INVISIBLES 14 SCHOOL

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Neutrino Physics
(BSM and phenomenological implications)

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Lecture 3
Neutrinos and LFV

LFV expected at some level

neutrino masses and $U_{PMNS} \neq 1$ ⇒ L_i violated ($i=e,\mu,\tau$)

evidence for lepton flavor conversion

direct	$\nu_e \to \nu_\mu, \nu_\tau$	sol, LBL exp
indirect	$\nu_\mu \to \nu_\tau$	atm

should show up in processes with charged leptons

Process	Relative probability	Present Limit	Experiment	Year	prospects
$\mu \to e\gamma$	1	5.7×10^{-13}	MEG	2012	6×10^{-14}
$\mu^- \mathrm{Ti} \to e^- \mathrm{Ti}$	$Z\alpha/\pi$	4.3×10^{-12}	SINDRUM II	2006	$10^{-15} \div 10^{-16}$
$\mu^- \mathrm{Au} \to e^- \mathrm{Au}$	$Z\alpha/\pi$	7×10^{-13}	SINDRUM II	2006	$10^{-15} \div 10^{-16}$
$\mu \to eee$	α/π	4.3×10^{-12}	SINDRUM	1988	$10^{-15} \div 10^{-16}$
$\tau \to \mu\gamma$	$(m_\tau/m_\mu)^{2\div4}$	3.3×10^{-8}	B-factories	2011	
$\tau \to e\gamma$	$(m_\tau/m_\mu)^{2\div4}$	4.5×10^{-8}	B-factories	2011	

Table 1: Relative sensitivities and experimental limits of the main CLFV processes.

here: focus on radiative decays of charged leptons in the SM, minimally extended to accommodate e.g. Dirac neutrinos

$$BR(\mu \to e\gamma) \approx \frac{3\alpha}{32\pi} \left| U^*_{\mu i} U_{ei} \frac{m_i^2}{m_W^2} \right|^2 \approx 10^{-53}$$

Exercise 10: reproduce this

[solution in Cheng and Li]

[unobservable also within type I see-saw] $m_i \approx 0.05\, eV \quad U_{fi} \approx O(1)$

depleted by
-- weak interactions
-- loop factor
-- GIM mechanism (mixing angle large, but neutrino masses tiny)

<-> GIM suppression for quarks:
small mixing angles
large top mass

a good place to look for BSM physics

general parametrization of LFV effects BSM

$$L = L_{SM} + \sum_i c_i^5 \frac{O_i^5}{\Lambda} + \sum_i c_i^6 \frac{O_i^6}{\Lambda^2} + \ldots$$

O^d_i gauge invariant operators dimension d

low-energy effective Lagrangian in the lepton sector

$$L = L_{SM} + i\frac{e}{\Lambda^2} e^c \left(\sigma^{\mu\nu} F_{\mu\nu}\right) \mathcal{Z} (\Phi^+ l) + \frac{1}{\Lambda^2} [\text{4-fermion}] + h.c. + \ldots$$

[relation between the scale Λ and new particle masses M' can be non-trivial in a weakly interacting theory g Λ/4π≈M']

$\mathcal{Z}_{ij}$ a matrix in flavour space

$$L_Y = -e^c\, y_e (\Phi^+ l) + h.c. + \ldots$$

in the basis where charged leptons are diagonal

$\mathrm{Im}\left[\mathcal{Z}\right]_{ii}$	d_i	electric dipole moments
$\mathrm{Re}\left[\mathcal{Z}\right]_{ii}$	$a_i = \dfrac{(g-2)_i}{2}$	anomalous magnetic moments
$\left\|\left[\mathcal{Z}\right]_{ij}\right\|^2 \quad (i \neq j)$	$R_{ij} = \dfrac{BR(l_i \to l_j \gamma)}{BR(l_i \to l_j \nu_i \bar{\nu}_j)}$	radiative decays $\mu \to e\gamma \quad \tau \to \mu\gamma \quad \tau \to e\gamma$
[4-fermion operators]	other LFV transitions	$\mu \to eee \quad \tau \to \mu\mu\mu \quad \tau \to eee \quad \ldots$

$$BR(\mu \to e\gamma) < 5.7 \times 10^{-13}$$

$$\frac{\mathcal{Z}_{\mu e}}{\Lambda^2} < 2 \times 10^{-9}\ \text{TeV}^{-2}$$

either the scale of new physics is very large or flavour violation from New Physics is highly non-generic

$$\Lambda > 2 \times 10^4 \left[\sqrt{\mathcal{Z}_{\mu e}}\right] TeV$$

not a specific problem of the lepton sector

here: constraints from flavour physics on d=6 |ΔF|=2 operators

FLAVOUR PROBLEM

Operator	Bounds on Λ in TeV ($c_{ij}=1$)		Bounds on c_{ij} ($\Lambda = 1$ TeV)		Observables
	Re	Im	Re	Im	
$(\bar{s}_L\gamma^\mu d_L)^2$	9.8×10^2	1.6×10^4	9.0×10^{-7}	3.4×10^{-9}	Δm_K; ϵ_K
$(\bar{s}_R\, d_L)(\bar{s}_L d_R)$	1.8×10^4	3.2×10^5	6.9×10^{-9}	2.6×10^{-11}	Δm_K; ϵ_K
$(\bar{c}_L\gamma^\mu u_L)^2$	1.2×10^3	2.9×10^3	5.6×10^{-7}	1.0×10^{-7}	Δm_D; $\lvert q/p\rvert, \phi_D$
$(\bar{c}_R\, u_L)(\bar{c}_L u_R)$	6.2×10^3	1.5×10^4	5.7×10^{-8}	1.1×10^{-8}	Δm_D; $\lvert q/p\rvert, \phi_D$
$(\bar{b}_L\gamma^\mu d_L)^2$	5.1×10^2	9.3×10^2	3.3×10^{-6}	1.0×10^{-6}	Δm_{B_d}; $S_{\psi K_S}$
$(\bar{b}_R\, d_L)(\bar{b}_L d_R)$	1.9×10^3	3.6×10^3	5.6×10^{-7}	1.7×10^{-7}	Δm_{B_d}; $S_{\psi K_S}$
$(\bar{b}_L\gamma^\mu s_L)^2$	1.1×10^2		7.6×10^{-5}		Δm_{B_s}
$(\bar{b}_R\, s_L)(\bar{b}_L s_R)$	3.7×10^2		1.3×10^{-5}		Δm_{B_s}

TABLE I: Bounds on representative dimension-six $\Delta F = 2$ operators. Bounds on Λ are quoted assuming an effective coupling $1/\Lambda^2$, or, alternatively, the bounds on the respective c_{ij}'s assuming $\Lambda = 1$ TeV. Observables related to CPV are separated from the CP conserving ones with semicolons. In the B_s system we only quote a bound on the modulo of the NP amplitude derived from Δm_{B_s} (see text). For the definition of the CPV observables in the D system see Ref. [15].

[Isidori, Nir, Perez, 2010]

Minimal Flavour Violation (quarks)

[Chivukula. Georgi 1987
D' Ambrosio, Giudice, Isidori, Strumia 2002]

- useful benchmark: a framework where the only source of flavour violation beyond the SM are the Yukawa coupling. Well-defined in the quark sector.

- in the limit $y_u = y_d = 0$, the SM lagrangian is invariant under a $U(3)^3$ flavour symmetry

$$G_q = SU(3)_{u^c} \times SU(3)_{d^c} \times SU(3)_q \times \ldots$$

$$q = (1,1,3) \quad u^c = (\bar{3},1,1) \; d^c = (1,\bar{3},1)$$

if the Yukawa couplings y_u and y_d are promoted to non-dynamical fields (spurions) transforming conveniently, the SM lagrangian remains formally invariant under the flavour group G_q

$$L_{SM} = \ldots - d^c y_d (\Phi^+ q) - u^c y_u (\tilde{\Phi}^+ q) + h.c.$$

$$y_u = (3,1,\bar{3}) \qquad y_d = (1,3,\bar{3})$$

- MFV assumes that new operators coming from New Physics do not involve any additional field/spurions and that they are still invariant under G_q
[additional assumption: no additional sources of CPV other than those in $y_{u,d}$]

Exercise 11: build the leading operator contributing to b -> s γ in MFV

a convenient basis:

$$y_d = \hat{y}_d \qquad y_u = \hat{y}_u V_{CKM} \qquad \hat{y}_{u,d} \text{ diagonal}$$

leading order MFV invariant

$$i\frac{e}{\Lambda^2} d^c \left(\sigma^{\mu\nu} F_{\mu\nu}\right) Z^d \left(\Phi^+ q\right) + h.c.$$

$$Z^d = y_d y_u^+ y_u = \frac{2\sqrt{2}}{v^3}\left(\hat{m}_d V_{CKM}^+ \hat{m}_u^2 V_{CKM}\right)$$

$$\hat{m}_u \approx \text{diag}(0,0,m_t)$$

$$b \to s\gamma \quad \Leftrightarrow \quad \left(Z^d\right)_{32}^*, \quad \left(Z^d\right)_{23}$$

$$\left(Z^d\right)_{32}^* = \frac{2\sqrt{2}}{v^3} m_b \left(m_t^2 V_{tb} V_{ts}^*\right)$$

$$\left(Z^d\right)_{23} = \frac{2\sqrt{2}}{v^3} m_s \left(m_t^2 V_{tb} V_{ts}^*\right)$$

MFV is nothing but the GIM mechanism extended to BSM contributions

$\left[b^c \left(\sigma F\right) s\right]^+$ dominates over by (m_t/m_b) $s^c \left(\sigma F\right) b$

$$BR(B \to X_s \gamma) = (3.55 \pm 0.24 \pm 0.09) \times 10^{-4} \quad \Rightarrow \quad \Lambda > 6.1\, TeV$$

Exercise 12: build the leading operator with $\Delta F=2$ in MFV

same basis as before:

$$y_d = \hat{y}_d \qquad y_u = \hat{y}_u V_{CKM} \qquad \hat{y}_{u,d} \text{ diagonal}$$

leading MFV invariant

$$\bar{q}_{Li}\gamma^\mu (y_u^+ y_u)_{ij} q_{Lj}\, \bar{q}_{Lk}\gamma_\mu (y_u^+ y_u)_{kl} q_{Ll}$$

looking at the down quark sector and selecting i=k=d,s and j=l=b
we get the MFV operator contributing to $\Delta B=2$

$$O_{MFV}(|\Delta B|=2) = \frac{c}{\Lambda_{NP}^2} y_t^4 (V_{tb}V_{tq}^*)^2\, \bar{q}_L\gamma^\mu b_L\, \bar{q}_L\gamma_\mu b_L \qquad (q=d,s)$$

where we used
$\hat{m}_u \approx \mathrm{diag}(0,0,m_t)$

again same CKM suppression as in the SM. Now the bound on the scale of New Physics reads

$$\Lambda_{NP} > 5.9\ TeV$$

define 2 New Physics parameters

$$\Delta_q \equiv \frac{M_{12}^q}{M_{12}^{q,SM}} \qquad (q=d,s)$$

[O_{MFV} modify M_{12} for B_d and B_s in the same way: i.e Δ_d and Δ_s are identical and real in MFV]

bound on the scale of New Physics in MFV

Operator	Bound on Λ	Observables
$H^\dagger\left(\overline{D}_R Y^{d\dagger} Y^u Y^{u\dagger} \sigma_{\mu\nu} Q_L\right)(eF_{\mu\nu})$	6.1 TeV	$B \to X_s\gamma$, $B \to X_s\ell^+\ell^-$
$\frac{1}{2}(\overline{Q}_L Y^u Y^{u\dagger}\gamma_\mu Q_L)^2$	5.9 TeV	ϵ_K, Δm_{B_d}, Δm_{B_s}
$H_D^\dagger\left(\overline{D}_R Y^{d\dagger} Y^u Y^{u\dagger} \sigma_{\mu\nu} T^a Q_L\right)(g_s G^a_{\mu\nu})$	3.4 TeV	$B \to X_s\gamma$, $B \to X_s\ell^+\ell^-$
$\left(\overline{Q}_L Y^u Y^{u\dagger}\gamma_\mu Q_L\right)\left(\overline{E}_R\gamma_\mu E_R\right)$	2.7 TeV	$B \to X_s\ell^+\ell^-$, $B_s \to \mu^+\mu^-$
$i\left(\overline{Q}_L Y^u Y^{u\dagger}\gamma_\mu Q_L\right) H_U^\dagger D_\mu H_U$	2.3 TeV	$B \to X_s\ell^+\ell^-$, $B_s \to \mu^+\mu^-$
$\left(\overline{Q}_L Y^u Y^{u\dagger}\gamma_\mu Q_L\right)\left(\overline{L}_L\gamma_\mu L_L\right)$	1.7 TeV	$B \to X_s\ell^+\ell^-$, $B_s \to \mu^+\mu^-$
$\left(\overline{Q}_L Y^u Y^{u\dagger}\gamma_\mu Q_L\right)(eD_\mu F_{\mu\nu})$	1.5 TeV	$B \to X_s\ell^+\ell^-$

TABLE II: Bounds on the scale of new physics (at 95% C.L.) for some representative $\Delta F = 1$ [27] and $\Delta F = 2$ [12] MFV operators (assuming effective coupling $\pm 1/\Lambda^2$), and corresponding observables used to set the bounds.

[Isidori, Nir, Perez, 2010]

Minimal Flavour Violation (leptons)

extension of MFV to leptons is ambiguous:
we can describe neutrino masses in several ways

1 B-L conserved, pure Dirac neutrino masses
just copy the quark sector

$$G_l = SU(3)_{\nu^c} \times SU(3)_{e^c} \times SU(3)_l \times \ldots$$

$$l = (1,1,3) \quad \nu^c = (\bar{3},1,1) \; e^c = (1,\bar{3},1)$$

$$y_\nu = (3,1,\bar{3})$$

$$y_e = (1,3,\bar{3})$$

$$i\,\frac{e}{\Lambda^2} e^c \left(\sigma^{\mu\nu} F_{\mu\nu}\right) \mathcal{Z}(\Phi^+ l) + h.c.$$

choose as basis:

$$y_e = \hat{y}_e \qquad y_\nu = \hat{y}_\nu U^+_{PMNS}$$

$$\mathcal{Z} = y_e y_\nu^+ y_\nu$$

$$= \frac{2\sqrt{2}}{v^3}\left(\hat{m}_e U_{PMNS} \hat{m}_\nu^2 U^+_{PMNS}\right)$$

dominant contribution to μ -> e γ

$$\left(\mathcal{Z}\right)^*_{21} = \frac{2\sqrt{2}}{v^3} m_\mu \left(U^*_{\mu i} U_{ei} m_i^2\right)$$

$$\approx 10^{-28}$$

μ -> e γ unobservable
even for $\Lambda \approx 1$ TeV

2 B-L violated, neutrino masses from d=5 operator

[Cirigliano, Grinstein, Isidori, Wise 2005]

$$L = \ldots + e^c y_e (\Phi^+ l) + \frac{1}{2\Lambda_L}\left(\tilde{\Phi}^+ l\right) w \left(\tilde{\Phi}^+ l\right) + h.c.$$

an important assumption: $\Lambda_L \neq \Lambda$

$$G_l = SU(3)_{e^c} \times SU(3)_l \times \ldots$$

$$l = (1,3) \qquad e^c = (\bar{3},1)$$

$$y_e = (3,\bar{3})$$

$$w = (1,\bar{6})$$

the only sources of G_l breaking

$$y_e = \sqrt{2}\frac{m_e^{diag}}{v} \qquad w = \frac{2\Lambda_L}{v^2} U^* m_\nu^{diag} U^+$$

spurions expressed in terms of known quantities and Λ_L

$$\mathcal{Z} = y_e w^+ w$$

$$= \frac{4\sqrt{2}}{v^3}\frac{\Lambda_L^2}{v^2}\left(\hat{m}_e U_{PMNS} \hat{m}_\nu^2 U^+_{PMNS}\right)$$

μ -> e γ dominated by

$$\left(\mathcal{Z}\right)^*_{21} = \frac{4\sqrt{2}}{v^3}\frac{\Lambda_L^2}{v^2} m_\mu \left(U^*_{\mu i} U_{ei} m_i^2\right)$$

enhancement factor can be huge $\frac{\Lambda_L^2}{v^2}$

experimental bound satisfied by $(\Lambda_L/\Lambda) < 10^9$

μ -> e γ observable if $\Lambda_L >> \Lambda$

[qualitatively similar conclusion when MFV extended to the type I see-saw case]

Exercise 13: show that

+ for normal hierarchy
- for inverted hierarchy

$$\mathcal{Z}_{ij} = \frac{4\sqrt{2}}{v^3}\frac{\Lambda_L^2}{v^4}\left[\Delta m^2_{sol} U_{i2} U^*_{j2} \pm \Delta m^2_{atm} U_{i3} U^*_{j3}\right]$$

and estimate

$$\frac{R_{\mu e}}{R_{\tau\mu}} = \frac{BR(\mu \to e\gamma)}{BR(\tau \to \mu\gamma)} \times \frac{BR(\tau \to \mu \nu_\tau \bar{\nu}_\mu)}{BR(\mu \to e \nu_\mu \bar{\nu}_e)}$$

solution

$$\frac{R_{\mu e}}{R_{\tau\mu}} \approx \left|\frac{2}{3} r \pm \sqrt{2} \sin\vartheta_{13} e^{i\delta}\right|^2 \approx (0.035 \div 0.055) \qquad r \equiv \frac{\Delta m^2_{sol}}{\Delta m^2_{atm}}$$

from present bound on μ -> e γ

$$\boxed{R_{\tau\mu} < (1.0 \div 1.6) \times 10^{-11}}$$

hints:

-- use unitarity relation for U_{PMNS}

-- use approximate values

$$U_{\mu 3} \approx U_{\tau 3} \approx 1/\sqrt{2}$$

$$U_{e2} \approx U_{\mu 2} \approx -U_{\tau 2} \approx 1/\sqrt{3}$$

LFV in the limit of vanishing neutrino masses

MFV extended to the lepton sector reproduces the GIM suppression
in particular LF is conserved when m_i=0

GIM suppression can be evaded in several models of fermion masses
e.g. in partial compositness where elementary fermions acquire a mass
through their mixing with a composite sector

a toy model

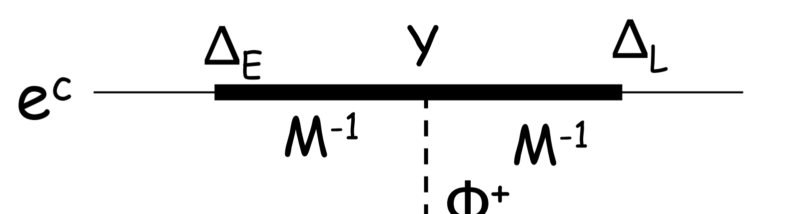
$$
\begin{aligned}
L_Y &= -e^c \Delta_E E - L^c \Delta_L l && \Leftrightarrow \text{ elementary-composite mixing} \\
&\quad - E^c M E - L^c M L && \Leftrightarrow \text{ Dirac masses for composite fermions} \\
&\quad - E^c Y (\Phi^+ L) - (L^c \tilde{\Phi}^+) \tilde{Y} E + h.c. && \Leftrightarrow \text{ Yukawa coupling of composite fermions}
\end{aligned}
$$

by integrating out the composite sector [Exercise 14]

$$L_Y = -e^c y_e (\Phi^+ l) + h.c.$$

$$y_e = (\Delta_E M^{-1}) Y (M^{-1} \Delta_L) + \ldots$$

higher-orders in (Φ/M)



Exercise 15

compute the corrections to previous LO relations by using the equation of motion for the composite sector. Start with 1 generation and then discuss the 3 generation case.

write L_Y in matrix notation

$$L_Y = -\begin{pmatrix} e^c & E^c & L^c \end{pmatrix} \begin{pmatrix} 0 & \Delta_E & 0 \\ 0 & M & Y\Phi^+ \\ \Delta_L & \tilde{\Phi}^+\tilde{Y} & M \end{pmatrix} \begin{pmatrix} l \\ E \\ L \end{pmatrix} + h.c.$$

write the e.o.m. for the composite fields (E^c,L^c) and (E,L) in the limit of negligible kinetic term and substitute them back into L_Y

$$L_Y = e^c \begin{pmatrix} \Delta_E & 0 \end{pmatrix} \begin{pmatrix} M & Y\Phi^+ \\ \tilde{\Phi}^+\tilde{Y} & M \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ \Delta_L \end{pmatrix} l + h.c.$$

expand this expression in powers of the Higgs field

At the LO $\quad y_e = F_{E^c} Y F_L \qquad F_{E^c} = \Delta_E M^{-1} \qquad F_L = M^{-1}\Delta_L$

an intriguing possibility (anarchic scenario):

-- Yukawa coupling Y in the composite sector are O(1)

-- fermion mass hierarchy entirely due to the amount of mixing F

it arises is many SM extensions

split fermions in an Extra Dimension

$$F_{X_i} = \sqrt{\frac{2\mu_i}{1-e^{-2\mu_i r}}}$$

ED	μ_i	r
Flat $[0,\pi R]$	M_i/Λ	$\Lambda\pi R$
Warped $[R,R']$	$1/2 - M_i R$	$\log R'/R$

M_i = bulk mass of fermion X_i

$Y_{u,d}$ = O(1) Yukawa couplings between bulk fermions and a Higgs localized at one brane

no symmetry:
hierarchy produced by geometry

fermion masses from abelian flavour symmetries $\quad Q(X_i) \geq 0$

$$F_{X_i} = \mathrm{diag}\left(\lambda^{Q(X_1)}, \lambda^{Q(X_2)}, \lambda^{Q(X_3)}\right) \qquad \lambda = \frac{\langle\varphi\rangle}{\Lambda}$$

chiral multiplets X_i of the MSSM coupled to a superconformal sector

[Nelson-Strassler 0006251]

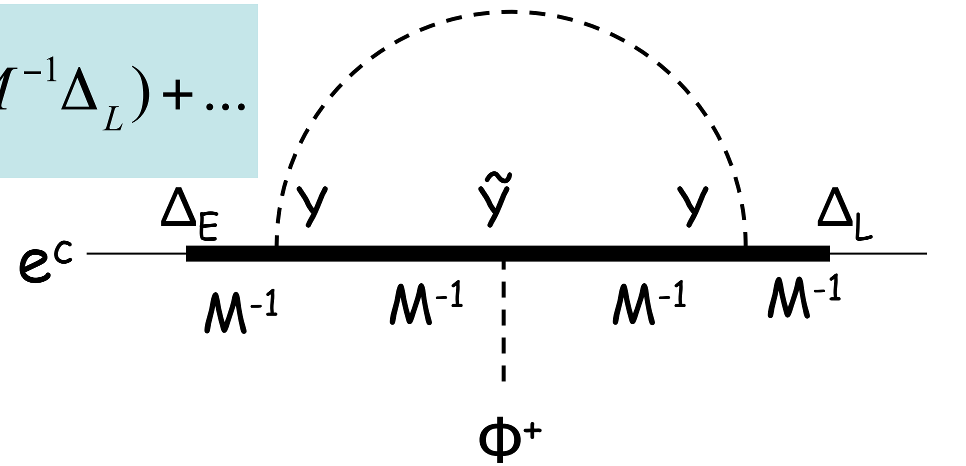
$$F_{X_i} = \left(\frac{\Lambda_c}{\Lambda}\right)^{\frac{\gamma_i}{2}} < 1$$

γ_i anomalous dimension of X_i

so far neutrino are massless
do we expect LFV in our toy model?

one-loop contribution to lepton dipole operator from Higgs exchange
(assuming M proportional to identity)

$$\frac{Z}{\Lambda^2} \approx \frac{1}{16\pi^2 M^2}(\Delta_E M^{-1}) Y \tilde{Y} Y (M^{-1}\Delta_L) + \ldots$$



in general these combinations
not diagonal in the same basis

$$y_e = (\Delta_E M^{-1}) Y (M^{-1}\Delta_L) + \ldots$$

LFV not suppressed by neutrino masses and unrelated to (B-L) breaking scale

rough estimate

$$\frac{Z_{\mu e}}{\Lambda^2} < 2\times 10^{-9}\ \text{TeV}^{-2}$$

$$M > 10\ \text{TeV}$$

$$\Delta_E \approx \Delta_L$$

$$\frac{\Delta_f}{M} \approx \sqrt{\frac{m_f}{v}}$$

$$Y \approx \tilde{Y} \approx O(1)$$

Exercise 16: reproduce flavour pattern of Z from a spurion analysis

$$\frac{Z}{\Lambda^2} \approx \frac{1}{16\pi^2 M^2}(\Delta_E M^{-1}) Y \tilde{Y} Y (M^{-1}\Delta_L) + \ldots$$

-- identify the maximal flavour symmetry G of our toy model

-- identify the transformation properties of the spurions Δ_L, Δ_E, Y, $\tilde{Y}$, that guarantee the invariance of L_y

-- using previous tools, build the relevant dipole operator invariant under G

summary

- LFV expected in charged leptons = CLFV

- CLFV probes physics beyond the νSM [=SM minimally extended to accommodate ν masses]

  observable rates for CLFV require new physics at a scale well below the GUT or the L-violation scales [$\Lambda \ll \Lambda_L$ in our example of MFV]

- GIM suppression in CLFV is a special feature of MFV: it can be violated in models of fermion masses and relation to neutrino masses and mixing angles can be more indirect