

# Phenomenology of the Accelerating Universe

Federico Piazza

Gubitosi, F. P., Vernizzi, 1210.0201  
Gleyzes, Langlois, F.P., Vernizzi, 1304.4840  
F. P., F. Vernizzi, 1307.4350  
F. P., C. Marinoni, H. Steigerwald 1312.6111  
Gleyzes, Langlois, F.P., Vernizzi, 1404.6495  
In progress...



**Paris Centre for  
Cosmological Physics**



# Nobel Prize in Physics 2011

The Universe is accelerating!

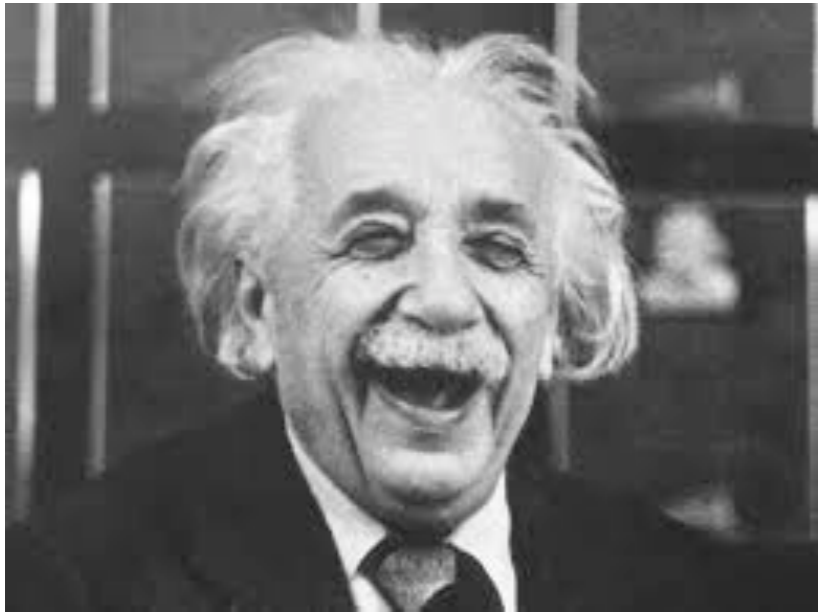


# Nobel Prize in Physics 2011

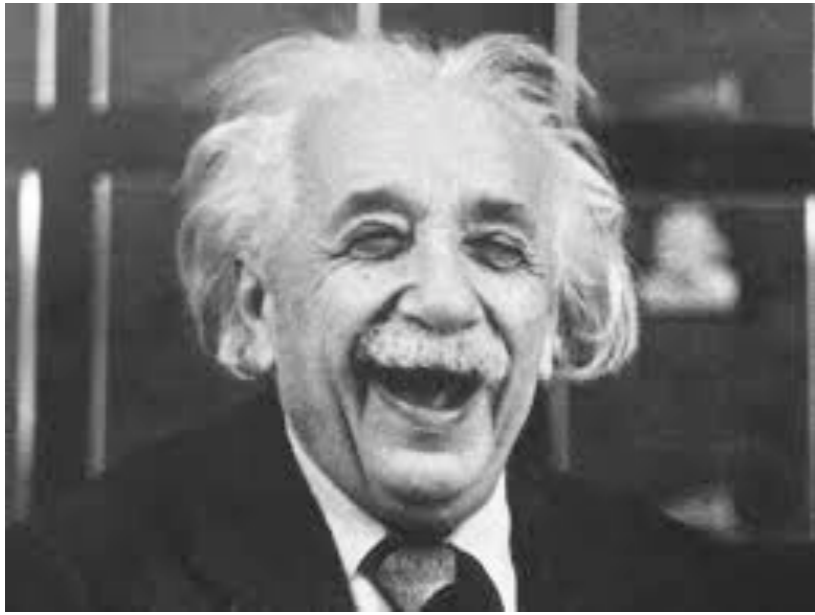
The Universe is accelerating!

...why?!





$\Lambda$ CDM



$\Lambda$ CDM

The only consistent  
low energy theory for  
a spin-two field  $g_{\mu\nu}$ .



# Beware of theoretical prejudice!

## MEASUREMENTS<sup>1</sup> OF THE COSMOLOGICAL PARAMETERS $\Omega$ AND $\Lambda$ FROM THE FIRST SEVEN SUPERNOVAE AT $z \geq 0.35$

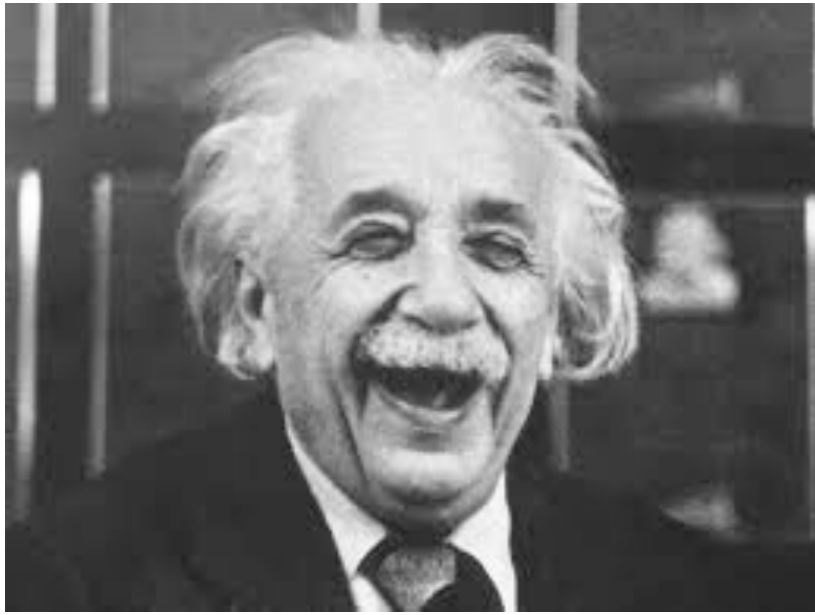
S. PERLMUTTER,<sup>2,3</sup> S. GABI,<sup>2,4</sup> G. GOLDBABER,<sup>2,3</sup> A. GOOBAR,<sup>2,3,5</sup> D. E. GROOM,<sup>2,3</sup> I. M. HOOK,<sup>3,6</sup>  
A. G. KIM,<sup>2,3</sup> M. Y. KIM,<sup>2</sup> J. C. LEE,<sup>2</sup> R. PAIN,<sup>2,7</sup> C. R. PENNYPACKER,<sup>2,4</sup> I. A. SMALL,<sup>2,3</sup>  
R. S. ELLIS,<sup>8</sup> R. G. MCMAHON,<sup>8</sup> B. J. BOYLE,<sup>9,10</sup> P. S. BUNCLARK,<sup>9</sup> D. CARTER,<sup>9</sup>  
M. J. IRWIN,<sup>9</sup> K. GLAZEBROOK,<sup>10</sup> H. J. M. NEWBERG,<sup>11</sup> A. V. FILIPPENKO,<sup>3,6</sup>  
T. MATHESON,<sup>6</sup> M. DOPITA,<sup>12</sup> AND W. J. COUCH<sup>13</sup>

(THE SUPERNOVA COSMOLOGY PROJECT)

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### ABSTRACT

We have developed a technique to systematically discover and study high-redshift supernovae that can be used to measure the cosmological parameters. We report here results based on the initial seven of more than 28 supernovae discovered to date in the high-redshift supernova search of the Supernova Cosmology Project. We find an observational dispersion in peak magnitudes of  $\sigma_{M_B} = 0.27$ ; this dispersion narrows to  $\sigma_{M_B, \text{corr}} = 0.19$  after “correcting” the magnitudes using the light-curve “width-luminosity” relation found for nearby ( $z \leq 0.1$ ) Type Ia supernovae from the Calán/Tololo survey (Hamuy et al.). Comparing light-curve width-corrected magnitudes as a function of redshift of our distant ( $z = 0.35$ – $0.46$ ) supernovae to those of nearby Type Ia supernovae yields a global measurement of the mass density,  $\Omega_M = 0.88^{+0.69}_{-0.60}$  for a  $\Lambda = 0$  cosmology. For a spatially flat universe (i.e.,  $\Omega_M + \Omega_\Lambda = 1$ ), we find  $\Omega_M = 0.94^{+0.34}_{-0.28}$  or, equivalently, a measurement of the cosmological constant,  $\Omega_\Lambda = 0.06^{+0.28}_{-0.34}$  ( $< 0.51$  at the 95% confidence level). For the more general Friedmann-Lemaître cosmologies with independent  $\Omega_M$  and  $\Omega_\Lambda$ , the results are presented as a confidence region on the  $\Omega_M$ - $\Omega_\Lambda$  plane. This region does not correspond to a unique value of the deceleration parameter  $q_0$ . We present analyses and checks for statistical and systematic errors and also show that our results do not depend on the specifics of the width-luminosity correction. The results for  $\Omega_\Lambda$ -versus- $\Omega_M$  are inconsistent with  $\Lambda$ -dominated, low-density, flat cosmologies that have been proposed to reconcile the ages of globular cluster stars with higher Hubble constant values.



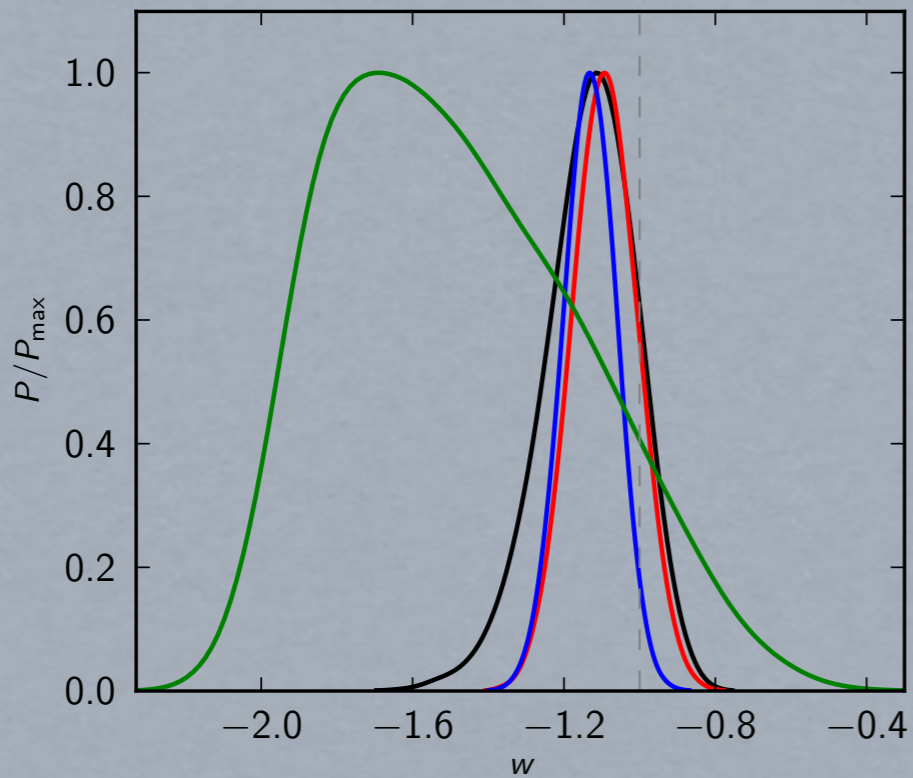
$\Lambda$ CDM

The only consistent  
low energy theory for  
a spin-two field  $g_{\mu\nu}$ .



$g_{\mu\nu} + \text{STUFF}$

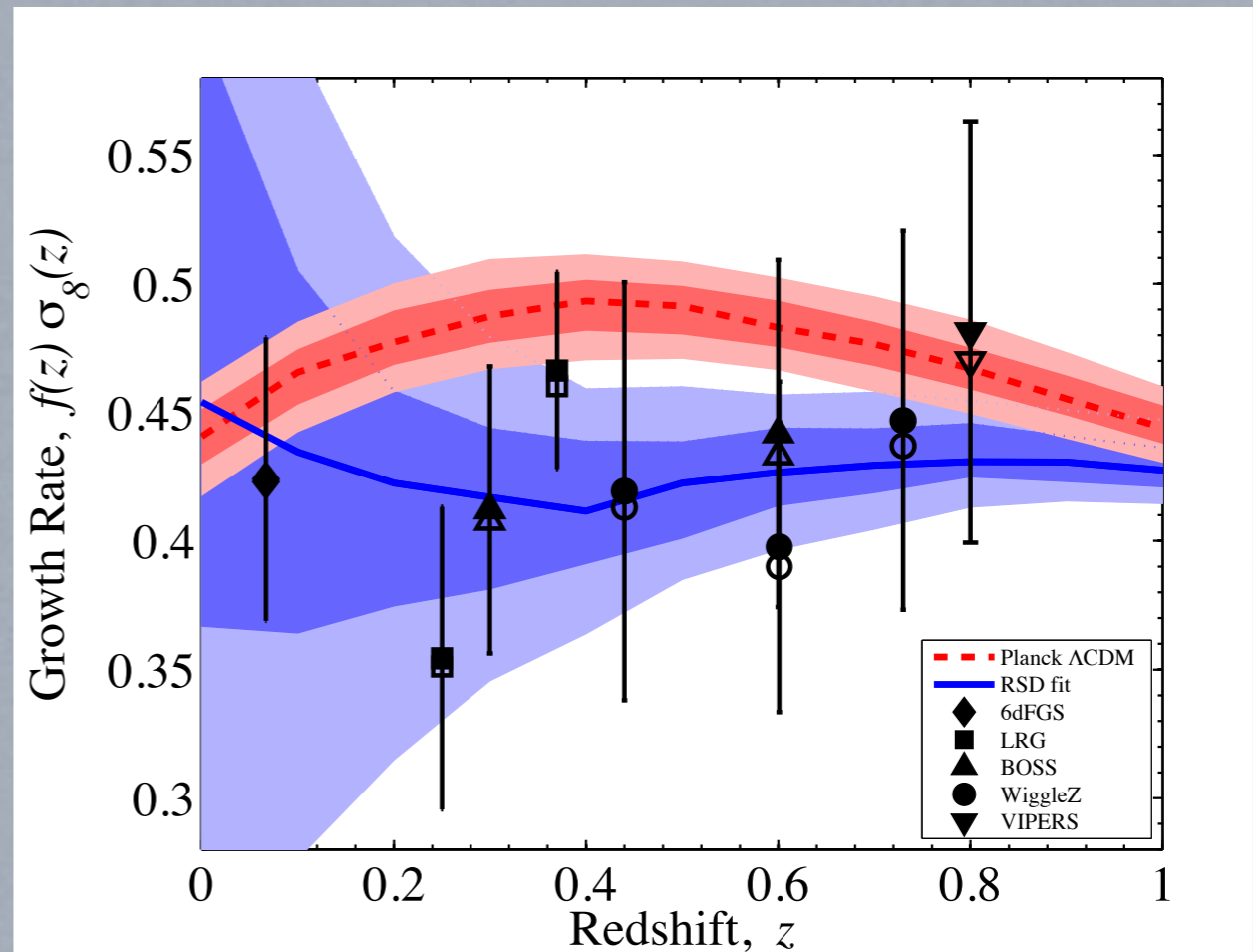
— *Planck*+WP+BAO      — *Planck*+WP+SNLS  
 — *Planck*+WP+Union2.1      — *Planck*+WP



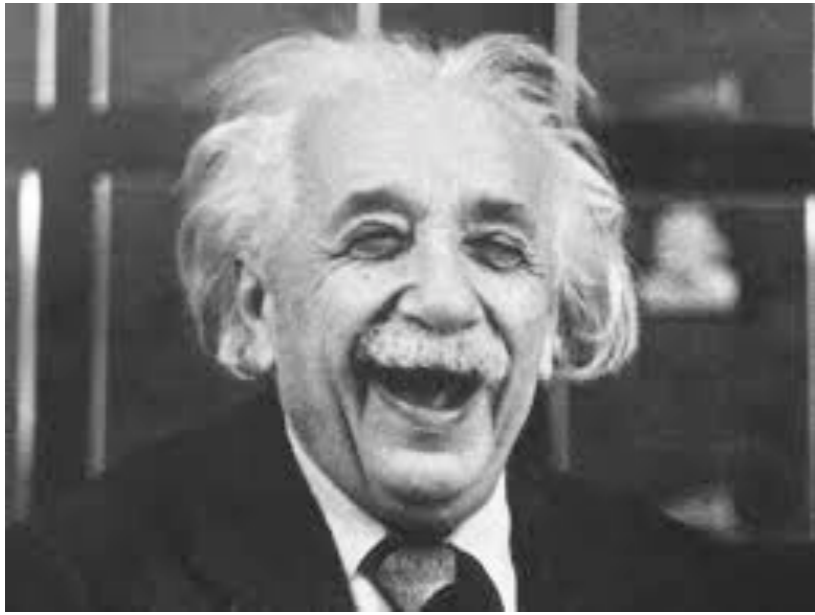
## Tensions with LCDM...?

$$w = -1.24^{+0.18}_{-0.19} \quad (95\%; \textit{Planck}+\textit{WP}+H_0)$$

Macaulay et al. '13



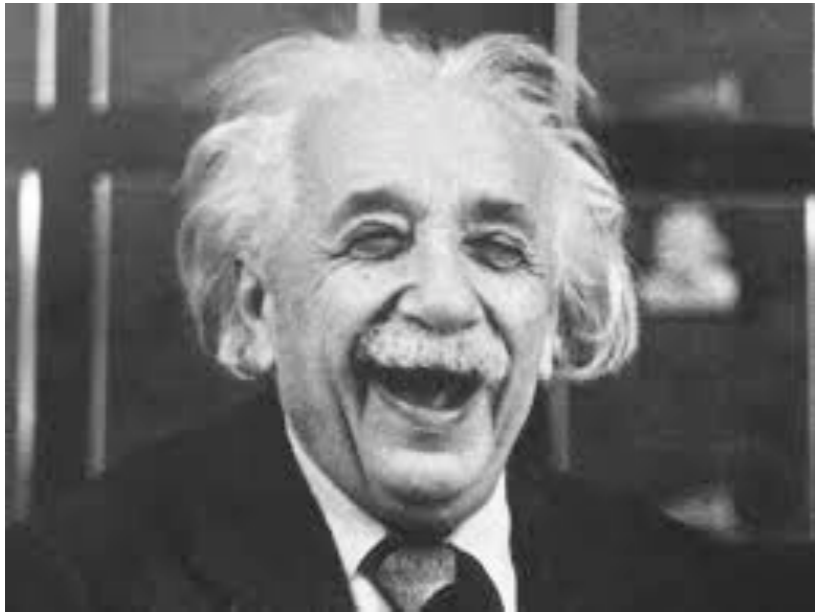




$\Lambda$ CDM



$g_{\mu\nu} + \text{STUFF}$



$\Lambda$ CDM



$g_{\mu\nu} + \text{STUFF}$



Other fundamental ingredients?



Dark Energy

Big Bang Cosmology

Inflation

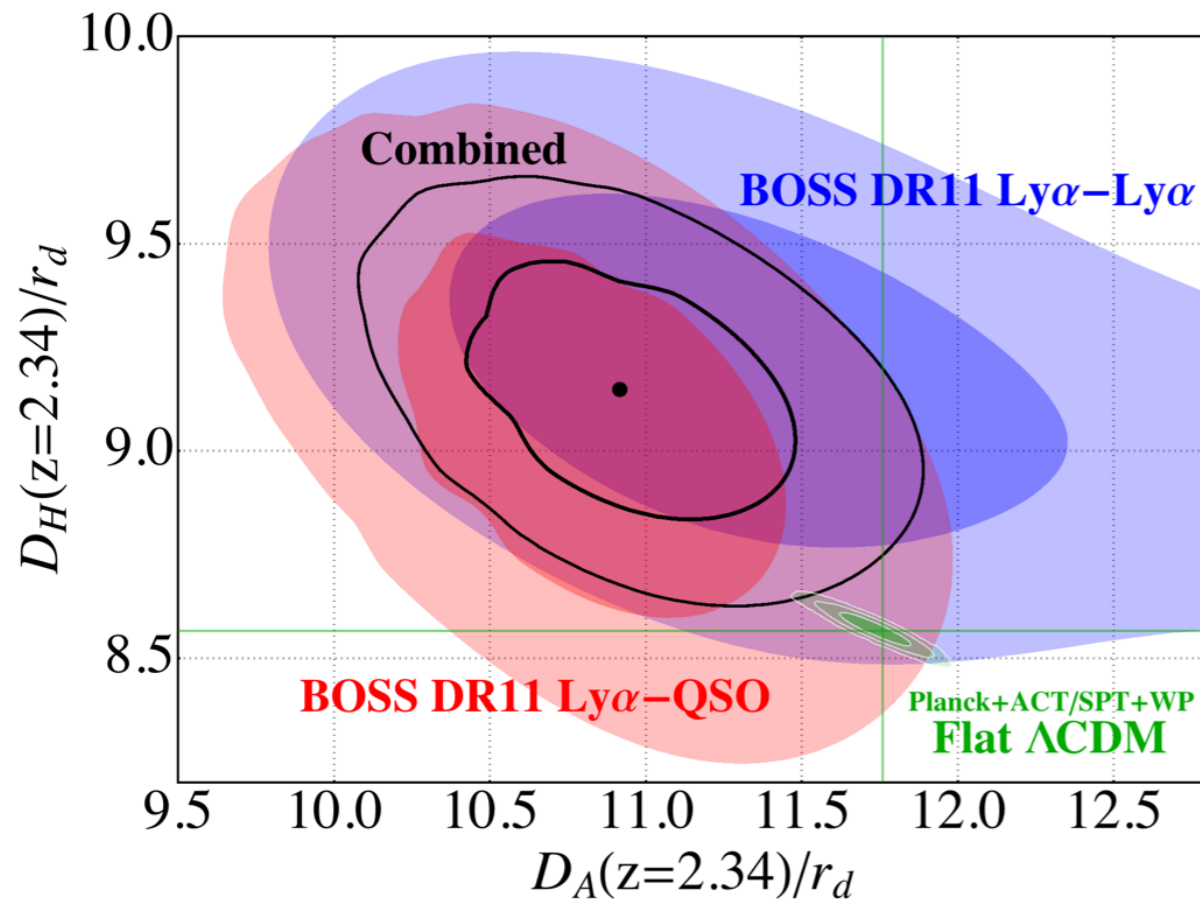
The history of the Universe

# The history of the Universe

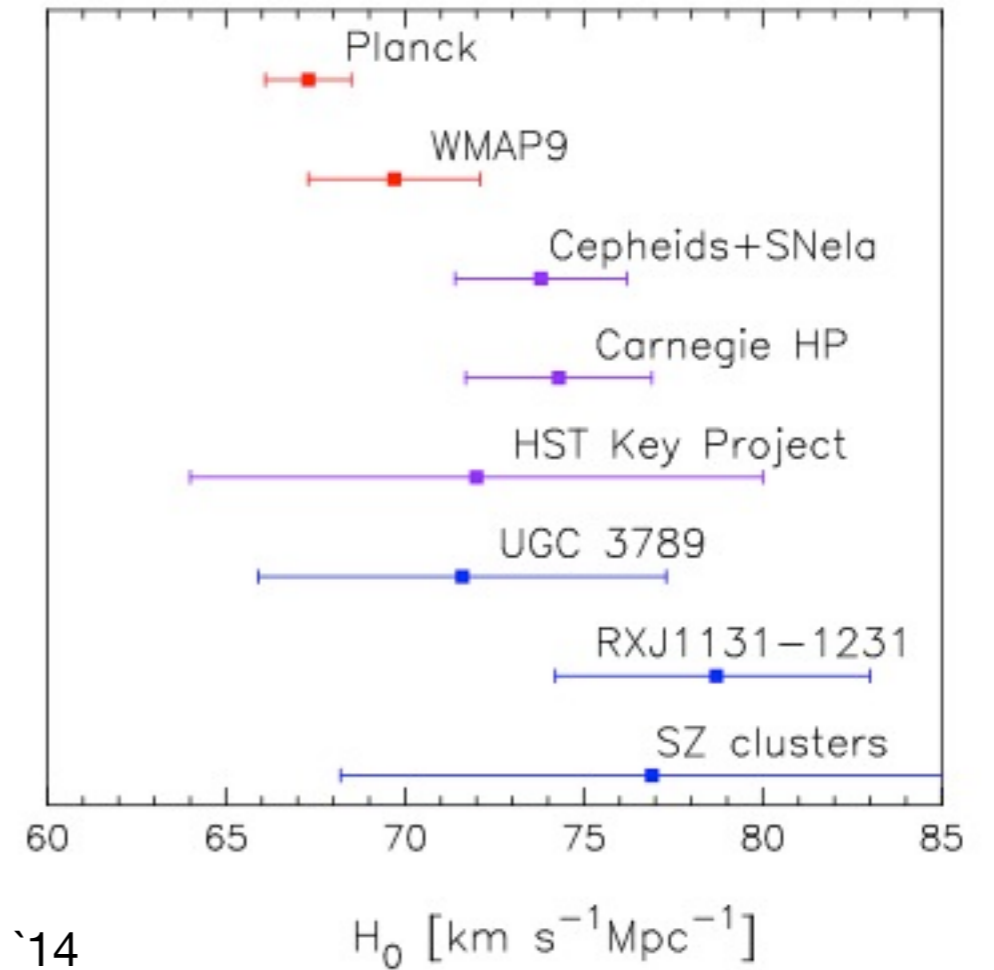
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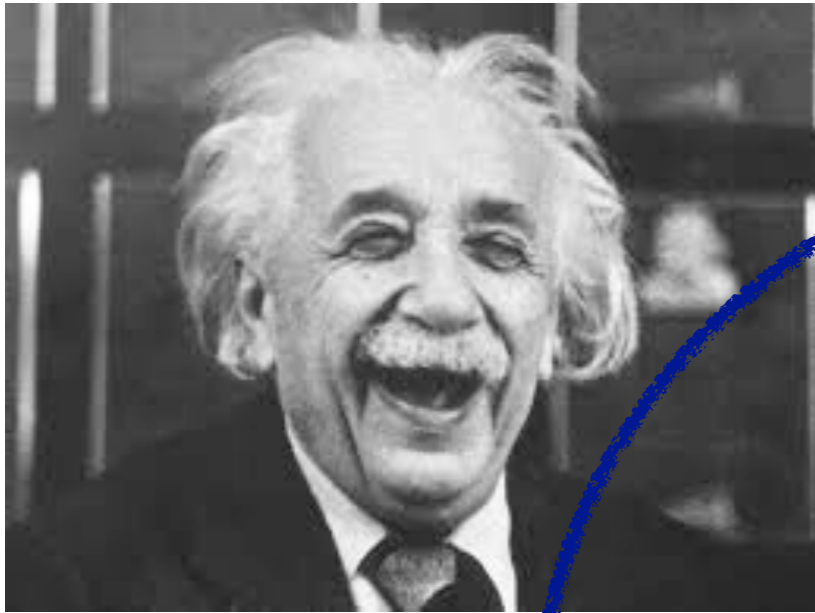


Planck paper



Delubac et al. '14

Geometrical anomalies?  
However: "small" effects...



$\Lambda$ CDM

This Talk



$g_{\mu\nu} + \text{STUFF}$



Other fundamental ingredients?

# Phenomenology of dark energy

- Two main classes of **observables**

# Phenomenology of dark energy

- Two main classes of **observables**

Background

$$\phi = \phi_0(t)$$

$$ds^2 = - dt^2 + a^2(t) dx^2$$

$$\rho_m = \rho_m(t)$$

equation of state  $w(z)$

$$S[\phi, g_{\mu\nu}, \Psi_m]$$

Perturbations

$$\delta\rho_m(t, \vec{x}) \longleftrightarrow \delta\phi(t, \vec{x})$$

growth rate, lensing potential, etc.

# Phenomenology of dark energy

- Two main classes of **observables**
- **EUCLID, DESI etc...** are specifically designed to target perturbation sector
- `No shortage' of dark energy models (>5000 papers on Spire)  
Need for a **Unifying** and **Effective** description

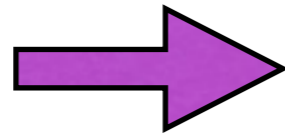
## Ideally...

- A limited number of effective operators, each one responsible for an observable dynamical feature (e.g. flavor-changing neutral currents in physics beyond Standard Model)



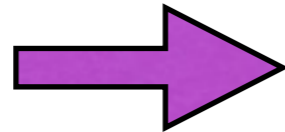
# What are the relevant degrees of freedom?

QCD



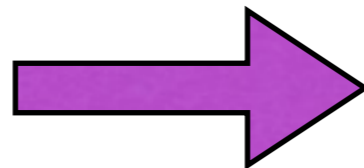
nucleons and pions

EW theory



3 massive vector bosons, 1 "Higgs"...

Cosmology



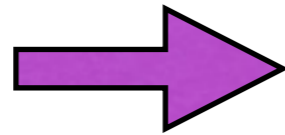
Cosmological Perturbations!

They (re-)enter the horizon

- 1) Small in amplitude (expansion in **number of cosmological perturbations**)
- 2) Large in size (expansion in number of derivatives)

# What are the relevant degrees of freedom?

QCD



nucleons and pions

EW theory

Cosmology

They (re-

As opposed to: Expansion in number of “fields” (Weinberg `08)

$$V = V_1\phi + V_2\phi^2 + V_3\phi^3 + V_4\phi^4$$

$$= V_2\delta\phi^2 + V_3\phi_0(t)\delta\phi^2 + 6V_4\phi_0^2(t)\delta\phi^2$$

All terms potentially important in cosmological perturbation theory!

- 1) Small in amplitude (expansion in **number of cosmological perturbations**)
- 2) Large in size (expansion in number of derivatives)

# Unitary Gauge in Cosmology

**The Effective Field Theory of Inflation** (Creminelli et al. '06, Cheung et al. '07)

Main idea: scalar degrees of freedom are 'eaten' by the metric. Ex:

$$\phi(t, \vec{x}) \rightarrow \phi_0(t) \quad (\delta\phi = 0) \quad -\frac{1}{2}\partial\phi^2 \rightarrow -\frac{1}{2}\dot{\phi}_0^2(t) g^{00}$$



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**Effective Field Theory of Dark Energy:** (Gubitosi, F.P., Vernizzi 2012)

- 1) Assume WEP (universally coupled metric  $S_m[g_{\mu\nu}, \Psi_i]$ )
- 2) Write the most generic action for  $g_{\mu\nu}$  compatible with the residual un-broken symmetries (3-diff).

# The Action

$$S = \int d^4x \sqrt{-g} \frac{M^2(t)}{2} [R - 2\lambda(t) - 2\mathcal{C}(t)g^{00}]$$

The most generic action written in terms of  $g_{\mu\nu}$  compatible with the residual symmetry of spatial diffeomorphisms

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Genuine 4-dim covariant terms are still allowed, but will in general be multiplied by functions of time cause time translations are broken

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General functions of time are allowed

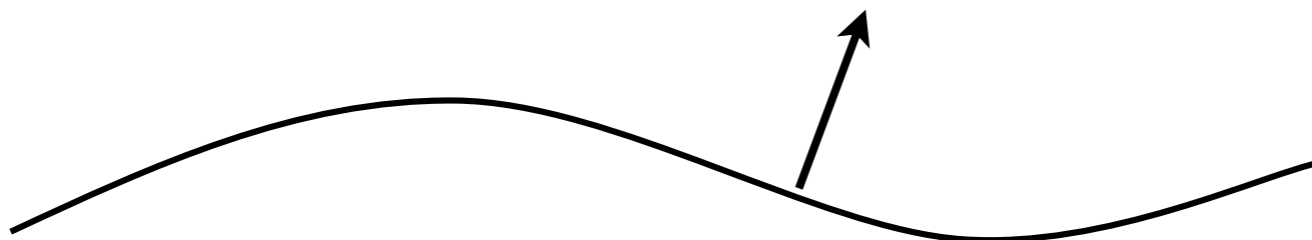
# The Action

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The most generic action written in terms of  $g_{\mu\nu}$  compatible with the residual symmetry of spatial diffeomorphisms

...as well as tensors with free '0' indices

Essentially: contractions with  $n_\mu = -\frac{\partial_\mu \phi}{\sqrt{-(\partial\phi)^2}}$





# The Action

$$S = \int d^4x \sqrt{-g} \frac{M^2(t)}{2} [R - 2\lambda(t) - 2\mathcal{C}(t)g^{00}]$$

Any arbitrarily complicated action with one scalar d.o.f. will reduce to **this** in Unitary gauge, plus **terms** that start explicitly quadratic in the perturbations

# The Action

$$S = \int d^4x \sqrt{-g} \frac{M^2(t)}{2} \left[ R - 2\lambda(t) - 2\mathcal{C}(t)g^{00} \right. \\ \left. + \mu_2^2(t)(\delta g^{00})^2 - \mu_3(t) \delta K \delta g^{00} + \epsilon_4(t) \left( \delta K^\mu_\nu \delta K^\nu_\mu - \delta K^2 + \frac{R^{(3)} \delta g^{00}}{2} \right) + \dots \right]$$

# The Action

Background (expansion history)

$$S = \int d^4x \sqrt{-g} \frac{M^2(t)}{2} \left[ R - 2\lambda(t) - 2\mathcal{C}(t)g^{00} \right. \\ \left. + \mu_2^2(t)(\delta g^{00})^2 - \mu_3(t)\delta K\delta g^{00} + \epsilon_4(t) \left( \delta K^\mu_\nu \delta K^\nu_\mu - \delta K^2 + \frac{R^{(3)}\delta g^{00}}{2} \right) + \dots \right]$$

(linear) perturbations

Time-dependent couplings

# The Power of the EFT of DE

$$S = \int d^4x \sqrt{-g} \frac{M^2(t)}{2} \left[ R - 2\lambda(t) - 2\mathcal{C}(t)g^{00} \right. \\ \left. + \mu_2^2(t)(\delta g^{00})^2 - \mu_3(t)\delta K\delta g^{00} + \epsilon_4(t) \left( \delta K^\mu{}_\nu \delta K^\nu{}_\mu - \delta K^2 + \frac{R^{(3)}\delta g^{00}}{2} \right) + \dots \right]$$

- Clear separation: background v.s. perturbed sector
- Expansion in number of cosmological perturbations
- Expansion in number of derivatives
- Observables, stability etc.

# Examples

$$S = \int d^4x \sqrt{-g} \frac{M^2(t)}{2} \left[ R - 2\lambda(t) - 2\mathcal{C}(t)g^{00} \right. \\ \left. + \mu_2^2(t)(\delta g^{00})^2 - \mu_3(t)\delta K\delta g^{00} + \epsilon_4(t) \left( \delta K^\mu{}_\nu \delta K^\nu{}_\mu - \delta K^2 + \frac{R^{(3)}\delta g^{00}}{2} \right) + \dots \right]$$

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const.

Quintessence

# Examples

$$S = \int d^4x \sqrt{-g} \frac{M^2(t)}{2} \left[ R - 2\lambda(t) - 2\mathcal{C}(t)g^{00} \right. \\ \left. + \mu_2^2(t)(\delta g^{00})^2 - \mu_3(t)\delta K\delta g^{00} + \epsilon_4(t) \left( \delta K^\mu_\nu \delta K^\nu_\mu - \delta K^2 + \frac{R^{(3)}\delta g^{00}}{2} \right) + \dots \right]$$

**Non-minimally coupled scalar field**  
(Brans-Dicke,  $f(R)$  etc.)

# Examples

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const.

**K-essence** (Amendariz-Picon et al., 2000)

$$S = \int d^4x \sqrt{-g} P(\phi, X) \quad X \equiv g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$$



# Examples

$$S = \int d^4x \sqrt{-g} \left[ \frac{M^2(t)}{2} [R - 2\lambda(t) - 2\mathcal{C}(t)g^{00}] \right. \\ \left. + \mu_2^2(t)(\delta g^{00})^2 - \mu_3(t)\delta K\delta g^{00} + \epsilon_4(t) \left( \delta K^\mu_\nu \delta K^\nu_\mu - \delta K^2 + \frac{R^{(3)}\delta g^{00}}{2} \right) + \dots \right]$$

“Galilean Cosmology” (Chow and Khoury, 2009)

$$S = \int d^4x \sqrt{-g} \left[ \frac{M^2}{2} e^{-2\phi/M} R - \frac{r_c^2}{M} (\partial\phi)^2 \square\phi \right]$$

# Examples

$$S = \int d^4x \sqrt{-g} \frac{M^2(t)}{2} \left[ R - 2\lambda(t) - 2\mathcal{C}(t)g^{00} + \mu_2^2(t)(\delta g^{00})^2 - \mu_3(t)\delta K\delta g^{00} + \epsilon_4(t) \left( \delta K^\mu_\nu \delta K^\nu_\mu - \delta K^2 + \frac{R^{(3)}\delta g^{00}}{2} \right) + \dots \right]$$

## “Generalized Galileons” ( $\equiv$ Horndeski)

(Deffayet et al., 2011)

$$\mathcal{L}_2 = A(\phi, X) ,$$

$$\mathcal{L}_3 = B(\phi, X)\square\phi ,$$

$$\mathcal{L}_4 = C(\phi, X)R - 2C_{,X}(\phi, X) [(\square\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2] ,$$

$$\mathcal{L}_5 = D(\phi, X)G^{\mu\nu}\nabla_\mu\nabla_\nu\phi + \frac{1}{3}D_{,X}(\phi, X) [(\square\phi)^3 - 3(\square\phi)(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3] ,$$

# Examples

$$\begin{aligned}
 S = & \int d^4x \sqrt{-g} \frac{M^2(t)}{2} [R - 2\lambda(t) - 2\mathcal{C}(t)g^{00} \\
 & + \mu_2^2(t)(\delta g^{00})^2 - \mu_3(t)\delta K\delta g^{00} + \epsilon_4(t) \left( \delta K^\mu_\nu \delta K^\nu_\mu - \delta K^2 + \frac{R^{(3)}\delta g^{00}}{2} \right) + \dots ]
 \end{aligned}$$

$\epsilon_4(t)$      $\tilde{\epsilon}_4(t)$

## Beyond Horndeski (linear)

The most general (linear) theory without higher derivatives on the propagating degree of freedom

# Beyond Horndeski (full, non-linear)

Standard Lore:

higher (time) derivatives: disaster!

New degrees of freedom AND ghost-like

## Equations of motion with higher derivatives BUT

- Not on the propagating degree of freedom once you solve the constraints
- Not if taken in appropriate combinations
- Not in some specific gauge (ex. the 3+1 ADM formalism)

# Beyond Horndeski (full, non-linear)

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# Beyond Horndeski (full, non-linear)

$$\mathcal{L}_4 = (2XC_X - C)(K^2 - K_{ij}K^{ij}) + C {}^{(3)}R + \dots$$

$$\mathcal{L}_4 = C(\phi, X)R - 2C_{,X}(\phi, X) [(\square\phi)^2 - (\nabla_\mu \nabla_\nu \phi)^2],$$

**3+1 decomposition**



# Beyond Horndeski (full, non-linear)

$$\mathcal{L}_4^{\text{new}} = A(N, t)(K^2 - K_{ij}K^{ij}) + B(N, t) {}^{(3)}R$$

see also X. Gao 1406.0822

**Generalization!**

$$\mathcal{L}_4 = (2XC_X - C)(K^2 - K_{ij}K^{ij}) + C {}^{(3)}R + \dots$$

$$\mathcal{L}_4 = C(\phi, X)R - 2C_{,X}(\phi, X)[(\square\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2],$$

**3+1 decomposition**

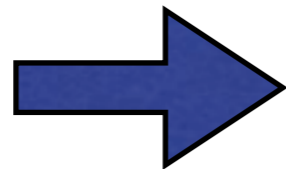


# Beyond Horndeski (full, non-linear)

$$\mathcal{L}_4^{\text{new}} = A(N, t)(K^2 - K_{ij}K^{ij}) + B(N, t) {}^{(3)}R$$

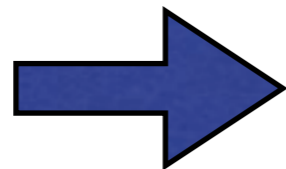
$$H_4^{\text{new}} = \int d^3x [N \mathcal{H}_0(N, h_{ij}, \pi^{ij}) + N^i \mathcal{H}_i(h_{ij}, \pi^{ij})]$$

$$\frac{\delta}{\delta N^i}$$



same as GR: 3 “first class” constraints

$$\frac{\delta}{\delta N}$$



fixes N in terms of the other variables

**6 - 3 = 3 d.o.f. As any healthy scalar tensor theory should have**

# Beyond Horndeski (full, non-linear)

- Equations of motion with higher derivatives
- Only two derivatives on the true propagating degree of freedom
- No ghosts
- Interesting phenomenology (modified Jeans phenomenon)

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$$(\partial_t^2 + c_m^2 k^2) \delta \rho_m \approx 0$$


$$(\partial_t^2 + c_s^2 k^2) \delta \phi - C_\phi \dot{\phi} \partial_t \delta \rho_m \approx 0,$$

$$(\partial_t^2 + c_m^2 k^2) \delta \rho_m - C_m k^2 \partial_t (\delta \phi / \dot{\phi}) \approx 0$$

# Stability and Observables

$$S = \int d^4x \sqrt{-g} \frac{M^2(t)}{2} [R - 2\lambda(t) - 2\mathcal{C}(t)g^{00} + \mu_2^2(t)(\delta g^{00})^2 - \mu_3(t)\delta K\delta g^{00} + \epsilon_4(t) \left( \delta K^\mu_\nu \delta K^\nu_\mu - \delta K^2 + \frac{R^{(3)}\delta g^{00}}{2} \right) + \dots]$$

$$\lambda(t), \mathcal{C}(t), \mu(t) \equiv \frac{dM^2(t)}{dt} \left\{ \begin{array}{l} \bar{w}(t) \\ \mu(t) \\ \mu_3(t) \\ \epsilon_4(t) \\ \mu_2^2(t) \end{array} \right. \begin{array}{l} \text{Expansion History} \\ \text{Growth rate, lensing etc.} \\ \text{Unconstrained} \end{array}$$

# Stability

$$S_\pi = \int a^3(t) M^2(t) \left[ A(\mu, \mu_2^2, \mu_3, \epsilon_4) \dot{\pi}^2 + B(\mu, \mu_3, \epsilon_4) \frac{(\vec{\nabla}\pi)^2}{a^2} \right] + \text{lower order in derivatives.}$$

No ghost:  $A > 0$

No gradient instabilities:  $B > 0$

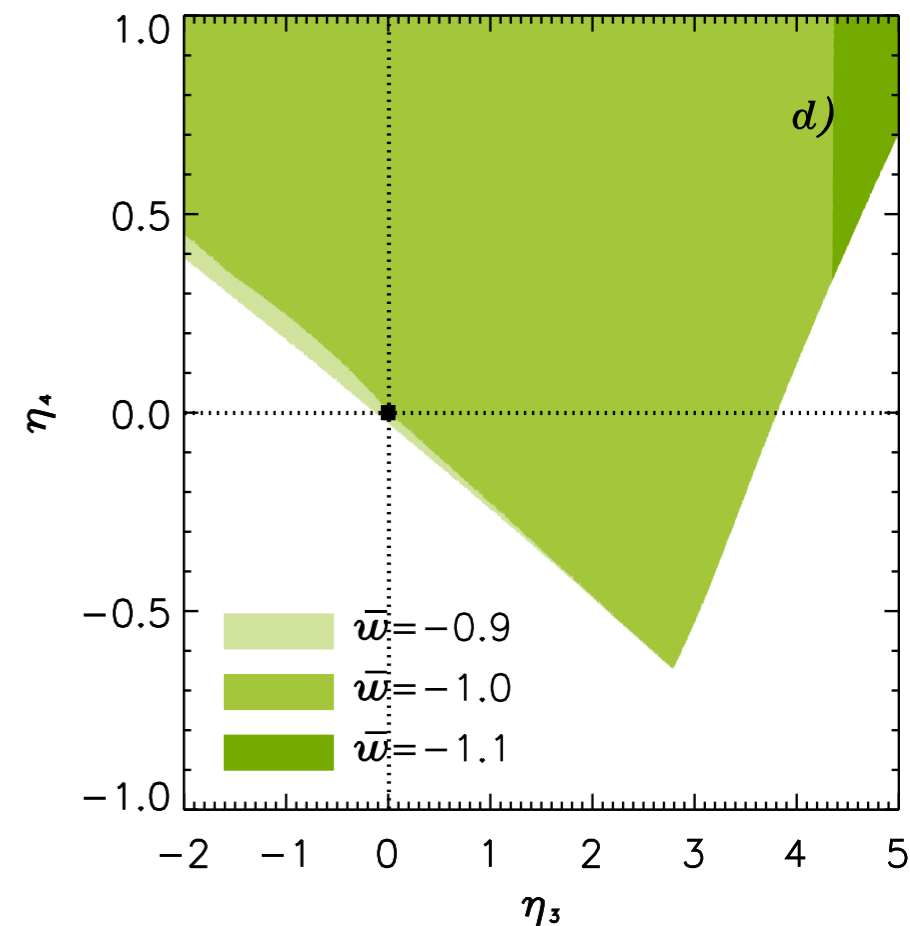
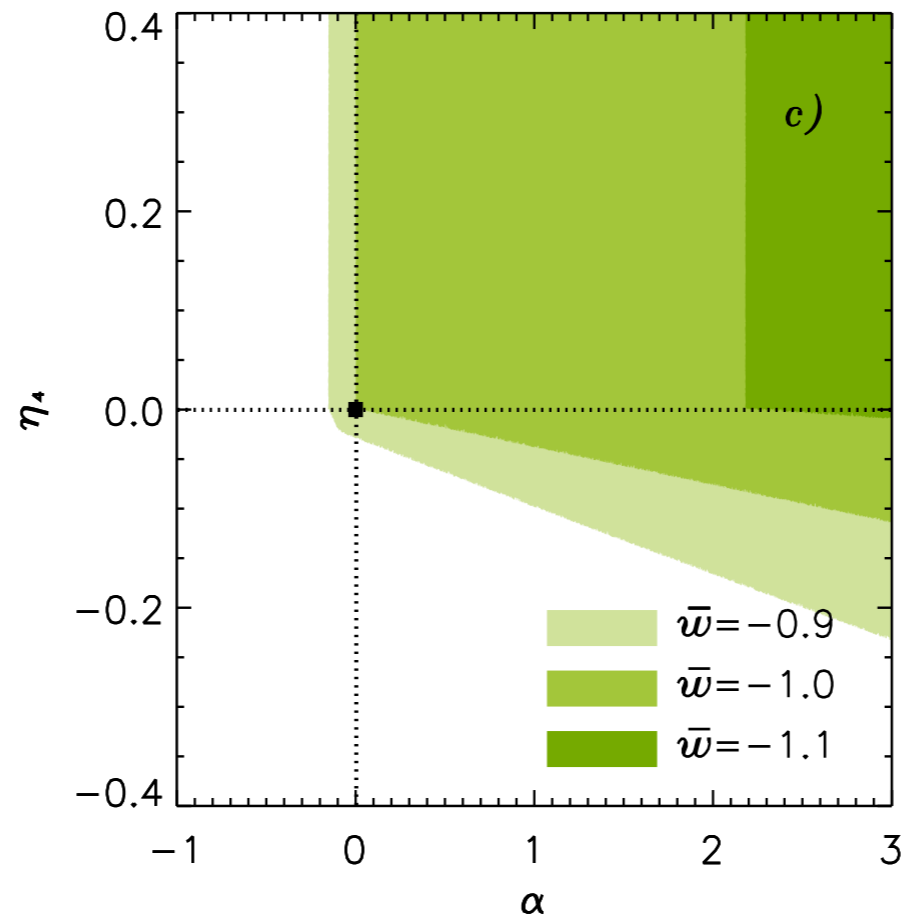
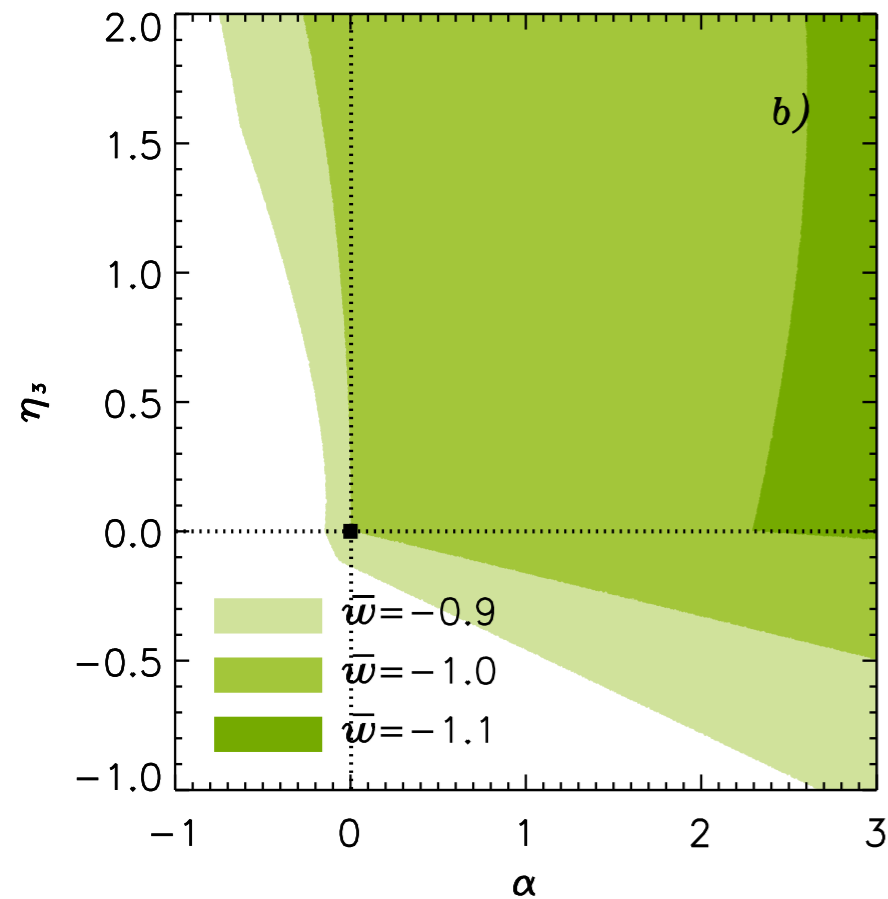
# Stability

$$S_\pi = \int a^3(t) M^2(t) \left[ A(\mu, \mu_2^2, \mu_3, \epsilon_4) \dot{\pi}^2 + B(\mu, \mu_3, \epsilon_4) \frac{(\vec{\nabla}\pi)^2}{a^2} \right] + \text{lower order in derivatives.}$$

No ghost:  $A > 0$

No gradient instabilities:  $B > 0$

$$\mu_2^2 = 0$$



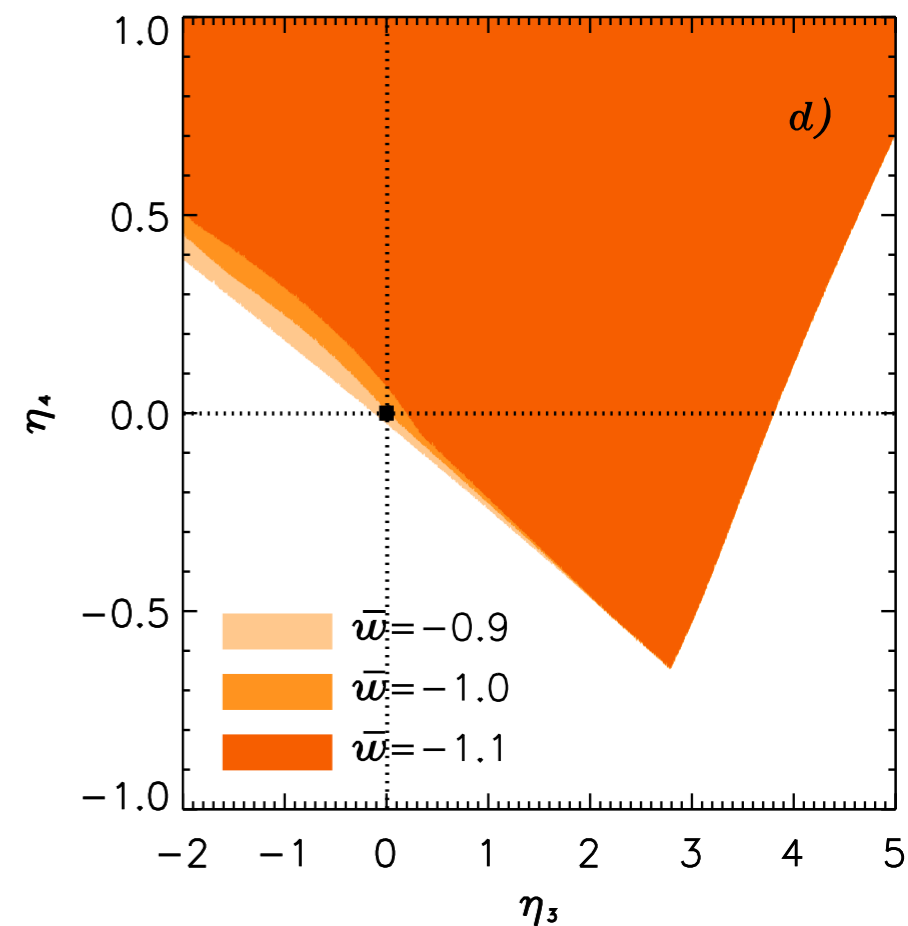
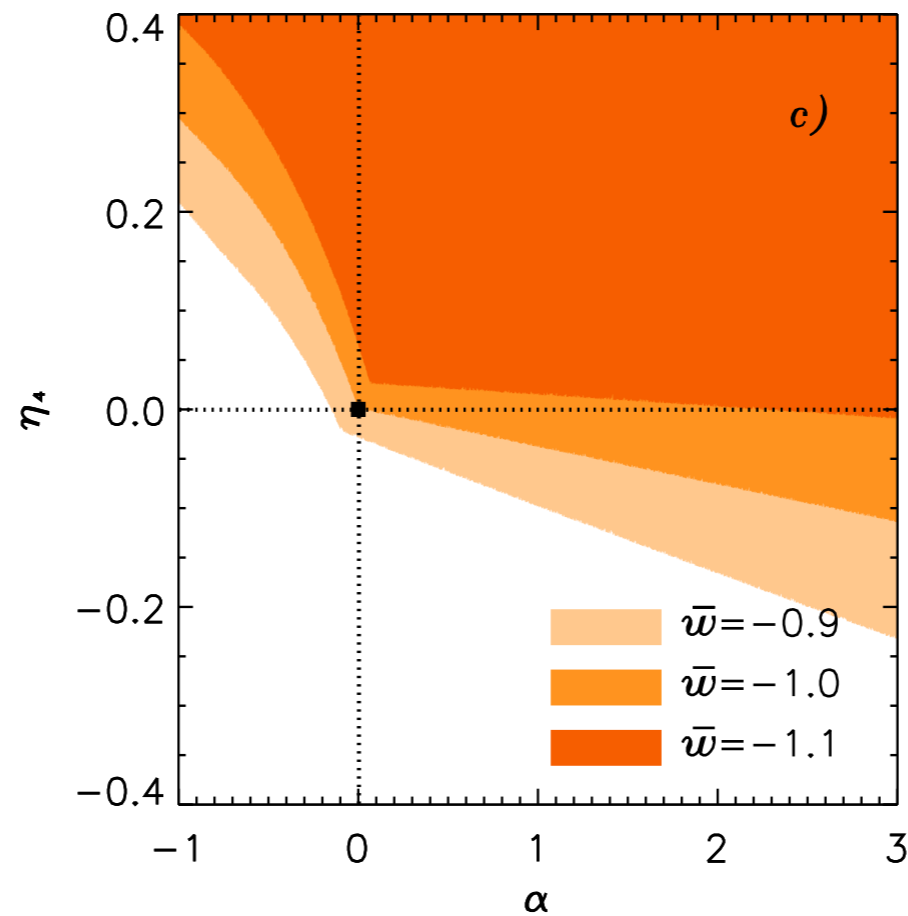
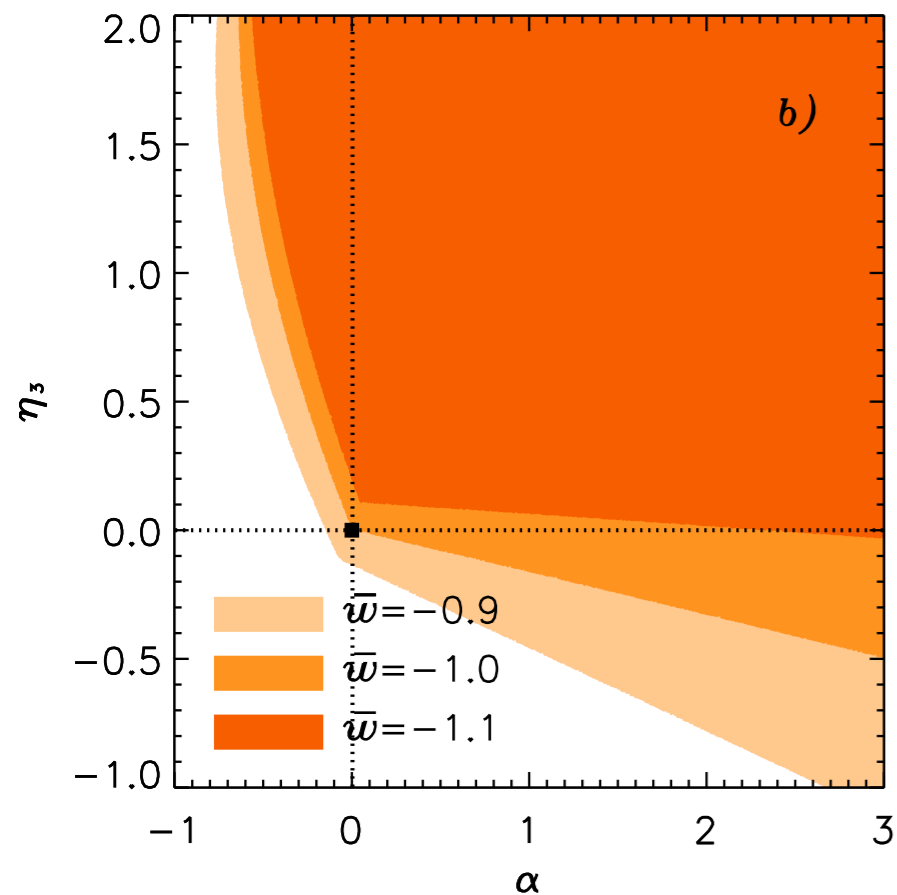
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No ghost:  $A > 0$

No gradient instabilities:  $B > 0$

$$\mu_2^2 \gg H^2$$





# Growth rate

$$G_{\text{eff}}(t) = G_{\text{eff}}(\mu, \mu_3, \epsilon_4)$$

$$f \equiv \frac{d \ln \delta}{d \ln a} = \Omega_m^{\gamma_0 + \gamma_1 \ln(\Omega_m)}$$

Steigerwald, Bel Marinoni 1403.0898

# Growth rate

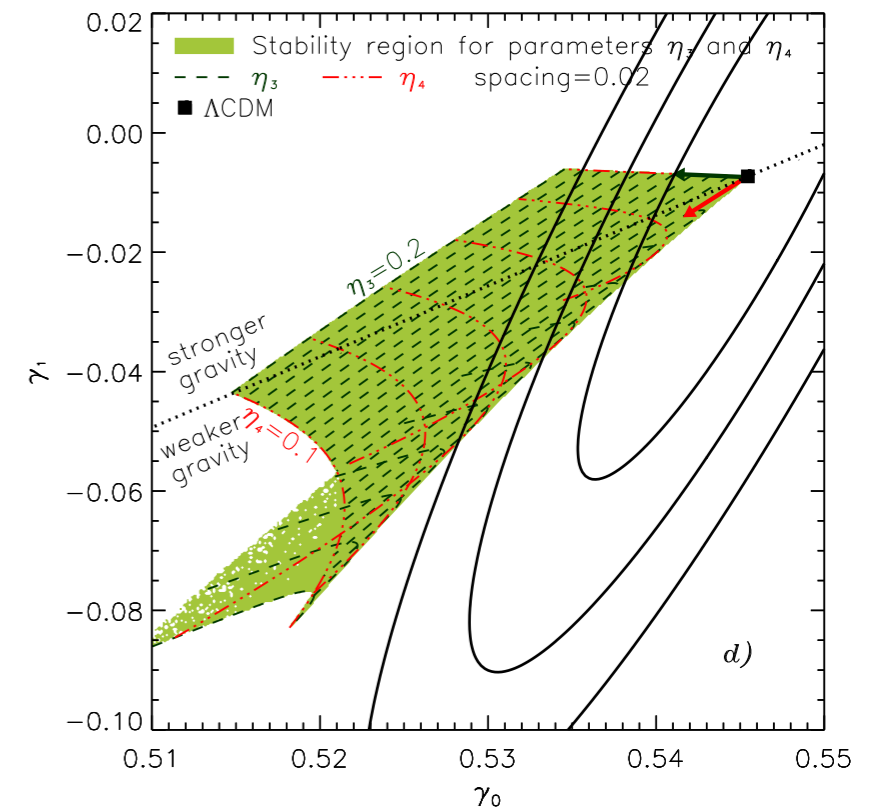
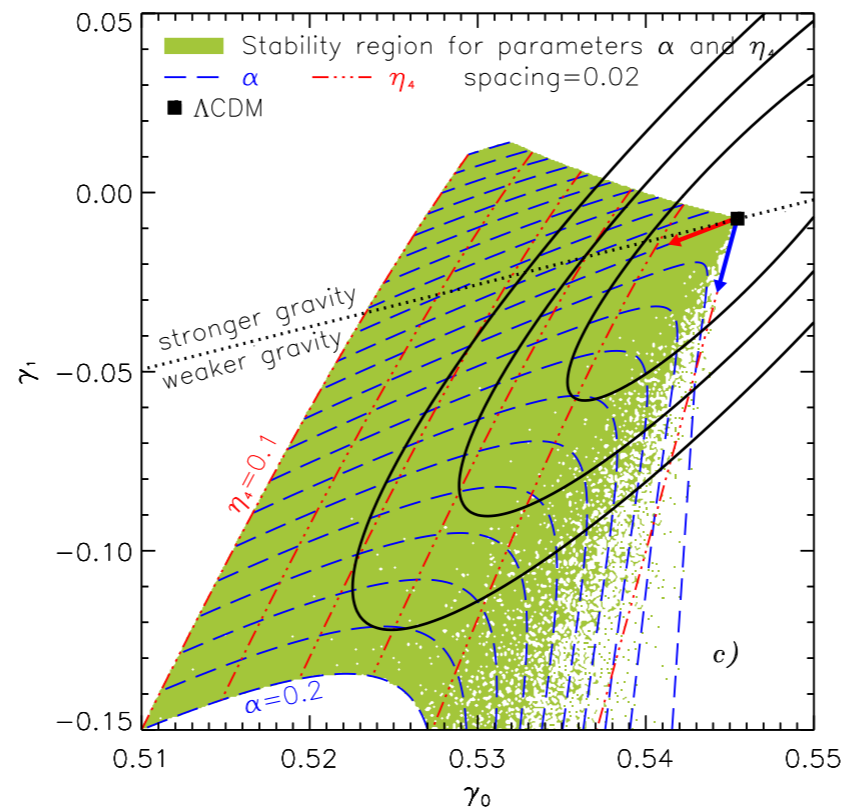
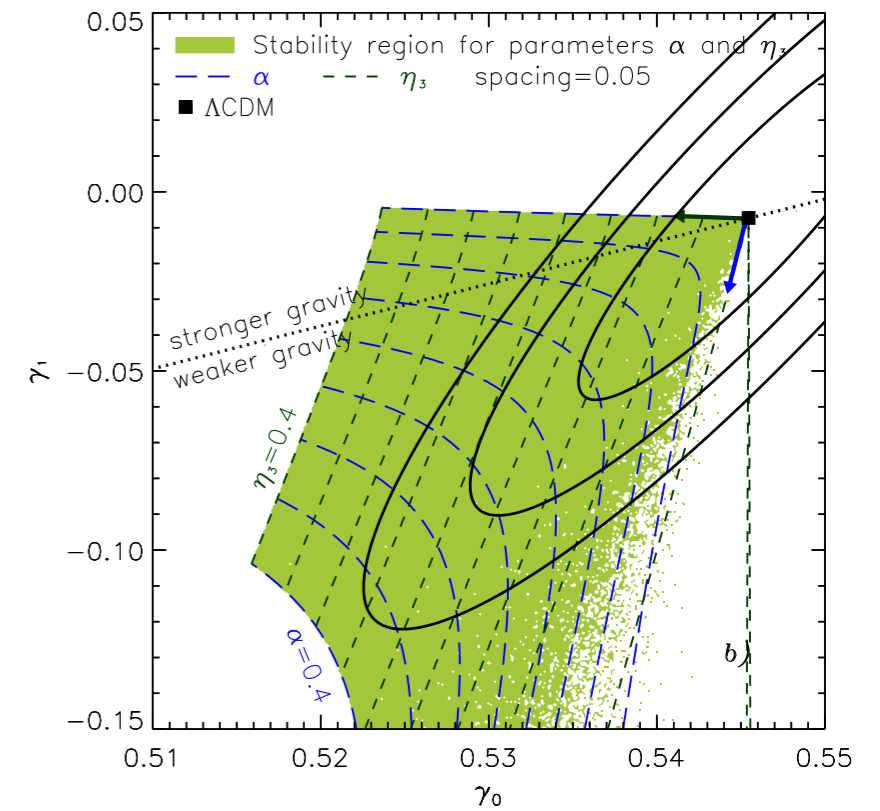
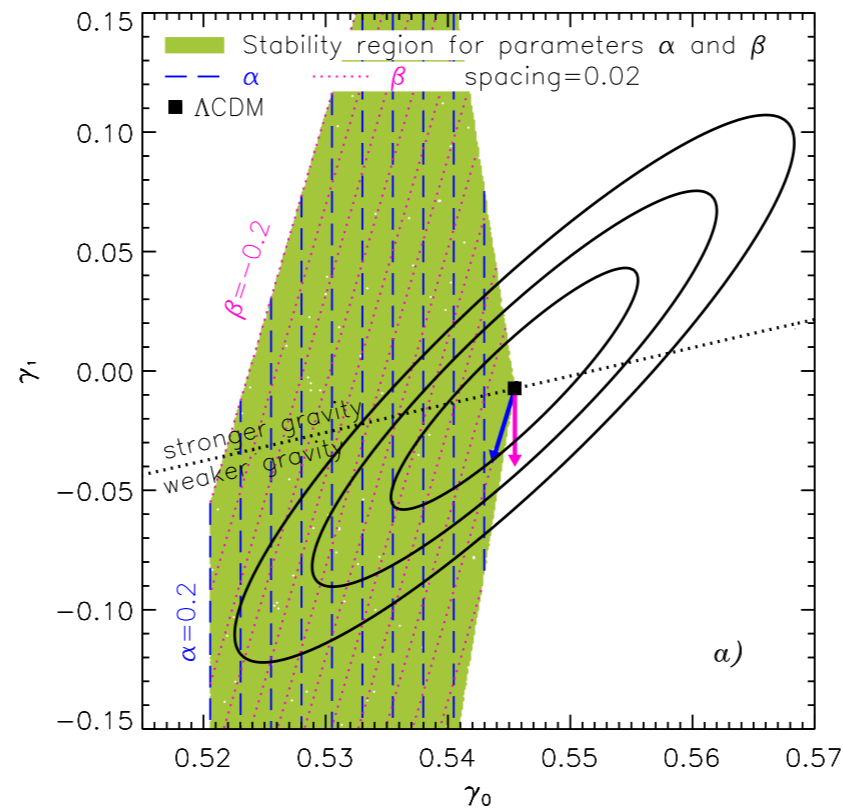
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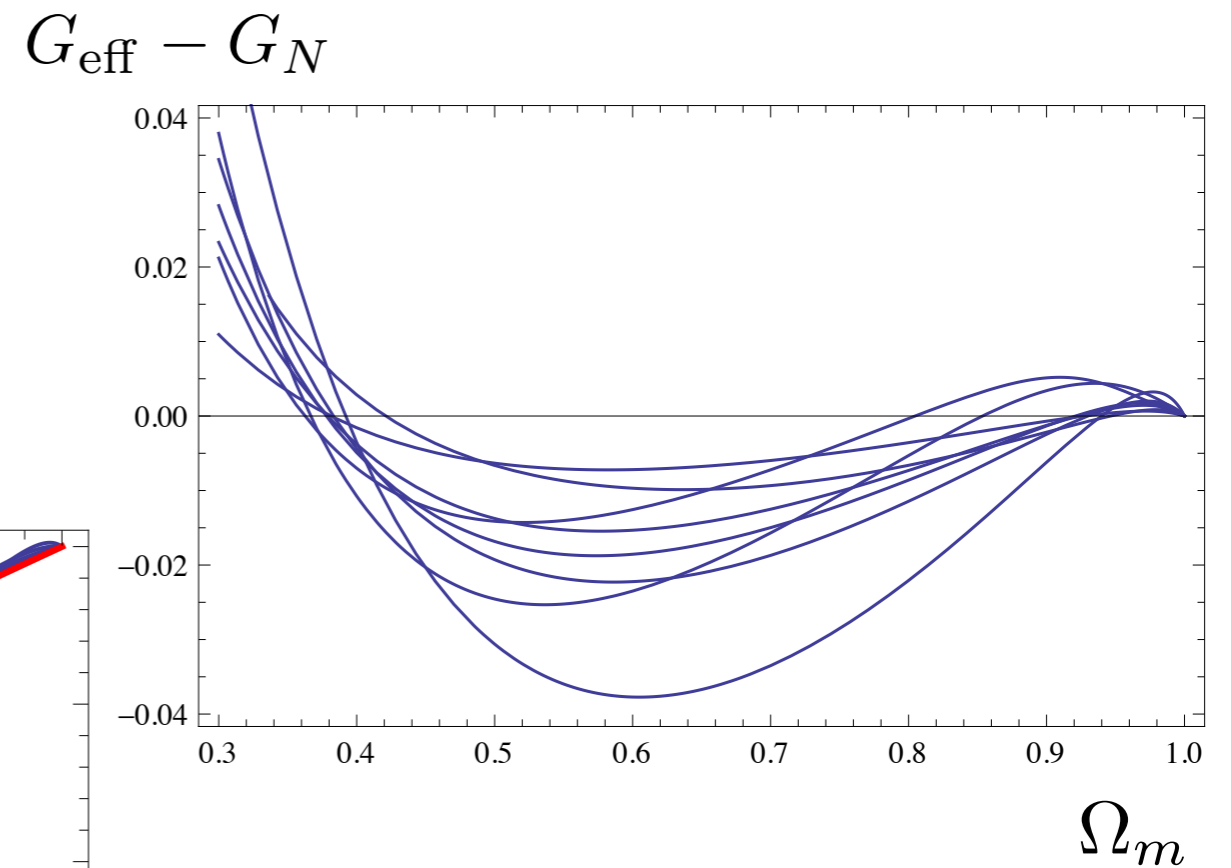
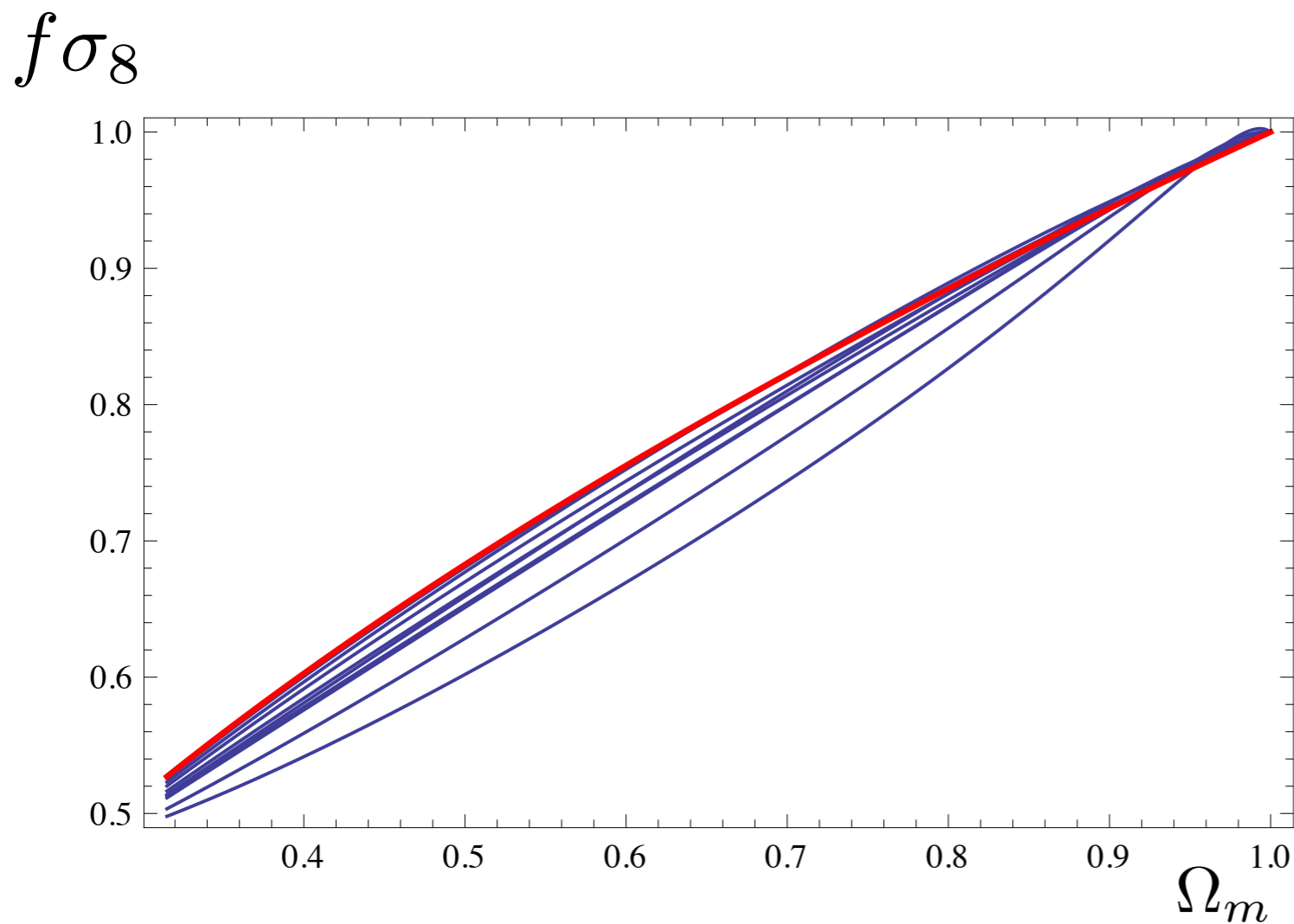
**Non trivial result:**

$$\gamma_0 < \gamma_0(\Lambda\text{CDM})$$



# Growth rate (preliminary)

## Modified gravity: less growth than LCDM?



# Conclusions

- Unifying framework for dark energy/modified gravity
- Effective language: cosmological perturbations as the relevant d.o.f.
- Systematic way to address stability (e.g. stable violations of NEC)
- Observational constraints and forecasts: much work in progress

$$S_\pi = \int a^3 M^2 \left\{ \left[ (\mathcal{C} + 2\mu_2^2)(1 + \epsilon_4) + \frac{3}{4}(\mu - \mu_3)^2 \right] \dot{\pi}^2 - \left[ (\mathcal{C} + \frac{\dot{\mu}_3}{2} - \dot{H}\epsilon_4 + H\dot{\epsilon}_4)(1 + \epsilon_4) - (\mu - \mu_3) \left( \frac{\mu - \mu_3}{4(1 + \epsilon_4)} - \mu - \dot{\epsilon}_4 \right) \right] \frac{(\vec{\nabla}\pi)^2}{a^2} \right\}$$


---


$$4\pi G_{\text{eff}} = \frac{1}{2M^2} \frac{2\mathcal{C} + 2(\mu + \dot{\epsilon}_4)^2 + \dot{\mu}_3 - 2\dot{H}\epsilon_4 + 2H\dot{\epsilon}_4 + 3(a/k)^2 \mathcal{A}}{(1 + \epsilon_4)^2 [2\mathcal{C} + \dot{\mu}_3 - 2\dot{H}\epsilon_4 + 2H\dot{\epsilon}_4] + 2(1 + \epsilon_4)(\mu + \dot{\epsilon}_4)(\mu - \mu_3) - (\mu - \mu_3)^2/2 + 3(a/k)^2 \mathcal{A}'}$$


---

$$\dot{\mu}_3 \equiv \dot{\mu}_3 + \mu\mu_3 + H\mu_3, \quad \dot{\epsilon}_4 \equiv \dot{\epsilon}_4 + \mu\epsilon_4 + H\epsilon_4$$

$$\mathcal{A} \equiv 2\dot{H}\mathcal{C} - \dot{H}\dot{\mu}_3 + \ddot{H}(\mu - \mu_3) - 2H\dot{H}\mu_3 - 2H^2(\mu^2 + \dot{\mu}), \quad \mathcal{A}' \equiv (1 + \epsilon_4)^2 \mathcal{A}$$

# The Action


$$S = \int d^4x \sqrt{-g} \left[ \frac{M^2}{2} f(t) R - \Lambda(t) - c(t) g^{00} \right] + S_{DE}^{(2)}$$


Enough for background equations:

$$c = \frac{1}{2}(-\ddot{f} + H\dot{f})M^2 + \frac{1}{2}(\rho_D + p_D)$$

$$\Lambda = \frac{1}{2}(\ddot{f} + 5H\dot{f})M^2 + \frac{1}{2}(\rho_D - p_D)$$

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Generally Related to post-newtonian parameters

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Generally Related to post-newtonian parameters

“Bare” Planck Mass

Defined by the modified Friedman equations

Matter + Dark matter (in practice  $\rho_m \propto a^{-3}$ )

$$H^2 = \frac{1}{3fM^2}(\rho_m + \rho_D)$$

$$\dot{H} = -\frac{1}{2fM^2}(\rho_m + \rho_D + p_m + p_D)$$

# Mixing with gravity 1: Brans-Dicke

$$S = \int \sqrt{-g} \left( \frac{M^2}{2} f R - \Lambda - c g^{00} + \frac{M_2^4}{2} (\delta g^{00})^2 - \frac{\bar{m}_1^3}{2} \delta g^{00} \delta K + \frac{1}{2} T^{\mu\nu} \delta g_{\mu\nu} \right)$$

Apply Stueckelberg and go to  
Newtonian Gauge

$$ds^2 = -(1 + 2\Phi)dt^2 + a^2(1 - 2\Psi)\delta_{ij}dx^i dx^j$$

$$S^{\text{kinetic}} \equiv \int M^2 f \left[ -3\dot{\Psi}^2 - 2\vec{\nabla}\Phi\vec{\nabla}\Psi + (\vec{\nabla}\Psi)^2 + c\dot{\pi}^2 - c(\vec{\nabla}\pi)^2 + 3(\dot{f}/f)\dot{\Psi}\dot{\pi} + (\dot{f}/f)\vec{\nabla}\pi(\vec{\nabla}\Phi - 2\vec{\nabla}\Psi) \right]$$

Mixing

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De-mixing = conformal transformation

Mixing

$$\Phi_E = \Phi + \frac{1}{2}(\dot{f}/f)\pi$$

$$\Psi_E = \Psi - \frac{1}{2}(\dot{f}/f)\pi$$

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$$1 - \gamma \equiv \frac{\Phi - \Psi}{\Phi} = \frac{M^2 \dot{f}^2 / f}{2(c + M^2 \dot{f}^2 / f)}$$

anisotropic stress

Newtonian  
limit



$$G_{\text{eff}} = \frac{1}{8\pi M^2 f} \frac{c + M^2 \dot{f}^2 / f}{c + \frac{3}{4} M^2 \dot{f}^2 / f}$$

dressed Newton constant

# Mixing with gravity 2:

(Cf. braiding: Deffayet et al., 2010)

$$S = \int \sqrt{-g} \left( \frac{M^2}{2} f R - \Lambda - c g^{00} + \frac{M_2^4}{2} (\delta g^{00})^2 - \frac{\bar{m}_1^3}{2} \delta g^{00} \delta K + \frac{1}{2} T^{\mu\nu} \delta g_{\mu\nu} \right)$$

$f(t) = 1$

Apply Stueckelberg and go to  
Newtonian Gauge

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$$S^{\text{kinetic}} \equiv \int M^2 \left[ -3\dot{\Psi}^2 - 2\vec{\nabla}\Phi\vec{\nabla}\Psi + (\vec{\nabla}\Psi)^2 \right] + c\dot{\pi}^2 - c(\vec{\nabla}\pi)^2 - 3\bar{m}_1^3\dot{\Psi}\dot{\pi} - \bar{m}_1^3\vec{\nabla}\Phi\vec{\nabla}\pi$$

De-mixing  $\neq$  conformal transformation

$$\Phi_E = \Phi + \frac{\bar{m}_1^3}{2M^2}\pi$$

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Mixing

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Mixing

Speed of Sound of DE

$$c_s^2 = \frac{c + \frac{1}{2}(H\bar{m}_1^3 + \dot{\bar{m}}_1^3) - \frac{1}{4}\bar{m}_1^6/M^2}{c + \frac{3}{4}\bar{m}_1^6/M^2}$$



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$$1 - \gamma = \frac{\Phi - \Psi}{\Phi} = 0$$

NO anisotropic stress

Newtonian  
limit



$$G_{\text{eff}} = \frac{1}{8\pi M^2 f} \left( 1 - \frac{\bar{m}_1^3}{4cM^2} \right)^{-1}$$

dressed Newton constant