# Physics of the B-Factories Book

## CKM sides: $V_{td}$ and $V_{ts}$


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# Chapter Outline

- Basic plan of the chapter is to describe extraction of |Vtd/Vts| from two sets of experimental results:
  - $\Delta m_d$ and $\Delta m_s$
    - Forward reference(s) to mixing chapter
      - HFAG plot of $\Delta m_d$ results is currently included here
    - Need Tevatron $\Delta m_s$ result, and a few other inputs
      - Agree on values/uncertainties with (most?) global fits folks
  - B->$X_{s,d}$ $\gamma$ branching fractions
    - Babar has done two correlated analyses (same dataset)
      - Exclusive B->K*$\gamma$ and B->rho/omega $\gamma$
      - Semi-inclusive B->$X_s\gamma$ and B->$X_d\gamma$
    - One Belle exclusive modes result
      - PRL 101, 111801 (2008)
    - Give Vtd/Vts for all three results separately
    - Final average will use Babar semi-incl plus Belle excl

## 14.2 $V_{td}$ and $V_{ts}$

*Editors:*
*Kevin Flood (BABAR)*
*Tobias Hurth (theory)*

The CKM matrix elements $V_{td}$ and $V_{ts}$ are fundamental parameters of the Standard Model (SM) which can only be determined experimentally using $\Delta F = 1$ loop-mediated $B$ or $K$ rare decays, or $\Delta F = 2$ box diagram processes involving top quarks. Measurement of the single top quark production cross-section allows for a model-independent direct determination of $V_{tb}$, but the magnitudes of $V_{td}$ and $V_{ts}$ cannot be similarly extracted from tree-level decays. Derivation of $V_{td}$ and $V_{ts}$ from the experimental observables necessarily assumes the SM although the FCNC observables used, e.g. from $B_{d,s}$ mixing, $B \to X(s,d)\gamma$, or $\epsilon$ in the kaon sector, may receive new physics contributions from unrelated sources. Independent determination of the magnitudes of $V_{td}$ and $V_{ts}$ from several different sources, along with $V_{tb}$ from single top cross-section measurements, can provide a robust model-independent check of the unitarity of the CKM matrix or, conversely, offer a powerful probe for the possible presence of new BSM physics.

In the past few years, the experimental and lattice QCD inputs necessary to calculate $V_{td}$ and $V_{ts}$ to good precision have become available. The B-Factories have contributed measurements of $\Delta m_d$, the mass difference between $B^0$ and $\overline{B}^0$, and branching fractions from the inclusive and exclusive one-loop radiative penguin processes $B \to X(s,d)\gamma$, while the Tevatron collaborations have measured $\Delta m_s$, the mass difference between $B_s$ and $\bar{B}_s$, to sub-percent precision. These results have been matched by progress in lattice QCD calculations leading to increased precision in the additional parameters which are required to extract $V_{td}$ and $V_{ts}$ from the experimental results. At the time of publication of all of the $\Delta m_d$ analyses from both Babar and Belle, knowledge of the required lattice inputs was so significantly worse than the experimental precision that no attempts were made to calculate $V_{td}$ at the time.

Equation ?? relates $\Delta m_d$ to $V_{td}$ [add cite]:

[eq 1 from Belle hep-ex/0211065 goes here]

where $m_t$, $m_B$ and $m_W$ are respectively the top quark, $B^0$ and W masses; $G_F$ is the Fermi constant; $\eta_t$ is a QCD correction [add cite]; S is a function of $m_t^2/m_W^2$ [add cite]; $f_B$ is the $B$-meson decay constant; and $\hat{B}_B$ is the $B$-meson bag parameter. Although most of these parameters are well-characterized, an unavoidable dependence on lattice QCD enters in the product $\hat{B}_B f_B^2$.

In order to extract $V_{td}$ using Eq. ??, we adopt the latest combination of unquenched lattice QCD results available from "www.latticeaverages.org" [add cite], who report $f_b\sqrt{\hat{B}_B} = 216 \pm 15$. This result is obtained by combining the average decay constant $f_b$ obtained from the MILC and HPQCD collaborations, along with the HPQCD determination of the bag parameter $\hat{B}_B$, which minimizes the total uncertainty with respect to taking the two parameters separately. Using the seven B-Factory results shown in Fig 1, which are averaged by the Heavy Flavor Averaging Group (HFAG) [cite] to obtain a final value of $\Delta m_d = 0.508 \pm 0.005$ ps$^{-1}$, we find $V_{td} = xxx \pm xxx$.

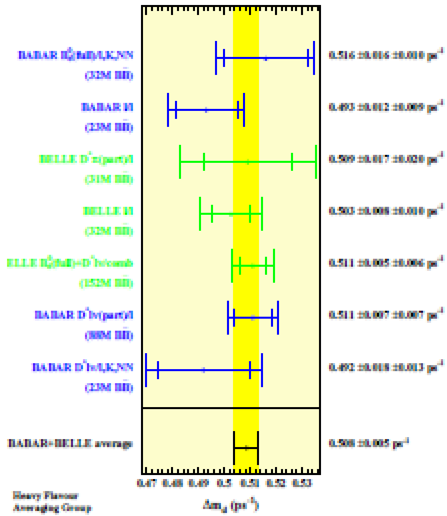



**Fig. 1.** Measurements and HFAG averaged $\Delta m_d$ from Babar and Belle.

The uncertainty in $V_{td}$ induced by the uncertainty in $f_b\sqrt{\hat{B}_B}$ can be reduced by rewriting this factor as $f_b\sqrt{\hat{B}_B} = f_s\sqrt{\hat{B}_Bs}/\xi$, where $\xi = f_s\sqrt{\hat{B}_Bs}/f_b\sqrt{\hat{B}_B}$. The factor $\xi$ can be more accurately determined in lattice QCD calculations than its individual terms because of the inclusion of $f_s\sqrt{\hat{B}_Bs}$, which is obtained directly at the physical strange quark mass rather than by extrapolation to the down quark mass, and approximate cancellation of some uncertainties in the ratio. Using the values $\xi = 1.243 \pm 0.028$ and $f_s\sqrt{\hat{B}_Bs} = 275 \pm 13$, we find $V_{td} = xxx \pm xxx$, a reduction of $\sim xx$% in the uncertainty relative to the result above based solely on $f_b\sqrt{\hat{B}_B}$. The lattice parameter uncertainties can be further reduced by taking the ratio $V_{td}/V_{ts}$, which directly uses $\xi^{-1}$, and incorporating the combined CDF/DO result for $\Delta m_s = 17.78 \pm 0.12$ ps$^{-1}$ [cite] and an expression for $V_{ts}$ analogous to Eq. ??, and we find $V_{td}/V_{ts} = xxx \pm xxx$.

### 14.2.1 $B \to X(s,d)\gamma$

Exclusive Babar and Belle BF results: $B \to K^*\gamma$ and $B \to \rho,\omega\gamma$

Semi-inclusive Babar BF results: $B \to X_{s,d}\gamma$

### 14.2.2 Summary

Recapitulation ...
Future prospects ...

# Vtd, Vtd/Vts from Δm

- Final result is ratio Vtd/Vts using both B(d,s) mass difference and B->X(s,d)gamma, but want also to discuss Vtd from Δm alone and would like to have agreement with others on values of inputs

$$\Delta m_d = \frac{G_F^2}{6\pi^2} f_B^2 m_B m_W^2 \eta_t S |V_{tb}^* V_{td}|^2 B_B \ ,$$

$$\frac{\Delta m_s}{\Delta m_d} = \frac{m_{B_s}}{m_{B_d}} \frac{f_{B_s}^2 B_{B_s}}{f_{B_d}^2 B_{B_d}} \left| \frac{V_{ts}}{V_{td}} \right|^2$$

- Need a table of non-lattice inputs agreed upon by all
  - $m_t$, $m_W$, etc. do not seem controversial . . .
  - Should be part of the book (perhaps an appendix?), but all needed inputs that can be trivially agreed upon should be memorialized soon
  - This task falls most naturally to global fit editors

# Vtd/Vts from Exclusive Radiative Penguins

- Babar and Belle use similar methods to extract Vtd/Vts from the exclusive B->(ρ/ω)γ and K* γ BFs

P. Ball, G.W. Jones, and R. Zwicky, J. High Energy Phys. 04 (2006) 046; P. Ball, G.W. Jones, and R. Zwicky, Phys. Rev. D **75**, 054004 (2007).

$$R_{\rm exp} \equiv \frac{\overline{\mathcal{B}}(B \to (\rho,\omega)\gamma)}{\overline{\mathcal{B}}(B \to K^*\gamma)}\,,$$

where $\overline{\mathcal{B}}(B \to (\rho,\omega)\gamma)$ is defined as the CP-average $\frac{1}{2}[\mathcal{B}(B \to (\rho,\omega)\gamma) + \mathcal{B}(\bar{B} \to (\bar{\rho},\omega)\gamma)]$ of

$$\mathcal{B}(B \to (\rho,\omega)\gamma) = \frac{1}{2}\left\{\mathcal{B}(B^+ \to \rho^+\gamma) + \frac{\tau_{B^+}}{\tau_{B^0}}\left[\mathcal{B}(B^0 \to \rho^0\gamma) + \mathcal{B}(B^0 \to \omega\gamma)\right]\right\},$$

and $\overline{\mathcal{B}}(B \to K^*\gamma)$ is the isospin- and CP-averaged branching ratio of the $B \to K^*\gamma$

# Vtd/Vts from Exclusive Radiative Penguins

Within QCD factorisation, and using the notations of Ref. [6], the amplitude for $B \to V\gamma$ can be written as

$$A(\bar{B} \to V\gamma) = \frac{G_F}{\sqrt{2}} \left[\lambda_u a_7^u(V\gamma) + \lambda_c a_7^c(V\gamma)\right] \langle V\gamma|Q_7|\bar{B}\rangle\,,$$

where $\lambda_q$ are products of CKM matrix elements and the factorisation coefficients $a_7^{u,c}$ consist of Wilson coefficients and non-factorisable corrections from hard scattering and annihilation; explicit expressions can be found in Ref. [6]. $a_7^{u,c}$ depends in particular on the form factor $T_1$ and the twist-2 DA $\phi_{V;\perp}$. The theoretical expression for $R$ is then given by

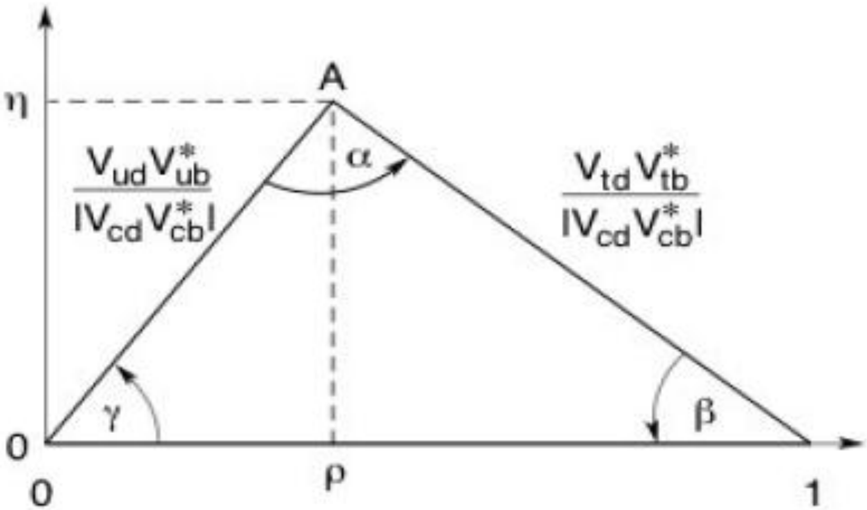
$$R_{\rm th} = \left|\frac{V_{td}}{V_{ts}}\right|^2 \frac{1}{\xi^2} \left(\frac{1-m_\rho^2/m_B^2}{1-m_{K^*}^2/m_B^2}\right)^3 \left|\frac{a_7^c(\rho\gamma)}{a_7^c(K^*\gamma)}\right|^2 \left(1 + \mathrm{Re}\,(\delta a_\pm + \delta a_0)\left[\frac{R_b^2 - R_b\cos\gamma}{1 - 2R_b\cos\gamma + R_b^2}\right]\right.$$
$$\left. + \frac{1}{2}\left(|\delta a_\pm|^2 + |\delta a_0|^2\right)\left\{\frac{R_b^2}{1-2R_b\cos\gamma + R_b^2}\right\}\right) \qquad (19)$$

with $\delta a_{0,\pm} = a_7^u(\rho^{0,\pm}\gamma)/a_7^c(\rho^{0,\pm}\gamma) - 1$. Here, $\gamma$ is one angle of the CKM unitarity triangle and $R_b$ one of its sides:

$$R_b = \left(1 - \frac{\lambda^2}{2}\right)\frac{1}{\lambda}\left|\frac{V_{ub}}{V_{cb}}\right|.$$

# Vtd/Vts from Semi-Incl Radiative Penguins

- Method for extracting Vtd/Vts from inclusive B-> X(s,d) gamma




Unitarity Triangle using Wolfenstein parameterisation

At $O(\lambda^4)$ $\boxed{\sqrt{\eta^2+(1-\rho)^2}=\frac{1}{\lambda}\left|\frac{V_{td}}{V_{ts}}\right|=\frac{1}{\lambda}X}$

At $O(\lambda^7)$ $\boxed{\sqrt{\bar{\eta}^2+(1-\bar{\rho})^2}}$

$$\boxed{R(d\gamma/s\gamma) = \frac{|\xi_t|^2}{|\lambda_t|^2} + \frac{D_u}{D_t}\frac{|\xi_u|^2}{|\lambda_t|^2} + \frac{D_r}{D_t}\frac{Re(\xi_t^*\xi_u)}{|\lambda_t|^2}}$$

**Ali et al., Phys.Lett. B429, 87 (1998)**

$\xi_q=V_{qb}V_{qd}^*$; $\lambda_q=V_{qb}V_{qs}^*$

D functions contain contributions from Wilson Coeffs, etc.

Evaluated numerically in paper

# Vtd/Vts from Semi-Incl Radiative Penguins

- Ali et al rewrite R as

$$R = \lambda^2[1+\lambda^2(1-2\bar{\rho})]\left[(1-\bar{\rho})^2+\bar{\eta}^2+\frac{D_u}{D_t}(\bar{\rho}^2+\bar{\eta}^2)+\frac{D_r}{D_t}(\bar{\rho}(1-\bar{\rho})-\bar{\eta}^2)\right]$$

- To remove dependence on earlier estimates of Vtd/Vts, rewrite as a function of Vtd/Vts and $\varphi_2$, and after some algebra end up with

$$R = \kappa_1 X^2 + \kappa_2 X + \kappa_3$$

$$\begin{aligned}
\kappa_1 &= 1+\frac{D_u}{D_t}\left(1-2\lambda^2\cos^2\beta\right)-\frac{D_r}{D_t}\left(\lambda^2\cos^2\beta+1\right),\\
\kappa_2 &= \lambda\cos\beta\left[\frac{D_u}{D_t}\left(3\lambda^2-2\right)+\frac{D_r}{D_t}\left(1+\frac{\lambda^2}{2}\right)\right],\\
\kappa_3 &= \lambda^2\frac{D_u}{D_t}\left(1-\lambda^2\right).
\end{aligned}$$

# Summary

- Draft of full text for Vtd,Vts from mass difference committed a few days ago
  - Clean up, then commit final draft in near future
  - Will use central values and uncertainties for lattice inputs from "latticeaverages.org", which UT and CKM global fit editors have also agreed to use
- Formalism for extracting Vtd/Vts from radiative penguins is well-defined, but needs updating for latest values and uncertainties of several input parameters
  - CKM, UT, Scan? I'll probably go with UT ...
- Would like to see non-controversial physics inputs globally defined and put into a table somewhere in the book
  - Perhaps in an appendix?
- Theory editor has agreed to review whatever I write ☺
- Belle (TB) has also agreed to review the text
- Final word will be comparison of mass diff vs radpen ratios