

# Jet Physics and the Small- $R$ Limit

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# Outline

## 1 Introduction

- Jet algorithms
- Perturbative properties of jets

## 2 Generating functionals

- Evolution equations

## 3 Observables

- Inclusive microjet observables
- Hardest microjet observables
- Microjet vetoes

## 4 Conclusion

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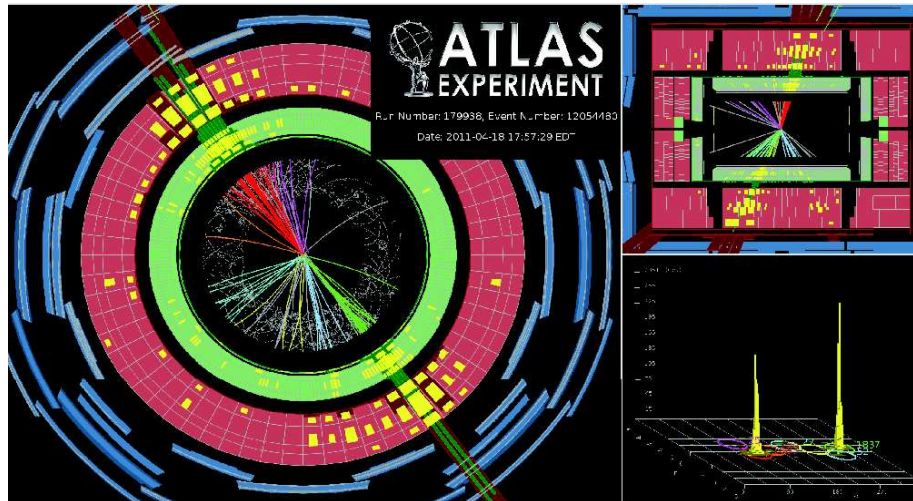
## 3 Observables

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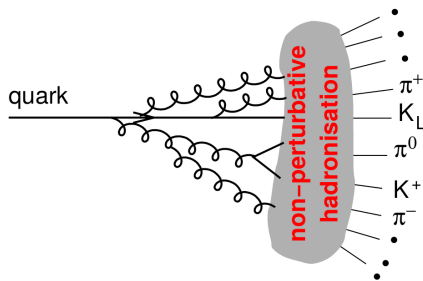
# What constitutes a Jet?

Jets are collimated bunches of particles produced by hadronization of a quark or gluon.



# What constitutes a Jet?

- Jets can emerge from a variety of processes
  - ▶ scattering of partons inside colliding protons,
  - ▶ hadronic decay of heavy particles,
  - ▶ radiative gluon emission from partons, ...
- We use jet algorithms to combine particles in order to retrieve information on what happened in the event.
- No unique or optimal definition of a jet, but jets are widely used at hadron colliders as a proxy for hard quarks and gluons.



*figure by G. Salam*

# Why are Jets important?

- QCD processes are at the heart of modern hadron colliders.
- Most of CMS and ATLAS searches make use of jets.
- The increase in energy and pileup at the LHC is raising the necessity for a deeper understanding of jet processes.

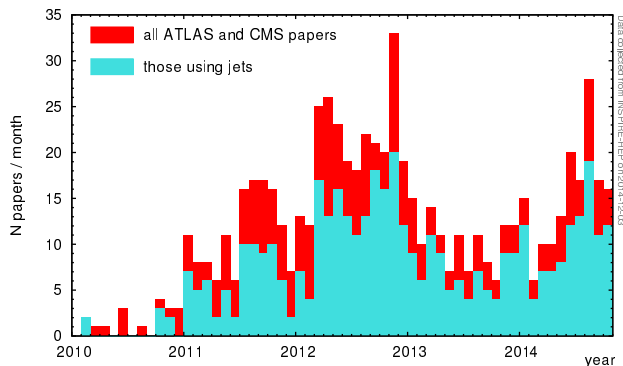
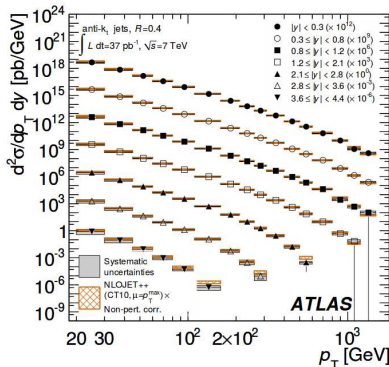
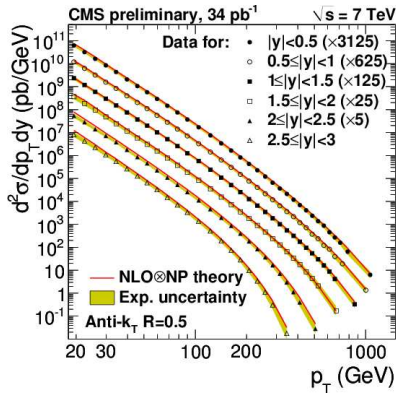


figure by G. Salam

# Example: Inclusive Jet Spectrum

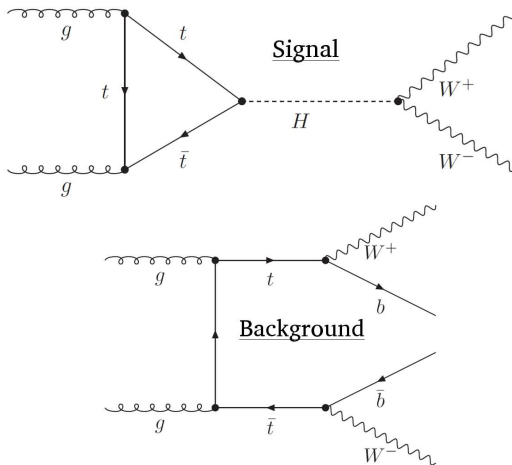
Agreement between experiment and theory over several orders of magnitude  $\Rightarrow$  precise probe of underlying interactions



## Example: Background discrimination in Higgs production

Main background to Higgs production via gluon fusion (with  $W^+W^-$  decay) is  $t\bar{t}$  production.

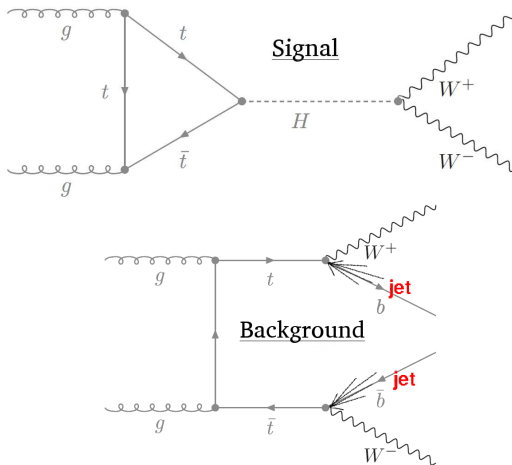
⇒ background can be separated with veto on hard jets.



## Example: Background discrimination in Higgs production

Main background to Higgs production via gluon fusion (with  $W^+W^-$  decay) is  $t\bar{t}$  production.

$\Rightarrow$  background can be separated with veto on hard jets.



# How are Jets defined?

A jet definition includes

- a jet algorithm mapping final state particle momenta to jet momenta,
- parameters required by the algorithm, typically the jet radius  $R$ ,
- a recombination scheme indicating what momentum to assign to the combination of two particles (eg. 4-vector sum)

A *good* jet definition should also

- be simple to implement in an experimental analysis,
- be simple to implement in theoretical calculations,
- yield cross sections that are finite at any order in perturbation theory and relatively insensitive to hadronization.

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# Jet algorithm classes

There are two main classes of jet algorithms

## Cone algorithms

A top-down approach, centered around the idea of finding stable cones from energy flow.

## Sequential recombination algorithms

A bottom-up approach, defined by an iterative recombination of particles using a distance measure.

At the LHC, most jet definitions used are based on sequential recombination algorithms, as they are simpler and closer to the underlying QCD branching picture.

# Jet algorithms and the jet radius

A jet algorithm maps final state particle momenta to jet momenta.

$$\underbrace{\{p_i\}}_{\text{particles}} \implies \underbrace{\{j_k\}}_{\text{jets}}$$

This requires an external parameter, the jet radius  $R$ , which specifies an angular scale.

The jet radius  $R$  defines up to which point separate partons are recombined into a single jet.

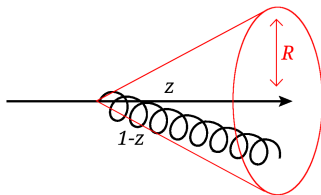


Figure : Gluon emission from a quark combined into a single jet.

# Cambridge/Aachen algorithm with incoming hadrons

The basic idea is to invert QCD branching process, clustering pairs which are closest in metric defined by the divergence structure of the theory

## Definition

- 1 For any pair of particles  $i, j$  find the minimum of

$$d_{ij} = \frac{\Delta R_{ij}^2}{R^2}$$

where  $\Delta R_{ij} = (y_i - y_j)^2 + (\phi_i - \phi_j)^2$ .

- 2 If the minimum distance  $d_{ij} > 1$  then the corresponding particle is removed from the list and defined as a jet, otherwise  $i$  and  $j$  are merged.
- 3 Repeat until no particles are left.

Most algorithms used nowadays at hadron colliders follow this pattern, with some variations in the distance measure (eg. the anti- $k_t$  algorithm).

# Perturbative properties

Jet properties will be affected by gluon radiation and  $g \rightarrow q\bar{q}$  splitting.

In particular, considering gluon emissions from an initial parton for a jet of radius  $R$ , then

- radiation at angles  $> R$  reduces the jet energy,
- radiation at angles  $< R$  generates a mass for the jet.

We will try to investigate the effects of perturbative radiation on a jet analytically, particularly in the small- $R$  limit.

## Example: Jet $p_t$ with emissions at angle $\theta > R$

We can calculate the average energy difference between the hardest final state jet and the initial quark, considering emissions beyond the reach of the jet. In the small- $R$  limit, we find

$$\begin{aligned}\langle \Delta z \rangle_q^{\text{hardest}} &= \int^{\mathcal{O}(1)} \frac{d\theta^2}{\theta^2} \int dz (\max[z, 1-z] - 1) \frac{\alpha_s}{2\pi} p_{qq}(z) \Theta(\theta - R) \\ &= \frac{\alpha_s}{\pi} C_F \left( 2 \ln 2 - \frac{3}{8} \right) \ln R + \mathcal{O}(\alpha_s)\end{aligned}$$

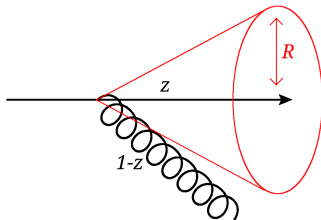


Figure : Gluon emission beyond the reach of the jet.

# Jet radius values

In recent years, jet radii have become ever smaller.

What are usual values for the jet radius  $R$  ?

- Most common choice of jet radius is  $R = 0.4$  (ATLAS) or  $R = 0.5$  (CMS).
- In some environments (eg. heavy ions), even smaller values down to  $R = 0.2$  are used.
- Many modern jet tools, such as trimming and filtering, resolve small subjets (typically with  $R_{\text{sub}} = 0.2 - 0.3$ ) within moderate  $R$  jets.

Perturbation theory breaks down when

$$\alpha_s \ln \frac{1}{R} \sim \mathcal{O}(1)$$

where the resummation of  $\alpha_s^n \ln^n R$  terms to all orders is required.

# Microjets

## Definition

Microjets are jets with small values for the jet radius

$$R \ll 1$$

Small- $R$  limit relevant in a number of contexts, e.g.

- In Higgs physics, where complicated dependence on the jet radius appears due to clustering, in particular in the resummation of jet veto logarithms.
- Decay of heavy particles to boosted  $W, Z$  bosons and top quarks.
- Heavy-ion physics where small values for  $R$  are used due to the large background.
- In high pileup environments, where use of smaller  $R$  might help mitigate adverse effects of pileup.

Theoretically interesting because  $\alpha_s \ln R \gg \alpha_s$ , therefore calculations simplify and one can investigate all-order structure.

# How relevant are small- $R$ effects?

We can evaluate numerically how important the effect of perturbative  $\ln R$  terms is on the microjet  $p_t$ .

Taking  $R = 0.2$  we find that

- quark-induced jets have a hardest microjet  $p_t \sim 5 - 10\%$  smaller than the original quark,
- gluon-induced jets have a hardest microjet  $p_t \sim 15 - 25\%$  smaller than the original gluon.

# Resummation of $(\alpha_s \ln R)^n$ terms

*“In the small  $R$  limit, new clustering logarithms [...] arise at each order and cannot currently be resummed.”*

— Tackmann, Walsh & Zuberi ([arXiv:1206.4312](https://arxiv.org/abs/1206.4312))

How important can contributions from higher orders be, e.g.  $(\alpha_s \ln R)^n$ , especially at smaller values of  $R$  ?

We aim to resum all leading logarithmic  $(\alpha_s \ln R)^n$  terms in the limit of small  $R$  for a wide variety of observables.

We will approach this question using generating functionals.

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## Evolution variable $t$

Start with a parton and consider emissions at successively smaller angular scales.

We introduce an evolution variable  $t$  corresponding to the integral over the collinear divergence weighted with  $\alpha_s$

$$t = \int_{R^2}^1 \frac{d\theta^2}{\theta^2} \frac{\alpha_s(p_t \theta)}{2\pi} = \frac{1}{b_0} \sum_{n=1}^{\infty} \frac{1}{n} \left( \frac{\alpha_s b_0}{2\pi} \ln \frac{1}{R^2} \right)^n$$

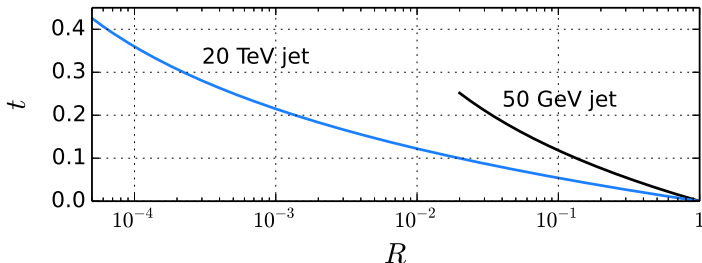


Figure : Plot of  $t$  as a function of  $R$  down to  $Rp_t = 1$  GeV for  $p_t = 0.01 - 20$  TeV.

# Generating functional

## Definition

$Q(x, t_1, t_2)$  is the generating functional encoding the parton content one would observe when resolving a quark with momentum  $x p_t$  at scale  $t_1$  on an angular scale  $t_2 > t_1$  (ie.  $R_1 \gg R_2$ ).

The mean number of quark microjets of momentum  $z p_t$  produced from a quark of momentum  $p_t$  are

$$\frac{dn_{q(z)}}{dz} = \frac{\delta Q(1, 0, t_2)}{\delta q(z)} \Big|_{\forall q(z)=1, g(z)=1}$$

We can formulate an evolution equation for the generating functionals

$$Q(x, 0, t) = Q(x, \delta_t, t) \left( 1 - \delta t \int dz p_{qq}(z) \right) + \delta t \int dz p_{qq}(z) \left[ Q(zx, \delta_t, t) G((1-z)x, \delta_t, t) \right].$$

The gluon generating functional  $G(x, t_1, t_2)$  is defined the same way.

# Quark evolution equation

We rewrite the equation on slide [17] in graphical form as an evolution equation for the quark generating functional,

$$\frac{d}{dt} \text{---} Q(x,t) = \int dz p_{qq}(z) \left[ \begin{array}{c} \text{---} Q(zx,t) \\ \text{---} G((1-z)x,t) \end{array} \right] - \text{---} Q(x,t)$$

Here the blobs represent the generating functionals at a scale  $t$ .

# Evolution equations

We can write the complete evolution equations as differential equations, for the quark the previous graph corresponds to

## Quark

$$\frac{dQ(x, t)}{dt} = \int dz p_{qq}(z) [Q(zx, t) G((1 - z)x, t) - Q(x, t)] .$$

In the gluon case we find,

## Gluon

$$\begin{aligned} \frac{dG(x, t)}{dt} = & \int dz p_{gg}(z) [G(zx, t) G((1 - z)x, t) - G(x, t)] \\ & + \int dz n_f p_{qg}(z) [Q(zx, t) Q((1 - z)x, t) - G(x, t)] . \end{aligned}$$

# Solving the evolution equations

We can solve these equations order by order as a power expansion in  $t$ , writing

$$Q(x, t) = \sum_n \frac{t^n}{n!} Q_n(x),$$

$$G(x, t) = \sum_n \frac{t^n}{n!} G_n(x).$$

Furthermore the evolution equations can be used to perform an all-order resummation of  $(\alpha_s \ln R)^n$  terms.

These methods allow us to calculate observables in the small- $R$  limit up to a fixed order in perturbation theory, or to resum them to all orders numerically.

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# Inclusive microjet observables

## Definition

Given a parton of flavour  $i$ , the inclusive microjet fragmentation function  $f_{j/i}^{\text{incl}}(z, t)$  is the inclusive distribution of microjets of flavour  $j$  carrying a momentum fraction  $z$ .

The inclusive microjet fragmentation function satisfies a DGLAP-like equation.

$$\frac{df_{j/i}^{\text{incl}}(z, t)}{dt} = \sum_k \int_z^1 \frac{dz'}{z'} P_{jk}(z') f_{k/i}^{\text{incl}}(z/z', t),$$

with initial condition

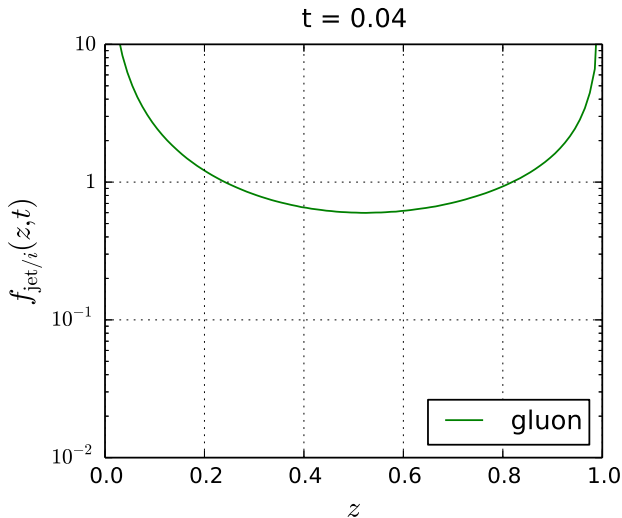
$$f_{j/i}^{\text{incl}}(z, 0) = \delta(1 - z) \delta_{ji}.$$

Momentum conservation ensures that

$$\sum_j \int dz z f_{j/i}^{\text{incl}}(z, t) = 1.$$

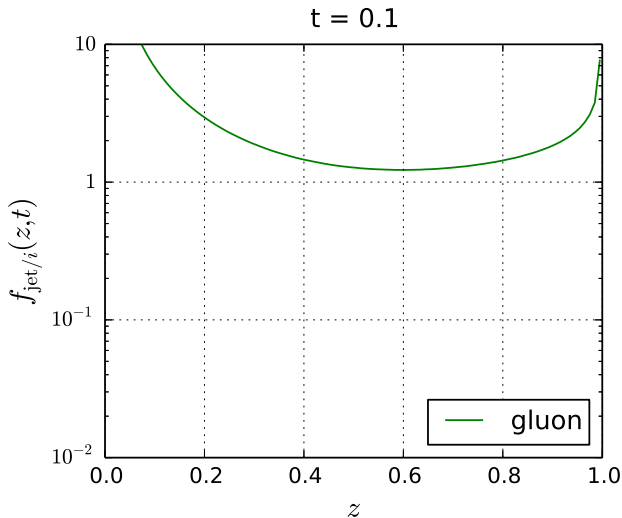
# Inclusive microjet fragmentation function

Peak at 1 is original parton, peak at 0 is soft gluon microjets.



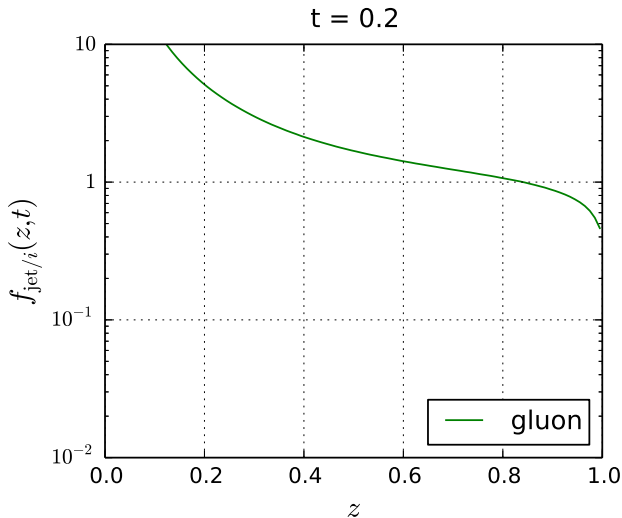
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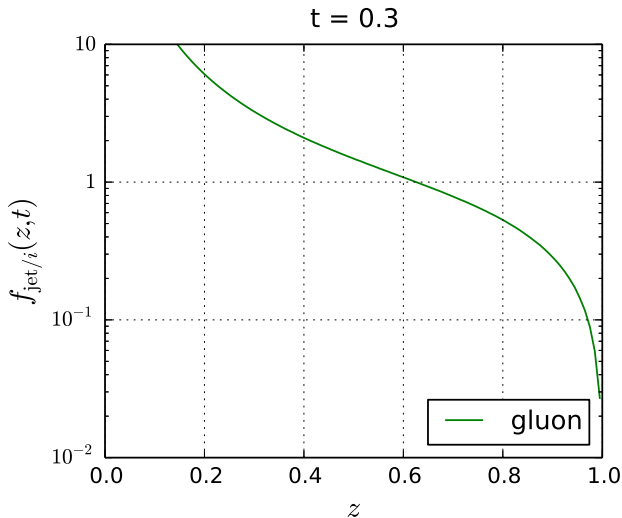
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# Jet spectrum from microjet fragmentation function

The jet spectrum can be obtained from the convolution of the inclusive microjet fragmentation function with the inclusive partonic spectrum from hard  $2 \rightarrow 2$  scattering

$$\frac{d\sigma_{\text{jet}}}{dp_t} = \sum_i \int_{p_t} \frac{dp'_t}{p'_t} \frac{d\sigma_i}{dp'_t} f_{\text{jet}/i}^{\text{incl}}(p_t/p'_t, t),$$

where  $f_{\text{jet}/i} \equiv \sum_j f_{j/i}$ .

If we assume that the partonic spectrum is dominated by a single flavour  $i$  and that its  $p_t$  dependence is  $d\sigma_i/dp_t \sim p_t^{-n}$  then

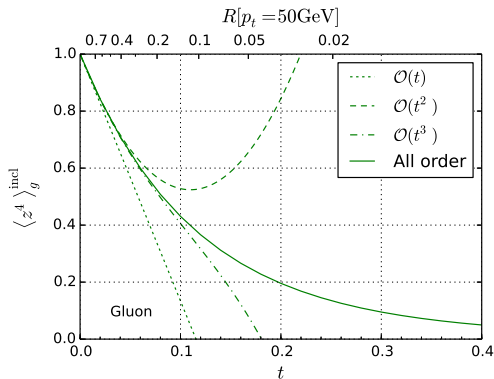
$$\frac{d\sigma_{\text{jet}}}{dp_t} \simeq \frac{d\sigma_i}{dp_t} \int_0^1 dz z^{n-1} f_{\text{jet}/i}^{\text{incl}}(z, t) \equiv \frac{d\sigma_i}{dp_t} \langle z^{n-1} \rangle_i^{\text{incl}}.$$

## Moment of inclusive microjet spectrum $\langle z^4 \rangle$

At the LHC typical  $n$  values for the partonic spectrum range from about 4 at low  $p_t$  to 7 or even higher at high  $p_t$ . We show results for  $n = 5$ .

Small- $R$  terms are important here, around **30 – 50% effect** on gluonic inclusive spectrum.

Convergence is slow: for gluon-initiated jets, the  $\mathcal{O}(t^2)$  corrections (ie. NNLO) deviate noticeably from all-orders results below  $R = 0.3$



# Hardest microjet observables

## Definition

$f^{\text{hardest}}(z)$  is the probability that the hardest microjet carries a momentum fraction  $z$ .

Probability conservation imposes

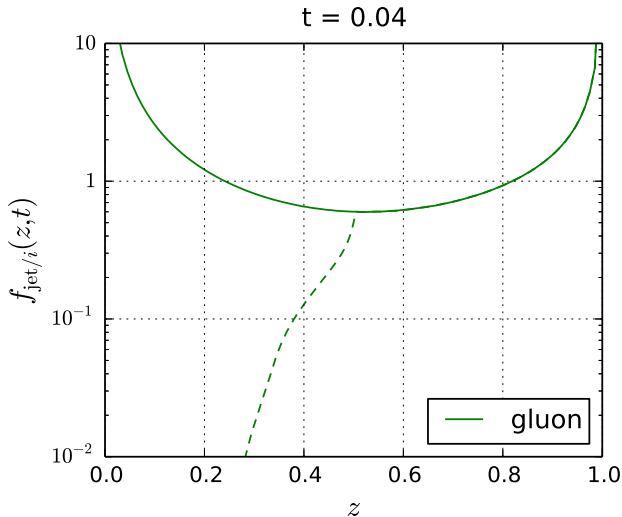
$$\int_0^1 dz f^{\text{hardest}}(z) = 1$$

No general DGLAP-like equation, but equal to the inclusive microjet fragmentation function for  $z > 0.5$ .

# Hardest microjet fragmentation function

Solid line: inclusive microjet fragmentation function.

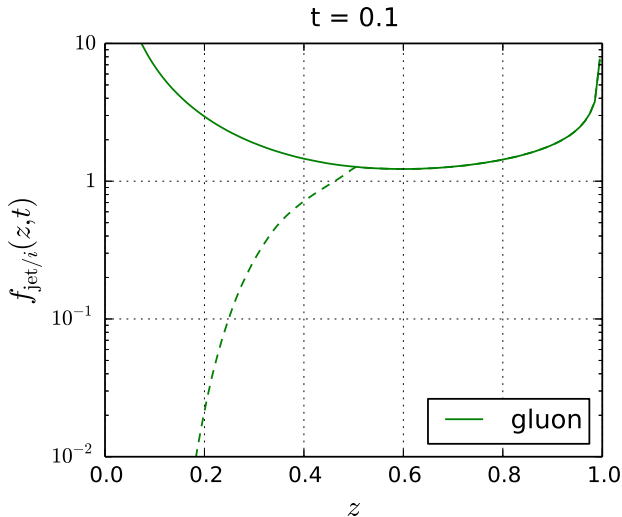
Dashed line: hardest microjet fragmentation function.



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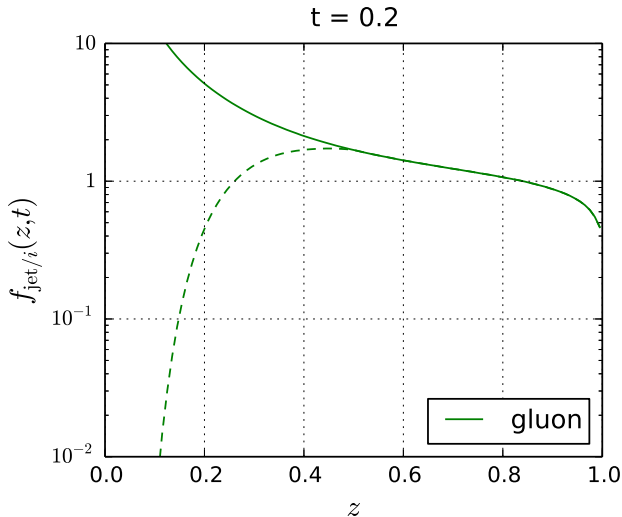
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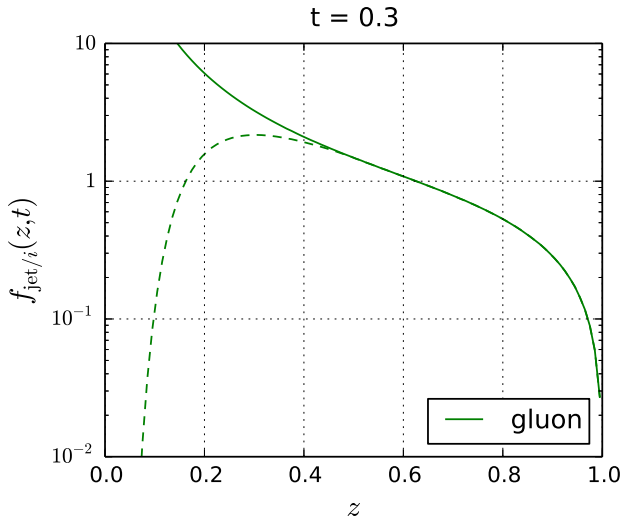
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# Hardest microjet fragmentation function

Solid line: inclusive microjet fragmentation function.

Dashed line: hardest microjet fragmentation function.



## Microjet vetoes

Jet veto resummations are a context where all-order small- $R$  corrections could be important.

Writing the probability of no gluon emissions above a scale  $p_t$  as

$$P(\text{no primary-parton veto}) = \exp \left[ - \int_{p_t}^Q \frac{dk_t}{k_t} \bar{\alpha}_s(k_t) 2 \ln \frac{Q}{k_t} \right],$$

one can show that including small- $R$  corrections and applying the veto on the hardest microjet, we have

$$\begin{aligned} \mathcal{U} &\equiv P(\text{no microjet veto}) / P(\text{no primary-parton veto}) \\ &= \exp \left[ - 2 \bar{\alpha}_s(p_t) \ln \frac{Q}{p_t} \int_0^1 dz f^{\text{hardest}}(z, t(R, p_t)) \ln z \right]. \end{aligned}$$

The  $R$ -dependent correction generates a series of terms

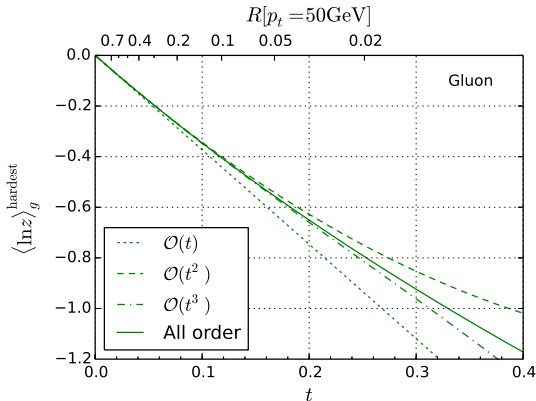
$$\alpha_s^{m+n}(Q) \ln^m(Q/p_t) \ln^n R.$$

## Logarithmic moment $\langle \ln z \rangle$

The logarithmic moment of  $f^{\text{hardest}}$  is, as seen previously

$$\langle \ln z \rangle^{\text{hardest}} \equiv \int_0^1 dz f^{\text{hardest}}(z) \ln z.$$

This seems to have a particularly stable perturbative expansion.



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- Using a generating-functional approach, we carried out numerical LL resummation of  $\ln R$  enhanced-terms in small- $R$  jets.
- Resummation complemented by analytical calculations of the LL expansion for the first few orders in perturbation theory.
- Studied inclusive microjet spectrum and identified the spectrum of the hardest microjet emerging from parton fragmentation.
- Calculated the logarithmic moment of hardest microjet spectrum, relevant in particular for jet vetoes in Higgs-boson production.
- Small- $R$  effects can be substantial, for example reducing the inclusive jet spectrum by 30 – 50% for gluon jets for  $R = 0.4 - 0.2$ .
- Study of phenomenological implications are forthcoming.

further reading on [arXiv:1411.5182](https://arxiv.org/abs/1411.5182)