Review of Recent Leading-Order Hadronic Vacuum Polarization Calculation

LABORATOIRE DE L'ACCÉLÉRATEUR LINÉAIRE

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UNIVERSITÉ PARIS SUD
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Outline

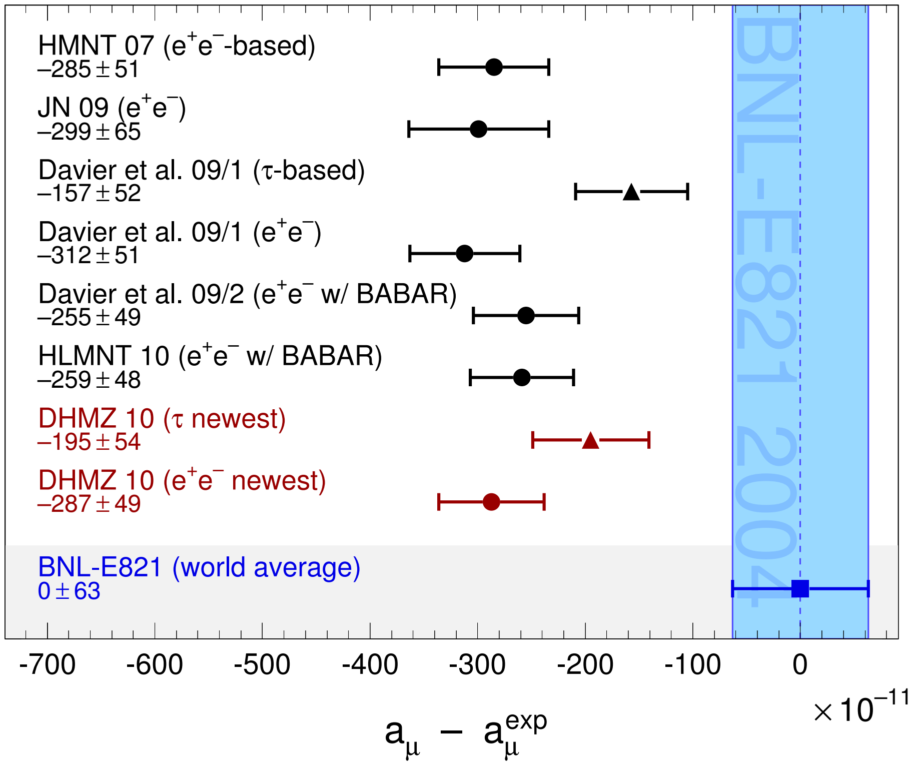
- Introduction
- Evaluation of hadronic contribution to a_μ
- A few open issues
- Conclusion & perspective

Introduction

The SM has been extremely successful:

- the only missing component Higgs now discovered (to be verified)
- all predictions verified often to extraordinary precision
- no sign of new physics with few exceptions, e.g. the muon g-2 anomaly

DHMZ 10 (arXiv:1010.4180v2)



Observed difference with experiment:

$$a_\mu^{exp} - a_\mu^{SM} = (29.6 \pm 8.1) \times 10^{-10}$$

➡ 3.6 "standard deviations"

Maybe the best hint for new physics

Why LO Hadronic Contribution?

All numbers shown in 10^{-10}

$$a_\mu^{SM} \equiv \left(\frac{g-2}{2}\right)_\mu = a_\mu^{QED} + a_\mu^{had,LO} + a_\mu^{had,NLO} + a_\mu^{weak}$$

$\sigma^{Exp} = 6.3$ | $\sigma_{QED}^{SM} \simeq 0.02$ | $\sigma_{had,LO}^{SM} \approx 4$ | $\sigma_{had,NLO}^{SM} \simeq \sigma_{had,LBLS}^{SM} \approx 3$ | $\sigma_{weak}^{SM} \approx 0.2$

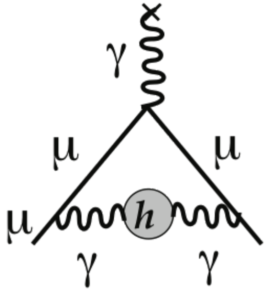
→ This & next talks

→ H. Spiesberger A. Vainshtein

→SM error on a_μ dominated by (LO) hadronic part, which is mostly related to the precision of experimental data

- either from e^+e^-
- or from tau

Hadronic Contribution a_μ^{had}



Leading-Order Higher-Order Light-By-Light

$$a_\mu^{\rm had} = a_\mu^{\rm had,LO} + a_\mu^{\rm had,HO} + a_\mu^{\rm had,LBL}$$
$$\simeq 700(\sim 7) - 9.79(0.09) + 10.5(2.6)$$
$$(\rightarrow \sim 4) \;\; - 9.84\ (0.07)$$

HO: Hagiwara et al., 2007
2011

LBL: Prades-de Rafael-Vainshtein, 2008

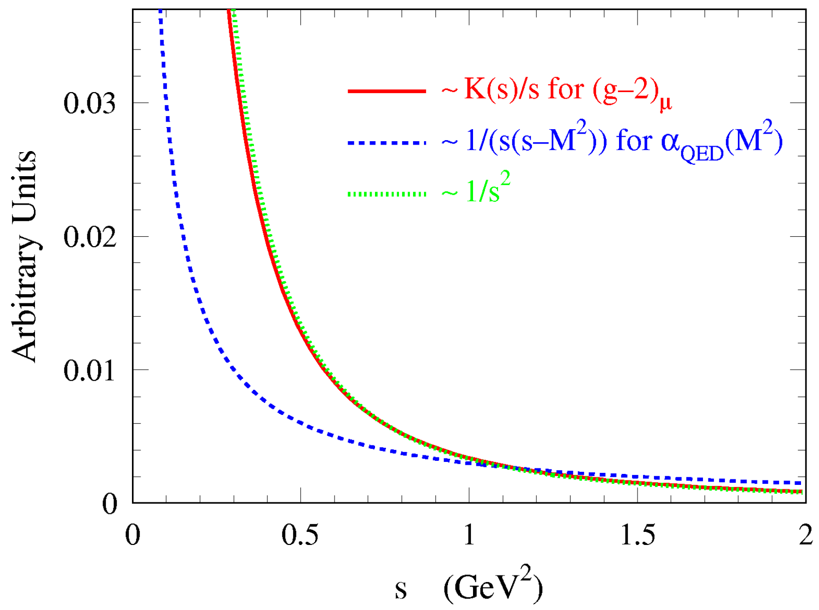
- Hadronic (q & g) loop contributions cannot reliably be calculated from perturbative QCD (pQCD)
- There are however lattice and model based attempts (→ M. Benayoun):

- S. Bodenstein, C. A. Dominguez, K. Schilcher, H. Spiesberger, arXiv:1302.1735v3 [hep-ph]
- M. Della Morte, B. Jäger, A. Jüttner and H. Wittig, JHEP 1203 (2012) 055, arXiv:1112.2894 [hep-lat]
- M. Benayoun, P. David, L. DelBuono, F. Jegerlehner, arXiv:1210.7184; Eur.Phys.J. C72 (2012) 1848 [arXiv:1106.1315]
- D. Greynat and E. de Rafael, JHEP 1207 (2012) 020 [arXiv:1204.3029]
- T. Goecke, C. Fischer, R. Williams, Phys. Lett. B704 (2011) 211 [arXiv:1107.2588]
- S. Bodenstein, C. A. Dominguez, and K. Schilcher, Phys. Rev. D85 (2012) 014029, arXiv:1106.0427

LO Hadronic Contribution a_μ^{had}

- We focus here on methods which provide the most precise estimate
- Use low energy e^+e^- data to calculate the dominant LO contributions:

$$a_\mu^{\rm had} = \frac{\alpha^2(0)}{3\pi^2}\int_{4\pi_\pi^2}^{\infty} ds \frac{K(s)}{s} R(s)\,, \qquad R(s) = \frac{\sigma(e^+e^- \to {\rm hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$



Dispersion relation: Bouchiat and Michel, 1961

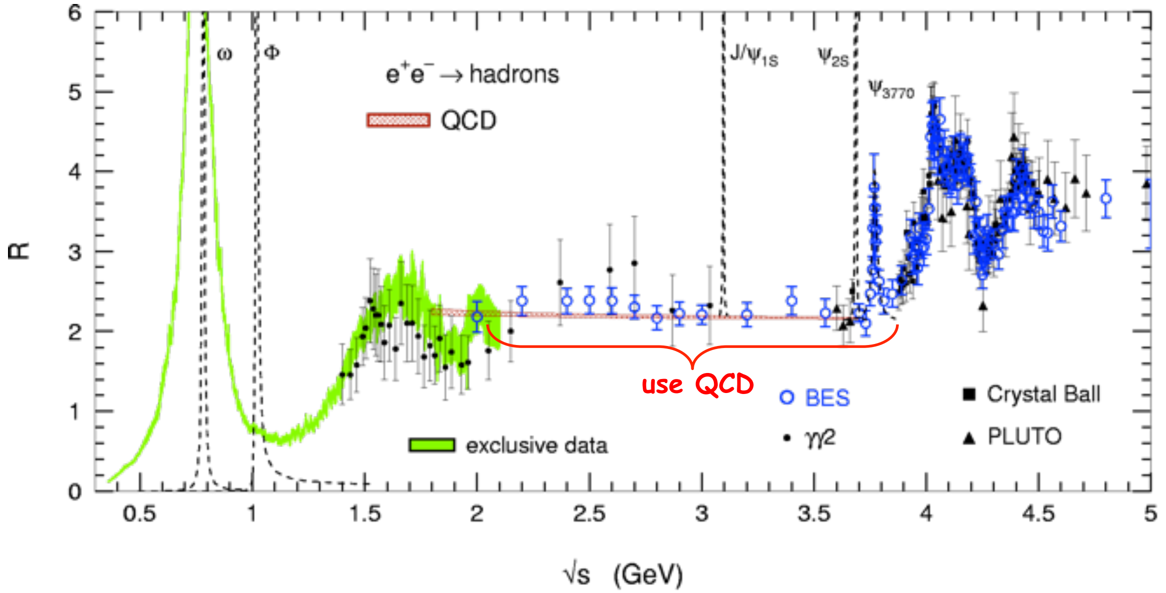
➔ The QED kernel K(s) has such an s dependence that low energy data contribute most

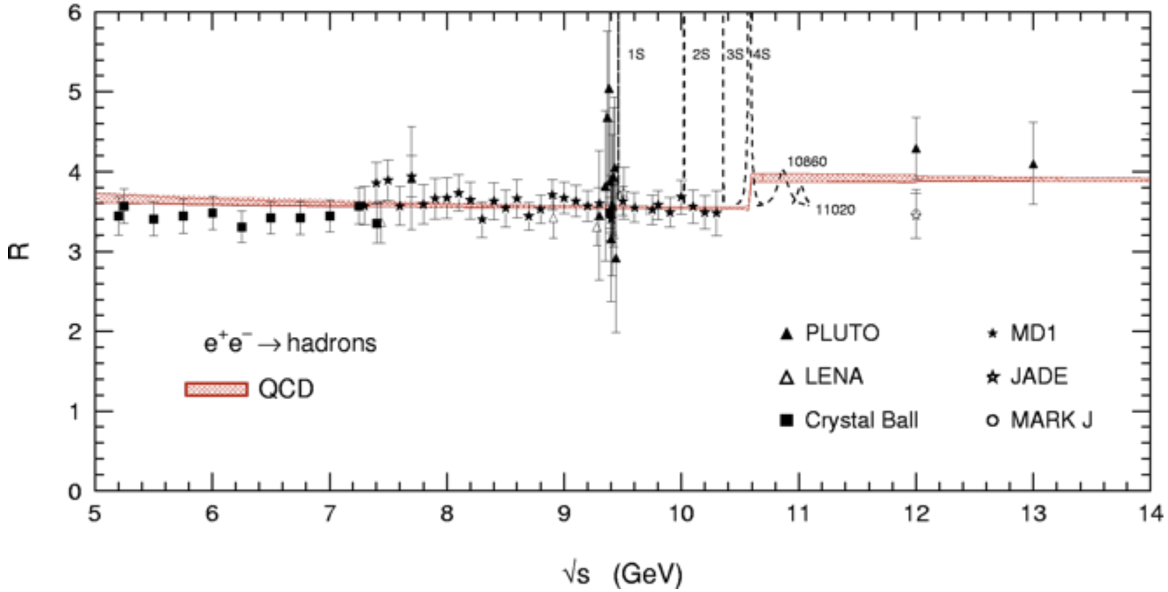
Brodsky, de Rafael, 1968

- Davier, Hoecker, Malaescu, and Zhang, Eur. Phys. J. *C71* (2011) 1, Erratum-ibid. *C72* (2012) 1874, arXiv: 1010.4180 [hep-ph], updated for FF workshop 2012
- Jegerlehner and Szafron, Eur. Phys. J. *C71*, 1632 (2011), 1101.2872
- Hagiwara, Liao, Martin, Nomura, and Teubner, J. Phys. *G* G38, 085003 (2011), 1105.3149

Input e^+e^- Data in Combination with pQCD

DHMZ 10



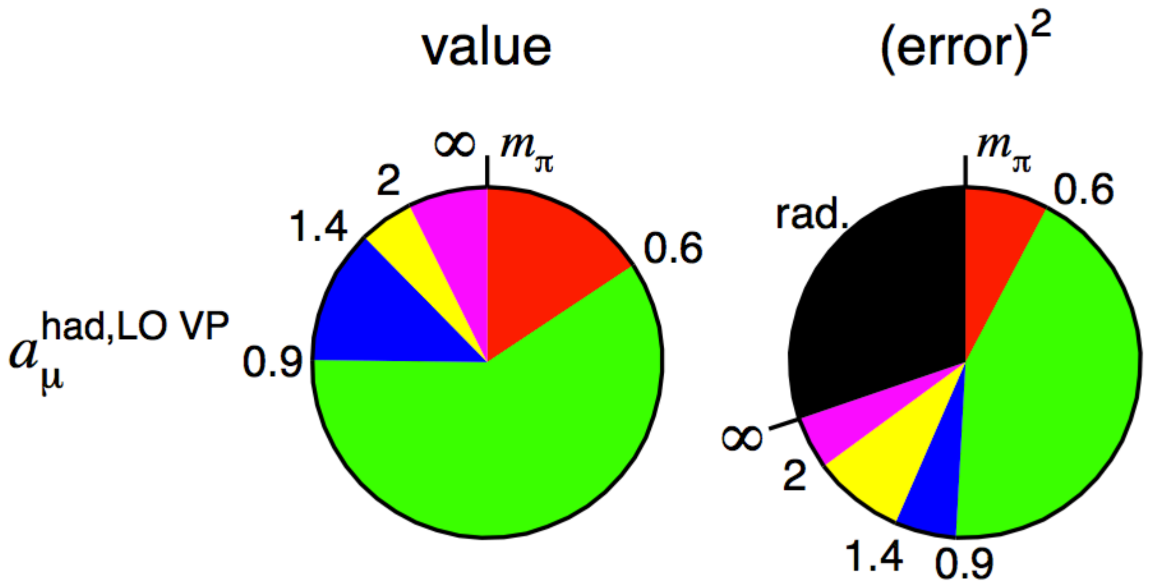


- $[\pi^0\gamma$-1.8GeV]
  - sum about 22 exclusive channels
  - estimate unmeasured using isospin relations
- [1.8-3.7] GeV
  - good agreement between data and pQCD calcultion → use 4-loop pQCD
  - J/ψ, $\psi(2s)$: Breit-Wigner integral
- [3.7-5] GeV
  use data
- >5GeV
  use 4-loop pQCD calculation

HLMNT 11 have similar treatment

Relative Contribution of Input Data vs Energy

HLMNT 11



➔ Energy region 0.6-0.9 GeV dominates in both value and uncertainty

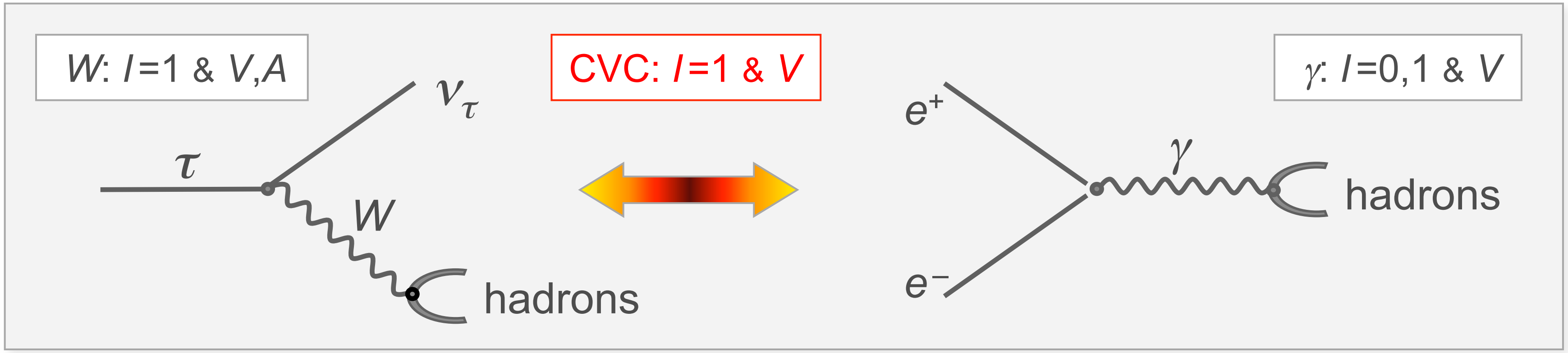
➔ 2π channel contributes more than 70%

➔ The e+e- data precision (was) limited

➔ Use (complement with) tau data

Alemany, Davier, Hoecker 1998

Connect τ and e^+e^- Data through CVC – SU(2)



Hadronic physics factorizes in Spectral Functions :

Isospin symmetry connects I=1 e^+e^- cross section to vector τ spectral functions:

$$\sigma^{(I=1)}\left[e^+e^- \to \pi^+\pi^-\right] = \frac{4\pi\alpha^2}{s}\upsilon\left[\tau^- \to \pi^-\pi^0\nu_\tau\right]$$

fundamental ingredient relating long distance (resonances) to short distance description (QCD)

$$\upsilon\left[\tau^- \to \pi^-\pi^0\nu_\tau\right] \propto \frac{\mathrm{BR}\left[\tau^- \to \pi^-\pi^0\nu_\tau\right]}{\mathrm{BR}\left[\tau^- \to e^-\bar{\nu}_e\nu_\tau\right]} \frac{1}{N_{\pi\pi^0}}\frac{dN_{\pi\pi^0}}{ds} \frac{m_\tau^2}{\left(1-s/m_\tau^2\right)^2\left(1+s/m_\tau^2\right)}$$

branching fractions | mass spectrum | kinematic factor (PS)

All known isospin breaking effects considered: Davier et al., Euro. Phys. J. C66 (2010) 127

What's New?

- KLOE has published new $e^+e^- \to \pi^+\pi^-$ cross section based on 2002 data.

  This time normalized to $e^+e^- \to \mu^+\mu^-$ cross section ➔ reduced syst errors

  → Talk of A. Passeri

- New data from Babar:
  - ✓ $e^+e^- \to 2\pi^+2\pi^-$ (arXiv:1201.5677)
  - ✓ $e^+e^- \to K^+K^-\pi^+\pi^-$, $K^+K^-\pi^0\pi^0$ (arXiv:1103.3001)

  → Talk of B. Malaescu

- Updated ALEPH tau data

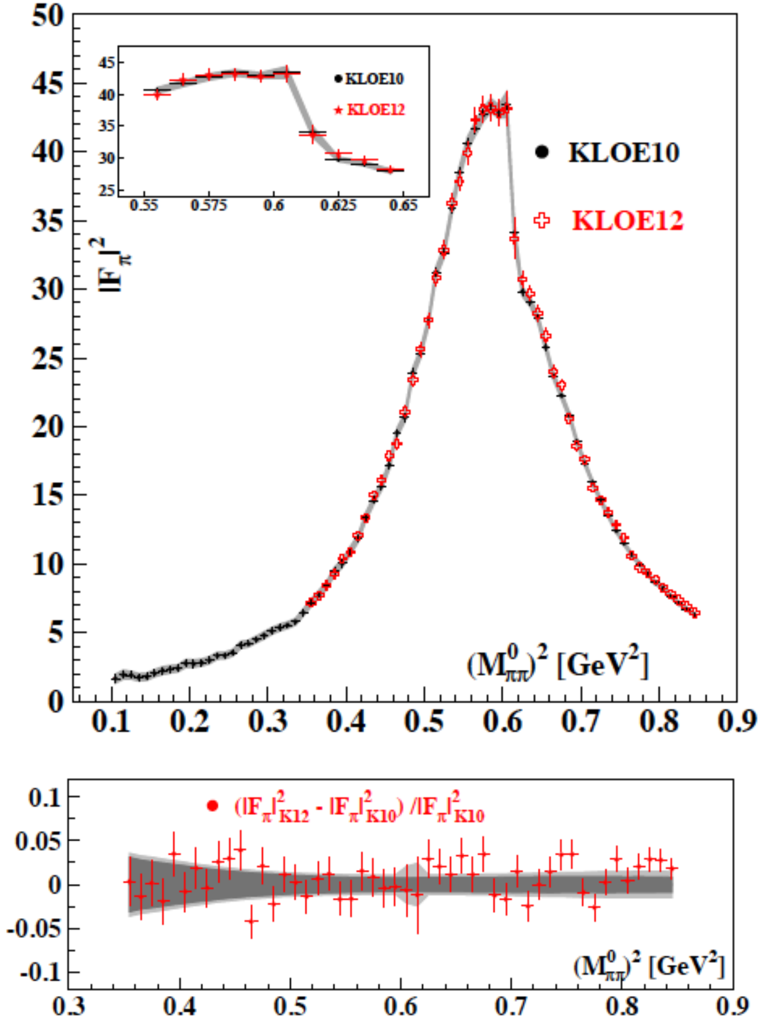
KLOE's New Measurements

- New measurement (Phys. Lett. B720 (2013) 336):
  KLOE 2012 (data of 2002, ~240pb^{-1}, 0.35-0.95GeV^2)

- Previous measurements (Phys. Lett. B 700 (2011) 102, Phys. Lett. B 670, (2009) 285):
  KLOE 2010 (large γ angle data 2006, ~233pb^{-1}, 0.1-0.85GeV^2)
  KLOE 2008 (small γ angle data 2002, ~240pb^{-1}, 0.35-0.95GeV^2)

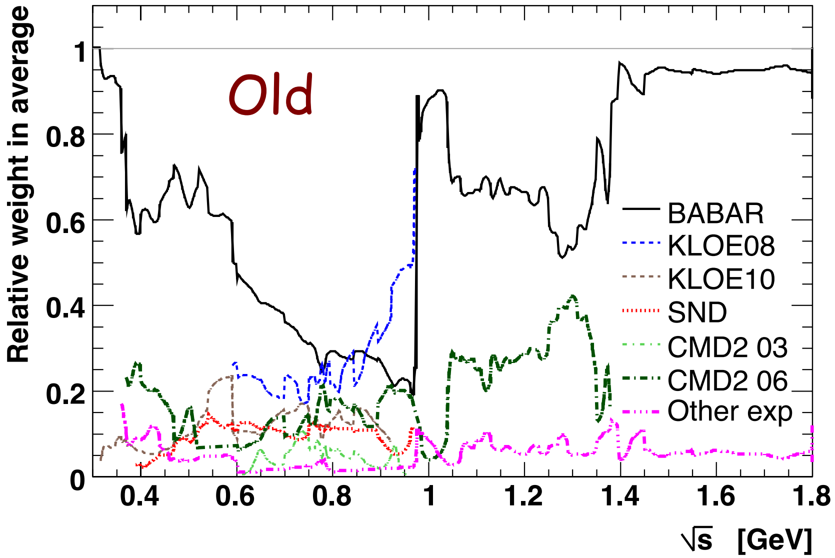
Measurement	$\Delta^{\pi\pi} a_\mu [0.35 - 0.85\ \mathrm{GeV}^2] \times 10^{10}$
KLOE 2012	$377.4 \pm 1.1_{\mathrm{stat}} \pm 2.7_{\mathrm{sys\&theo}}$
KLOE 2010	$376.6 \pm 0.9_{\mathrm{stat}} \pm 3.3_{\mathrm{sys\&theo}}$
	$\Delta^{\pi\pi} a_\mu [0.35 - 0.95\ \mathrm{GeV}^2] \times 10^{10}$
KLOE 2012	$385.1 \pm 1.1_{\mathrm{stat}} \pm 2.7_{\mathrm{sys\&theo}}$
KLOE 2008	$387.2 \pm 0.5_{\mathrm{stat}} \pm 3.3_{\mathrm{sys\&theo}}$

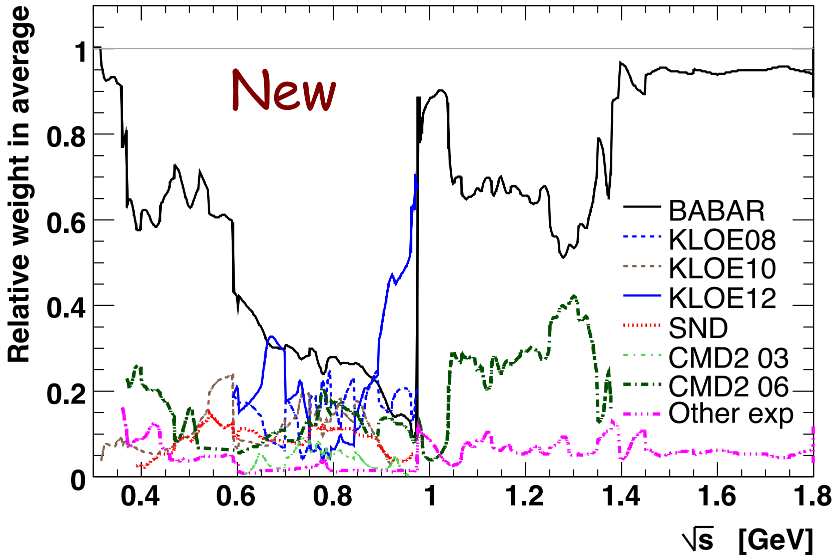
KLOE 2012 vs. 2010



Impact of KLOE12 Data in New Combination

A new combination performed including KLOE12 data by taking its correlation with previous data to our best knowledge

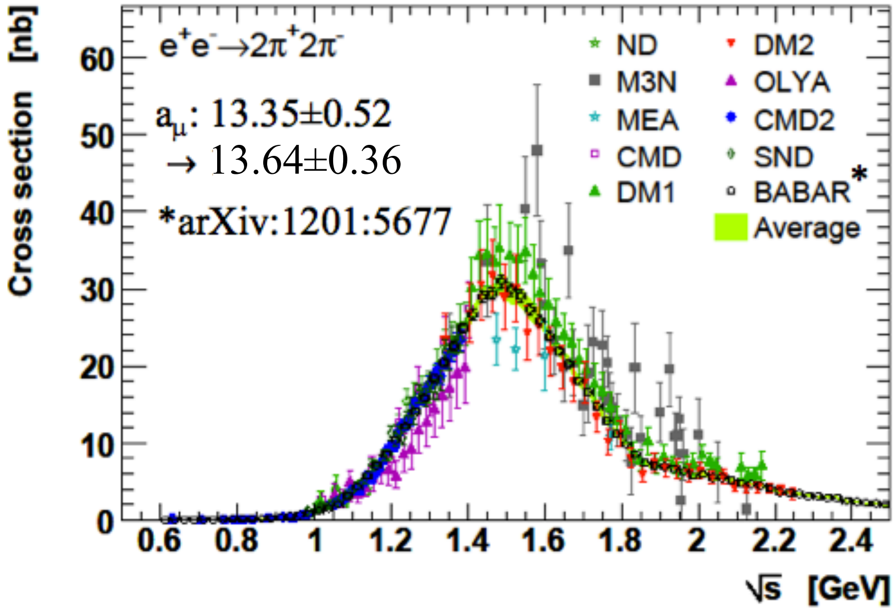


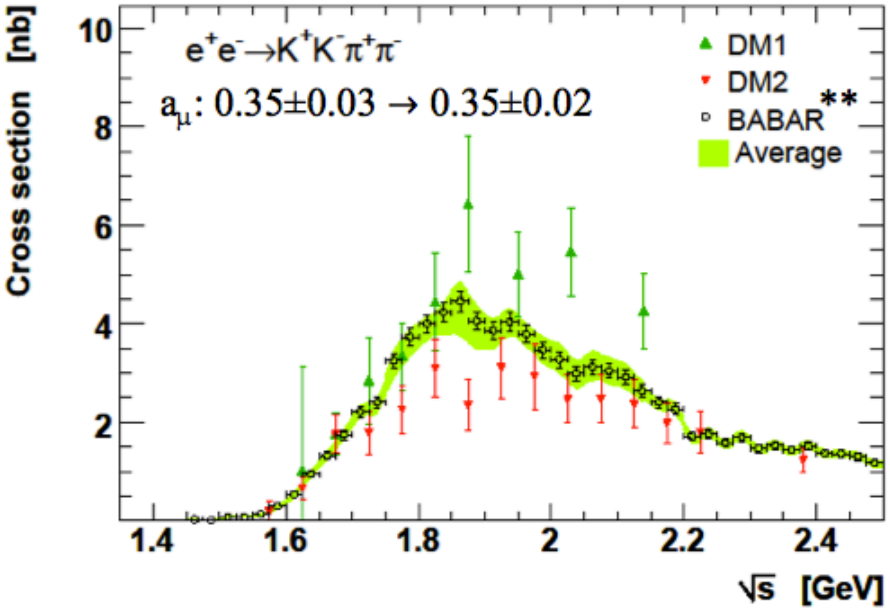


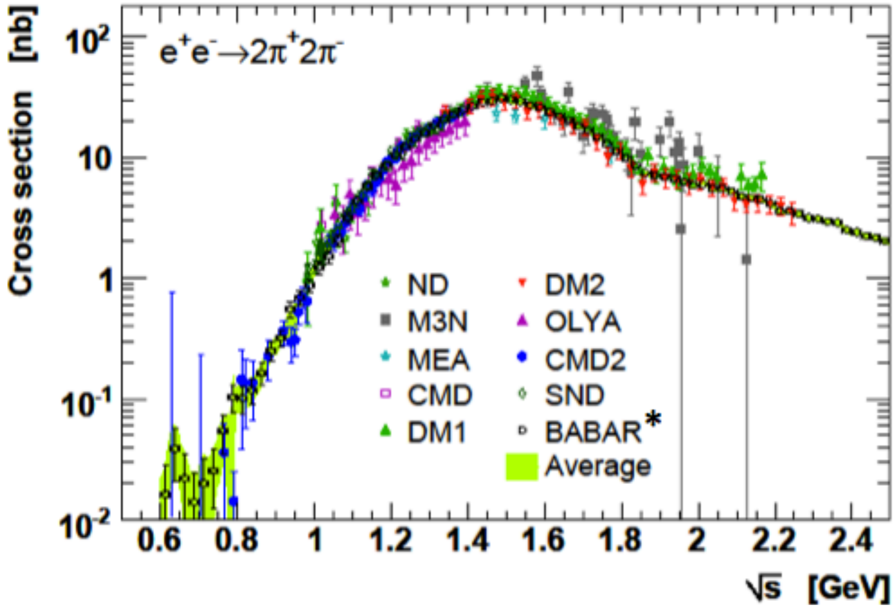
➔ KLOE12 takes over KLOE08 at 0.9-0.975GeV, the rest hardly changed

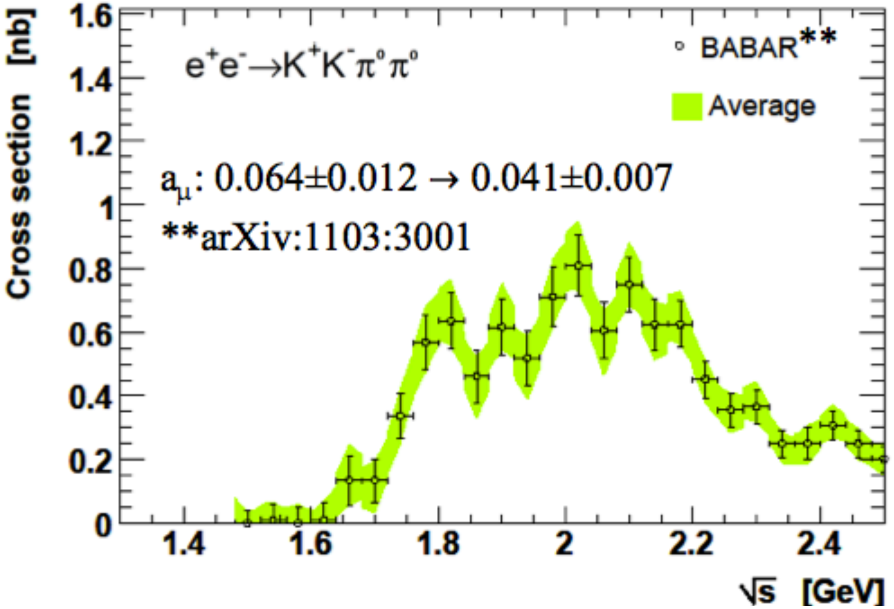
$a_\mu(2\pi, ee)$ 10^-10	0.3-1.8GeV
Old	$507.24 \pm 2.87_{stat} \pm 2.56_{syst}$
New	$506.56 \pm 2.61_{stat} \pm 2.38_{syst}$

Updated Channels – Latest Babar Results









Updated ALEPH Spectral Function (SF)

- One problem identified for the publicly available ALEPH $\pi\pi^0$ covariance matrix: treatment of statistical correlations from the SVD unfolding

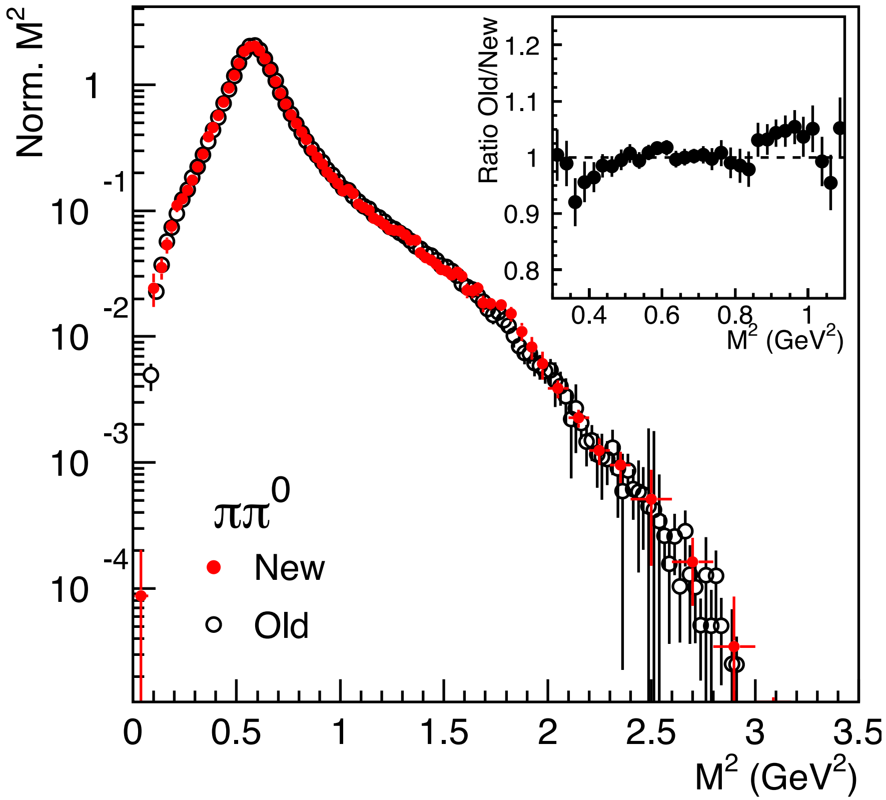
  Thanks to D. Boito for bringing this issue to our attention!

- Redone unfolding of the $\pi\pi^0$, $\pi2\pi^0$, $\pi3\pi^0$, $3\pi\pi^0$ and 3π spectral functions, using a new Iterative, Dynamically Stabilized (IDS) method (arXiv:0907.3791)

- Effects on rho parameter fit to $\pi\pi^0$, α_s determination, *g*-2 calculation are checked and found small

- Updated spectral functions to be released soon

Comparison of New and OLD SF: $\pi\pi^0$



The difference on the peak is within 2%, slightly larger off the peak

At low and high mass tails, larger bin size used in the new unfolding

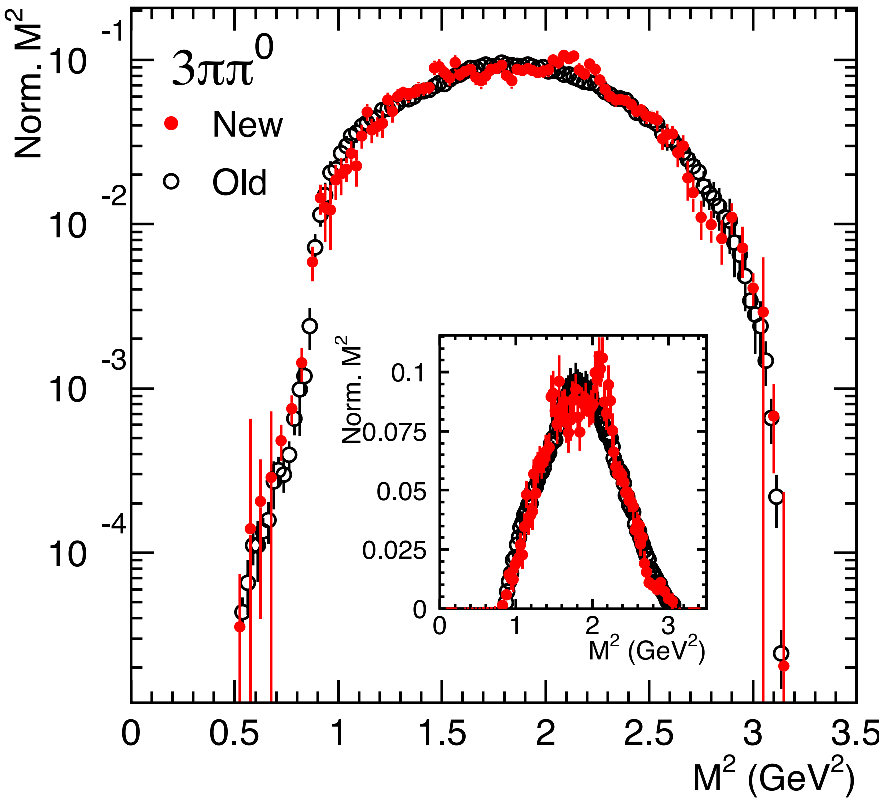
Old (Euro. Phys. J. C66 (2010) 127)

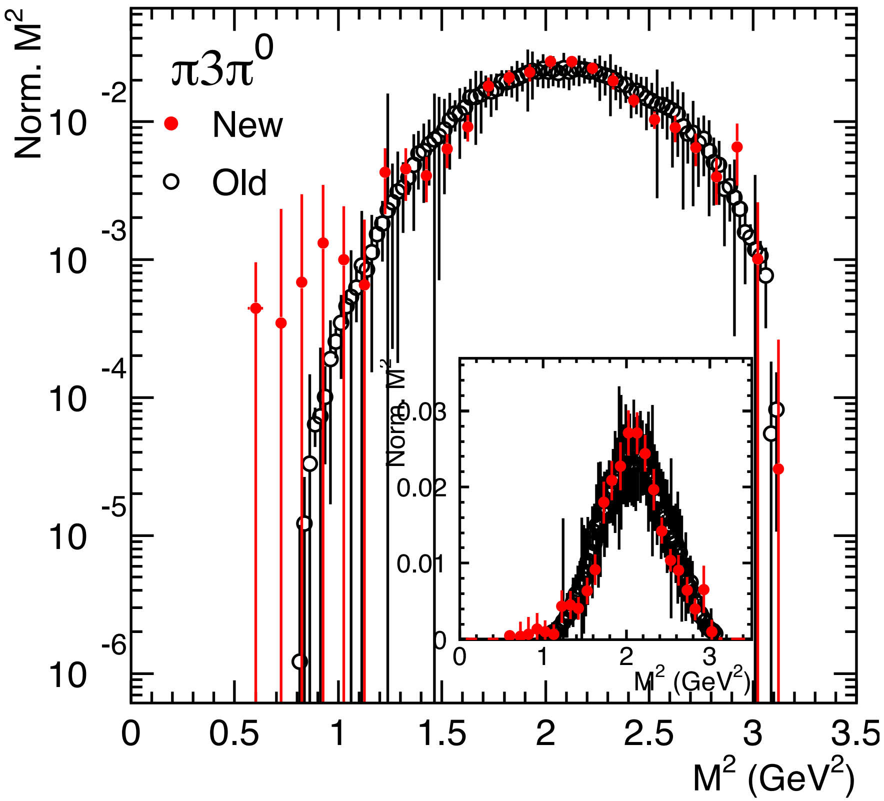
$a_\mu(\pi\pi)$ 10^-10	$2m_\pi$-0.36GeV	0.36-1.8GeV
ALEPH	$9.46\pm0.33_{exp}$	$499.19\pm5.20_{exp}$
Combined tau	$9.76\pm0.14_{exp}$	$505.46\pm1.97_{exp}$

New

$2m_\pi$-0.36GeV	0.36-1.8GeV
$9.80\pm0.40_{exp}$	$501.22\pm4.48_{exp}$
$9.82\pm0.13_{exp}$	$506.35\pm1.86_{exp}$

Comparison of New and OLD SF: $3\pi\pi^0$, $\pi 3\pi^0$





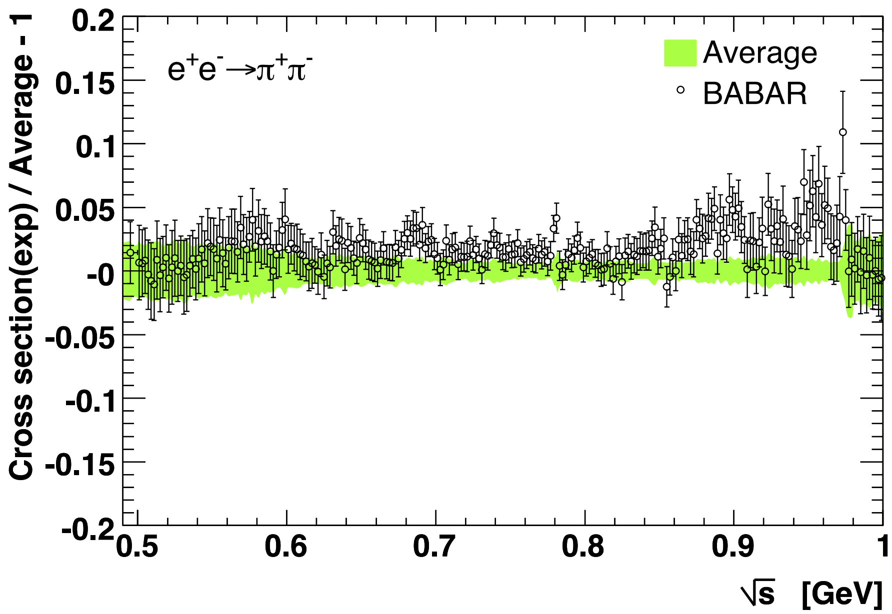
$a_\mu(2\pi 2\pi^0)$ 10^-10	Thre.-1.5GeV
Old	$14.87\pm1.22_{exp}\pm1.02_{Br}$
New	$14.70\pm0.28_{exp}\pm1.01_{Br}$

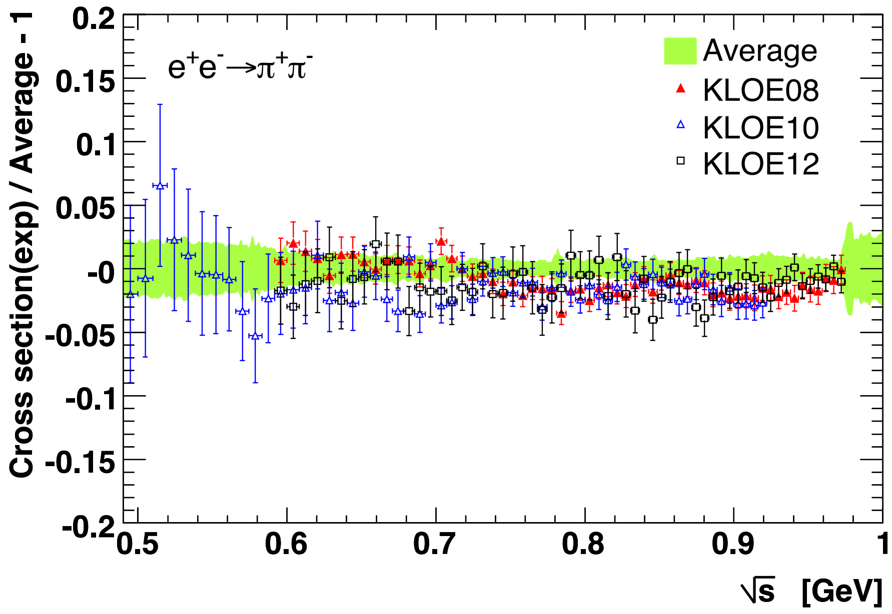
$a_\mu(4\pi)$ 10^-10	Thre.-1.5GeV
Old	$6.31\pm1.32_{exp}\pm0.42_{Br}$
New	$7.07\pm0.41_{exp}\pm0.48_{Br}$

Note: the old "exp" error contains normalization uncertainty, redundant with "Br" error, → fixed

Open Issue in $\pi^+\pi^-$ Channel (e^+e^-)

Average (green error band) obtained with HVPTools package (DHMYZ 09)





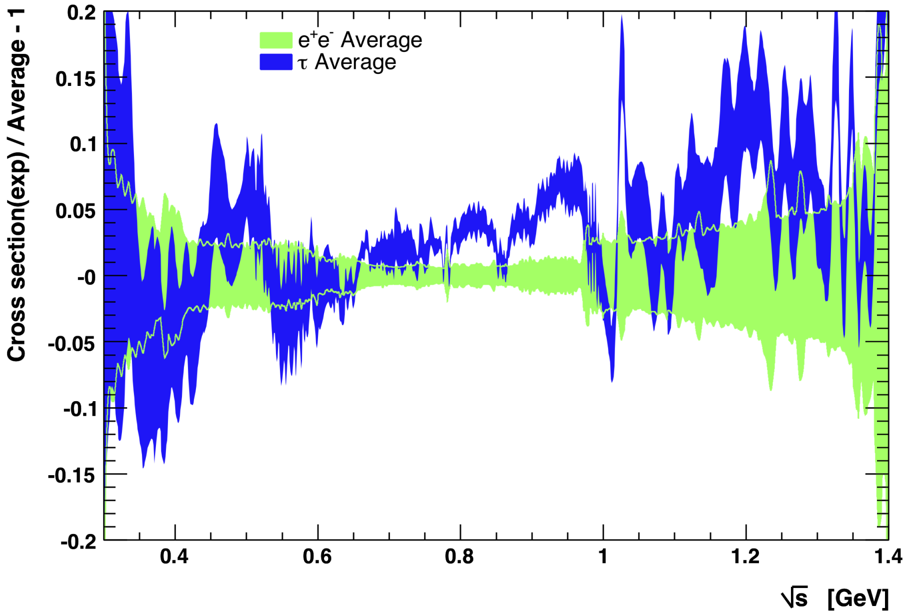
Shape difference between BaBar and KLOE:

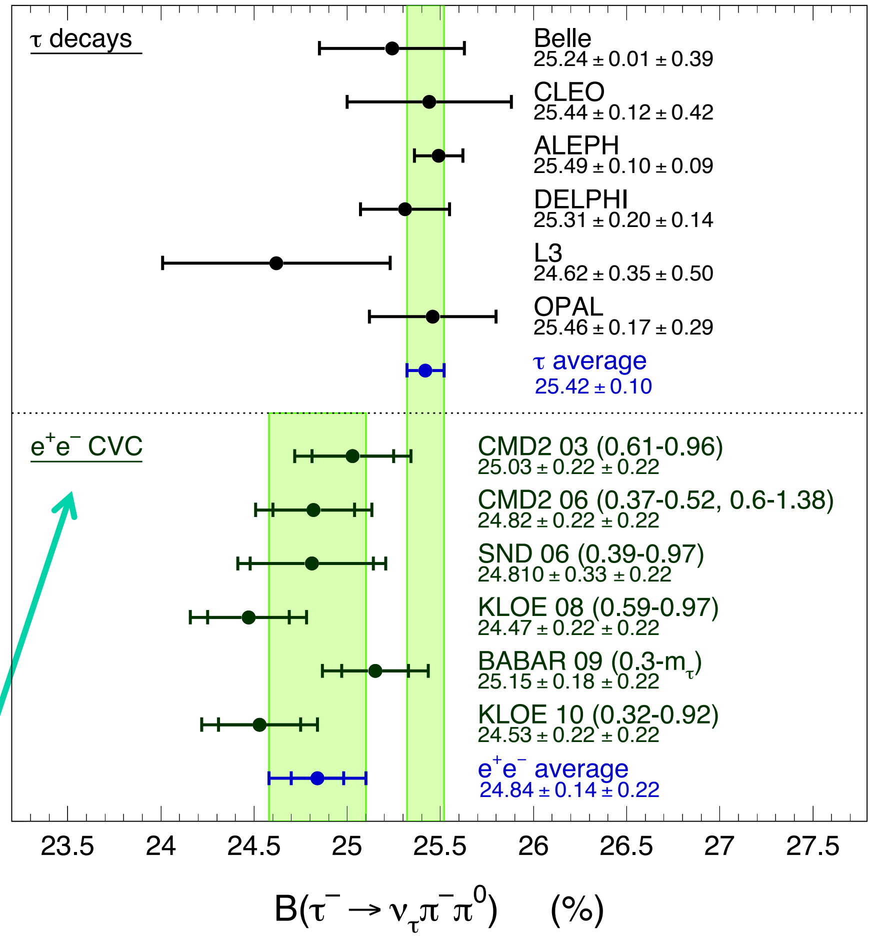
→ inflated error in the data combination (PDG prescription)

→ need new measurements to resolve the difference:

- KLOE provided data normalized to $\mu^+\mu^-\gamma$ instead of MC but these data are highly correlated with KLOE08 (→ talk by A. Passeri)
- VEPP-2000 in Novosibirsk (→ talk by S. Eidelman)

Open Issue in $\pi^+\pi^-$ Channel (e^+e^- vs. τ)





Good agreement between BABAR vs. Belle and CLEO
Conflict between KLOE and Tau data

$$\mathcal{B}_X^{\rm CVC} = \frac{3}{2}\frac{\mathcal{B}_e|V_{ud}|^2}{\pi\alpha^2 m_\tau^2}\int_{s_{\rm min}}^{m_\tau^2} ds\, s\, \sigma_{X^0}^I \left(1-\frac{s}{m_\tau^2}\right)^2 \left(1+\frac{2s}{m_\tau^2}\right)$$

All known isospin breaking effects studied/taken into account in M. Davier et al., Eur. Phys. J. C66, 127 (2010), [arXiv:0906.5443].

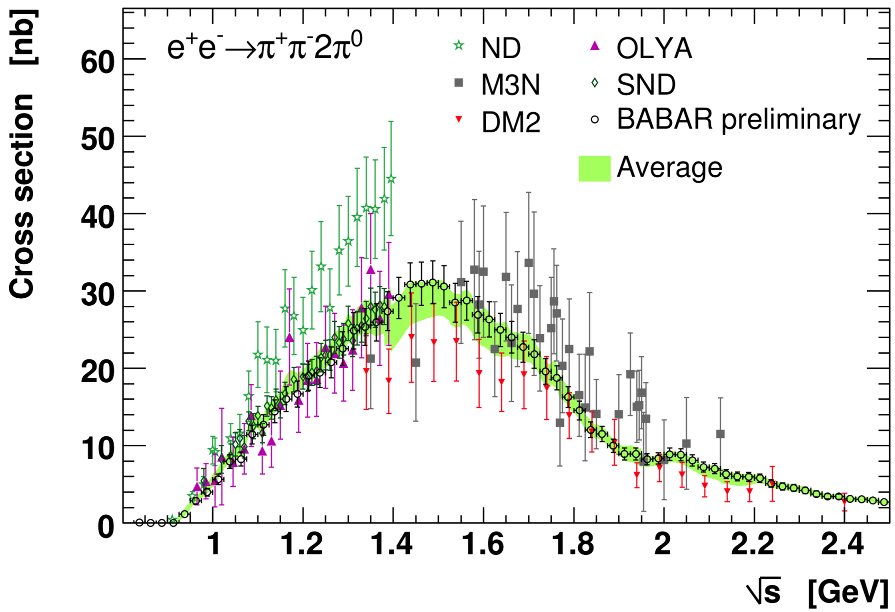
➔ 2.9σ (2006) → 2.4σ

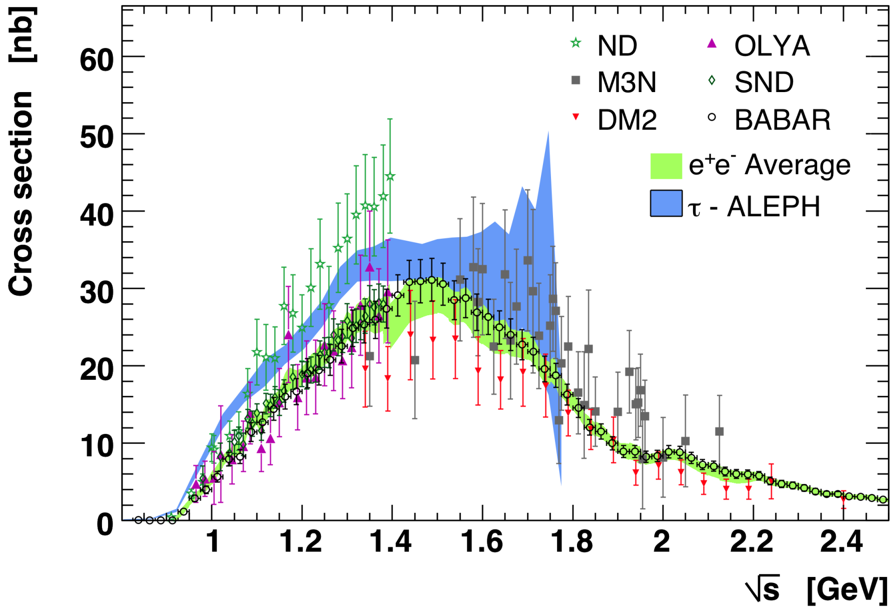
Jegerlehner and Szafron claim in Eur. Phys. J. C71, 1632 (2011), [arXiv:1101.2872] that missing ρ^0-γ mixing in τ data can explain e^+e^- and τ difference

Problematic $\pi^+\pi^-2\pi^0$ Channel

Old contribution: 16.8 ± 1.3 (Davier-Eidelman-Hoecker-Zhang, 2006)
Update 2009: 18.0 ± 1.2 including preliminary ISR BaBar data:
A. Petzold, EPS-HEP (2007)

τ [thre.-1.8GeV] (2009): 21.4 ± 1.4





→ Large scattering among e^+e^- data sets

→ (Normalization) difference e^+e^- vs. τ needs clarification/cross-check

Comparison HLMNT ('11) and DHMZ ('10)

Channel	HLMNT 11	DHMZ (10) [10]	Difference
$\eta\pi^+\pi^-$	0.88 ± 0.10	1.15 ± 0.19	-0.27
K^+K^-	22.09 ± 0.46	21.63 ± 0.73	0.46
$K^0_S K^0_L$	13.32 ± 0.16	12.96 ± 0.39	0.36
$\omega\pi^0$	0.76 ± 0.03	0.89 ± 0.07	-0.13
$\pi^+\pi^-$	505.65 ± 3.09	507.80 ± 2.84	-2.15
$2\pi^+2\pi^-$	13.50 ± 0.44	13.35 ± 0.53	0.15
$3\pi^+3\pi^-$	0.11 ± 0.01	0.12 ± 0.01	-0.01
$\pi^+\pi^-\pi^0$	47.38 ± 0.99	46.00 ± 1.48	1.38
$\pi^+\pi^-2\pi^0$	18.62 ± 1.15	18.01 ± 1.24	0.61
$\pi^0\gamma$	4.54 ± 0.14	4.42 ± 0.19	0.12
$\eta\gamma$	0.69 ± 0.02	0.64 ± 0.02	0.05
$\eta 2\pi^+2\pi^-$	0.02 ± 0.00	0.02 ± 0.01	0.00
$\eta\omega$	0.38 ± 0.06	0.47 ± 0.06	-0.09
$\eta\phi$	0.33 ± 0.03	0.36 ± 0.03	-0.03
$\phi(\rightarrow$ unaccounted)	0.04 ± 0.04	0.05 ± 0.00	-0.01
Sum of isospin channels	5.98 ± 0.42	6.06 ± 0.46	-0.08
Total	634.28 ± 3.53	633.93 ± 3.61	0.35

Table 4 from HLMNT 11

➔ Different data combination and error treatment

Comparing Measurements with Predictions

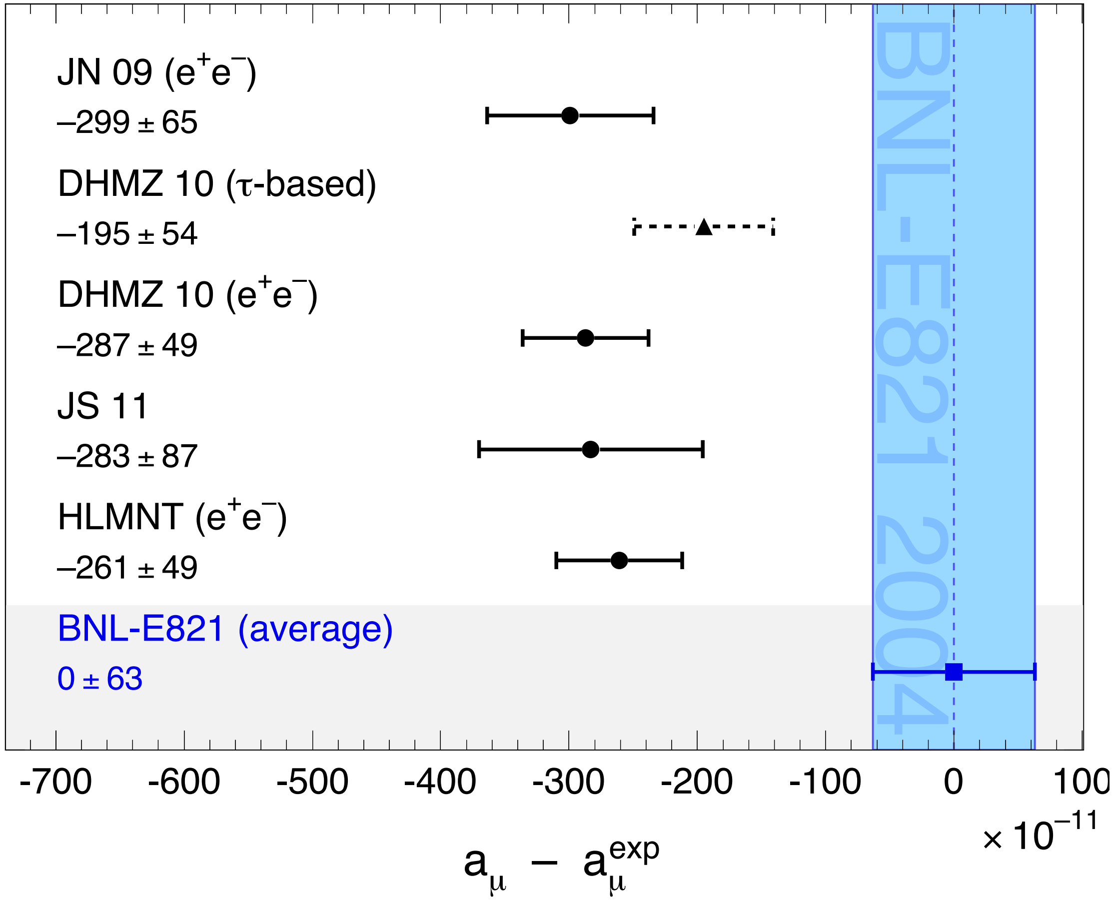
Measurement (BNL-E821)

PRD73(06)072003, hep-ex/0602035 — $11\,659\,208.9 \pm 5.4_{stat} \pm 3.3_{syst}$ $[10^{-10}]$

SM predictions:

QED	$11\,658\,471.809 \pm 0.014_{5th\ order} \pm 0.008_{\delta\alpha}$ Improved (Kinoshita et al.)	$[10^{-10}]$
HAD		
- LO	DHMZ10 e^+e^-: $692.3 \pm 4.2 \pm 0.2_{\psi} \pm 0.3_{QCD}$	$[10^{-10}]$
	HLMNT11 e^+e^-: $694.9 \pm 3.7 \pm 2.1_{rad}$	$[10^{-10}]$
	DHMZ10 τ: $701.5 \pm 4.2 \pm 0.3_{rad} \pm 1.9_{SU(2)}$	$[10^{-10}]$
- HO	-9.8 ± 0.1	$[10^{-10}]$
- LBL	10.5 ± 2.6	$[10^{-10}]$
Weak	15.4 ± 0.2	$[10^{-10}]$

Deviation Measurement & Prediction



DHMZ 10:

$692.3 \pm 1.4_{stat} \pm 3.1_{sys} \pm 2.4_{cor} \pm 0.2_{\psi} \pm 0.3_{QCD}$

Update FF workshop 12 (including latest BaBar 4π; 2K2π and 2K2π^0 results in the e^+e^- combination + latest QED calculation (Kinoshita et al.)):

$692.4 \pm 1.3_{stat} \pm 3.1_{sys} \pm 2.3_{cor} \pm 0.2_{\psi} \pm 0.3_{QCD}$

→ Deviation ee: $(28.5 \pm 8.0)\ 10^{-10}$ (3.6 σ)

Deviation τ: $(19.5 \pm 8.3)\ 10^{-10}$ (2.4 σ)

Conclusion & Perspective

Deviation 3.6σ is significant but not sufficient for claiming new physics
(the a_μ deviation and large H→$\gamma\gamma$ rate can however be explained by a stau, → Giudice, Paradisi and Strumia, arXiv:1207.6393)

On the prediction side:

- the tension in $\pi^+\pi^-$ channel between Babar (CMD2, SND) and KLOE prevents further error reduction
  ➔ New measurements (KLOE, VEPP-2000) timely
- the LBL has the subleading error contribution

On the experimental side:

- the experimental uncertainty currently dominates over prediction
  ➔ looking forward to a factor of 4 reduction from Fermilab & JPARC
  (→talk Mark Lancaster)