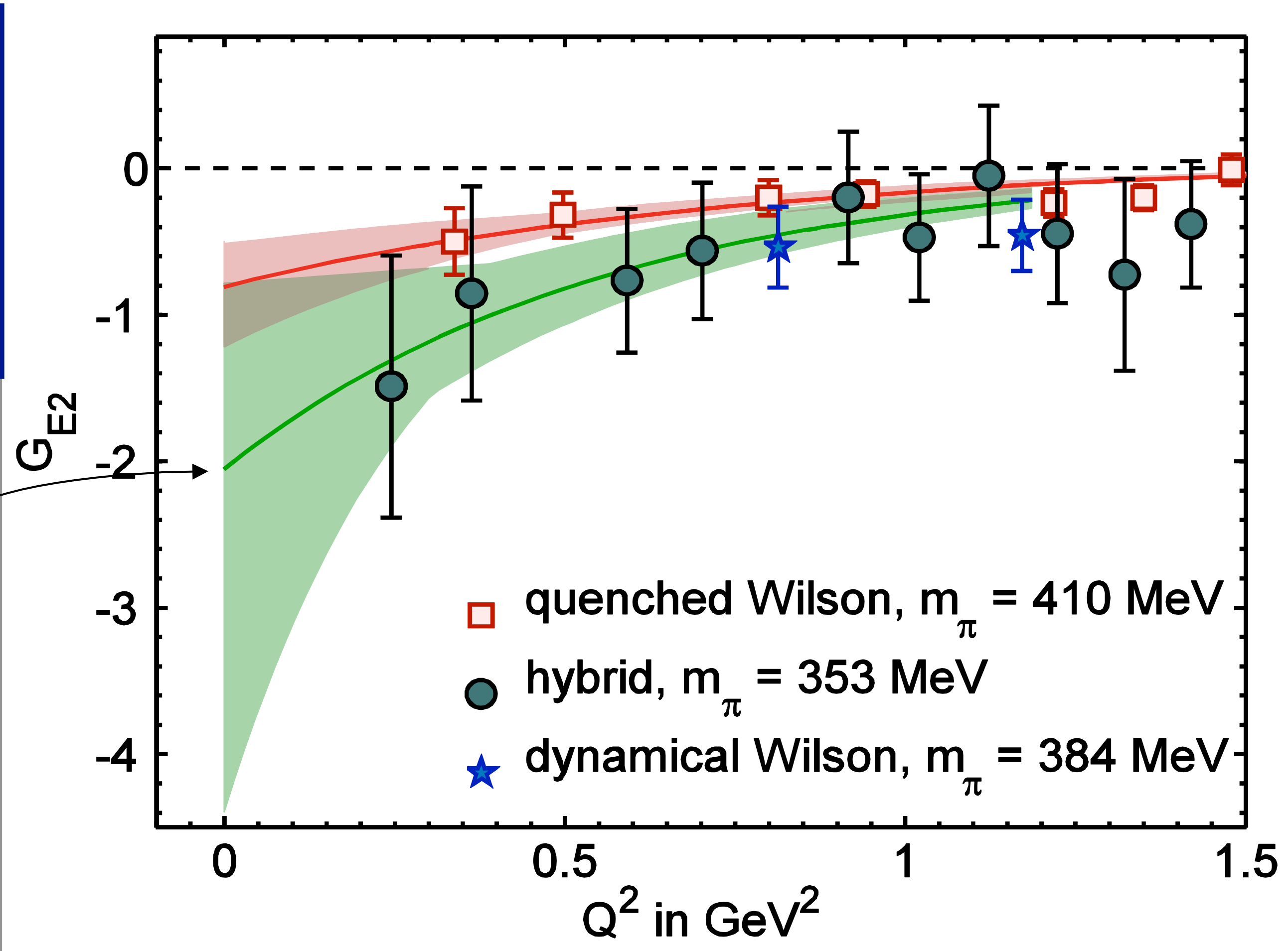
hadron shape : e.m. Δ to Δ transition



C0 , M1,

C2 , M3

transitions



lattice analysis :

Alexandrou, Korzec, Koutsou, Leontiou, Lorcé, Negele, Pascalutsa, Tsapalis, Vdh (2008)

transverse charge densities in Δ(1232)

$$\rho^{\Delta}_{T\,s_\perp}(\vec{b}) \equiv \int \frac{d^2\vec{q}_\perp}{(2\pi)^2}\, e^{-i\,\vec{q}_\perp\cdot\vec{b}}\, \frac{1}{2P^+}\langle P^+, \frac{\vec{q}_\perp}{2}, s_\perp \,|\, J^+(0)\,|\, P^+, -\frac{\vec{q}_\perp}{2}, s_\perp\rangle$$

$$Q^{\Delta}_{s_\perp} \equiv e\int d^2\vec{b}\,(b_x^2 - b_y^2)\,\rho^{\Delta}_{T\,s_\perp}(\vec{b})$$

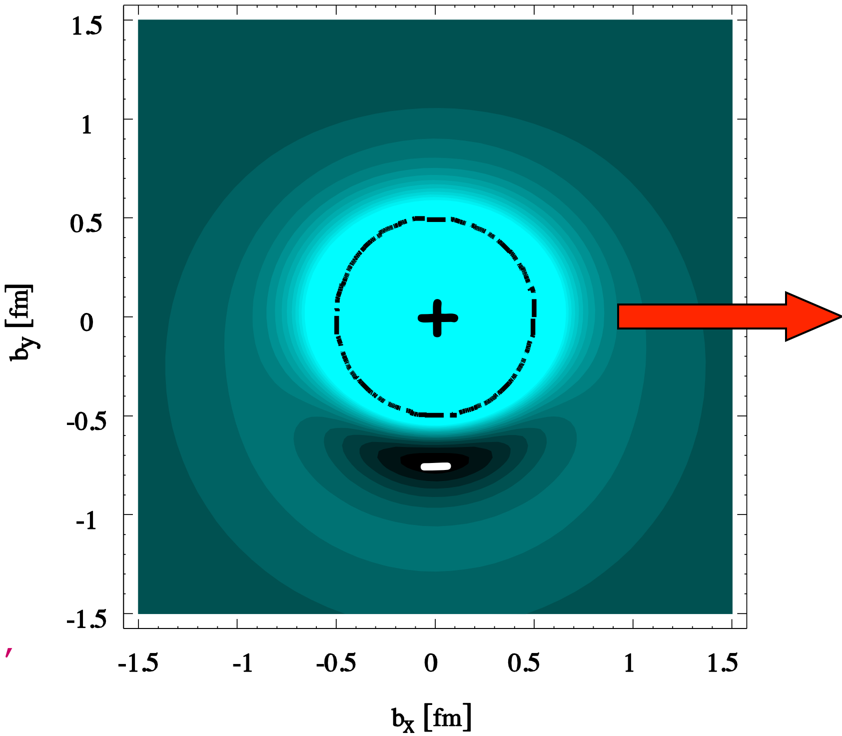
$$Q^{\Delta}_{\frac{3}{2}} = -Q^{\Delta}_{\frac{1}{2}} = \frac{1}{2}\{2\,[G_{M1}(0) - 3e_\Delta] + [G_{E2}(0) + 3e_\Delta]\}\left(\frac{e}{M_\Delta^2}\right)$$

$s_\perp = +3/2$

for spin-3/2 point particle

GM1 (0) = 3eΔ and GE2(0) = -3eΔ

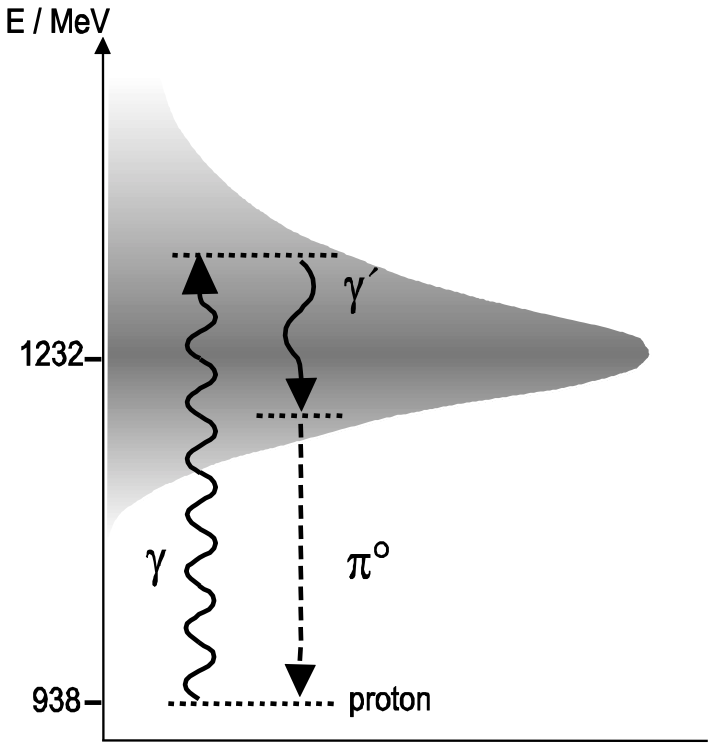
transverse charge densities depend only on anomalous values of e.m. moments -> reflect internal structure

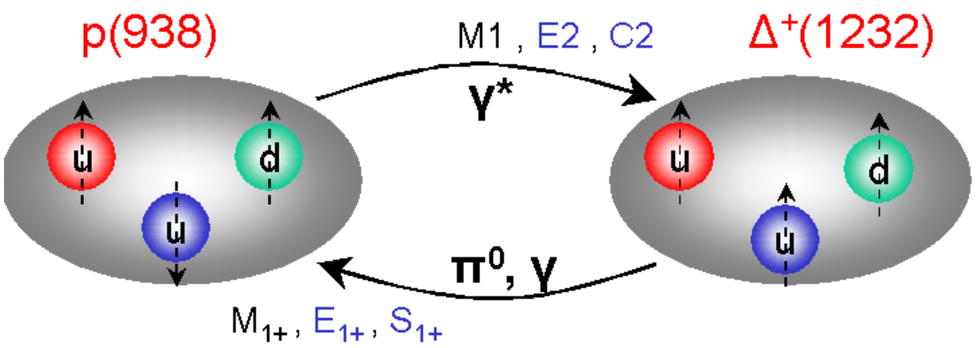


lattice analysis :

Alexandrou, Korzec, Koutsou, Leontiou, Lorcé, Negele, Pascalutsa, Tsapalis, Vdh (2008)

Transition charge densities for N -> N*, Δ





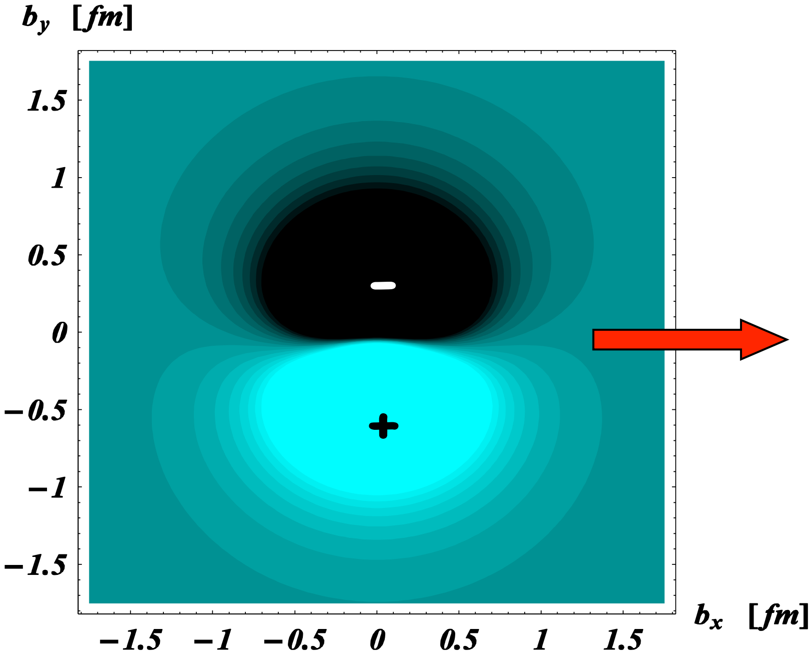
p -> Δ(1232) transition densities in transverse spin state

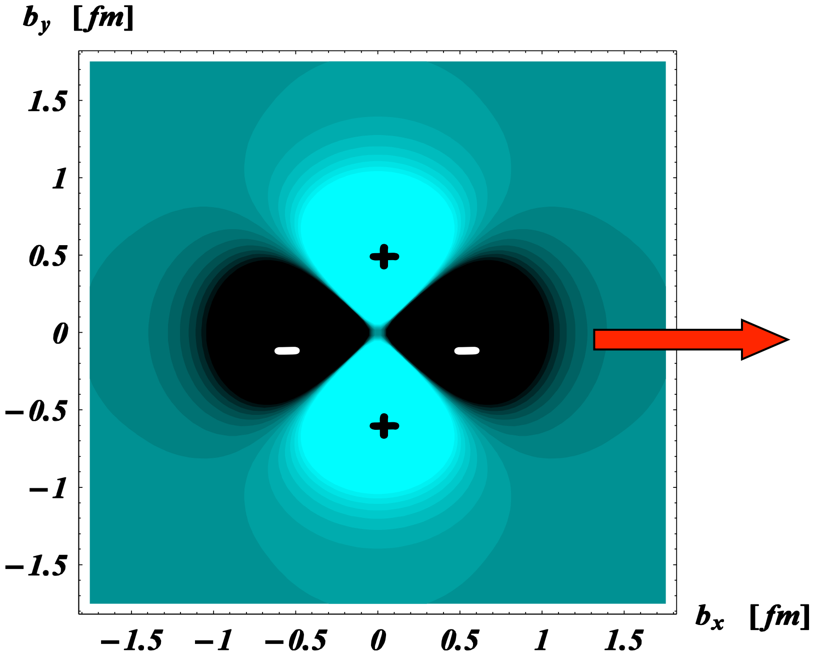
$$\rho_T^{N\Delta}(\vec{b}) \equiv \int \frac{d^2\vec{q}_\perp}{(2\pi)^2}\, e^{-i\,\vec{q}_\perp\cdot\vec{b}}\, \frac{1}{2P^+} \langle P^+, \frac{\vec{q}_\perp}{2}, s_\perp^\Delta = +\frac{1}{2}\,|\,J^+(0)\,|\,P^+, -\frac{\vec{q}_\perp}{2}, s_\perp^N = +\frac{1}{2}\rangle$$

$$= \int_0^\infty \frac{dQ}{2\pi}\,\frac{Q}{2}\,\Big\{J_0(b\,Q)\,G^+_{+\frac{1}{2}\,+\frac{1}{2}}$$ → monopole

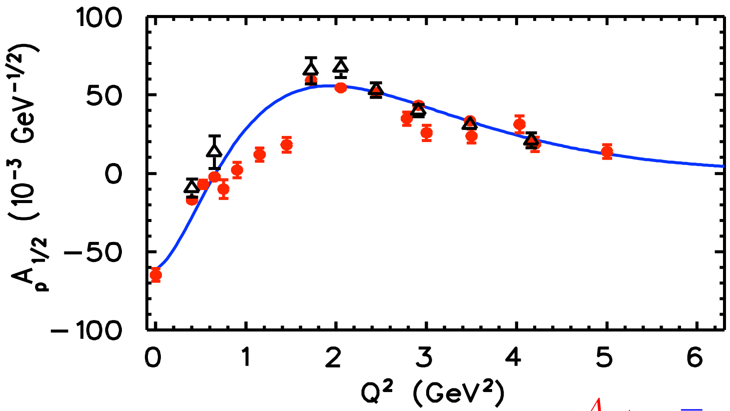
$$-\sin(\phi_b-\phi_S)\,J_1(b\,Q)\left[\sqrt{3}G^+_{+\frac{3}{2}\,+\frac{1}{2}} + G^+_{+\frac{1}{2}\,-\frac{1}{2}}\right]$$ → dipole

$$-\cos 2(\phi_b-\phi_S)\,J_2(b\,Q)\sqrt{3}\,G^+_{+\frac{3}{2}\,-\frac{1}{2}}\Big\}$$ → quadrupole





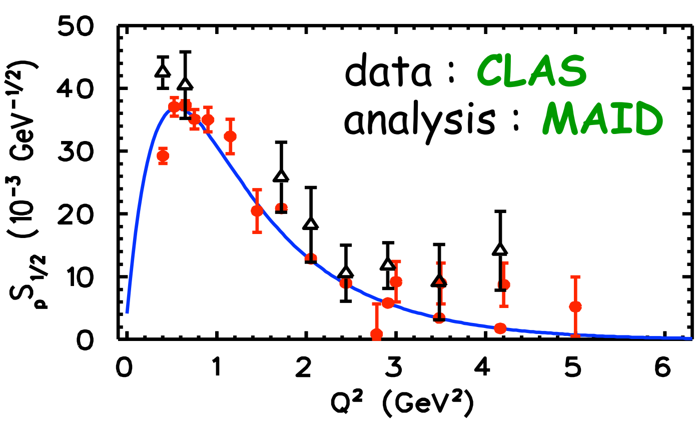
empirical e.m. transition FFs for p -> N*(1440) excitation

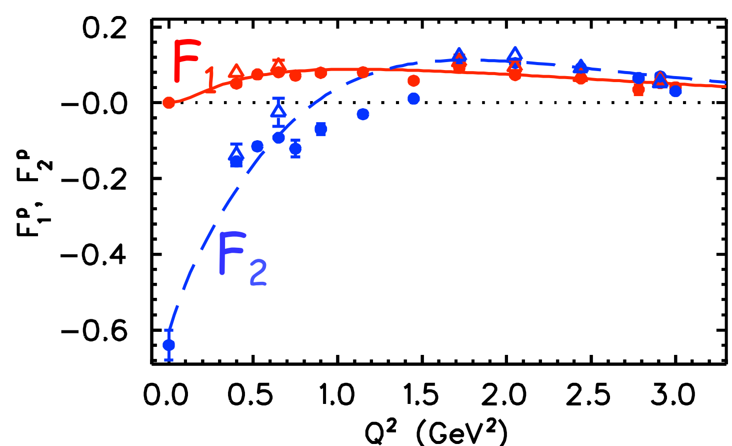


$$
\langle N^*(p',\lambda') \,|\, J^\mu(0) \,|\, N(p,\lambda) \rangle
= \bar{u}(p',\lambda') \left\{ F_1^{NN^*}(Q^2) \left(\gamma^\mu - \gamma \cdot q \, \frac{q^\mu}{q^2} \right) + F_2^{NN^*}(Q^2) \frac{i\sigma^{\mu\nu} q_\nu}{(M^* + M_N)} \right\} u(p,\lambda)
$$

$$
A_{1/2} = e \, \frac{Q_-}{\sqrt{K}\,(4 M_N M^*)^{1/2}} \left\{ F_1^{NN^*} + F_2^{NN^*} \right\}
$$

$$
S_{1/2} = e \, \frac{Q_-}{\sqrt{2K}\,(4 M_N M^*)^{1/2}} \left(\frac{Q_+ Q_-}{2M^*} \right) \frac{(M^* + M_N)}{Q^2} \left\{ F_1^{NN^*} - \frac{Q^2}{(M^* + M_N)^2} F_2^{NN^*} \right\}
$$

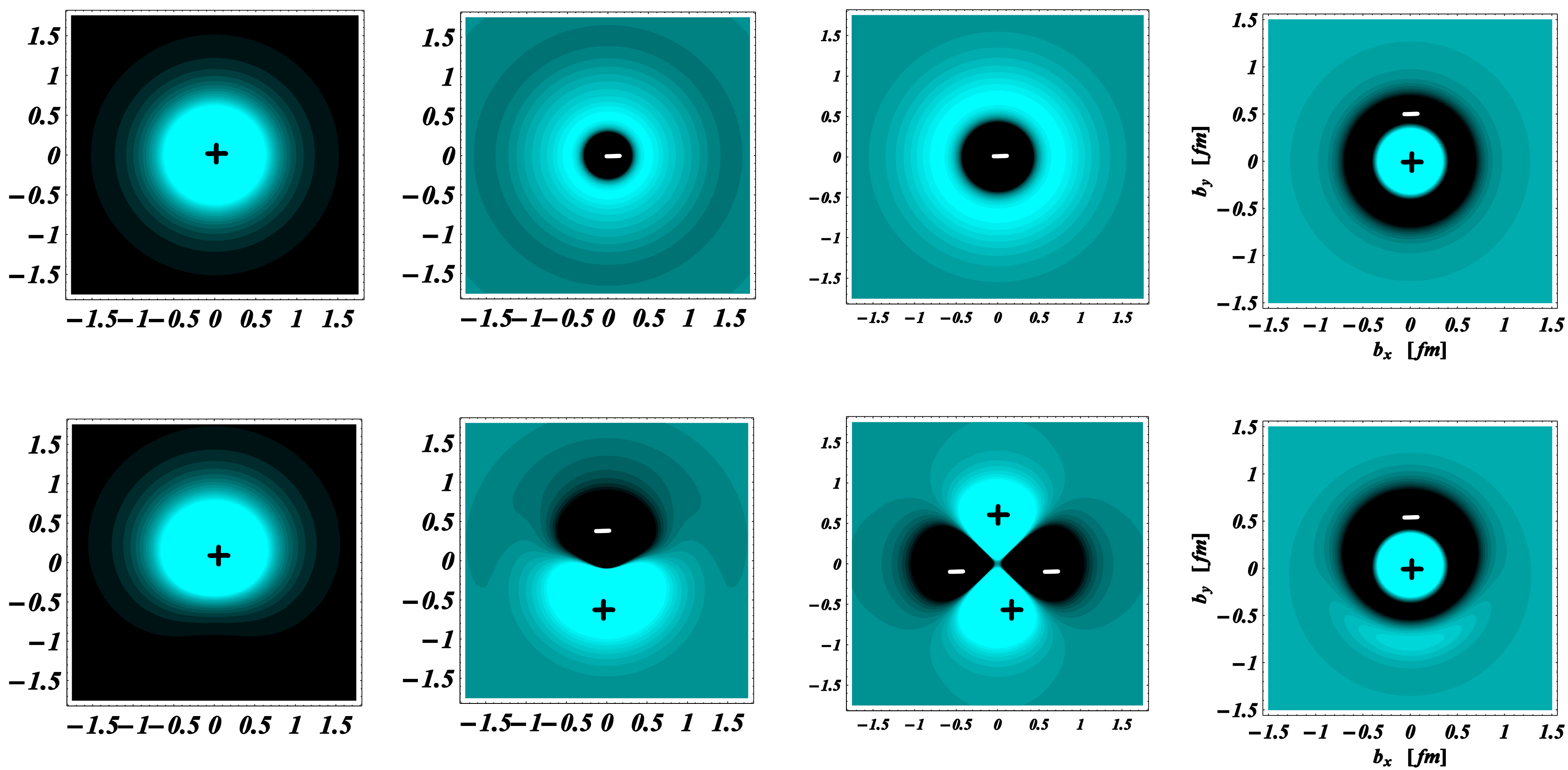




Tiator, Vdh (2008)

empirical transverse transition densities

p | n | p -> Δ^+(1232) | p -> N^*(1440)



Carlson, Vdh (2007)

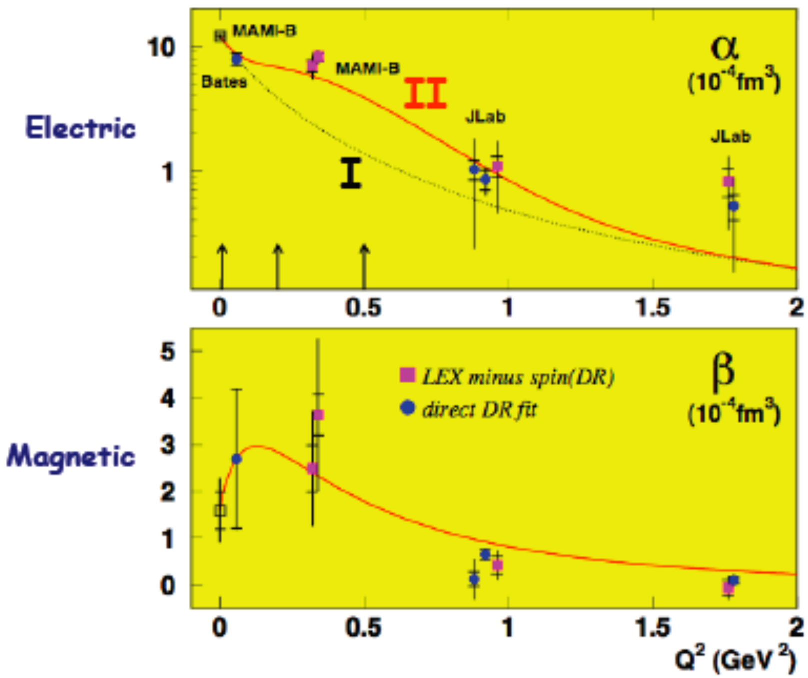
quadrupole pattern

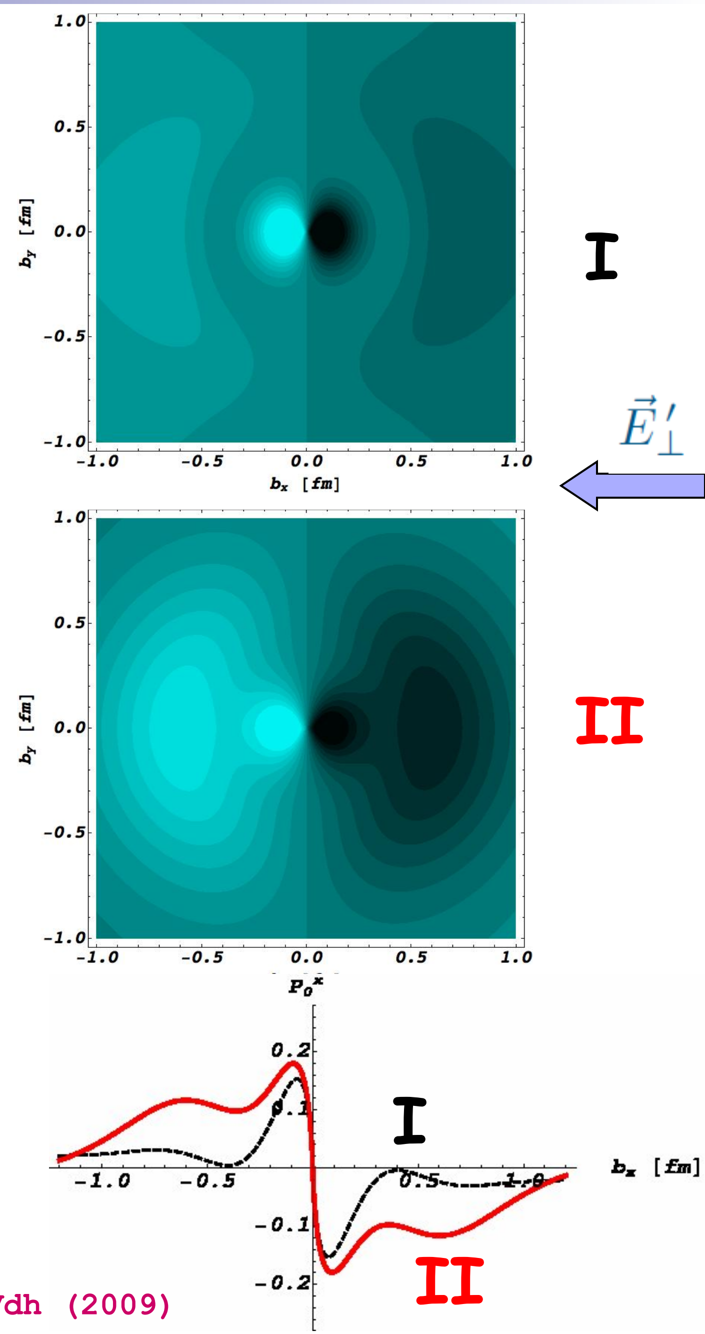
Tiator, Vdh (2008)

induced polarization in proton

$$\vec{P}_0(\vec{b}) = \int \frac{d^2\vec{q}_\perp}{(2\pi)^2} e^{-i\vec{q}_\perp\cdot\vec{b}} \vec{P}_0(\vec{q}_\perp)$$

$$= \hat{b} \int_0^\infty \frac{dQ}{(2\pi)} Q\, J_1(b\,Q)\, A(Q^2)$$





Gorchtein, Lorcé, Pasquini, Vdh (2009)