Factorization of Hard Processes

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Orsay

- I. Origins of factorization: the parton model for DIS and elastic scattering
- II. Quantum field theory: finding out where perturbation theory works
  - Renormalization, the running coupling and the search for infrared safety
  - Landau equations, physical pictures and power counting
  - Fixed-angle elastic amplitudes in ϕ^4: collinear singularities
  - Fixed-angle elastic amplitudes in gauge theories: soft singularities and longitudinal polarizations
  - Ward identities, gauge links and matrix elements

- **III. Factorization and evolution**
  - **Unitarity, infrared safety, and jets**
  - **Factorization and evolution in DIS**
  - **Heuristics of hadron-hadron scattering**
  - **The nature of factorization proofs**
- **IV. Hadron-hadron inclusive cross sections and exclusive amplitudes**
  - **Drell-Yan inclusive and Q_T cross sections: collinear and TMD factorization, Sudakov resummation**
  - **Crossed TMD factorization and its limitations**
  - **Factorizations for fixed-angle and deeply-virtual Compton scattering**
  - **Generalizations: what factorizes and what doesn't?**

The Context of QCD: "Fundamental Interactions"

- Electromagnetic
- + Weak Interactions ⇒ Electroweak
- + Strong Interactions (QCD) ⇒ Standard Model
- + ... = Gravity and the rest?
- QCD: A theory "off to a good start". Think of ...

  - $\vec{F}_{12} = -GM_1M_2\hat{r}/R^2 \Rightarrow$ elliptical orbits
    ... 3-body problem ...

  - $L_{\rm QCD} = \bar{q}\ \not{D} q - (1/4)F^2 \Rightarrow$ asymptotic freedom
    ... confinement ...

I. The Parton Model and Deep-inelastic Scattering

IA. Nucleons to Quarks

IB. DIS: Structure Functions and Scaling

IC. Getting at the Quark Distributions

ID. Classic Parton Model Extensions:
Fragmentation and Drell-Yan

Introduce concepts and results that predate QCD, led to QCD and were incorporated and explained by QCD.

IA. From Nucleons to Quarks

- **The pattern: nucleons, pions and isospin:**

$$\begin{pmatrix} p \\ n \end{pmatrix}$$

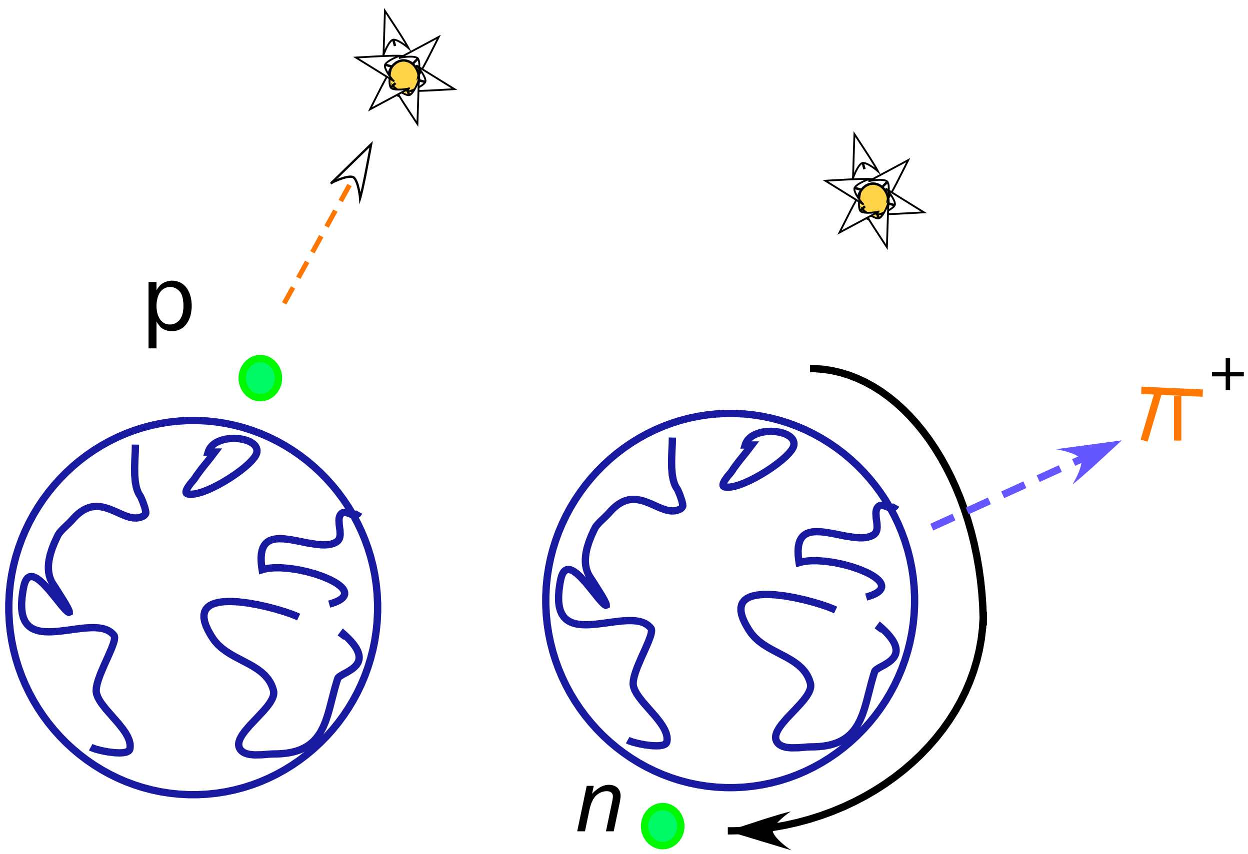
  - **p: m=938.3 MeV, $S = 1/2$, $I_3 = 1/2$**
  - **n: m=939.6 MeV, $S = 1/2$, $I_3 = -1/2$**

$$\begin{pmatrix} \pi^+ \\ \pi^0 \\ \pi^- \end{pmatrix}$$

  - **$\pi^\pm$: m=139.6 MeV, $S = 0$, $I_3 = \pm 1$**
  - **π^0: m=135.0 MeV, $S = 0$, $I_3 = 0$**

• **Isospin space . . .**

• **Globe with a "north star" set by electroweak interactions:**



Analog: the rotation group (more specifically, $SU(2)$).

- **Explanation: π, N common substructure: quarks (Gell Mann, Zweig 1964)**

- **spin $S = 1/2$, $I = 1/2$ (u, d) & $I = 0$ (s) with approximately equal masses (s heavier);**

$$\begin{pmatrix} u\ (Q = 2e/3, I_3 = 1/2) \\ d\ (Q = -e/3, I_3 = -1/2) \\ s\ (Q = -e/3, I_3 = 0) \end{pmatrix}$$

$$\pi^+ = (u\bar{d})\ , \quad \pi^- = (\bar{u}d)\ , \quad \pi^0 = \frac{1}{\sqrt{2}}\left(u\bar{u} + d\bar{d}\right)\ ,$$

$$p = (uud)\ , \quad n = (udd)\ , \quad K^+ = (u\bar{s})\ldots$$

This is the quark model

- Quark model nucleon has symmetric spin/isospin wave function (return to this later)
- Early success: μ_p/μ_n = -3/2 (good to %)
- And now, six: 3 'light' (u, d, s), 3 'heavy': (c, b, t)
- Of these all but t form bound states of quark model type.

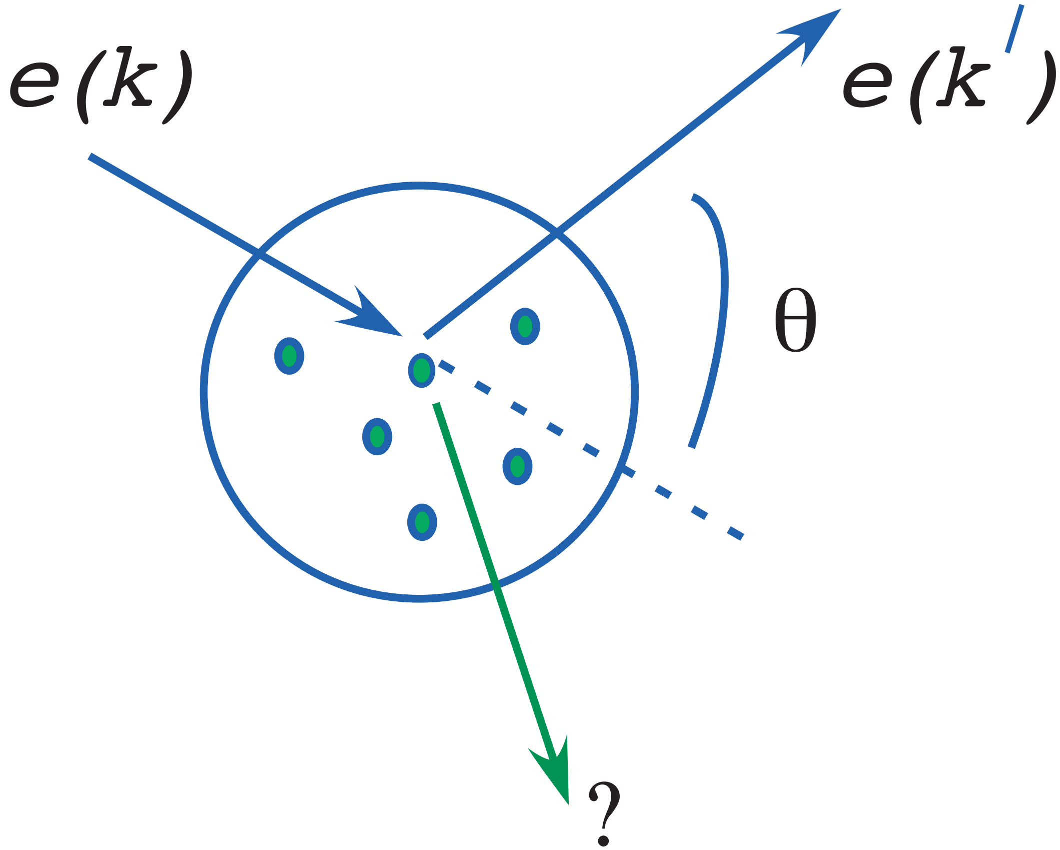
- **Quarks as Partons: “Seeing” Quarks.**

No isolated fractional charges seen (“confinement.”)

Can such a particle be detected? (SLAC 1969)

Look closer: do high energy electrons
bounce off anything hard? (‘Rutherford-prime’)

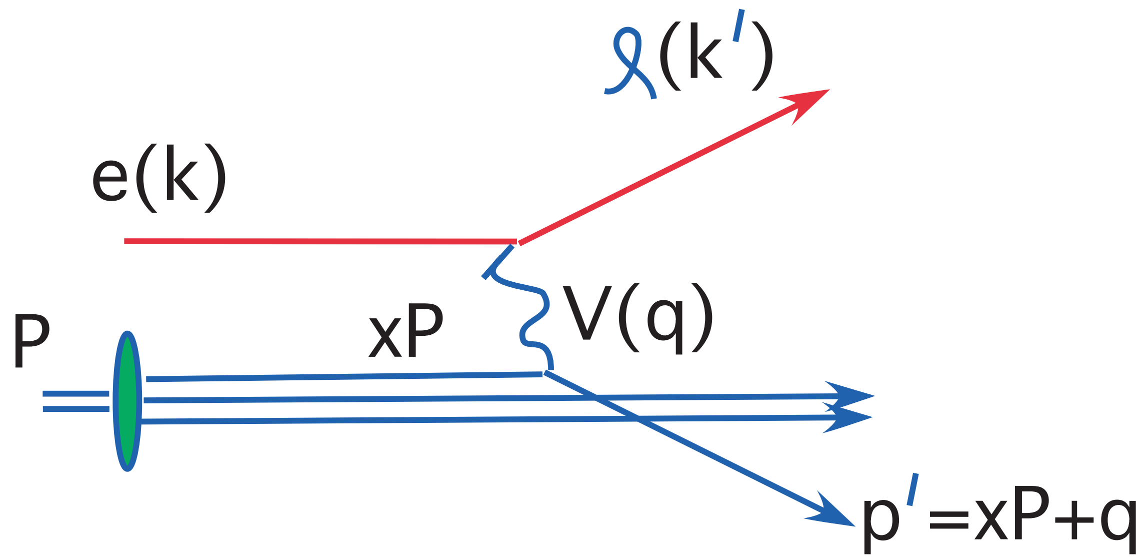
- So look for:



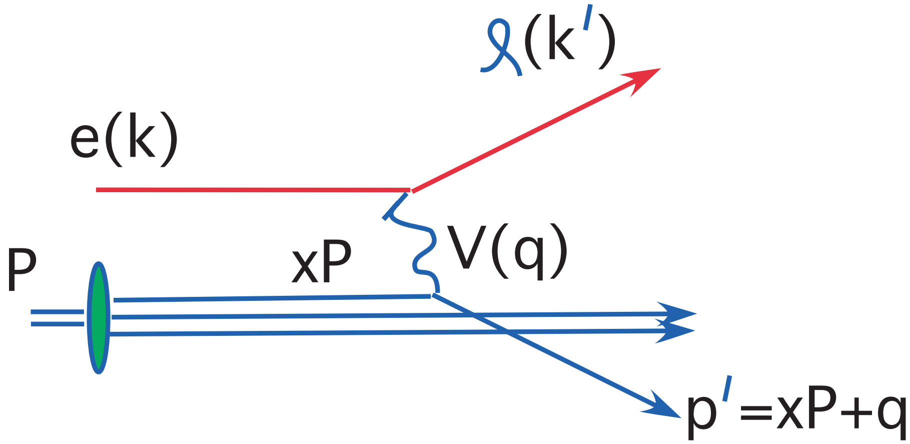
"Point-like' constituents.

The angular distribution gives information on the constituents.

Kinematics ($e + N(P) \to \ell + X$)



- $V = \gamma, Z_0 \Rightarrow \ell = e,\ \mu$, **"neutral current" (NC).**
- $V = W^-(e^-, \nu_e)$, $V = W^+(e^+, \bar{\nu}_e)$, **or** $e \to \mu$ **"charged current" (CC).**
- $W^2 \equiv (p+q)^2 \gg m^2_{\text{proton}}$: **Deep-inelastic scattering (DIS)**



$Q^2 = -q^2 = -(k-k')^2$ **momentum transfer.**

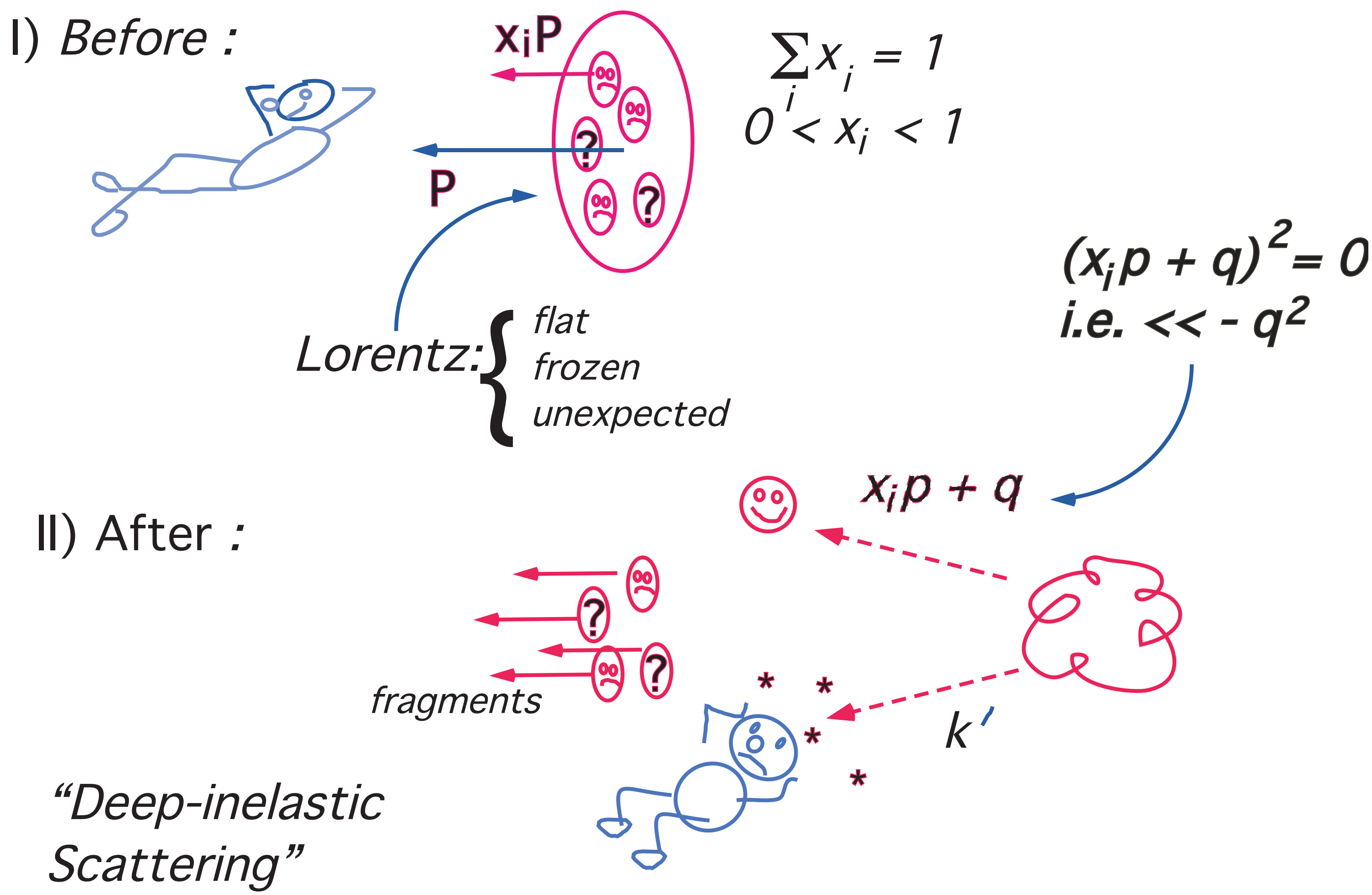
$x \equiv \frac{Q^2}{2p\cdot q}$ **momentum fraction (from** $p'^2 = (xp+q)^2 = 0$**).**

$y = \frac{p\cdot q}{p\cdot k}$ **fractional energy transfer.**

$W^2 = (p+q)^2 = \frac{Q^2}{x}(1-x)$ **squared final-state mass of hadrons.**

$$xy = \frac{Q^2}{S}$$

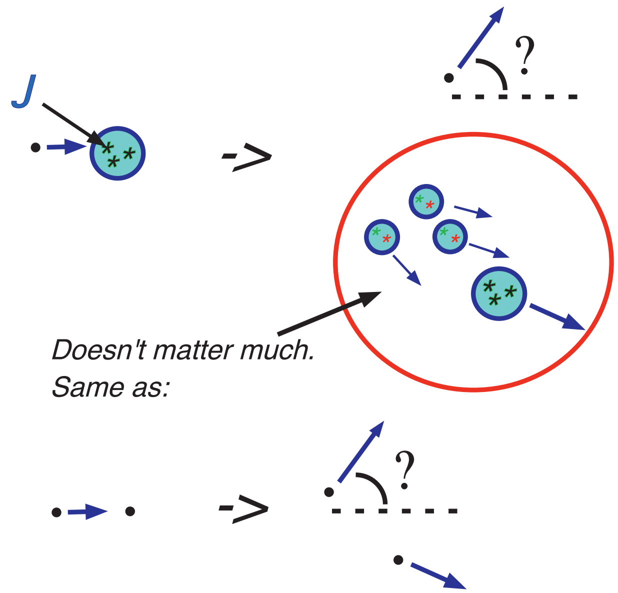
Parton Interpretation (Feynman 1969, 72)
Look in the electron's rest frame . . .



- **Surprise: scaling in inclusive deep inelastic scattering**

  In inclusive ep inelastic scattering

  Electron sees $\star\star$'s as spin-1/2 point particles



- **The "strong" force seems weak, almost irrelevant to the electron**

- **The "quark-parton" model: Ignore $\star\star$ interactions**

  – $\frac{d\sigma_{\text{ep inclusive}}(Q)}{dQ^2}$

  $\sim \frac{d\sigma_{\text{e}\star\to\text{e}\star}(Q)}{dQ^2}$ **x (probability to a find parton)**

- Basic Parton Model Relation: factorization

$$\sigma_{\rm eh}(p,q) = \sum_{\text{partons } a} \int_0^1 d\xi \; \hat{\sigma}^{\rm el}_{ea}(\xi p, q) \; \phi_{a/h}(\xi)\,,$$

- where: σ_{eh} is the cross section for
  $e(k) + h(p) \to e(k' = k - q) + X(p + q)$

- and $\hat{\sigma}^{\rm el}_{ea}(xp, q)$ is the elastic cross section for
  $e(k) + a(\xi p) \to e(k' - q) + a(\xi p + q)$ which sets
  $(\xi p + q)^2 = 0 \to \xi = -q^2/2p \cdot q \equiv x$.

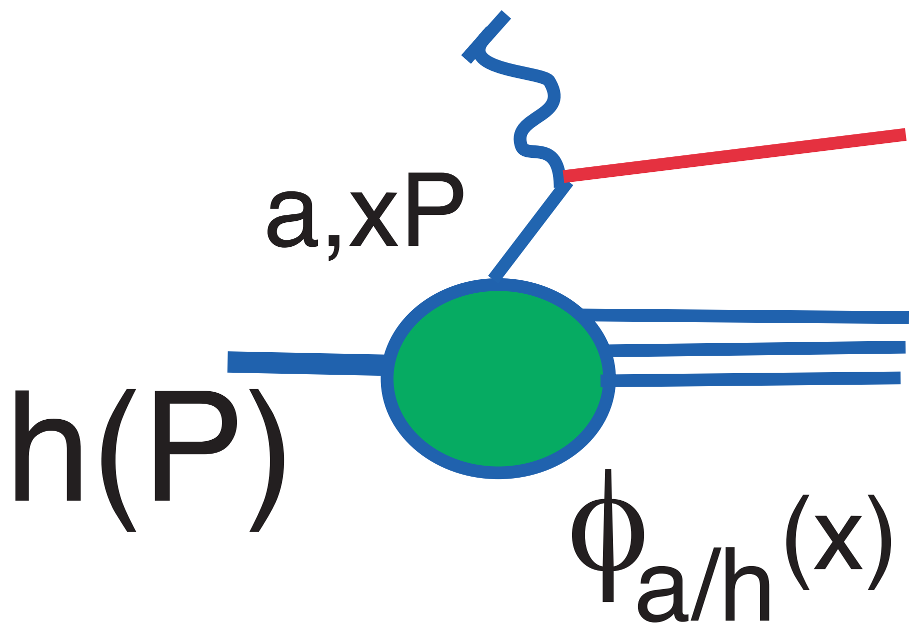
- and $\phi_{a/h}(x)$ is the distribution of parton a in hadron h,
  the "probability for a parton of type a to have momentum xp".
  Has a meaning independent of the details of the hard scattering.
  The hallmark of "factorization".

- **in words: Hadronic INELASTIC cross section is the sum of convolutions of partonic ELASTIC cross sections with the hadron's parton distributions.**

- **The nontrivial assumption: quantum mechanical incoherence of large-q scattering and the partonic distributions. Multiply probabilities without adding amplitudes.**

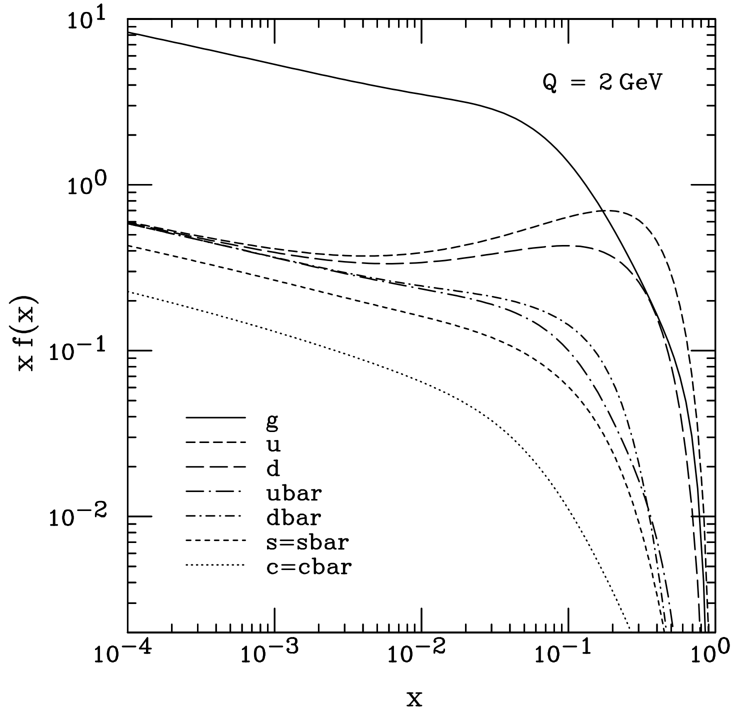
- **Heuristic justification: the binding of the nucleon involves long-time processes that do not interfere with the short-distance scattering. Later we'll see how this works in QCD.**

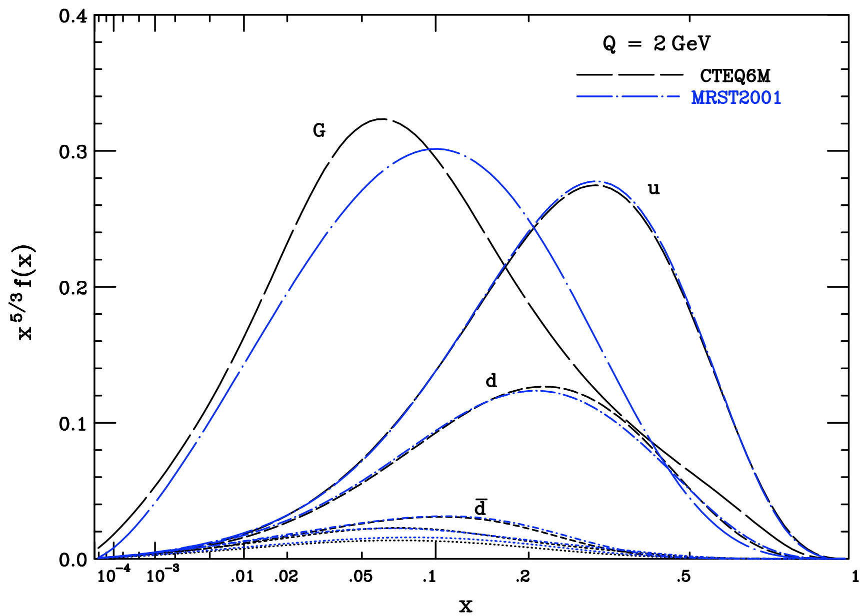
- **The familiar picture**



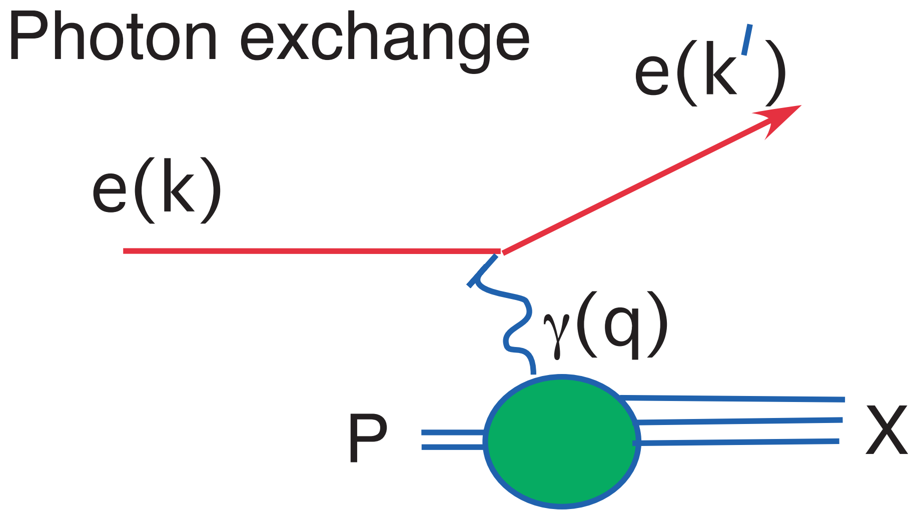
- **"QM incoherence" ⇔ no interactions between of the outgoing scattered quark and the rest.**

- **Two modern parton distribution sets at moderate momentum transfer (note different weightings with x):**





- **We'll see where these come from.**



$$A_{e+N\to e+X}(\lambda,\lambda',\sigma;q) = \bar{u}_{\lambda'}(k')(-ie\gamma_\mu)u_\lambda(k) \times \frac{-ig^{\mu\mu'}}{q^2} \times \langle X|\, eJ^{\text{EM}}_{\mu'}(0)\, |p,\sigma\rangle$$

- **Historically** an assuption that the photon couples to hadrons by point-like current operator. Now, built into the Standard Model.

- **The cross section:**

$$d\sigma_{\mathrm{DIS}} = \frac{1}{2^2}\frac{1}{2s}\frac{d^3k'}{(2\pi)^3 2\omega_{k'}} \sum_X \sum_{\lambda,\lambda',\sigma} |A|^2$$
$$\times\ (2\pi)^4\,\delta^4(p_X + k' - p - k)$$

In $|A|^2$, separate the known leptonic part from the "unknown" hadronic part:

- **The leptonic tensor:**

$$L^{\mu\nu} = \frac{e^2}{8\pi^2}\sum_{\lambda,\lambda'} (\bar{u}_{\lambda'}(k')\gamma^\mu u_\lambda(k))^*\,(\bar{u}_{\lambda'}(k')\gamma^\nu u_\lambda(k))$$
$$= \frac{e^2}{2\pi^2}\,(\,k^\mu k'^{\,\nu} + k'^{\,\mu}k^\nu - g^{\mu\nu}k\cdot k'\,)$$

- **Leaves us with the hadronic tensor:**

$$W_{\mu\nu} = \frac{1}{8\pi} \sum_{\sigma,X} \langle X|J_\mu|p,\sigma\rangle^* \langle X|J_\nu|p,\sigma\rangle$$

- **And the cross section:**

$$2\omega_{k'} \frac{d\sigma}{d^3k'} = \frac{1}{s(q^2)^2} L^{\mu\nu} W_{\mu\nu}$$

- $W_{\mu\nu}$ **has sixteen components,**
  but known properties of the strong interactions constrain $W_{\mu\nu}$ **. . .**

- **An example: current conservation,**

$$\begin{aligned}
\partial^\mu J_\mu^{\mathrm{EM}}(x) &= 0 \\
&\Rightarrow \langle X|\, \partial^\mu J_\mu^{\mathrm{EM}}(x)\, |p\rangle = 0 \\
&\Rightarrow (p_X - p)^\mu \langle X|\, J_\mu^{\mathrm{EM}}(x)\, |p\rangle = 0 \\
&\Rightarrow q^\mu W_{\mu\nu} = 0
\end{aligned}$$

- **With parity, time-reversal, etc . . .**

$$W_{\mu\nu} = -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) W_1(x, Q^2)$$
$$+ \left(p_\mu - q_\mu \frac{p \cdot q}{q^2}\right) \left(p_\nu - q_\nu \frac{p \cdot q}{q^2}\right) W_2(x, Q^2)$$

- Often given in terms of the dimensionless structure functions,

$$F_1 = W_1 \qquad F_2 = p \cdot q W_2$$

- Note that if there is no other mass scale, the F's cannot depend on Q except indirectly through x.

- **Structure functions in the Parton Model:**
  The Callan-Gross Relation

From the "basic parton model formula":

$$\frac{d\sigma_{eh}}{d^3k'} = \sum_{\text{quarks } f} \int d\xi \; \frac{d\sigma^{\text{el}}_{ef}(\xi)}{d^3k'} \; \phi_{f/h}(\xi) \qquad (1)$$

Can treat a quark of "flavor" f just like any hadron and get

$$\omega_{k'} \; \frac{d\sigma^{\text{el}}_{ef}(\xi)}{d^3k'} = \frac{1}{2(\xi s)Q^4} \; L^{\mu\nu} \, W^{ef}_{\mu\nu}(k + \xi p \to k' + p')$$

Let the charge of f be e_f.

Exercise 1: Compute $W^{ef}_{\mu\nu}$ to find:

$$W^{ef}_{\mu\nu} = -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)\, \delta\left(1 - \frac{x}{\xi}\right) \frac{e_f^2}{2} + \left(\xi p_\mu - q_\mu \frac{\xi p \cdot q}{q^2}\right) \left(\xi p_\nu - q_\nu \frac{\xi p \cdot q}{q^2}\right) \delta\left(1 - \frac{x}{\xi}\right) \frac{e_f^2}{\xi p \cdot q}$$

Ex. 2: by substituting in (1), find the Callan-Gross relation,

$$F_2(x) = \sum_{\text{quarks} f} e_f^2 x\, \phi_{f/p}(x) = 2x F_1(x)$$

And Ex. 3: that this relation is quite different for scalar quarks.

- **The Callan-Gross relation shows the compatibility of the quark and parton models.**

- **In addition: parton model structure functions are independent of Q^2, a property called "scaling".**

- **With massless partons, there is no other massive scale. Then the F's must be Q-independent; see above.**

- **Approximate properties of the kinematic region explored by SLAC in late 1960's – early 1970's.**

- **Explore corrections to this picture in QCD "evolution".**

IC. Getting at the Quark Distributions

- **Relating the parton distributions to experiment**
- **Simplifying assumptions that illustrate the general approach.**

$$\phi_{u/p} = \phi_{d/n} \qquad \phi_{d/p} = \phi_{u/n} \qquad \text{isospin}$$

$$\phi_{\bar{u}/p} = \phi_{\bar{u}/n} = \phi_{\bar{d}/p} = \phi_{\bar{d}/n} \qquad \text{symmetric sea}$$

$$\phi_{c/p} = \phi_{b/N} = \phi_{t/N} = 0 \qquad \text{no heavy quarks}$$

- **Adequate to early experiments, but no longer.**

- **With assumptions above, find for e, ν and $\bar{\nu}$ scattering (see appendix)**

$$
\begin{aligned}
F_2^{(eN)}(x) &= 2xF_1^{(eN)}(x) = \sum_{f=u,d,s} e_F^2 x \phi_{f/N}(x) \\
F_2^{(W^+N)} &= 2x\left(\sum_{D=d,s,b} \phi_{D/N}(x) + \sum_{U=u,c,t} \phi_{\bar{U}/N}(x)\right) \\
F_2^{(W^-N)} &= 2x\left(\sum_D \phi_{\bar{D}/N}(x) + \sum_U \phi_{U/N}(x)\right) \\
F_3^{(W^+N)} &= 2\left(\sum_D \phi_{D/N}(x) - \sum_U \phi_{\bar{U}/N}(x)\right) \\
F_3^{(W^-N)} &= 2\left(-\sum_D \phi_{\bar{D}/N}(x) + \sum_U \phi_{U/N}(x)\right)
\end{aligned}
$$

- **Exercise: derive some of these for yourself.**

- **The distributions are actually overdetermined with these assumptions, which checks the consistency of the picture.**

- **Further consistency checks: Sum Rules.**

$$N_{u/p} = \int_0^1 dx \left[\phi_{u/p}(x) - \phi_{\bar{u}/p}(x) \right] = 2$$

etc. for $N_{d/p} = 1$.

The most famous ones make predictions for structure functions. Two examples . . .

- **The Adler Sum Rule:**

$$
\begin{aligned}
1 &= N_{u/p} - N_{d/p} \\
&= \int_0^1 dx \, \left[\phi_{d/n}(x) - \phi_{\bar{u}/p}(x) - \Big(\phi_{d/p}(x) - \phi_{\bar{u}/n}(x)\Big) \right] \\
&= \int_0^1 dx \, \left[\sum_D \phi_{D/n}(x) + \sum_U \phi_{\bar{U}/n}(x) \right] \\
&\quad - \int_0^1 dx \, \left[\sum_D \phi_{D/p}(x) + \sum_U \phi_{\bar{U}/p}(x) \right] \\
&= \int_0^1 dx \, \frac{1}{2x} \left[F_2^{(\nu n)} - F_2^{(\nu p)} \right]
\end{aligned}
$$

In the second equality, we've used isospin invariance, in the third, all the extra terms from heavy quarks $D = s$, b, $U = c, t$ cancel.

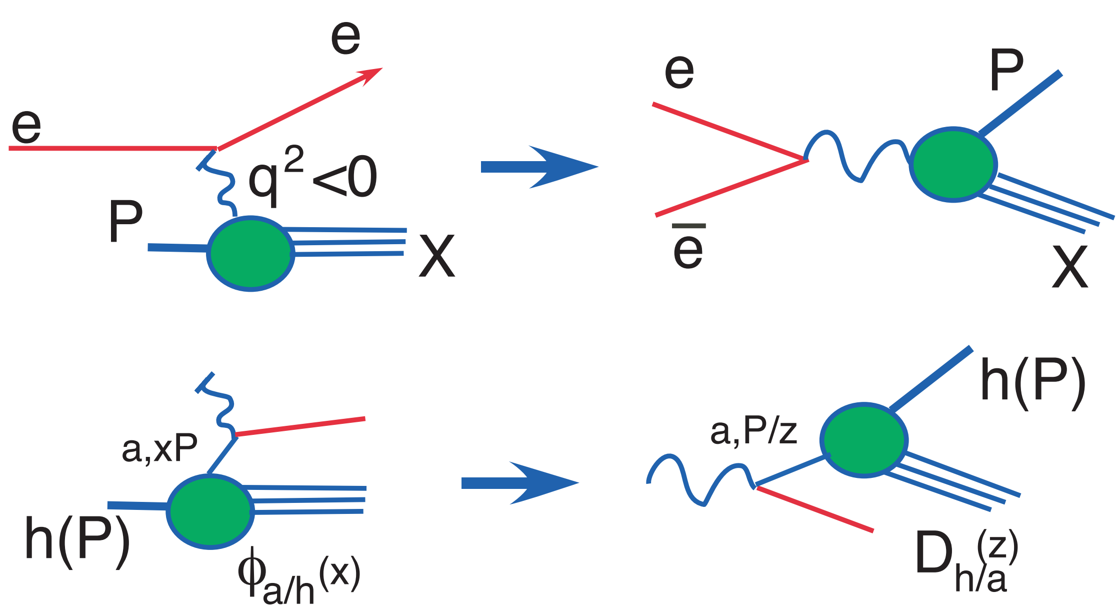
- **And similarly, the Gross-Llewellyn-Smith Sum Rule:**

$$
3 = N_{u/p} + N_{d/p} = \int_0^1 dx \, \frac{1}{2x} \left[xF_3^{(\nu n)} + xF_3^{(\nu p)} \right]
$$

ID. Classic Parton Model Extensions: Fragmentation and Drell Yan

- **Fragmentation functions**
- **"Crossing" applied to DIS: "Single-particle inclusive" (1PI)**
  From scattering to pair annihilation.

Parton distributions become "fragmentation functions".



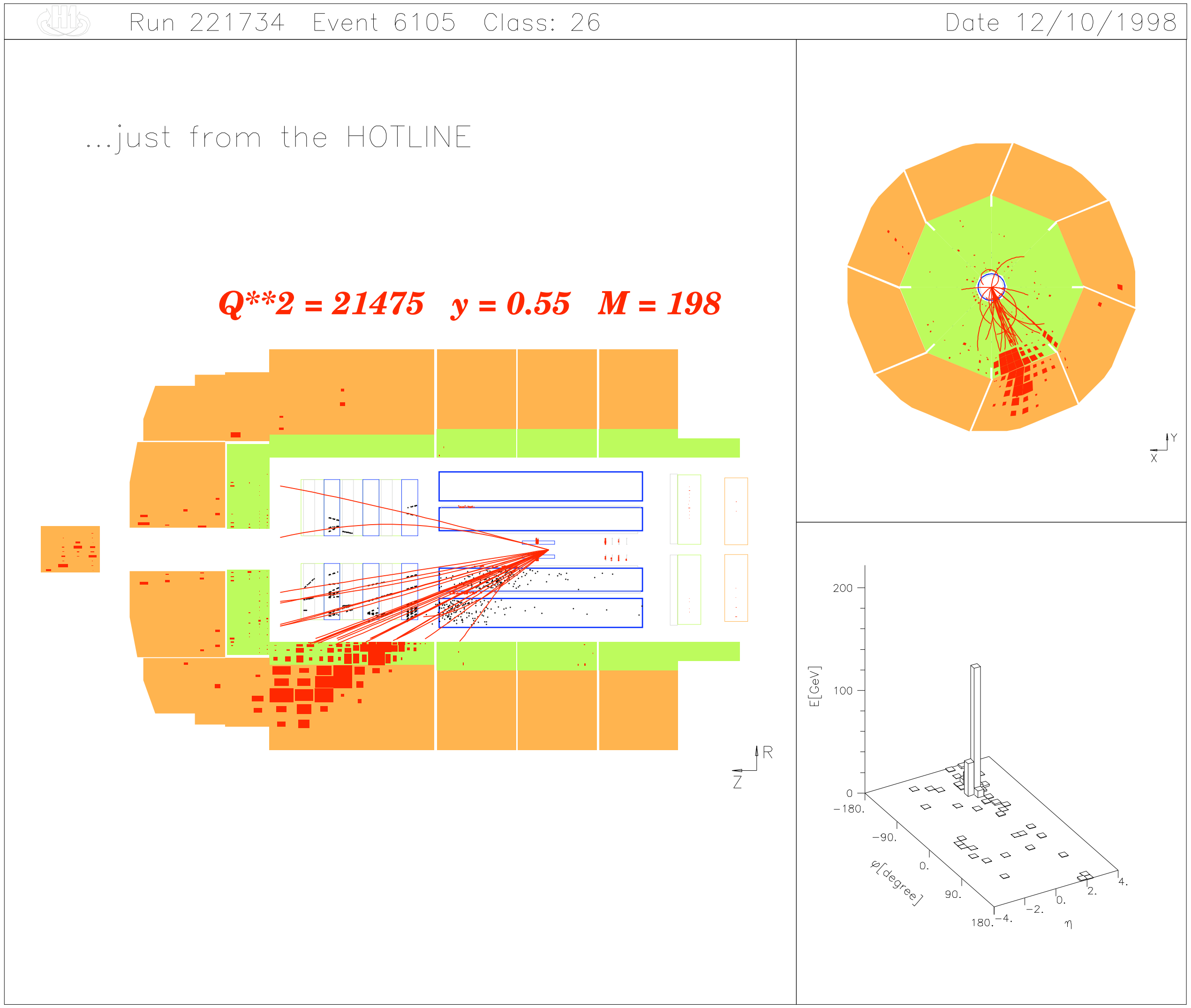
- **Parton model relation for 1PI cross sections**

$$\boldsymbol{\sigma_h(P,q) = \sum_a \int_0^1 dz\ \hat{\sigma}_a(P/z,q)\ D_{h/a}(z)}$$

- **Heuristic justification: Formation of hadron C from parton a takes a time τ_0 in the rest frame of a, but much longer in the CM frame – this "fragmentation" thus decouples from $\hat{\sigma}_a$, and is independent of q (scaling).**

- **Fragmentation picture suggests that hadrons are aligned along parton direction $\Rightarrow$ jets. And this is what happens.**

- **For DIS:**



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Date 12/10/1998
...just from the HOTLINE
Q**2 = 21475 y = 0.55 M = 198
R
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E[GeV]
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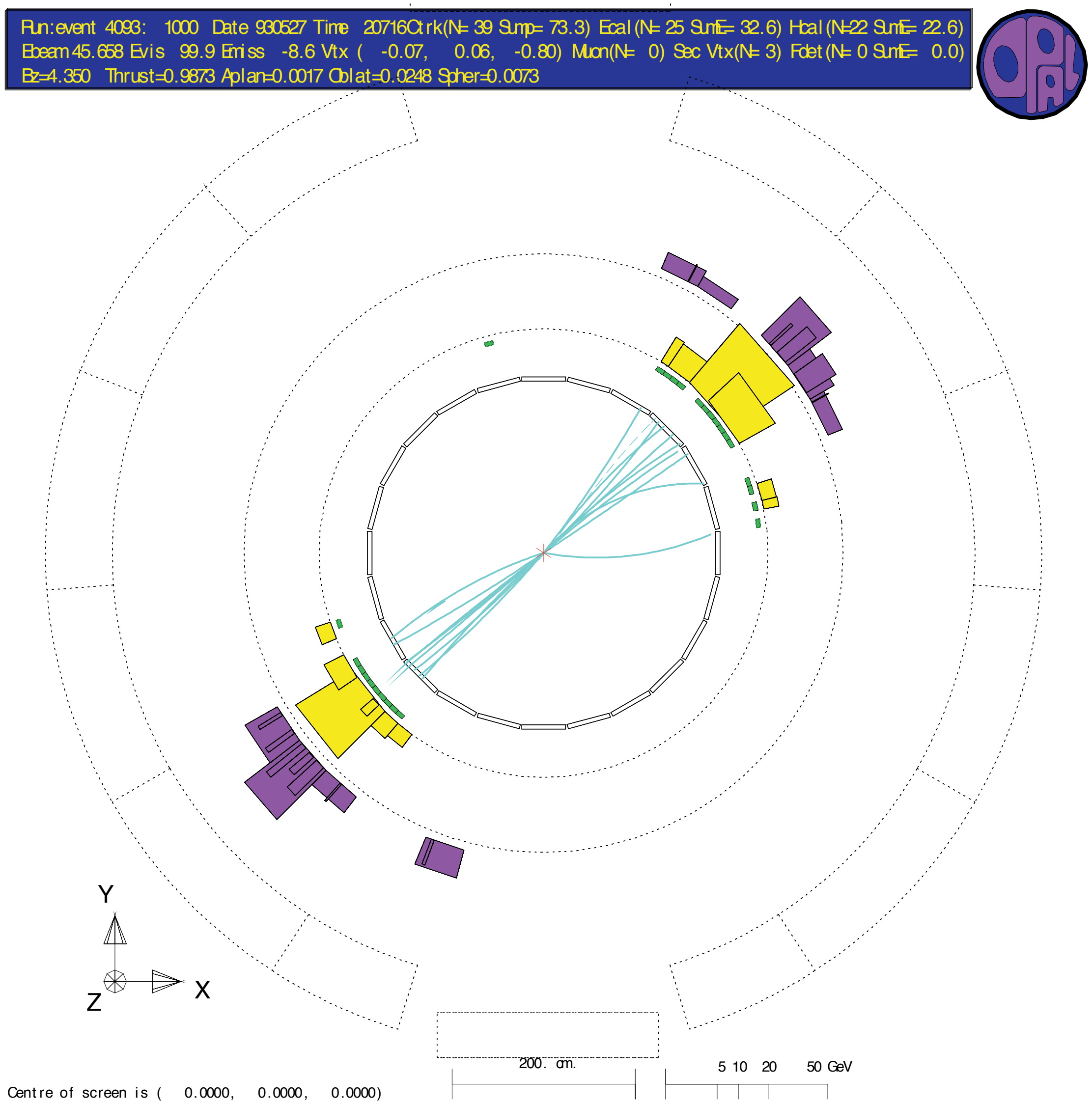
- **For** e^+e^-**:**



Run:event 4093: 1000 Date 930527 Time 20716Ctrk(N= 39 Sump= 73.3) Ecal(N= 25 SumE= 32.6) Hcal(N=22 SumE= 22.6)
Ebeam 45.658 Evis 99.9 Emiss -8.6 Vtx (-0.07, 0.06, -0.80) Muon(N= 0) Sec Vtx(N= 3) Fdet(N= 0 SumE= 0.0)
Bz=4.350 Thrust=0.9873 Aplan=0.0017 Oblat=0.0248 Spher=0.0073
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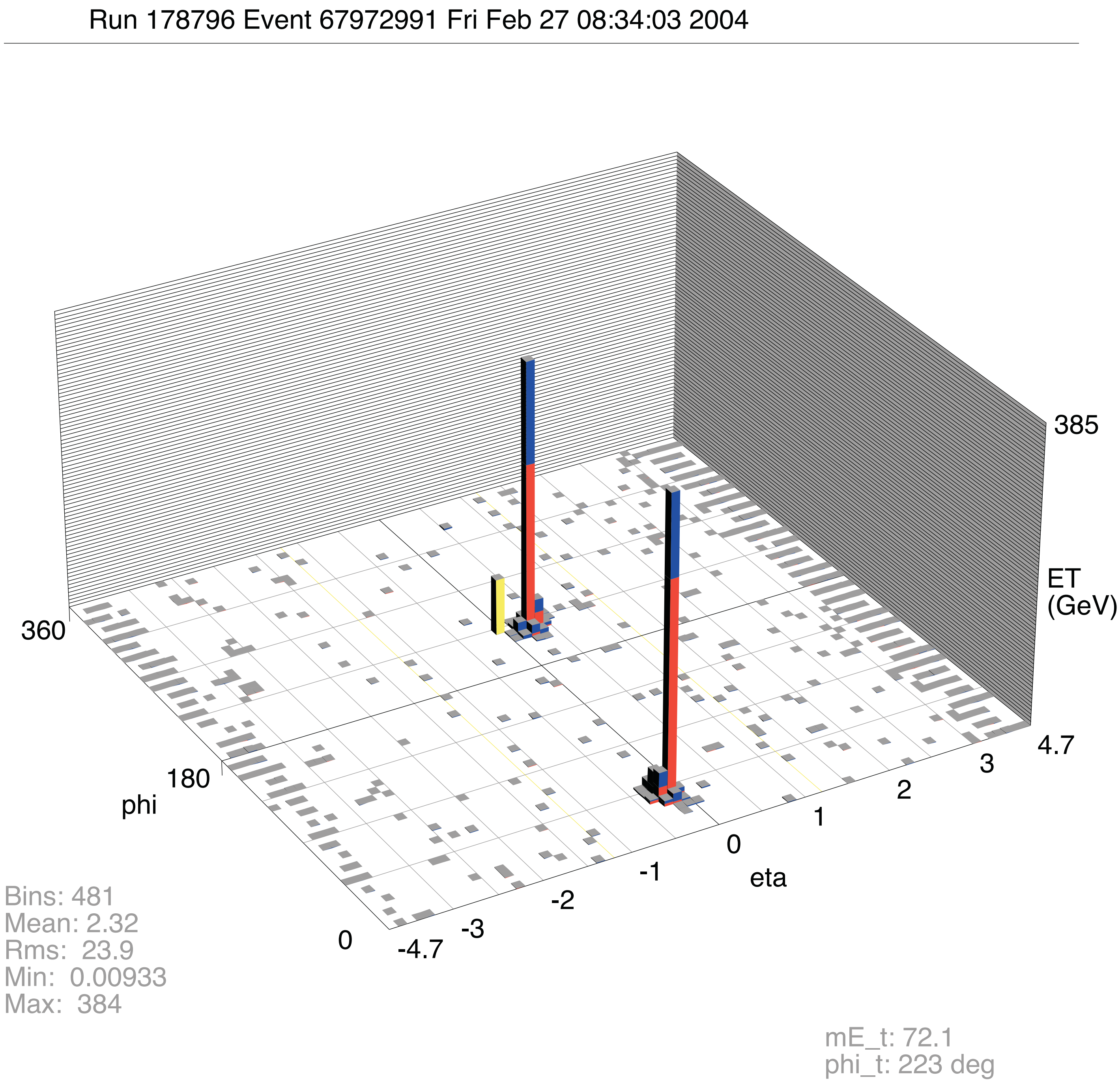
Centre of screen is (0.0000, 0.0000, 0.0000)

- **And in nucleon-nucleon collisions:**



- Finally: the Drell-Yan process

- **In the parton model (1970).**
  Drell and Yan: look for the annihilation of quark pairs into virtual photons of mass Q ... any electroweak boson in NN scattering.

$$\frac{d\sigma_{NN\to\mu\bar{\mu}+X}(Q,p_1,p_2)}{dQ^2 d\ldots} \sim$$

$$\int d\xi_1 d\xi_2 \sum_{a=\mathrm{q}\bar{\mathrm{q}}} \frac{d\sigma^{\mathrm{EW,\,Born}}_{\mathrm{a}\bar{\mathrm{a}}\to\mu\bar{\mu}}(Q,\xi_1 p_1,\xi_2 p_2)}{dQ^2 d\ldots}$$

$$\times(\text{probability to find parton a}(\xi_1)\text{ in }N)$$

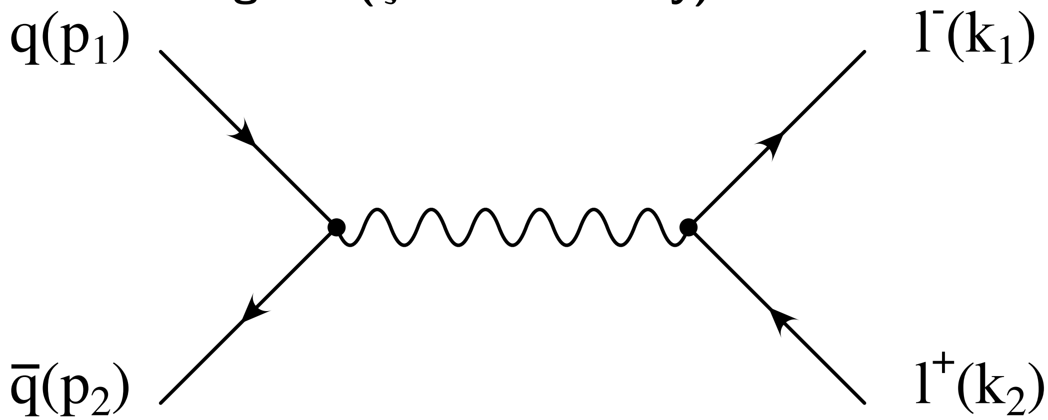
$$\times(\text{probability to find parton }\bar{\mathrm{a}}(\xi_2)\text{ in }N)$$

The probabilities are $\phi_{q/N}(\xi_i)$'s from DIS!

How it works (with colored quarks) ...

- **The Born cross section**

$\boldsymbol{\sigma}^{\mathrm{EW,Born}}$ **is all from this diagram ($\boldsymbol{\xi}$'s set to unity):**



With this matrix element:

$M = e_q \frac{e^2}{Q^2} \overline{u}(k_1)\gamma_\mu v(k_2) \overline{v}(p_2)\gamma^\mu u(p_1)$

- **First square and sum/average M. Then evaluate phase space.**

- Total cross section at pair mass Q

$$\sigma^{\mathrm{EW,\,Born}}_{q\bar{q}\to\mu\bar{\mu}}(x_1p_1, x_2p_2) = \frac{1}{2\hat{s}}\int \frac{d\Omega}{32\pi^2}\frac{e_q^2 e^4}{3}(1+\cos^2\theta)$$
$$= \frac{4\pi\alpha^2}{9Q^2}\sum_q e_q^2 \equiv \sigma_0(M)$$

With Q the pair mass and 3 for color average.

- **And measured rapidity:**

  Pair mass (Q) and rapidity

$$\eta \equiv (1/2) \ln \left(\frac{Q^+}{Q^-} \right) = (1/2) \ln \left(\frac{Q^0 + Q^3}{Q^0 - Q^3} \right)$$

- **ξ's are overdetermined $\rightarrow$ delta functions in the Born cross section**

$$\begin{aligned} \frac{d\sigma^{(PM)}_{NN \rightarrow \mu\bar{\mu}+X}(Q, p_1, p_2)}{dQ^2 d\eta} = & \int_{\xi_1, \xi_2} \sum_{a=q\bar{q}} \sigma^{\mathrm{EW,\,Born}}_{a\bar{a} \rightarrow \mu\bar{\mu}}(\xi_1 p_1, \xi_2 p_2) \\ & \times \delta\left(Q^2 - \xi_1 \xi_2 S\right) \delta\left(\eta - \frac{1}{2} \ln\left(\frac{\xi_1}{\xi_2}\right)\right) \\ & \times \phi_{a/N}(\xi_1)\, \phi_{\bar{a}/N}(\xi_2) \end{aligned}$$

- and integrating over rapidity, back to $d\sigma/dQ^2$,

$$\frac{d\sigma}{dQ^2} = \left(\frac{4\pi\alpha_{\rm EM}^2}{9Q^4}\right)\int_0^1 d\xi_1\, d\xi_2\, \delta\left(\xi_1\xi_2 - \tau\right) \times \sum_a \lambda_a^2\, \phi_{a/N}(\xi_1)\, \phi_{\bar{a}/N}(\xi_s)$$

Found by Drell and Yan in 1970 (aside from 1/3 for color).

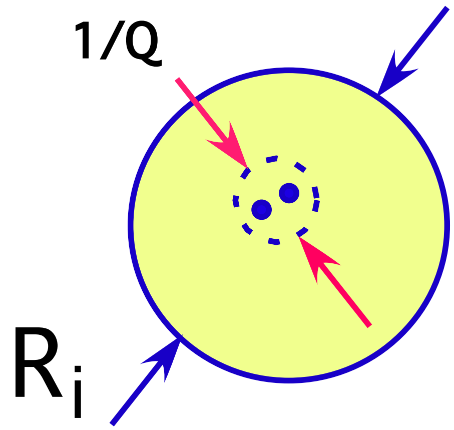
Analog of DIS scaling in x is DY scaling in $\tau = Q^2/S$.

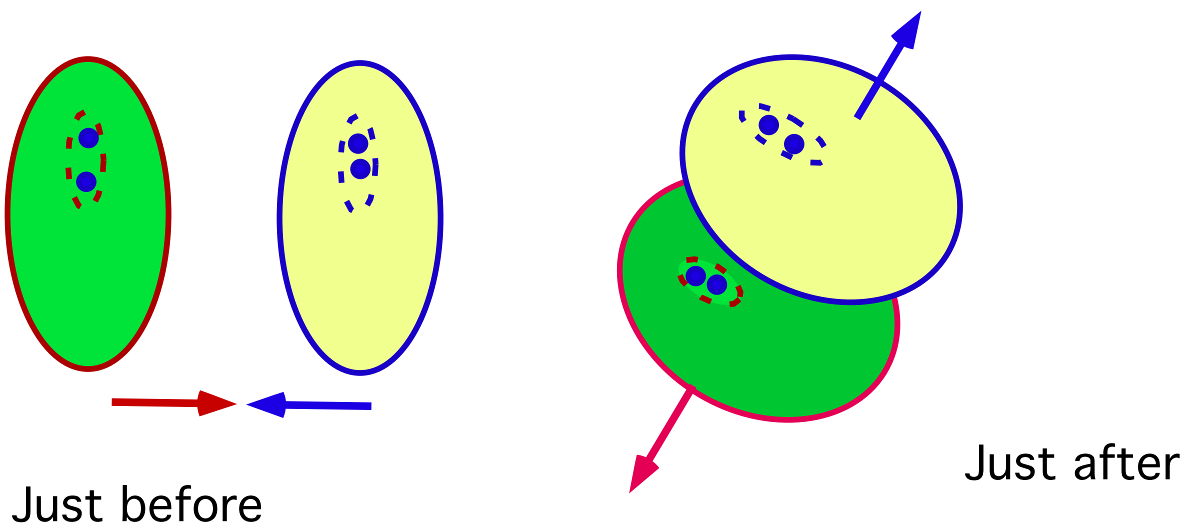
- Template for all hard inclusive hadron-hadron scattering

- **Exclusive reactions: Quark counting, the valence state and geometric counting**
  - Parton model applied to high-energy elastic scattering (1973: Brodsky, Farrar; Matveev, Muradyan, Tavkhelidze)
  - Elastic scattering is through the valence state:
  - Parton picture: in c.m., wave functions are Lorentz-contracted.
  - large t requires all n_i valence (anti-)quarks of hadron i in a region of area $1/Q^2$ for both incoming hadrons.
  - Likelihood is $\sim \left(\frac{1}{Q^2} \times \frac{1}{\pi R_H^2}\right)^{n_H - 1}$ for each hadron.
  - Geometric picture: Must be true of both incoming and outgoing states, for overlap of wave functions.
  - Scaling: assume that otherwise the amplitude is a function only of the scattering angle.
  - The result, at fixed s/t (c.m. scattering angle):

$$\frac{d\sigma}{dt} = \frac{f(s/t)}{s^2} \left(\frac{m^2}{s}\right)^{\sum_{i=1}^{4}(n_i - 1)}$$

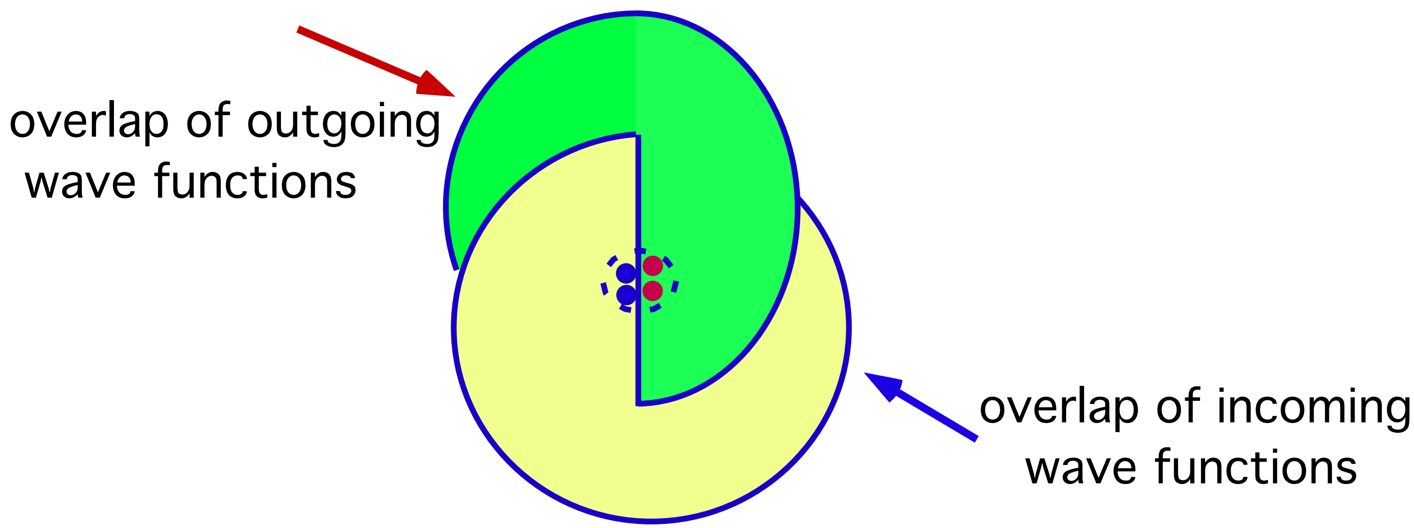
How it looks:





And also:

Quark counting picture just at the moment of collision for mesons



– The corresponding elastic amplitude
(1979: Brodsky and Lepage, Efremov and Radyushkin)

$$\mathcal{M}(s,t;h_i) \;=\; \int \prod_{i=1}^{4} \,[dx]\; \phi(x_{m,i},\lambda_{m,i},h_i;\mu) \times M_H\left(\frac{x_{n,i}x_{m,j}p_i\cdot p_j}{\mu^2};\lambda_{n,i},h_i\right)$$

with factorized & evolved valence (light-cone) wave functions $\phi(x_{m,i},\lambda_{m,i},h_i;\mu)$, and with

$$[dx] = dx_{1,i}dx_{2,i}dx_{3,i}\,\delta\left(1-\sum_{n=1}^{3}x_{n,i}\right)$$

for helicities: h_i (hadron) $\lambda_{n,i}$ (quarks)

.

– Template for all hard exclusive hadron-hadron scattering

– Next, the quantum field theory of all this . . . QCD

- **Appendix I: Quarks in the Standard Model**
  Electroweak interactions of quarks: SU(2)$_L$ $\times$ U(1). Their non-QCD interactions.

- **Quark and lepton fields: L(eft) and R(ight)**
  - $\psi = \psi^{(L)} + \psi^{(R)} = \frac{1}{2}(1-\gamma_5)\psi + \frac{1}{2}(1+\gamma_5)\psi$; $\psi = q, \ell$
  - **Helicity: spin along $\vec{p}$ (R=right handed) or opposite (L=left handed) in solutions to Dirac equation**
  - $\psi^{(L)}$: **expanded only in L particle solutions to Dirac eqn. R antiparticle solutions**
  - $\psi^{(R)}$: **only R particle solutions, L antiparticle**
  - **An essential feature: L and R have different interactions in general!**

– L quarks come in "weak $SU(2)$" = "weak isospin" pairs:

$$q_i^{(L)} = \begin{pmatrix} u_i \\ d'_i = V_{ij} dj \end{pmatrix} \qquad u_i^{(R)},\ d_i^{(R)}$$

$$(u, d') \qquad (c, s') \qquad (t, b')$$

$$\ell_i^{(L)} = \begin{pmatrix} \nu_i \\ e_i \end{pmatrix} \qquad e_i^{(R)},\ \nu_i^{(R)}$$

$$(\nu_e, e) \qquad (\nu_\mu, \mu) \qquad (\nu_\tau, \tau)$$

(We've neglected neutrino masses.)

– V_{ij} is the "CKM" matrix.

– The electroweak interactions distinguish L and R.

- **Weak vector bosons: electroweak gauge groups**

  - **SU(2): three vector bosons B_i with coupling g**

  - **U(1); one vector boson C with coupling g'**

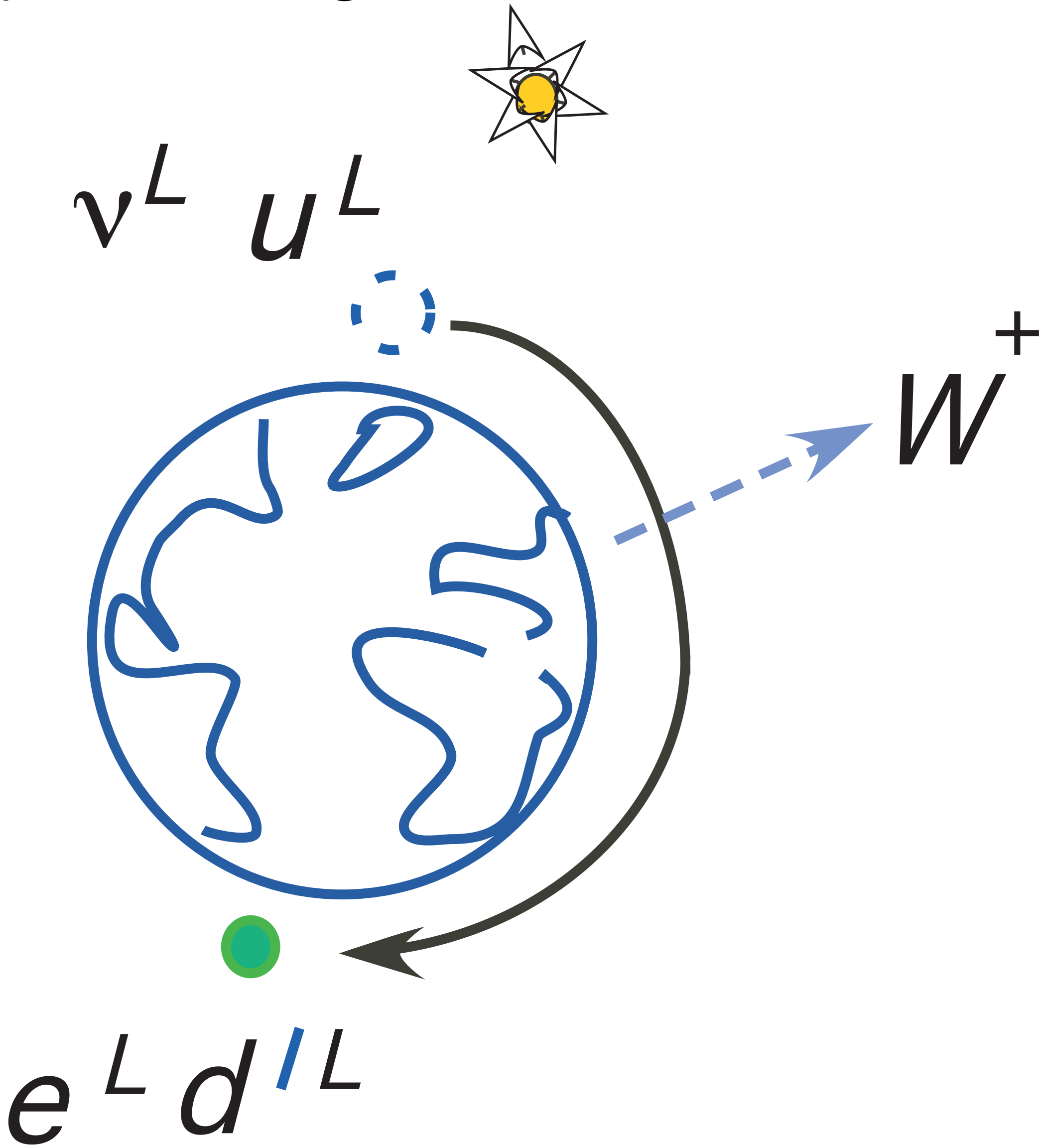
  - **The physical bosons:**

$$W^{\pm} = B_1 \pm iB_2$$
$$Z = -C \sin\theta_W + B_3 \cos\theta_W$$
$$\gamma \equiv A = C \cos\theta_W + B_3 \sin\theta_W$$

$$\sin\theta_W = g'/\sqrt{g^2 + g'^2} \qquad M_W = M_Z/\cos\theta_W$$

$$e = gg'/\sqrt{g^2 + g'^2} \qquad M_W \sim g/\sqrt{G_F}$$

- **Weak isospin space: connecting u with d'**



- **Only left handed fields move around this globe.**

– The interactions of quarks and leptons with the photon, W, Z

$$\mathcal{L}_{\mathrm{EW}}^{(fermion)} = \sum_{\text{all } \psi} \bar{\psi}\,(i\not\partial - e\lambda_\psi \not A - (g m_\psi 2 M_W) h)\,\psi$$
$$-(g/\sqrt{2}) \sum_{q_i, e_i} \bar{\psi}^{(L)} \left(\sigma^+ \not W^+ + \sigma^- \not W^-\right) \psi^{(L)}$$
$$-(g/2\cos\theta_W) \sum_{\text{all } \psi} \bar{\psi}\,(v_f - a_f \gamma_5)\,\not Z\;\psi$$

– Interactions with W are through ψ_L's only.

– Neutrino Z exchange depends on $\sin^2\theta_W$ even at low energy.

– This observation made it clear by early 1970's that $M_W \sim g/\sqrt{G_F}$ is large $\rightarrow$ a need for colliders.

– Coupling to the Higgs $h \propto$ mass (special status of t).

- Symmetry violations in the standard model:

  – W's interact through $\psi^{(L)}$ only, $\psi = q, \ell$.

  – These are left-handed quarks & leptons;
  right-handed antiquarks, antileptons.

  – Parity (P) exchanges L/R; Charge conjugation (C)
  exchanges particles, antiparticles.

  – CP combination OK ($L \to_P R \to_C L$) if all else equal,
  but it's not (quite) . . .

  Complex phases in CKM V result in CP violation.

- **Appendix II: Structure Functions and Photon Polarizations**

In the P rest frame can take

$$q^\mu = \left(\nu; 0, 0, \sqrt{Q^2 + \nu^2}\right) , \qquad \nu \equiv \frac{p \cdot q}{m_p}$$

In this frame, the possible photon polarizations $(\epsilon \cdot q = 0)$**:**

$$\epsilon_R(q) = \frac{1}{\sqrt{2}}(0; 1, -i, 0)$$
$$\epsilon_L(q) = \frac{1}{\sqrt{2}}(0; 1, i, 0)$$
$$\epsilon_{\rm long}(q) = \frac{1}{Q}\left(\sqrt{Q^2 + \nu^2}, 0, 0, \nu\right)$$

- **Alternative Expansion**

$$W^{\mu\nu} = \sum_{\lambda=L,R,long} \epsilon_\lambda^{\mu *}(q)\epsilon_\lambda^{\nu}(q)\, F_\lambda(x, Q^2)$$

- **For photon exchange (Exercise 4):**

$$F^{\gamma e}_{L,R} = F_1$$
$$F_{\rm long} = \frac{F_2}{2x} - F_1$$

- **So $F_{\rm long}$ vanishes in the parton model by the C-G relation.**

- **Generalizations: neutrinos and polarization**

- **Neutrinos: flavor of the "struck" quark is changed when a $W^{\pm}$ is exchanged. For W^{+}, a d is transformed into a linear combination of u, c, t, determined by CKM matrix (and momentum conservation).**

- **Z exchange leaves flavor unchanged but still violates parity.**

- **The Vh structure functions for $= W^+, W^-, Z$:**

$$W_{\mu\nu}^{(Vh)} - \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) W_1^{(Vh)}(x, Q^2)$$
$$+ \left(p_\mu - q_\mu \frac{p \cdot q}{q^2}\right) \left(p_\nu - q_\nu \frac{p \cdot q}{q^2}\right) \frac{1}{m_h^2} W_2(x, Q^2)$$
$$-i\epsilon_{\mu\nu\lambda\sigma} p^\lambda q^\sigma \frac{1}{m_h^2} W_3^{(Vh)}(x, Q^2)$$

- **with dimensionless structure functions:**

$$F_1 = W_1\,, \qquad F_2 = \frac{p \cdot q}{m_h^2} W_2\,, \qquad F_3 = \frac{p \cdot q}{m_h^2} W_3$$

- $F_i^{(\nu h)}$ **gives** $W^+ h$ **scattering,** $F_i^{(\bar{\nu} h)}$ **gives** $W^- h$

- **And with spin (for the photon).**

$$
\begin{aligned}
W^{\mu\nu} \;&=\; \frac{1}{4\pi} \int d^4z\, e^{iq\cdot z} \;\langle h(P,S)\,|\, J^\mu(z) J^\nu(0)\,|\, h(P,S)\rangle \\
&=\; \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right)\; F_1(x,Q^2) \\
&\qquad + \left(P^\mu - q^\mu\,\frac{P\cdot q}{q^2}\right)\left(P^\nu - q^\nu\,\frac{P\cdot q}{q^2}\right) F_2(x,Q^2) \\
&+\; i m_h\, \epsilon^{\mu\nu\rho\sigma} q_\rho \left[\frac{S_\sigma}{P\cdot q}\, g_1(x,Q^2) + \frac{S_\sigma(P\cdot q) - P_\sigma(S\cdot q)}{(P\cdot q)^2}\, g_2(x,Q^2)\right]
\end{aligned}
$$

- **Parton model structure functions:**

$$F_2^{(eh)}(x) = \sum_f e_f^2 \, x \, \phi_{f/h}(x)$$

$$g_1^{(eh)}(x) = \frac{1}{2} \sum_f e_f^2 \, \left(\Delta\phi_{f/n}(x) + \Delta\bar{\phi}_{f/h}(x)\right)$$

- **Notation:** $\Delta\phi_{f/h} = \phi^+_{f/h} \; - \; \phi^-_{f/h}$ **with** $\phi^{\pm}_{f/h}(x)$ **probability for struck quark** f **to have momentum fraction** x **and helicity with (+) or against (−)** h**'s helicity.**