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7 Mixing and time-dependent analyses

Editors:

Adrian Bevan (BABAR) Some Clever Person (Belle) No one (theory)

Things assumed to have been defined before this section:

- Symmetries of CP, CPT and the concept of symmetry violation.
- Tagging formalism
- B-Flav (B reconstruction) technique

If it turns out that something has not been defined from the above, will have to define, or in special circumstances forward reference.

Figures to be included: either 2 or 3 depending on how information is presented. (i) Plot the physical Δt distribution, along with the same distribution with resolution, and dilution. The same plot for the asymmetry distribution should also be included. This could be one half page set of plots in a single figure, or it could be two figures containing the same plots. (ii) Cartoon of B meson decay, illustrating the concept of a Tag B and a Rec B.

This Chapter focuses on the introduction of the formalism for neutral meson mixing, and the principles underlying time-dependent analysis in B meson decays. A detailed discussion of experimental concerns for a time-dependent analysis follows on from a theoretical introduction of mixing and time-dependent formalism. The experimental aspects discussed include the use of flavour tagging methods introduced in Section 5.3 and the inevitable dilution of information when the tagging assignment is incorrect, interference effects of the B meson used for the flavour tag assignment, and the impact of resolution effects on the measurement of physical observables. The final part of this Chapter discusses how parameters required to describe mixing and time-evolution of B mesons in the detector can be extracted from the data.

Mixing in the neutral B meson system was discovered by the ARGUS Collaboration, Albrecht et al. (1987), and Chapter 14.5 summarises the measurements of B mixing performed by BABAR and Belle. An understanding of mixing in B mesons is one of the crucial ingredients in the study of time-dependent CP asymmetries, and tests of CPT using B meson decays at the $\Upsilon(4S)$. In particular this is crucial for the measurement of the angles of the Unitarity Triangle introduced in Chapter 13 and discussion of measurements of the angles can be found in Chapters 14.6 through 14.8. Tests of the CPT symmetry using neutral B mesons discussed in Chapter 14.9 also rely on a good

understanding of mixing. Mixing in the neutral charm system was discovered only recently at the B factories, and this is discussed in Section 16.2.

Neutral meson mixing

Meson mixing is a phenomenon that only occurs for neutral K, D and B mesons. Collectively we can refer to these mesons as P when describing formalism common to all three systems. The effective Hamiltonian describing neutral meson mixing is given by

$$\mathcal{H}_{eff} = \mathbf{M} + \frac{i\mathbf{\Gamma}}{2},\tag{1}$$

where **M** and Γ are two-by-two Hermitian matrices describing the mass and decay rate components of \mathcal{H}_{eff} , respectively. Thus neutral meson mixing is described by

$$\begin{pmatrix} |P_1\rangle \\ |P_2\rangle \end{pmatrix} = \left[\begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} + \frac{i}{2} \begin{pmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{21} & \Gamma_{22} \end{pmatrix} \right] \begin{pmatrix} |P^0\rangle \\ |\overline{P}^0\rangle \end{pmatrix}, (2)$$

where $|P^0\rangle$ and $|\overline{P}^0\rangle$ are strong eigenstates of neutral B, D, or K mesons, and $|P_{1,2}\rangle$ are the corresponding weak eigenstates. CP or CPT symmetry imposes that the matrix elements in Eq. 1 satisfy $M_{11}=M_{22}$ and $\Gamma_{11}=\Gamma_{22}$. In the limit of CP or T invariance, $\Gamma_{12}/M_{12}=\Gamma_{21}/M_{21}$ is real.

Weak eigenstates can be represented as an admixture of the strong eigenstates via

$$|P_{1,2}\rangle = p|P^0\rangle \pm q|\overline{P}^0\rangle,$$
 (3)

where $q^2 + p^2 = 1$ to normalize the wave function, and

$$\frac{q}{p} = \sqrt{\frac{M_{12}^* - i\Gamma_{12}^*/2}{M_{12} - i\Gamma_{12}/2}},\tag{4}$$

so the magnitude of p/q is very nearly one in the Standard Model (SM). If one considers the weak eigenstates under the CP operator, it follows that $|P_1\rangle$ is CP even (with an eigenvalue of +1), and $|P_2\rangle$ is CP odd (with an eigenvalue of -1). It is possible to generalize Eq. (3) to the scenario where CPT is violated. In this case the coefficients of the strong eigenstates are modified, and an additional complex parameter z is introduced as a measure of CPT violation. The resulting weak eigenstates are given by

$$|P_{1,2}\rangle = p\sqrt{1-z}|P^0\rangle \pm q\sqrt{1+z}|\overline{P}^0\rangle.$$
 (5)

In B_d^0 decays the eigenstates P_1 and P_2 are referred to as B_L and B_H to indicate that there is a measurable mass difference between the two states parameterized by

$$\Delta m_d = m_H - m_L \simeq 2|M_{12}|. \tag{6}$$

The corresponding decay rate difference is

$$\Delta\Gamma = \Gamma_H - \Gamma_L \simeq 2|M_{12}|Re(\Gamma_{12}/M_{12}). \tag{7}$$

Experimentally we know that $\Delta m_d = 5.02 \pm 0.007$ ps [REF PDG], while $\Delta \Gamma$ is measured to be compatible with zero. A priori the sign of $\Delta \Gamma$ is unknown, and unless otherwise stated in the rest of this Book, it is assumed that $\Delta \Gamma = 0$. A detailed discussion of the measurement of neutral B meson mixing is given in Section 14.5.

Experimental evidence for D mixing is relatively recent, and as the effect is small, the experimental precision of measured mixing parameters is not as good as in the case of B mesons. For neutral D mixing the weak states P_1 and P_2 are referred to as D_1 and D_2 , so that D_1 is CP even. When discussing measurements of charm mixing, one normally refers to the parameters x and y where

$$x = \frac{\Delta m_D}{\Gamma},\tag{8}$$

$$y = \frac{\Delta \Gamma}{\Gamma},\tag{9}$$

where Γ is $(\Gamma_1 + \Gamma_2)/2$, and Δm_D and $\Delta \Gamma$ are mass and width differences between D_1 and D_2 defined in such a way that Δm_D remains positive. Some measurements of charm mixing use a rotated basis of parameters (x', y') related to (x, y). A detailed discussion of the measurement of neutral B meson mixing is given in Section 16.2.

Time-dependent evolution

This will be 1-2 pages long.

The following papers will be cited in this section

- BABAR CPT (hadronic) paper (Aubert, 2004).

At BABAR and Belle neutral B mesons are produced via $e^+e^- \to \Upsilon(4S) \to B^0 \overline{B}{}^0$. The wave function for the final state B pair is prepared in a coherent P-wave (L=1) state. The B mesons remain in a coherent state until one of them decays. When the first B meson decays, the wave function collapses into a decoherent state. The remaining un-decayed B meson will continue to propagate through space-time and mix with a frequency Δm_d as described above, until this too finally decays.

If one of the B mesons decays into a final state that can be used to unambiguously determine the flavour of the B at the time it decayed, we refer to that as a $B_{\rm tag}$. Similarly if the other B decayed into a CP-eigenstate or admixture we refer to that as the $B_{\rm rec}$. Events with one $B_{\rm tag}$ and one $B_{\rm rec}$ are of interest in the study of time-dependent CP violation.

To be completed.

Fig. 1. An illustration of B meson pair decaying in the laboratory frame of reference. One of these decays into a $B_{\rm tag}$ final state, and the other into a $B_{\rm rec}$ final state.

- Introduction of time evolution of the wave function
- Identify CP asymmetry parameters

Lead through to result: TDCP asymmetry measurements: (S, C), (S, -A).

From the rho+rho- PRD (BaBar)

The signal decay-rate distribution of a CP-eigenstate decay, $f_+(f_-)$ for $B_{\text{tag}} = B^0$ (\overline{B}^0), is given by:

$$f_{\pm}(\Delta t) = \frac{e^{-|\Delta t|/\tau}}{4\tau} [1 \pm S \sin(\Delta m_d \Delta t) \mp C \cos(\Delta m_d \Delta t)] 0$$

where $\tau=1.536\pm0.014\,\mathrm{ps}$ is the mean B^0 lifetime and $\Delta m_d=0.502\pm0.007\,\mathrm{ps^{-1}}$ is the B^0 - \overline{B}^0 mixing frequency?. This assumes that there is no difference between B^0 lifetimes, $\Delta\Gamma=0$. The parameters S and C are defined as:

$$S = \frac{2Im\lambda}{1+|\lambda|^2}, \qquad C = \frac{1-|\lambda|^2}{1+|\lambda|^2}, \qquad (11)$$

where $\lambda=\frac{q}{p}\overline{A}$ is related to the level of B^0 - \overline{B}^0 mixing (q/p), and the ratio of amplitudes of the decay of a \overline{B}^0 or B^0 to the final state under study (\overline{A}/A) . CP violation is probed by studying the time-dependent decay-rate asymmetry

$$\mathcal{A} = \frac{R(\Delta t) - \overline{R}(\Delta t)}{R(\Delta t) + \overline{R}(\Delta t)},\tag{12}$$

where $R(\overline{R})$ is the decay-rate for B^0 $(\overline{B}{}^0)$ tagged events. This asymmetry has the form

$$\mathcal{A} = S\sin(\Delta m_d \Delta t) - C\cos(\Delta m_d \Delta t). \tag{13}$$

Belle use a different convention to BABAR with $C = -A_{CP}$.

Use of flavour tagging

This will be 2 pages long, covering the two sub-sub-sections.

 Overview: tagging techniques used as an input to effectively weight events

It would be very helpful to see an early draft of the tagging section before writing this, as we need to draw on the technical details introduced earlier in the book.

Dilution

 mistag probabilities, amnd how this modifies the physical performance.

It would be very helpful to see an early draft of the tagging section before writing this, as we need to draw on the technical details introduced earlier in the book.

Tag side interference

discuss how this affects the measurements of S and C, how one can estimate the effect from monte carlo, and how one might eliminate the effect altogether for a systematics limited measurement.

The following papers will be cited in this section

- Refer to the paper on tag side interference by Long et al., (Long, Baak, Cahn, and Kirkby, 2003).

Resolution on Δt

This will be 2 pages long.

- Detector resolution issues: revisit the physical time dependence to add dilution and resolution effects.
- Solution used for BaBar (Ref long s2b PRD)
- Solution used for Belle (Ref NIM paper)
- Asymmetry plot to illustrate physics, physics
 + resolution, Physics + resolution + dilution?

The following papers will be cited in this section

- Belle detector resolution paper by (Tajima, 2004).
- BABAR detector resolution information from the following paper by (Aubert, 2002).

The effect of detector resolution, and the finite lifetime of some intermediate particles in the decay chain of B_{tag} and B_{rec} need to be considered when trying to understand the resolution on Δt . There are a number of physical effects that are relevant when considering the detector resolution on Δt . These include

- B_{tag} vertex resolution, which is a complicated superposition of tracking effects and the finite lifetime of D mesons for a sub-sample of B_{tag} mesons.
- $B_{\rm rec}$ vertex resolution, which is a superposition of tracking effects. If there are long lived intermediate particle states in the $B_{\rm rec}$ decay chain, then these may also have an effect on the resolution of the $B_{\rm rec}$ vertex.
- Resolution on the measurement of the boost factor $\beta\gamma$ determined from the energy of the e^+ and e^- beams.

The reason why understanding and control of resolution effects on the measurement of Δt is so important is that the average separation between a $B_{\rm rec}$ and $B_{\rm tag}$ at the B factories in a typical event is $250\mu m$ [CHECK THIS NUMBER], whereas the resolution is of the order of $160\mu m$ [CHECK TH IS NUMBER].

There are two approaches taken to understand resolution effects, Belle characterise resolution effects according to the physical source, whereas BABAR adopt a parametric approach to describe the Δt resolution. Both approaches work well, and provide a good description of resolution for use in time-dependent analyses.

The Belle Δt resolution function describes four different effects

- $-B_{\text{tag}}$ vertex resolution
- $-B_{\rm rec}$ vertex resolution
- finite D meson flight length for the tag-side decays
- XYZ

To be completed.

The BABAR Δt resolution function has a triple Gaussian form, where the mean μ_i and width of the two central Gaussian components are scaled by $\sigma_{\Delta t}$, the measured uncertainty on Δt , on an event-by-event basis. The three Gaussians G_i are referred to as the core, tail, and outlier, in order of increasing width, and the resolution function is given by

$$\mathcal{R}_{\text{sig}}(\Delta t, \sigma_{\Delta t}) = f_{\text{core}}G(\Delta t, \mu_{\text{core}}\sigma_{\Delta t}, \sigma_{\text{core}}\sigma_{\Delta t}) + f_{\text{tail}}G(\Delta t, \mu_{\text{tail}}\sigma_{\Delta t}, \sigma_{\text{tail}}\sigma_{\Delta t}) + f_{\text{outlier}}G(\Delta t, \mu_{\text{outlier}}, \sigma_{\text{outlier}}) , \quad (14)$$

where μ_i and σ_i are the mean and width of the i^{th} Gaussian, with i= core, tail, and outlier. The parameters σ_{tail} , σ_{outlier} and μ_{outlier} are set to 3.0 ps, 8.0 ps and 0.0 ps, respectively, and the other parameters are determined from reference samples of fully reconstructed B meson decays as described in the following section. As the physical tagging category for BABAR have different purities and dilutions, the values of μ_i and σ_i for the core Gaussian contribution to the resolution function depend on the flavor tagging category of an event. This difference is taken into account when analysing data.

To be completed.

Fig. 2. Distributions of (top) Δt for (solid) B^0 and (dashed) \overline{B}^0 tagged events for perfectly reconstructed decays, and (bottom) the corresponding distributions after taking into account resolution effects.

To be completed.

Fig. 3. Distributions of the time-dependent *CP* asymmetry for (solid) perfectly reconstructed decays, and (dashed) the corresponding distributions after taking into account dilution and resolution effects.

Parameter extraction from data

This will be 2 pages long.

 B-Flav sample (related to full B reconstruction), note that this will be used in different ways for different measurements.

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