Flavor physics:
Precision as an avenue to discovery

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LAPTh Annecy

Outline

- ☑ **Why flavor physics**
- ☑ **CP violation in $K^0 - \overline{K}^0$ oscillations**
- ☑ **The very rare decay $B_s \to \mu\mu$**

SM flavor violation: Within the SM, all of (quark) flavor violation is ruled by

two renormal. "Yukawa" interactions with one scalar doublet

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- The only entry in the Y's that is O(1) (in units of the SM Higgs vev and assuming one single vev) is $(Y_u)_{33}$
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The SM pattern of breaking of the "flavor" symmetry is highly peculiar

No apparent reason why it should hold for physics beyond the SM, whatever its scale

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- So, the more the high-p_T picture at LHC is SM-like,
  the higher the relative weight of indirect observables in defining future strategies

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- *In 1967, Sakharov recognized CP violation as one of the building blocks of the matter-antimatter asymmetry of the universe.*

On fundamental questions, we should never give up.

D. Guadagnoli, HDR seminar

CP violation

in the $K^0 - \overline{K}^0$ system

A SM test of CP violation

- CP violation is today measured in several observables. Especially well-known are:

ϵ_K		$K^0 - \overline{K}^0$	
$\sin 2\beta$	measuring (indirect) CPV in the	$B_d - \overline{B}_d$	system
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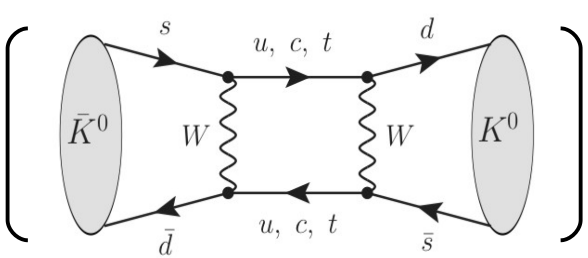
$\epsilon_K \sim \mathrm{Im}$ 

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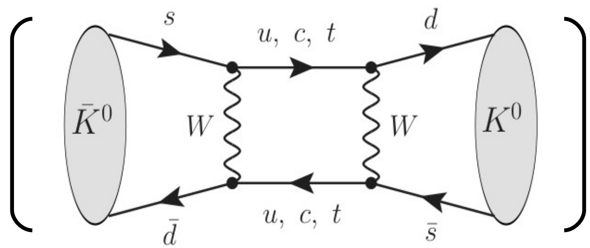

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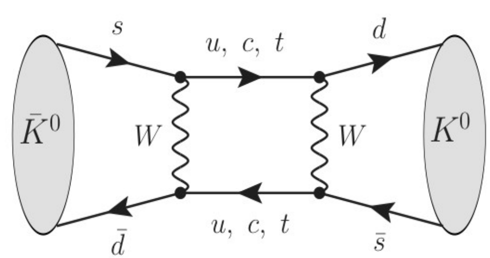

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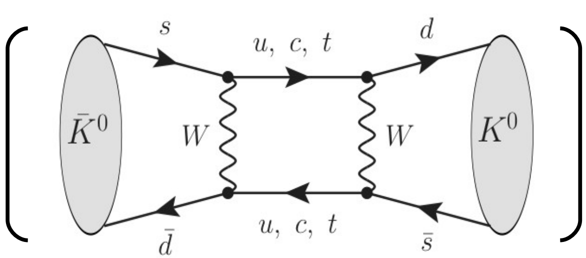

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Long-distance effects to ϵ_K

Buras, DG
PRD 08 & 09

- We saw that $\epsilon_K \simeq C \cdot \hat{B}_K \cdot \sin 2\beta$ by just using short-distance contributions to ϵ_K

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- take $\sin 2\beta = S_{\psi K_s} \simeq 0.68$ (experimental average)

$\Rightarrow$ $|\epsilon_K|_{\mathrm{SM}} \simeq 1.78 \cdot 10^{-3}$ vs. $|\epsilon_K|_{\exp} = 2.232(7) \cdot 10^{-3}$

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In either case: O(20%) discrepancy!

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$K_S \to 2\pi$ (and occasionally to 3π)

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From Cronin & Fitch's experiment, we learned that

occasionally $\simeq$ 1/350 $\simeq$ 3×10^{-3}

Experimental observables

Recall: $|K_{S,L}\rangle \propto |K_{\pm}\rangle + \bar{\epsilon}|K_{\mp}\rangle$

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The κ_ϵ correction

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The ϵ_K magnitude obeys the following theoretical formula

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Using $|\epsilon'/\epsilon_K|_{exp} = 1.66(26) \cdot 10^{-3}$
(with namely 15% accuracy)

we get ξ with $20\% \oplus 15\% = 25\%$ error

A closer look at the OPE for ϵ_K

Buras, DG, Isidori
PLB 10

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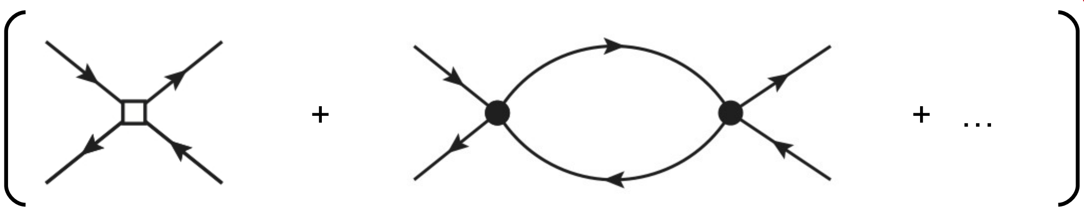
Im 

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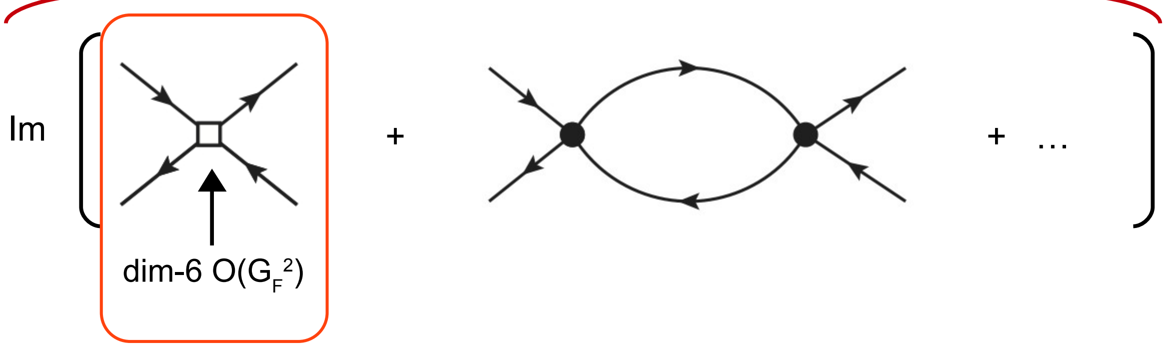

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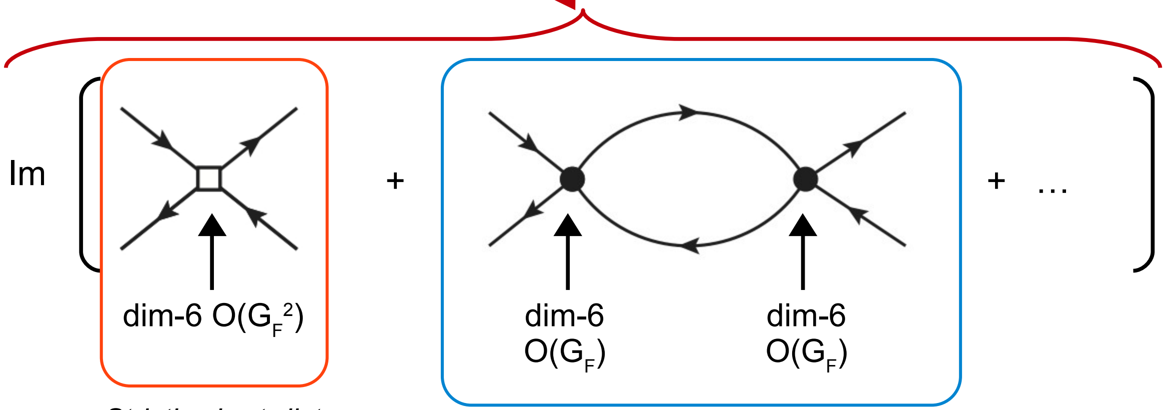

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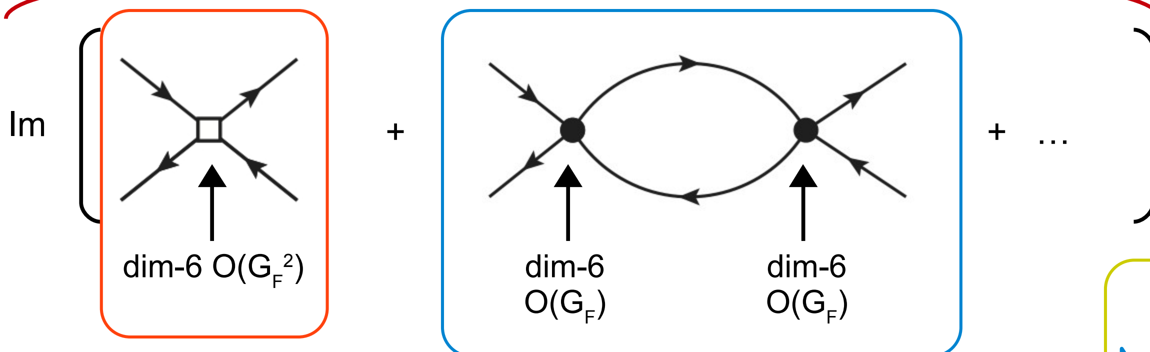

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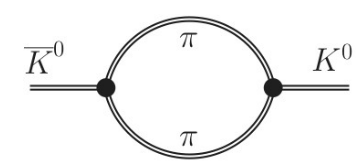

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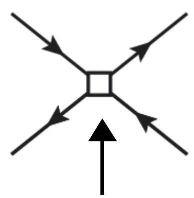
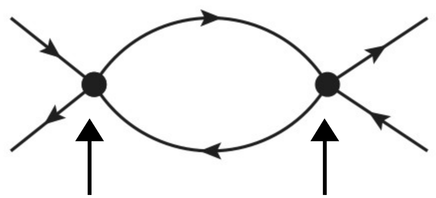
Im [ +  + …]

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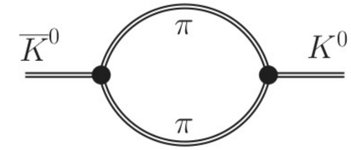

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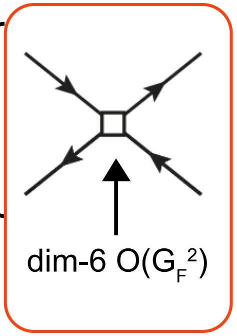
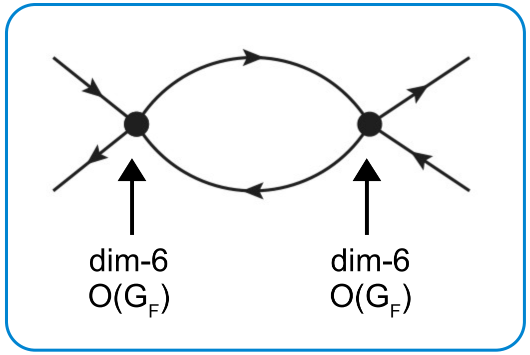
Im [ +  + …]

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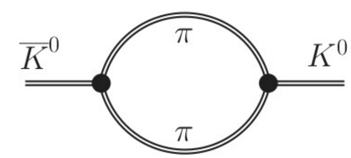

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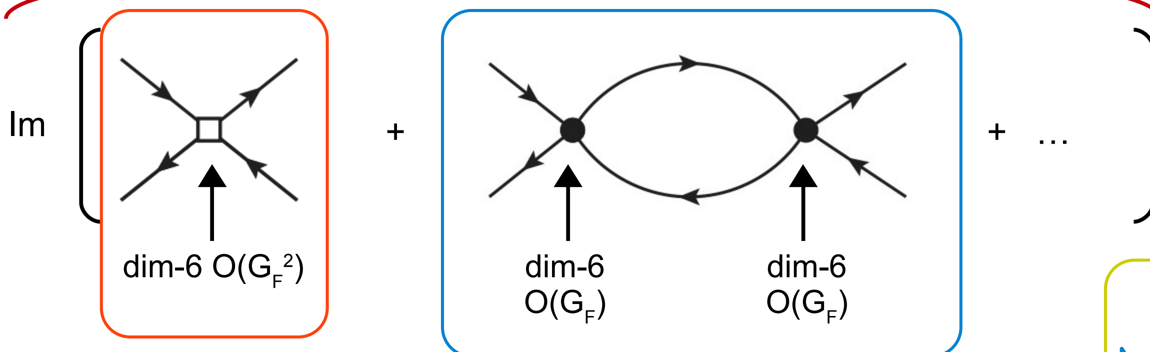

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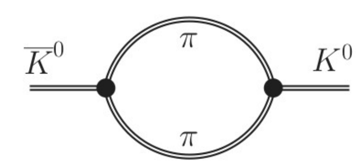

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⇨ *since we argued about the necessity of including ξ, for consistency we should also include Im $M_{12}^{non-local}$*

Calculation of Im $M_{12}^{\text{non-local}}$

Buras, DG, Isidori
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Main observation

Within ChPT, Im $M_{12}^{\text{non-local}}$ is dominated by <u>one single operator</u>.

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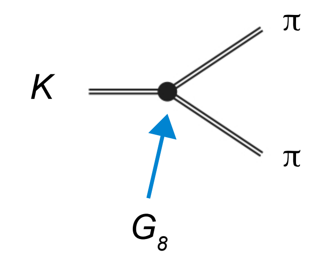

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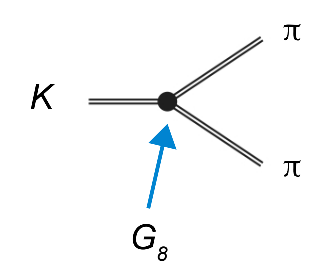

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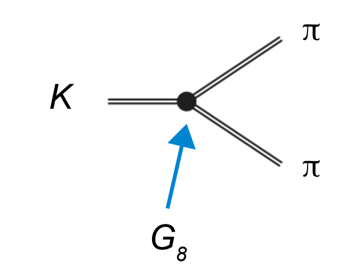
$$a_0 \equiv A_W(K^0 \to (\pi\pi)_0) \propto G_8$$



Calculation of Im $M_{12}^{\text{non-local}}$

Buras, DG, Isidori
PLB 10

Main observation

Within ChPT, Im $M_{12}^{\text{non-local}}$ is dominated by <u>one single operator</u>.

This operator must also be responsible for the large $K \to (\pi\pi)_0$ amplitude with respect to $K \to (\pi\pi)_2$ fact known as the $\Delta I = ½$ "rule"

Therefore we can determine the (complex) coupling G_8 of this "octet" operator entirely <u>from data:</u>

- *the magnitude can be measured from the $K^0 \to (\pi\pi)_0$ rate:*

$$|G_8| \simeq 9 \times 10^{-6}\ \text{GeV}^{-2}$$

i.e. $|G_8|$ has
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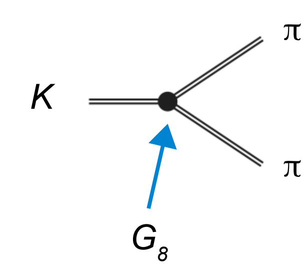

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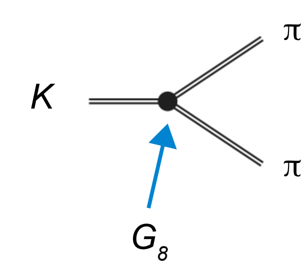

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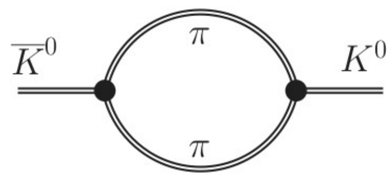

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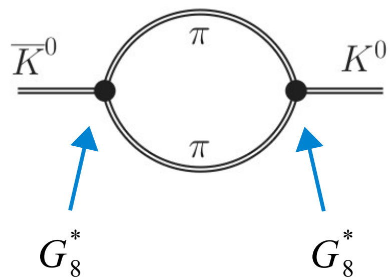

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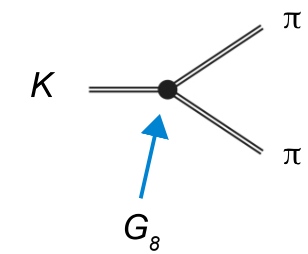

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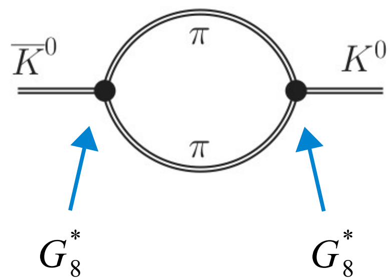

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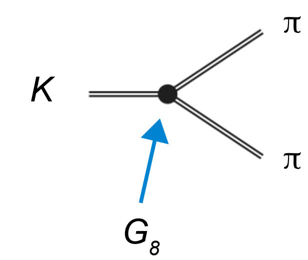

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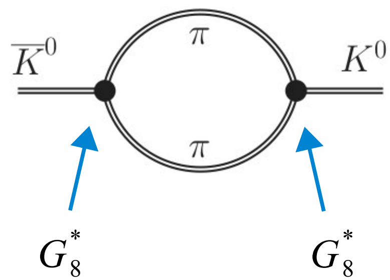

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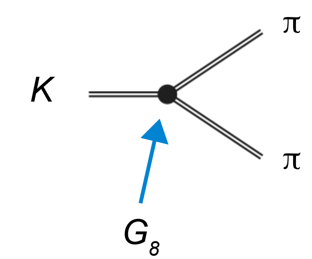

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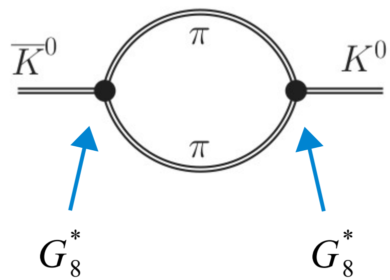

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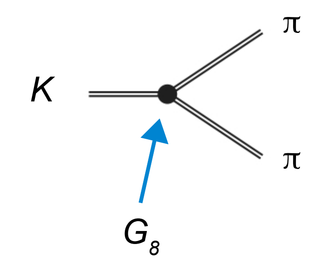

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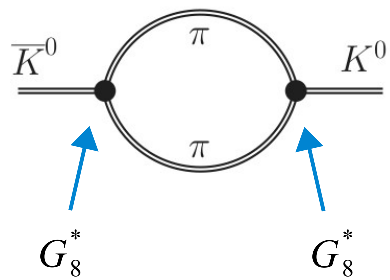

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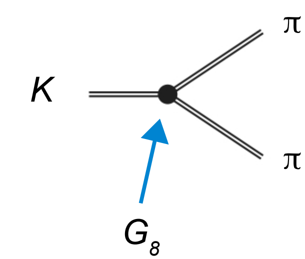

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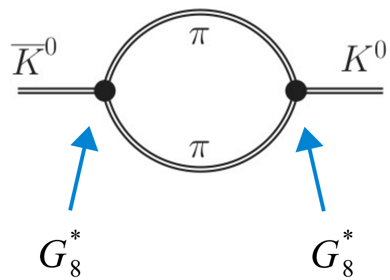

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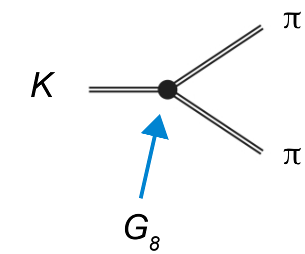

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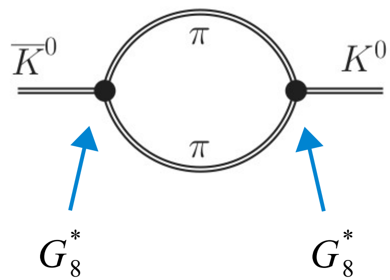

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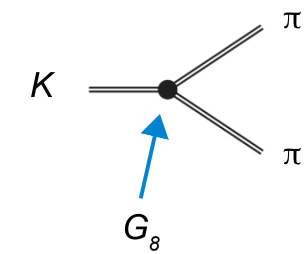

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Bottom line

$$|\epsilon_K| \equiv |\epsilon_K|_{\text{approx}} \cdot \kappa_\epsilon$$

$$\Rightarrow \quad \kappa_\epsilon = 0.94 \pm 0.02$$

The very rare decay

$B_s \rightarrow \mu\mu$

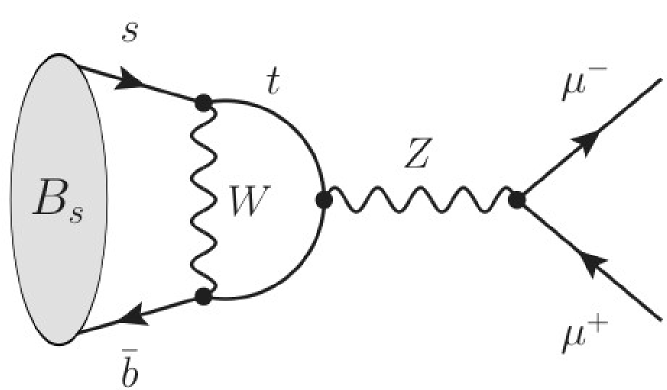
See: Buchalla, Buras '93; Misiak, Urban '99

BR[$B_s \to \mu\mu$] has the following structure

$$BR[B_s \to \mu^+\mu^-] \simeq \frac{1}{\Gamma_s} \times \left(\frac{G_F^2 \alpha_{\text{e.m.}}^2}{16\pi^3 s_W^4} \right) \cdot \left| V_{tb}^* V_{ts} \right|^2 \cdot f_{B_s}^2 \; m_{B_s} \cdot m_\mu^2 \cdot Y^2 (m_t^2 / M_W^2)$$

Main diagram: Z-penguin

contributes 80% of the Y function



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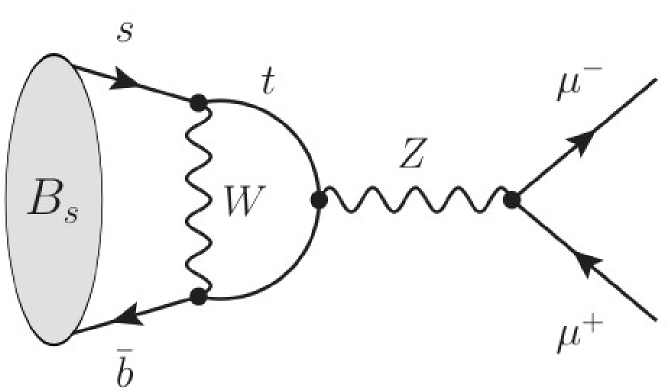

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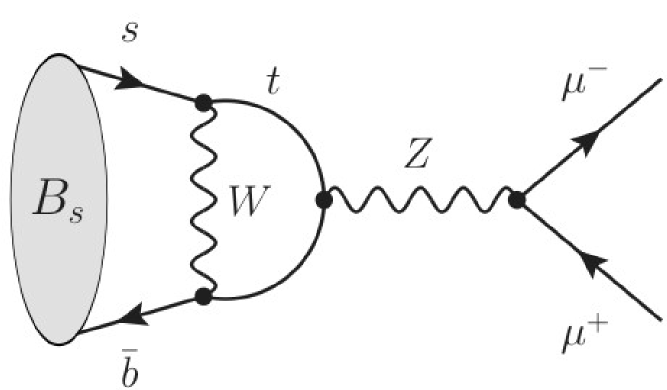

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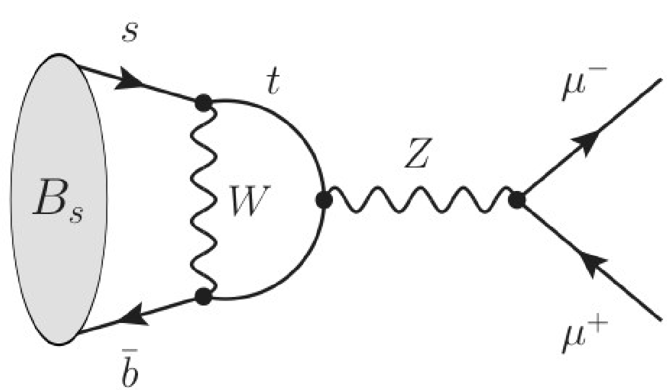

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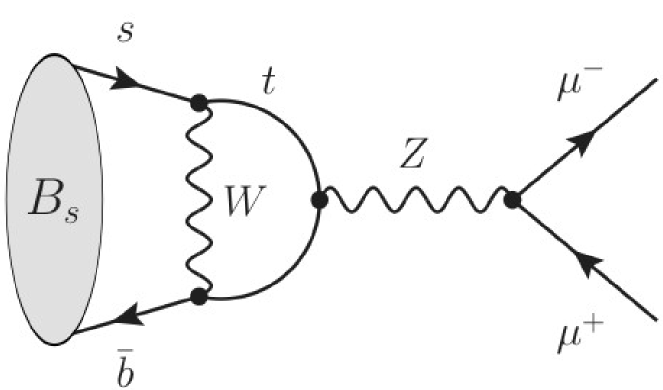

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hadronic matrix element

Recall: the final state is purely leptonic

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- f_{Bs} is among the simplest quantities for lattice QCD
- high-precision calculations possible, and in part already reality

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Additional "chiral" suppression:
relative 4×10^{-4} factor

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top “pole” mass here

$$BR[B_s \to \mu^+\mu^-] = 3.23 \cdot 10^{-9} \cdot \left(\frac{\tau_{B_s}}{1.466\,\text{ps}} \right) \cdot \left(\frac{\text{Re}\left(V_{tb}^* V_{ts} \right)}{4.05 \cdot 10^{-2}} \right)^2 \cdot \left(\frac{f_{B_s}}{227\,\text{MeV}} \right)^2 \cdot \left(\frac{M_t}{173.2\,\text{GeV}} \right)^{3.07}$$

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- Using this expression, one can easily work out the main error components as follows

	pdgLive
Input	$\tau_{B_s} = 1.466(31)\,\mathrm{ps}$
Contribution to BR relative error	2%

$BR[B_s \to \mu\mu]$ error: parametric

- The main sources of error within the BR formula are:

$$BR[B_s \to \mu^+\mu^-] \simeq \frac{1}{\Gamma_s} \times \left(\frac{G_F^2 \alpha_{\text{e.m.}}^2}{16 \pi^3 s_W^4} \right) \cdot \left| V_{tb}^* V_{ts} \right|^2 \cdot f_{B_s}^2 m_{B_s} \cdot m_\mu^2 \cdot Y^2 \left(m_t^2 / M_W^2 \right)$$

- Thus, one can write the following phenomenological expression for the BR

top "pole" mass here

$$BR[B_s \to \mu^+\mu^-] = 3.23 \cdot 10^{-9} \cdot \left(\frac{\tau_{B_s}}{1.466\,\text{ps}} \right) \cdot \left(\frac{\text{Re}(V_{tb}^* V_{ts})}{4.05 \cdot 10^{-2}} \right)^2 \cdot \left(\frac{f_{B_s}}{227\,\text{MeV}} \right)^2 \cdot \left(\frac{M_t}{173.2\,\text{GeV}} \right)^{3.07}$$

- Using this expression, one can easily work out the main error components as follows

	pdgLive	CKMfitter or UTfit
Input	$\tau_{B_s} = 1.466(31)\,\text{ps}$	$\text{Re}(V_{tb}^* V_{ts}) = 4.05(8) \cdot 10^{-2}$
Contribution to BR relative error	2%	4%

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	pdgLive	CKMfitter or UTfit	LQCD average (central value from C. Davies)
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Contribution to BR relative error	2%	4%	7%

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- Using this expression, one can easily work out the main error components as follows

	pdgLive	CKMfitter or UTfit	LQCD average (central value from C. Davies)	Tevatron average on 5.8/fb: 1107.5255
Input	$\tau_{B_s} = 1.466(31)\,\rm ps$	${\rm Re}(V_{tb}^* V_{ts}) = 4.05(8) \cdot 10^{-2}$	$f_{B_s} = 227(8)\,\rm MeV$	$M_t = 173.2(0.9)\,\rm GeV$
Contribution to BR relative error	2 %	4 %	7 %	1.6 %

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Contribution to BR relative error	2%	4%	7%	1.6%

Total relative error expected for BR[$B_s \to \mu\mu$]: **about 8.5%**

D. Guadagnoli, HDR seminar

A qualification about the f_{Bs} error

	pdgLive	CKMfitter or UTfit	LQCD average (central value from C. Davies)	Tevatron average on 5.8/fb: 1107.5255
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Contribution to BR relative error	2%	4%	7%	1.6%

☑ Actually, there are different schools of thought as to whether the above f_{Bs} error is "the right choice"

- The FLAG (Flavor Lattice Averaging Group) collab. quotes as reference error the weighted average among the most recent (= unquenched) lattice calculations: 4.5 MeV

  ☞ This average is however dominated by one determination (HPQCD collab.), that has about half the error of the other ones.

  In BR[$B_s \to \mu\mu$], this choice makes the f_{Bs} error subleading with respect to the CKM error.

- We adopted the more conservative approach of estimating the error from the spread of the central values.

This issue is still debatable to some extent

(or at least it would be so in case of a SM vs. exp discrepancy)

BR[$B_s \to \mu\mu$] systematics 1: the initial state oscillates

Dunietz, Fleischer, Nierste, PRD 01;
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- The $B_s \to \mu\mu$ rate is measured as follows:

$b\,\bar{b}$ production $\Rightarrow$ the b hadronizes into a $\bar{B}_s$ or the $\bar{b}$ hadronizes into a B_s at $t = 0$

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This is what theory calculates:
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☑ **How are BR_{th} and BR_{exp} connected**

$$\frac{BR_{th}}{1 - y_s} = BR_{exp}$$

with $y_s = \Delta\Gamma_s / (2\,\Gamma_s) \simeq 0.088$

See:

- LHCb 1212.4140
- latest HFAG average: 1207.1158

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☑ **Intuitive picture of this correction**

Recall: $BR_{th} \propto \dfrac{1}{\Gamma_s}$

D. Guadagnoli, HDR seminar

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Recall: $\mathrm{BR}_{\mathrm{th}} \propto \dfrac{1}{\Gamma_s}$

Then one finds: $\dfrac{1}{\Gamma_s} \times \dfrac{1}{1-\Delta\Gamma_s/(2\Gamma_s)} = \dfrac{1}{\Gamma_s}\,\dfrac{\Gamma_s}{\Gamma_{\mathrm{long}}}$

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Then one finds: $\frac{1}{\Gamma_s} \times \frac{1}{1 - \Delta\Gamma_s/(2\Gamma_s)} = \frac{1}{\Gamma_s}\,\frac{\Gamma_s}{\Gamma_{\mathrm{long}}}$

Namely the $1/(1-y_s)$ factor just "renormalizes" BR_{th} to the width of the long-lived B_s eigenstate

BR[$B_s \to \mu\mu$] systematics 2: soft radiation

Buras, Girrbach, DG, Isidori, EPJC 13

$$\text{Measured } BR = BR(B_s \to \mu\mu) + BR(B_s \to \mu\mu + n\gamma)\big|_{n \neq 0}$$

an arbitrary number of soft, undetected photons

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Main physics argument

- The correction is *multiplicative*, and of the form

$$BR(B_s \to \mu\mu[+n\gamma])\big|_{\sum E_{\gamma i} \leq E_{\text{cut}}} = \left(\frac{E_{\text{cut}}}{m_{B_s}/2}\right)^{\frac{\alpha_{\text{e.m.}}}{\pi}\#} \cdot BR(B_s \to \mu\mu)_{\text{th}}$$

with E_{cut} the minimum energy that one or more γ have to have to be detected.

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taking E_{cut} = 60 MeV [LHCb]
correction = 0.89

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- Multiplicative corr.: arises as the resummation of log-like corr's to an arbitrary number of photons

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- Multiplicative corr.: arises as the resummation of log-like corr's to an arbitrary number of photons
- The two scales in the ratio, E_{cut} and $m_{B_s}/2$, are the UV cutoffs of the two contributions to the corr:

  real emitted soft γ + virtual soft γ

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- The two scales in the ratio, $E_{\rm cut}$ and $m_{B_s}/2$, are the UV cutoffs of the two contributions to the corr:
  real emitted soft γ + virtual soft γ
- Only adding up these two contrib's yields an IR-safe result

BR[$B_s \to \mu\mu$] systematics 2: soft radiation vs. other sources of radiation

☑ The discussed radiation is from final-state bremsstrahlung.

What about emission of photons from the initial-state quarks?

This contribution is not helicity-suppressed, so it may be important. In $B \to \mu \nu (\gamma)$ it is.

See:
Becirevic, Haas, Kou
PLB 09

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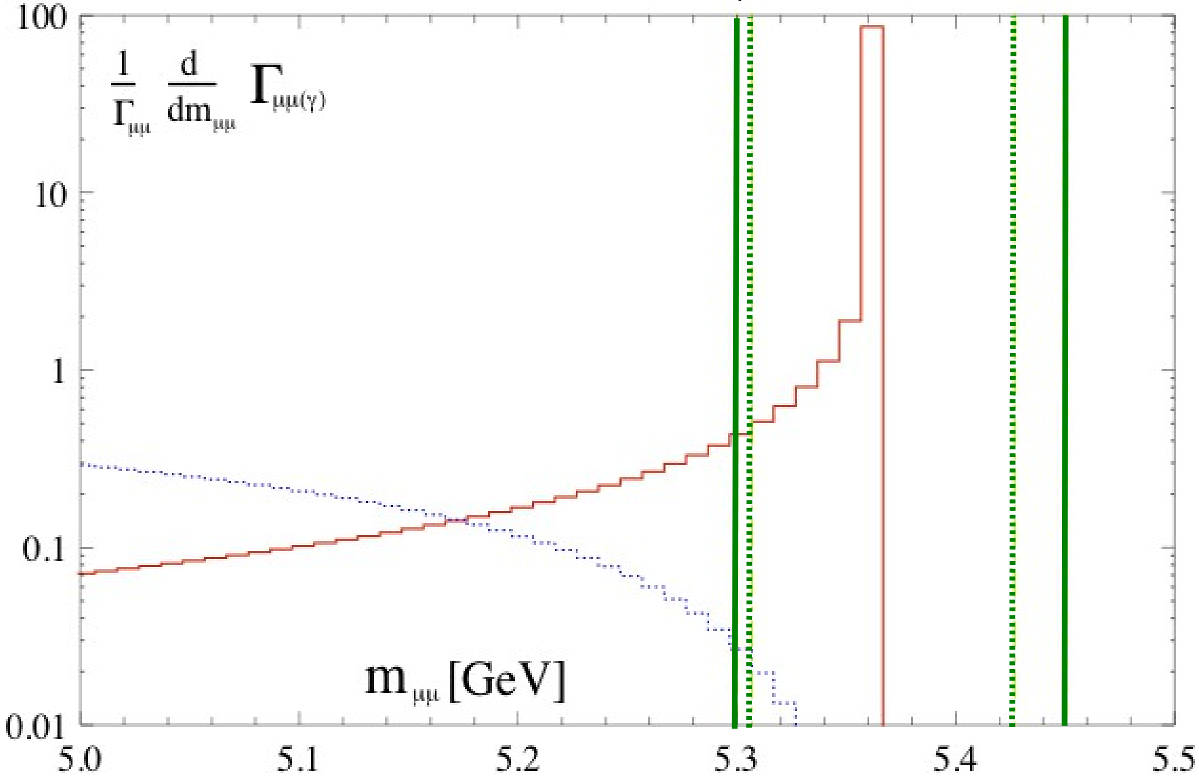

soft final-state bremsstrahlung
[Buras, Girrbach, DG, Isidori, EPJC 12]

"structure-dependent" emission
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signal window: LHCb

signal window: CMS

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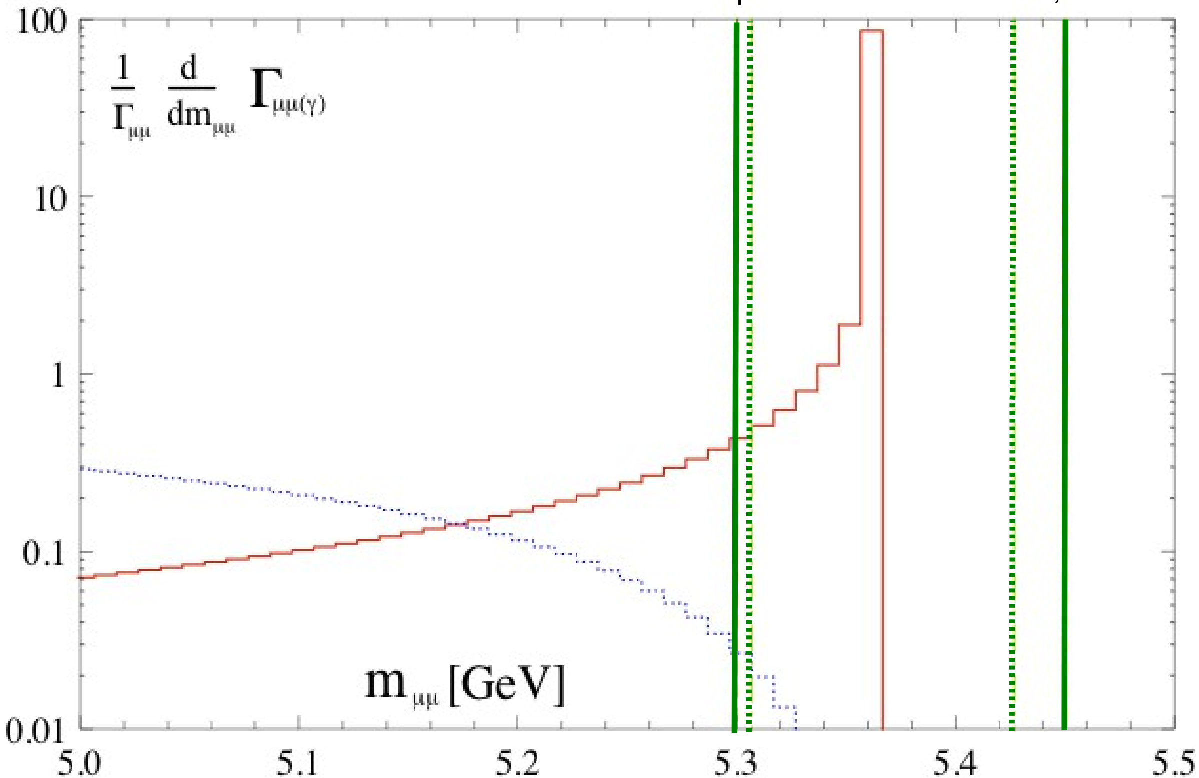

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Note that:

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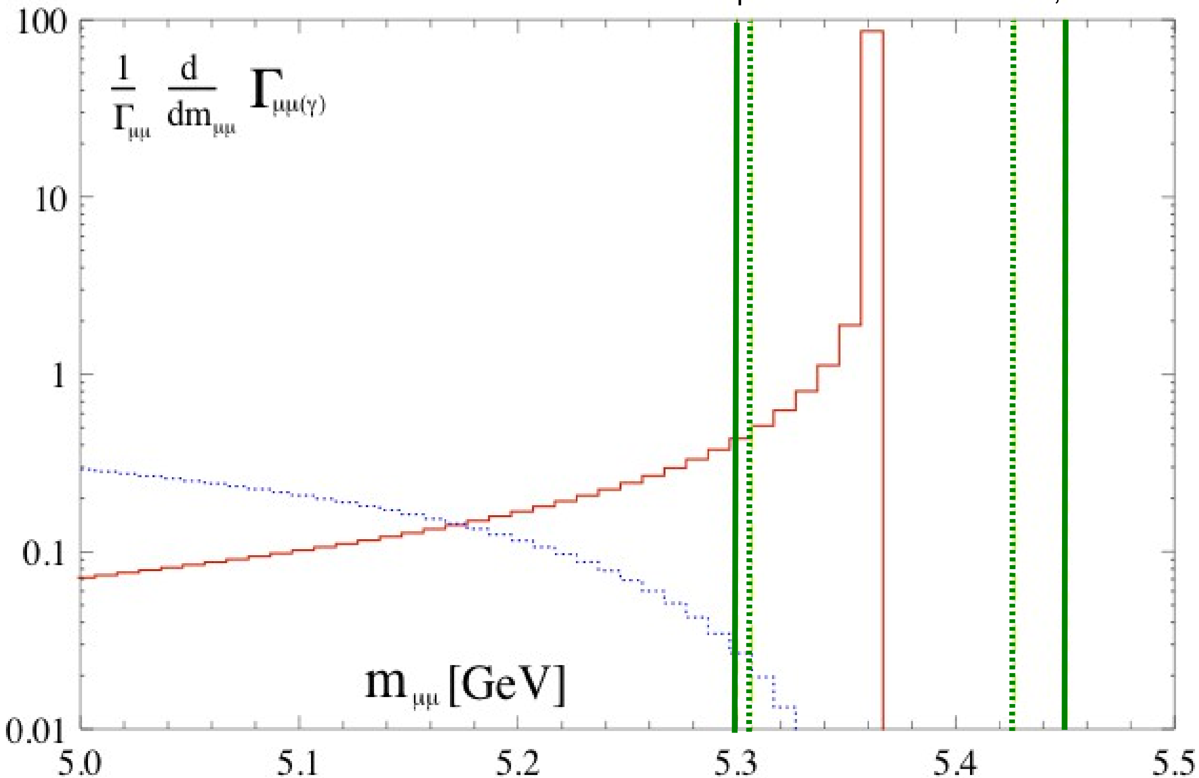

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Note that:

- the structure-dep. contribution must go to zero in the soft limit, because the initial state is neutral
- in fact, it goes to zero as E_γ^3 (Low's theorem), i.e. fastly enough to be negligible in the (narrow) signal region

$BR[B_s \to \mu\mu]$ error: recap of systematics

Initial-state effect

- **Effect of $B_s - \overline{B}_s$ oscillations:**

De Bruyn *et al.*, PRL 12 & PRD 12

$$BR_{\mathrm{exp}} = BR_{\mathrm{th}} \frac{1}{1 - \Delta\Gamma_s / 2\Gamma_s} = BR_{\mathrm{th}} \times 1.09$$

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- **Incomplete knowledge of NLO EW corrections:**

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EW short-dist. effect

- **Incomplete knowledge of NLO EW corrections:**

- *Implied systematic error comparable to f_{Bs} error*

  Albeit impact arguably small (~ O(1%)) in appropriate scheme [see Buras, Girrbach, DG, Isidori, EPJC 13]

BR[$B_s \to \mu\mu$] error: recap of systematics

Initial-state effect

- **Effect of $B_s - \overline{B}_s$ oscillations:**

  De Bruyn *et al.*, PRL 12 & PRD 12

$$BR_{\text{exp}} = BR_{\text{th}} \frac{1}{1 - \Delta\Gamma_s / 2\Gamma_s} = BR_{\text{th}} \times 1.09$$

EW final-state effect

- **Effect of soft undetected photons in the final state:**

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$$BR_{\text{exp}} = BR_{\text{th}} \times 0.89$$

Taken into account by exp

EW short-dist. effect

- **Incomplete knowledge of NLO EW corrections:**

- *Implied systematic error comparable to f_{Bs} error*

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- *Final answer: full calculation*
  - ***NLO EW:*** *Bobeth et al., 1311.1348, PRD14*
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  - *See also* ***NNLO QCD:*** *Hermann et al., 1311.1347, JHEP13*

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All in all, theory (SM) ready to match expected experimental accuracy

$B_s \to \mu\mu$

beyond the SM

BR[$B_s \to \mu\mu$]: a multi-faceted new-physics probe

- From an effective-theory point of view, 6 operators built out of SM fields, can contribute to this decay

- (One may write also two tensor operators, but their matrix elements vanish for this process.)

SM operator

$$\boxed{O_A \equiv (\bar{b}\gamma_L^\alpha s)(\bar{\mu}\gamma_\alpha\gamma_5\mu)} \qquad O'_A \equiv (\bar{b}\gamma_R^\alpha s)(\bar{\mu}\gamma_\alpha\gamma_5\mu)$$

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- **Main observation:**
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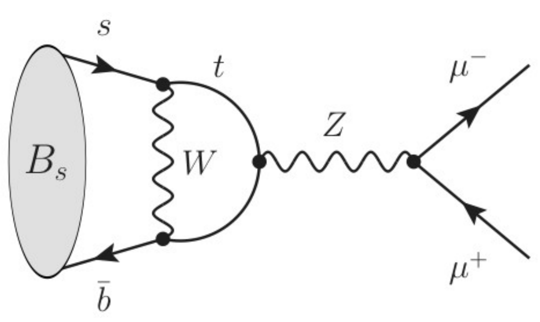
Z-penguin contribution to $B_s \to \mu\mu$



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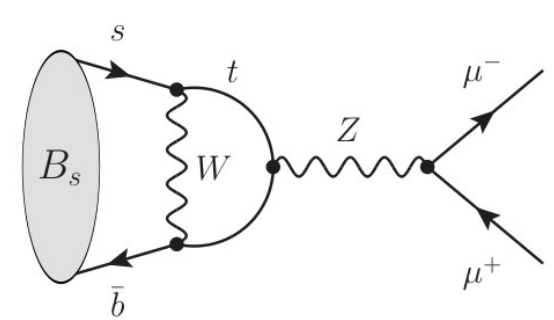

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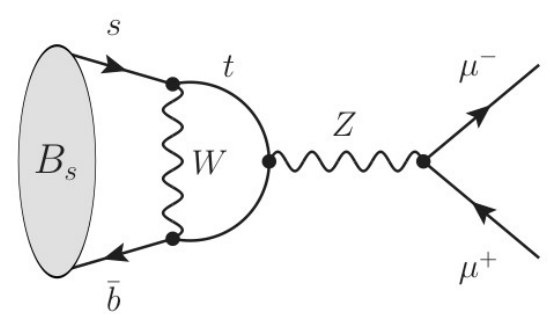

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2. $B_s \to \mu\mu$ as a probe of anomalous Z-quark couplings

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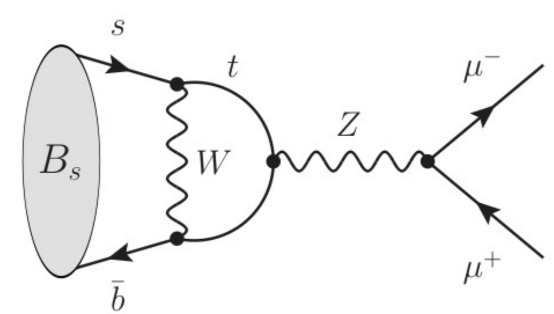

D. Guadagnoli, HDR seminar

BR[$B_s \to \mu\mu$] as an EW precision test

DG, Isidori, PLB 13

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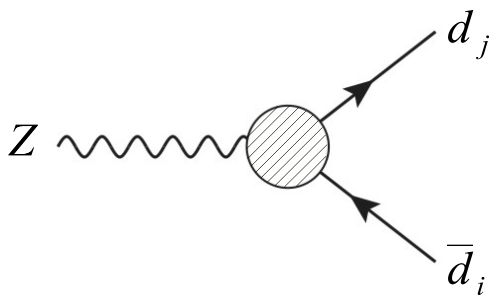

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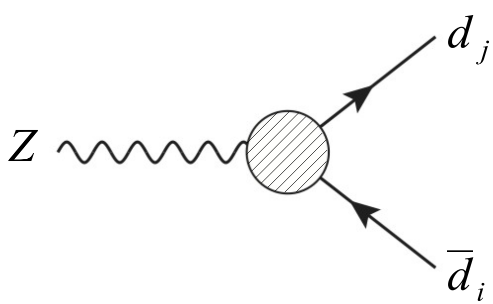

Flavor-diag: *$i = j$ (= 3)*

Affects LEP-measured
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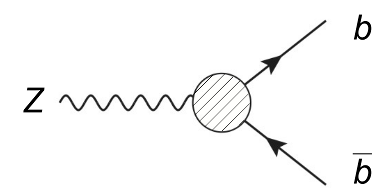

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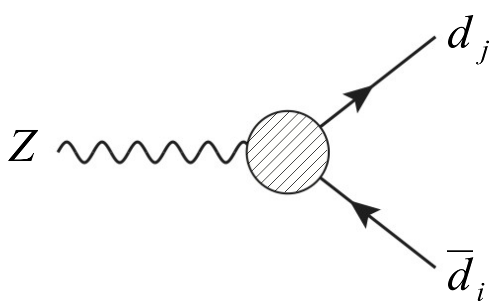

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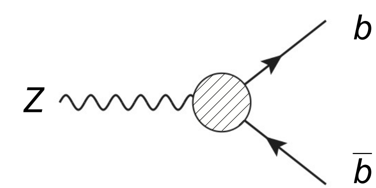

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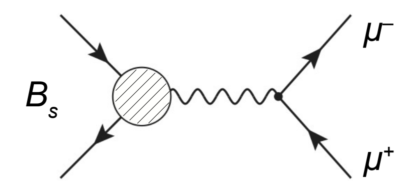

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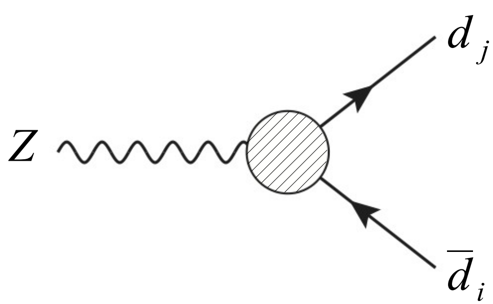

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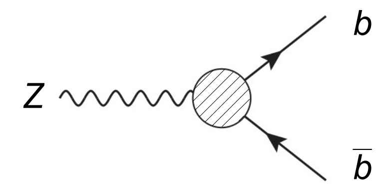

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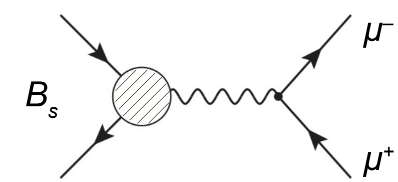

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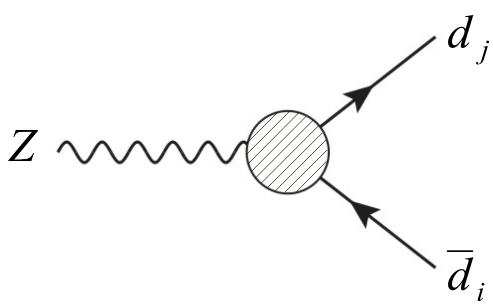

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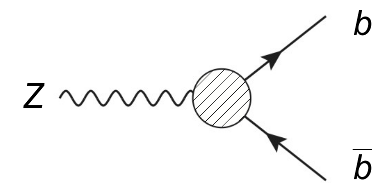

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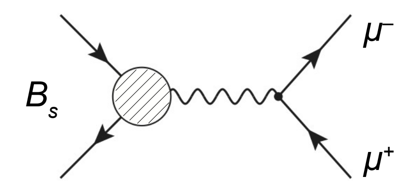

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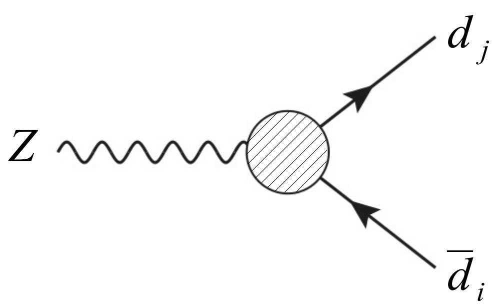

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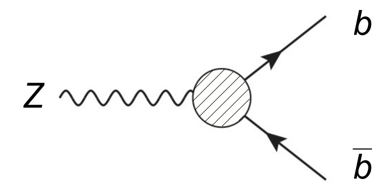

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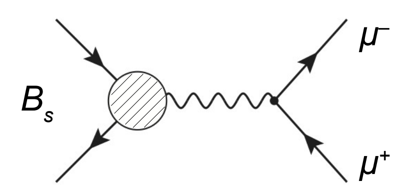

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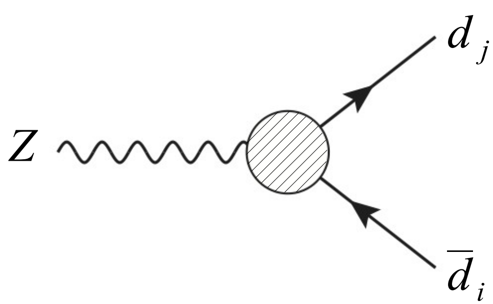

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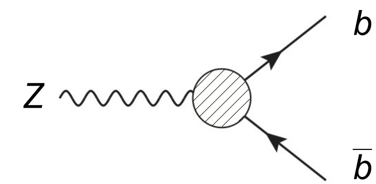

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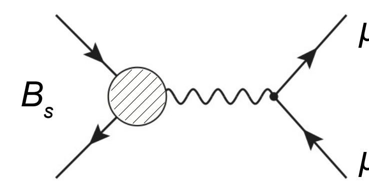

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This can be done within general and motivated frameworks such as:

- MFV
- Partial Compositeness

In either case, FV and FC couplings will be proportional to two universal shifts:
δg_L & δg_R

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DG, Isidori, PLB 13

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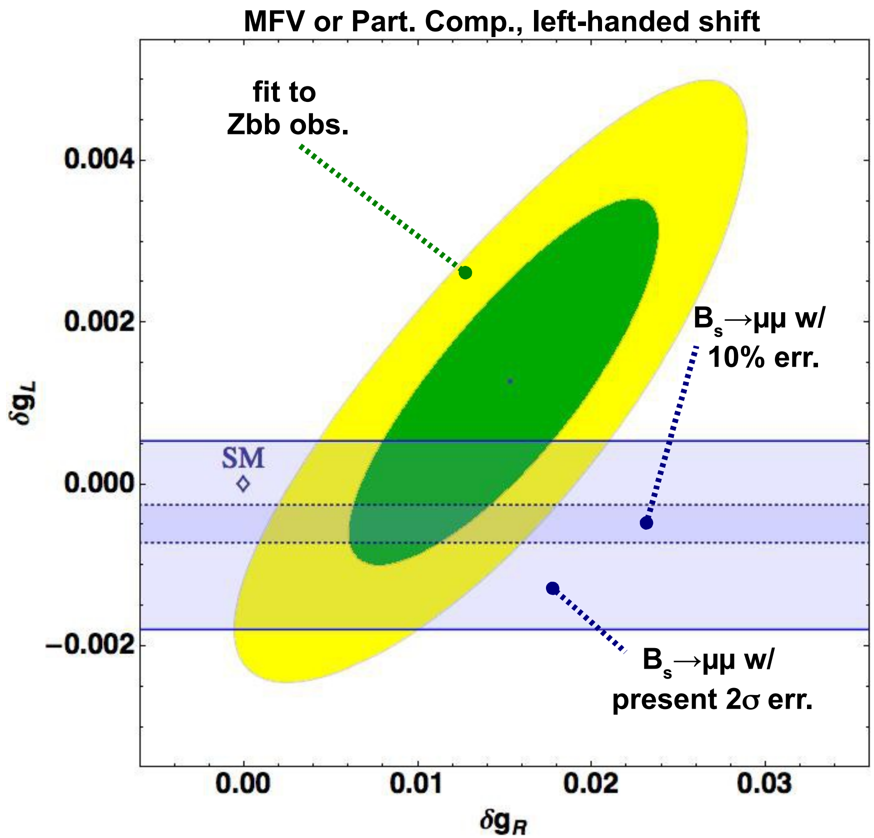

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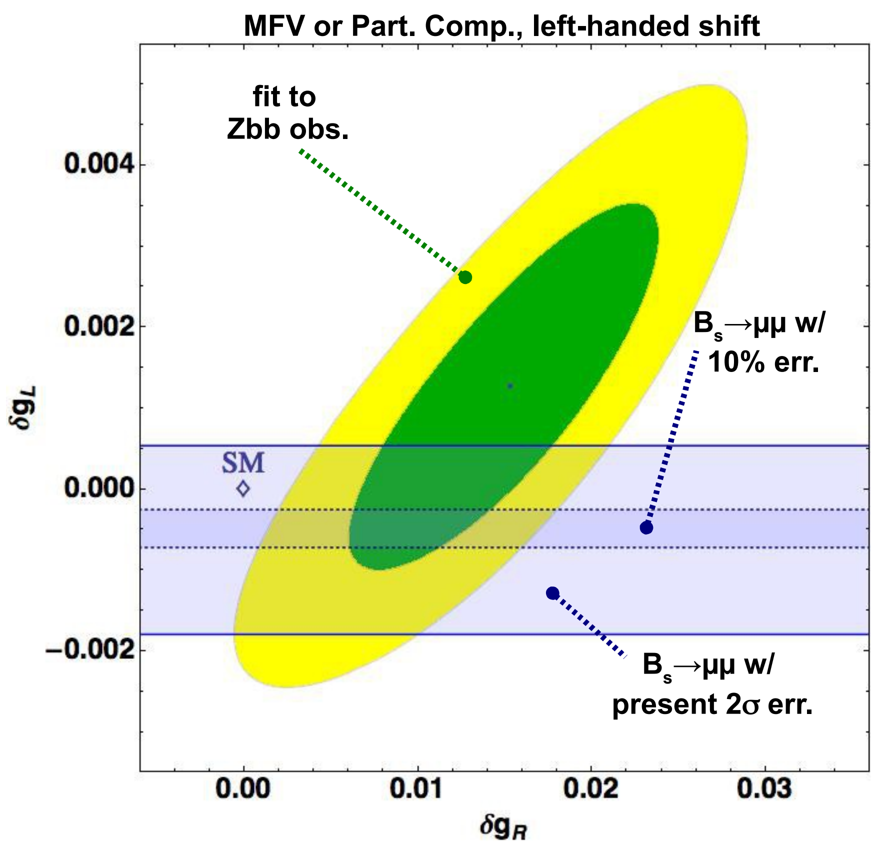

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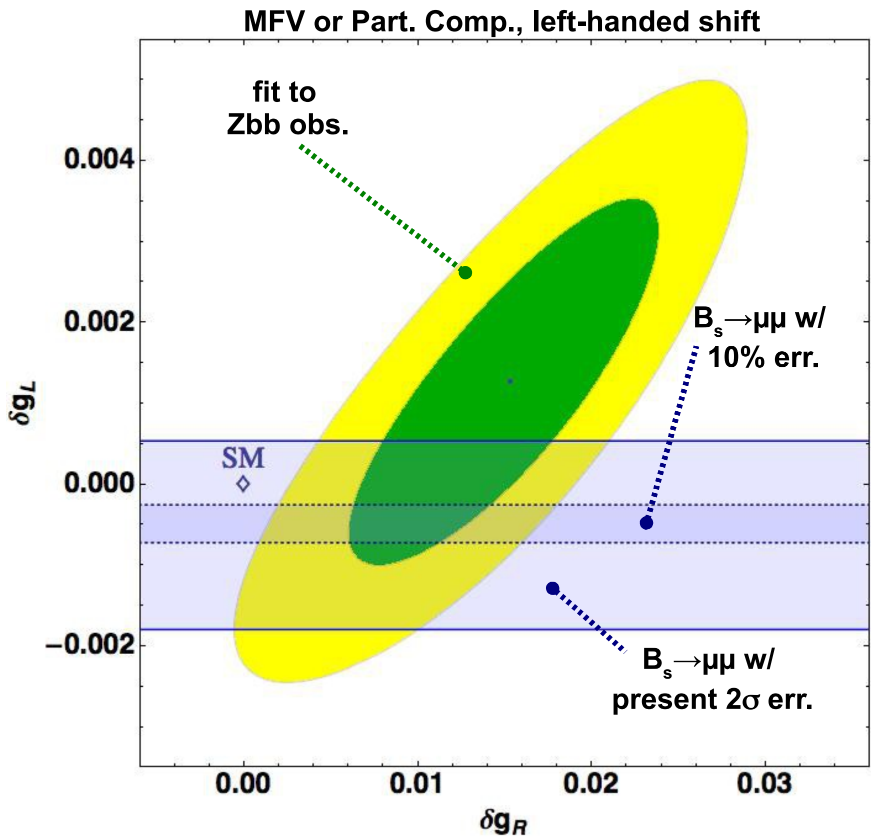

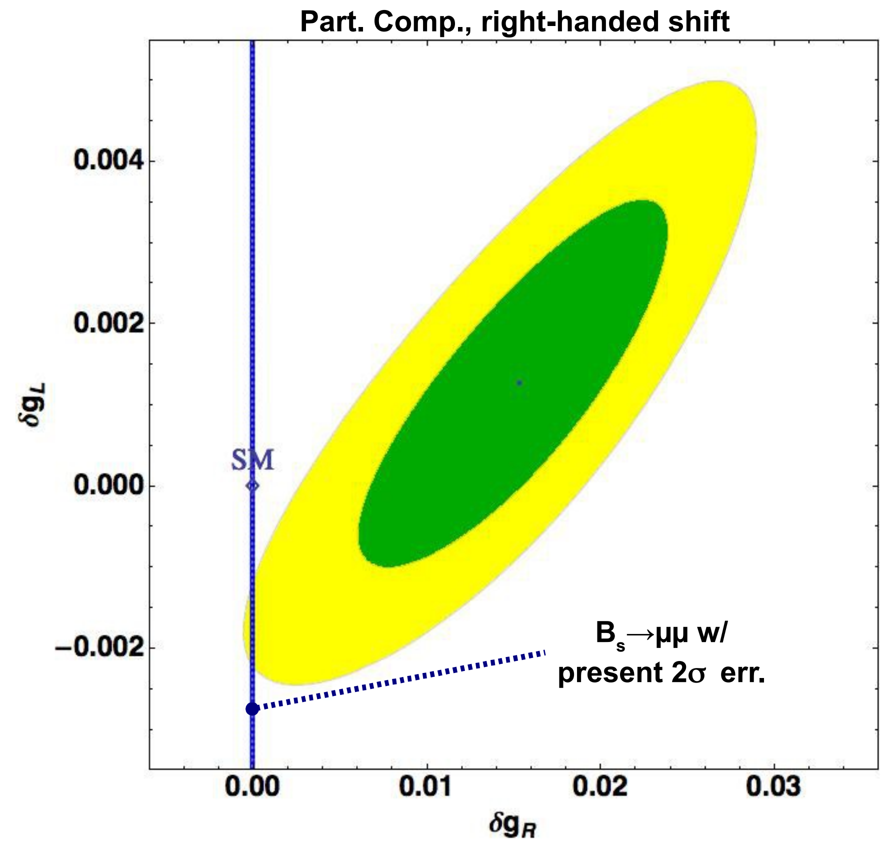

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  *crucial to ϵ_K: **LHCb and its upgrade***

☑ **$B_s \to \mu\mu$ and related decays**

- *Error will always be dominated by exp one.*

- *Focus on $B_d \to \mu\mu$ and $B_s \to \tau\tau$*

- *More $B_s \to \mu\mu$ statistics: time-dependent measurement.*
  Able to potentially probe even CP violation in $B_s \to \mu\mu$ [De Bruyn et al., PRL 12]