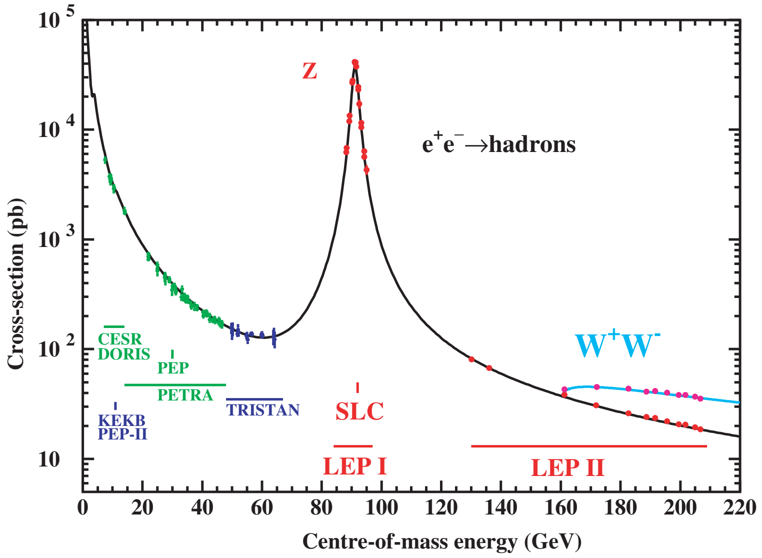
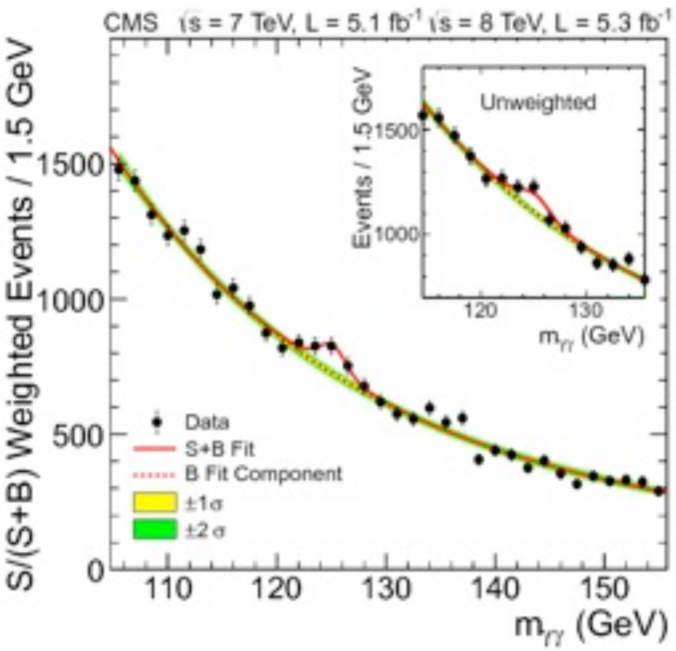
The electroweak SM fit



Roman Kogler
(University of Hamburg)

for the Gfitter group

Rencontres de Moriond
La Thuile, March 2-9, 2013

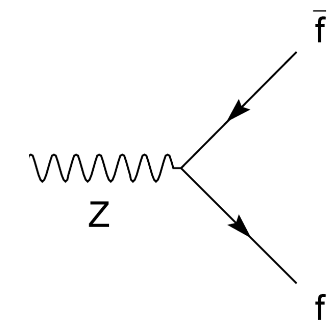


The Gfitter group: M. Baak (CERN), J. Haller (Univ. Hamburg), A. Höcker (CERN), R. K. (Univ. Hamburg), K. Mönig (DESY), M. Schott (CERN) J. Stelzer (DESY)

Predictive Power of the SM

Tree level relations for $Z \rightarrow f\,\bar{f}$

$$g^{(0)}_{V,f} \equiv g^{(0)}_{L,f} + g^{(0)}_{R,f} = I_3^f - 2Q^f \sin^2\theta_W$$
$$g^{(0)}_{A,f} \equiv g^{(0)}_{L,f} - g^{(0)}_{R,f} = I_3^f$$



$$i\bar{f}\gamma^\mu\left(g_{V,f} - g_{A,f}\gamma_5\right) f Z_\mu$$

- Unification connects the electromagnetic and the weak couplings
- M_W can be expressed in terms of M_Z and G_F

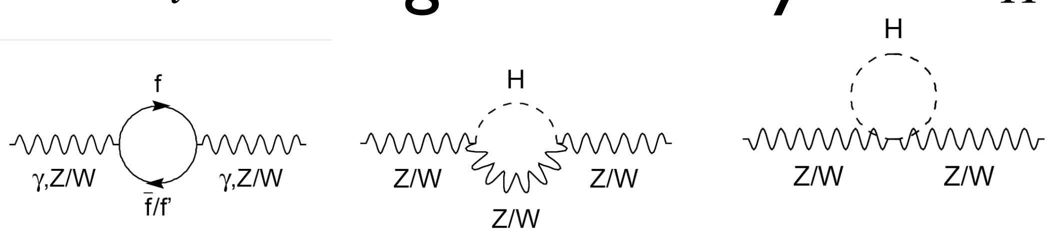
Radiative corrections

- Parametrisation through electroweak form factors ρ, κ, Δr
- Effective couplings at the Z-pole
- ρ, κ, Δr depend nearly quadratically on m_t and logarithmically on M_H

$$\sin^2\theta^f_{\text{eff}} = \kappa^f_Z \sin^2\theta_W$$

$$g_{V,f} = \sqrt{\rho^f_Z}\left(I_3^f - 2Q^f \sin^2\theta^f_{\text{eff}}\right)$$
$$g_{A,f} = \sqrt{\rho^f_Z}\, I_3^f$$

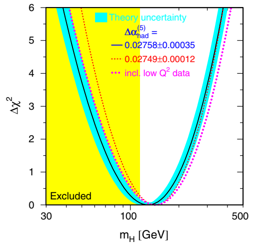
$$M_W^2 = \frac{M_Z^2}{2}\left(1 + \sqrt{1 - \frac{\sqrt{8}\pi\alpha(1+\Delta r)}{G_F M_Z^2}}\right)$$

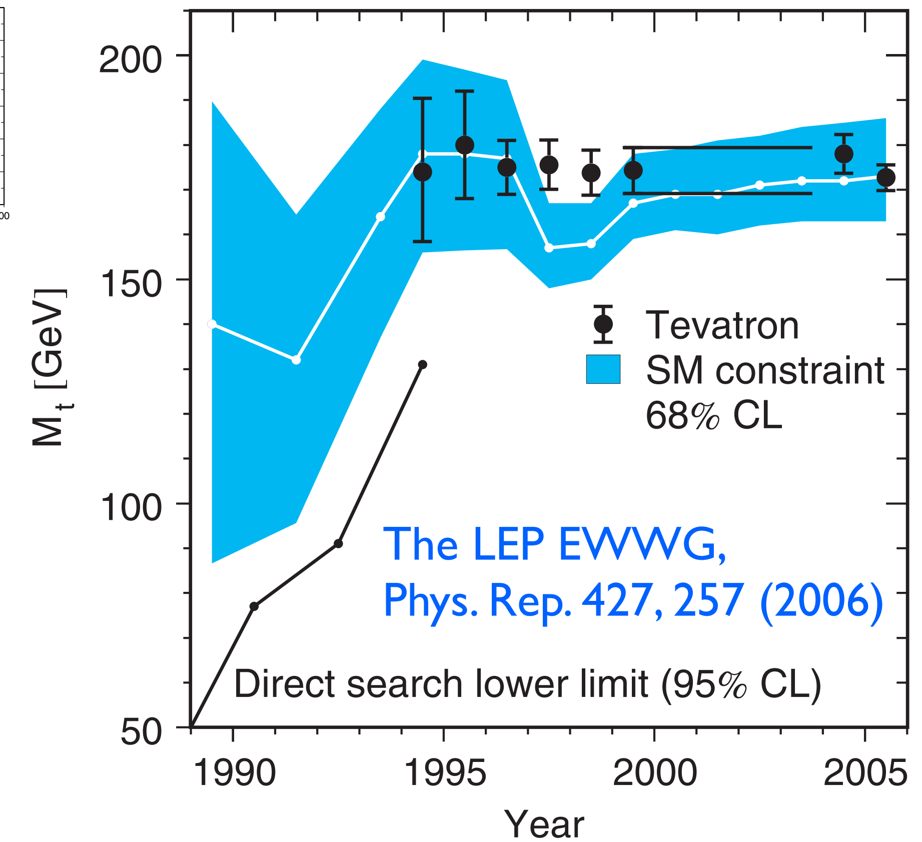


Electroweak Fits

A long tradition

- Huge amount of pioneering work to precisely understand loop corrections
- Observables known at least in two-loop order, sometimes higher orders available
- Precision measurements crucial, after the LEP/SLC era results from Tevatron and LHC become available

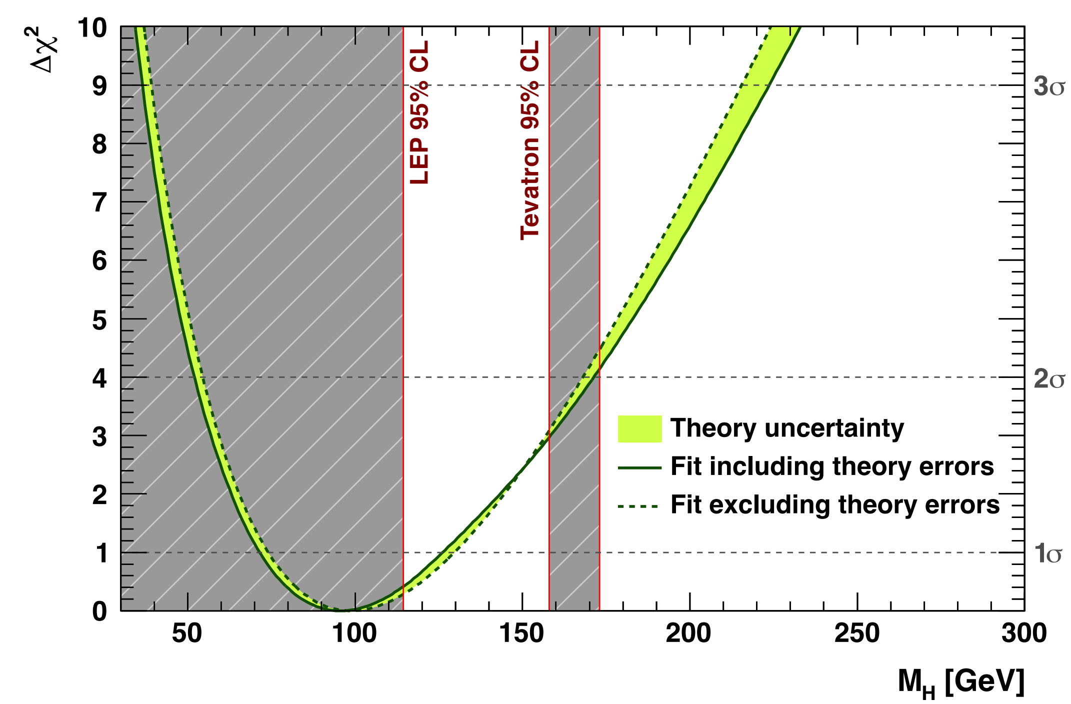




Higgs Hunting

- M_H last missing parameter
- Indirect determination (2011): $M_H = 96\,^{+31}_{-24}$ GeV
- Exclusion limits incorporated in EW fits

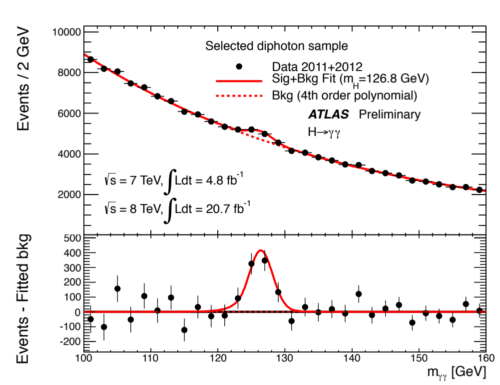
Gfitter group, EPJC 72, 2003 (2012)

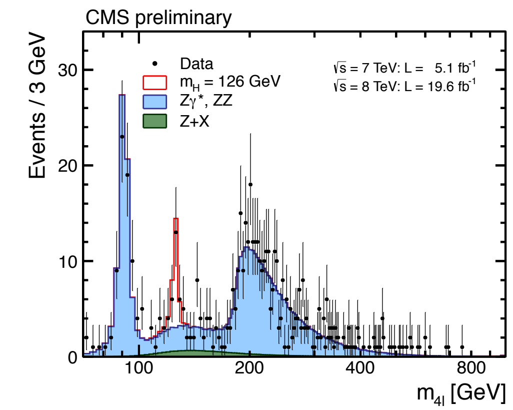


The SM Fit with Gfitter

The Discovery of a new boson

- The cross section and branching ratios are compatible with the SM scalar boson
- We assume that the boson is the SM scalar: M_H = 125.7 ± 0.4 GeV
- Change between fully uncorrelated and fully correlated systematic uncertainties: δM_H : 0.4 → 0.5 GeV





The SM is for the first time fully overconstrained: test its consistency

Calculations used

- M_W mass of the W boson [M. Awramik et al., Phys. Rev. D69, 053006 (2004)]
- Γ_Z, Γ_W partial and total widths of the Z and W [Cho et. al, arXiv:1104.1769]
- $\sin^2\theta^l_{\rm eff}$ effective weak mixing angle [M. Awramik et al., JHEP 11, 048 (2006), M. Awramik et al., Nucl.Phys.B813:174-187 (2009)]
- $\Gamma_{\rm had}$ QCD Adler functions at N3LO [P.A. Baikov et al., Phys.Rev.Lett. 108, 222003 (2012)]
- R_b partial width of Z→b$\bar{\rm b}$ [Freitas et al., JHEP08, 050 (2012)] ⟵ NEW!

The Global EW Fit

“There's two possible outcomes: if the result confirms the hypothesis, then you've made a discovery. If the result is contrary to the hypothesis, then you've made a discovery.”

(Enrico Fermi)

Experimental Input

Observables:

- Z-pole observables: LEP/SLD results
  [ADLO+SLD, Phys. Rept. 427, 257 (2006)]
- M_W and Γ_W: LEP/Tevatron [arXiv:1204:0042]
- m_t : Tevatron [arXiv:1207:1069]
- $\Delta\alpha_{\rm had}^{(5)}(M_Z)$ [M. Davier et al., EPJC 71, 1515 (2011)]
- $\overline{m}_c$, $\overline{m}_b$: world averages
  [PDG, J. Phys. G33, 1 (2006)]
- M_H : LHC [arXiv:1207.7214 , arXiv:1207.7235]

Free fit parameters:

- M_Z, M_H, $\Delta\alpha_{\rm had}^{(5)}(M_Z)$, $\alpha_s(M_Z)$, $\overline{m}_c$, $\overline{m}_b$, m_t
- Scale parameters for theoretical uncertainties
  δM_W (4 MeV), $\delta\sin^2\theta^l_{\rm eff}$ ($4.7\cdot10^{-5}$)

M_H [GeV]$^{(\circ)}$	125.7 ± 0.4	LHC
M_W [GeV]	80.385 ± 0.015	Tevatron
Γ_W [GeV]	2.085 ± 0.042	
M_Z [GeV]	91.1875 ± 0.0021	
Γ_Z [GeV]	2.4952 ± 0.0023	
$\sigma^0_{\rm had}$ [nb]	41.540 ± 0.037	LEP
R^0_ℓ	20.767 ± 0.025	
$A^{0,\ell}_{\rm FB}$	0.0171 ± 0.0010	
A_ℓ $^{(\star)}$	0.1499 ± 0.0018	SLC
$\sin^2\theta^\ell_{\rm eff}(Q_{\rm FB})$	0.2324 ± 0.0012	
A_c	0.670 ± 0.027	SLC
A_b	0.923 ± 0.020	
$A^{0,c}_{\rm FB}$	0.0707 ± 0.0035	
$A^{0,b}_{\rm FB}$	0.0992 ± 0.0016	LEP
R^0_c	0.1721 ± 0.0030	
R^0_b	0.21629 ± 0.00066	
$\overline{m}_c$ [GeV]	$1.27^{+0.07}_{-0.11}$	
$\overline{m}_b$ [GeV]	$4.20^{+0.17}_{-0.07}$	
m_t [GeV]	173.18 ± 0.94	Tevatron
$\Delta\alpha^{(5)}_{\rm had}(M^2_Z)$ $^{(\triangle\triangledown)}$	2757 ± 10	

[The Gfitter Group, EPJC 72, 2205 (2012)]

	Input value	Free in fit	Fit result incl. M_H	Fit result not incl. M_H	Fit result incl. M_H but not exp. input in row
M_H [GeV]$^{(\circ)}$	125.7 ± 0.4	yes	125.7 ± 0.4	94^{+25}_{-22}	94^{+25}_{-22}
M_W [GeV]	80.385 ± 0.015	–	80.367 ± 0.007	80.380 ± 0.012	80.359 ± 0.011
Γ_W [GeV]	2.085 ± 0.042	–	2.091 ± 0.001	2.092 ± 0.001	2.091 ± 0.001
M_Z [GeV]	91.1875 ± 0.0021	yes	91.1878 ± 0.0021	91.1874 ± 0.0021	91.1983 ± 0.0116
Γ_Z [GeV]	2.4952 ± 0.0023	–	2.4954 ± 0.0014	2.4958 ± 0.0015	2.4951 ± 0.0017
$\sigma^0_{\rm had}$ [nb]	41.540 ± 0.037	–	41.479 ± 0.014	41.478 ± 0.014	41.470 ± 0.015
R^0_ℓ	20.767 ± 0.025	–	20.740 ± 0.017	20.743 ± 0.018	20.716 ± 0.026
$A^{0,\ell}_{\rm FB}$	0.0171 ± 0.0010	–	0.01627 ± 0.0002	0.01637 ± 0.0002	0.01624 ± 0.0002
A_ℓ $^{(\star)}$	0.1499 ± 0.0018	–	$0.1473^{+0.0006}_{-0.0008}$	0.1477 ± 0.0009	$0.1468 \pm 0.0005^{(\dagger)}$
$\sin^2\theta^\ell_{\rm eff}(Q_{\rm FB})$	0.2324 ± 0.0012	–	$0.23148^{+0.00011}_{-0.00007}$	$0.23143^{+0.00010}_{-0.00012}$	0.23150 ± 0.00009
A_c	0.670 ± 0.027	–	$0.6680^{+0.00025}_{-0.00038}$	$0.6682^{+0.00042}_{-0.00035}$	0.6680 ± 0.00031
A_b	0.923 ± 0.020	–	$0.93464^{+0.00004}_{-0.00007}$	0.93468 ± 0.00008	0.93463 ± 0.00006
$A^{0,c}_{\rm FB}$	0.0707 ± 0.0035	–	$0.0739^{+0.0003}_{-0.0005}$	0.0740 ± 0.0005	0.0738 ± 0.0004
$A^{0,b}_{\rm FB}$	0.0992 ± 0.0016	–	$0.1032^{+0.0004}_{-0.0006}$	0.1036 ± 0.0007	0.1034 ± 0.0004
R^0_c	0.1721 ± 0.0030	–	0.17223 ± 0.00006	0.17223 ± 0.00006	0.17223 ± 0.00006
R^0_b	0.21629 ± 0.00066	–	0.21474 ± 0.00003	0.21475 ± 0.00003	0.21473 ± 0.00003
$\overline{m}_c$ [GeV]	$1.27^{+0.07}_{-0.11}$	yes	$1.27^{+0.07}_{-0.11}$	$1.27^{+0.07}_{-0.11}$	–
$\overline{m}_b$ [GeV]	$4.20^{+0.17}_{-0.07}$	yes	$4.20^{+0.17}_{-0.07}$	$4.20^{+0.17}_{-0.07}$	–
m_t [GeV]	173.18 ± 0.94	yes	173.52 ± 0.88	173.14 ± 0.93	$175.8^{+2.7}_{-2.4}$
$\Delta\alpha^{(5)}_{\rm had}(M^2_Z)$ $^{(\triangle\bigtriangledown)}$	2757 ± 10	yes	2755 ± 11	2757 ± 11	2716^{+49}_{-43}
$\alpha_s(M^2_Z)$	–	yes	0.1191 ± 0.0028	0.1192 ± 0.0028	0.1191 ± 0.0028
$\delta_{\rm th} M_W$ [MeV]	$[-4, 4]_{\rm theo}$	yes	4	4	–
$\delta_{\rm th} \sin^2\theta^\ell_{\rm eff}$ $^{(\triangle)}$	$[-4.7, 4.7]_{\rm theo}$	yes	-1.4	4.7	–

[The Gfitter Group, EPJC 72, 2205 (2012)]

	Input value	Free in fit	Fit result incl. M_H	Fit result not incl. M_H	Fit result incl. M_H but not exp. input in row
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M_Z [GeV]	91.1875 ± 0.0021	yes	91.1878 ± 0.0021	91.1874 ± 0.0021	91.1983 ± 0.0116
Γ_Z [GeV]	2.4952 ± 0.0023	–	2.4954 ± 0.0014	2.4958 ± 0.0015	2.4951 ± 0.0017
$\sigma^0_{\rm had}$ [nb]	41.540 ± 0.037	–	41.479 ± 0.014	41.478 ± 0.014	41.470 ± 0.015
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$\sin^2\theta^\ell_{\rm eff}(Q_{\rm FB})$	0.2324 ± 0.0012	–	$0.23148^{+0.00011}_{-0.00007}$	$0.23143^{+0.00010}_{-0.00012}$	0.23150 ± 0.00009
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$\overline{m}_c$ [GeV]	$1.27^{+0.07}_{-0.11}$	yes	$1.27^{+0.07}_{-0.11}$	$1.27^{+0.07}_{-0.11}$	–
$\overline{m}_b$ [GeV]	$4.20^{+0.17}_{-0.07}$	yes	$4.20^{+0.17}_{-0.07}$	$4.20^{+0.17}_{-0.07}$	–
m_t [GeV]	173.18 ± 0.94	yes	173.52 ± 0.88	173.14 ± 0.93	$175.8^{+2.7}_{-2.4}$
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$\alpha_s(M^2_Z)$	–	yes	0.1191 ± 0.0028	0.1192 ± 0.0028	0.1191 ± 0.0028
$\delta_{\rm th} M_W$ [MeV]	$[-4, 4]_{\rm theo}$	yes	4	4	–
$\delta_{\rm th} \sin^2\theta^\ell_{\rm eff}$ $^{(\triangle)}$	$[-4.7, 4.7]_{\rm theo}$	yes	-1.4	4.7	–

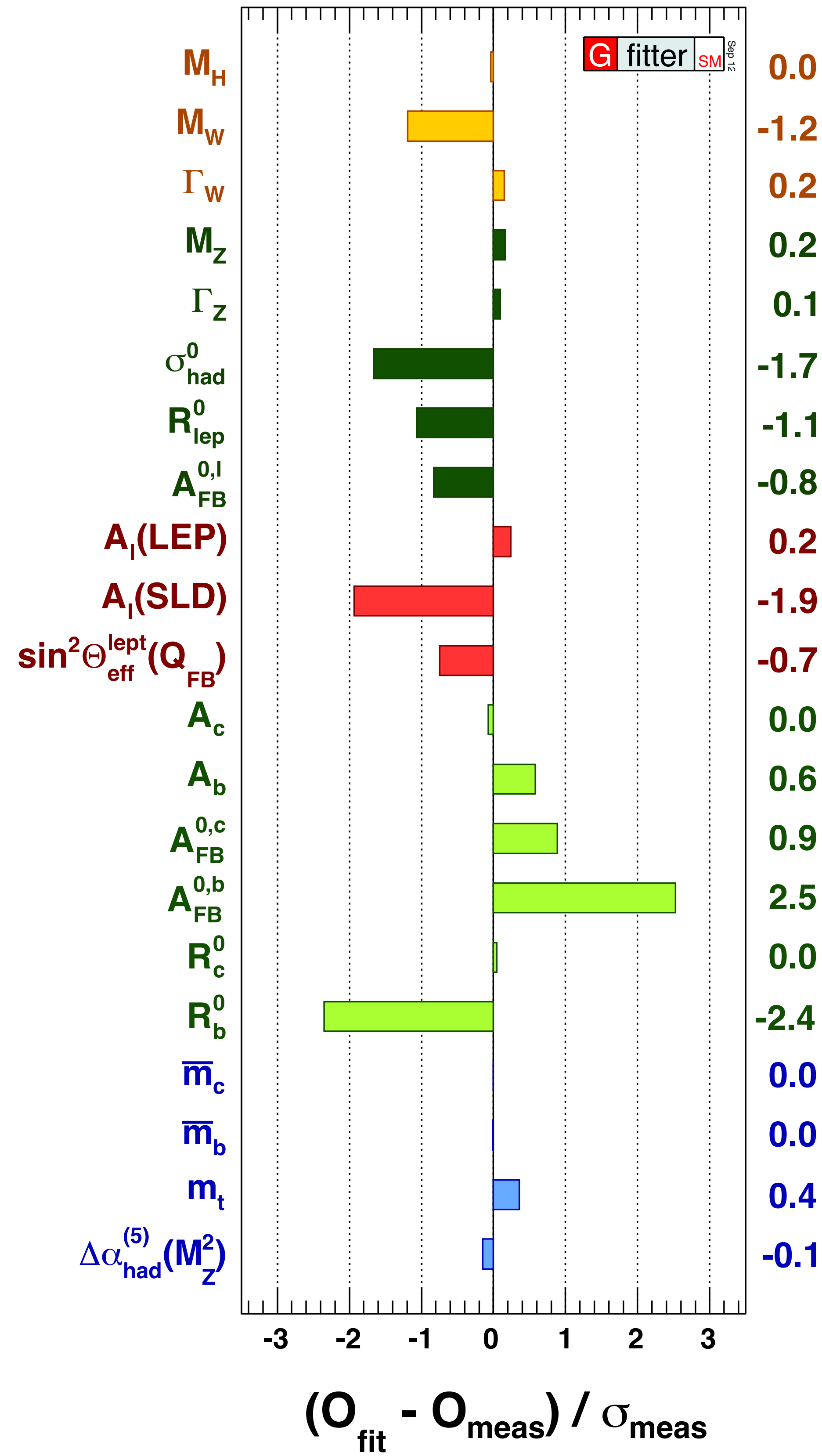
Global Fit: Results

χ^2_{min}/ndf = 21.8/14 → p-value = 0.08

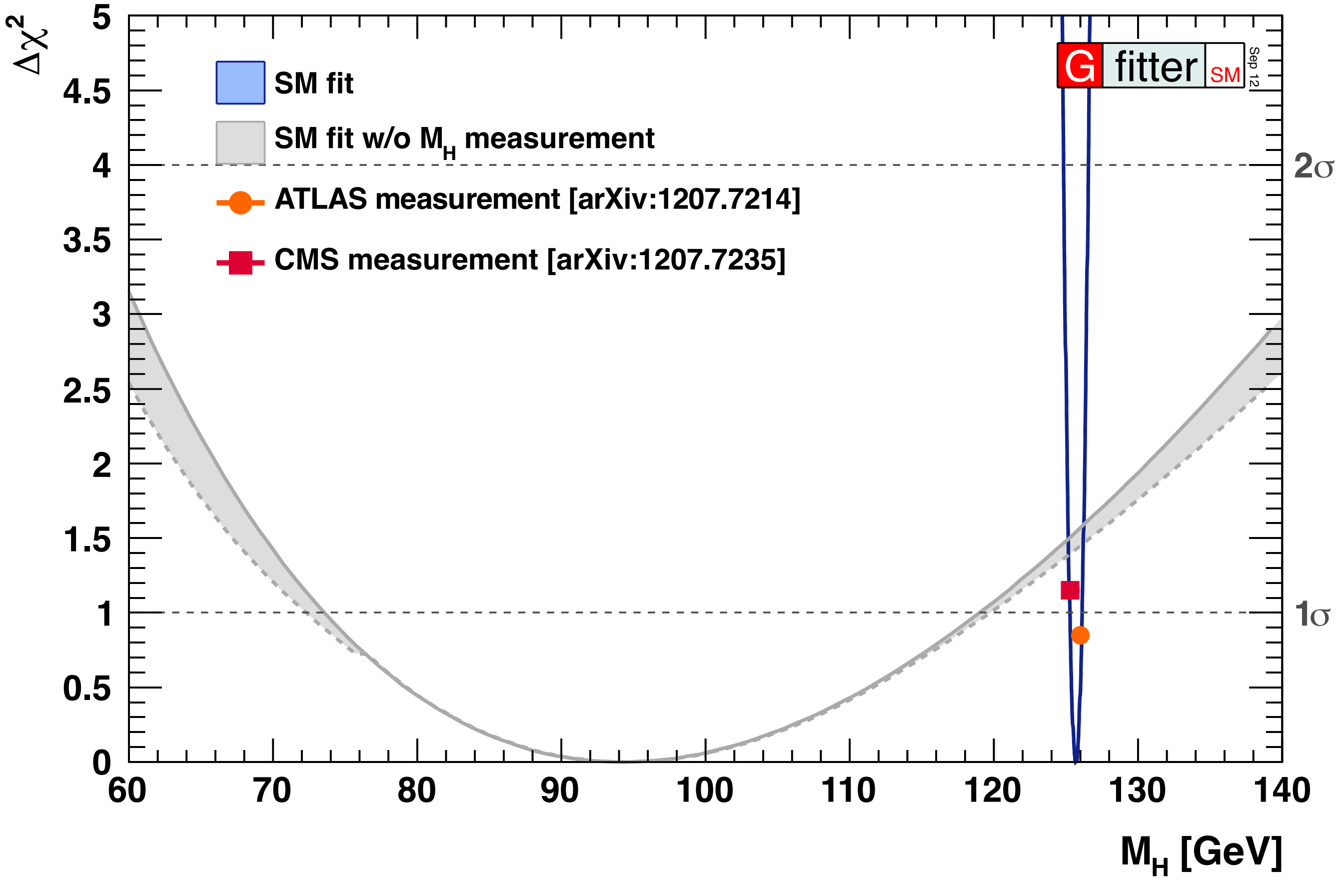
- large value of χ^2_{min} not due to inclusion of M_H measurement
- without M_H measurement: χ^2_{min} /ndf = 20.3/13 → naive p-value = 0.09

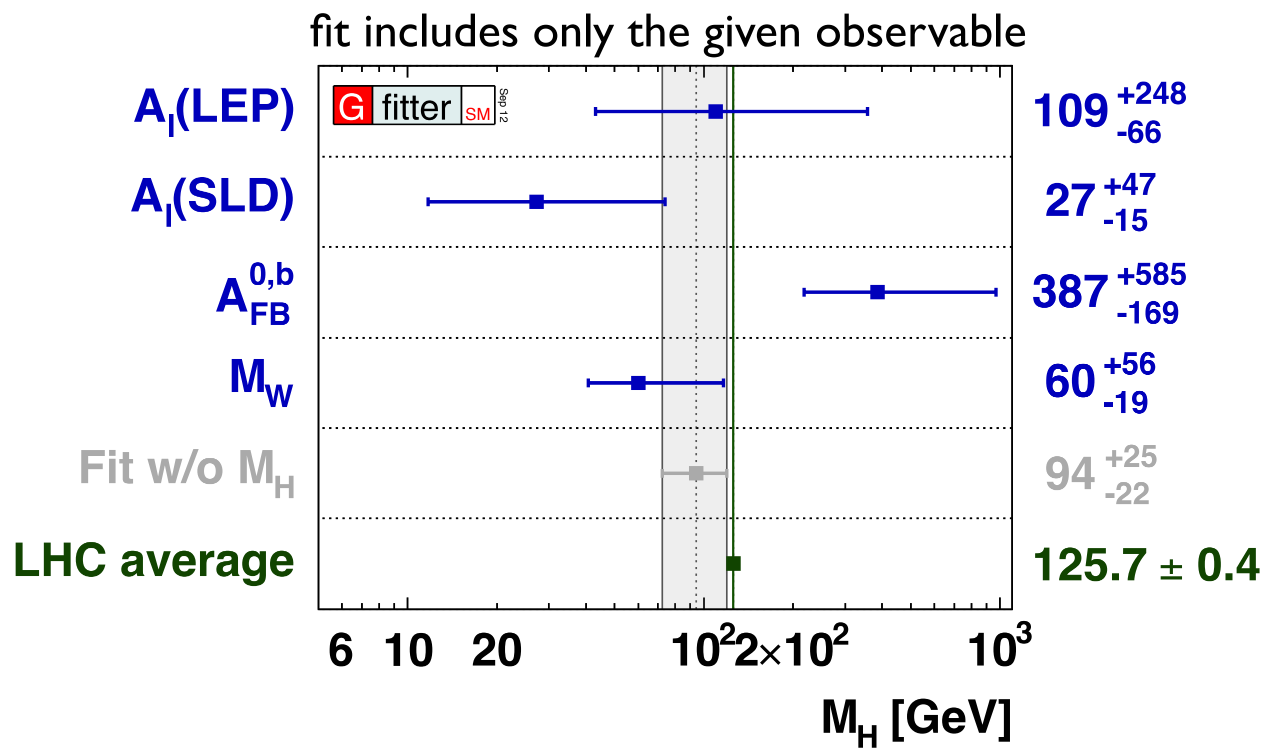
Pull values after the fit

- No pull value exceeds deviations of more than 3σ (consistency of SM)
- Small values for M_H, A_c, $R^0{}_c$, m_c and m_b indicate that their input accuracies exceed the fit requirements
- Largest deviations in the b-sector: $A^{0,b}{}_{FB}$ and $R^0{}_b$ with 2.5σ and -2.4σ (little dependence on M_H)
- $R^0{}_b$ using one-loop calculation: 0.8σ



Global Fit: Results





Scan of the $\Delta\chi^2$ profile versus M_H

- blue line: full SM fit
- grey band: fit without M_H measurement
- fit without M_H input gives
  $M_H = 94^{+25}_{-22}$ GeV
- consistent within 1.3σ with measurement

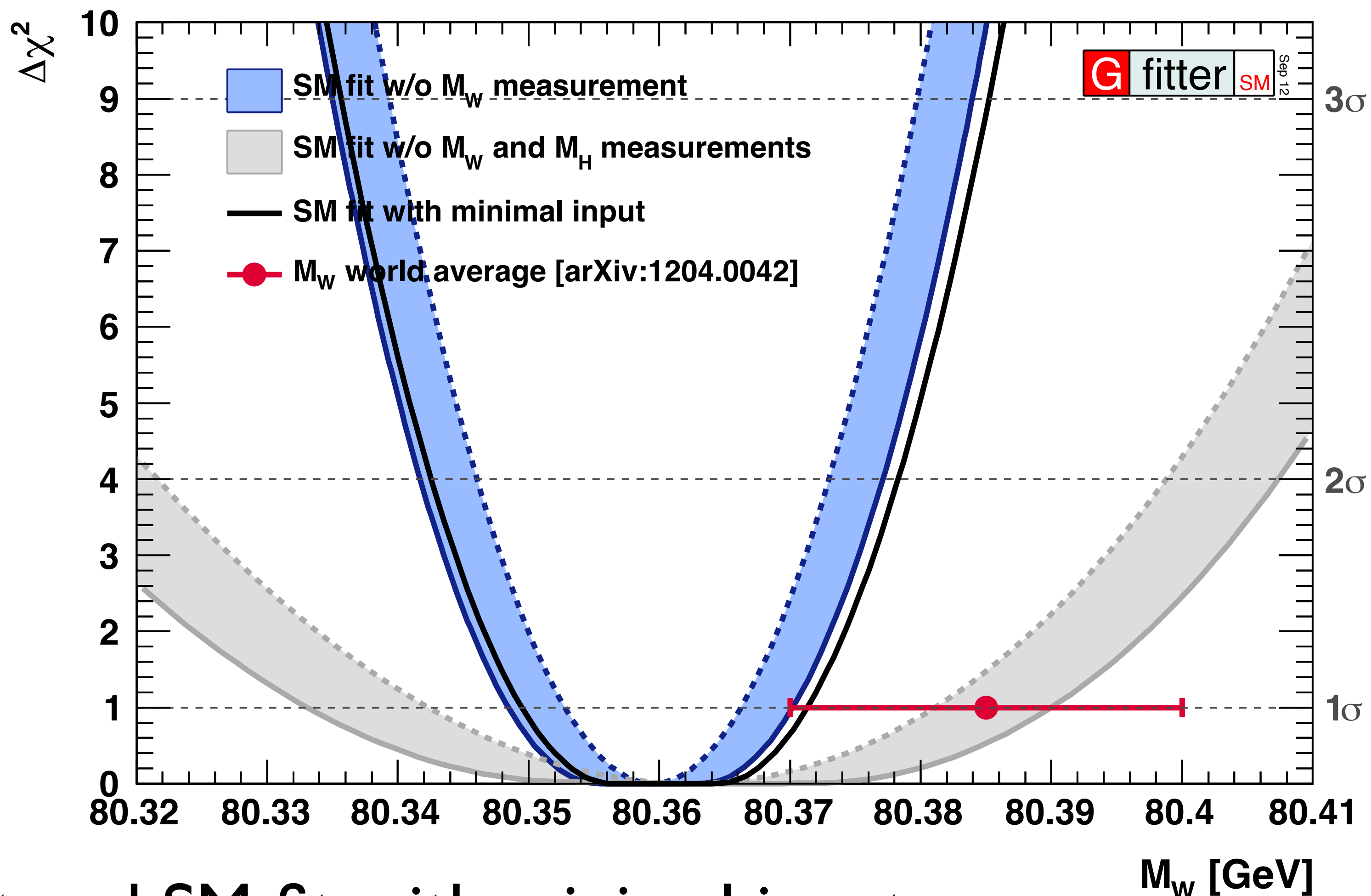
Determination of M_H removing all sensitive observables except the given one:

Tension (2.5σ) between $A^{0,b}{}_{FB}$, A_{lep}(SLD) and M_W visible

Indirect Determination: W Mass

Scan of the $\Delta\chi^2$ profile versus M_W

- M_H measurement allows for precise constraint of M_W
- also shown: SM fit with minimal input: M_Z, G_F, $\Delta\alpha_{had}^{(5)}(M_Z)$, $\alpha_s(M_Z)$, M_H and fermion masses



- Consistency between total fit and SM fit with minimal input
- Fit result for the indirect determination of M_W:

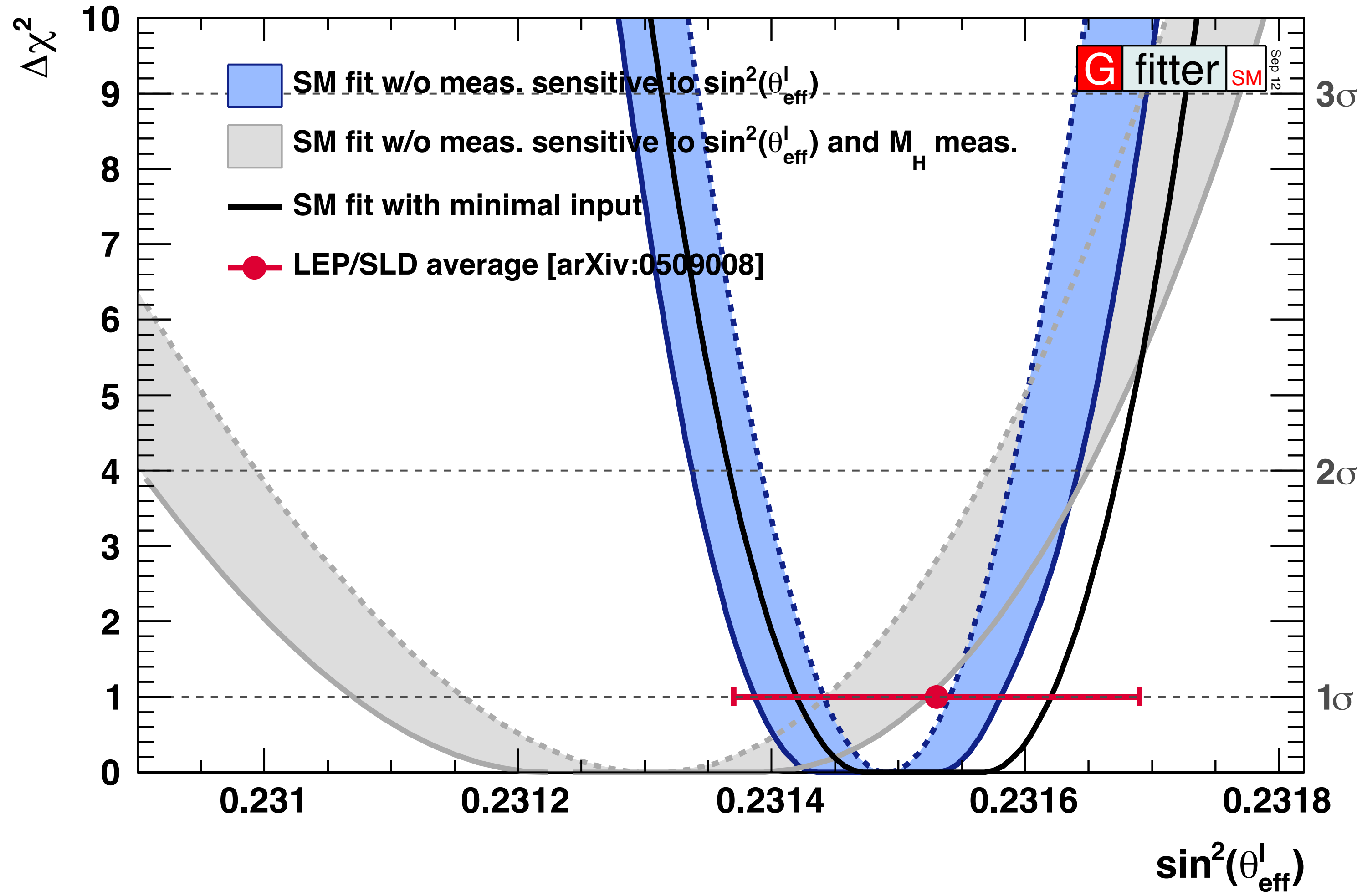
$$\begin{aligned} M_W &= 80.3593 \pm 0.0056_{m_t} \pm 0.0026_{M_Z} \pm 0.0018_{\Delta\alpha_{\rm had}} \\ &\qquad \pm 0.0017_{\alpha_S} \pm 0.0002_{M_H} \pm 0.0040_{\rm theo} \\ &= 80.359 \pm 0.011_{\rm tot} \end{aligned}$$

$\boxed{\text{More precise than the direct measurements}}$

The Effective Weak Mixing

Scan of the $\Delta\chi^2$ profile versus $\sin^2\theta^l_{eff}$

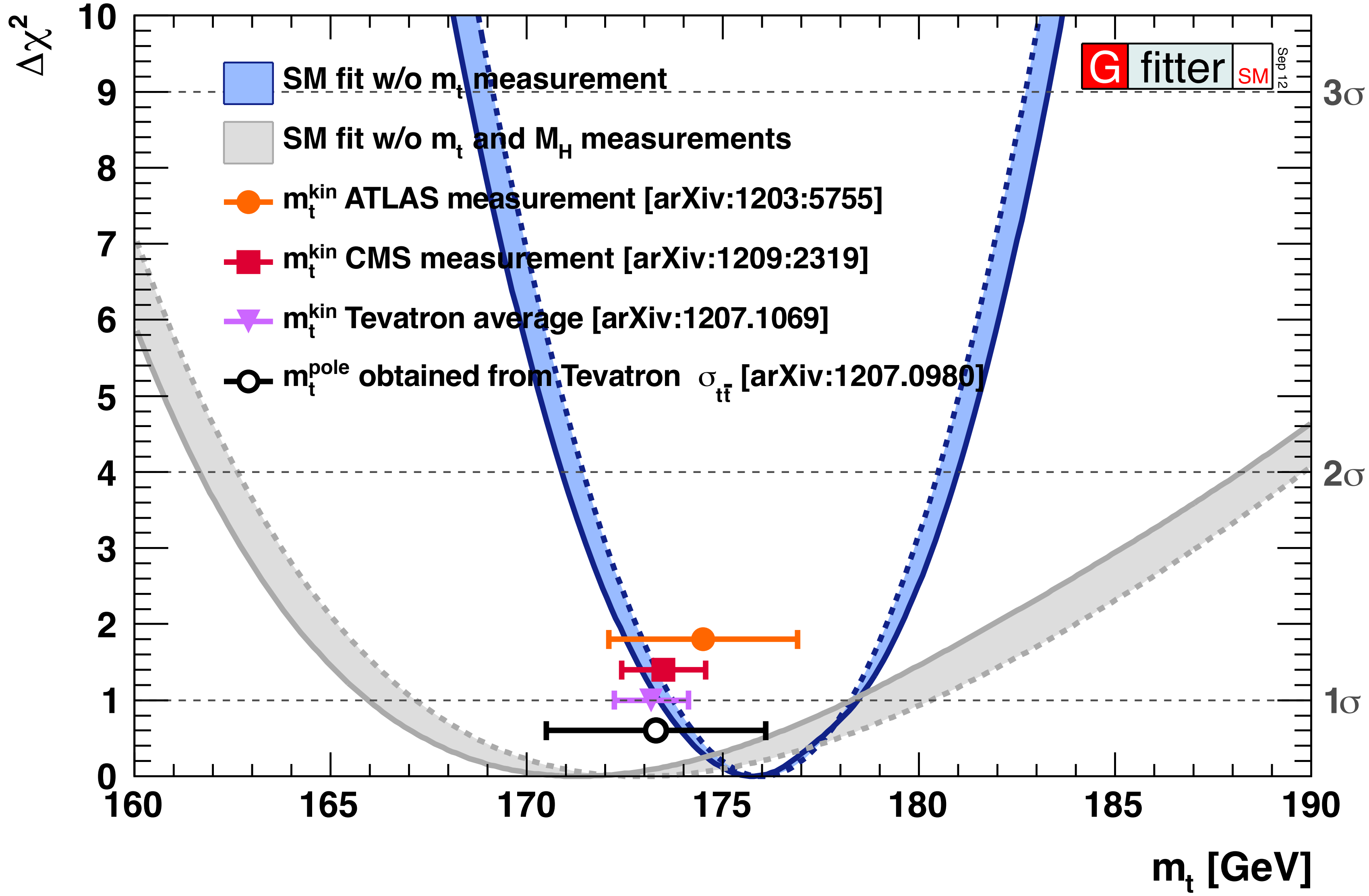
- all observables sensitive to $\sin^2\theta^l_{eff}$ removed from fit
- M_H measurement allows for precise constraint of $\sin^2\theta^l_{eff}$
- also shown: SM fit with minimal input



$$\begin{aligned}\sin^2\theta^\ell_{\rm eff} &= 0.231496 \pm 0.000030_{m_t} \pm 0.000015_{M_Z} \pm 0.000035_{\Delta\alpha_{\rm had}} \\ &\quad \pm 0.000010_{\alpha_S} \pm 0.000002_{M_H} \pm 0.000047_{\rm theo}, \\ &= 0.23150 \pm 0.00010_{\rm tot}\end{aligned}$$

More precise than the direct determination from LEP/SLD measurements

Indirect Determination: Top Mass



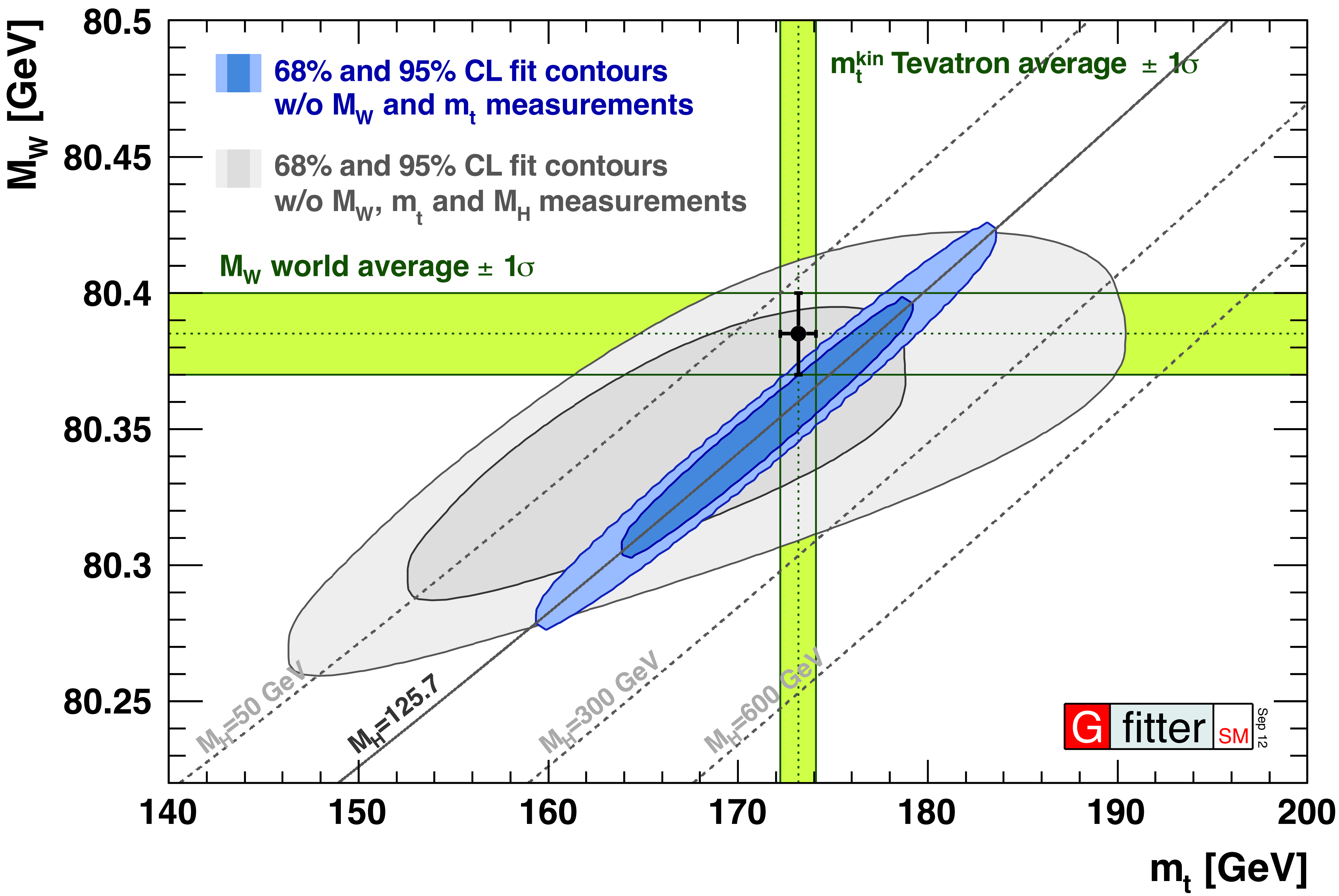
Scan of the $\Delta\chi^2$ profile versus m_t

- consistency with direct measurements
- M_H measurement allows for better constraint of m_t

$m_t = 175.8^{+2.7}_{-2.4}\,\mathrm{GeV}$ (Tevatron average: m_t = 173.2 ± 0.9 GeV)

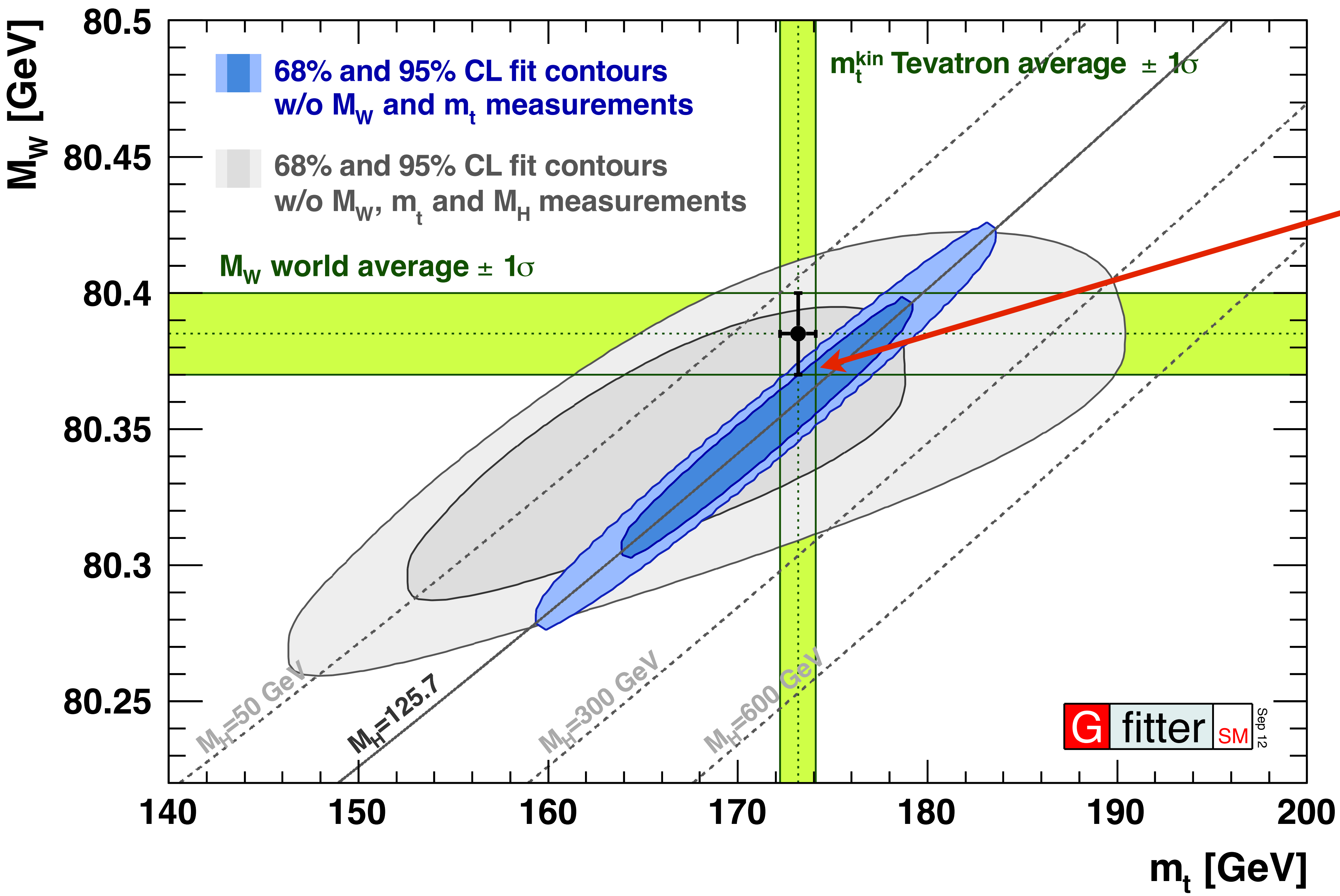
W and Top Mass

Impressive consistency of the SM



W and Top Mass

Impressive consistency of the SM



Once M_H is fixed, we cornered the SM!

Effects of new physics through loop corrections!

⇒ improve measurements of EW precision observables

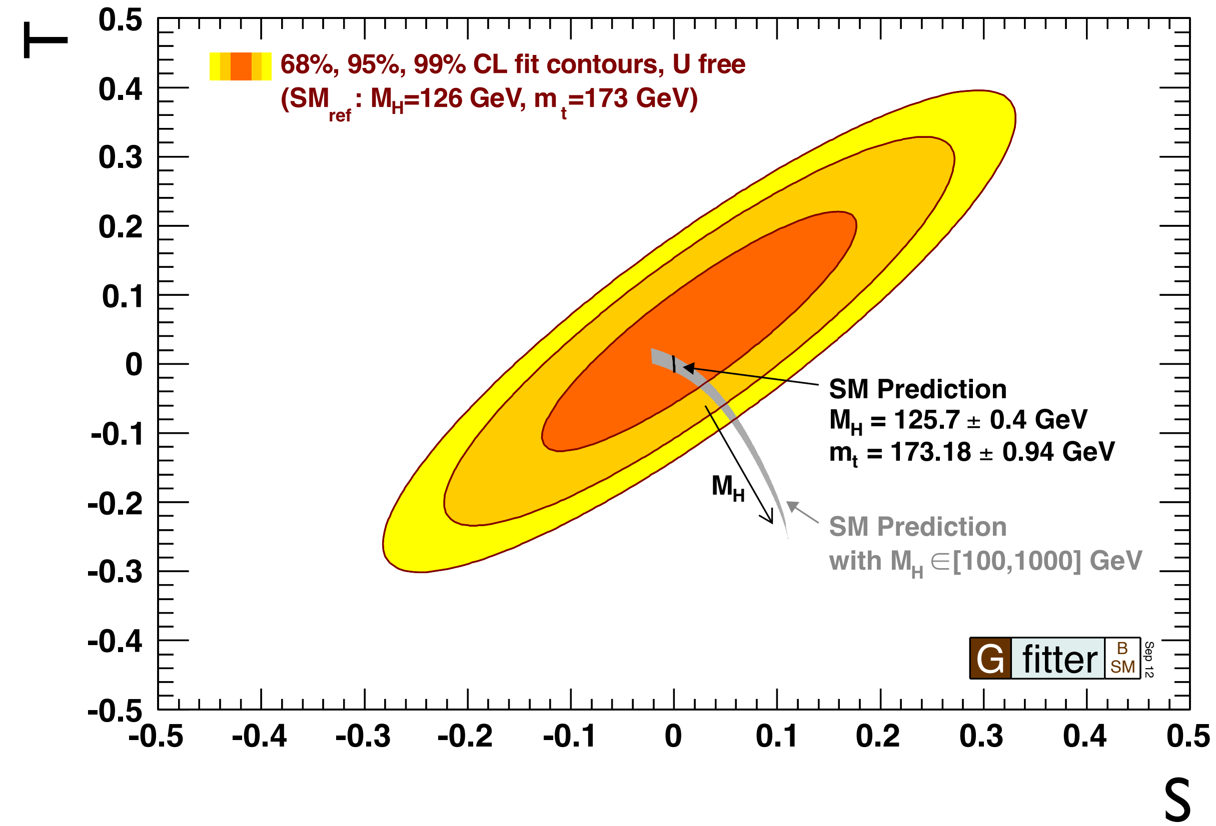
Constraints on S, T and U

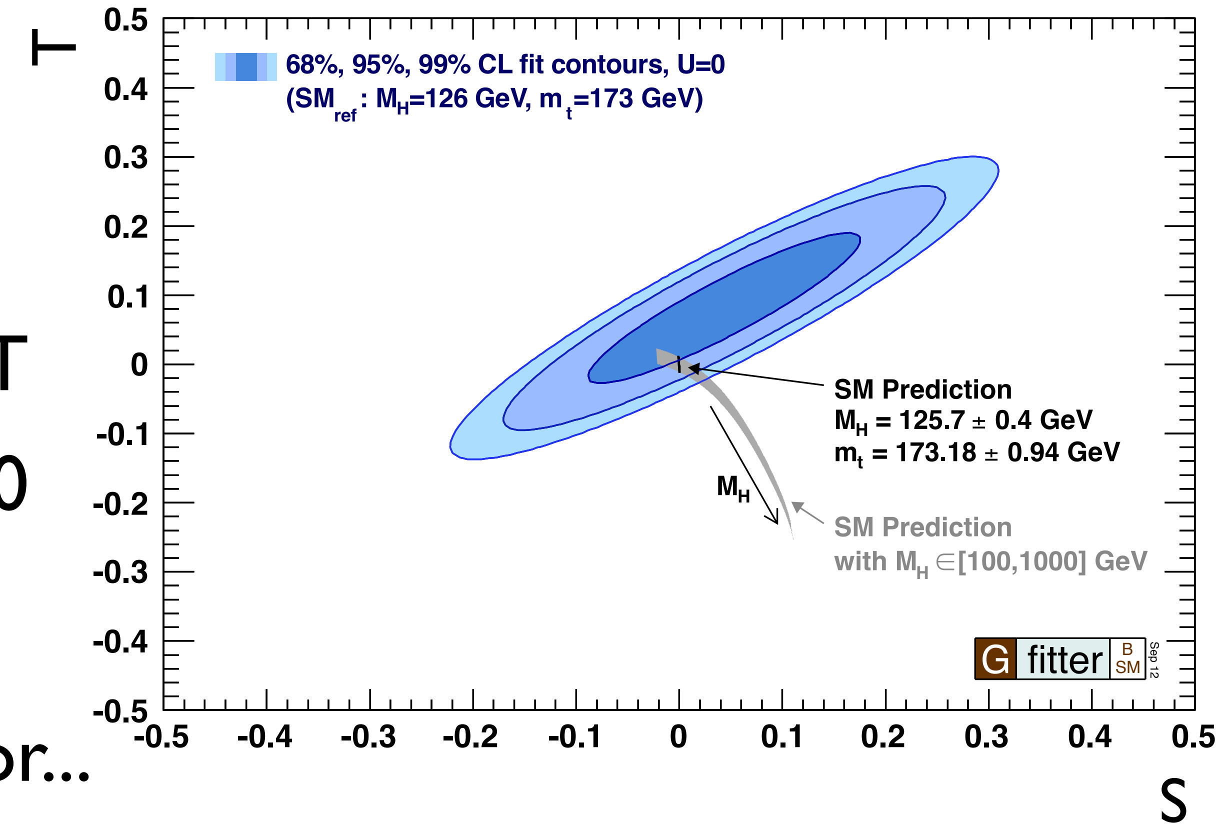
Parametrise contributions from vacuum polarisations

- sensitivity to new physics
- SM reference chosen to be
  $\left.\begin{array}{l} M_{H,\mathrm{ref}} = 126\ \mathrm{GeV} \\ m_{t,\mathrm{ref}} = 173\ \mathrm{GeV} \end{array}\right]$ defines (0, 0, 0)
  - S,T depend logarithmically on M_H
- Fit result:
  S = 0.03 ± 0.10
  T = 0.05 ± 0.12
  U = 0.03 ± 0.10
  with large correlation between S and T
- Stronger constraints from fit with U=0

No indication of new physics

- Constrains on 2HDM, LED, Technicolor...





The Future

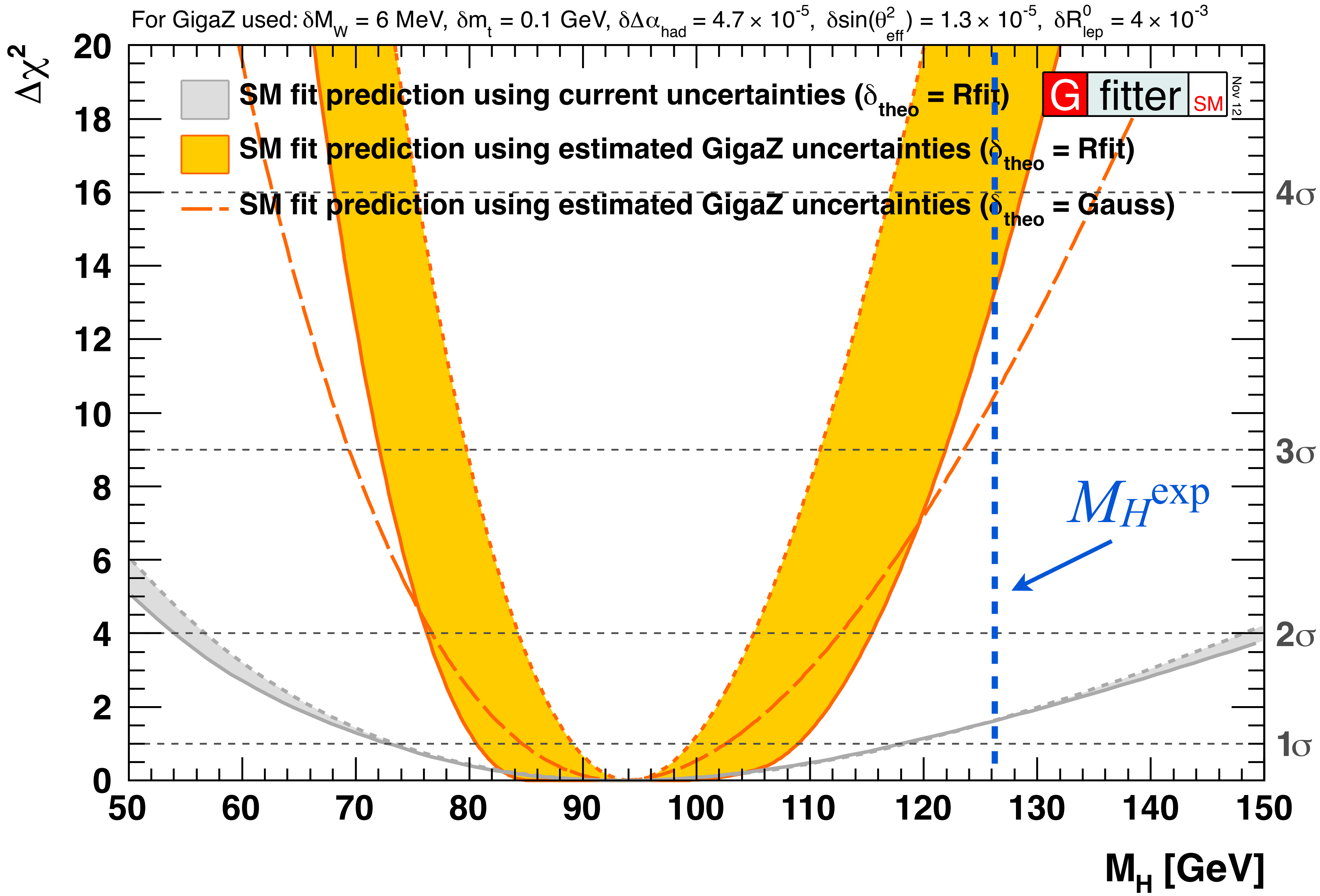
“The future you have tomorrow will not be the same future you had yesterday.”
(Chuck Palahniuk)

ILC with GigaZ

A future linear collider would tremendously improve the precision of electroweak observables

- $t\bar{t}$ threshold
  - obtain m_t indirectly from production cross section: δm_t = 1 → 0.1 GeV
- Z peak measurements
  - polarised beams, uncertainty $\delta A^{0,f}_{LR}$: $10^{-3} \to 10^{-4}$ translates to $\delta \sin^2\theta^l_{\mathrm{eff}}$: $10^{-4} \to 1.3 \cdot 10^{-5}$
  - high statistics: 10^9 Z decays: $\delta R^0_{\mathrm{lep}}$: $2.5 \cdot 10^{-2} \to 4 \cdot 10^{-3}$
- WW threshold
  - from threshold scan: δM_W = 15 → 6 MeV
- Low energy data
  - $\Delta\alpha_{\mathrm{had}}$: more precise cross section data for low energy ($\sqrt{s}$ < 1.8 GeV) and around $c\bar{c}$ resonance (BES-III), improved α_s, improvements in theory: $10^{-4} \to 5 \cdot 10^{-5}$

Prospects for ILC with GigaZ

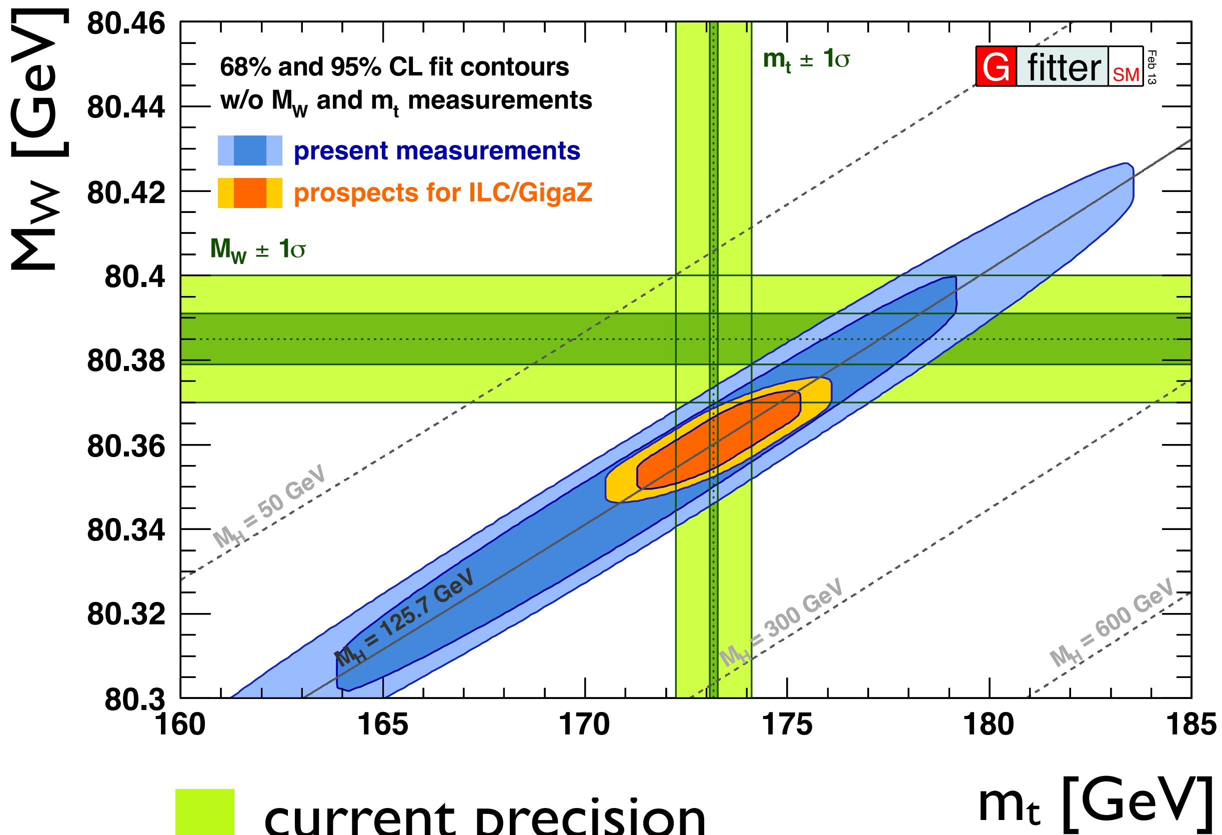


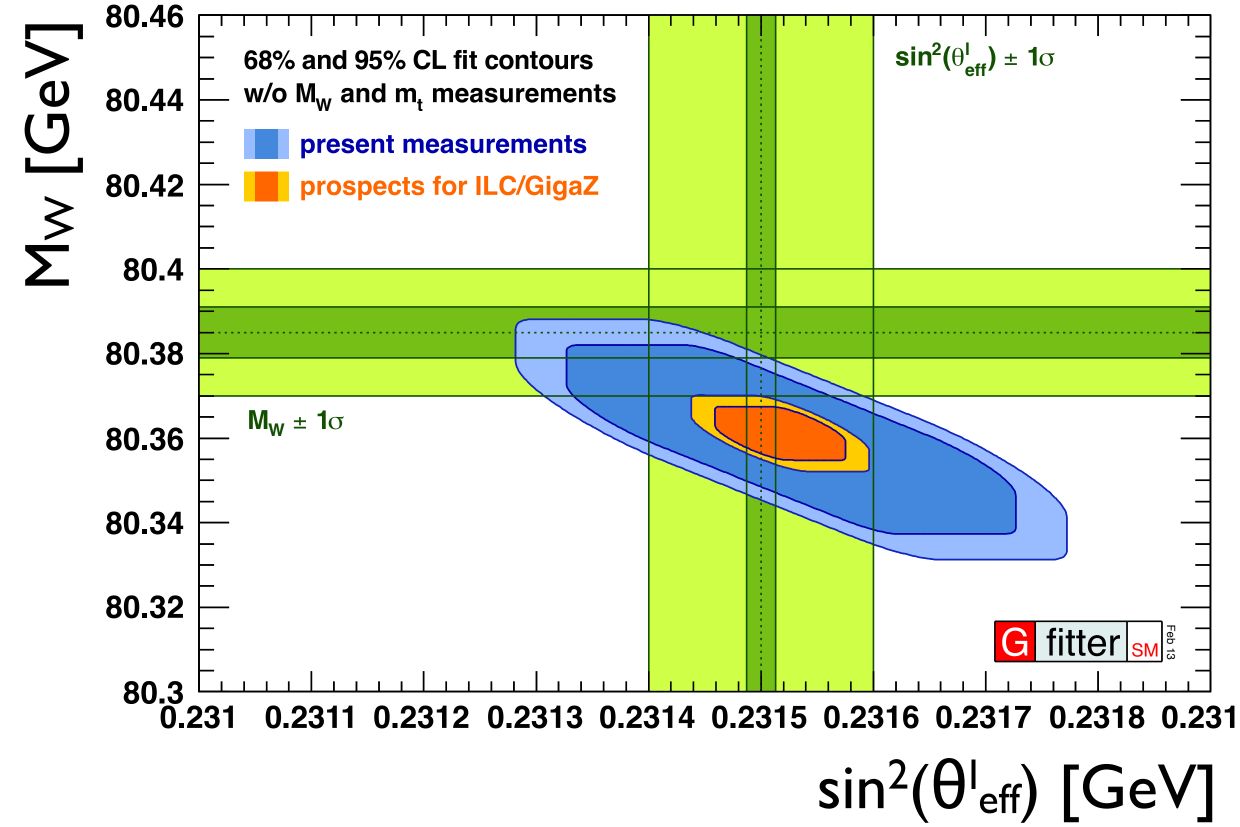
- no theory uncertainty: $M_H = 94.2\,^{+5.3}_{-5.0}\,(^{+22.7}_{-18.7})$ GeV
- Rfit scheme: $M_H = 92.3\,^{+16.6}_{-11.6}\,(^{+36.3}_{-23.3})$ GeV

in brackets the 4σ values

- strong coupling: $\alpha_s(M_Z) = 0.1190 \pm 0.0005(\text{exp}) \pm 0.0001(\text{theo})$

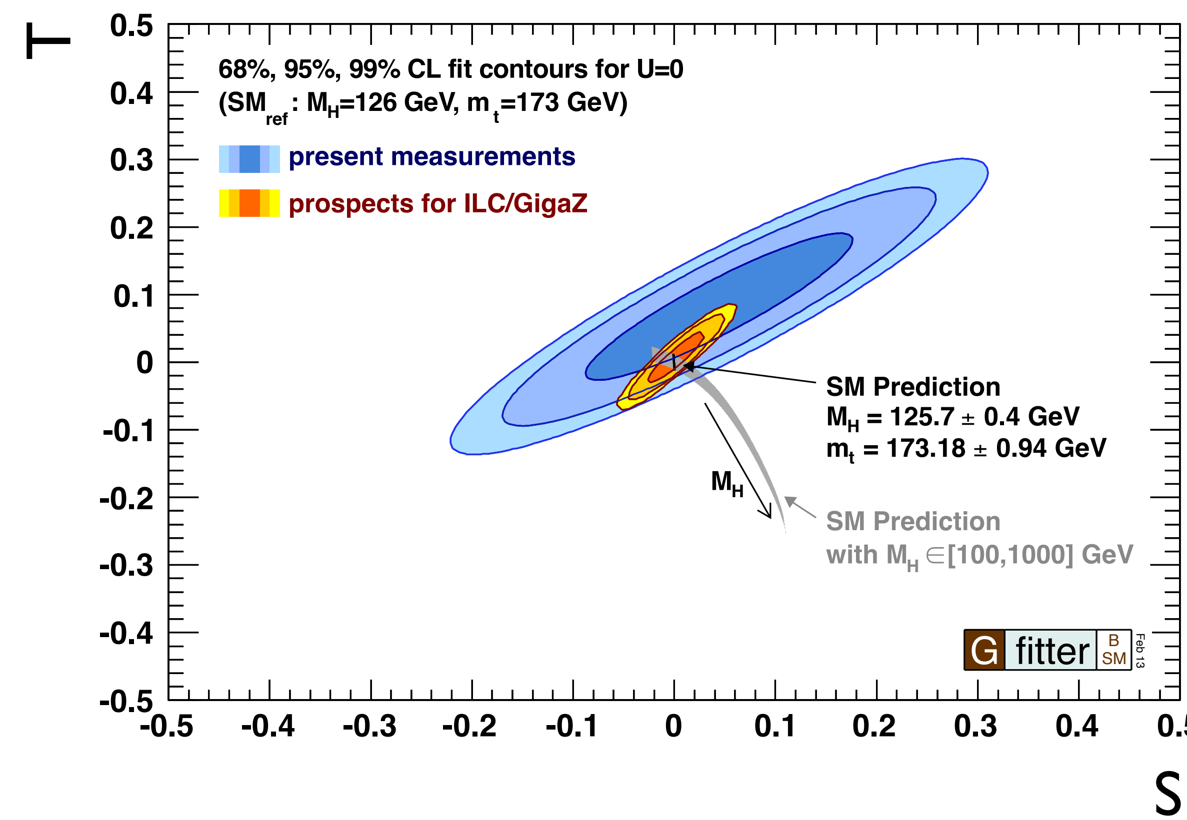
Prospects for ILC with GigaZ





current precision
prospects for direct measurements

- Assume 50% of today's theoretical uncertainty (implies three-loop EW calculations)
- Huge reduction of uncertainty for indirect determinations
- Strong constraints on S, T, U



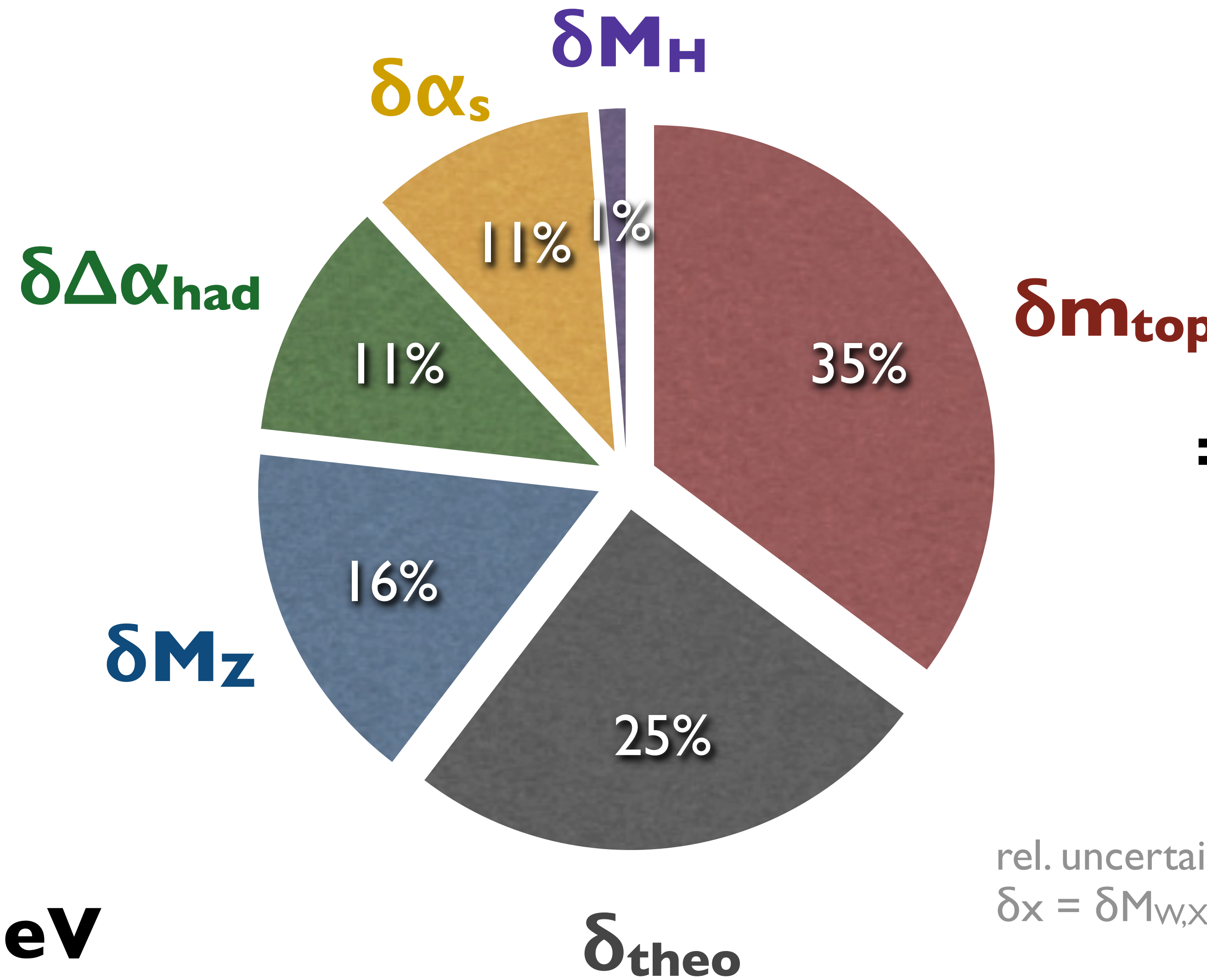
Summary

SM Fit with p-value of 0.07

- incentive to revisit $Z \rightarrow b\bar{b}$ experimentally and theoretically !

Consistency of the SM:

δM_W (indirect) = **= 11 MeV**



rel. uncertainty:
$\delta x = \delta M_{W,x} / \delta M_{W,tot}$

δM_W (exp) = 15 MeV

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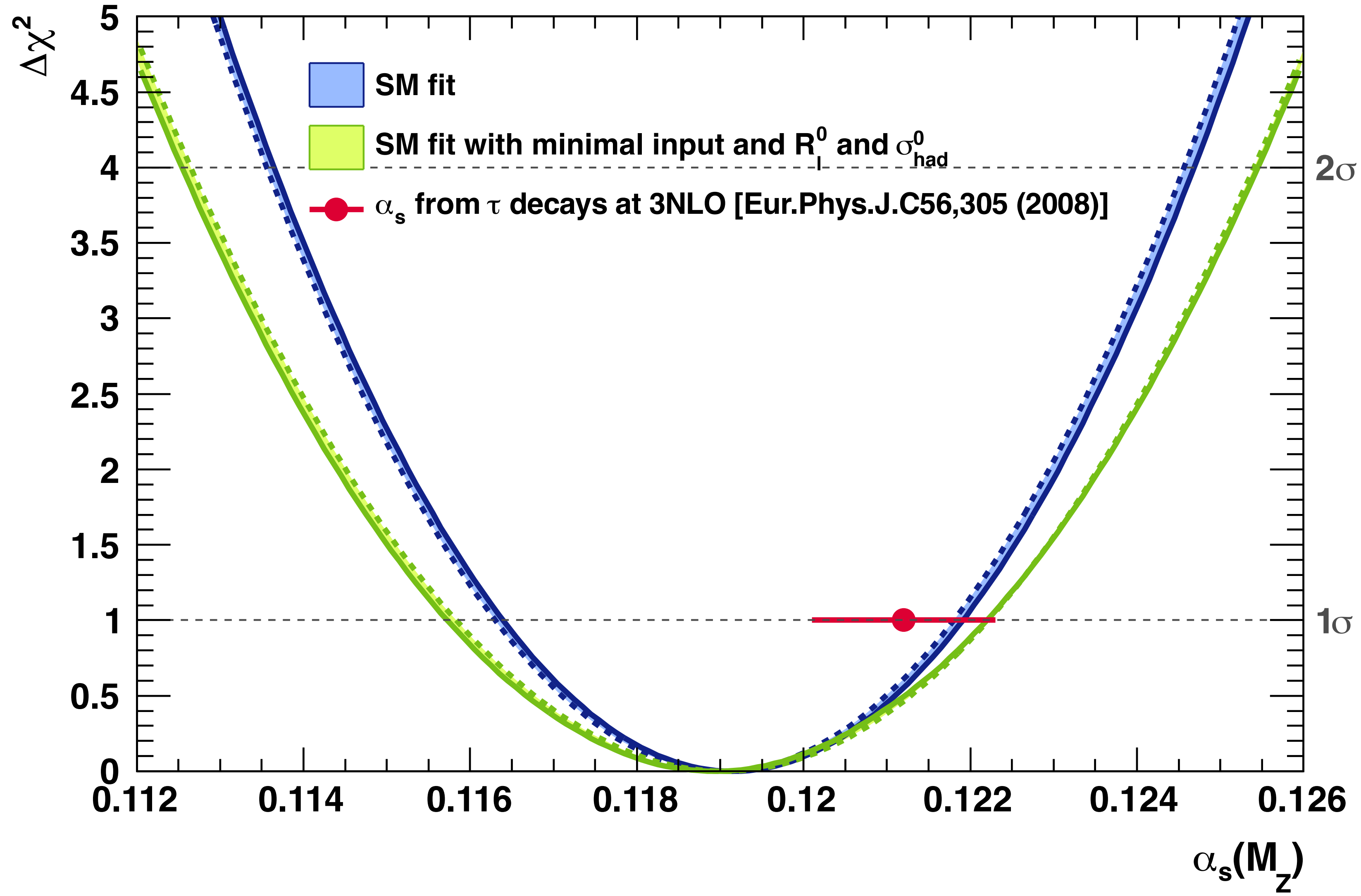
Additional Material

$\alpha_s(M_Z)$ from Z→hadrons

- Determination of α_s at NNNLO
- most sensitivity through total hadronic cross section σ^0_{had} and the partial leptonic width R^0_l
- Theory uncertainty obtained by scale variation, per-mille level



$$\alpha_s(M_Z) = 0.1191 \pm 0.0028\,(\text{exp.}) \pm 0.0001\,(\text{theo.})$$

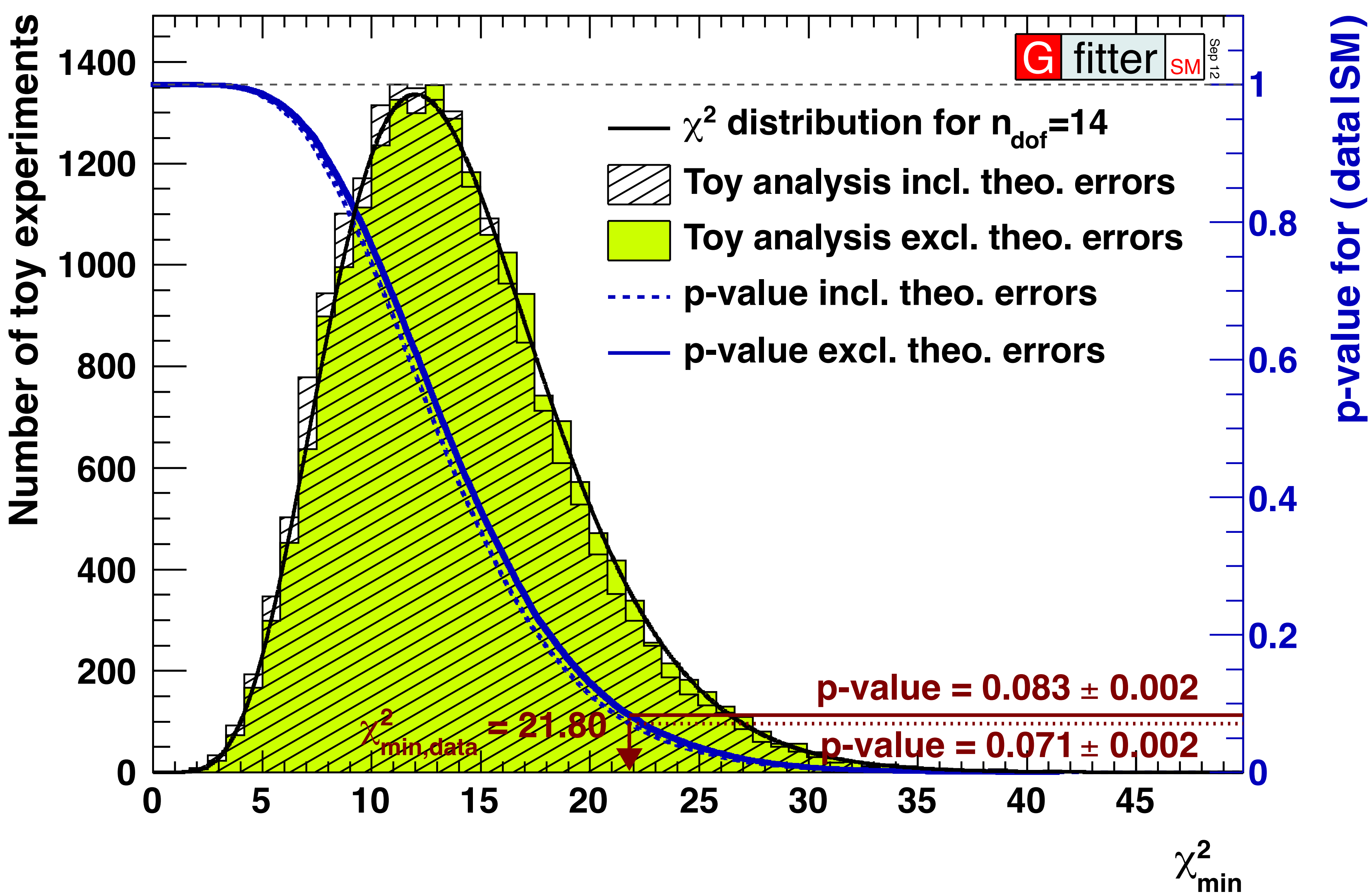
- Good agreement with value from τ decays, also at N³LO

Improvement in precision only with ILC/GigaZ expected

Goodness of Fit

Toy analysis with 20000 toy experiments

- p-value: probability for getting $\chi^2_{\text{min, toy}}$ larger than χ^2_{min} from data
- p-value: probability for wrongly rejecting the SM: 0.07 ± 0.01 (theo)



Oblique Parameters

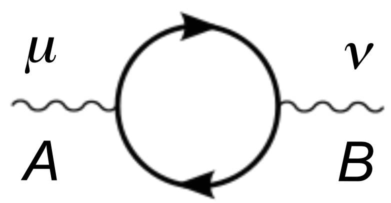
"A man should look for what is, and not for what he thinks should be."

(Albert Einstein)

Gfitter BSM — Beyond the SM

At low energies, BSM physics appears dominantly through vacuum polarisation

- Aka, *oblique corrections*



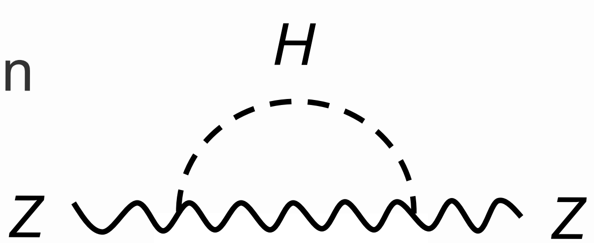
$$= i\Pi^{\mu\nu}_{AB=\{W,Z,\gamma\}}(q)$$

- Direct corrections (vertex, box, bremsstrahlung) generally suppressed by m_f / Λ

Oblique corrections reabsorbed into electroweak parameters $\Delta\rho$, $\Delta\kappa$, Δr

Electroweak fit sensitive to BSM physics through oblique corrections

- In direct competition with Higgs loop corrections



- Oblique corrections from New Physics described through ***STU* parameters**
  [Peskin-Takeuchi, Phys. Rev. D46, 381 (1992)]

$$\boxed{O_{\text{meas}} = O_{\text{SM,ref}}(M_H, m_t) + c_S S + c_T T + c_U U}$$

S : (S+U) New Physics contributions to neutral (charged) currents

T : Difference between neutral and charged current processes – sensitive to weak isospin violation

U : Constrained by M_W and Γ_W. Usually very small in NP models (often: U=0)

- Also considered: correction to $Z \to bb$ coupling, and extended parameters (VWX)
  [Burgess et al., PLB 326, 276 (1994), PRD 49, 6115 (1994)]

Measurements at the Z-Pole

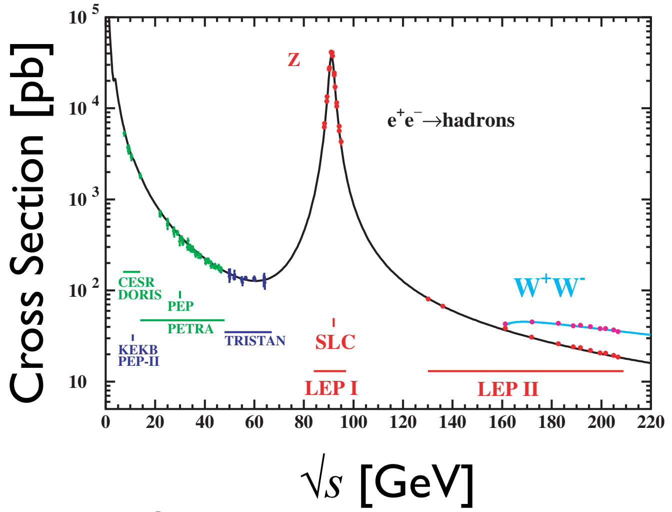
Total cross section

- Express in terms of partial decay width of initial and final state

$$\sigma^Z_{f\bar{f}} = \sigma^0_{f\bar{f}} \frac{s\Gamma_Z^2}{(s-M_Z^2)^2 + s^2\Gamma_Z^2/M_Z^2} \frac{1}{R_{\mathrm{QED}}} \quad \text{with} \quad \sigma^0_{f\bar{f}} = \frac{12\pi}{M_Z^2} \frac{\Gamma_{ee}\Gamma_{f\bar{f}}}{\Gamma_Z^2}$$

Corrected for QED radiation

- Full width: $\Gamma_Z = \Gamma_{ee} + \Gamma_{\mu\mu} + \Gamma_{\tau\tau} + \Gamma_{\mathrm{had}} + \Gamma_{\mathrm{inv}}$
- Highly correlated set of parameters



Less correlated set of parameters

- Z mass and width: $\boxed{M_Z}$, $\boxed{\Gamma_Z}$
- Hadronic pole cross section $\boxed{\sigma^0_{\mathrm{had}}} = 12\pi/M_Z^2 \cdot \Gamma_{ee}\Gamma_{\mathrm{had}}/\Gamma_Z^2$
- Three leptonic ratios (lepton univ.) $\boxed{R^0_\ell} = R^0_e = \Gamma_{\mathrm{had}}/\Gamma_{ee}\ (= R^0_\mu = R^0_\tau)$
- Hadronic width ratios $\boxed{R^0_b}$, $\boxed{R^0_c}$

Measurements at the Z-Pole

Definition of Asymmetry

- Distinguish axial and axial-vector couplings of the Z

$$A_f = \frac{g_{L,f}^2 - g_{R,f}^2}{g_{L,f}^2 + g_{R,f}^2} = \frac{2 g_{V,f}\, g_{A,f}}{g_{V,f}^2 + g_{A,f}^2}$$

- Directly related to $\sin^2 \theta_{\mathrm{eff}}^{f\bar{f}} = \frac{1}{4Q_f}\left(1 + \mathcal{R}e\left(\frac{g_{V,f}}{g_{A,f}}\right)\right)$

Observables

- In case of no beam polarisation (LEP) use final state angular distribution to define forward/backward asymmetry

$$A_{FB}^{f} = \frac{N_F^f - N_B^f}{N_F^f + N_B^f} \qquad A_{FB}^{0,f} = \frac{3}{4} A_e A_f$$

- Polarised beams (SLC): define left/right asymmetry

$$A_{LR}^{f} = \frac{N_L^f - N_R^f}{N_L^f + N_R^f} \frac{1}{\langle |P|_e \rangle} \qquad A_{LR}^{0} = A_e$$

- Measurements: $\boxed{A_{FB}^{0,\ell},}$ $\boxed{A_{FB}^{0,c},}$ $\boxed{A_{FB}^{0,b}}$ $\boxed{A_\ell,}$ $\boxed{A_c,}$ $\boxed{A_b}$

The Electromagnetic Coupling

Running of the EM coupling

- The EW fit requires precise knowledge of $\alpha(M_Z)$ (better than 1%)
- Conventionally parametrised as ($\alpha(0)$ = fine structure constant)

$$\alpha(s) = \frac{\alpha(0)}{1 - \Delta\alpha(s)}$$

- Evolution with renormalisation scale

$$\Delta\alpha(s) = \Delta\alpha_{\mathrm{lep}}(s) + \Delta\alpha^{(5)}_{\mathrm{had}}(s) + \Delta\alpha_{\mathrm{top}}(s)$$

- Leptonic term known up to three loops for $q^2 \gg m_l$ [M. Steinhauser, Phys. Lett. B429, 158 (1998)]
- Top quark contribution known up to two loops, small: $-0.7 \cdot 10^{-4}$
- Hadronic contribution difficult, cannot be obtained from pQCD alone
  - analysis of low energy e^+e^- data
  - usage of pQCD if lack of data

$$\Delta\alpha_{\mathrm{had}}(M_Z^2) = (274.2 \pm 1.0) \cdot 10^{-4}$$

[M. Davier et al., Eur. Phys. J. C71, 1515 (2011)]

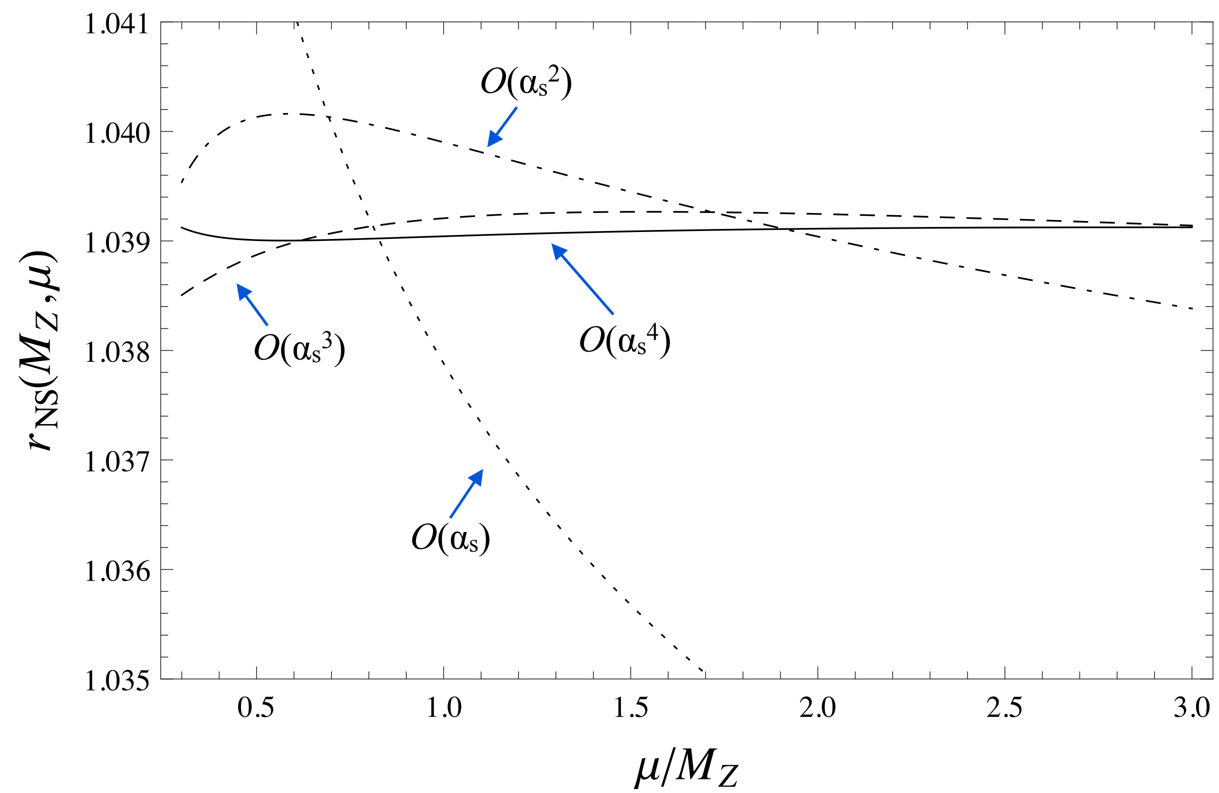
Radiator Functions

- Partial widths are defined inclusively: they contain QCD and QED contributions
- Corrections can be expressed as radiator functions $R_{A,f}$ and $R_{V,f}$

$$\Gamma_{f\bar{f}} = N_c^f \frac{G_F M_Z^3}{6\sqrt{2}\pi} \left(|g_{A,f}|^2 R_{A,f} + |g_{V,f}|^2 R_{V,f} \right)^2$$

[D. Bardin, G. Passarino, "The Standard Model in the Making", Clarendon Press (1999)]

- High sensitivity to the strong coupling α_s
- Recently full four-loop calculation of QCD Adler function became available (N^{3}LO)
- Much reduced scale dependence
- Theoretical uncertainty of 0.1 MeV, compare to experimental uncertainty of 2.0 MeV



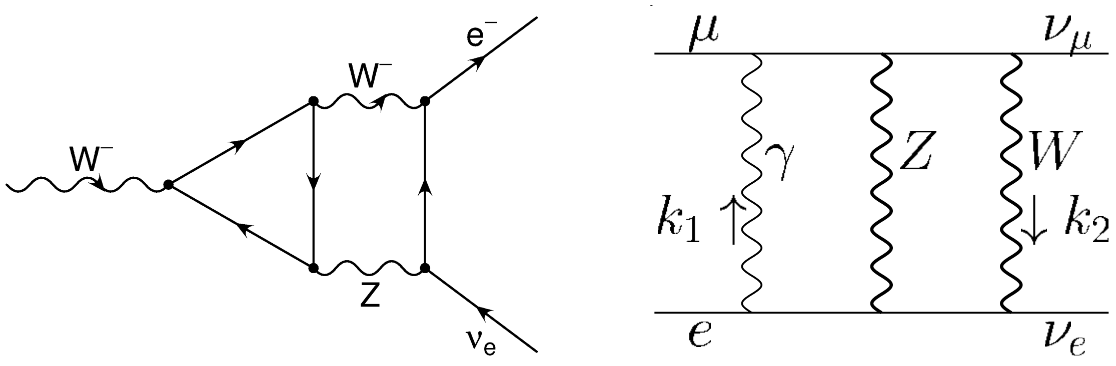
[P. Baikov et al., Phys. Rev. Lett. 108, 222003 (2012)]
[P. Baikov et al Phys. Rev. Lett. 104, 132004 (2010)]

Calculation of M_W

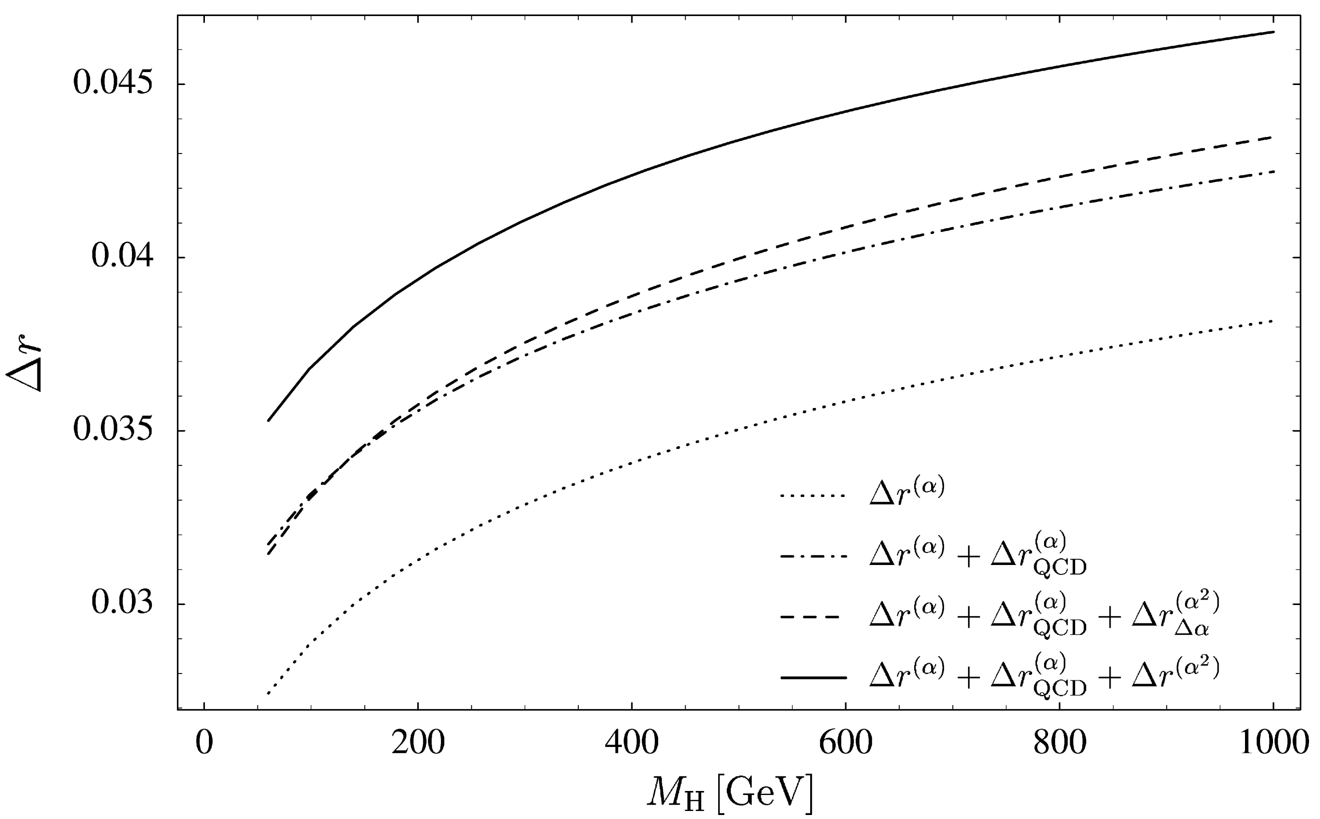
- Full EW one- and two-loop calculation of fermionic and bosonic contributions
- One- and two-loop QCD corrections and leading terms of higher order corrections
- Results for Δr include terms of order $O(\alpha)$, $O(\alpha\alpha_s)$, $O(\alpha\alpha_s^2)$, $O(\alpha^2_{\text{ferm}})$, $O(\alpha^2_{\text{bos}})$, $O(\alpha^2\alpha_s m_t^4)$, $O(\alpha^3 m_t^6)$
- Uncertainty estimate:
  - missing terms of order $O(\alpha^2\alpha_s)$: about 3 MeV (from $O(\alpha^2\alpha_s m_t^4)$)
  - electroweak three-loop correction $O(\alpha^3)$: < 2 MeV
  - three-loop QCD corrections $O(\alpha\alpha_s^3)$: < 2 MeV
  - Total: $\delta M_W \approx$ 4 MeV

[M Awramik et al., Phys. Rev. D69, 053006 (2004)]

[M Awramik et al., Phys. Rev. Lett. 89, 241801 (2002)]



A Freitas et al., Phys. Lett. B495, 338 (2000)]



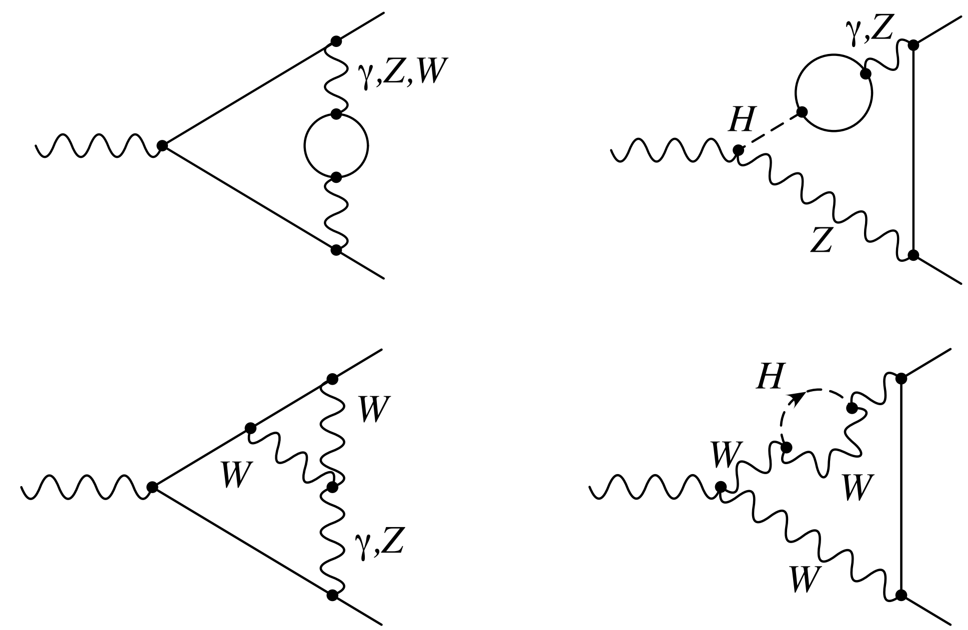
Calculation of $\sin^2(\theta^l_{eff})$

- Effective mixing angle:

$$\sin^2\theta_{\text{eff}}^{\text{lept}} = \left(1 - M_W^2/M_Z^2\right)(1+\Delta\kappa)$$

[M Awramik et al, Phys. Rev. Lett. 93, 201805 (2004)]

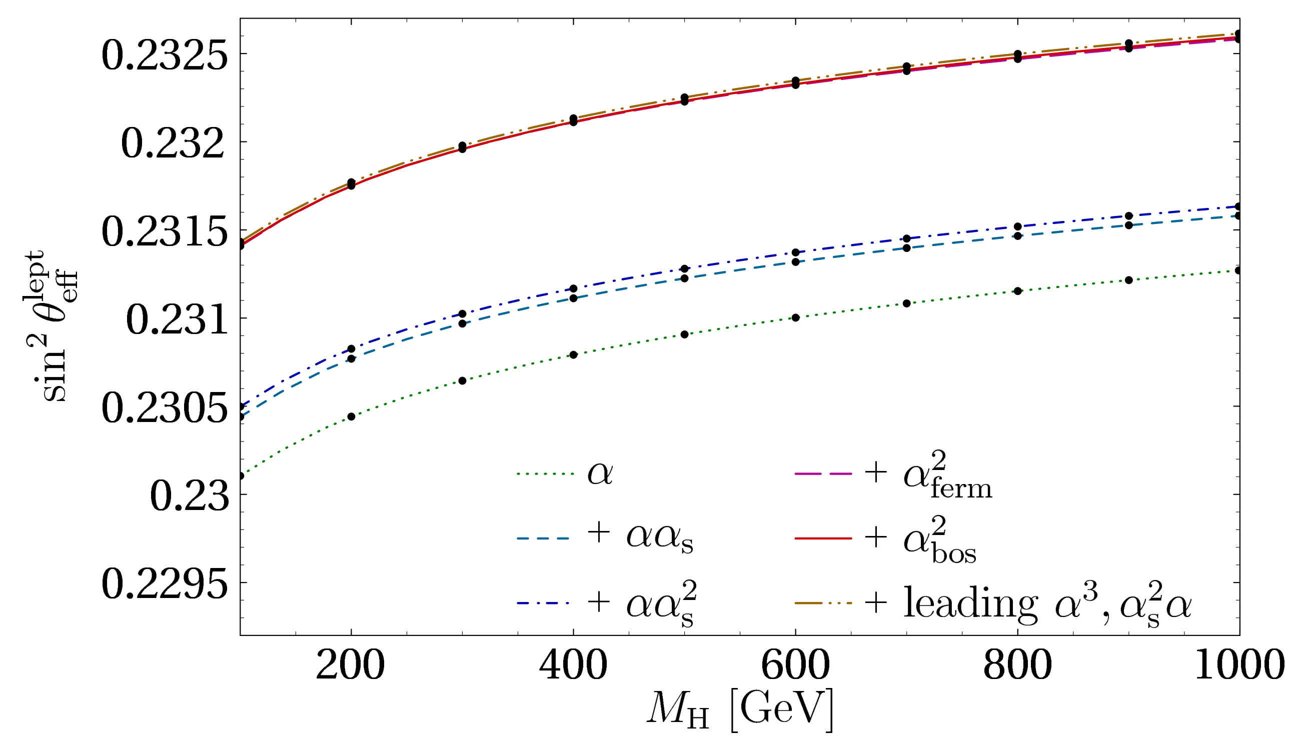
[M Awramik et al., JHEP 11, 048 (2006)]



- Two-loop EW and QCD correction to $\Delta\kappa$ known, leading terms of higher order QCD corrections
- fermionic two-loop correction about 10^{-3}, whereas bosonic one 10^{-5}
- Uncertainty estimate obtained with different methods, geometric progression:

$$\mathcal{O}(\alpha^2\alpha_s) = \frac{\mathcal{O}(\alpha^2)}{\mathcal{O}(\alpha)}\,\mathcal{O}(\alpha\alpha_s).$$

$\mathcal{O}(\alpha^2\alpha_s)$ beyond leading m_t^4	$3.3\ldots2.8\times10^{-5}$
$\mathcal{O}(\alpha\alpha_s^3)$	$1.5\ldots1.4$
$\mathcal{O}(\alpha^3)$ beyond leading m_t^6	$2.5\ldots3.5$



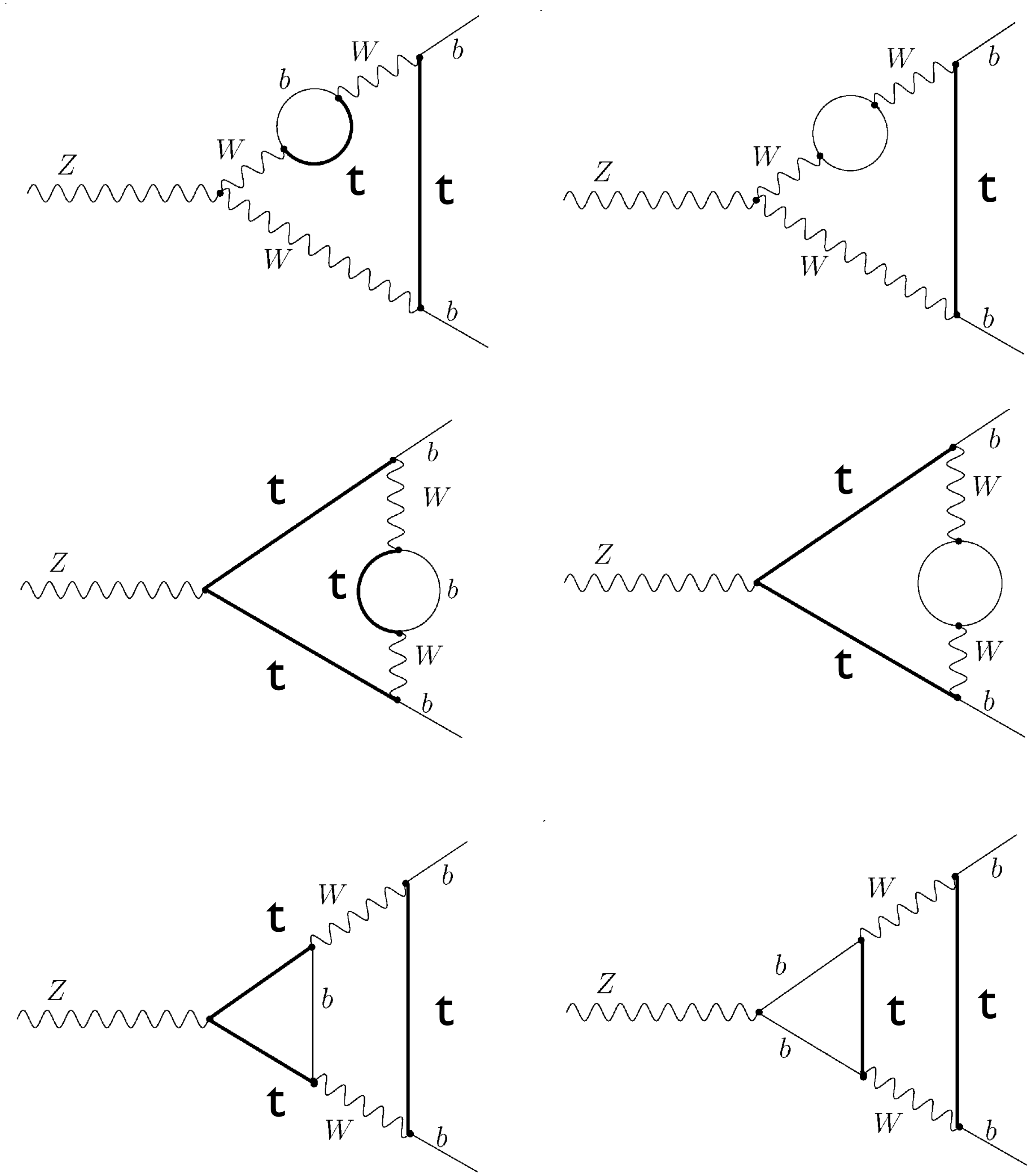
Total: $\delta\sin^2\theta^l_{eff} \approx 4.7\ 10^{-5}$

New Calculation of $\sin^2(\theta^{bb}_{eff})$

[M Awramik et al, Nucl. Phys. B813, 174 (2009)]

- Calculation of $\sin^2\theta_{eff}$ for b-quarks more involved, because of top quark propagators in the $Z \rightarrow b\bar{b}$ vertex
- Investigation of known discrepancy between $\sin^2\theta_{eff}$ from leptonic and hadronic asymmetry measurements
- Two-loop EW correction only recently completed, effect of $O(10^{-4})$
- Now $\sin^2\theta^{bb}_{eff}$ known at the same order as $\sin^2\theta_{eff}$ for leptons and light quarks
- Uncertainty assumed to be of same size as for $\sin^2\theta_{eff}$:

$$\delta\sin^2\theta^{bb}_{eff} \approx 4.7\ 10^{-5}$$



New Calculation of R^0_b

[A. Freitas et al., JHEP 1208, 050 (2012)]

Full two-loop calculation of $Z \rightarrow b\bar{b}$

- The branching ratio $R^0{}_b$: partial decay width of $Z \rightarrow b\bar{b}$ and $Z \rightarrow q\bar{q}$

$$R_b \equiv \frac{\Gamma_b}{\Gamma_{\rm had}} = \frac{\Gamma_b}{\Gamma_d + \Gamma_u + \Gamma_s + \Gamma_c + \Gamma_b} = \frac{1}{1 + 2(\Gamma_d + \Gamma_u)/\Gamma_b}$$

- Contribution of same terms as in the calculation of $\sin^2\theta^{bb}{}_{\rm eff}$
  → cross-check the two results, found good agreement
- Two-loop corrections are comparable to experimental uncertainty ($6.6 \cdot 10^{-4}$)

	1-loop EW and QCD correction to FSR	2-loop EW correction	2-loop EW and 2+3-loop QCD correction to FSR	1+2-loop QCD correction to gauge boson selfenergies
$M_{\rm H}$ [GeV]	$\mathcal{O}(\alpha) + {\rm FSR}_{1-{\rm loop}}$ $[10^{-3}]$	$\mathcal{O}(\alpha^2_{\rm ferm})$ $[10^{-4}]$	$\mathcal{O}(\alpha^2_{\rm ferm}) + {\rm FSR}_{>1-{\rm loop}}$ $[10^{-4}]$	$\mathcal{O}(\alpha\alpha_{\rm s}, \alpha\alpha^2_{\rm s})$ $[10^{-4}]$
100	-3.632	-6.569	-9.333	-0.404
200	-3.651	-6.573	-9.332	-0.404
400	-3.675	-6.581	-9.331	-0.404