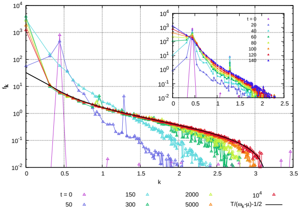
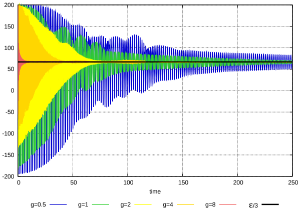
Thermalization in Scalar Field Theories





Orsay, June 2012

Thomas Epelbaum
IPhT

Outline

1. **Resummation scheme**
2. **Fixed volume**
3. **Expanding volume**

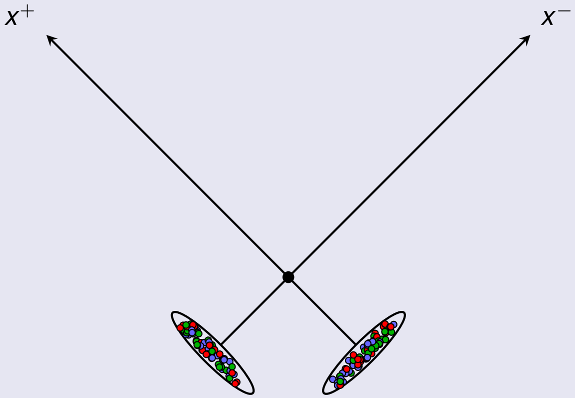
1 Resummation scheme

2 Fixed volume
 - Toy model
 - Energy-momentum tensor
 - Distribution function
 - Bose-Einstein Condensation

3 Expanding volume
 - The model
 - Numerical results

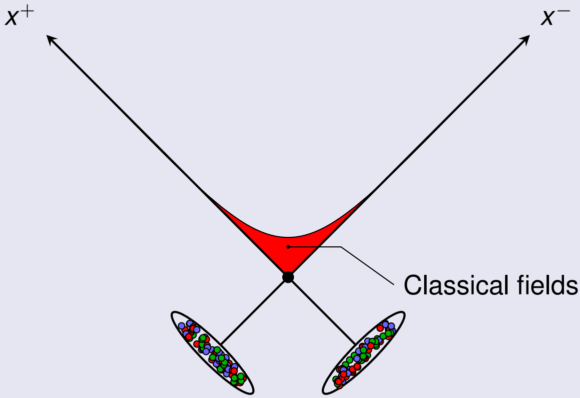
The general picture

Heavy Ion Collision



The general picture

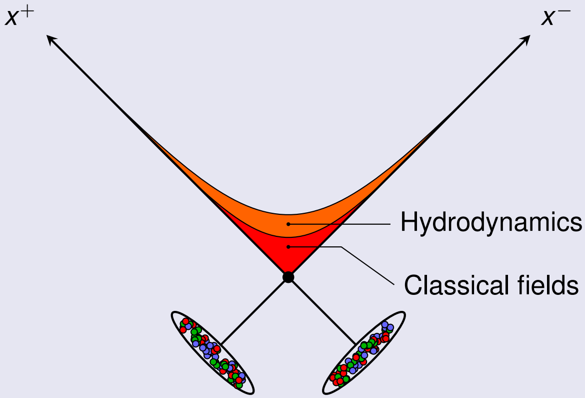
Heavy Ion Collision



Thomas EPELBAUM

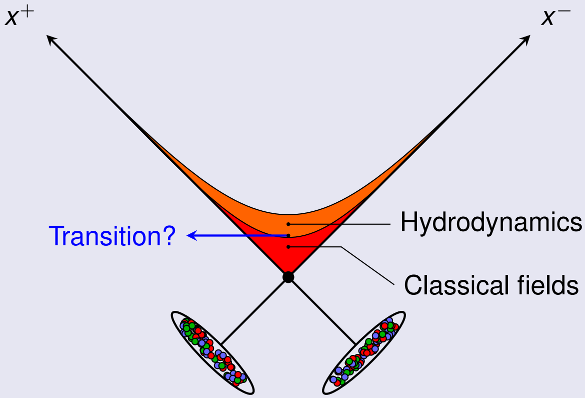
The general picture

Heavy Ion Collision



The general picture

Heavy Ion Collision



The general picture

Hydro prerequisites

- Thermal equilibrium (Equation of state, BOSE-EINSTEIN distribution fonction...)
- Initial conditions: energy density, pressure
- Transport coefficients: viscosity...

Theoretical framework

- Color Glass Condensate (CGC) semi classical effective theory (unphysical momentum cutoff Λ between color fields and classical sources)
- JIMWLK equation (renormalization group equation for the evolution with Λ)

1 Resummation scheme

2 Fixed volume
Toy model
Energy-momentum tensor
Distribution function
Bose-Einstein Condensation

3 Expanding volume
The model
Numerical results

Resummation formula [GELIS, LAPPI, VENUGOPALAN (2008)]

Secular divergences

- LO $\mapsto$ Finite results, but no equilibration
- NLO $\mapsto$ Secular divergences $\mapsto$ NLO $>$ LO

Resummation formula [GELIS, LAPPI, VENUGOPALAN (2008)]

Secular divergences

- LO $\mapsto$ Finite results, but no equilibration
- NLO $\mapsto$ Secular divergences $\mapsto$ NLO $>$ LO

Energy-momentum tensor at LO

$$T^{\mu\nu}_{\text{LO}} = (\partial^\mu \varphi)(\partial^\nu \varphi) - g^{\mu\nu}\mathcal{L}(\varphi)\,,$$

$$\partial_\mu \frac{\delta \mathcal{L}}{\delta(\partial_\mu \varphi)} = \frac{\delta \mathcal{L}}{\delta \varphi}$$

$\varphi = \varphi_0$ at t_0

Resummation formula [Gelis, Lappi, Venugopalan (2008)]

Energy-momentum tensor at NLO

$$T^{\mu\nu}_{\text{NLO}} = \widehat{O}\; T^{\mu\nu}_{\text{LO}}[\varphi_0]$$

$$\widehat{O} = \frac{1}{2} \int_{u,v\in\Sigma} G(u,v) \frac{\delta}{\delta\varphi_0(u)} \frac{\delta}{\delta\varphi_0(v)}$$

Resummation formula [GELIS, LAPPI, VENUGOPALAN (2008)]

Resummed Energy-momentum tensor

$$T^{\mu\nu}_{\text{resum}} = e^{\widehat{O}}\, T^{\mu\nu}_{\text{LO}}[\varphi_0] = T^{\mu\nu}_{\text{LO}} + T^{\mu\nu}_{\text{NLO}} + \cdots$$

$$\widehat{O} = \frac{1}{2} \int_{u,v\in\Sigma} G(u,v) \frac{\delta}{\delta\varphi_0(u)} \frac{\delta}{\delta\varphi_0(v)}$$

Resummation formula [Gelis, Lappi, Venugopalan (2008)]

Resummed Energy-momentum tensor

$$T^{\mu\nu}_{\text{resum}} = e^{\widehat{O}}\, T^{\mu\nu}_{\text{LO}}[\varphi_0] = T^{\mu\nu}_{\text{LO}} + T^{\mu\nu}_{\text{NLO}} + \cdots$$

$$\widehat{O} = \frac{1}{2} \int_{u,v\in\Sigma} G(u,v) \frac{\delta}{\delta\varphi_0(u)} \frac{\delta}{\delta\varphi_0(v)}$$

Equivalent formulation

$$e^{\frac{\gamma}{2}\partial_x^2} f(x) = \int_{-\infty}^{+\infty} \frac{\mathrm{d}z}{\sqrt{2\pi\gamma}}\, e^{-\frac{z^2}{2\gamma}}\, f(x+z)$$

Resummation formula [GELIS, LAPPI, VENUGOPALAN (2008)]

Resummed Energy-momentum tensor

$$T^{\mu\nu}_{\text{resum}} = e^{\widehat{O}}\, T^{\mu\nu}_{\text{LO}}[\varphi_0] = T^{\mu\nu}_{\text{LO}} + T^{\mu\nu}_{\text{NLO}} + \cdots$$

$$\widehat{O} = \frac{1}{2}\int_{u,v\in\Sigma} G(u,v)\frac{\delta}{\delta\varphi_0(u)}\frac{\delta}{\delta\varphi_0(v)}$$

Equivalent formulation

$$T^{\mu\nu}_{\text{resum}} = \int [Da(u)]\, e^{-\frac{1}{2}\int_{u,v\in\Sigma} a(u)\, G^{-1}(u,v)\, a(v)}\; T^{\mu\nu}_{\text{LO}}[\varphi_0 + a]$$

- Solve the EOM for the initial condition φ_0 with random gaussian fluctuations a on top of it.
- Semi-classical calculation that takes into account some quantum corrections.
- Functional integral done by Monte Carlo sampling.

1 Resummation scheme

2 Fixed volume

- Toy model
- Energy-momentum tensor
- Distribution function
- Bose-Einstein Condensation

3 Expanding volume

- The model
- Numerical results

2 Fixed volume

Toy model

Energy-momentum tensor

Distribution function

Bose-Einstein Condensation

Scalar field theory

Lagrangian of the theory

$$\mathcal{L}(\phi) = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \underbrace{\frac{g^2}{4!}\phi^4}_{V(\phi)} + J\phi$$

where

$$J \propto \theta(-x^0)$$

Why do we use this model?

- Scale invariance in $3+1$ dimensions
- Parametric resonance
- A lot simpler!

Form of the solution

Initial condition of the EOM

$$\phi_{\rm i}(t,\boldsymbol{x}) = \varphi_0(\boldsymbol{x}) + \sum_{\boldsymbol{k}} \mathrm{Re}\left[c_{\boldsymbol{k}}\, e^{i\omega_{\boldsymbol{k}} t}\, v_{\boldsymbol{k}}(\boldsymbol{x})\right]$$

with

$$\left[-\Delta + V''(\varphi_0)\right] v_{\boldsymbol{k}}(\boldsymbol{x}) = \omega_{\boldsymbol{k}}^2 v_{\boldsymbol{k}}(\boldsymbol{x})$$
$$\langle c_{\boldsymbol{k}}\ c_{\boldsymbol{l}}^*\rangle = \delta(\boldsymbol{k}-\boldsymbol{l})$$

2 Fixed volume

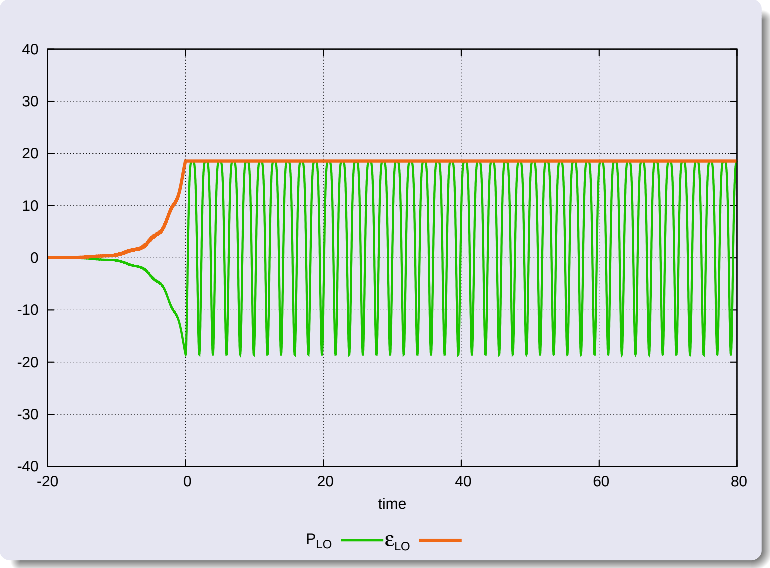
Toy model

Energy-momentum tensor

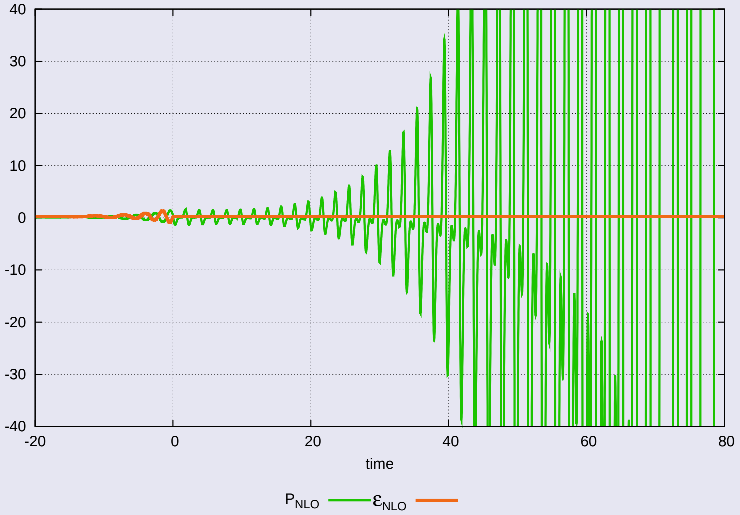
Distribution function

Bose-Einstein Condensation

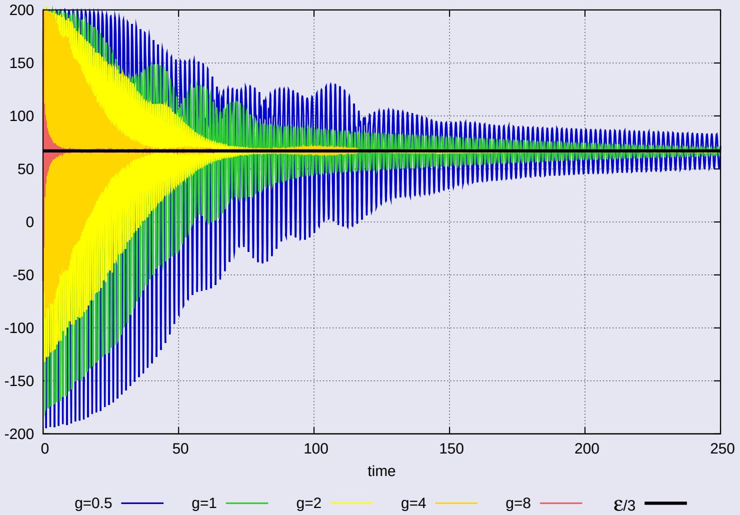
$T^{\mu\nu}_{\mathbf{LO}}$ [DUSLING, TE, GELIS, VENUGOPALAN (2010)]



$T^{\mu\nu}_{\mathrm{NLO}}$: secular divergences



$T^{\mu\nu}_{\mathrm{resum}}$: Pressure equilibration



2 Fixed volume

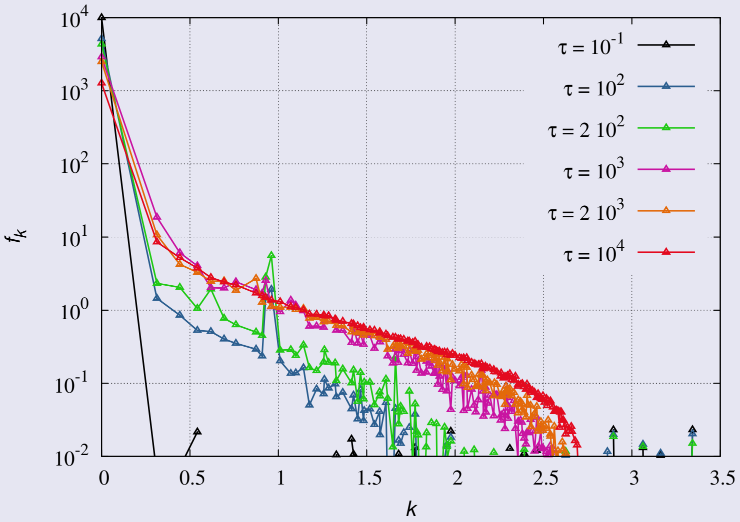
Toy model

Energy-momentum tensor

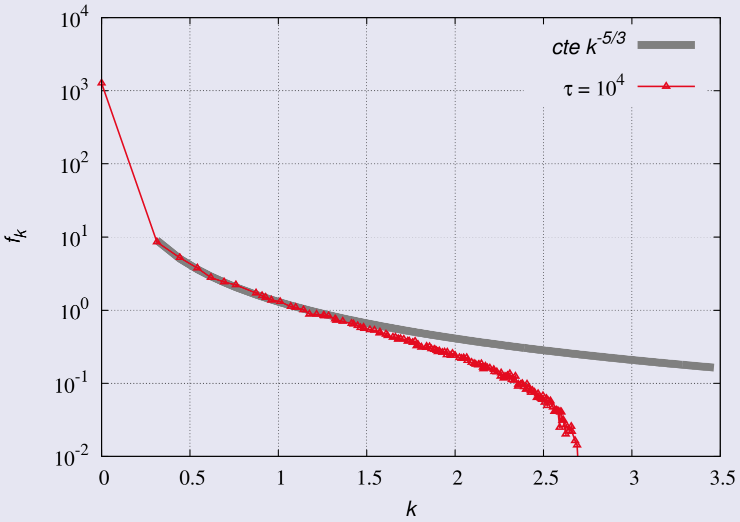
Distribution function

Bose-Einstein Condensation

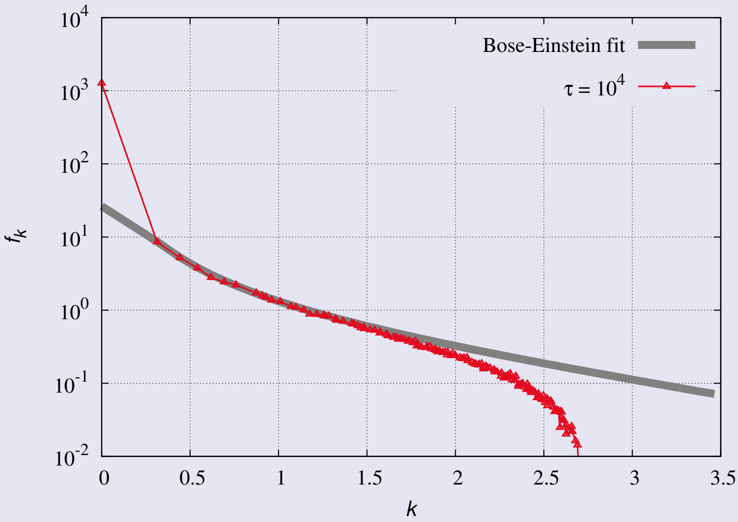
Time evolution of the occupation number [TE, GELIS (2011)]



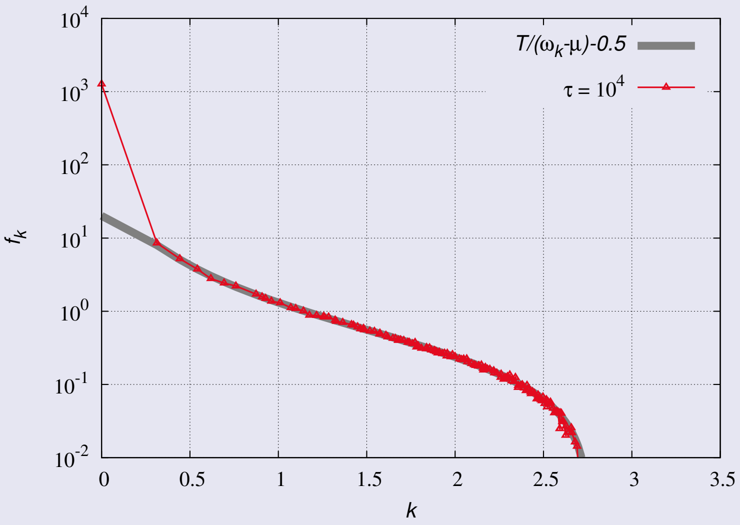
Kolmogorov scaling at late times?



BOSE-EINSTEIN equilibrium distribution?



"Classical" equilibrium distribution



2 Fixed volume

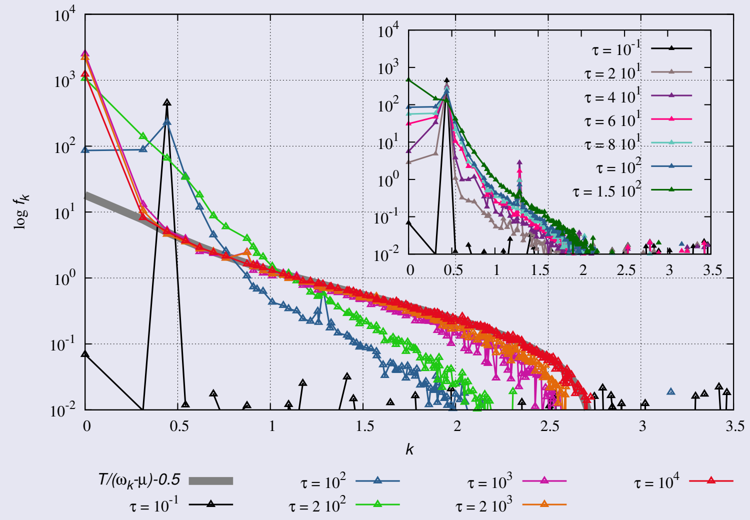
Toy model

Energy-momentum tensor

Distribution function

Bose-Einstein Condensation

Non-zero initial mode: $\varphi_0 \sim \cos \boldsymbol{k}.\boldsymbol{x}$



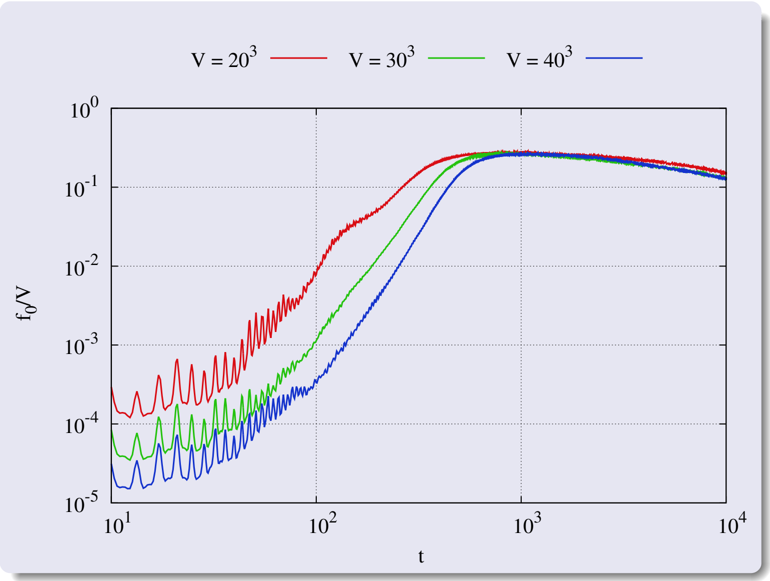
Evolution of the condensate

$$f_{\boldsymbol{k}} = \frac{T}{\omega_{\boldsymbol{k}} - \mu} - \frac{1}{2} + n_0 \delta(\boldsymbol{k})$$

implies

$$\frac{f_0}{V} = \text{cte}$$

Evolution of the condensate



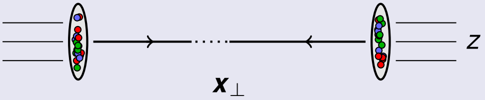
3 Expanding volume

The model

Numerical results

Scalar field theory

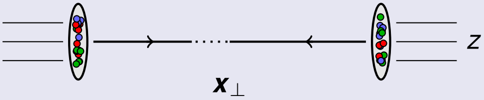
Adapted coordinate system to describe a Heavy Ion Collision?



System boost invariant in z direction

Scalar field theory

Adapted coordinate system to describe a Heavy Ion Collision?



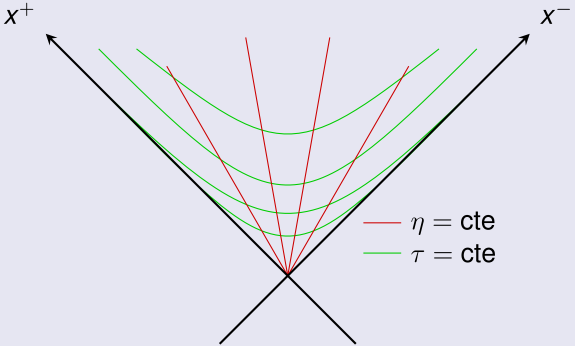
System boost invariant in z direction

Proper time/rapidity coordinate system

$$\eta = \frac{1}{2} \ln \frac{t+z}{t-z} \qquad \tau = \sqrt{t^2 - z^2}$$

Scalar field theory

Proper time/rapidity coordinate system

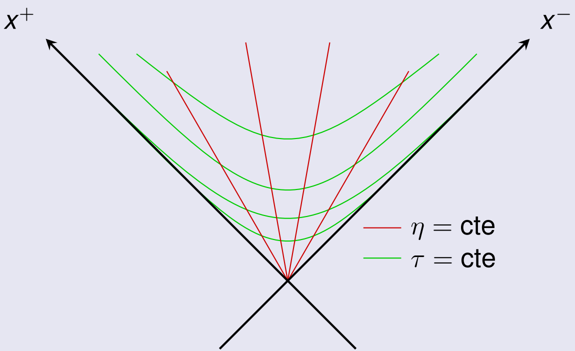


Proper time/rapidity coordinate system

$$\eta = \frac{1}{2} \ln \frac{t+z}{t-z} \qquad \tau = \sqrt{t^2 - z^2}$$

Scalar field theory

Proper time/rapidity coordinate system

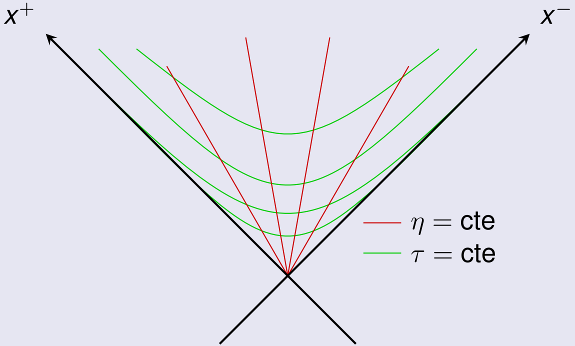


EOM for a boost-invariant field

$$\left[\frac{\partial^2}{\partial\tau^2} + \frac{1}{\tau}\frac{\partial}{\partial\tau} - \boldsymbol{\nabla}_\perp^2\right]\varphi + V'(\varphi(\tau, \boldsymbol{x}_\perp)) = 0$$

Scalar field theory

Proper time/rapidity coordinate system



EOM for a small fluctuation a

$$\left[\frac{\partial^2}{\partial\tau^2} + \frac{1}{\tau}\frac{\partial}{\partial\tau} - \nabla_\perp^2 - \frac{1}{\tau^2}\frac{\partial^2}{\partial\eta^2} + V''(\varphi)\right] a(\tau,\eta,\boldsymbol{x}_\perp) = 0$$

Form of the solution

Initial condition of the EOM

$$\phi_\text{i}(\tau, \boldsymbol{x}_\perp, \eta) = \varphi_0(\boldsymbol{x}_\perp) + \sum_{\boldsymbol{k}_\perp, \nu} \text{Re}\left[c_{\boldsymbol{k}_\perp, \nu} H^{(2)}_{i\nu}\left(\omega_{\boldsymbol{k}_\perp}\tau\right) e^{i\nu\eta}\, v_{\boldsymbol{k}_\perp}(\boldsymbol{x}_\perp)\right]$$

with the Hankel functions $H^{(2)}_{i\nu}$ being the equivalent of the plane waves in the fixed volume case , and

$$\left[-\Delta_\perp + V''(\varphi_0)\right] v_{\boldsymbol{k}_\perp}(\boldsymbol{x}_\perp) = \omega^2_{\boldsymbol{k}_\perp} v_{\boldsymbol{k}_\perp}(\boldsymbol{x}_\perp)$$

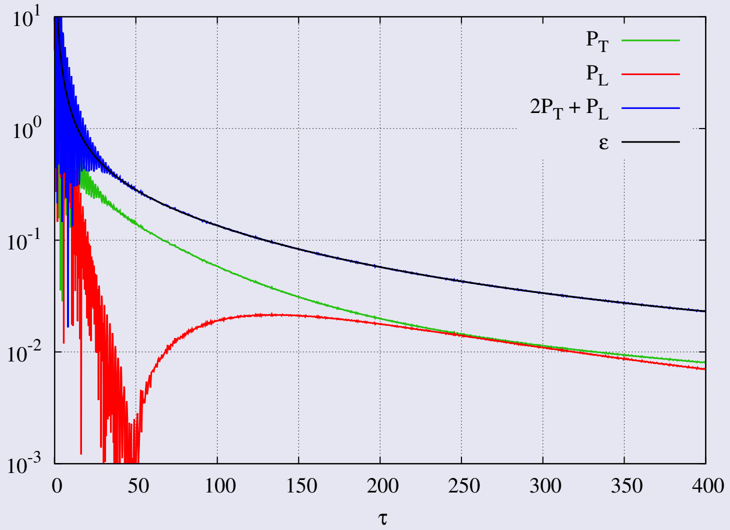
$$\left\langle c_{\boldsymbol{k}_\perp, \nu}\, c^*_{\boldsymbol{l}_\perp, \mu}\right\rangle = \delta(\boldsymbol{k}_\perp - \boldsymbol{l}_\perp)\delta(\nu - \mu)$$

3 Expanding volume

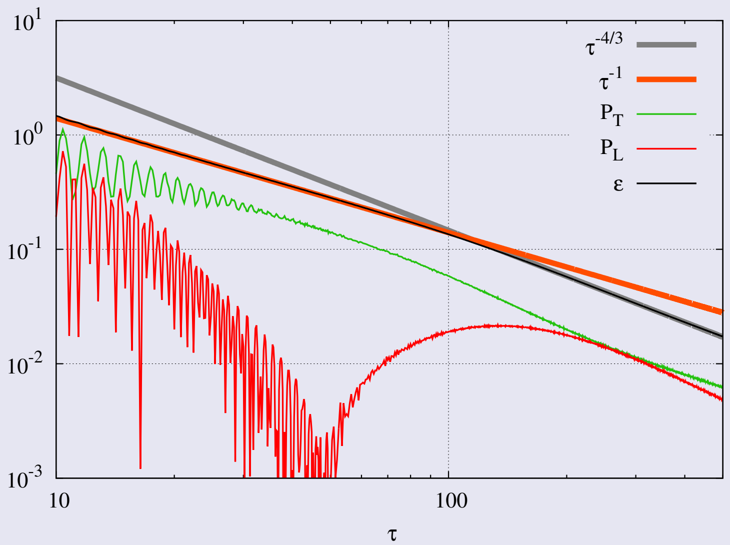
The model

Numerical results

$T^{\mu\nu}_{\text{resum}}$ [Work in progress]

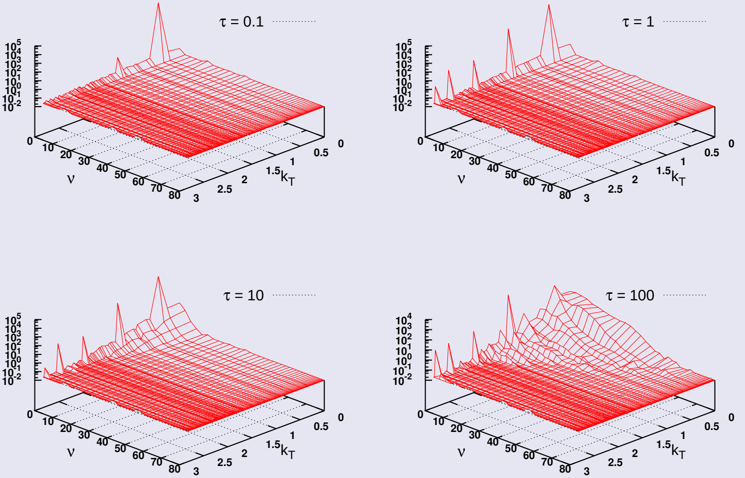


ϵ behaviour [Work in progress]

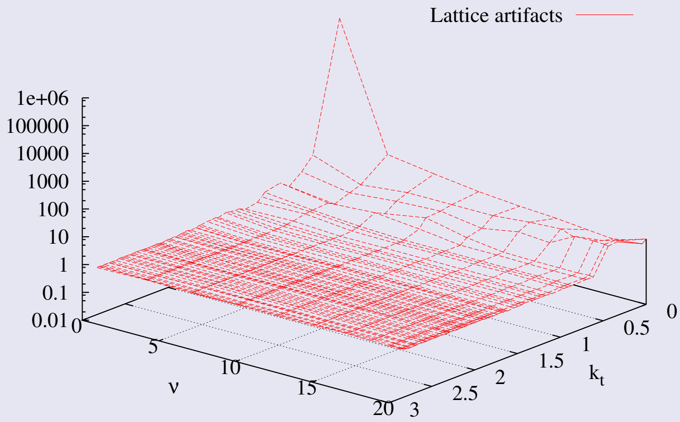


Bjorken Law: $\dot{\epsilon} + \frac{\epsilon + P_L}{\tau} = 0$

Evolution of the distribution function [Work in progress]

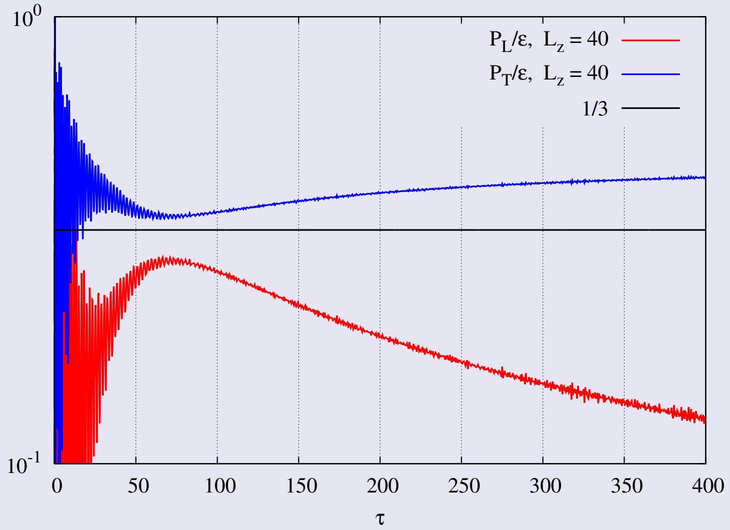


Lattice artifacts [Work in progress]

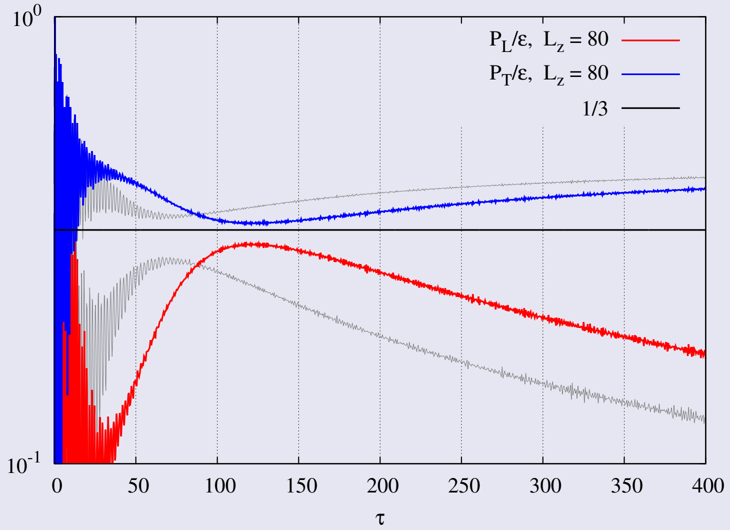


In thermal equilibrium $\nu \propto \tau^{\frac{2}{3}}$

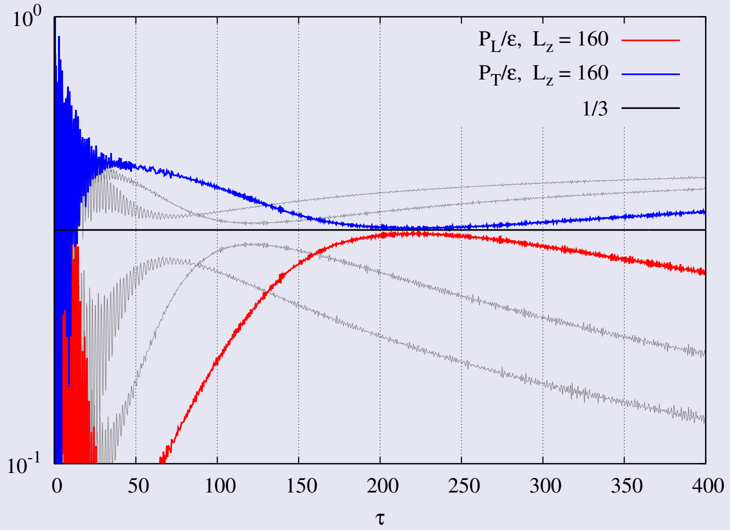
$T^{\mu\nu}_{\text{resum}}$: Time evolution of $\frac{P_{L,T}}{\epsilon}$ [Work in progress]



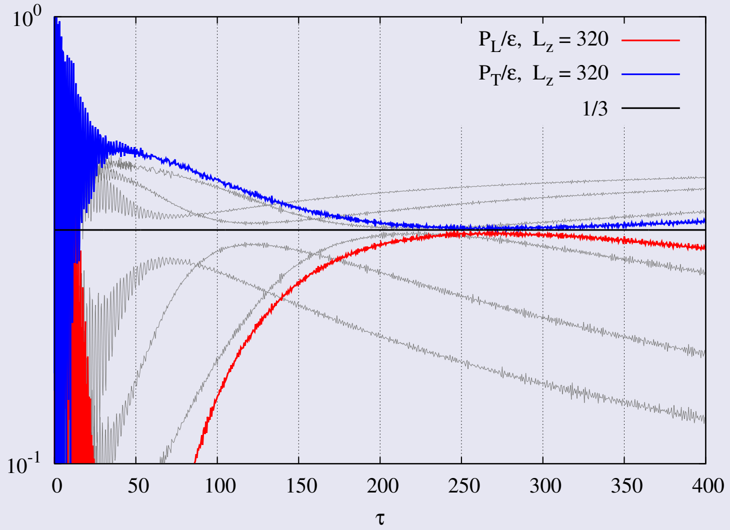
$T^{\mu\nu}_{\text{resum}}$: Time evolution of $\frac{P_{L,T}}{\epsilon}$ [Work in progress]



$T^{\mu\nu}_{\text{resum}}$: Time evolution of $\frac{P_{L,T}}{\epsilon}$ [Work in progress]



$T^{\mu\nu}_{\text{resum}}$: Time evolution of $\frac{P_{L,T}}{\epsilon}$ [Work in progress]



Conclusion

Principal results for the fixed volume

- Equation of state $\epsilon = 3P$
- $P_x = P_y = P_z$
- $f_{\boldsymbol{k}} \propto \frac{T}{\omega_{\boldsymbol{k}} - \mu} - \frac{1}{2}$ at late times
- BOSE-EINSTEIN condensate

Principal results for the exanding volume

- Equation of state
- BOSE-EINSTEIN condensate?
- $P_L = P_T$?