Brane SUSY Breaking and Inflation – implications for scalar fields and CMB distorsion

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- E. Dudas, N. Kitazawa, AS, *P.L.* ***B*694** (2010) 80 [arXiv:1009.0874 [hep-th]].
- E. Dudas, N. Kitazawa, S. Patil, AS, *JCAP* **1205** (2012) 012 [arXiv:1202.6630 [hep-th]]
- P. Fré, A.S., A.S. Sorin, to appear
- E. Dudas, N. Kitazawa, S. Patil, AS, in progress

SUPERFIELDS
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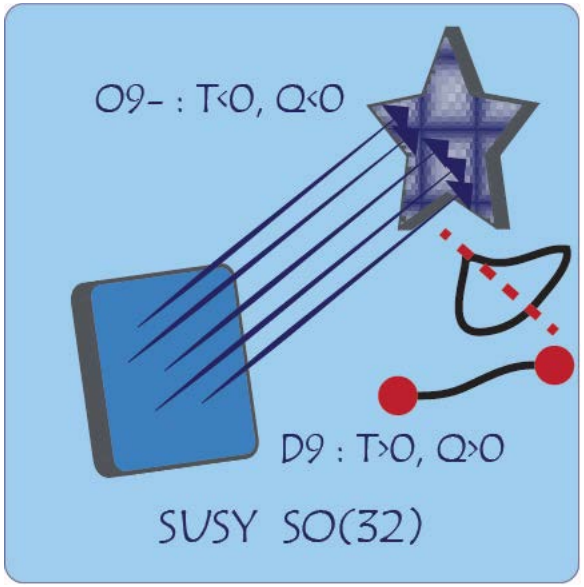
Brane SUSY Breaking

(Sugimoto, 1999)
(Antoniadis, Dudas, AS, 1999)
(Aldazabal, Uranga, 1999)
(Angelantonj, 1999)

- **Dualities :** link different strings
- **Orientifolds :** link closed and open strings



- SUSY : D9 ($T > 0, Q > 0$) + $O9_-$ ($T < 0, Q < 0$) → SO(32)

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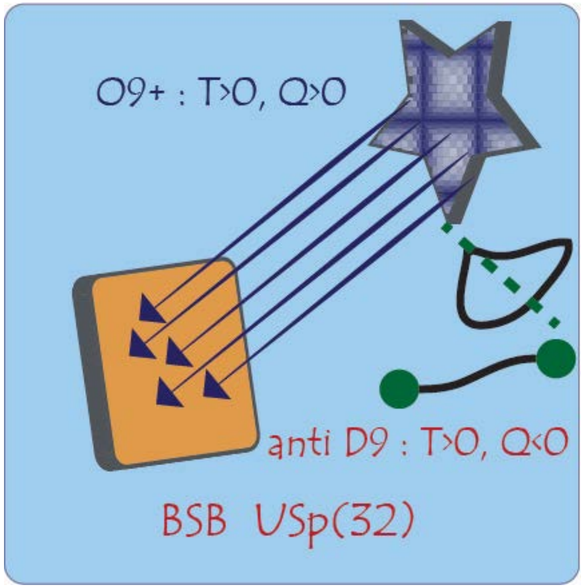

- BSB : anti-D9($T > 0$, $Q < 0$) + $O9_+$ ($T > 0$, $Q > 0$) $\rightarrow$ USp(32)

Brane SUSY Breaking

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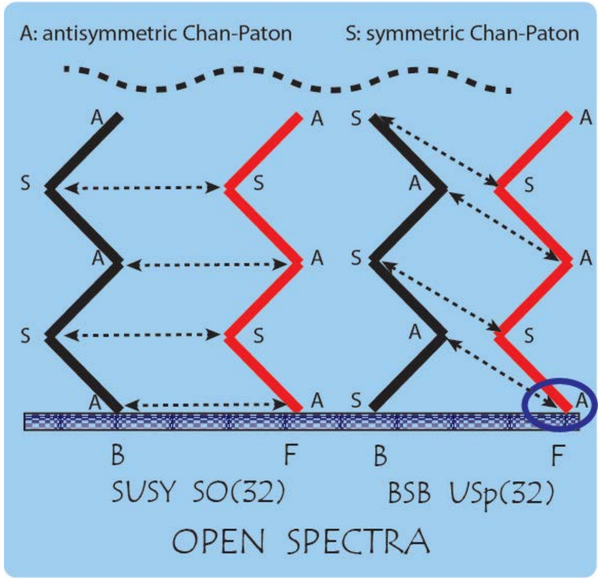
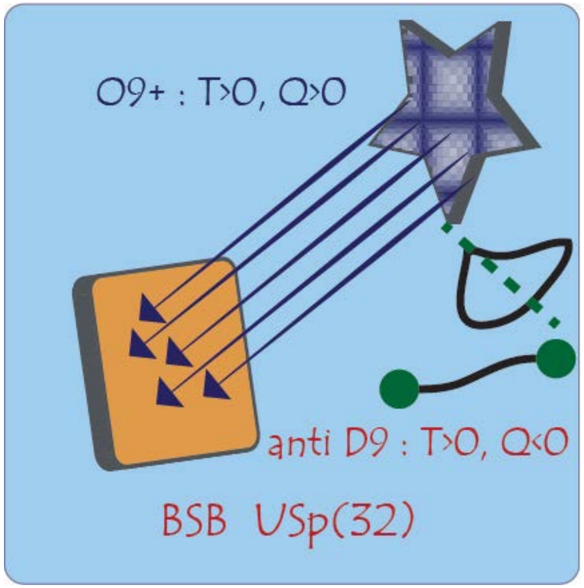


Tree – level BSB

- SUSY broken at string scale in open sector, exact in closed
- Stable vacuum
- Goldstino in open sector



BSB: Tension unbalance → exponential potential

$$S_{10} = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-g}\ \{ e^{-2\phi} \left(- R + 4 (\partial \phi)^2 \right) - T e^{-\phi} + \ldots \}$$

- Flat space : runaway behavior
- String-scale breaking : early-Universe Cosmology ?

A climbing scalar in d dim's

- Consider the action for gravity and a scalar ϕ :

$$\boxed{S \;=\; \frac{1}{2\kappa^2}\int d^D x\,\sqrt{-g}\left[R \,-\, \frac{1}{2}\,(\partial\phi)^2 \,-\, V(\phi) \,+\, \ldots\right]}$$

- Look for **cosmological solutions** of the type

$$\boxed{ds^2 \;=\; -\,e^{\,2\mathcal{B}(t)}\,dt^2 \,+\, e^{\,2A(t)}\,d\mathbf{x}\cdot d\mathbf{x}}$$

(Halliwell, 1987)
………………
(Dudas,,Mourad, 2000)
(Russo, 2004)
…………………

- Make the convenient **gauge choice** $\boxed{V(\phi)\,e^{2\mathcal{B}} \;=\; M^2}$

- Let :

$$\boxed{\beta \;=\; \sqrt{\frac{d-1}{d-2}}\;, \qquad \tau \;=\; M\,\beta\,t\;, \qquad \varphi \;=\; \frac{\beta\,\phi}{\sqrt{2}}\;, \qquad \mathcal{A} \;=\; (d-1)\,A}$$

- In expanding phase :

$$\boxed{\ddot{\varphi} \,+\, \dot{\varphi}\,\sqrt{1 \,+\, \dot{\varphi}^{\,2}} \,+\, \left(1+\dot{\varphi}^{\,2}\right)\,\frac{1}{2V}\,\frac{\partial V}{\partial\varphi} \;=\; 0}$$

- OUR CASE :

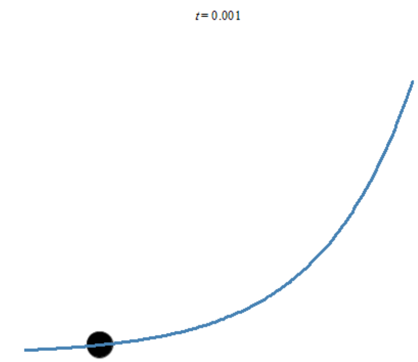
$$\boxed{V \;=\; \exp(\,2\,\gamma\,\varphi) \;\longrightarrow\; \frac{1}{2\,V}\,\frac{\partial V}{\partial\,\varphi} \;=\; \gamma}$$

A climbing scalar in d dim's

- $\gamma < 1$? Both signs of speed

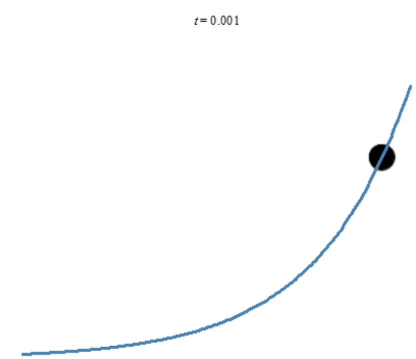
a. "Climbing" solution (φ climbs, then descends):

$$\boxed{\dot{\varphi} = \frac{1}{2}\left[\sqrt{\frac{1-\gamma}{1+\gamma}}\,\coth\left(\frac{\tau}{2}\sqrt{1-\gamma^2}\right) - \sqrt{\frac{1+\gamma}{1-\gamma}}\,\tanh\left(\frac{\tau}{2}\sqrt{1-\gamma^2}\right)\right]}$$



b. "Descending" solution (φ only descends):

$$\boxed{\dot{\varphi} = \frac{1}{2}\left[\sqrt{\frac{1-\gamma}{1+\gamma}}\,\tanh\left(\frac{\tau}{2}\sqrt{1-\gamma^2}\right) - \sqrt{\frac{1+\gamma}{1-\gamma}}\,\coth\left(\frac{\tau}{2}\sqrt{1-\gamma^2}\right)\right]}$$



NOTE : only φ_o . Early speed → singularity time !

Limiting τ- speed (LM attractor): $\quad v_l = -\frac{\gamma}{\sqrt{1-\gamma^2}}$

$\gamma \to 1$: LM attractor & descending solution disappear

- $\gamma \geq 1$? Climbing ! E.g. for γ=1 : $\quad \dot{\varphi} = \frac{1}{2\tau} - \frac{\tau}{2}$

CLIMBING : in ALL asymptotically exponential potentials with $\gamma \geq 1$!

String Realizations

NOTE :
a. Two - derivative couplings : α′ corrections ? *(Condeescu, Dudas, in progress)*
b. [BUT: climbing → weak string coupling]

Dimensional reduction of (critical) 10-dimensional low-energy EFT:

$$\boxed{S_D = \frac{1}{2\kappa_{10}^2} \int d^{10}x\sqrt{-g}\,\{ e^{-2\phi}\left(-R + 4(\partial\phi)^2\right) - T\,e^{-\phi} + \ldots\}}$$

$$\boxed{ds^2 = e^{-\frac{(10-d)}{(d-2)}\sigma}\, g_{\mu\nu}\,dx^\mu\,dx^\nu + e^{\sigma}\,\delta_{ij}\,dx^i\,dx^j}$$

$$\boxed{S_d = \frac{1}{2\kappa_d^2}\int d^dx\sqrt{-g}\left\{ - R - \frac{1}{2}(\partial\phi)^2 - \frac{2(10-d)}{(d-2)}(\partial\sigma)^2 - T\,e^{\frac{3}{2}\phi - \frac{(10-d)}{(d-2)}\sigma} + \ldots\right\}}$$

- Two scalar combinations (Φ_s and Φ_t). Focus on Φ_t :

$$\boxed{S_d = \frac{1}{2\kappa_d^2}\int d^dx\sqrt{-g}\left\{ - R - \frac{1}{2}(\partial\Phi_s)^2 - \frac{1}{2}(\partial\Phi_t)^2 - T\,e^{\Delta\,\Phi_t}\right\}}$$

$$\boxed{\Delta = \sqrt{\frac{2(d-1)}{(d-2)}}} \quad \Longrightarrow \quad \gamma = 1 \quad \forall d < 10!$$

Climbing with a SUSY Axion

(Kachru, Kallosh, Linde, Trivedi, 2003)

- No-scale reduction + 10D tadpole → KKLT uplift

(Cremmer, Ferrara, Kounnas, Nanopoulos, 1983)
(Witten, 1985)

$$T \;=\; e^{-\frac{\Phi_t}{\sqrt{3}}} \;+\; i\,\frac{\theta}{\sqrt{3}}$$

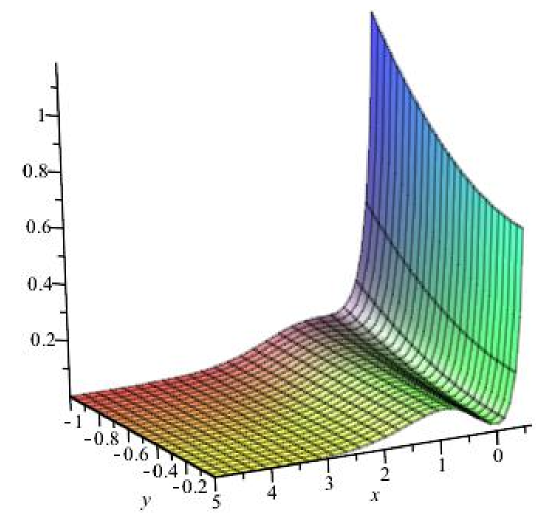
$$S_4 \;=\; \frac{1}{2\kappa_4^2}\int d^4x\,\sqrt{-g}\,\left\{R \;-\; \frac{1}{2}\,(\partial\Phi_t)^2 \;-\; \frac{1}{2}\,e^{\frac{2}{\sqrt{3}}\,\Phi_t}\,(\partial\theta)^2 \;-\; V(\Phi_t,\theta) + \cdots\right\}$$

$$V(\Phi_t,\theta) \;=\; \frac{c}{(T+\bar{T})^3} + V_{(non\ pert.)}$$

$$\Phi_t = \frac{2}{\sqrt{3}}\,x\;,\quad \theta = \frac{2}{\sqrt{3}}\,y$$

$$\frac{d^2x}{d\tau^2} \;+\; \frac{dx}{d\tau}\sqrt{1+\left(\frac{dx}{d\tau}\right)^2 + e^{\frac{4x}{3}}\left(\frac{dy}{d\tau}\right)^2} \;+\; \frac{1}{2\,V}\,\frac{\partial V}{\partial x}\left[1+\left(\frac{dx}{d\tau}\right)^2\right]$$
$$+\;\frac{1}{2\,V}\,\frac{\partial V}{\partial y}\,\frac{dx}{d\tau}\,\frac{dy}{d\tau} \;-\; \frac{2}{3}\,e^{\frac{4x}{3}}\left(\frac{dy}{d\tau}\right)^2 \;=\; 0\;,$$

$$\frac{d^2y}{d\tau^2} \;+\; \frac{dy}{d\tau}\sqrt{1+\left(\frac{dx}{d\tau}\right)^2 + e^{\frac{4x}{3}}\left(\frac{dy}{d\tau}\right)^2} \;+\; \left(\frac{1}{2\,V}\,\frac{\partial V}{\partial x}+\frac{4}{3}\right)\frac{dx}{d\tau}\,\frac{dy}{d\tau}$$
$$+\;\frac{1}{2\,V}\,\frac{\partial V}{\partial y}\left[e^{-\frac{4x}{3}}+\left(\frac{dy}{d\tau}\right)^2\right] \;=\; 0$$

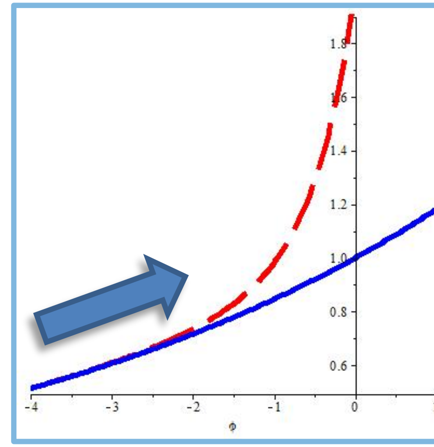


AXION INITIALLY "FROZEN" → CLIMBING !

Climbing can inject Inflation

a. "Hard" exponential of Brane SUSY Breaking

b. "Soft" exponential ($\gamma < 1/\sqrt{3}$):



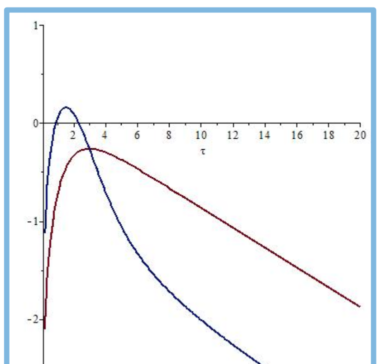
Would need : $\gamma \approx \frac{1}{12}$

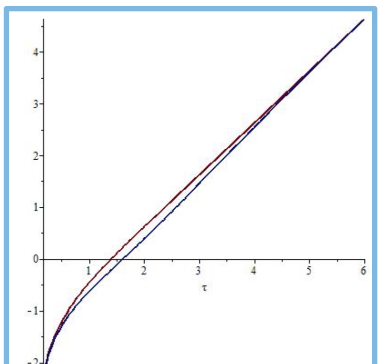
$$V(\phi) = \overline{M}^{\,4} \left(e^{2\varphi} + e^{2\gamma\varphi}\right)$$

Non-BPS D3 brane gives $\gamma = 1/2$
[+ stabilization of Φ_s]

(Sen , 1998)
(Dudas, Mourad, AS 2001)

- BSB "Hard exponential" → makes initial **climbing** phase **inevitable**
- "Soft exponential" → drives **inflation** during subsequent descent





φ_0 : "hardness" of kick !

Mukhanov – Sasaki Equation

Schroedinger-like equation for scalar (or tensor) fluctuations :

$$\frac{d^2 v_k(\eta)}{d\eta^2} + \left[k^2 - W_s(\eta)\right] v_k(\eta) = 0$$

$$ds^2 = a^2(\eta)\left(-d\eta^2 + d\mathbf{x}\cdot d\mathbf{x}\right)$$

$$Scalar : z(\eta) = a^2(\eta)\,\frac{\phi_0'(\eta)}{a'(\eta)}$$

$$Tensor : z(\eta) = a$$

$$W_s = \frac{1}{z}\,\frac{d^2 z}{d\eta^2}$$

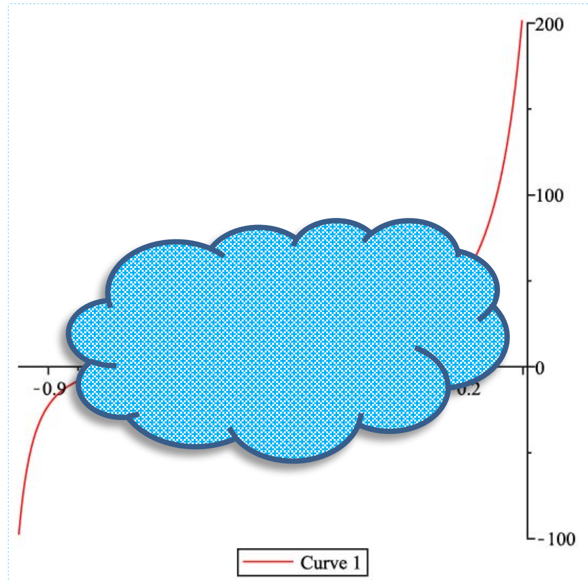
"MS Potential" : determined by the background

$$\textbf{Initial Singularity} : W_s \underset{\eta\to-\eta_0}{\sim} -\frac{1}{4}\,\frac{1}{(\eta+\eta_0)^2}$$

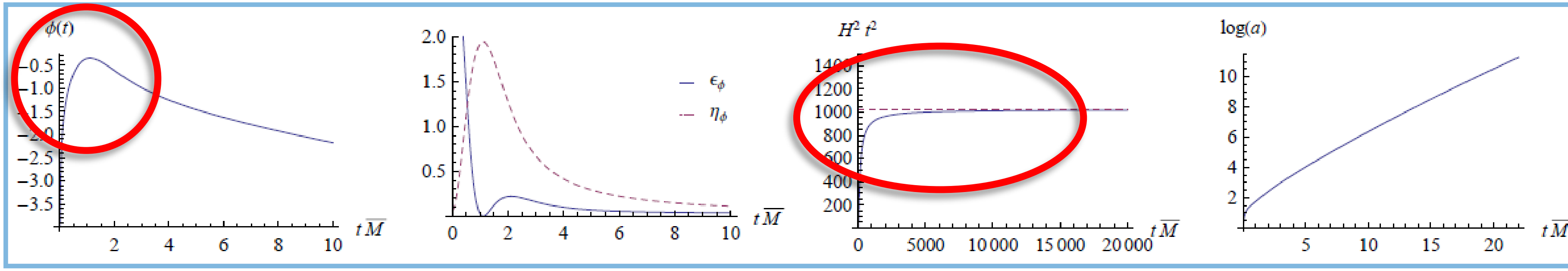
$$\textbf{LM Inflation} : W_s \underset{\eta\to 0}{\sim} \frac{\nu^2-\frac{1}{4}}{\eta^2}$$

$$\left[\nu = \frac{3}{2}\,\frac{1-\gamma^2}{1-3\gamma^2}\right]$$

$$P(k) \sim k^3 \left|\frac{v(-\epsilon)}{z(-\epsilon)}\right|^2$$



Numerical Power Spectra



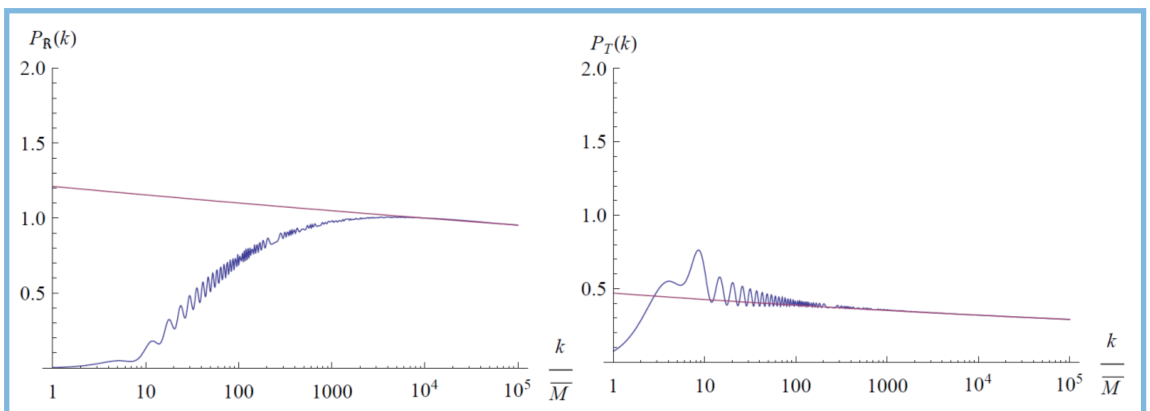
Key features:

1. **Harder "kicks"** make φ reach **later** the attractor
2. Even with mild kicks the **time scale** is 10^3– 10^4 in t M !
3. **η re-equilibrates slowly**

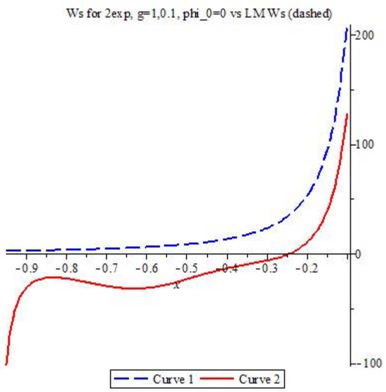
$$\epsilon_\phi \equiv - \frac{\dot H}{H^2} \ , \qquad \eta_\phi \equiv \frac{V_{\phi\phi}}{V}$$

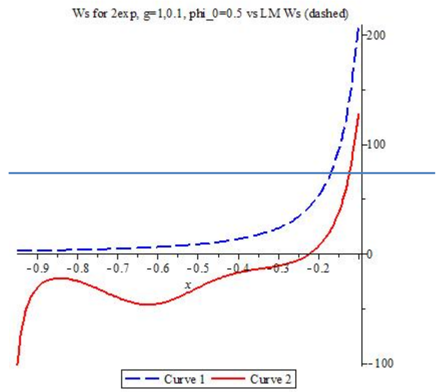
$$P_{S,T} \ \sim \ \int \frac{dk}{k} \ k^{n_{S,T}-1}$$

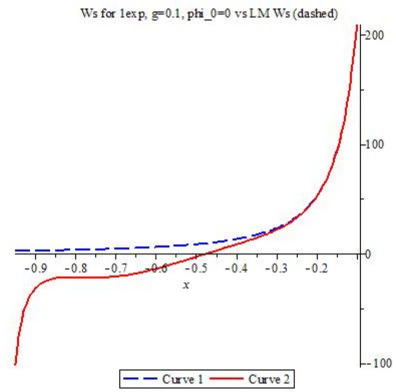
$$\begin{aligned} n_S \ - \ 1 \ &= \ 2(\,\eta_\phi \ - \ 3\,\epsilon_\phi) \ , \\ n_T \ - \ 1 \ &= \ - \ 2\,\epsilon_\phi \end{aligned}$$

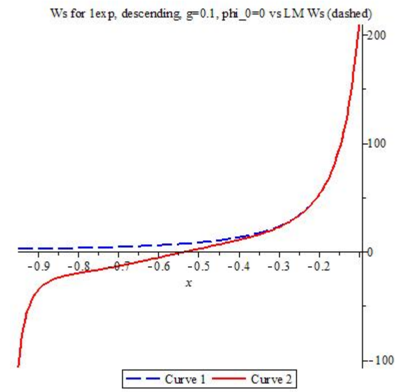


Analytic Power Spectra





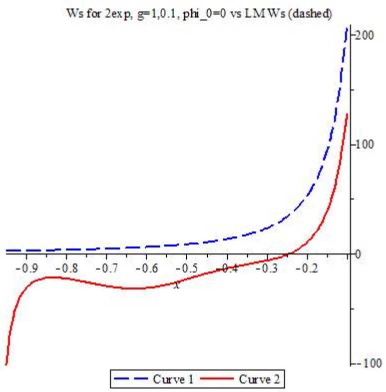


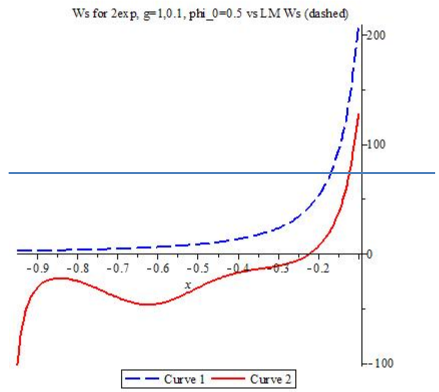


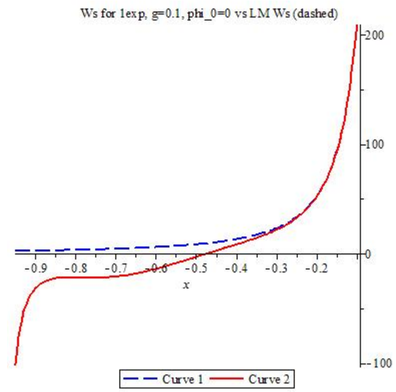
WKB:

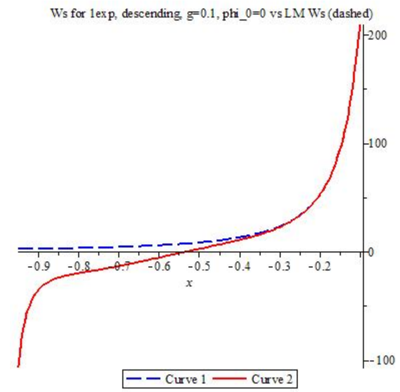
$$\boxed{v_k(-\epsilon) \sim \frac{1}{\sqrt[4]{|W_s(-\epsilon) - k^2|}} \exp\left(\int_{-\eta^\star}^{-\epsilon} \sqrt{|W_s(y) - k^2|}\, dy\right)}$$

Analytic Power Spectra









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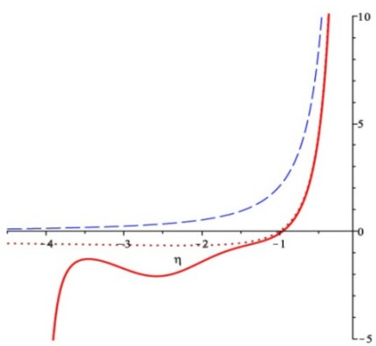
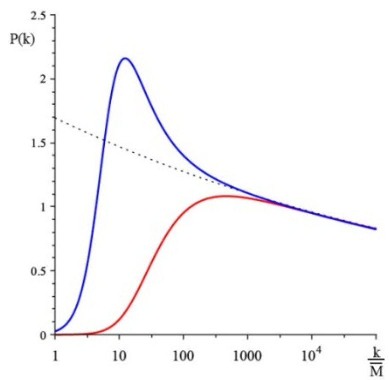
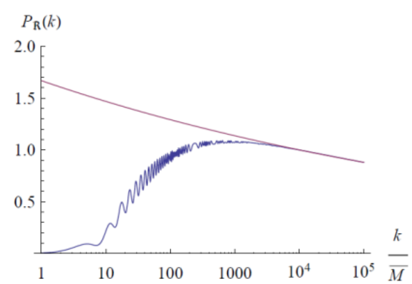
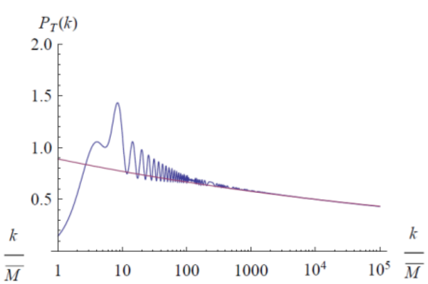




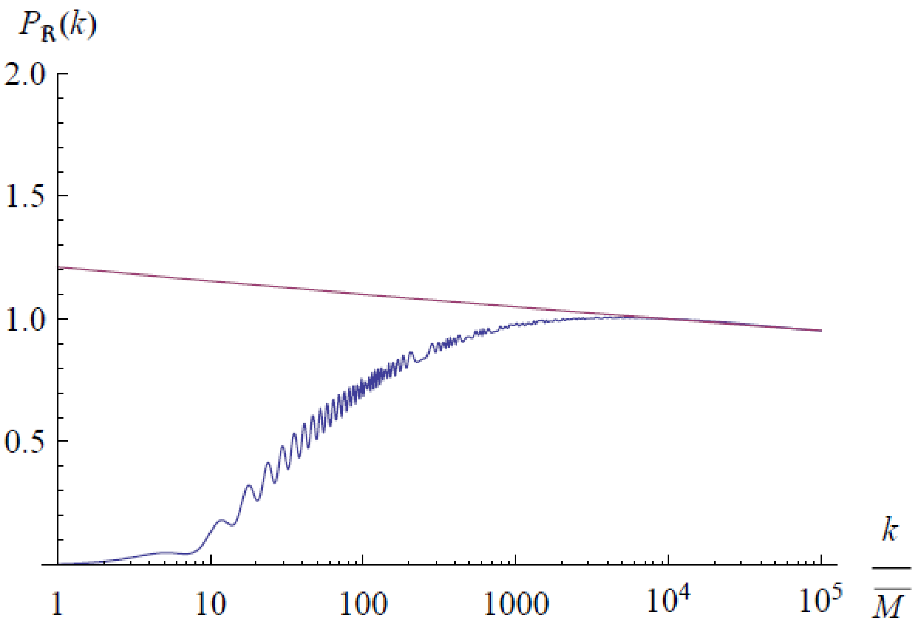


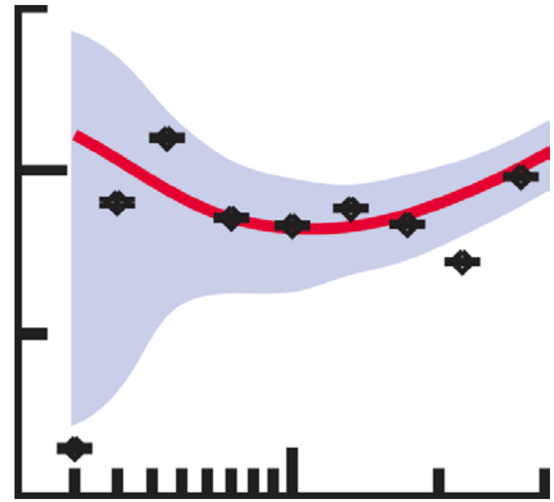


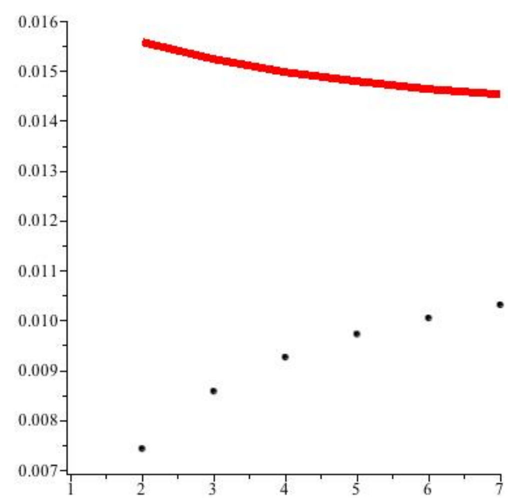
WIGGLES : cfr. Q.M. resonant transmission

An Observable Window ?

WMAP9 power spectrum :







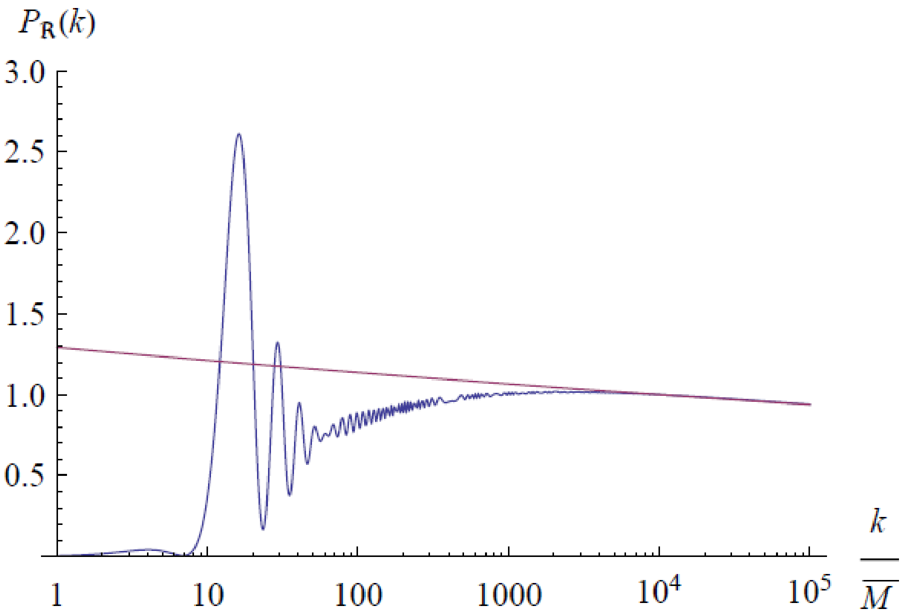
NOTE :

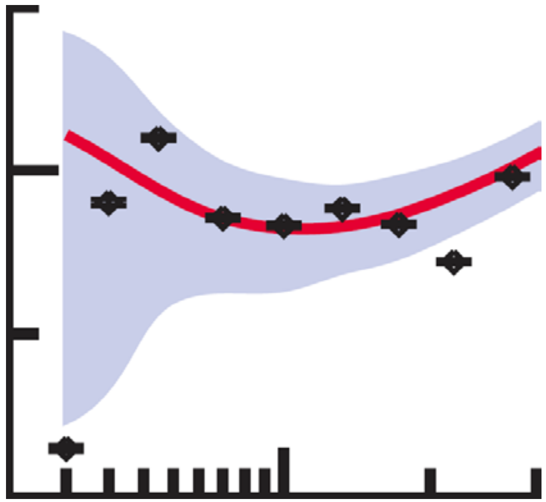
$$\left(\left| \frac{\Delta C_\ell}{C_\ell} \right| = \sqrt{\frac{2}{2\ell+1}} \right)$$

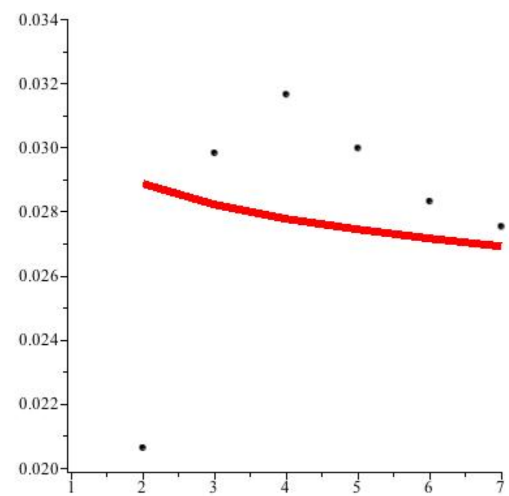
(Dudas, Kitazawa, Patil, AS, 2013, in progress)

An Observable Window ?

WMAP9 power spectrum :



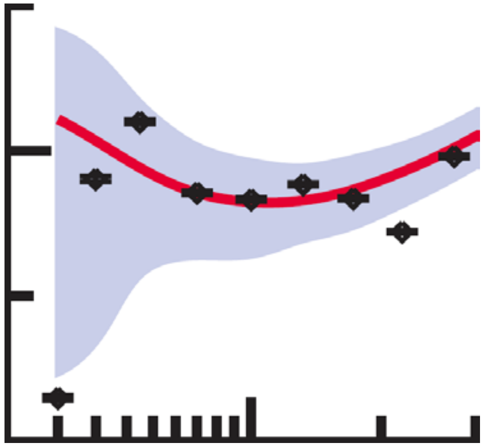


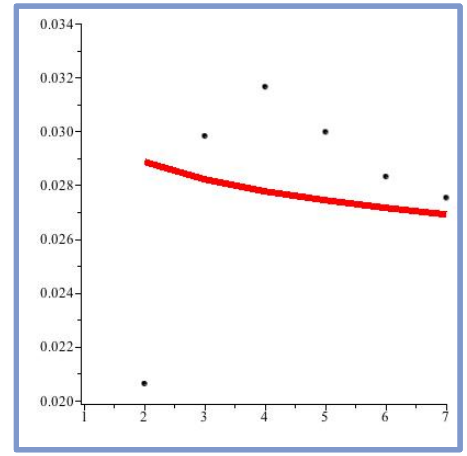


- **But with a harder "kick"…**

Qualitatively the low-k tail

(Dudas, Kitazawa, Patil, AS, 2013, in progress)





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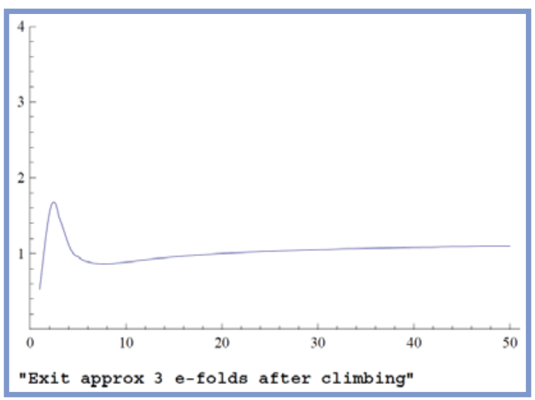
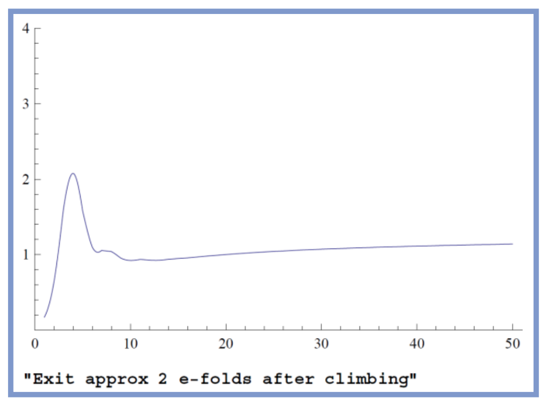
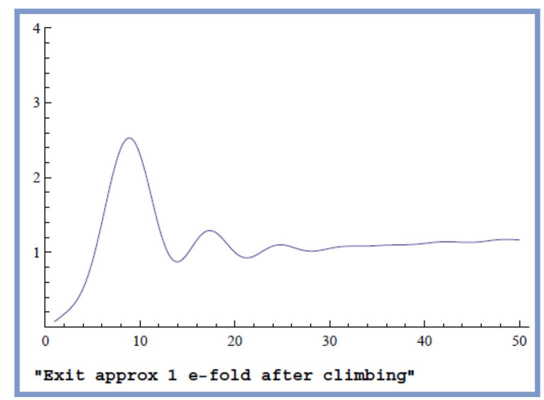






Another way of presenting the results in slide 15
2 parameters to adjust : "hardness" of kick & time of horizon exit

Summary & Outlook

- BRANE SUSY BREAKING ($d \leq 10$) : "critical" exponential potentials
- "HARD" exponential of BSB + "MILD" exponential (for inflation) :

❖ WITH "short" inflation (~ 60 e-folds) :

  - WIDE IR depression of scalar spectrum (~ 6 e-folds)
  - *[MILDER IR enhancement of tensor spectrum]*
  - LARGE quadrupole depression & qualitatively next few multipoles !
  - *[LARGE CLASS of integrable potentials with climbing (to appear)]*

BISPECTRUM ?

Extra slides

Scales

- BSB potential:

$$T_{10} = \frac{1}{(\alpha')^5} \quad \rightarrow T_4 = \frac{1}{(\alpha')^2} \left(\frac{R}{\sqrt{\alpha'}} \right)^6 \; = \; (\overline{M})^4$$

- Attractor Power spectra:

$$\begin{aligned} P_S(k) &= \frac{1}{16\pi G_N\, \epsilon} \left(\frac{H_\star}{2\pi} \right)^2 \left(\frac{k}{a\, H_\star} \right)^{n_S - 1} \\ P_T(k) &= \frac{1}{\pi G_N\, \epsilon} \left(\frac{H_\star}{2\pi} \right)^2 \left(\frac{k}{a\, H_\star} \right)^{n_s - 1} \\ n_S &= 1 - 6\epsilon + 2\eta \qquad n_T \; = \; -\,2\,\epsilon \\ \epsilon &= 8\pi G_N \left(\frac{V'}{V} \right)^2 \,, \qquad \eta \; = \; 16\pi G_N \left(\frac{V''}{V} \right)^2 \end{aligned}$$

- COBE normalization & bounds on ϵ:

$$H_\star \approx 10^{15} \; \times \; (\epsilon)^{\frac{1}{2}} \; GeV$$

$$\bar{M} \approx 6.5 \; 10^{16} \; \times \; (\epsilon)^{\frac{1}{4}} \; GeV$$

$$10^{-4} \; < \; \frac{P_T}{P_S} \; < \; 1.28 \; \rightarrow \; 10^{-5} < \epsilon < 0.08$$

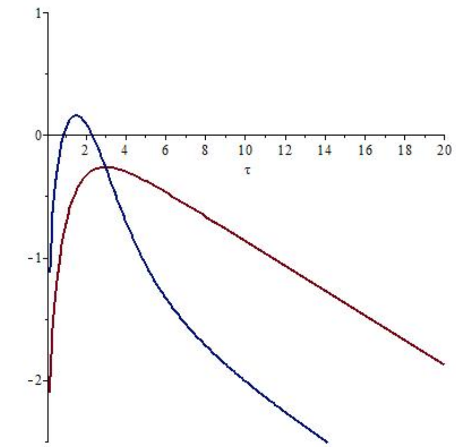
$$\boxed{\begin{aligned} 3.5 \; 10^{15} \; GeV \; &< \; \overline{M} \; < \; 3 \; 10^{16} \; GeV \\ 3 \; 10^{12} \; GeV \; &< \; H_\star \; < \; 3.4 \; 10^{14} \; GeV \end{aligned}}$$

Integrable Scalar Potentials, I (climbing+barrier)

(Fré, A.S., Sorin, to appear)

A number of exactly solvable (exp-like) potentials can be identified with techniques drawn from the theory of integrable systems. For instance, if

$$V(\phi) = \lambda \left(e^{\frac{2}{\gamma}\,\varphi} + e^{2\,\gamma\,\varphi} \right)$$



$$ds^2 = -\,e^{2\,\mathcal{A}}\,d\tau^2 + e^{\frac{2\,\mathcal{A}}{d-1}}\,d\mathbf{x}\cdot d\mathbf{x}$$

Letting :

$$\omega^2 = \frac{\lambda}{\gamma}\,\sqrt{1-\gamma^2}\;e^{2\,\mathcal{A}_0\,\sqrt{1-\gamma^2}}$$

$$e^{\varphi} = e^{\varphi_0}\left[\frac{\sinh(\omega\gamma\tau)}{\cosh\omega(\tau-\tau_0)}\right]^{\frac{\gamma}{1-\gamma^2}} \qquad e^{\mathcal{A}} = e^{\mathcal{A}_0}\left[\frac{\cosh^{\gamma^2}\omega(\tau-\tau_0)}{\sinh(\omega\gamma\tau)}\right]^{\frac{1}{1-\gamma^2}}$$

Qualitatively : as in the figure

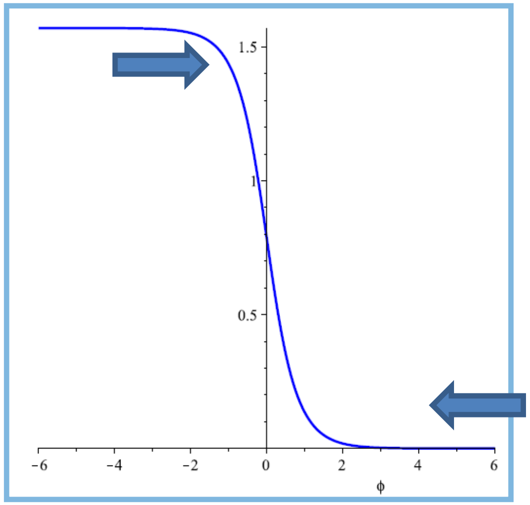
Integrable Scalar Potentials, II (graceful exit)

(Fré, A.S., Sorin, to appear)

An integrable potential where climbing is followed by a graceful exit.

$$ds^2 \;=\; -\, e^{-\,2\,\mathcal{A}}\, d\tau^2 \;+\; e^{\frac{2\,\mathcal{A}}{d-1}}\, d\mathbf{x}\cdot d\mathbf{x}$$

$$\begin{aligned}
&\mathcal{V}(\varphi) \;=\; \arctan\left(e^{-\,2\,\varphi}\right)\\
&\mathcal{A} \;=\; \log\left(x\,y\right)\,, \qquad \varphi \;=\; \log\left(\frac{x}{y}\right)\\
&z \;=\; x \;+\; i\,y\\
&\mathcal{L} \;=\; 2\,Im\Big[-\,\dot{z}^2 \;-\; 8\,C\,\log z \;-\; 8\,D\Big]\\
&Im\Big[\dot{z}^2 \;-\; 8\,C\,\log z \;-\; 8\,D\Big] \;=\; 0 \quad (\mathrm{H.\,C.})
\end{aligned}$$



System solved by a contour integral, and the shape of the contour is determined so that t is REAL

$$t \;=\; \int_a^z \frac{dz}{\sqrt{\log\left(\frac{z}{z_0}\right)}}$$