Higgs inflation and large tensor-to-scalar ratio – status and predictivity

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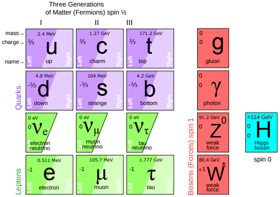
Outline

Standard Model – describes nearly everything



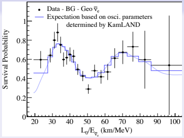
+ Einstein gravity

Describes

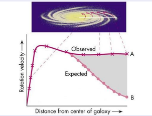
- all laboratory experiments – electromagnetism, nuclear processes, etc.
- all processes in the evolution of the Universe after the Big Bang Nucleosynthesis ($T < 1$ MeV, $t > 1$ sec)

Experimental problems:

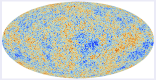
- Laboratory
  - ? Neutrino oscillations



- Cosmology
  - ? Baryon asymmetry of the Universe
  - ? Dark Matter



  - ? Inflation



  - ? Dark Energy

Minimal extensions of the SM to account for everything

Should explain everything

- Neutrino oscillations
- Dark Matter
- Baryon asymmetry of the Universe
- Inflation

(Neutrino oscillations, Dark Matter, Baryon asymmetry of the Universe) } νMSM

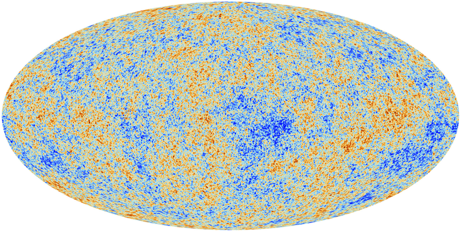
(Inflation) } this talk – with Higgs

in a minimal way

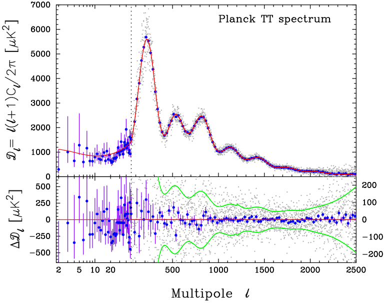
- Introduce minimal amount of new particle/parameters
  - Simple
  - Predictive
- No new scales up to gravity/inflation
  - With scale invariance – removes hierarchy problem
  - Allows to make relations between inflation and particle physics

Perturbations at inflation are observable in CMB

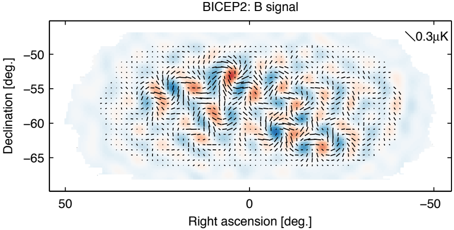
Temperature fluctuations (PLANCK)



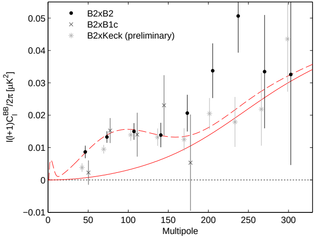
CMB temperature spectrum



B-mode Polarization (BICEP2)

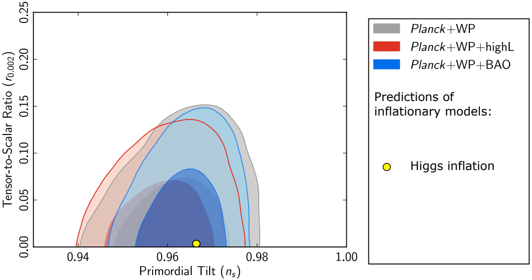


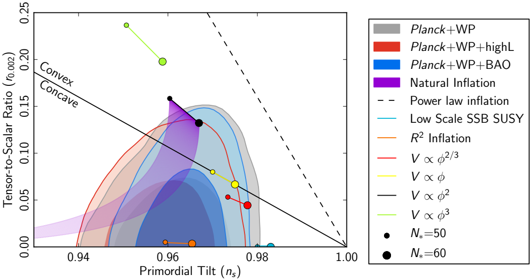
B-mode polarization spectrum



Inflationary parameters from CMB

$$U_{\text{inflation}}^{1/4} \sim 1.9 \times 10^{16}\ \text{GeV} \left(\frac{r}{0.1}\right)^{1/4}$$





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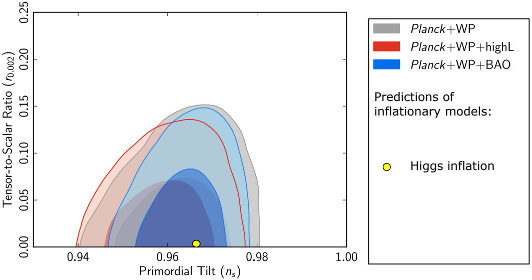
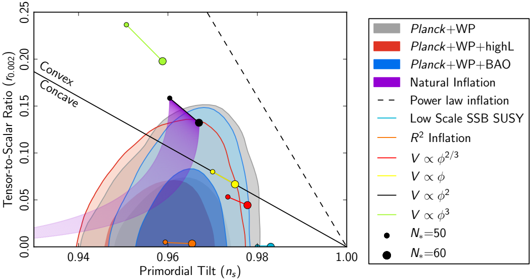
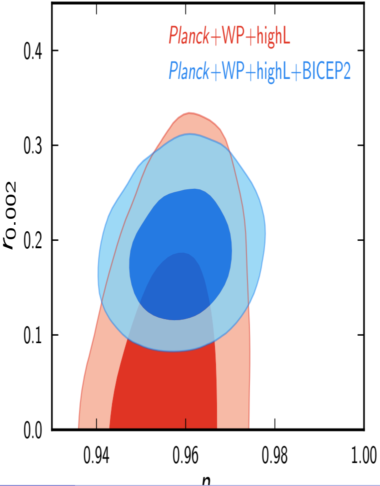
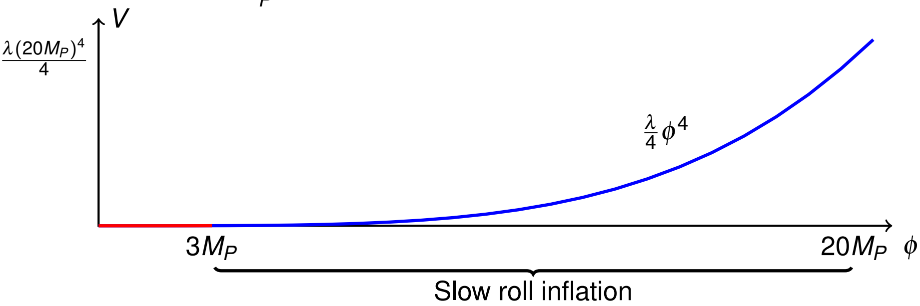






Chaotic inflation–a scalar field

$$\mathscr{H}^2 \simeq \frac{1}{3M_P^2}\left(V(\phi)+\dot{\phi}^2/2\right), \qquad \ddot{\phi}+3\mathscr{H}\dot{\phi}+V'(\phi)=0$$



$\delta T/T \sim 10^{-5}$ normalization

quartic coupling: $\lambda \sim 10^{-13}$ (or mass: $m \sim 10^{13}$ GeV)

Can not be the SM Higgs field?

Non-minimal coupling to gravity solves the problem

Quite an old idea

For a scalar field coupling to the Ricci curvature is possible (actually *required* by renormalization)

- A.Zee'78, L.Smolin'79, B.Spokoiny'84
- D.Salopek J.Bond J.Bardeen'89

Scalar part of the (Jordan frame) action

$$S_J = \int d^4x \sqrt{-g} \left\{ -\frac{M_P^2}{2} R - \xi \frac{h^2}{2} R + g_{\mu\nu} \frac{\partial^\mu h \partial^\nu h}{2} - \frac{\lambda}{4} (h^2 - v^2)^2 \right\}$$

- h is the Higgs field; $M_P \equiv \frac{1}{\sqrt{8\pi G_N}} = 2.4 \times 10^{18}\text{GeV}$
- SM higgs vev $v \ll M_P/\sqrt{\xi}$ – can be neglected in the early Universe
- At $h \gg M_P/\sqrt{\xi}$ *all* masses are proportional to h – scale invariant spectrum!

[FB, Shaposhnikov'08]

Conformal transformation – nice way to calculate

It is possible to get rid of the non-minimal coupling by the conformal transformation (change of variables)

$$\hat{g}_{\mu\nu} = \Omega^2 g_{\mu\nu} , \qquad \Omega^2 \equiv 1 + \frac{\xi h^2}{M_P^2}$$

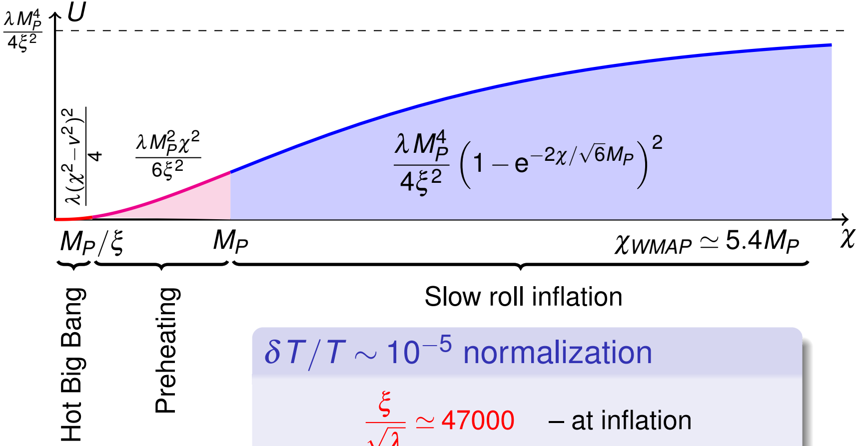
Redefinition of the Higgs field to get canonical kinetic term

$$\frac{d\chi}{dh} = \sqrt{\frac{\Omega^2 + 6\xi^2 h^2/M_P^2}{\Omega^4}} \quad \Longrightarrow \begin{cases} h \simeq \chi & \text{for } h < M_P/\xi \\ \Omega^2 \simeq \exp\left(\frac{2\chi}{\sqrt{6}M_P}\right) & \text{for } h > M_P/\xi \end{cases}$$

Resulting action (Einstein frame action)

$$S_E = \int d^4x \sqrt{-\hat{g}} \left\{ -\frac{M_P^2}{2}\hat{R} + \frac{\partial_\mu \chi \partial^\mu \chi}{2} - \frac{\lambda}{4}\frac{h(\chi)^4}{\Omega(\chi)^4} \right\}$$

Potential – different stages of the Universe

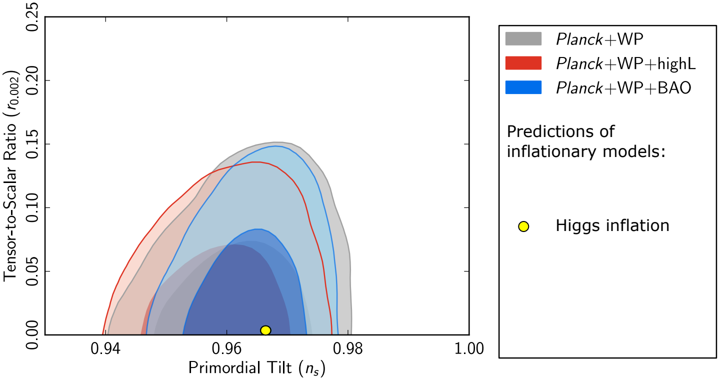


$\delta T/T \sim 10^{-5}$ normalization

$$\frac{\xi}{\sqrt{\lambda}} \simeq 47000 \quad \text{– at inflation}$$

Small λ is traded for large ξ

CMB parameters are predicted



spectral index $n \simeq 1 - \frac{8(4N+9)}{(4N+3)^2} \simeq 0.97$

tensor/scalar ratio $r \simeq \frac{192}{(4N+3)^2} \simeq 0.0033$

$$\delta T/T \sim 10^{-5} \implies \frac{\xi}{\sqrt{\lambda}} \simeq 47000$$

Higgs decouples from all fields during inflation

Action for the gauge fields and fermions is invariant under conformal transformations ($A_\mu \mapsto A_\mu$, $\psi \mapsto \Omega^{3/2}\psi$) except for the mass terms

$$\mathscr{L}_A^J = g^2 h^2 A_\mu A_\mu \quad \mapsto \quad \mathscr{L}_A^E = g^2 \frac{h^2}{\Omega^2} A_\mu A_\mu = g^2 \frac{M_P^2}{\xi}\left(1 - e^{-\frac{2\chi}{\sqrt{6}M_P}}\right) A_\mu A_\mu$$

$$\mathscr{L}_Y^J = yh\bar{\psi}\psi \quad \mapsto \quad \mathscr{L}_Y^E = y\frac{h}{\Omega}\bar{\psi}\psi = y\frac{M_P}{\sqrt{\xi}}\left(1 - e^{-\frac{2\chi}{\sqrt{6}M_P}}\right)^{1/2} \bar{\psi}\psi$$

In inflationary region $h > M_P/\sqrt{\xi}$:

$$\Omega^2 \equiv 1 + \frac{\xi h^2}{M_P^2} \simeq \exp\left(\frac{2\chi}{\sqrt{6}M_P}\right)$$

Exponentially weak coupling of χ to other matter

Non-minimal coupling made the Higgs potential flat and at the same time took care of the corrections from the other fields

Consistency

Up to now we assumed that the model is a full model, and anything beyond it does not spoil the story.

Is this really the case?

Cut off scale today

Let us work in the Einstein frame for simplicty

Change of variables: $\frac{d\chi}{dh} = \frac{M_P\sqrt{M_P^2+(\xi+6\xi^2)h^2}}{M_P^2+\xi h^2}$ leads to the higher order terms in the potential (expanded in a power law series)

$$V(\chi) = \lambda\frac{h^4}{4\Omega^4} \simeq \lambda\frac{h^4}{4} \simeq \lambda\frac{\chi^4}{4} + \#\frac{\chi^6}{(M_P/\xi)^2} + \cdots$$

Unitarity is violated at tree level

in scattering processes (eg. $2 \to 4$) with energy above the "cut-off"

$$E > \Lambda_0 \sim \frac{M_P}{\xi}$$

Hubble scale at inflation is $H \sim \lambda^{1/2}\frac{M_P}{\xi}$ – not much smaller than the today cut-off Λ_0 :(

[Burgess, Lee, Trott'09, Barbon, Espinosa'09, Hertzberg'10]

"Cut off" is background dependent!

Classical background → $\chi(x,t) = \bar{\chi}(t) + \delta\chi(x,t)$ ← Quantum perturbations

leads to background dependent suppression of operators of dim $n > 4$

$$\frac{\mathscr{O}_{(n)}(\delta\chi)}{[\Lambda_{(n)}(\bar{\chi})]^{n-4}}$$

Example

Potential in the inflationary region $\chi > M_P$: $U(\chi) = \frac{\lambda M_P^4}{4\xi^2}\left(1 - e^{-\frac{2\chi}{\sqrt{6}M_P}}\right)^2$

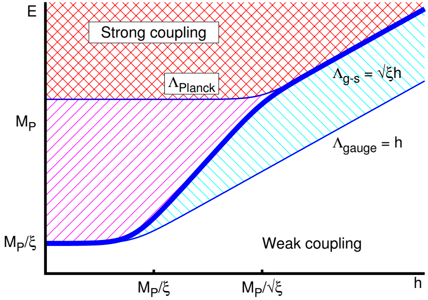
leads to operators of the form: $\frac{\mathscr{O}_{(n)}(\delta\chi)}{[\Lambda_{(n)}(\bar{\chi})]^{n-4}} = \frac{\lambda M_P^4}{\xi^2} e^{-\frac{2\bar{\chi}}{\sqrt{6}M_P}} \frac{(\delta\chi)^n}{M_P^n}$

Leading at high n to the "cut-off"

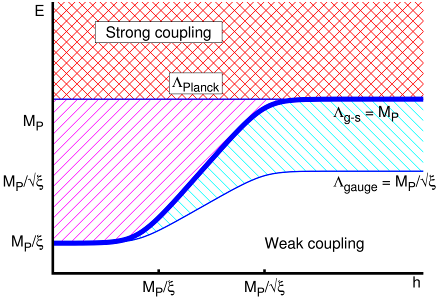
$$\Lambda \sim M_P$$

Cut-off grows with the field background

Jordan frame



Einstein frame



Relation between cut-offs in different frames:

$$\Lambda_{\text{Jordan}} = \Lambda_{\text{Einstein}}\Omega$$

Relevant scales

Hubble scale $H \sim \lambda^{1/2}\frac{M_P}{\xi}$

Energy density at inflation

$$V^{1/4} \sim \lambda^{1/4}\frac{M_P}{\sqrt{\xi}}$$

Reheating temperature $M_P/\xi < T_{\text{reheating}} < M_P/\sqrt{\xi}$

[FB, Magnin, Shaposhnikov,, Sibiryakov'11]

- Running cut-off above typical energy scales – all our calculations of perturbations on top of background solutions are ok (i.e. no need to worry about higher order corrections here)
- But what can we say about the background solutions itself?

RG improved effective potential

$$U(\phi) = \frac{\lambda(\mu)}{4}\phi^4 + \sum_i \frac{m_i^4(\phi)}{64\pi^2}\left(\ln\frac{m_i^2(\phi)}{\mu^2} + \text{const}_i\right) + \cdots$$

with $m_i(\phi) = g\phi,\ \frac{y}{\sqrt{2}}\phi$, so that $m_i^4 \propto \phi^4$

- U should be independent on non-physical parameter μ – leads to RG equation for λ

$$\frac{\partial\lambda}{\partial\ln\mu} = \beta_\lambda$$

- At the same time, one can choose $\mu \simeq m(\phi) \simeq y_t\phi$ to minimize the logarithms

$$U_{\text{RG improved}} \simeq \frac{\lambda(\mu(\phi))}{4}\phi^4$$

$$\mu^2 \simeq \alpha^2\frac{y_t}{2}\phi^2$$

α is of order one

RG improved potential for Higgs inflation

$$U_{\text{RG improved}}(\chi) = \frac{\lambda(\mu)}{4}\frac{M_P^4}{\xi^2}\left(1 - e^{-\frac{2\chi}{\sqrt{6}M_P}}\right)^2$$

with

$$\mu^2 = \alpha^2 m_t^2(\chi) = \alpha^2 \frac{y_t^2(\mu)}{2}\frac{M_P^2}{\xi}\left(1 - e^{-\frac{2\chi}{\sqrt{6}M_P}}\right)$$

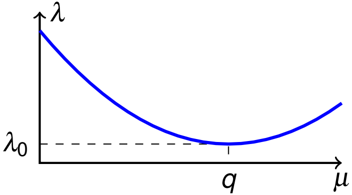
- Large λ – slow (logarithmic) running, no noticeable change compared to tree level potential
- Small λ – may give interesting “features” in the potential
- Strictly speaking ξ is also running – not relevant for the current discussion for a set of reasons, especially in the region of small λ

RG running indicates small λ at Planck scale

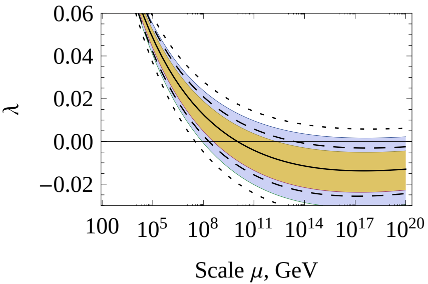
Renormalization evolution of the Higgs self coupling λ

$$\lambda \simeq \lambda_0 + b \ln^2 \frac{\mu}{q}$$

$b \simeq 0.000023$
$\left.\begin{array}{l}\lambda_0 \text{ – small} \\ q \text{ of the order } M_P\end{array}\right\}$ depend on $M_h^*,\ m_t^*$



Higgs mass M_h=125.3±0.6 GeV



$$(4\pi)^2 \frac{\partial \lambda}{\partial \ln \mu} = 24\lambda^2 - 6y_t^4 + \frac{3}{8}(2g_2^4 + (g_2^2 + g_1^2)^2) + (-9g_2^2 - 3g_1^2 + 12y_t^2)\lambda$$

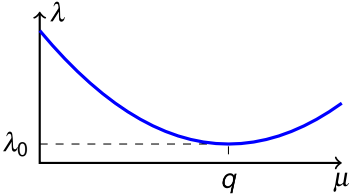
RG running indicates small λ at Planck scale

Potentials in different regimes

$$\lambda \simeq \lambda_0 + b \ln^2 \frac{\mu}{q}$$

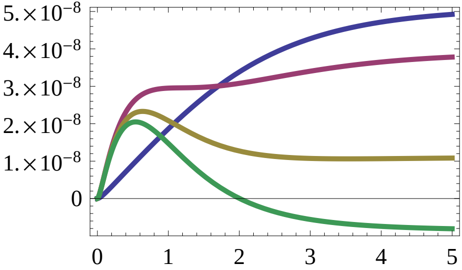
$b \simeq 0.000023$

λ_0 – small
q of the order M_p } depend on $M_h^*, \; m_t^*$

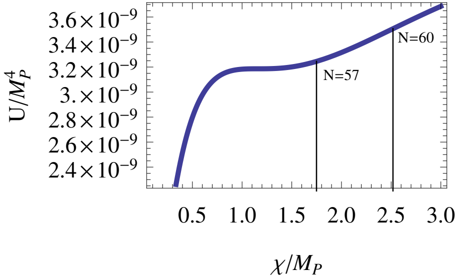


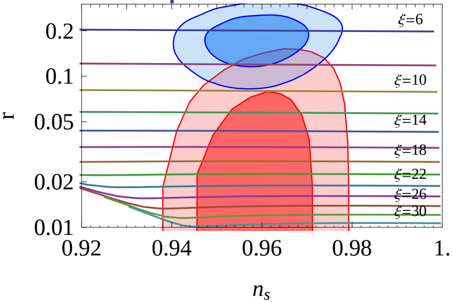
$$U(\chi) \simeq \frac{\lambda(\mu) M_P^4}{4\xi^2}\left(1 - e^{-\frac{2\chi}{\sqrt{6}M_P}}\right)^2$$

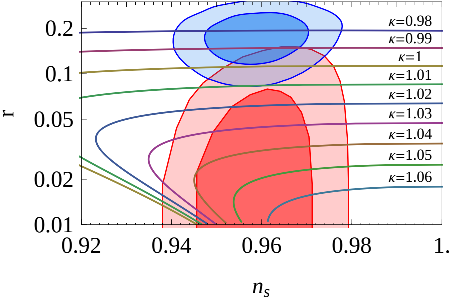
$$\mu^2 = \alpha^2 \frac{y_t(\mu)^2}{2} \frac{M_P^2}{\xi}\left(1 - e^{-\frac{2\chi}{\sqrt{6}M_P}}\right)$$



Interesting inflation near to the critical point







Parameters in
particle physics: λ_0, q, ξ
cosmology: $\mathscr{P}_R$, r, n_s

$\kappa \sim q\frac{\sqrt{\xi}}{M_P}\frac{\sqrt{2}}{y_t}$

For given r (or ξ) very small change of κ (or M_h^*) gives any n_s

[Hamada, Kawai, Oda„ Park'14, FB, Shaposhnikov'14]

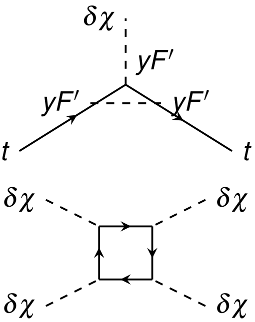
Connection with the low energy physics

Inflationary "masses" m_t^*, M_h^* differ from physical m_t, M_h

Let us analyse counterterms generated by

$$\mathscr{L}_t = \frac{y_t}{\sqrt{2}}\bar\psi_t\psi_t F(\chi), \quad F(\chi) = \frac{h(\chi)}{\Omega(h(\chi))}$$

Low energy $F'(0)=1$, at inflation $F'(\infty)=0$



$$y_t \to y_t + \frac{y_t^3}{16\pi^2}\left(\frac{9}{4\varepsilon}+C_t\right)F'^2$$

$$m_t^* = m_t\left(1-\frac{y_t^2 C_t}{16\pi^2}\right)$$

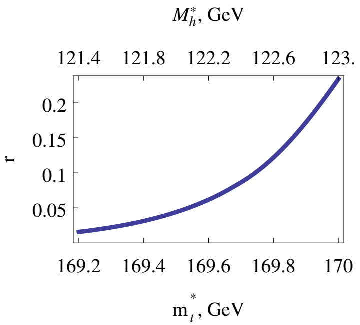
$$\lambda \to \lambda - \frac{y_t^4}{16\pi^2}\left(\frac{3}{\varepsilon}-C_\lambda\right)F'^4$$

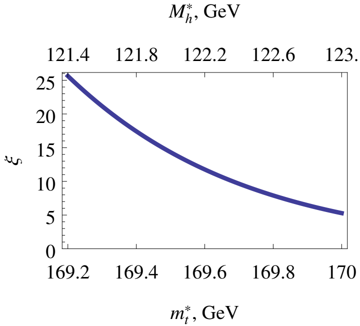
$$M_h^* = M_h\left(1-\frac{y_t^4 C_\lambda}{16\pi^2}\frac{v^2}{M_h^2}\right)$$

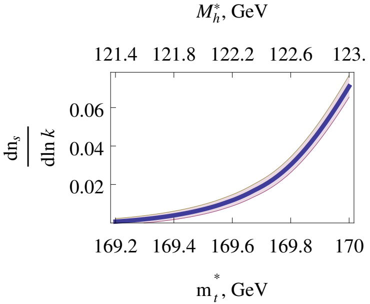
Inflation-particle mass difference $m^* - m$ of several GeV for $C \sim 1$

[FB, Magnin, Shaposhnikov„ Sibiryakov'11, FB, Shaposhnikov'14]

Cosmological parameters for critical point HI







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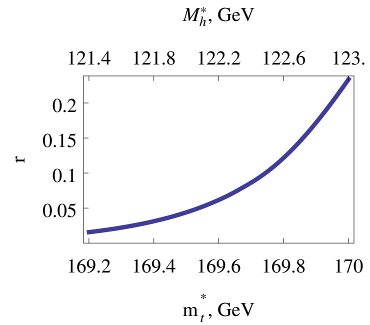
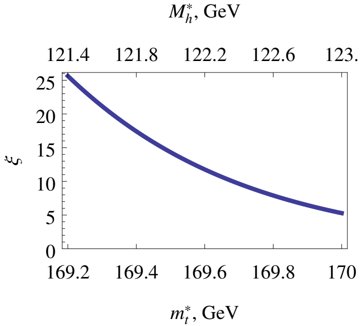
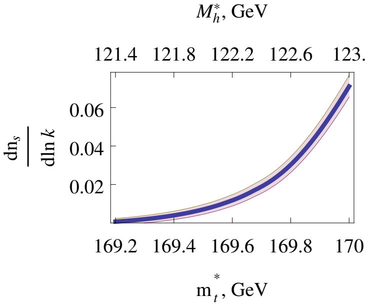
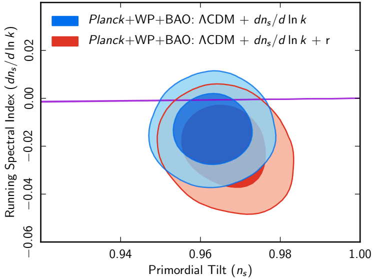


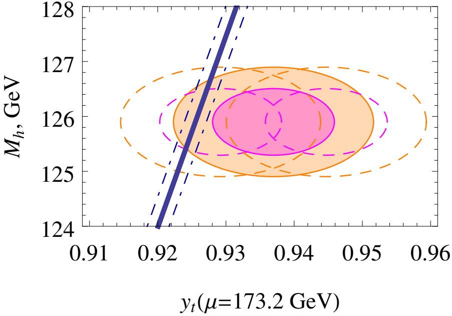






Electroweak vacuum should be stable

2σ compatible with the observations



Main uncertainties

- Determination of $\overline{\text{MS}}$ y_t
  - Experimental M_t
  - Extraction of $\overline{\text{MS}}$ mass/Yukawa
- Strong coupling constant
- Higgs mass

$$y_t^{\text{crit}} = 0.9268 + 0.0057 \times \left[\frac{M_h - 125.9}{0.4} \times 0.2 + \frac{\alpha_s(M_Z) - 0.1184}{0.0007} \times 0.28\right]$$

[FB, Kalmykov, Kniehl,, Shaposhnikov'12, Buttazzo, Degrassi, Giardino, Giudice, Sala, *et al.*'13], Pikelner'QUARKS 14

Conclusions

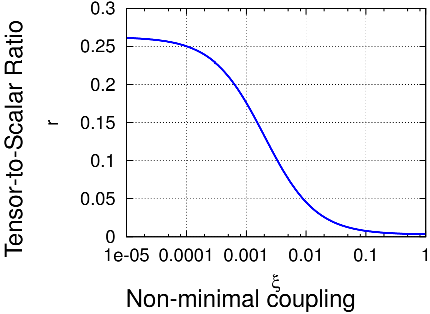
Higgs inflation as the minimal inflationary model:

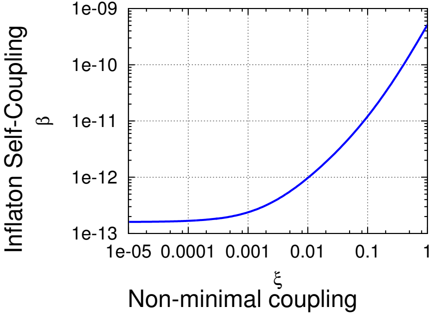
- Large ξ regime
  - Cosmology: $n_s \simeq 0.97$, $r \simeq 0.0033$
  - Particle physics: rather generic
- Small ξ regime
  - Cosmology:
    - any n_s, r
    - predicts positive $dn_s/d\ln k$,...
  - Particle physics:
    - Higgs and top masses correspond to absolute vacuum stability
    - High (inflationary) and low (particle physics) scale coupling constants are rather close when matched over the $h \sim M_P/\xi$ region

Is any of this true?

- Measure $\overline{\text{MS}}$ top quark Yukawa – lepton collider, better theoretical analysis on hardon collider
- Measure CMB properties!

The tensor perturbations are suppressed, inflaton self-coupling β is increased

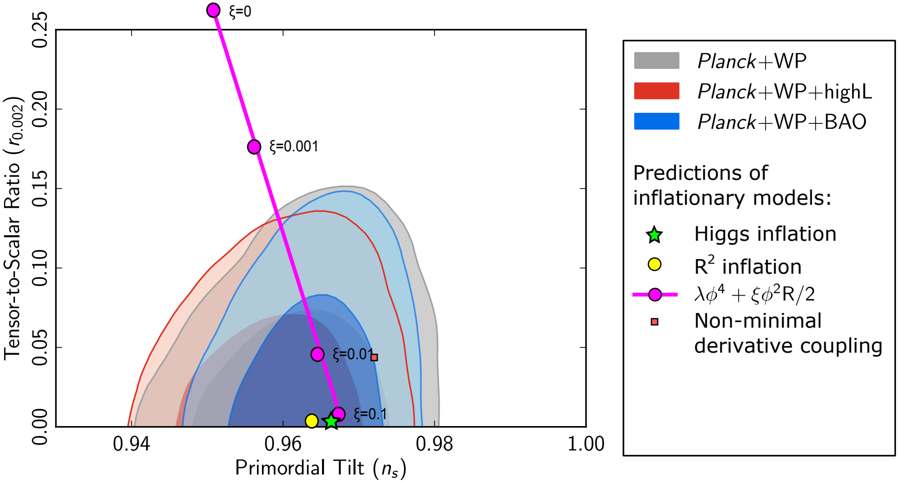




For each ξ the self-coupling β is fixed by $\delta T/T \simeq 10^{-5}$ requirement.

[Tsujikawa, Gumjudpai'04, FB'08, Okada, Rehman, Shafi'10]

Inflationary predictions are ok for $\xi \gtrsim 0.003$



Shift symmetric UV completion allows to have a form of effective theory during inflation

$$\mathscr{L} = \frac{(\partial_\mu \chi)^2}{2} - U_0 \left(1 + \sum u_n e^{-n\cdot\chi/M}\right)$$
$$= \frac{(\partial_\mu \chi)^2}{2} - U_0 \left(1 + \sum \frac{1}{k!} \left[\frac{\delta\chi}{M}\right]^k \sum n^k u_n e^{-n\cdot\bar{\chi}/M}\right)$$

Effective action (from quantum corrections of loops of $\delta\chi$)

$$\mathscr{L}_{\text{eff}} = f^{(1)}(\chi)\frac{(\partial_\mu \chi)^2}{2} - U(\chi) + f^{(2)}(\chi)\frac{(\partial^2 \chi)^2}{M^2} + f^{(3)}(\chi)\frac{(\partial \chi)^4}{M^4} + \cdots$$

All the divergences are absorbed in u_n and in $f^{(n)} \sim \sum f_l e^{-n\chi/M}$

UV completion requirement

Shift symmetry (or scale symmetry in the Jordan frame) is respected

$$\chi \mapsto \chi + \text{const}$$

Connection of inflationary and low energy physics requires more assumptions on the UV theory

$$\lambda U(\bar{\chi}+\delta\chi) = \lambda \left(U(\bar{\chi}) + \frac{1}{2} U''(\bar{\chi})(\delta\chi)^2 + \frac{1}{3!} U'''(\bar{\chi})(\delta\chi)^3 + \cdots \right)$$

in one loop: $\lambda U''(\bar{\chi})\bar{\Lambda}^2,\ \lambda^2 (U''(\bar{\chi}))^2 \log\bar{\Lambda}$,

in two loops: $\lambda U^{(IV)}(\bar{\chi})\bar{\Lambda}^4,\ \lambda^2 (U''')^2\bar{\Lambda}^2,\ \lambda^3 U^{(IV)}(U'')^2(\log\bar{\Lambda})^2$,

If no power law divergences are generated

then the loop corrections are arranged in a series in λ

$$U(\chi) = \lambda U_1(\chi) + \lambda^2 U_2(\chi) + \lambda^3 U_3(\chi) + \cdots$$

A rule to fix the finite parts of the counterterm functions $U_i(\chi)$

Example – dimensional regularisation + $\overline{\text{MS}}$

Summary on radiative corrections

- The tree level calculations can be ok, as far as the cut-off is background dependent
- Underlying theory respects shift (scale) invariance – effective (order by order) analysis of the inflationary potential is possible
- Underlying theory respects scale invariance and does not generates any quadratic contributions – calculations are fully possible provided the action *and* the subtraction rules are specified.

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