From Gribov copies to the mass of gluon

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Strong interaction and color degrees of freedom

- Hadrons (proton,...) made of **quarks** ($\sim$ electrons)
- Intermediate bosons: **Gluons** ($\sim$ photons)
- Gluons and quarks carry **color charge** ($\sim$ electric charge)
- Quantum chromodynamic (QCD) describes their interactions

Confinement

No colored states seen in nature $\Rightarrow$ **Physical states are colorless bound states**.

- Confinement is a large scale property $\rightarrow$ Infra-red (IR) regime
- Dynamic of Gluons is believed to be responsible of the confinement

Study simpler case of pure Gluon-dynamic = **Yang-Mills (YM)** theories

Yang-Mills (YM) theories and non-abelian gauge symmetry

YM Lagrangian: $\mathcal{L}_{YM}\left[A_\mu(x), g\right],$

$A_\mu(x)$ Gluon (matrix) field, g coupling constant

Describes **massless** self interacting Gluons.

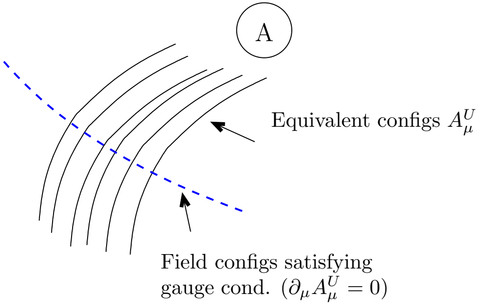
- Presents Gauge symmetry:

$$\begin{aligned} A_\mu(x) \to A_\mu^U(x) &= U(x)A_\mu(x)U^\dagger(x) + \frac{i}{g}U(x)\partial_\mu U^\dagger(x) \\ U(x) \quad &, \quad \text{a local element of } SU(N_c) \end{aligned}$$

- Physical observables are gauge-invariant: $\mathcal{O}_{phys}\left[A_\mu\right] = \mathcal{O}_{phys}\left[A_\mu^U\right]$

Gauge fixing

- Need to fix the gauge in order to quantize the theory:
  1. **gauge orbit** $\left\{A_\mu^U\right\}_U$: Set of field configurations linked by a gauge transformation
  2. **choose a gauge** : select one representative per gauge orbit with additional constrain
     e.g. $\partial_\mu A_\mu^U = 0$



  3. **gauge-fixing procedure** Include this condition in the initial Lagrangian:
     $\mathcal{L}_{YM} \to \mathcal{L}_{YM} + \mathcal{L}_{GF}$ (Faddeev-Popov (FP) procedure)

Perturbation theory and confinement

Compute quantities in order to probe the IR physic and features of confinement.

Building blocks are Green functions (correlation functions) e.g. propagator: $\left\langle A_\mu^a(x) A_\nu^b(y) \right\rangle$

Standard analytic technique is perturbation theory: expansion in power of the coupling constant g, Feynman diagrams machinery,...

Non-abelian gauge symmetry and perturbation theory

Non-abelian gauge symmetry implies that:

1. coupling constant vanishes at high energy, **asymptotic freedom**
2. coupling constant diverges at finite energy, **Landau pole** in the IR

Confinement

It is believed that confinement is a **non-perturbative feature** of YM theories.

Need non-perturbative techniques, e.g. lattice simulations.

(gauge-fixed) Lattice simulations

$\left\langle A^a_\mu A^b_\nu \right\rangle (p) = G(p)(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2})\delta^{ab}$

Lattice simulations in the Landau gauge: $\partial_\mu A_\mu = 0$ were performed

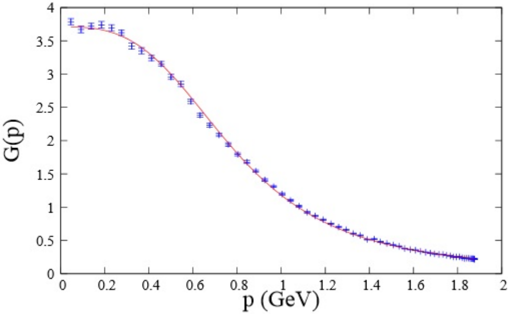
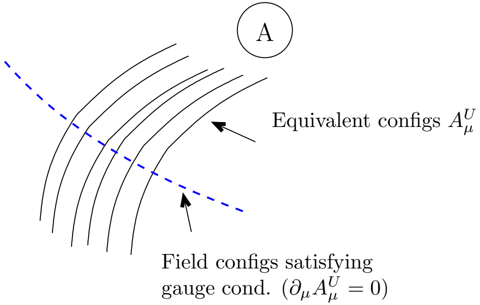


FIGURE: Gluon propagator. Bleu: lattice simulations

Propagator is not divergent at zero momenta $\rightarrow$ **massive gluon**

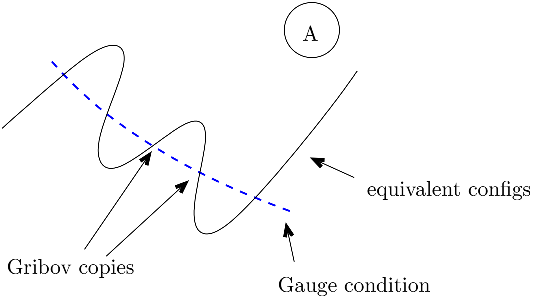
Gribov copies

- **gauge orbit** $\left\{A_\mu^U\right\}_U$: Set of field configurations linked by a gauge transformation
- **choose a gauge** : select one representative per gauge orbit with additional constrain
  e.g. $\partial_\mu A_\mu^U = 0$



Gribov copies

- **But** : Infinitely many U that satisfy $\partial_\mu A_\mu^U = 0 \rightarrow$ **Gribov copies**

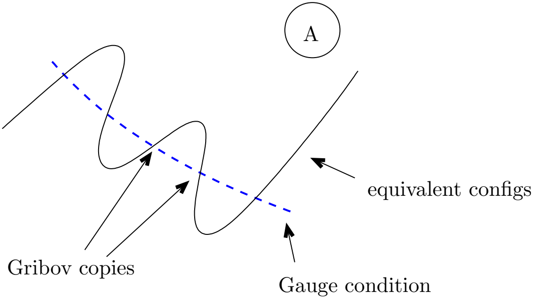


Gribov copies not taken into account in analytical gauge-fixing procedure!

- Assumed to be irrelevant in the UV
- Influence for the IR physics?

How consistently take into account Gribov copies?

1. "standard" procedure (pint point a copy):
   - analytic: Gribov-Zwanziger proposal $\rightarrow$ not enough!
   - Lattice simulations: use numerical algorithms $\rightarrow$ non tractable analytically



2. Serreau-Tissier proposal:
   - take into account all the Gribov copies
   - averaging over them in defining vaccum expectation values
   - lift their degeneracy in the averaging procedure

Lifting the Gribov ambiguity

$\rightarrow$ new gauge fixing Lagrangian $\mathcal{L}^{'}_{GF}$ different from the one obtained through the FP procedure.

New properties of the such as gauge fixed YM theories

1. Massive Gluons: $\mathcal{L}_{m^2} \in \mathcal{L}^{'}_{GF} \rightarrow$ **Gluons mass appears as a consequence of Gribov copies**

Presents good theoretical properties

1. Computations of physical observables are those of standard YM
2. Infra-red safe trajectories (no Landau pole) $\rightarrow$ IR physics accessible through perturbation theory
3. Renormalizable
4. Theory remains assymptotically free

First results 1

Computed all the propagators and renormalization group (RG) trajectories of the parameter at one-loop order.

- check of the proof of renormalizability

- very good agreement with lattice simulation in the Landau gauge

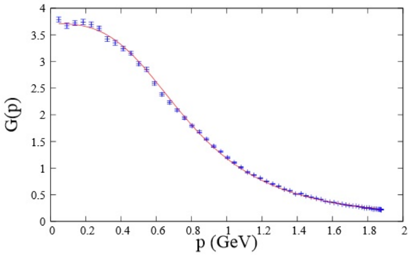


FIGURE: Gluon propagator. Red: perturbative calculations Bleu: lattice simulations

- IR-safe RG trajectories in different gauge

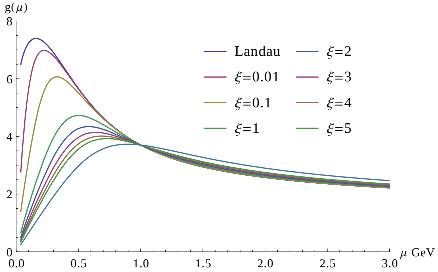


FIGURE: RG flow of the coupling constant in different gauges indexed by ξ

First results 2

Generation of a mass gap.

- Massive gluon

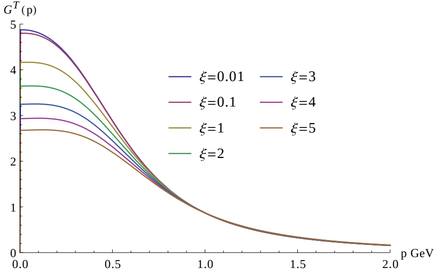


FIGURE: Gluon propagator

- Massive ghosts

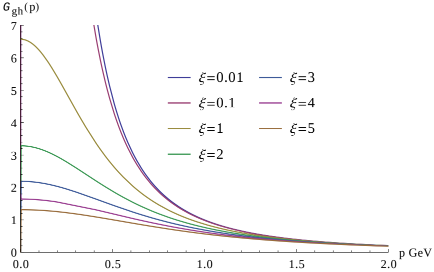


FIGURE: Ghost propagator

Outlook

CONCLUSION

1. **Gauges where gluon are massive**
2. **IR Yang-Mills physics can be probed perturbatively !**

Up to now

1. Prove the Renormalizability, asymptotic freedom is conserved, gluon mass vanishes in the UV
2. Computations of the propagators
3. Study of the infra-red safe trajectories

Future

1. Computations of physical observables
2. Link between Gribov copies and confinement?
3. Non-zero temperature, confining-deconfining phase transition (already done)

Boum 1

- average over Gribov copies

$$\langle \mathcal{O}[A] \rangle = \frac{\int \mathcal{D}\eta \mathcal{P}[\eta] \sum_i \mathcal{O}[A^{U_i}] s(i) e^{-\beta_0 \mathcal{H}[A,\eta,U_i]}}{\int \mathcal{D}\eta \mathcal{P}[\eta] \sum_i s(i) e^{-\beta_0 \mathcal{H}[A,\eta,U_i]}},$$

- average over YM weight

$$\overline{\langle \mathcal{O}[A] \rangle} = \frac{\int \mathcal{D}A \, \langle \mathcal{O}[A] \rangle \, e^{-S_{\mathrm{YM}}[A]}}{\int \mathcal{D}A \, e^{-S_{\mathrm{YM}}[A]}}.$$

Boum 2

$$
\begin{aligned}
\mathcal{L} \quad = \quad & \frac{1}{4}\left(F^a_{\mu\nu}\right)^2 + \partial_\mu \bar{c}^a D_\mu c^a + \beta_0 \left(\frac{1}{2}(A^a_\mu)^2 + \xi_0 \bar{c}^a c^a\right) + i h^a \partial_\mu A^a_\mu \\
& + \xi_0 \left[\frac{(h^a)^2}{2} - \frac{g_0}{2} f^{abc} i h^a \bar{c}^b c^c - \frac{g_0^2}{4}\left(f^{abc}\bar{c}^b c^c\right)^2\right] \\
& \underbrace{+ \frac{1}{g_0^2}\sum_{k=2}^{n}\int_{\underline{\theta}_k} \mathrm{tr}\left\{D_\mu \mathcal{V}_k^\dagger D_\mu \mathcal{V}_k + \frac{\xi_0}{2} g^{MN} \partial_N \mathcal{V}_k^\dagger \partial_M \mathcal{V}_k\right\}}_{\mathcal{L}_{SUSY}} .
\end{aligned}
$$

with

$$
\mathcal{V}(x,\theta,\bar{\theta}) = \exp\left\{ i g_0 \left(\bar{\theta} c + \bar{c}\theta + \bar{\theta}\theta \hat{h}\right)\right\} U,
$$