Muon $g - 2$ and QCD Sum Rules

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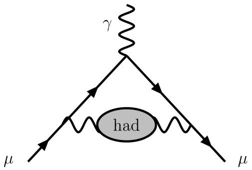
Anomalous magnetic moment of the muon

$a_\mu^{EXP} = 11\,659\,208.9\,(6.3) \times 10^{-10}$		
$a_\mu^{SM} = 11\,659\,180.2\,(4.2)\,(2.6)\,(0.2) \times 10^{-10}$		[1]
$\Delta a_\mu = 28.7\,(8.0) \times 10^{-10}$	$3.6\,\sigma$	
$a_\mu^{SM} = 11\,659\,182.8\,(4.9) \times 10^{-10}$		[2]
$\Delta a_\mu = 26.1\,(8.0) \times 10^{-10}$	$3.3\,\sigma$	

[1] Davier, Hoecker, Malaescu, Zhang, 2010
[2] Hagiwara et al. 2011

Dominating source of uncertainty:
Lowest order hadronic contribution



- Theoretical framework:
  Perturbative Quantum ChromoDynamics and
  Operator Product Expansion ➜ QCD sum rules
- Input: e^+e^- data and condensates
- Results, assumptions and uncertainties

Talk based on arXiv:1302.1735, work with
S. Bodenstein, C. A. Dominguez and K. Schilcher

Current-current correlator

$$\Pi^{JJ}_{\mu\nu}(q^2) = i \int d^4x \; e^{iqx} \; \langle 0 | T(J^\dagger_\mu(x) \, J_\nu(0)) | 0 \rangle$$
$$= \left(-q^2 g_{\mu\nu} + q_\mu q_\nu\right) \; \Pi^{(0+1)}_{JJ}(q^2)$$

$$J^{\mathrm{em}}_\mu(x) = \sum_f Q_f \bar{q}_f(x) \gamma^\mu q_f(x)$$

Spectral function $\mathrm{Im}\,\Pi^{\mathrm{em}}(s)$ from data on e^+e^- annihilation to hadrons
Unitarity (optical theorem)

$$R(s) = \frac{\sigma_{\mathrm{tot}}(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

$$\sigma(e^+e^- \to \mu^+\mu^-) = \frac{4\pi\alpha^2_{\mathrm{em}}}{3s}$$

From QCD, valid at large s:

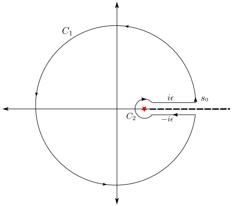
$$R(s) = 12\,\pi \; \mathrm{Im}\,\Pi^{\mathrm{em}}(s) = 3 \sum_{f=u,d,s} Q_f^2 \left(1 + \frac{\alpha_s}{\pi} + \ldots\right)$$

$$a_\mu^{had,LO} = \int_{s_{th}}^{\infty} \widetilde{K}(s)R(s)ds$$

$\widetilde{K}(s) \sim 1/s^2$ ➜ about 92 % from low-energy region below $\sqrt{s_0} = 1.8$ GeV

Cauchy's theorem: for any analytic function $p(s)$:

$$\int_{s_{th}}^{s_0} p(s)R(s)ds = 6\pi i \oint_{|s|=s_0} p(s)\Pi(s)ds$$



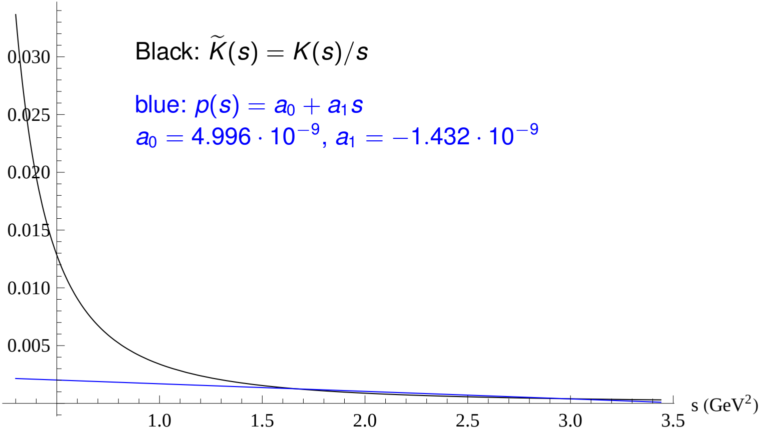
Quark-hadron duality: replace $\Pi(s)$ by $\Pi_{OPE}(s)$ in the integral around the circle

➜ Low-energy part of $a_\mu^{had,LO}(s_0) = \int_{s_{th}}^{s_0} \widetilde{K}(s)R(s)ds$:

$$\tilde{a}_\mu^{had,LO}(s_0) = \int_{s_{th}}^{s_0} \left(\widetilde{K}(s) - p(s)\right) R(s)ds + 6\pi i \oint_{|s|=s_0} p(s)\Pi_{OPE}(s)ds$$

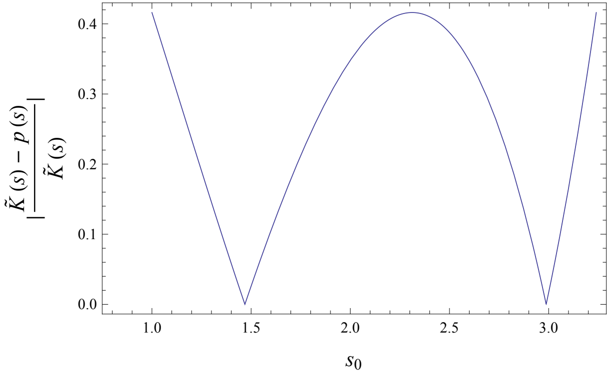
Choose $p(s)$ to suppress the contribution of e^+e^- data in $R(s)$

Pinched kernel



- Optimize suppression in 1 GeV $< \sqrt{s} <$ 1.8 GeV
- No power s^n with $n \geq 2$ to avoid higher dimension condensates

Pinched kernel



Suppression factor: smaller than 0.4 in 1 GeV $< \sqrt{s} <$ 1.8 GeV

$$\Pi_{OPE}(s) = C_0 + \sum_{N=0} \frac{C_{2N+2}}{s^{N+1}} \left\langle \mathcal{O}_{2N+2}(\mu^2) \right\rangle$$

C_0: from perturbative QCD

$$\Pi_{pQCD}(s) = \frac{1}{16\pi^2} \sum Q_f^2 \left[\frac{20}{3} + \ln\left(-\frac{s}{\mu^2} \right) + \ldots + O(\alpha_s^5) \right]$$

Gorishnii, Surguladze, Chetyrkin, Dine, Celmaster; Chetyrkin, Kniehl, Steinhauser, ...

Dimension 2: no non-perturbative condensate,
but $O(m_f^2/s)$ contributions from pertrubative QCD:
known to 3-loop order

Chetyrkin 1997

$$C_2 \langle \mathcal{O}_2 \rangle = \sum_f \frac{Q_f^2}{4\pi^2} \frac{\overline{m}_f^2(\mu)}{s} \left[6 + O(\alpha_s) \right]$$

PDG values:

$$\overline{m}_u(2\,\mathrm{GeV}) = (2.3 \pm 0.7)\ \mathrm{MeV}$$
$$\overline{m}_d(2\,\mathrm{GeV}) = (4.8 \pm 0.7)\ \mathrm{MeV}$$
$$\overline{m}_s(2\,\mathrm{GeV}) = (95 \pm 5)\ \mathrm{MeV}$$

Dimension 4:

$$C_4\langle\mathcal{O}_4\rangle = \frac{1}{s^2}\sum_f Q_f^2 \left\{\left[\frac{1}{12} - \frac{11}{216}\frac{\alpha_s(\mu)}{\pi}\right]\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle + \left[2 - \frac{2}{3}\frac{\alpha_s}{\pi}\right]\overline{m}_f(\mu)\langle q_f\bar{q}_f\rangle(\mu)\right\}$$

$\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle = 0.04 \pm 0.01\ \mathrm{GeV}^4$ from lattice QCD

$\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle = 0.01 \pm 0.01\ \mathrm{GeV}^4$ from QCD sum rules using ALEPH τ decay data

We use $\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle = 0.015 \pm 0.015\ \mathrm{GeV}^4$

Uncertainty from gluon condensate dominates, quark condensate small
(and known from the Gell Mann-Oakes-Renner relation)

Also include QED corrections to the pQCD correlator:

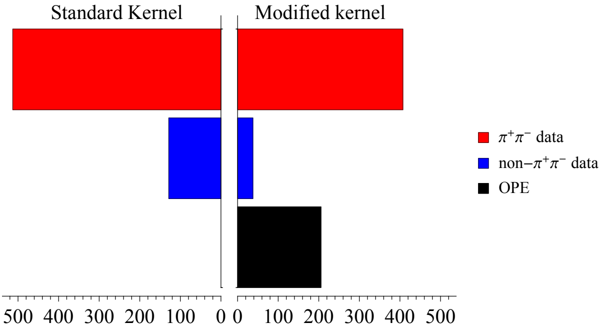
$$\Pi_{QED}(s) = \frac{3}{16\pi^2} \sum_f Q_f^4 \left[\frac{55}{12} - 4\zeta_3 - \ln\left(-\frac{s}{\mu^2}\right)\right] \frac{\alpha_{em}}{\pi}$$

On the data: FSR corrections, treated as additional 1 % systematic error

- Use only most recent data for a given exclusive hadronic final state in a given energy range
  ➜ No overlapping data sets, no averaging over (sometimes inconsistent) data
- $\pi^+\pi^-$ data from BaBar (2009) with (conservative) 100 % correlations
- Above $\sqrt{s} = 2$ GeV, use BES inclusive data
- For some channels used estimates based on isospin arguments
- Corrected for vacuum polarization, if necessary
- Assume no correlations for statistical, and 100 % correlation for systematic errors
- Verified against Davier, Hoecker, Malaescu, Zhang, EPJC71

For details see
Bodenstein, Dominguez, Eidelman, Schilcher, H.S., JHEP 1, 39 (2012)

Contribution to $a_\mu^{had,LO}/10^{-10}$, $\sqrt{s} < 1.8$ GeV



Our result for the standard kernel: $a_\mu^{had,LO}(s_0) = (640.7 \pm 6.5_{data}) \times 10^{-10}$

Fixed Order Perturbation Theory (FOPT)

$$\begin{aligned}\tilde{a}_\mu^{had,LO}(s_0) &= (651.0 \pm 3.1_{data} \pm 1.9_{conv} \pm 1.0_{\alpha_s} \pm 1.3_{\langle G^2\rangle}) \times 10^{-10} \\ &= (651.0 \pm 4.0) \times 10^{-10}\end{aligned}$$

contribution from the data reduced to 445.1×10^{-10} (from 640.7×10^{-10})

Contour Improved Perturbation Theory (CIPT)

$$\begin{aligned}\tilde{a}_\mu^{had,LO}(s_0) &= (649.4 \pm 3.1_{data} \pm 1.4_{conv} \pm 0.8_{\alpha_s} \pm 1.3_{\langle G^2\rangle}) \times 10^{-10} \\ &= (649.4 \pm 3.8) \times 10^{-10}\end{aligned}$$

Average of the FOPT and CIPT:

$$\begin{aligned}\tilde{a}_\mu^{had,LO}(s_0) &= (650.2 \pm 3.1_{data} \pm 1.7_{conv} \pm 0.9_{\alpha_s} \pm 1.3_{\langle G^2\rangle} \pm 0.8_{int}) \times 10^{-10} \\ &= (650.2 \pm 4.0) \times 10^{-10}\end{aligned}$$

(for NLO: $a_\mu^{had,NLO}(s_0) = (-10.1 \pm 0.6) \times 10^{-10}$, $\tilde{a}_\mu^{had,LO} - a_\mu^{had,NLO} = -0.2 \times 10^{-10}$)

$\alpha_s(M_Z) = 0.1184 \pm 0.0007$, $\quad \sqrt{s_0} = 1.8\,\text{GeV}$,
for $s > s_0$ use pQCD, see e.g. Bodenstein, Dominguez, Schilcher, PRD85

Standard kernel: $a_\mu^{had,LO}(s_0) = (640.7 \pm 6.5) \times 10^{-10}$

FOPT and CIPT average: $= (650.2 \pm 4.0) \times 10^{-10}$

➜ Shift of $+9.5 \times 10^{-10}$

Applied to a full calculation, e.g. of Davier et al.:
the discrepancy to experimental value

$$\Delta a_\mu = a_\mu^{EXP} - a_\mu^{SM} = (28.7 \pm 8.0) \times 10^{-10} \quad (3.6\,\sigma)$$

is reduced to

$$\Delta \tilde{a}_\mu = a_\mu^{EXP} - \tilde{a}_\mu^{SM} = (19.2 \pm 8.0) \times 10^{-10} \quad (2.4\,\sigma)$$

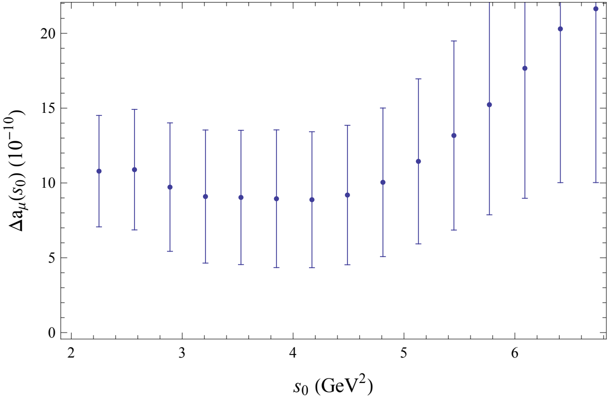
The analysis above includes errors from

- data: $\Delta a_\mu^{had,LO}(s_0) = \pm 3.1 \times 10^{-10}$
- estimate of perturbative expansion of QCD: $\pm 1.7 \times 10^{-10}$
- α_s: $\pm 0.9 \times 10^{-10}$
- gluon condensate: $\pm 1.3 \times 10^{-10}$
- difference of FOPT and CIPT: $\pm 0.8 \times 10^{-10}$

Now discuss

- Dependence on s_0
- Global quark-hadron duality: duality violations?
- Absence of dimension $d = 2$ term in the OPE?
- Data selection

Difference between standard and pinched kernels: $\delta a_\mu = \tilde{a}_\mu^{had,LO} - a_\mu^{had,LO}$



Both smaller or larger s_0 would increase a_μ, i.e. a smaller discrepancy

$$\Delta_{DV} = \Pi(s) - \Pi_{OPE}(s)$$

$$\oint_{|s|=s_0} p(s)\Pi(s)ds = \oint_{|s|=s_0} p(s)\left[\Pi_{OPE}(s) + \Delta_{DV}\right] ds$$

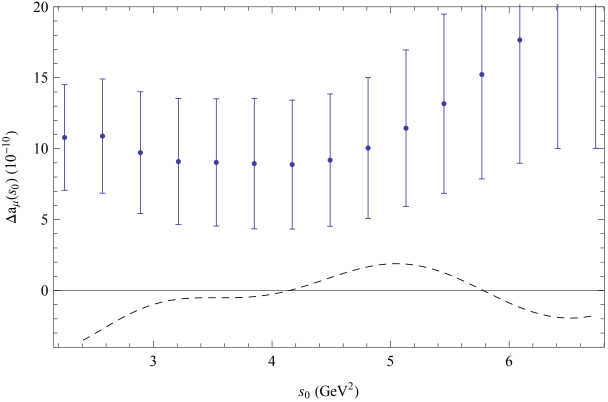
$$\frac{1}{\pi} Im\,\Delta_{DV}(s) = \sum_f Q_f^2 \left[\frac{5}{6}\kappa_V e^{-\gamma_V s}\sin(\alpha_V + \beta_V s) + \frac{1}{6}\kappa_V e^{-\gamma_V s}\sin(\alpha'_V + \beta_V s)\right]$$

Catà, Golterman, Peris

$$6\pi i \oint_{|s|=s_0} p(s)\,\Delta_{DV}\,ds = -12\pi \int_{s_0}^{\infty} p(s) Im\,\Delta_{DV}(s)\,ds = (-0.59 \pm 0.59) \times 10^{-10}$$

(Error from uncertainties in the model parameters)

Duality violations not large enough to bring $\Delta\tilde{a}_\mu$ in agreement with Δa_μ



Dashed line: contribution from duality violations

Renormalons, small-size strings give rise to a static potential $\propto kr$
may create an effective, tachyonic gluon mass ?

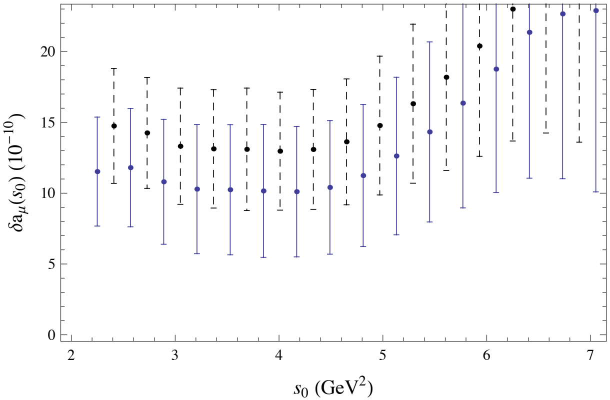
Chetyrkin, Narison, Zakharov, 1999

$$C_2 \langle \mathcal{O}_2 \rangle = \sum_f Q_f^2 \frac{1}{16\pi^2} \frac{\alpha_s}{\pi} \lambda^2 \left(\frac{128}{3} - 32\zeta_3 \right)$$

λ^2: tachyonic gluon mass,
$\lambda^2 < 0$, estimated in the range $-0.085\ \mathrm{GeV}^2 < \frac{\alpha_s}{\pi}\lambda^2 < -0.034\ \mathrm{GeV}^2$

➜ would increase a_μ by $(3.6 - 8.9) \times 10^{-10}$, i.e., decrease the discrepancy Δa_μ

$e^+e^- \to \pi^+\pi^-$ data: dashed lines using KLOE 2010, blue: using BaBar 2009



(From Davier et al EPJC71, 2011: $\pi^+\pi^-$ contribution to $a_\mu^{had,LO}/10^{-10}$:
514.1 ± 3.8 (BaBar), 503.1 ± 7.1 (KLOE), 506.6 ± 3.9 (CMD2), 505.1 ± 6.7 (SND),
507.8 ± 2.8 (average) ➜ 3.6σ discrepancy for average reduced to 2.8σ for BaBar)

- In the framework of perturbative QCD
  $+$ operator product expansion
  $+$ sum rules,
- assuming
  - quark-hadron duality,
  - absence of gauge non-invariant dimension 2 OPE term,
  - recent e^+e^- data,

  we find a
- reduction of the discrepancy between a_μ^{EXP} and $\tilde{a}_\mu^{SM}$ by 9.5×10^{-10}, i.e. to 2.4σ
- Further reduction would follow from
  - increasing s_0,
  - a different choice of data,
  - a hypothetical dimension-2 condensate

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Hadronic contributions to the muon anomalous magnetic moment:
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Coordinator:
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