# The Effective Action Approach to the High Energy Limit of QCD and $\mathcal{N} = 4$ SYM

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**QCD Prospects for Future *ep* and *eA* Colliders**

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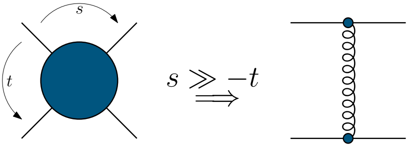
† *Based on work in collaboration with M. Angioni, G. Chachamis, M. Hentschinski & A. Sabio Vera*

# Outline

# Why to Study the High Energy Limit of QCD?




- Purely Phenomenological Interest
- Emergence of the Pomeron and Reggeization
- Insight into the Non-Perturbative Regime: Saturation
- Role of Conformal Symmetry. Integrability
- Similarity with $\mathcal{N} = 4$ SYM. Connection with AdS/CFT

(...) *"the small-$x$* [i.e. high-energy] *problem in QCD is, except for the understanding of confinement, the most interesting problem in QCD."*

*Mueller '90*

# The $S$-Matrix Approach

## The Idea

Amplitudes are extremely constrained by general properties of the $S$-matrix:

- Lorentz invariance
- Unitarity
- Analyticity & Crossing $\mathcal{A}_{a\bar{c}\to\bar{b}d}(s,t,u) = \mathcal{A}_{ab\to cd}(t,s,u)$

## The Tools

Partial Wave Expansion

$$\mathcal{A}_{a\bar{c}\to\bar{b}d}(s,t) = \sum_{\ell=0}^{\infty} a_\ell(s) P_\ell\left(1+\frac{2s}{t}\right)$$

Sommerfeld-Watson Transform

$$\mathcal{A}(s,t) = \frac{1}{2i}\oint d\ell(2\ell+1)\frac{a(\ell,t)}{\sin\pi\ell} P_\ell\left(1+\frac{2s}{t}\right)$$

## The Result

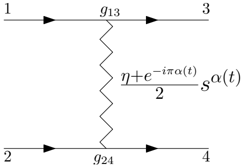
Each pole $\alpha(t)$ of the partial wave amplitude gives a contribution

$$\mathcal{A}(s,t) \xrightarrow{s\gg -t} \frac{\eta+e^{-i\pi\alpha(t)}}{2}\beta(t)s^{\alpha(t)}$$

# Regge Phenomenology

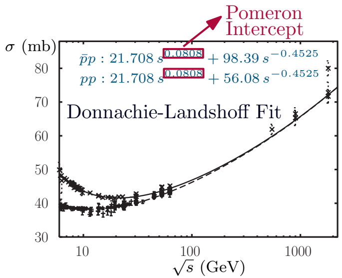
(...) *"Regge theory remains one of the great truths of particle physics."*

*Donnachie & Landshoff '90*




Regge-pole behaviour associated to *Reggeon* (~ bound state of particles with complex $t$-dependent spin) exchange

$$\sigma_{\text{tot}} \underset{s\to\infty}{\simeq} \frac{1}{s} \Im m\, \mathcal{A}(s, t=0) \sim \sum_i A_i s^{\alpha_i(0)-1}$$




**Regge trajectory**: $\alpha(t) = \alpha(0) + \alpha' t$

$t > 0$ Physical particles with distinct values of masses and spins

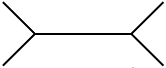
$t < 0$ Asymptotic Energy Behaviour

Rising Cross Section: Vacuum Quantum Numbers

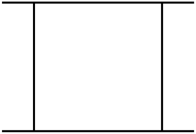
# Regge Poles in Field Theory

*All particles involved in hadronic processes have been found to lie in (approximately straight) Regge trajectories* $\Longrightarrow$ Elementary Particle Reggeization by Radiative Corrections

$\sim$ means *as* $s \to \infty$

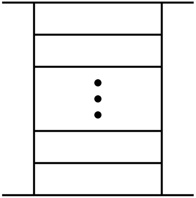


$$A^{(1)} \sim \frac{g^2}{s}$$



$$A^{(2)} \sim \frac{g^2}{s} K(t) \ln s$$

$$K(t) \sim g^2 \int \frac{d^2\mathbf{k}_\perp}{(\mathbf{k}_\perp^2 + m^2)[(\mathbf{k}_\perp + \mathbf{q}_\perp)^2 + m^2]}$$



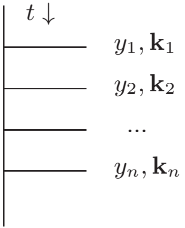
$$A^{(n)} \sim \frac{g^2}{s} \frac{(K(t) \ln s)^{n-1}}{(n-1)!}$$

$$A(s,t) = \sum_{n=1}^{\infty} A^{(n)} \sim \sum_{n=1}^{\infty} \frac{g^2}{s} \frac{(K(t) \ln s)^{n-1}}{(n-1)!} \simeq \frac{g^2}{s} e^{K(t) \ln s} \simeq g^2 s^{\alpha(t)}$$

# The Kinematics of the Ladder: Small-$x$ Regime

**Proposal**: Reggeized Gluon Built by $\ln s$-Enhanced Corrections to All Orders $(\alpha_s \ln s)^m$, $\alpha_s \ln s \sim 1$

*Regge limit* $s/|t| \gg 1 \iff$ *Small-*$x$ *in DIS; Large Rapidity Separation*

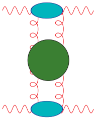



$$Y \sim \ln s$$

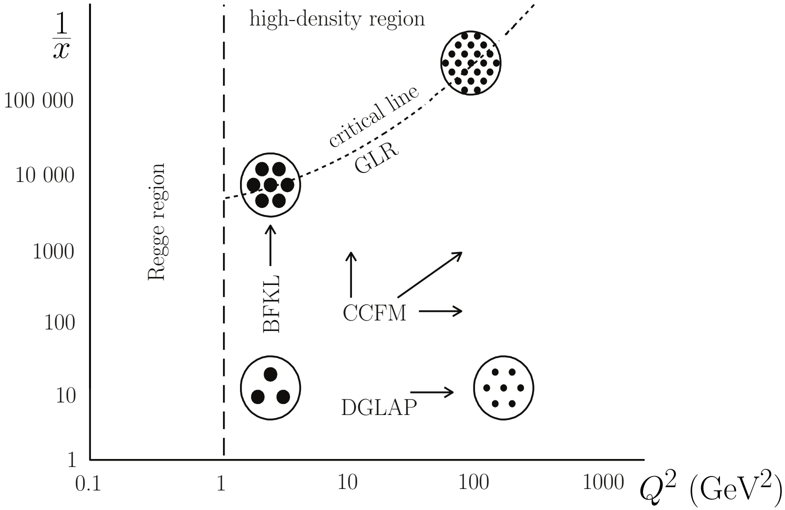
$$\mathbf{k}_i \sim \mathbf{k}_{i+1}$$

$$y_i << y_{i+1}$$

$$\alpha_s^n \int_0^Y dy_1 \int_0^{y_1} dy_2 ... \int_0^{y_{n-1}} dy_n \sim \frac{(\alpha_s Y)^n}{n!}$$



Compare with DGLAP, where strong ordering in transverse momenta holds


$\frac{1}{x}$
100 000
10 000
1000
100
10
1
0.1
1
10
100
1000
$Q^2$ (GeV$^2$)
high-density region
Regge region
critical line
GLR
BFKL
CCFM
DGLAP

# Non-Triviality of the Reggeization Ansatz

*Lipatov '76*

**Lipatov Ansatz** (from knowledge of first orders in perturbation theory)

***The effect of incorporating virtual corrections to all orders is reggeizing the intermediate $t$-channel gluons of the ladder:***

$$\frac{-i}{k_i^2} \to \frac{-i}{k_i^2}\left(-\frac{s_i}{k_i^2}\right)^{\alpha(-\mathbf{k}_i^2)}$$

- Reggeized gluon is an extremely non-trivial construction
- **Consistency**: Reggeizing intermediate gluons gives an overall reggeized-gluon exchange in $t$-channel when projecting in the octet
- **Bootstrap**: Gluon reggeization compatible with an infinity of non-linear constraints coming from $s$-channel unitarity

# BFKL Equation. Emergence of the (Hard) Pomeron

- **Pomeron** (singlet) exchange emerges as a **bound state of two reggeized gluons**
- Lipatov's Ansatz gives the amplitude $\mathcal{A}_{2\to n+2}$ in closed form
- Then the amplitude for pomeron exchange can be reconstructed **using unitarity** and summing in the number of steps of the ladder
- Mellin transforming (working with partial wave amplitudes) phase space integrations are disentangled and the **pomeron amplitude** is given as the **solution of an integral equation**

*Fadin, Kuraev & Lipatov '75,'76,'77*
*Lipatov '76*
*Balitskiĭ & Lipatov '78,'79*

## BFKL Equation

$$\omega f_\omega(\mathbf{k},\mathbf{k}') = \delta^2(\mathbf{k}-\mathbf{k}') + \int d^2\boldsymbol{\kappa}\,\mathcal{K}(\mathbf{k},\boldsymbol{\kappa}) f_\omega(\mathbf{k},\mathbf{k}')$$

$$\mathcal{K}(\mathbf{k},\boldsymbol{\kappa}) = 2\alpha(-\mathbf{k}^2)\delta^2(\mathbf{k}-\boldsymbol{\kappa}) + \frac{N_c\alpha_s}{\pi^2}\frac{1}{(\mathbf{k}-\boldsymbol{\kappa})^2}$$

$$\alpha(-\mathbf{k}^2) = \frac{N_c\alpha_s}{4\pi^2}\int d^2\boldsymbol{\kappa}\frac{-\mathbf{k}^2}{\boldsymbol{\kappa}^2(\boldsymbol{\kappa}-\mathbf{k})^2}$$

## Forward Solution and Total Cross Section

$$\sigma_{\mathrm{tot}}^{qq} = 4\alpha_s^2\mathcal{G}\iint d^2\mathbf{k}\,d^2\mathbf{k}'\,\frac{f(s,\mathbf{k},\mathbf{k}')}{\mathbf{k}^2\mathbf{k}'^2} \sim \frac{s^\lambda}{\ln s}$$

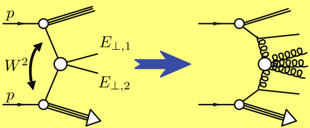
Pomeron Intercept $\lambda = \frac{N_c\alpha_s}{\pi}4\ln 2 \underset{\alpha_s\simeq 0{,}2}{\simeq} 0{,}5$

# The Problems with BFKL

- **Violation of Unitarity** (Froissart Bound): $\lambda > 0$
- **Diffusion** At asymptotic energies BFKL Green's function $f$ unavoidably receives contributions form non-perturbative region
- Convolution with PDFs in **hadron-pomeron vertices** limits predictability
- Collinear contamination. **DGLAP can mimic BFKL-like** behaviour for most processes $\Longrightarrow$ Need to suppress DGLAP evolution (use of similar transverse scales)
- **Need to Go Beyond LL Approximation**: to set the scales of validity and running

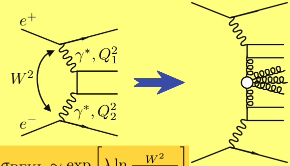
# Avoiding DGLAP Mimicking: Golden Signatures

**Dijets in $pp$ Collisions (Mueller-Navelet)**



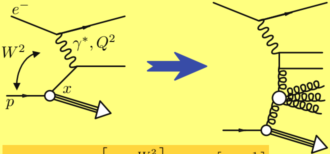

$\sigma_{\text{BFKL}} \sim \exp\left[\lambda \ln \frac{W^2}{E_{\perp,1}E_{\perp,2}}\right]$

**Total $\gamma^*\gamma^*$ Cross-Section**



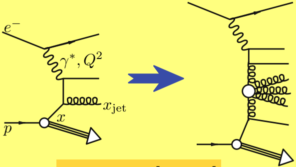

$\sigma_{\text{BFKL}} \sim \exp\left[\lambda \ln \frac{W^2}{\sqrt{Q_1^2 Q_2^2}}\right]$

**DIS: $F_2$ at Small-$x$**




$\sigma_{\text{BFKL}} \sim \exp\left[\lambda \ln \frac{W^2}{Q^2}\right] = \exp\left[\lambda \ln \frac{1}{x}\right]$

**Forward Jets in DIS**




$\sigma_{\text{BFKL}} \sim \exp\left[\lambda \ln \frac{x_{\text{jet}}}{x}\right]$

# Going Beyond Leading Order: Problems

**NLL BFKL Equation**

*Fadin & Lipatov '98*
*Ciafaloni & Camici '98*
*Kotikov & Lipatov '00*

$$\omega f(\mathbf{q}_1^2,\mathbf{q}_2^2,\omega)=\delta^2(\mathbf{q}_1^2-\mathbf{q}_2^2)+\int d^2\boldsymbol{\kappa}\,\mathcal{K}_{\rm NLL}(\mathbf{q}_1,\boldsymbol{\kappa})f(\boldsymbol{\kappa},\mathbf{q}_2,\omega)$$

(LL) Eigenfunctions

$$\langle\mathbf{q}|n,\nu\rangle=\frac{1}{\pi\sqrt{2}}(\mathbf{q}^2)^{i\nu-\frac{1}{2}}e^{in\vartheta}$$

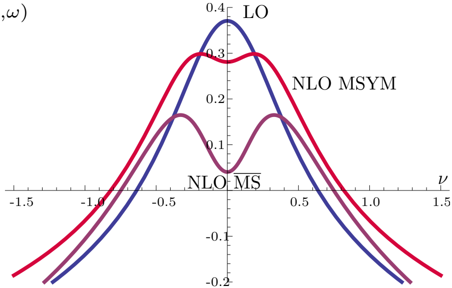
NLL 'Eigenvalues'

$$\langle n,\nu|\mathcal{K}|\nu',n'\rangle$$

$$=\bar{\alpha}_{s,\overline{\rm MS}}\Big[\chi_0\Big(|n'|,\frac{1}{2}+i\nu'\Big)+\bar{\alpha}_{s,\overline{\rm MS}}\chi_1\Big(|n'|,\frac{1}{2}+i\nu'\Big)$$

$$-\frac{\bar{\alpha}_{s,\overline{\rm MS}}\beta_0}{8N_c}\chi_0\Big(|n'|,\tfrac{1}{2}+i\nu'\Big)\Big\{-i\frac{\partial}{\partial\nu'}+i\frac{\partial}{\partial\nu}-2\ln\mu^2\Big\}$$

$$+i\frac{\bar{\alpha}_{s,\overline{\rm MS}}\beta_0}{8N_c}\frac{\chi_0\Big(|n'|,\frac{1}{2}+i\nu'\Big)}{\partial\nu'}\Big]\delta_{n,n'}\delta(\nu-\nu'),$$




$$\text{LO:}\bar{\alpha}_s\chi_0(n=0,\nu);\qquad \bar{\alpha}_s\equiv\frac{\alpha_s N_c}{\pi}$$

$$\text{NLO:}\bar{\alpha}_s\chi_0+\bar{\alpha}_s^2\chi_1(n=0,\nu)$$

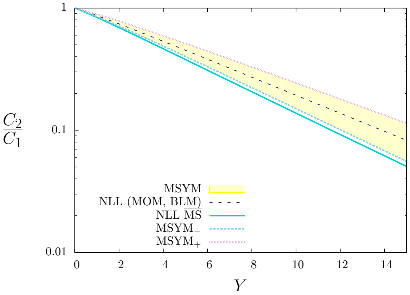
# Going Beyond Leading Order: Solutions

**Huge NLL Corrections**: Is after all $1/\ln s$ not a good expansion parameter?

**Situation is not hopeless: 3 vias for solution envisaged**

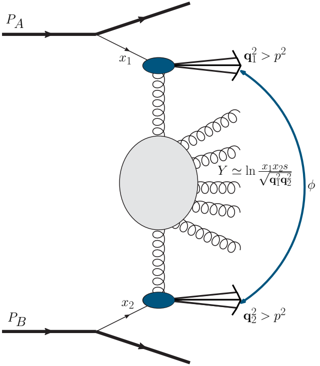
- Collinear Resummation to All Orders
- Rapidity Veto
- BLM Renormalization Scale Setting

# Effect of BLM Renormalization Scale Setting




[Angioni, Chachamis, JDM & Sabio Vera'11]

$$\frac{\mathcal{C}_m}{\mathcal{C}_n} = \frac{\langle\cos(m\phi)\rangle}{\langle\cos(n\phi)\rangle}$$




BLM results systematically closer than other renormalization schemes to $\mathcal{N} = 4$ SYM predictions for ratios of angular correlations of Mueller-Navelet jets

[Mueller & Navelet'87]; [Del Duca & Schmidt'94]; [Schwennsen & Sabio Vera'06]; [Marquet & Royon'06, '08]; [Colferai, Schwennsen, Szymanowski & Wallon'10]

# Lipatov's High-Energy Effective Action

## Effective Field Theory (EFT) and High-Energy Limit (HEL)

- Effective field theory: powerful tool for multi-scale problems
- Semihard processes in Regge limit: $s \gg -t \gg \mu^2$
- Unitarity directly restored in EFT
- Takes the reggeized gluon as the relevant degree of freedom: captures simplicity of HEL
- Very powerful to compute reggeon vertices for NLO and NNLO BFKL (*tree-level*) [Kniehl, Basin & Saleev'06; Braun, Lipatov, Salykin & Vyazovsky'11...]

★ Lipatov's EFT can be derived (at LO) by integrating out heavy modes

[Kirschner, Lipatov & Szymanowski'94,'95]

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[Lipatov'95]

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⇒ and now also available for computing at loop level!

[Hentschinski & Sabio Vera'11; Chachamis, Hentschinski, JDM & Sabio Vera'12]

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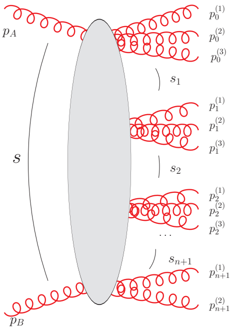
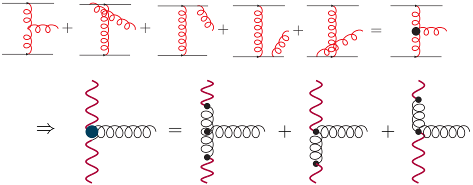
Generalized **Quasi-Multi-Regge Kinematics** (QMRK) [Fadin&Lipatov'89]




Clusters strongly ordered in rapidity: $y_0 \gg y_1 \gg \cdots \gg y_{n+1}$, $y_k = \frac{1}{2} \ln \frac{k^+}{k^-}$

- Strong rapidity ordering simplifies polarization tensor of $t$-channel reggeons: $g_{\mu\nu} \to \frac{1}{2}(n^+)_\mu (n^-)^\nu + \mathcal{O}(1/s)$
- Reggeized gluons couple to quarks and gluons through effective vertices local in rapidity: Effective vertex=Light-Cone Projection+Induced Contributions



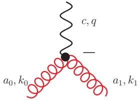
- Reggeon propagators are essentially transverse: $q_i^2 = -\boldsymbol{q}_i^2$

$p_a + p_b \to p_1 + p_2; \quad n^{+,-} = 2p_{a,b}/\sqrt{s}$,
$k = k^+ \frac{n^-}{2} + k^- \frac{n^+}{2} + \boldsymbol{k}$

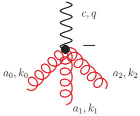
# Feynman Rules for Lipatov's Effective Action


$$= \Delta^{\nu_0 -}_{a_0 c} = -i\boldsymbol{q}^2 \delta^{a_0 c} (n^-)^{\nu_0},$$

[Antonov, Cherednikov, Kuraev & Lipatov'05]




$$= g\boldsymbol{q}^2 f^{a_0 a_1 c} \frac{1}{k_0^-} (n^-)^{\nu_0} (n^-)^{\nu_1},$$




$$= \Delta^{\nu_0 \nu_1 \nu_2 -}_{a_0 a_1 a_2 c} = ig^2 \boldsymbol{q}^2 \left( \frac{f^{a_2 a_1 a} f^{a_0 a c}}{k_2^- k_0^-} + \frac{f^{a_2 a_0 a} f^{a_1 a c}}{k_2^- k_1^-} \right) (n^-)^{\nu_0} (n^-)^{\nu_1} (n^-)^{\nu_2},$$


$$= \frac{i}{2\boldsymbol{q}^2}.$$

$$S_{\text{eff}} = S_{\text{QCD}} + S_{\text{ind}};$$

$$S_{\text{ind}} = \int d^4x \, \text{Tr} \left[ \left( W_+[v(x)] - \mathscr{A}_+(x) \right) \partial_\perp^2 \mathscr{A}_-(x) \right] + \int d^4x \, \text{Tr} \left[ \left( W_-[v(x)] - \mathscr{A}_-(x) \right) \partial_\perp^2 \mathscr{A}_+(x) \right];$$

$$W_\pm[v] = v_\pm \frac{1}{D_\pm} \partial_\pm = v_\pm - g v_\pm \frac{1}{\partial_\pm} v_\pm + \cdots$$

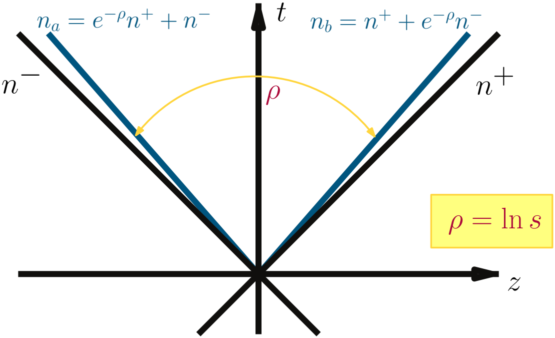
$\mathscr{A}_\pm$: reggeons, $v_\mu$: gluons

**Kinematical Constraints**

$\partial_\pm \mathscr{A}_\mp(x) = 0, \quad \sum_{i=0}^r k_i^\pm = 0$

- Reggeon fields invariant under *local* gauge transformations

# The Light-Cone Regularization




Tilting the light-cone vectors appearing in the induced vertices

[Collins & Soper'81,'82]
[Korchemsky & Radyushkin'87]
[Hentschinski & Sabio Vera'11]

- Regularization needed to make sense of non-local operators $\frac{1}{\partial_\pm}$
- Rest of divergences managed with dimensional regularization
- $\rho \to \infty$ in the high-energy limit
- Pole prescription: principal value [Hentschinski'11]

# The Gluon Regge Trajectory

**Amplitudes in Multi-Regge Kinematics: Reggeization**

$\mathcal{M}^{\text{LLA}}_{2\to2+n} = \mathcal{M}^{\text{tree}}_{2\to2+n} \prod_{i=1}^{n+1} s_i^{\omega(t_i)}$ [Lipatov'76] $\omega(t)$ =Regge Trajectory

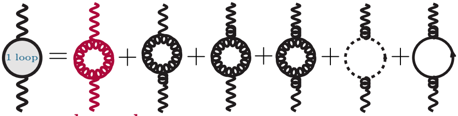
- Regge trajectory describes virtual contributions in the BFKL equation [Fadin, Kuraev & Lipatov'75,'77]; [Balitsky & Lipatov'78]

$$\omega \tilde{f}_\omega(\boldsymbol{q}_1, \boldsymbol{q}_2) = \delta^2(\boldsymbol{q}_1 - \boldsymbol{q}_2) + \int d^2\boldsymbol{\kappa}\, \mathcal{K}(\boldsymbol{q}_1, \boldsymbol{\kappa}) \tilde{f}_\omega(\boldsymbol{\kappa}, \boldsymbol{q}_2)$$

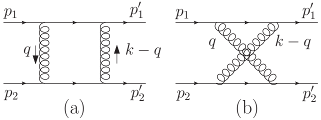
**One-Loop Trajectory**

***Effective Action Diagrams***

Enhanced means $\propto \rho = \ln s$

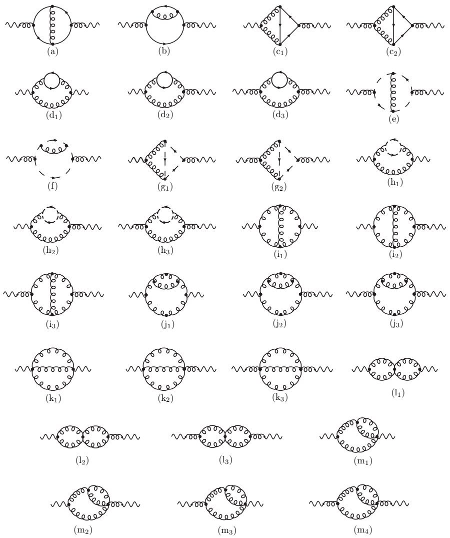



***Usual QCD Diagrams***




(+ non-enhanced contributions)

## 2-Loop Effective Action Diagrams




The **Regge trajectory** is an extremely important quantity:

- BFKL equation controls asymptotic rising of cross-sections at very high energies
- Includes as a piece the **cusp anomalous dimension**

$$\omega(-t) = \frac{1}{2}\int_{-t}^{\mu_{\rm IR}^2} \frac{d\boldsymbol{k}^2}{\boldsymbol{k}^2}\Gamma_{\rm cusp}(\alpha_s(\boldsymbol{k}^2)) + \Gamma_R(\alpha_s(-t)) + \text{poles in } (1/\epsilon_{\rm IR})$$

It is known

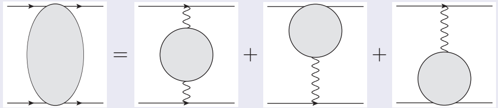
- at NLO in QCD

  [Fadin, Fiore & Kotsky'96]
- to all orders in $\mathcal{N}=4$ SYM

  [Kotikov & Lipatov'00; Beisert, Eden & Staudacher'07; Bartels, Lipatov & S.Vera'09]

## THE RECIPE to Compute the 2-Loop Gluon Trajectory $\omega^{(2)}$

1. Determine the high energy limit of the 2-loop parton-parton scattering amplitude by dropping terms not $\rho$-enhanced (remember, $\rho = \ln s$)
2. Subtract non-local contributions to reggeized gluon self-energy to avoid double-counting
3. Divide by the tree-level HEL result
4. Remove all terms corresponding to combinations of 1-loop trajectory and 1-loop impact factors (reggeon-parton scattering vertices)

$i\mathcal{M}^{(1)}_{q_a q_b \to q_1 q_2} =$ 

Cancellation of $\rho$-divergences in full amplitude [High-Energy Factorization]

5. Remove a term $\frac{1}{2}\ln^2(s/s_0)[\omega^{(1)}(t)]^2$ (logs arise from $s^\omega = 1 + \omega \ln s + \frac{1}{2!}\omega^2 \ln^2 s + \cdots, \quad \omega(t) = \omega^{(1)}(t) + \omega^{(2)}(t) + \cdots$)

**This is an example of a general procedure**

# The Subtraction Procedure

- In Lipatov's action, interactions between partons and reggeons assumed to occur at $\Delta y < \eta \ll \ln s$ (locality in rapidity)
- QMRK clusters connected by reggeon propagators (non-local in rapidity)

**However, when considering loops...**

☐ The constraint has to be enforced in the momentum integrals, e.g. with a cutoff in rapidity [Bartels, Hentschinski & Lipatov'07]

☐ Alternatively, subtract non-local contributions, mediated by a reggeized gluon. In our case...

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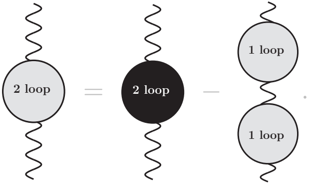

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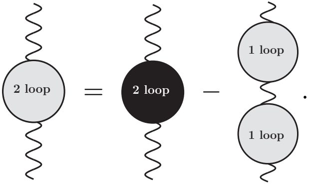
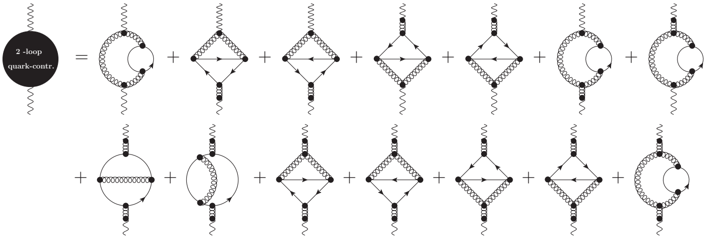


# 2-Loop Gluon Trajectory: Quark Part

***Contributions to Unsubtracted 2-Loop Gluon Self-Energy***




(only first diagram $\rho$-enhanced)

***Subtractions***

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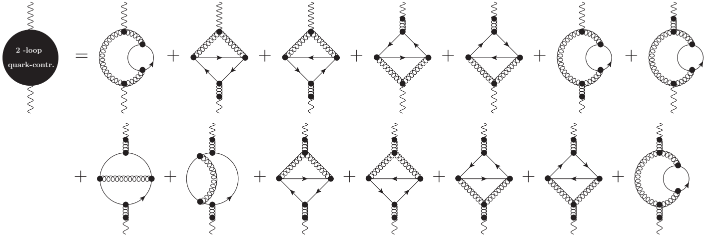


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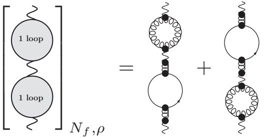

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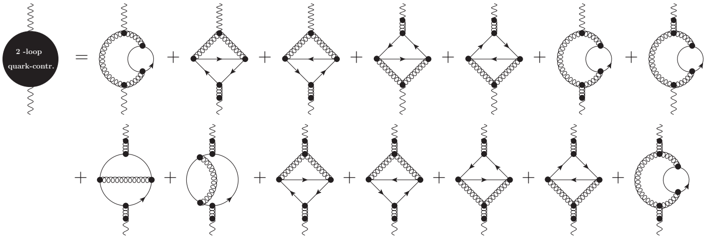


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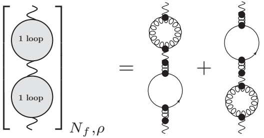

# Result of the Computation

## Exact Agreement with Previous Two-Loop Computation

[Fadin, Fiore & Kotsky'96; Fadin, Fiore & Quartarolo'96; Blümlein, Ravindran & van Neerven'98; Del Duca & Glover'01]

$$\omega_{n_f}^{(2)}\left(\epsilon,\frac{\boldsymbol{q}^2}{\mu^2}\right)=\bar{g}^4\left(\frac{\boldsymbol{q}^2}{\mu^2}\right)^{2\epsilon}\frac{4n_f}{\epsilon N_c}\frac{\Gamma^2(2+\epsilon)}{\Gamma(4+2\epsilon)}$$

$$\times\left[\frac{2\Gamma^2(1+\epsilon)}{\epsilon\,\Gamma(1+2\epsilon)}-\frac{3\Gamma(1-2\epsilon)\Gamma(1+\epsilon)\Gamma(1+2\epsilon)}{\epsilon\,\Gamma^2(1-\epsilon)\Gamma(1+3\epsilon)}\right];$$

$$\bar{g}^2=\frac{g^2N_c\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}},\quad d=4+2\epsilon$$

And now the computation of the rest of the trajectory is almost done... [Chachamis, Hentschinski, JDM & Sabio Vera, to appear soon]

- More difficult contributions, require a more powerful strategy
  1. Reduction to master integrals using integration by parts codes e.g. [Smirnov & Smirnov'08]
  2. Obtention of Mellin-Barnes representations and computation of residues relevant in Regge limit [Smirnov'99]
- General powerful procedure, which can be automatized

# Conclusions

☐ Lipatov's effective action is a very powerful tool for computations in the high-energy limit
☐ The proposed regularization-subtraction procedure gives a systematic way to employ this action for loop computations
☐ Quark piece for 2-loop trajectory: exact agreement. Agreement also found for 1-loop jet vertex

**Yet to be done...**

★ Check further the procedure (e.g. computation of gluon jet vertex)
★ Automatization of the computation