The cost of gauge coupling unification in the SU(5) at 3 loops

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Outline

- Motivation
- Georgi-Glashow SU(5) model
- A minimal extension: SU(5)+ $24_{\mathbf{F}}$
- Unification at 3-loop accuracy
- Results

Motivation

The SM is amazingly successful, but it has many open questions

- Why are there so many free parameters ?
- What is the origin of the mass spectrum?
- Why is charge quantized?
- Are the fundamental forces unified?
- What is the origin of the neutrino masses and mixing?
- What is the dark matter?
- Why is there matter-antimatter asymmetry?
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Possible answers in physics Beyond the Standard Model (BSM)

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Possible answers in physics Beyond the Standard Model (BSM)

Here a GUT prototype: a minimal extension of the SU(5) model

Georgi-Glashow Model

- Particle content of minimal SU(5) and SM embedding [Georgi and Glashow ’74]

  - Gauge sector:
    $$24_{\mathbf{V}} = B(1,1,0) \oplus W(1,3,0) \oplus G(8,1,0) \oplus X(3,2,-\tfrac{5}{6}) \oplus \overline{X}(\overline{3},2,+\tfrac{5}{6})$$
  - SM fermions: $\overline{5}_{\mathbf{F}} = \underbrace{(\overline{3},1,+\tfrac{1}{3})}_{d^c} \oplus \underbrace{(1,2,-\tfrac{1}{2})}_{\ell}$ and
    $$10_{\mathbf{F}} = \underbrace{(\overline{3},1,-\tfrac{2}{3})}_{u^c} \oplus \underbrace{(3,2,+\tfrac{1}{6})}_{q} \oplus \underbrace{(1,1,+1)}_{e^c}$$
  - Scalar sector: $5_{\mathbf{H}} = \underbrace{(3,1,-\tfrac{1}{3})_H}_{\mathcal{T}} \oplus \underbrace{(1,2,+\tfrac{1}{2})_H}_{h}$ and

$$24_{\mathbf{H}} = \underbrace{(1,1,0)_H}_{S_H} \oplus \underbrace{(1,3,0)_H}_{T_H} \oplus \underbrace{(8,1,0)_H}_{O_H} \oplus \underbrace{(3,2,-\tfrac{5}{6})_H}_{X_H} \oplus \underbrace{(\overline{3},2,+\tfrac{5}{6})_H}_{\overline{X}_H}$$

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  $24_{\mathbf{V}}\,,\quad \overline{5}_{\mathbf{F}}\,,\quad 10_{\mathbf{F}}\,,\quad 5_{\mathbf{H}}\,,\quad 24_{\mathbf{H}}$

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- SU(5) breaking:

  $$\mathrm{SU}(5) \quad \xrightarrow{\langle 24_{\mathbf{H}}\rangle} \quad \mathrm{SU}(3)_C \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y \quad \xrightarrow{\langle 5_{\mathbf{H}}\rangle} \quad \mathrm{SU}(3)_C \otimes \mathrm{U}(1)_Q$$

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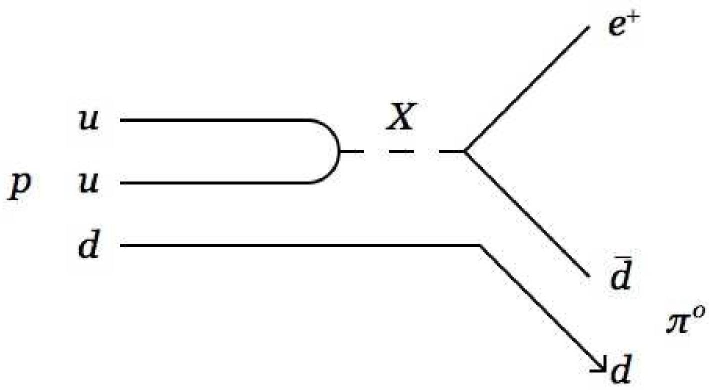
- Proton become unstable:



$$\tau_p^{\mathrm{th}} \sim \frac{1}{\alpha_G^2}\frac{M_X^4}{m_p^5}$$

$$\tau_p^{\mathrm{exp}} \geq 10^{33}\,\mathrm{yr}$$

$$\Rightarrow M_X \geq 10^{15.5}\mathrm{GeV}$$

Minimal SU(5) model is ruled out

- Gauge couplings do not unify

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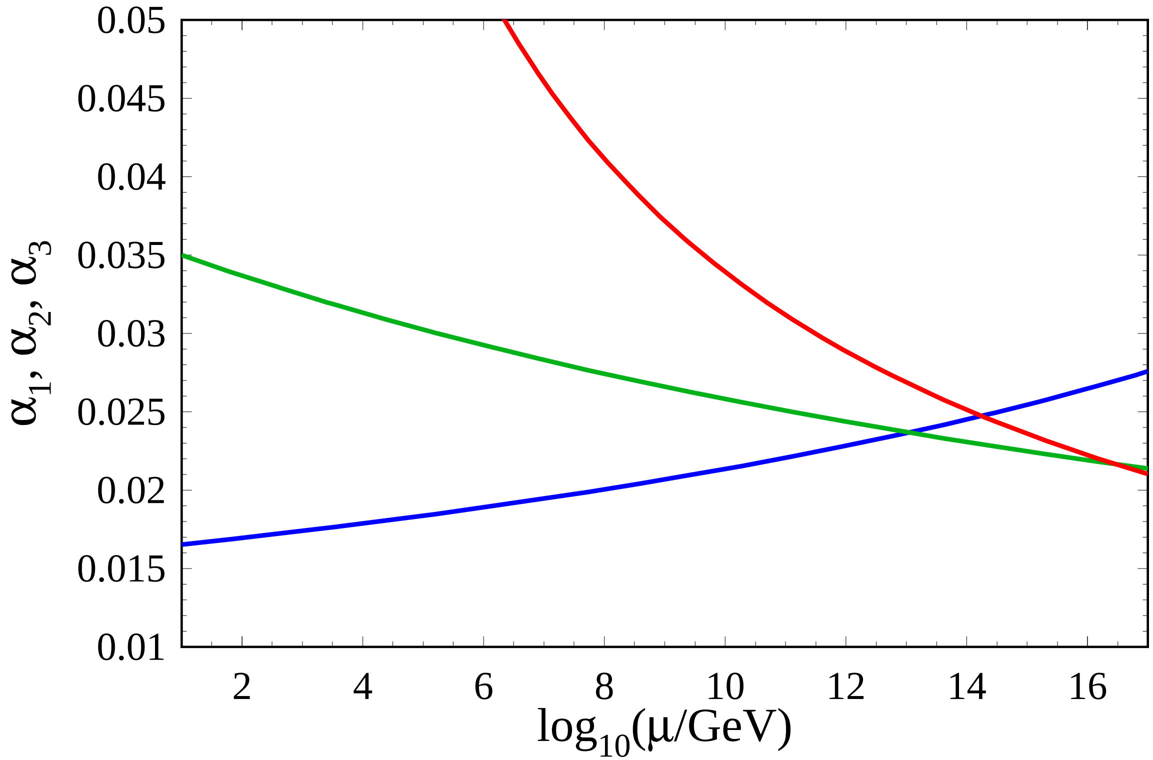
3-loop running: [L.M., J. Salomon, M. Steinhauser '12]

SM input parameters:

$$\alpha_1^{\overline{\text{MS}}}(M_Z) = 0.0169225 \pm 0.0000039\,,$$
$$\alpha_2^{\overline{\text{MS}}}(M_Z) = 0.0033735 \pm 0.000020\,,$$
$$\alpha_3^{\overline{\text{MS}}}(M_Z) = 0.1173 \pm 0.00069\,,$$
$$\alpha_t^{\overline{\text{MS}}}(M_Z) = 0.07514\,.$$

[PDG 2012]



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$m_\nu \sim Y_\nu \frac{v^2}{\Lambda} \quad \Rightarrow \quad$ for $\Lambda \approx 10^{17}$GeV: $\quad m_\nu \approx 10^{-4}\text{eV}$

$\Rightarrow \quad$ much too light!

SU(5)+ $24_{\mathbf{F}}$

- Add an adjoint fermionic multiplet: $24_{\mathbf{F}}$

  [Bajc, Senjanovic ’06], [Bajc,Nemevsek, Senjanovic ’07]

$$24_{\mathbf{F}} = \underbrace{(1,1,0)_F}_{S_F} \oplus \underbrace{(1,3,0)_F}_{T_F} \oplus \underbrace{(8,1,0)_F}_{O_F} \oplus \underbrace{(3,2,-\tfrac{5}{6})_F}_{X_F} \oplus \underbrace{(\overline{3},2,+\tfrac{5}{6})_F}_{\overline{X}_F}$$

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  New Yukawa interactions: $\delta\mathcal{L}_Y = y_\nu \overline{5}_{\mathbf{F}} 24_{\mathbf{F}} 5_{\mathbf{H}} \quad \Rightarrow$

  after EW breaking:

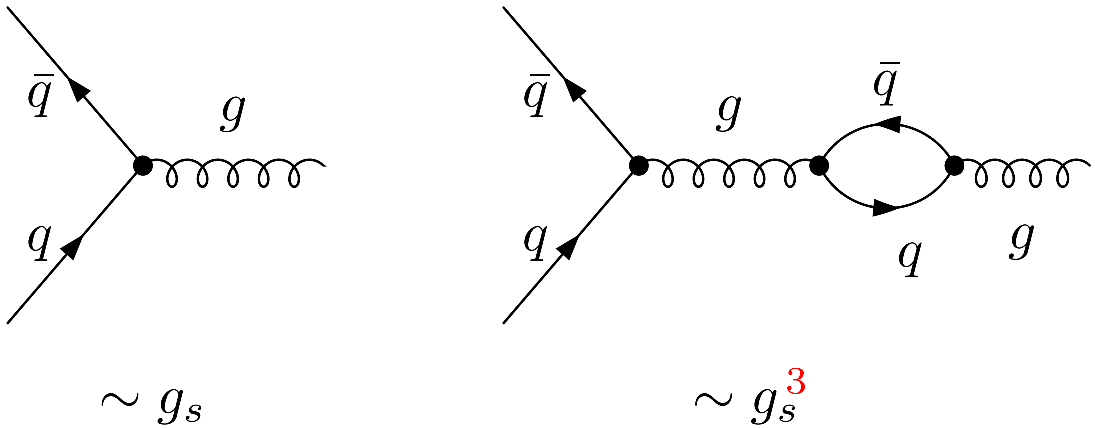
$$m^\nu_{ij} = -\frac{v^2}{2}\left(\frac{y^i_T y^j_T}{m_{T_F}} + \frac{y^i_S y^j_S}{m_{S_F}}\right)$$

  2 neutrinos are massive

Unification Pattern

Renormalization Group Equations

$$\mu^2 \frac{d}{d\mu^2}\frac{\alpha_i}{\pi} \quad = \quad \beta_i(\{\alpha_j\}) = -\left(\frac{\alpha_i}{\pi}\right)^2 \left[b_0^{(i)} + \sum_j \frac{\alpha_j}{\pi} b_1^{(ij)} + \ldots\right]$$



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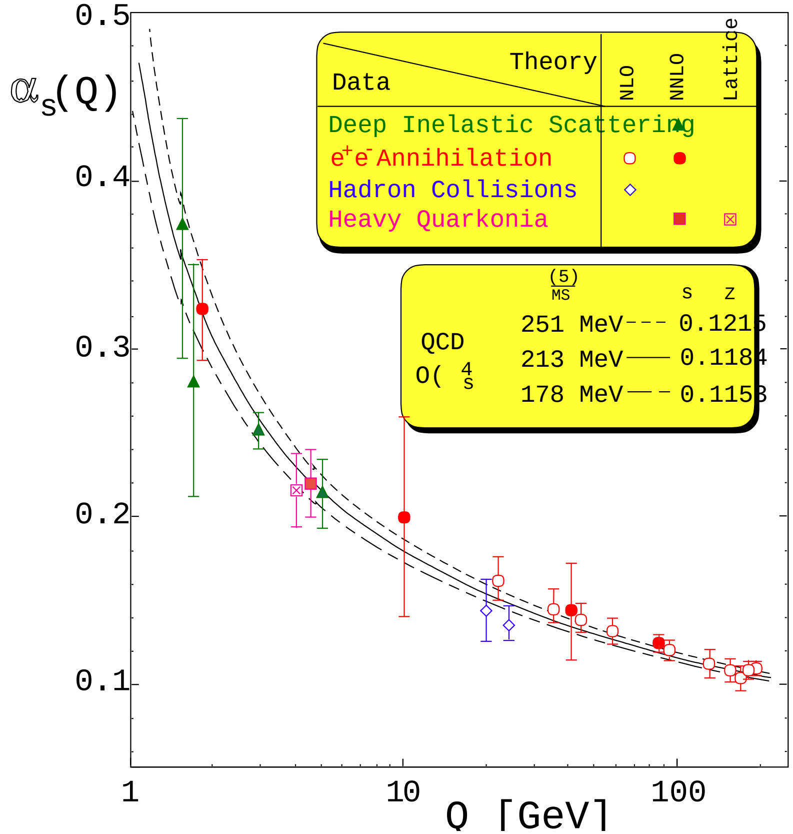

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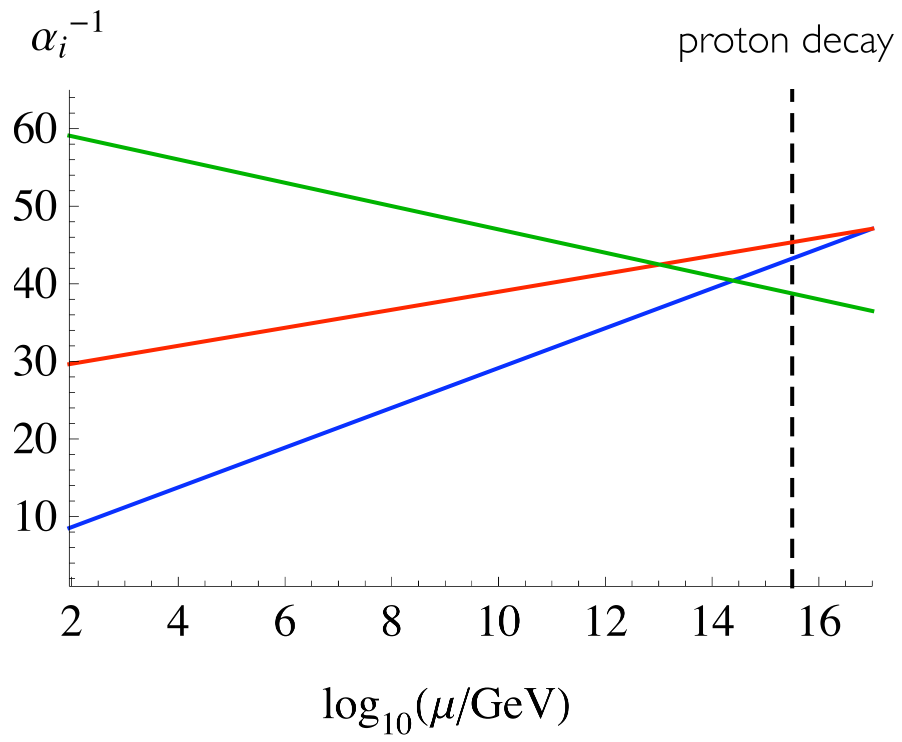
New particles w.r.t. SM that can contribute to the gauge coupling running (1-loop)

$SU(5)$	State	Δb_0
5_H	$\mathcal{T}$	$(\frac{1}{6}, 0, \frac{1}{15})$
24_H	T_H	$(0, \frac{1}{3}, 0)$
24_H	O_H	$(\frac{1}{2}, 0, 0)$
24_F	T_F	$(0, \frac{4}{3}, 0)$
24_F	O_F	$(2, 0, 0)$
24_F	X_F	$(\frac{4}{3}, 2, \frac{10}{3})$



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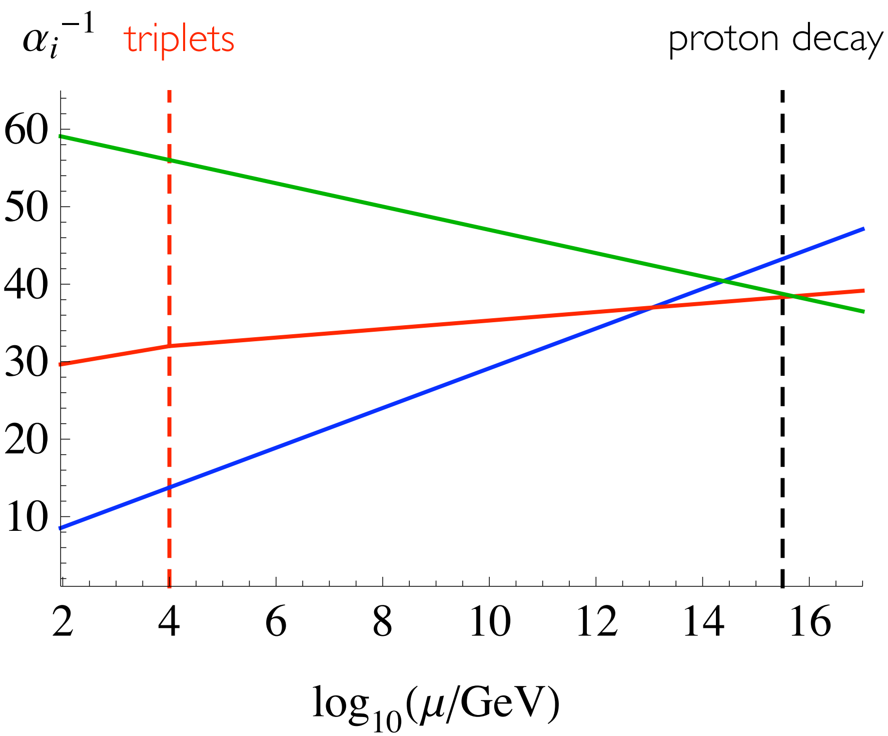

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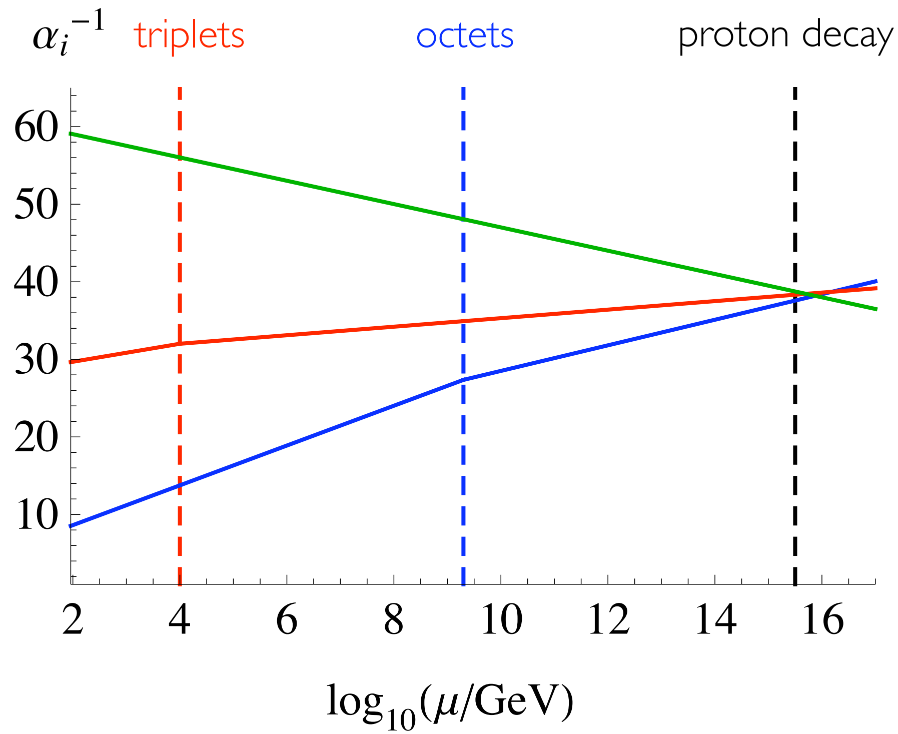

Mass Spectrum of SU(5) + 24_F

- Unification requires: $M_T \approx 1$ TeV, $M_O \approx 10^8$ GeV
- Unification constraints max. value of $m_3 \equiv (M_{T_F}^4 M_{T_H})^{1/5}$

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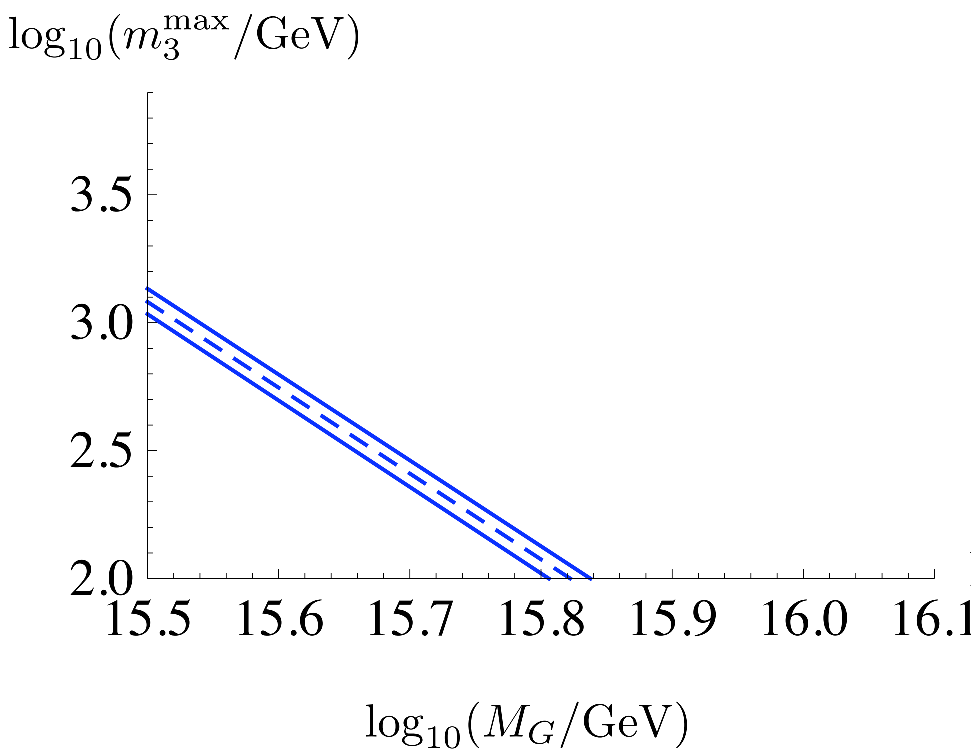

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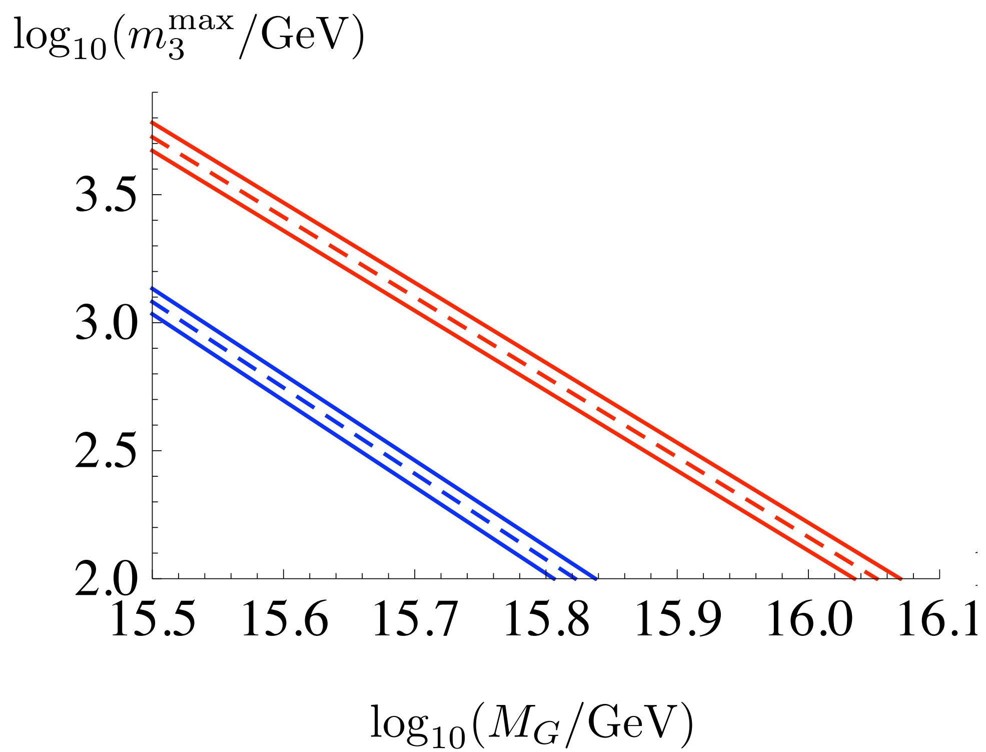
  - 2-loop running analysis



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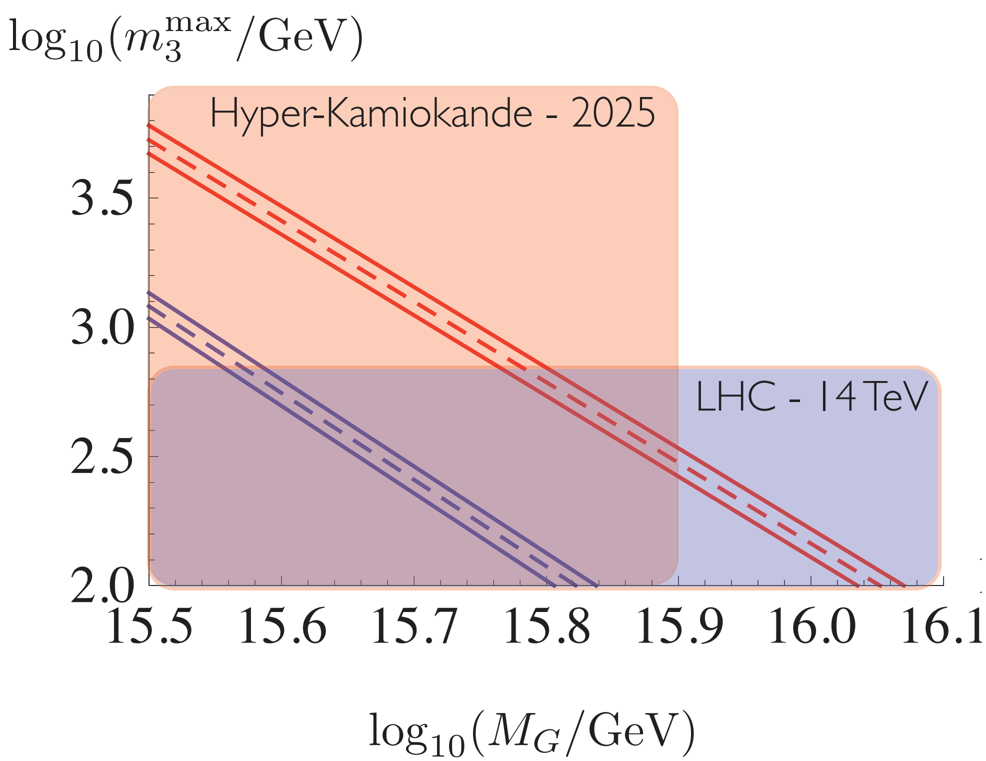
  - Interplay between LHC and HK to check the model



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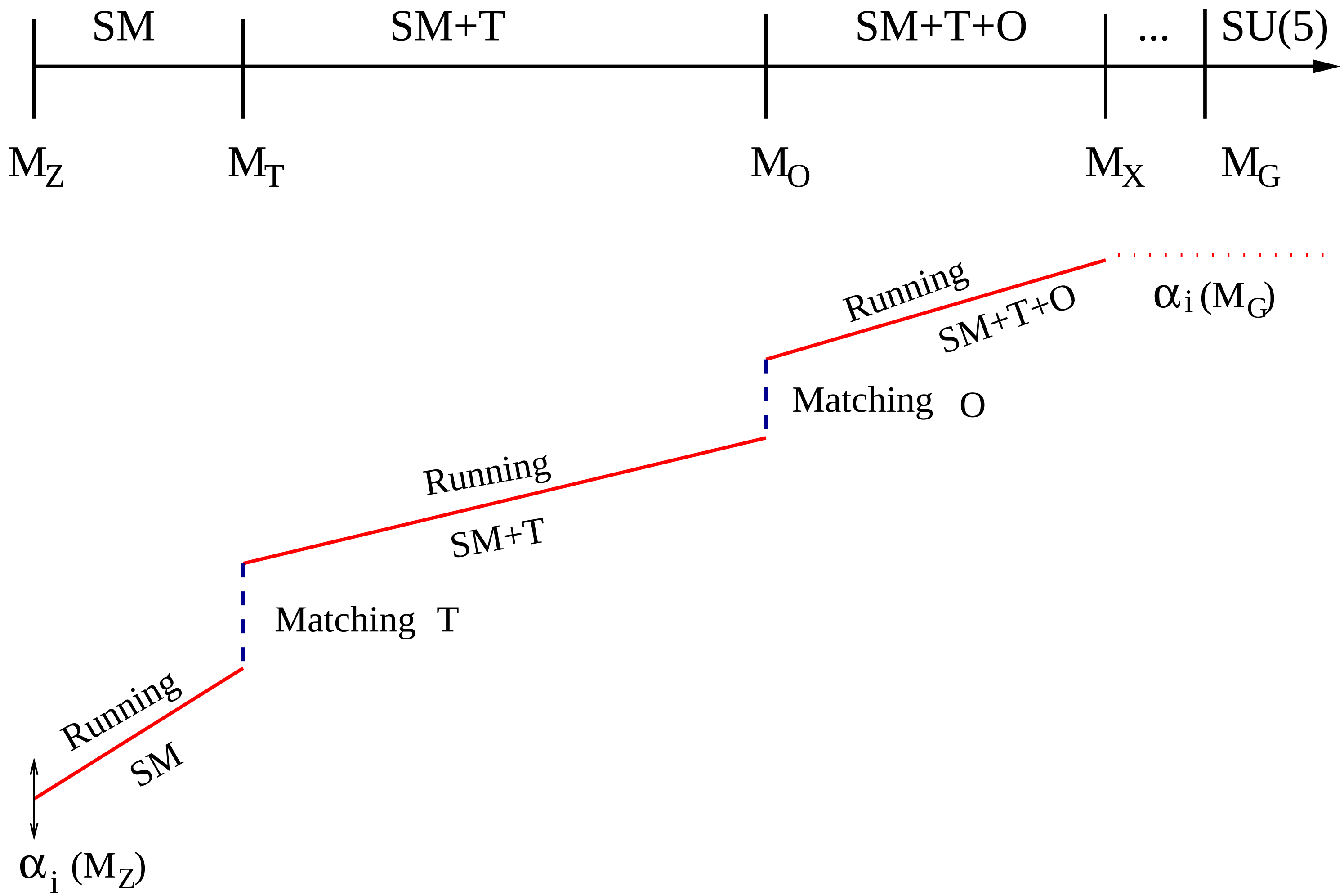
Theory side: 3-loop analysis is required !

Effective Field Theory Approach



Effective Field Theory Approach

3-loop running analysis for the gauge couplings requires:

- 3-loop gauge beta functions of SM

  [L.M., Salomon, Steinhauser '12], [Bednyakov, Pikelner, Velizhanin'12]
- 2-loop matching coefficients for T and O (here)
- 3-loop gauge beta functions of SM+T , SM+T+O, ... (here)
- 2-loop matching coefficients for super heavy particles $\approx M_G$ (missing)

3-loop gauge β functions

Calculation of Z_{g_i} to 3 loops:

$$Z_{g_i} = \frac{Z_{\mathrm{V}}}{\prod_k \sqrt{Z_{k,\mathrm{WF}}}} = \frac{Z_{1,g_i c_i \bar{c}_i}}{Z_{2,c_i}\sqrt{Z_{3,g_i}}} = \frac{Z_{1,g_i g_i g_i}}{(\sqrt{Z_{3,g_i}})^3} = \dots$$

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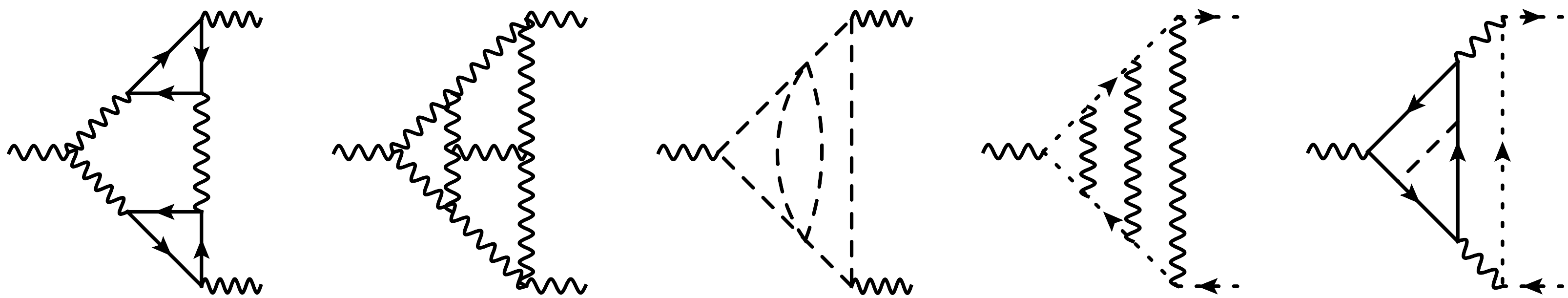
- $\mathcal{O}(10^5)$ Feynman diagrams



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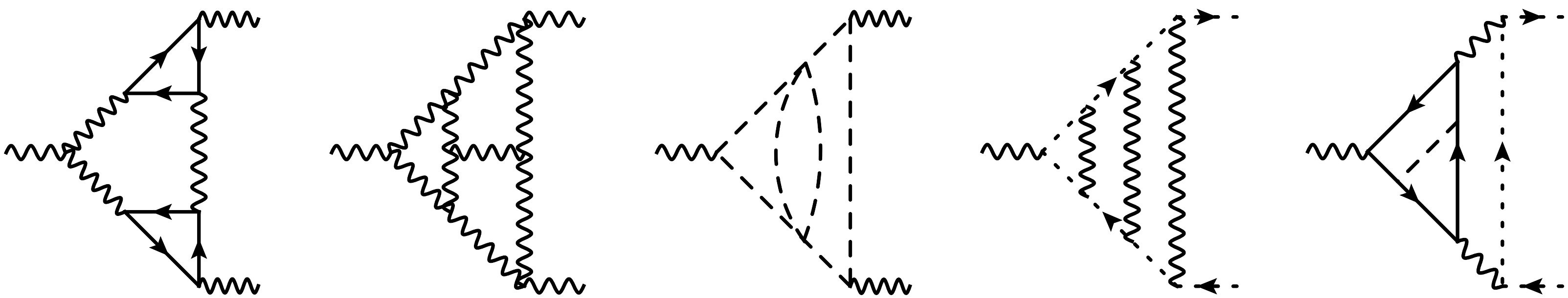

- $\overline{\mathrm{MS}}$ scheme
- 1 non zero external momentum & all masses set to zero
  $\Rightarrow$ MINCER [Larin, Tkachov, Vermaseren'91]

2-loop matching coefficients

- Effective Field Theory:

$$\mathcal{L}_{\rm full}(\alpha_i^{(\rm full)}, \ldots) \quad \rightarrow \qquad \mathcal{L}_{\rm eff}(\alpha_i^{(\rm eff)}, \ldots) \quad \text{at energy } \mu$$

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- “Matching” : low energy physics must be unchanged !!

$$\begin{aligned} \alpha_i^{(\text{eff})} &= \zeta_i \, \alpha_i^{(\text{full})} \\ &\vdots \\ \zeta_i &= \zeta_i(\alpha_i, M_{\text{heavy}}, \mu) \end{aligned}$$

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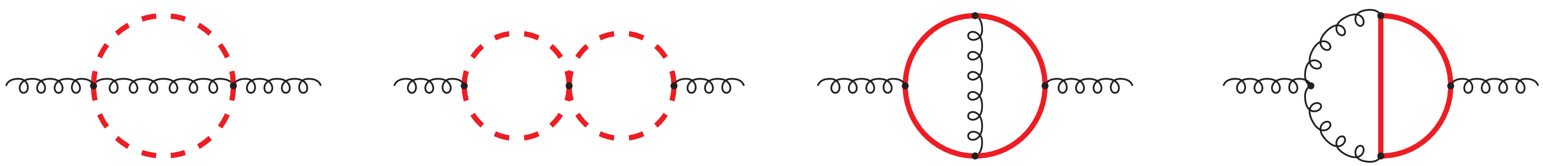
- An example: matching of SM+T to SM+T+O



3-loop results

- Input: SM low-energy gauge couplings $\alpha_i(M_Z)\,, i = 1, 2, 3$
- Assumptions: $M_{T_F} = M_{T_H}$, $M_{O_F} = M_{O_H} \approx 10^{7.5}$GeV

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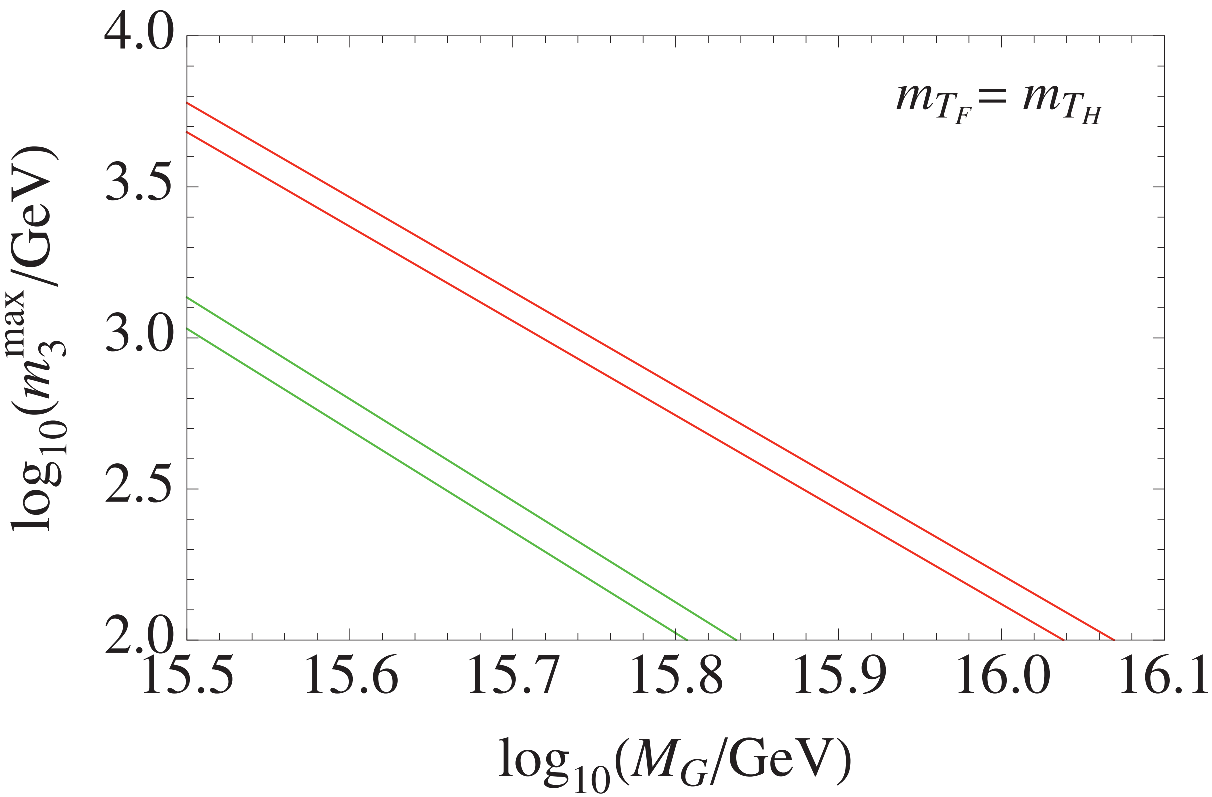
1- and 2-loop correlation



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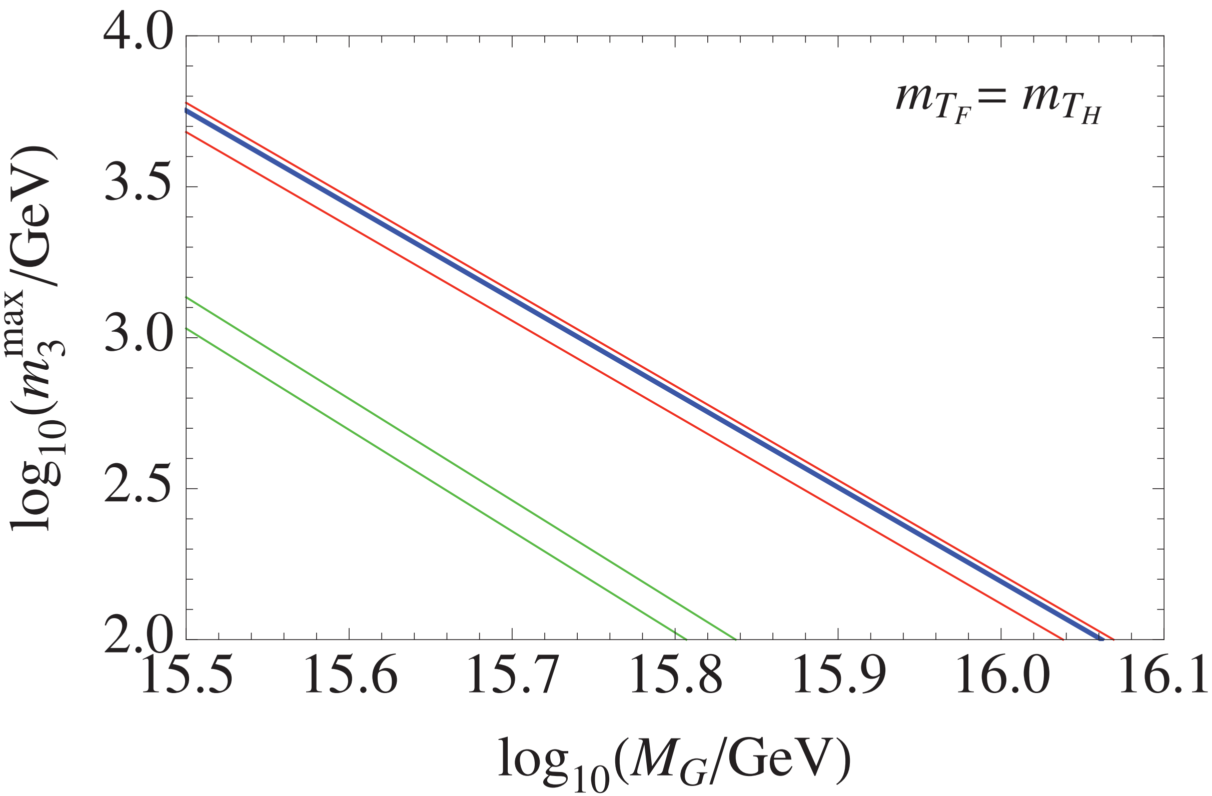

Conclusions

- Minimal extension of SU(5)
  - Unification possible at $\mathcal{O}(10^{16}\,\mathrm{GeV})$
  - Light electroweak triplets at $\mathcal{O}(1\,\mathrm{TeV})$
- Interplay between LHC and Hyper-Kamiokande to constraint the parameter space
- Theory: precise (3-loop) running analysis required