

NONCOMMUTATIVE GEOMETRY IN THE LHC-ERA

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Noncommutative geometry allows to unify the basic building blocks of particle physics, Yang-Mills-Higgs theory and General relativity, into a single geometrical framework. The resulting effective theory constrains the couplings of the particle model and reduces the number of degrees of freedom. After briefly introducing the basic ideas of noncommutative geometry, I will present its predictions for the Standard Model (SM) and the few known models beyond the Standard Model. Most of these models, including the Standard Model, are now ruled out by LHC data. But interesting extensions of the Standard Model which agree with the presumed Standard Model Scalar (SMS) mass and predict new particles are still very much alive and await further experimental data.

1 Introduction

The aim of noncommutative geometry is to unify General Relativity and the Standard Model (or suitable extensions of the Standard Model) on the same geometrical footing. This means to describe gravity and the electro-weak and strong forces as gravitational forces of a unified space-time. A first observation on the structure of General Relativity shows that gravity emerges as a pseudo-force associated to the space-time (= a manifold M) symmetries, i.e. the diffeomorphisms on M . If one tries to put the Standard Model into the same scheme, one cannot find a manifold within classical differential geometry which could be the equivalent of space time. A second observation is that one can find an equivalent description of space(-time) when trading the differential geometric description of the Euclidean space(-time) manifold M with metric g for the algebraic description of a spectral triple $(C^\infty(M), \not{D}, \mathcal{H})$. A spectral triple³ consists of the following entities:

- an algebra $\mathcal{A}$ ($= C^\infty(M)$), the equivalent of the topological space M
- a Dirac operator $\mathcal{D}$ ($= \not{D}$), the equivalent of the metric g on M
- a Hilbert space $\mathcal{H}$, on which the algebra is faithfully represented and on which the Dirac operator acts. It contains the fermions, i.e. in the space(-time) case, it is the Hilbert space of Dirac 4-spinors.
- a set of axioms⁴ to ensure a consistent description of the geometry

This setup allows to describe General Relativity in terms of spectral triples. Space(-time) is replaced by the algebra of C^∞ -functions over the manifold and the Dirac operator plays a double role: it is the algebraic equivalent of the metric and it gives the dynamics of the Fermions. The Einstein-Hilbert action is replaced by the spectral action⁶, which is the simplest invariant action

given by the number of eigenvalues of the Dirac operator up to a cut-off energy. For readers interested in a more thorough introduction to the field we recommend [22](#) and [12](#).

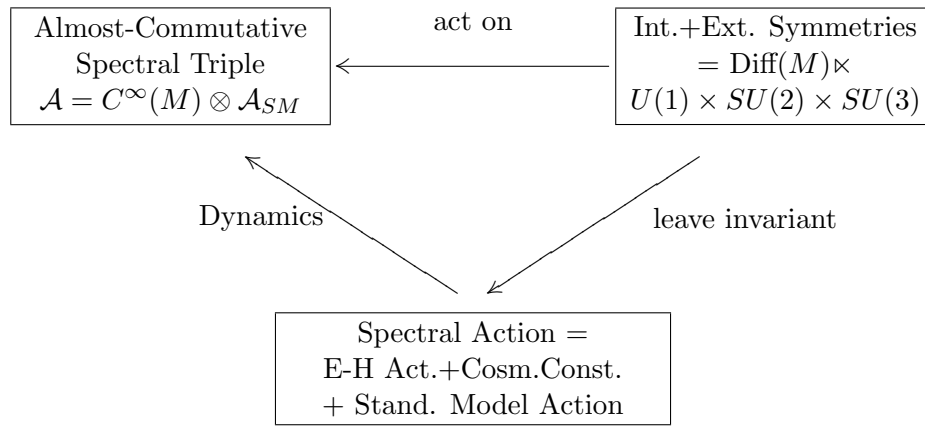
Connes' key observation is that the geometrical notions of the spectral triple remain valid, even if the algebra is *noncommutative*. A natural way of achieving noncommutativity is done by multiplying the function algebra $C^\infty(M)$ with a sum of real, complex or quaternionic matrix algebras

$$\mathcal{A}_f = M_1(\mathbb{K}) \oplus M_2(\mathbb{K}) \oplus \dots \quad (1)$$

These algebras represent *internal spaces* which have the $O(N)$ -, $U(N)$ - or $SU(N)$ -type Lie groups as their symmetries that one would like to obtain as gauge groups in particle physics. The choice

$$\mathcal{A}_{SM} = \mathbb{C} \oplus M_2(\mathbb{C}) \oplus M_3(\mathbb{C}) \oplus \mathbb{C} \quad (2)$$

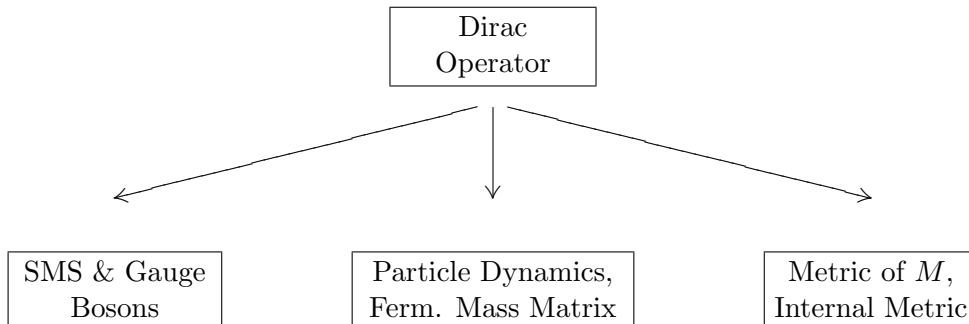
allows to construct the Standard Model and can be justified by different classification approaches [14,16,7](#). The combination of space(-time) and internal space into a product space, together with the spectral action [6](#), unifies General Relativity and the Standard Model as classical field theories:



The Standard Model in the noncommutative geometry setting automatically produces:

- The combined General Relativity and Standard (Particle) Model action
- A cosmological constant
- The SMS boson with the correct quartic SMS potential

The Dirac operator turns out to be one of the central objects and plays a multiple role:



Up to now one could conclude that noncommutative geometry consists merely in a fancy, mathematically involved reformulation of the Standard Model. But the choices of possible Yang-Mills-Higgs models that fit into the noncommutative geometry framework are limited. Indeed the geometrical setup leads already to a set of restrictions on the possible particle models:

- mathematical axioms => restrictions on particle content
- symmetries of finite space => determines gauge group
- representation of matrix algebra => representation of gauge group
(only fundamental and adjoint representations)
- Dirac operator => allowed mass terms / scalar fields

Further constraints come from the Spectral Action which results in an effective action valid at a cut-off energy Λ ⁶. This effective action comes with a set of constraints on the particle model couplings, also valid at Λ , which reduce the number of free parameters in a significant way.

The Spectral Action contains two parts. A fermionic part $(\Psi, \mathcal{D}\Psi)$ which is obtained by inserting the Dirac operator into the scalar product of the Hilbert space and a bosonic part $S_{\mathcal{D}}(\Lambda)$ which is just the number of eigenvalues of the Dirac operator up to the cut-off:

- $(\Psi, \mathcal{D}\Psi)$ = fermionic action includes Yukawa couplings & fermion–gauge boson interactions
- $S_{\mathcal{D}}(\Lambda)$ = the bosonic action given by the number of eigenvalues of $\mathcal{D}$ up to cut-off Λ
= the Einstein-Hilbert action + a Cosmological Constant
+ the full bosonic particle model action + constraints at Λ

The bosonic action $S_{\mathcal{D}}(\Lambda)$ can be calculated explicitly using the well known heat kernel expansion⁶. Note that $S_{\mathcal{D}}(\Lambda)$ is manifestly gauge invariant and also invariant under the diffeomorphisms of the underlying space-time manifold.

For the Standard Model the internal space is taken to be the matrix algebra $\mathcal{A}_f = \mathbb{C} \oplus M_2(\mathbb{C}) \oplus M_3(\mathbb{C}) \oplus \mathbb{C}$. The symmetry group of this discrete space is given by the group of (non-abelian) unitaries of $\mathcal{A}_f$: $U(2) \times U(3)$. This leads, when properly lifted to the Hilbert space, to the Standard Model gauge group $U(1)_Y \times SU(2)_w \times SU(3)_c$. It is a remarkable fact that the Standard Model fits so well into the noncommutative geometry framework!

Calculating from the geometrical data the Spectral Action leads to the following boundary conditions on the Standard Model parameters at the cut-off Λ

$$\frac{5}{3}g_1^2 = g_2^2 = g_3^2 = \frac{Y_2^2}{G_2} \frac{\lambda}{24} = \frac{1}{4}Y_2 \quad (3)$$

Where g_1 , g_2 and g_3 are the $U(1)_Y$, $SU(2)_w$ and $SU(3)_c$ gauge couplings, λ is the quartic scalar coupling, Y_2 is the sum of all Yukawa couplings squared and G_2 is the sum of all Yukawa couplings to the fourth power.

Assuming the Big Desert and the stability of the theory under the flow of the renormalisation group equations we can deduce that $g_2^2(\Lambda) = g_3^2(\Lambda)$ at $\Lambda = 1.1 \times 10^{17}$ GeV. Having thus fixed the cut-off scale Λ we can use the remaining constraints to determine the low-energy value of the quartic SMS coupling and the top quark Yukawa coupling (assuming that it dominates all Yukawa couplings). This leads to the following conclusions:

- $\frac{5}{3}g_1^2 \neq g_2^2$ at Λ
- $m_{\text{SMS}} = 168.3 \pm 2.5$ GeV
- $m_{\text{top}} < 190$ GeV
- no 4th Standard Model generation

So the boundary conditions (3) of the Standard Model cannot be fulfilled for the gauge couplings. This is of course a well known fact, since the conditions coincide with the $SU(5)$ -grand unified case. Note that the prediction for the SMS mass of ~ 170 GeV is also ruled out. This value has recently been shown to be too high^{1,10} since the SMS has a mass of ~ 125 GeV. Thus it seems plausible to consider also models beyond the Standard Model.

Although the Standard Model takes a prominent place^{14,16,7} within the possible models of almost-commutative geometries one can go further and construct models beyond the Standard Model. The techniques from the classification scheme developed in¹⁴ were used to enlarge the Standard Model^{24,25,23,26}, but most of these models^{24,25,23} suffer from a similar shortcoming as the Standard Model: The mass of the SMS is in general too high compared to the experimental value. Here the model in²⁶ will be of central interest, since it predicted approximately the correct SMS mass.

In the case of finite spectral triples of KO-dimension six^{2,5} a different classification leads to more general versions of the Standard Model algebra⁷, under some extra assumptions. Considering the first order axiom as being dynamically imposed on the spectral triple one finds a Pati-Salam type model⁹. From the same geometrical basis one can promote the Majorana mass of the neutrinos to a scalar field^{8,11} which allows to lower the SMS mass to its experimental value.

2 The model

The model we are investigating extends the Standard Model⁶ by N generations of chiral X_1 - and X_2 -particles and vectorlike $V_{c/w}$ -particles. It is a variation of the model in²⁶ and the model in²⁷. For details of the following calculations as well as the construction of the spectral triple we refer the reader to²⁷. In particular its Krajewski diagram is depicted in figure 4[?]. The necessary computational adaptations to the model in this publication are straightforward. The gauge group of the Standard Model is enlarged by an extra $U(1)_X$ subgroup, so the total group is $G = U(1)_Y \times SU(2)_w \times SU(3)_c \times U(1)_X$. The Standard Model particles are neutral with respect to the $U(1)_X$ subgroup while the X -particles are neutral with respect to the Standard Model subgroup $G_{SM} = U(1)_Y \times SU(2)_w \times SU(3)_c$. Furthermore the model contains two scalar fields: a scalar field in the SMS representation and a new scalar field carrying only a $U(1)_X$ charge. They induce a symmetry breaking mechanism $G \rightarrow U(1)_{em} \times SU(3)_c$.

The Hilbert spaces of the new fermions and the new scalar field expressed in terms of their representations are

$$\begin{aligned}\mathcal{H}_{X_1,l}^p \oplus \mathcal{H}_{X_2,l}^p &= \bigoplus_1^N [(0, 1, 1, +1) \oplus (0, 1, 1, 0)], \\ \mathcal{H}_{X_1,r}^p \oplus \mathcal{H}_{X_2,r}^p &= \bigoplus_1^N [(0, 1, 1, 0) \oplus (0, 1, 1, +1)], \\ \mathcal{H}_{V_{c,l}}^p \oplus \mathcal{H}_{V_{w,l}}^p &= \mathcal{H}_{V_{c,r}}^p \oplus \mathcal{H}_{V_{w,r}}^p = \bigoplus_1^N [(-\frac{1}{3}, 1, \bar{3}, \frac{1}{2}) \oplus (0, \bar{2}, 1, \frac{1}{2})] \\ \mathcal{H}_\varphi &= (0, 1, 1, -1)\end{aligned}\tag{4}$$

$$\tag{5}$$

Here we chose the standard normalisations of^{18,19,20} for the representation, i.e. the right-handed electron has hypercharge $Y_e = -1$.

The Lagrangians for the X -particles,

$$\begin{aligned}\mathcal{L}_{\nu X_1} &= ((g_{\nu X_1}) \psi_{\nu,r}^* \gamma_5 \varphi \psi_{X_1,l} + \text{h.c.}) \\ \mathcal{L}_{X_i} &= (\psi_{X_i,l}^* \psi_{X_i,r}^*) D_{X_i} \begin{pmatrix} \psi_{X_i,l} \\ \psi_{X_i,r} \end{pmatrix} + (\psi_{X_i,l}^* \gamma_5 g_{X_i} \psi_{X_i,r} + \text{h.c.}) \\ \mathcal{L}_{X_2,\Gamma} &= \psi_{X_2,r}^* \gamma_5 \Gamma_{X_2} \psi_{X_2,r}\end{aligned}\tag{6}$$

contain the term $\mathcal{L}_{\nu X_1}$ coupling X_1 to the right-handed neutrinos, the ordinary Dirac and Yukawa term $\mathcal{L}_{X_i}$ for both $X_{1/2}$ -particle species and a Majorana mass term $\mathcal{L}_{X_2, \Gamma}$ for the right-handed X_2 -particles. The fermionic Lagrangian of the V_c - and the V_w - particles is

$$\mathcal{L}_{V_{c/w}} = (\psi_{V_{c/w}, l}^*, \psi_{V_{c/w}, r}^*) D_{V_{c/w}} \begin{pmatrix} \psi_{V_{c/w}, l} \\ \psi_{V_{c/w}, r} \end{pmatrix} + (\psi_{V_{c/w}, l}^* \gamma_5 \mathcal{M}_{V_{c/w}} \psi_{V_{c/w}, r} + \text{h.c.}). \quad (7)$$

Together with usual fermionic Lagrangian of the Standard Model, $\mathcal{L}_{SM, f}$ they give the fermionic part of the theory. The Dirac operators of the form D_X and D_V in (6) and (7) are the Dirac operators with the respective gauge covariant derivatives.

The Yang-Mills Lagrangian for the Standard Model subgroup G_{SM} takes again its usual form and for $U(1)_X$ we have the standard Yang-Mills-Lagrangian

$$\mathcal{L}_{U(1)_X} = \frac{1}{4g_4^2} \sum_{i,j=1}^4 (B^X)_{ij} (B^X)_{ij} \quad (8)$$

where B^X is the field strength tensor for the $U(1)_X$ -covariant derivative. The Lagrangian of the scalar fields

$$\mathcal{L}_{H, \varphi} = |\nabla^H H|^2 + |\nabla^\varphi \varphi|^2 - \mu_H^2 |H|^2 - \mu_\varphi^2 |\varphi|^2 + \frac{\lambda_1}{6} |H|^4 + \frac{\lambda_2}{6} |\varphi|^4 + \frac{\lambda_3}{3} |H|^2 |\varphi|^2 \quad (9)$$

contains an interaction term of the H -field and the φ -field. The normalisation is again chosen according to [18,19,20](#) in order to obtain Lagrangians of the form $(1/2)(\partial\varphi)^2 - (\mu^2/2)\varphi^2 + (\lambda/24)\varphi^4$ for the real valued fields. Putting everything together the Dirac inner product $\langle \psi, D_\Phi \psi \rangle$ and the Spectral Action provide the Lagrangian

$$\mathcal{L}_{full} = \mathcal{L}_{SM, f} + \mathcal{L}_{SM, YM} + \mathcal{L}_{U(1)_X} + \mathcal{L}_{H, \varphi} + \mathcal{L}_{\nu X_1} + \mathcal{L}_{X_1} + \mathcal{L}_{X_2} + \mathcal{L}_{X_2, \Gamma} + \mathcal{L}_{V_c} + \mathcal{L}_{V_w}. \quad (10)$$

The Spectral Action does not only supply the bosonic part of the Lagrangian (10) but also relations among the free parameters of the model. These relations serve as boundary conditions for the renormalisation group flow at the cut-off scale Λ . The relation for the gauge couplings g_1, g_2, g_3 and g_4 at the cut-off scale Λ are:

$$\boxed{\sqrt{\frac{80 + \frac{4}{3}N}{48 + 8N}} g_1(\Lambda) = g_2(\Lambda) = g_3(\Lambda) = \sqrt{\frac{36N}{48 + 8N}} g_4(\Lambda)} \quad (11)$$

We notice the deviation of these relations compared to the case of the Standard Model (3) [6](#). For the Yukawa couplings the Spectral Action implies the boundary conditions [28](#)

$$\boxed{Y_2(\Lambda) = Y_X(\Lambda) = (4 + \frac{2}{3}N) g_2(\Lambda)^2} \quad (12)$$

where we have introduced the squared traces

$$\begin{aligned} Y_2 &:= 3 \text{tr}(g_u^* g_u) + 3 \text{tr}(g_d^* g_d) + \text{tr}(g_e^* g_e) + \text{tr}(g_\nu^* g_\nu) \\ Y_X &:= \text{tr}(g_{\nu X_1}^* g_{\nu X_1}) + \text{tr}(g_{X_1}^* g_{X_1}) + \text{tr}(g_{X_2}^* g_{X_2}). \end{aligned} \quad (13)$$

in terms of the Yukawa coupling matrices. With the traces of Yukawa matrices to the fourth power,

$$\begin{aligned} G_2 &:= 3 \text{tr}[(g_u^* g_u)^2] + 3 \text{tr}[(g_d^* g_d)^2] + \text{tr}[(g_e^* g_e)^2] + \text{tr}[(g_\nu^* g_\nu)^2] \\ G_X &:= \text{tr}[(g_{\nu X_1}^* g_{\nu X_1})^2] + \text{tr}[(g_{X_2}^* g_{X_2})^2] + \text{tr}[(g_{X_3}^* g_{X_3})^2] + 2 \text{tr}(g_{X_1}^* g_{X_1} g_{\nu X_1}^* g_{\nu X_1}) \\ G_{\nu X_1} &:= \text{tr}(g_\nu^* g_\nu g_{\nu X_1}^* g_{\nu X_1}). \end{aligned} \quad (14)$$

the boundary conditions of the quartic couplings read

$$\lambda_1(\Lambda) = g_2(\Lambda)^2(24 + 4N) \frac{G_2(\Lambda)}{Y_2(\Lambda)^2}, \quad \lambda_2(\Lambda) = g_2(\Lambda)^2(24 + 4N) \frac{G_X(\Lambda)}{Y_X(\Lambda)^2} \quad (15)$$

and

$$\lambda_3(\Lambda) = g_2(\Lambda)^2(24 + 4N) \frac{G_{\nu X_1}(\Lambda)}{Y_2(\Lambda)Y_X(\Lambda)} \quad (16)$$

3 A numerical example

We assume that we have $N = 3$ generations of new particles since there are three known Standard Model generations. This fixes the conditions of the gauge couplings at the cut-off scale Λ in (11). We also assume that the Standard Model Yukawa couplings are dominated by the top-quark coupling y_t and the τ -neutrino coupling y_{ν_τ} . Assuming that the Yukawa couplings that involve the X -particles are dominated by the coupling of the τ -neutrino to one generation of the X_1 -particles $y_{\nu_\tau X_1^1}$ we get for (13) and (14)

$$\begin{aligned} Y_2 &\approx 3y_t^2 + y_{\nu_\tau}^2, & Y_X &\approx y_{\nu_\tau X_1^1}^2, \\ G_2 &\approx 3y_t^4 + y_{\nu_\tau}^4, & G_X &\approx y_{\nu_\tau X_1^1}^4, & G_{\nu X_1} &\approx y_{\nu_\tau}^2 y_{\nu_\tau X_1^1}^2. \end{aligned}$$

The Majorana masses of the neutrinos¹⁵ and the Dirac masses of the X_1 -particles we put to $m_{X_1} \sim 10^{14}$ GeV. The Majorana masses of the X_2 -particles play no essential rôle for this numerical analysis. It triggers a seesaw mechanism for the X_2 -particles but has no effect on the masses of the scalar fields since we neglect the Yukawa matrix g_{X_2} . For the masses of the three V_c -particles we choose $m_{V_c} \sim 5.5 \times 10^{15}$ GeV. This particular value of the V_c -particle masses allows to fulfill the boundary conditions (11).

Defining $x := g_{\nu_\tau}/g_t$ the boundary relations (11), (12), (15) and (16) among the dimensionless parameters at the cut-off scale Λ simplify to

$$\frac{7}{6} g_1^2 = g_2^2 = g_3^2 = \frac{3}{2} g_4^2 = \frac{1}{6} g_t^2(3 + x^2) = \frac{1}{6} y_{\nu_\tau X_1^1}^2 = \frac{1}{36} \frac{(3 + x^2)^2}{3 + x^4} \lambda_1 = \frac{1}{36} \lambda_2 = \frac{1}{36} \frac{3 + x^2}{x^2} \lambda_3 \quad (17)$$

So the remaining free parameter for this particular point in parameter space, the ratio x , has to be chosen such that the low energy value of g_t coincides with its experimental value given by the experimental value of the top-quark mass. All normalisations are chosen as in¹⁵, i.e. the Standard Model fermion masses are given by $m_f = \sqrt{2}(g_f/g_2)m_{W^\pm}$.

Let us determine the cut-off scale Λ one-loop renormalisation group equations^{18,19,20} for g_1 , g_2 and g_3 . As experimental low-energy values we take²¹

$$g_1(m_Z) = 0.3575, \quad g_2(m_Z) = 0.6514, \quad g_3(m_Z) = 1.221.$$

The one-loop renormalisation group analysis shows that the high-energy boundary conditions for the gauge couplings in (17)

$$\frac{7}{6} g_1(\Lambda)^2 = g_2(\Lambda)^2 = g_3(\Lambda)^2$$

can be met for $\Lambda \approx 2.6 \times 10^{18}$ GeV. The running of the gauge couplings is shown in figure 1. Both scalar fields have nonzero vacuum expectation values, $|\langle H \rangle| = v_H/\sqrt{2} \neq 0$ and $|\langle \varphi \rangle| = v_\varphi/\sqrt{2} \neq 0$. With the $W^\pm$ boson mass, $m_{W^\pm} = (g_2/2) v_H$, we can determine the vacuum expectation value v_H of the first scalar field and obtain with $m_{W^\pm} = 80.34$ GeV²¹ its experimental value $v_H = 246.8$ GeV. In the numerical analysis it turns out that the errors are dominated by

the experimental uncertainties of the top quark mass and the SMS mass. Therefore we neglect the experimental uncertainties of the $W^\pm$ bosons and of the gauge couplings g_1 , g_2 and g_3 . The vacuum expectation value of φ is a free parameter and is determined by μ_φ . The mass matrix is not diagonal in the weak basis, but the mass eigenvalues can be calculated to be¹³

$$m_{\phi_H, \phi_\varphi} = \frac{\lambda_1}{6} v_H^2 + \frac{\lambda_2}{6} v_\varphi^2 \pm \sqrt{\left(\frac{\lambda_1}{6} v_H^2 - \frac{\lambda_2}{6} v_\varphi^2\right)^2 + \frac{\lambda_3^2}{9} v_H^2 v_\varphi^2}. \quad (18)$$

To obtain the low energy values of the quartic couplings λ_1 , λ_2 and λ_3 we evolve their values at the cut-off Λ fixed by the boundary conditions (17). The top-quark mass has the experimental value $m_t = 173.5 \pm 1.4$ GeV²¹ which determines the parameter $x = 2.145 \pm 0.065$. As a very recent input we have the SMS mass $m_{\Phi_H} = 125.6 \pm 1.2$ GeV with the maximal combined experimental uncertainties of ATLAS¹ and CMS¹⁰. We identify the SMS mass with the lower mass eigenvalue in (18). As it turns out, this is a reasonable assumption as long as the vacuum expectation value v_φ is larger than the v_H .

With the above values of the top quark and the $W^\pm$ boson masses and the previous results on the gauge couplings we obtain from the running of the couplings, from the whole set of boundary conditions (17) at Λ and from the mass eigenvalue equation (18) that $v_\varphi = 595 \pm 159$ GeV. The error in the vacuum expectation value is rather large due to the flat slope of the curve of the light eigenvalue, see figure 1. The flat slope amplifies the experimental uncertainties of the top quark mass and the SMS mass. We notice that v_φ is larger than v_H but of the same order of magnitude. This justifies the assumption to run all three quartic couplings down to low energies and also ensures that the vacuum is stable up to the cut-off scale $\Lambda \approx 2.6 \times 10^{18}$ GeV, see¹⁷ and references therein. For the second mass eigenvalue we find $m_{\Phi_\varphi} = 381 \pm 90$ GeV. For the mass of the new gauge boson Z_X associated to the broken $U(1)_X$ subgroup we get $m_{Z_X} = 209 \pm 56$ GeV where we took the value of $g_4(250 \text{ GeV}) \approx 0.351$ as a very good approximation and neglected further uncertainties of g_4 . Note that the masses of the gauge and the scalar sector are lower than in the related model²⁷.

To get a rough estimate of the masses of the X_1 - and X_2 -particles we note that the $g_{\nu_\tau X_1^1} \sim 1$. So if we assume that $g_{\nu_\tau X_1^1}$ dominates the sum of the squared Yukawa coupling Y_X with an accuracy of a few percent, it is reasonable to take $g_{X_{1/2}^i} \leq 0.1$ for the remaining Yukawa couplings. This leads to X_1 -particle masses of the order 50 GeV, or less. If we assume that the Majorana masses of the X_2 -particles $(\Gamma_{X_2})_{ij} \gg v_\varphi$ then we expect three mass eigenstates of order $|(\Gamma_{X_2})|$ and three of order $v_\varphi^2/|(\Gamma_{X_2})|$. These particles could therefore populate lower mass scales.

Let us end this short note with some open questions. We have only performed a one-loop renormalisation group analysis. Yet the two-loop effects on the running of the top-quark mass are of the order of 15%. In view of the sensibility of the scalar masses to the top-quark mass, how stable are the numerical results with respect to two-loop effects? Furthermore how stable are the results with respect to variations in the parameter space? From the experimental point of view the question is of course the detectability of the new particles (at the LHC?) and whether the dark sector contains suitable dark matter candidates. One would expect more kinetic mixing of $U(1)_X$ to the hypercharge group $U(1)_Y$ compared to the model²⁷ due to the fact that here the V_c -particles may have a different masses and therefore mix the abelian groups. This poses the question how Z' -like the Z_X -boson are and whether they are already excluded (at least for the mass range explored in this publication). The model certainly has a rich phenomenology and is one of the few known models to be consistent with (most) experiments, with the axioms of noncommutative geometry and with the boundary conditions imposed by the Spectral Action.

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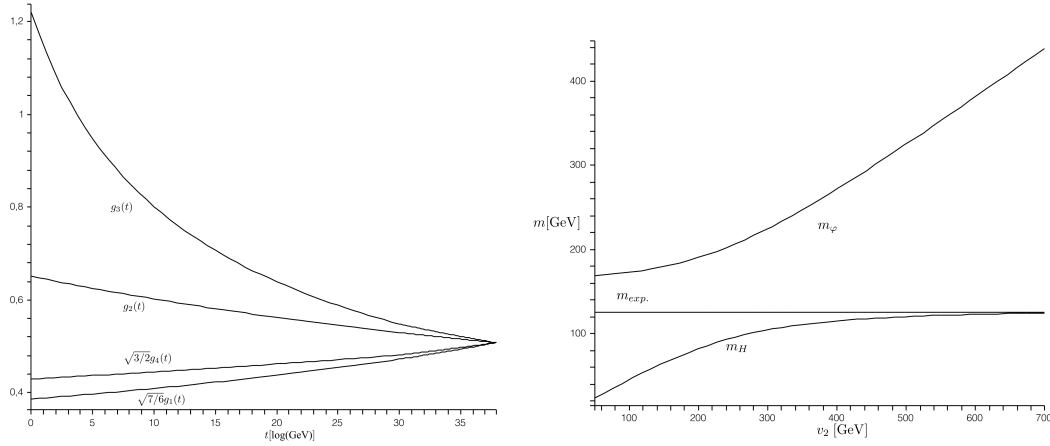


Figure 1: Left: Running of the gauge couplings with normalisation factors according to the high-energy boundary conditions (17). Right: Example for the mass eigenvalues m_H and m_φ of the scalar fields H and φ with respect to v_2 . Here the top quark mass is taken to be $m_t(m_Z) = 173.5$ GeV and the experimental SMS mass $m_{exp.} = 125.5$ GeV. We obtain a heavy scalar with $m_\varphi = 320$ GeV and a $U(1)_X$ -scalar boson mass $m_{Z_X} = 172$ GeV.

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