



Flavour and Lattice

FLAG activities

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Albert Einstein Center for Fundamental Physics

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Outline

Introduction: hadronic effects in flavour physics

FLAG

$f_+(0)$ and f_K/f_π : determination of V_{us} and V_{ud}

FLAG-2

Conclusions

Precision experiments

Interpretation of precision experiments:
hadronic contributions can be

- ▶ irrelevant

discovery of symmetry violations (e.g. $\mu \rightarrow e\gamma$ or $\mathbf{d}_n$)

- ▶ under good theoretical control

$K \rightarrow \pi \nu \bar{\nu}$

- ▶ measured indirectly elsewhere

hvp in $(g - 2)_\mu$

- ▶ important and not easily calculable $V_{us}/V_{ud}, \epsilon_K, \epsilon'/\epsilon, D \rightarrow KK/\pi\pi, \dots$

→ talks by U. Haisch, A. Lenz

Precision experiments

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Systematic, first-principle methods in hadronic physics:
Lattice, effective theories of QCD, dispersion relations

Precision experiments

Interpretation of precision experiments:
hadronic contributions can be

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Some claims of “tensions” of data vs. CKM interpretation are based on **lattice** calculations of hadronic matrix elements

Status of lattice calculations

Lattice calculations are not free from systematic effects:

- ▶ unphysical quark masses (not always) → talk of C. Hölbling
- ▶ finite lattice spacing
- ▶ finite volume
- ▶ operator renormalization (for specific quantities)
- ▶ isospin breaking

Status of lattice calculations

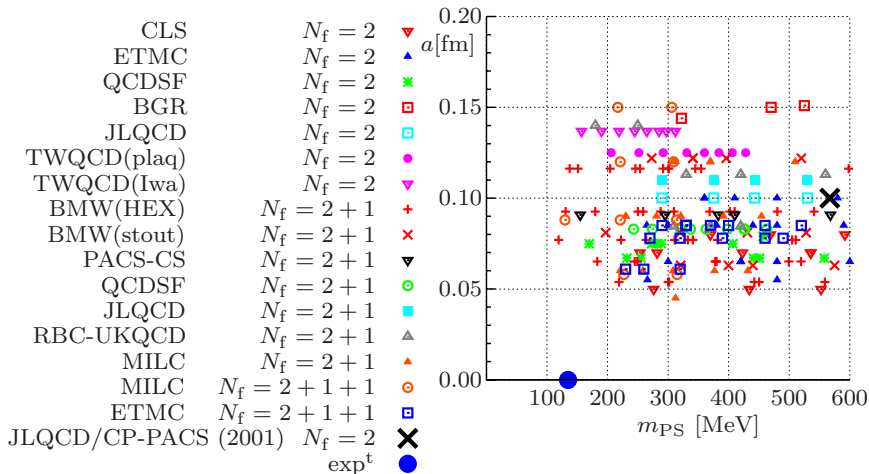
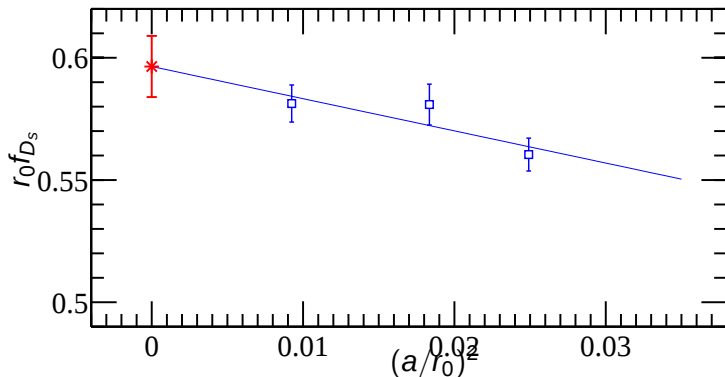


Figure courtesy of Gregorio Herdoiza

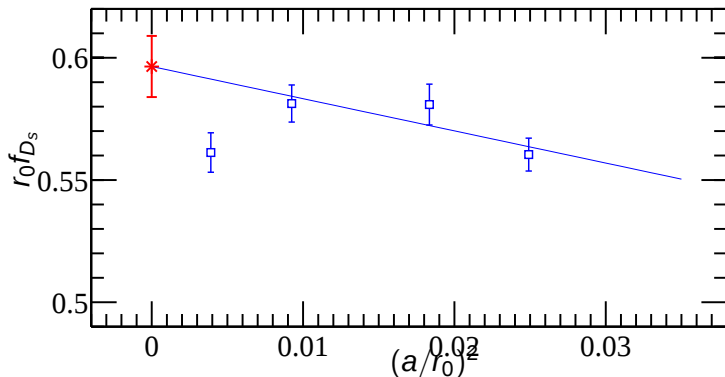
Status of lattice calculations

the **continuum** extrapolation can be tricky



Status of lattice calculations

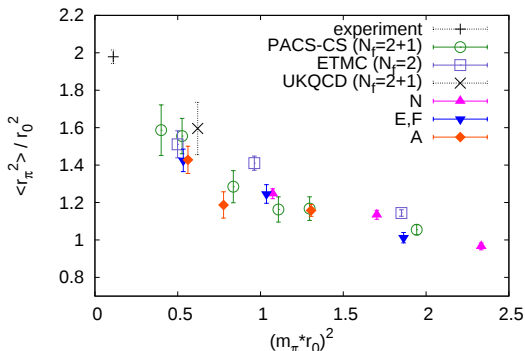
the **continuum** extrapolation can be tricky



Status of lattice calculations

the **chiral** extrapolation can be tricky

$$\langle r^2 \rangle = \frac{1}{16\pi^2 F_\pi^2} \ln \frac{M_\pi^2}{\Lambda^2} + \mathcal{O}(M_\pi^2)$$



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Users of lattice results should be (made) aware of potential systematic effects

⇒ FLAG

A similar initiative is due to Laiho, Lunghi and van de Water

What/Who is FLAG?

FLAG = **F**LAVIANet **L**attice **A**veraging **G**roup

Members:

Gilberto Colangelo (Bern)

Stephan Dürr (Jülich, BMW)

Andreas Jüttner (CERN, RBC/UKQCD)

Laurent Lellouch (Marseille, BMW)

Heiri Leutwyler (Bern)

Vittorio Lubicz (Rome 3, ETM)

Silvia Necco (CERN, Alpha)

Chris Sachrajda (Southampton, RBC/UKQCD)

Silvano Simula (Rome 3, ETM)

Tassos Vladikas (Rome 2, Alpha and ETM)

Urs Wenger (Bern, ETM)

Hartmut Wittig (Mainz, Alpha)

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History and status:

- ▶ Beginning: FLAVIANet meeting, Orsay, November 2007
- ▶ Start of the actual work: Bern, March 2008
- ▶ ...
- ▶ first paper appeared in November 2010
updated and published in May 2011 on EPJC
- ▶ webpage made public in 2011:
`http://itpwiki.unibe.ch/flag`
- ▶ future: **FLAG-2** (see below)

[arXiv.1011.4408](https://arxiv.org/abs/1011.4408)

What exactly does FLAG offer?

An answer to the questions

- ▶ what is the current lattice value for quantity X ?
- ▶ what is a reliable estimate of the uncertainty?

in a way easily accessible to non-experts

Quantities considered in the first edition:

- ▶ light quark masses
- ▶ LEC's
- ▶ decay constants
- ▶ form factors (of pions and kaons)
- ▶ B_K

→ talk of C. Hölbling

→ talk of C. Hölbling

What exactly does FLAG offer?

For each quantity we provide:

- ▶ complete list of references
- ▶ summary of relevant formulae and notation
- ▶ summary of the essential aspects of each calculation:
 - ▶ lattice action
 - ▶ number of dynamical quarks (N_f)
 - ▶ minimal value and range of quark masses
 - ▶ minimal value and range of lattice spacing
 - ▶ maximal value and range of lattice volumes
 - ▶ renormalization method (where applicable)

in a unified and easy to read (color coding) manner

- ▶ averages (if sensible)
- ▶ and a “lattice dictionary” for non-experts (details of lattice actions, etc.)

Color coding – our present definition

[subject to change before each new edition]

► chiral extrapolation

- ★ $M_{\pi,\min} < 250 \text{ MeV}$
- $250 \text{ MeV} \leq M_{\pi,\min} \leq 400 \text{ MeV}$
- $M_{\pi,\min} > 400 \text{ MeV}$

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- ▶ continuum extrapolation
 - ★ 3 or more lattice spacings, at least 2 points below 0.1 fm
 - 2 or more lattice spacings, at least 1 point below 0.1 fm
 - otherwise

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- ▶ finite volume effects
 - ★ $(M_{\pi}L)_{\min} > 4$ or at least 3 volumes
 - $(M_{\pi}L)_{\min} > 3$ and at least 2 volumes
 - otherwise

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- ▶ finite volume effects
 - ★ $(M_{\pi}L)_{\min} > 4$ or at least 3 volumes
 - $(M_{\pi}L)_{\min} > 3$ and at least 2 volumes
 - otherwise
- ▶ renormalization (where applicable)
 - ★ non-perturbative
 - 2-loop perturbation theory (converging series)
 - otherwise

Averages

Different lattice results will be averaged if

- ▶ published
[lattice proceedings not enough]
- ▶ no red tags
- ▶ same N_f
[no average of $N_f = 2$ and $N_f = 3$ calculations]

Final FLAG number:

- ▶ average or single *no-red-tag* $N_f = 3$ number (if available)
- ▶ average or single *no-red-tag* $N_f = 2$ number (if available)

If *both* $N_f = 3$ *and* $N_f = 2$ numbers available:

agreement $\Rightarrow$ more confidence in the final number

How to determine V_{ij}^{CKM}

Effective Lagrangian for semileptonic transitions

$$\mathcal{L}_{\text{eff}}^{\text{SM}} \sim G_F V_{ij} J_\mu^{\ell \dagger} J_{ij}^{\mu h} + \text{h.c.}$$

Matrix elements for $A_h A_\ell \rightarrow B_h B_\ell$

$$\langle B_h B_\ell | \mathcal{L}_{\text{eff}}^{\text{SM}} | A_h A_\ell \rangle \sim G_F \langle B_\ell | J_\mu^{\ell \dagger} | A_\ell \rangle V_{ij} \langle B_h | J_{ij}^{\mu h} | A_h \rangle$$

- ▶ measure the semileptonic transition
- ▶ calculate $\langle B_h | J_{ij}^{\mu h} | A_h \rangle$
- ▶ $\Rightarrow$ determine V_{ij}

How to determine V_{us} and V_{ud}

	process	theory
V_{ud}	superallowed Fermi decays neutron β -decay $\pi_{\ell 3}$ decay	CVC (rad. corr.) $V - A$ -theory, need g_A/g_V χ PT
V_{us}	$K_{\ell 3}$ decay τ hadronic decays	χ PT, lattice ($f_+(0)$) pert. QCD, sum rules
V_{us}/V_{ud}	$K_{\ell 2}$ and $\pi_{\ell 2}$ decay	lattice (F_K/F_π)

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Lattice calculations of $f_+(0)$ and f_K/f_π

$f_+(0)$	N_f		publication status	chiral extrapol.	finite volume	continuum extrapol.	action
0.9599(34)($^{+31}_{-47}$)(14)	2+1	RBC/UKQCD 10	A	●	★	■	DWF
0.9644(33)(34)(14)	2+1	RBC/UKQCD 07	A	●	★	■	DWF
0.9544(68) _{stat}	2	ETM 10D	C	●	★	●	max. tmQCD
0.9560(57)(62)	2	ETM 09A	P	●	●	●	max. tmQCD
0.9647(15) _{stat}	2	QCDSF 07	C	■	★	■	clover (NP)
0.968(9)(6)	2	RBC 06	A	■	★	■	DWF
0.967(6)	2	JLQCD 05	C	■	★	■	clover (NP)

Legenda publication status:

A = published article

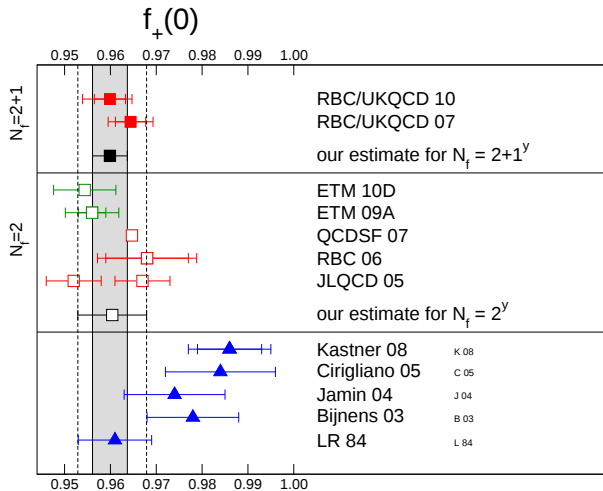
P = preprint

C = conference proceedings

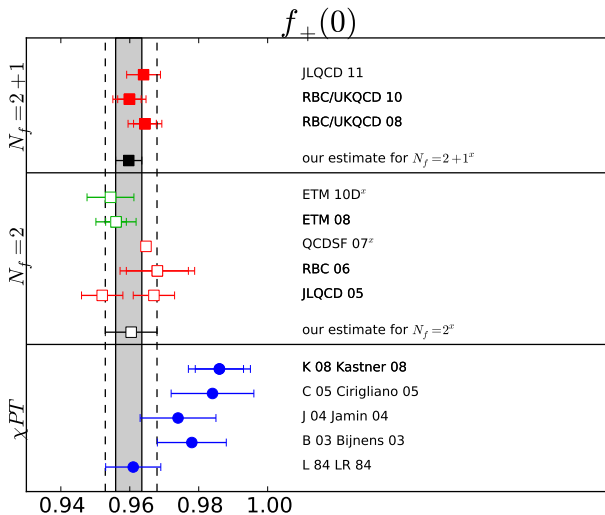
Lattice calculations of $f_+(0)$ and f_K/f_π

f_K/f_π	N_f		publication status	chiral extrapol.	finite volume	continuum extrapol.	action
1.197(2)($^{+3}_{-7}$)	2+1	MILC 10	C	●	★	★	KS _{MILC} ^{MILC}
1.204(7)(25)	2+1	RBC/UKQCD 10A	P	●	●	★	DWF
1.192(7)(6)	2+1	BMW 10	A	★	★	★	tlSW
1.210(12) _{stat}	2+1	JLQCD/TWQCD	C	●	■	■	overlap
1.198(1)($^{+6}_{-8}$)	2+1	MILC 09A	C	★	★	★	KS _{MILC} ^{MILC}
1.191(16)(16)	2+1	ALVdW 08	C	★	●	●	KS _{MILC} /DWF
1.189(20)	2+1	PACS-CS 08	P	★	■	■	clover (NP)
1.18(1)(1)	2+1	BMW 08	C	★	★	★	impr. Wilson
1.189(2)(7)	2+1	HPQCD/UKQCD 08	A	★	●	★	KS _{MILC} ^{HISQ}
1.205(18)(62)	2+1	RBC/UKQCD 07	A	●	★	■	DWF
1.218(2)($^{+11}_{-24}$)	2+1	NPLQCD 07	A	●	■	■	KS _{MILC} /DWF
1.210(6)(15)(9)	2	ETM 09	A	●	●	★	max. tmQCD
1.21(3)	2	QCDSF/UKQCD 07	C	●	★	●	clover (NP)

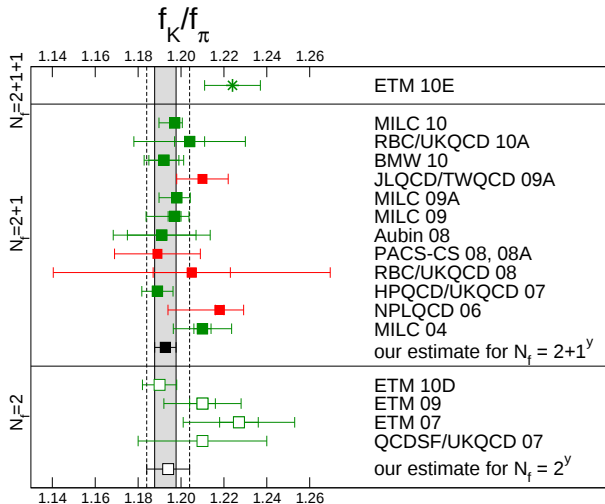
Lattice calculations of $f_+(0)$ and f_K/f_π



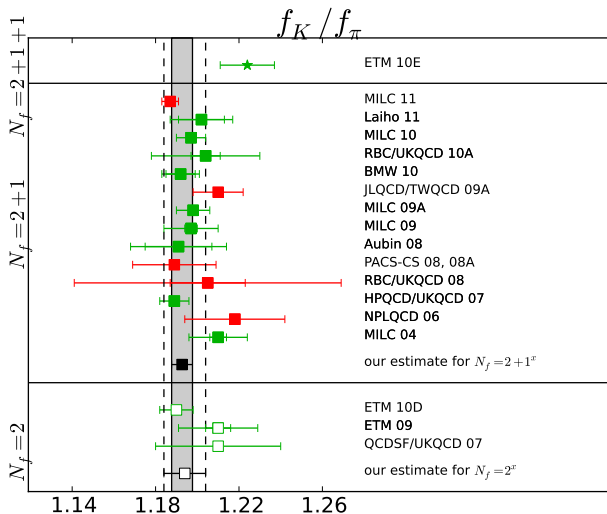
Lattice calculations of $f_+(0)$ and f_K/f_π



Lattice calculations of $f_+(0)$ and f_K/f_π



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Lattice calculations of $f_+(0)$ and f_K/f_π

Direct determinations of

$f_+(0)$:

$$f_+(0) = 0.9599(34)({}^{+31}_{-47})(14) \quad (N_f = 2 + 1)$$

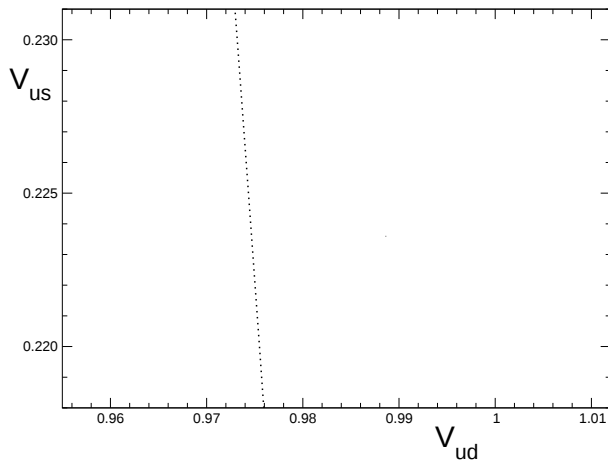
$$f_+(0) = 0.956(6)(6) \quad (N_f = 2)$$

f_K/f_π :

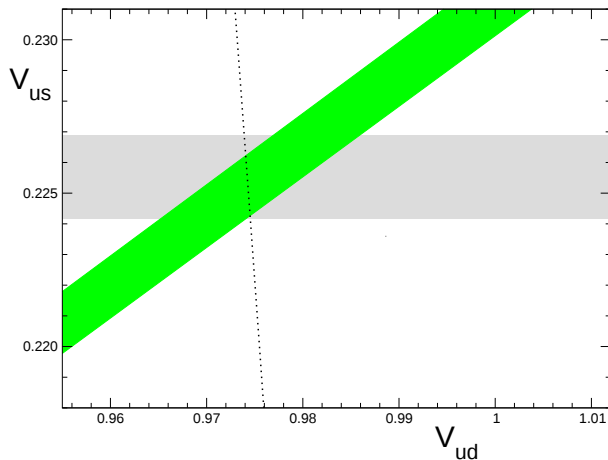
$$f_K/f_\pi = 1.193(6) \quad (N_f = 2 + 1)$$

$$f_K/f_\pi = 1.210(6)(17) \quad (N_f = 2)$$

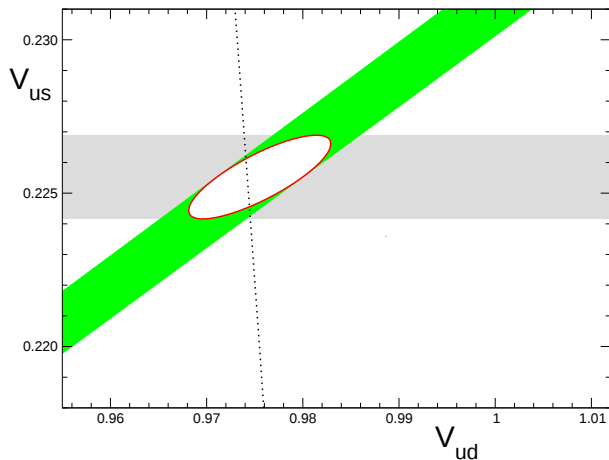
Unitarity test of the Standard Model



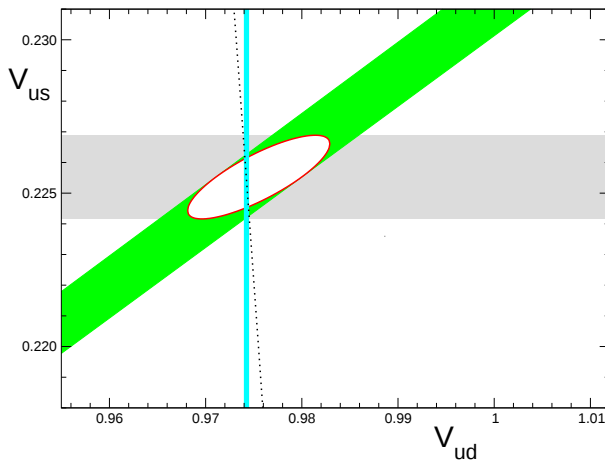
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Unitarity test of the Standard Model

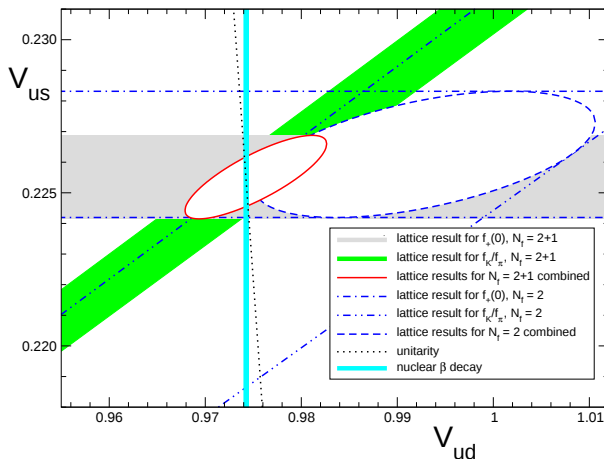


Unitarity test of the Standard Model



Strong constraint on right-handed currents

Unitarity test of the Standard Model



Strong constraint on right-handed currents

Analysis within the Standard Model

Unitarity + experiment:

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 \quad \left[|V_{ub}| = 3.89(44) \cdot 10^{-3}, \text{PDG (10)} \right]$$

Experiment:

FLAVIANet Kaon WG (10)

$$|V_{us}f_+(0)| = 0.2163(5)$$

$$\left| \frac{V_{us}f_K}{V_{ud}f_\pi} \right| = 0.2758(5)$$

3 relations and 4 unknowns

determine anyone of V_{ud} , V_{us} , $f_+(0)$ or f_K/f_π

$\Rightarrow$ get the other three

Analysis within the Standard Model

collaboration	$ V_{us} $	N_f	from
MILC 10	$0.2245(5)(^{+12}_{-5})$	2+1	f_K/f_π
RBC/UKQCD 10A	$0.2233(13)(44)$	2+1	f_K/f_π
RBC/UKQCD 10	$0.2253(10)(^{+12}_{-8})$	2+1	$f_+(0)$
BMW 10	$0.2254(13)(11)$	2+1	f_K/f_π
MILC 09A	$0.2243(5)(^{+14}_{-11})$	2+1	f_K/f_π
HPQCD/UKQCD 08	$0.2260(5)(13)$	2+1	f_K/f_π
ETM 09	$0.2222(11)(31)$	2	f_K/f_π
ETM 09A	$0.2263(14)(15)$	2	$f_+(0)$

Other sources of information on V_{ud} and V_{us}

Super-allowed nuclear β decays

$$|V_{ud}| = 0.97425(22) \quad \text{Hardy \& Towner 08}$$

$$\Rightarrow |V_{us}| = 0.22544(95) \quad f_+(0) = 0.9608(46) \quad f_K/f_\pi = 1.1927(59)$$

$\tau \rightarrow [\text{hadrons}(S=1)] + \nu$ decays

$$|V_{us}| = 0.2165(26)_{\text{exp}}(5)_{\text{th}} \quad \text{Gamiz et al. 07}$$

$$\Rightarrow |V_{ud}| = 0.9763(6) \quad f_+(0) = 0.999(12) \quad f_K/f_\pi = 1.244(16)$$

Problematic data: $\sum$ exclusive channels $\neq$ inclusive

Other sources of information on V_{ud} and V_{us}

Super-allowed nuclear β decays

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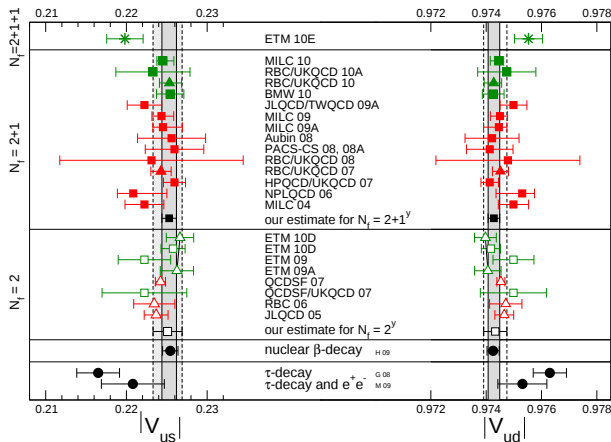
$\tau \rightarrow [\text{hadrons}(S=1)] + \nu$ decays + data on J_{em}

$$|V_{us}| = 0.2208(39) \quad \text{Maltman 09}$$

$$\Rightarrow |V_{ud}| = 0.9753(9) \quad f_+(0) = 0.981(17) \quad f_K/f_\pi = 1.219(23)$$

Problematic data: $\sum$ exclusive channels $\neq$ inclusive

Comparison between lattice and other determinations



Assuming unitarity lattice **predicts** $|V_{ud}|$ with the **same precision** as super-allowed Fermi β -decays

FLAG-2

FLAG = **F**lavour **L**attice **A**veraging **G**roup

has now entered its **phase 2** and has been extended in various directions

- ▶ quantities to be reviewed

main extension:

light quarks $\rightarrow$ + **heavy quarks**

- ▶ represented lattice collaborations:

Alpha, BMW, ETMC, RBC/UKQCD $\rightarrow$ + **CLS**, **Fermilab**, **HPQCD**, **JLQCD**, **MILC**, **PACS-CS**, **SWME**

- ▶ represented world regions: Europe $\rightarrow$ + **Japan and US**

- ▶ number of people: 12 $\rightarrow$ **28**

FLAG-2 organization

► Advisory Board:

S. Aoki (J), C. Bernard (US), C. Sachrajda (EU)

► Editorial Board:

GC, H. Leutwyler, T. Vladikas, U. Wenger

► Working Groups

► Quark masses

► V_{us}, V_{ud}

► LEC

► B_K

► α_S

► f_B, B_B

► $B \rightarrow H \ell \nu$

T. Blum, L. Lellouch, V. Lubicz

A. Jüttner, T. Kaneko, S. Simula

S. Dür, H. Fukaya, S. Necco

J. Laiho, S. Sharpe, H. Wittig

T. Onogi, J. Shigemitsu, R. Sommer

Y. Aoki, M. Della Morte, A. El Khadra

E. Lunghi, C. Pena, R. Van de Water

FLAG-2 plans and rules

- ▶ next review: end 2012
- ▶ regularly update the webpage
- ▶ new published review: every 2nd year
- ▶ some internal FLAG rules
 - ▶ members of the advisory board have a 4-year mandate
 - ▶ AB = EU+J+US
 - ▶ regular members can stay longer
 - ▶ replacements must keep/improve the balance of FLAG
 - ▶ WG members belong to 3 different lattice coll.
 - ▶ a paper is not reviewed (color-coded) by an author

Conclusions

- ▶ lattice plays an essential role in fully exploiting the potential of flavour physics
- ▶ the FLAG initiative aims to review lattice determinations of phenomenologically relevant quantities for non-experts
- ▶ as an example I have discussed the lattice determinations of $f_+(0)$ and f_K/f_π and of V_{ud} , V_{us}
- ▶ FLAG-2 has just started: it involves physicists from [EU+Japan+US](#) and will review also heavy-quark related quantities