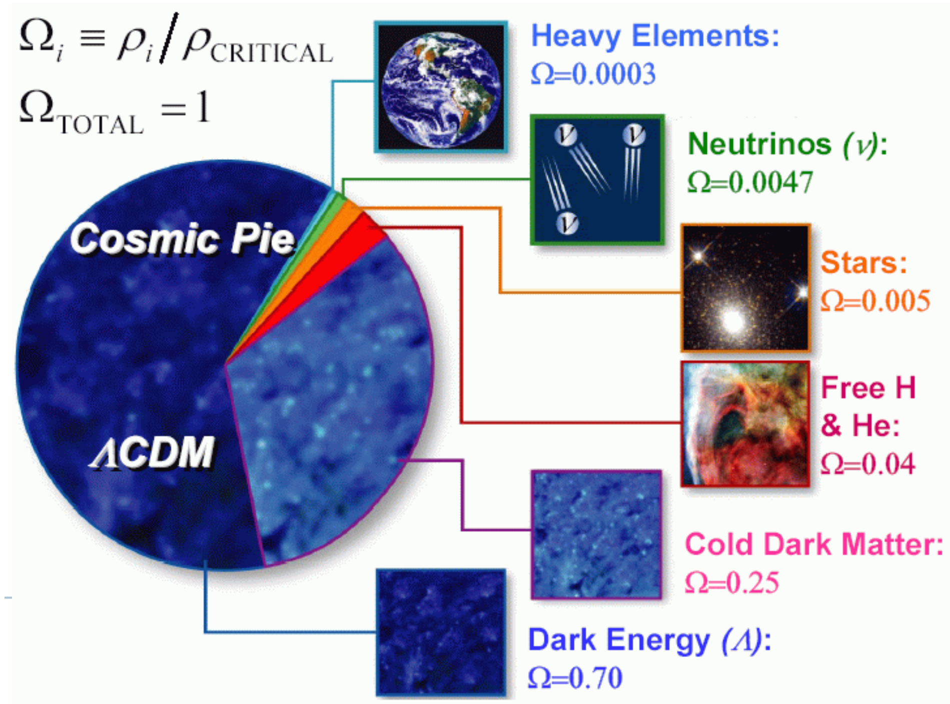
Non-linear structure formation in modified gravity models

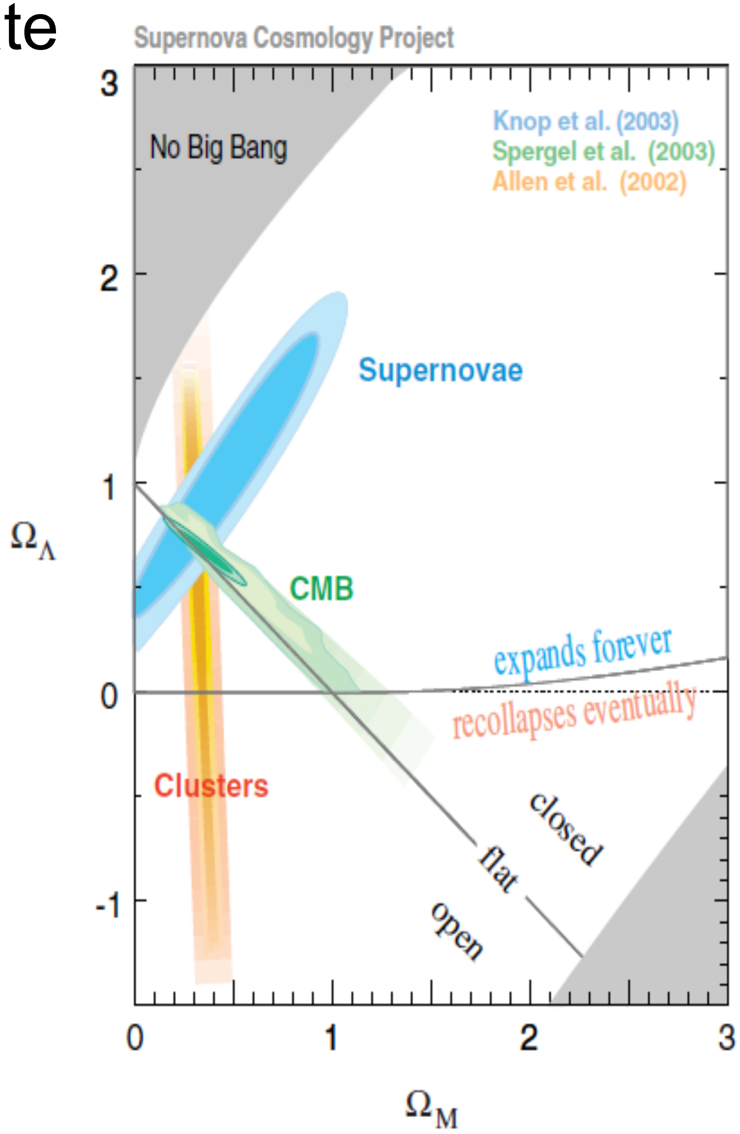
Kazuya Koyama University of Portsmouth

Cosmic acceleration

- Many independent data sets indicate the expansion of the Universe is accelerating

- Standard cosmology requires





Dark energy v Dark gravity

- Cosmological constant is the simplest candidate

  but the theoretical prediction is more than 50 orders of magnitude larger than the observed values

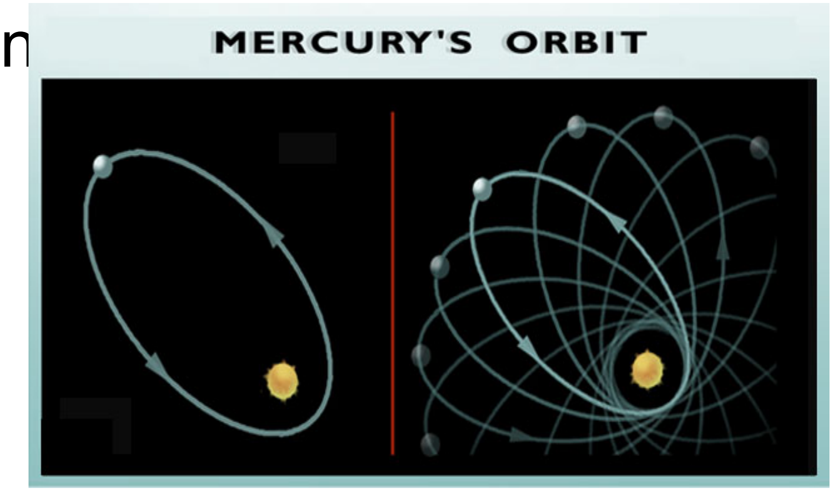
  - Most embarrassing observation in physics

- Standard model of cosmology is based on GR

  but we have never tested GR on cosmological scales

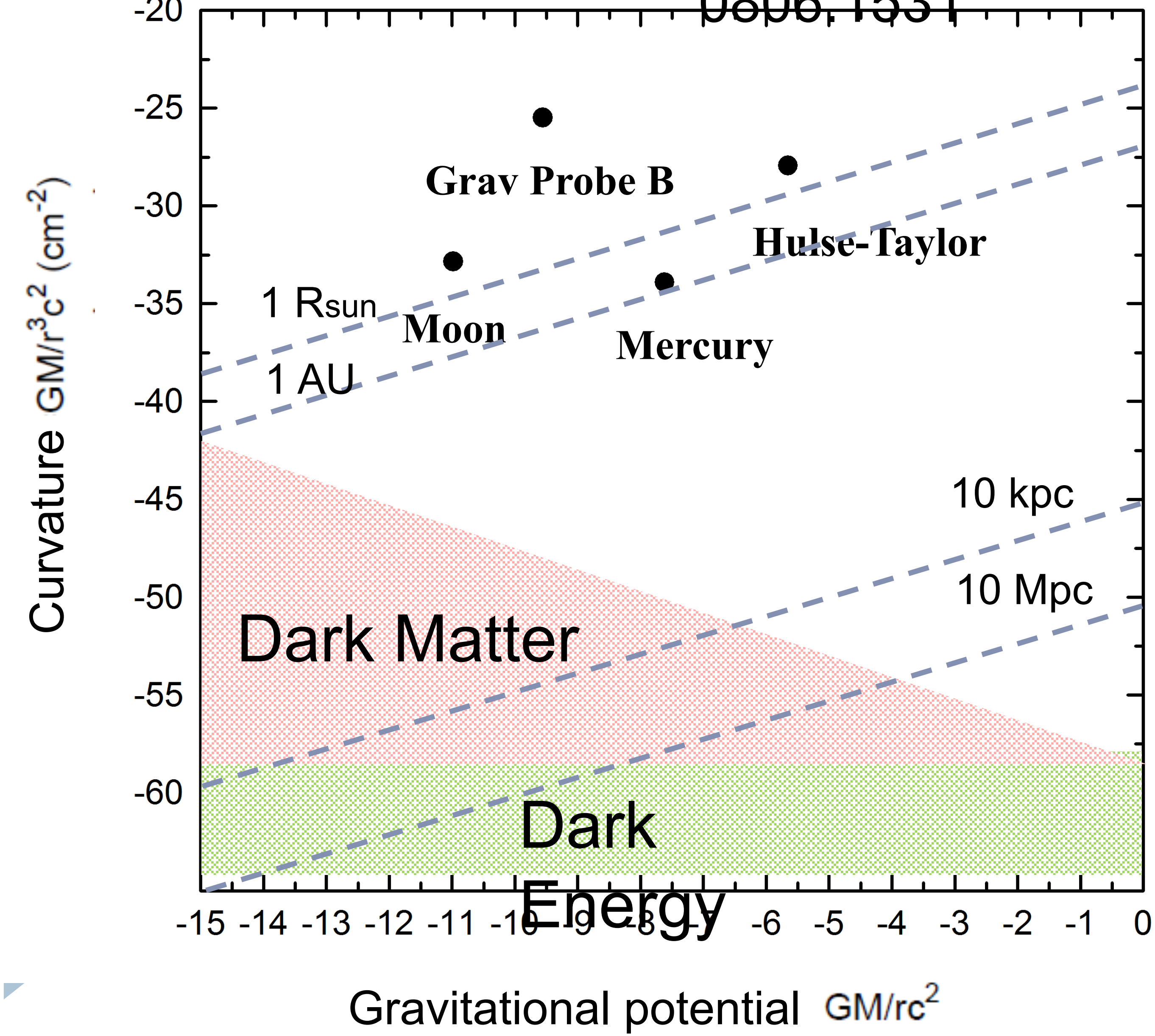
  cf. precession of perihelion

  dark planet v GR



Assuming GR

Psaltis 0806.1531

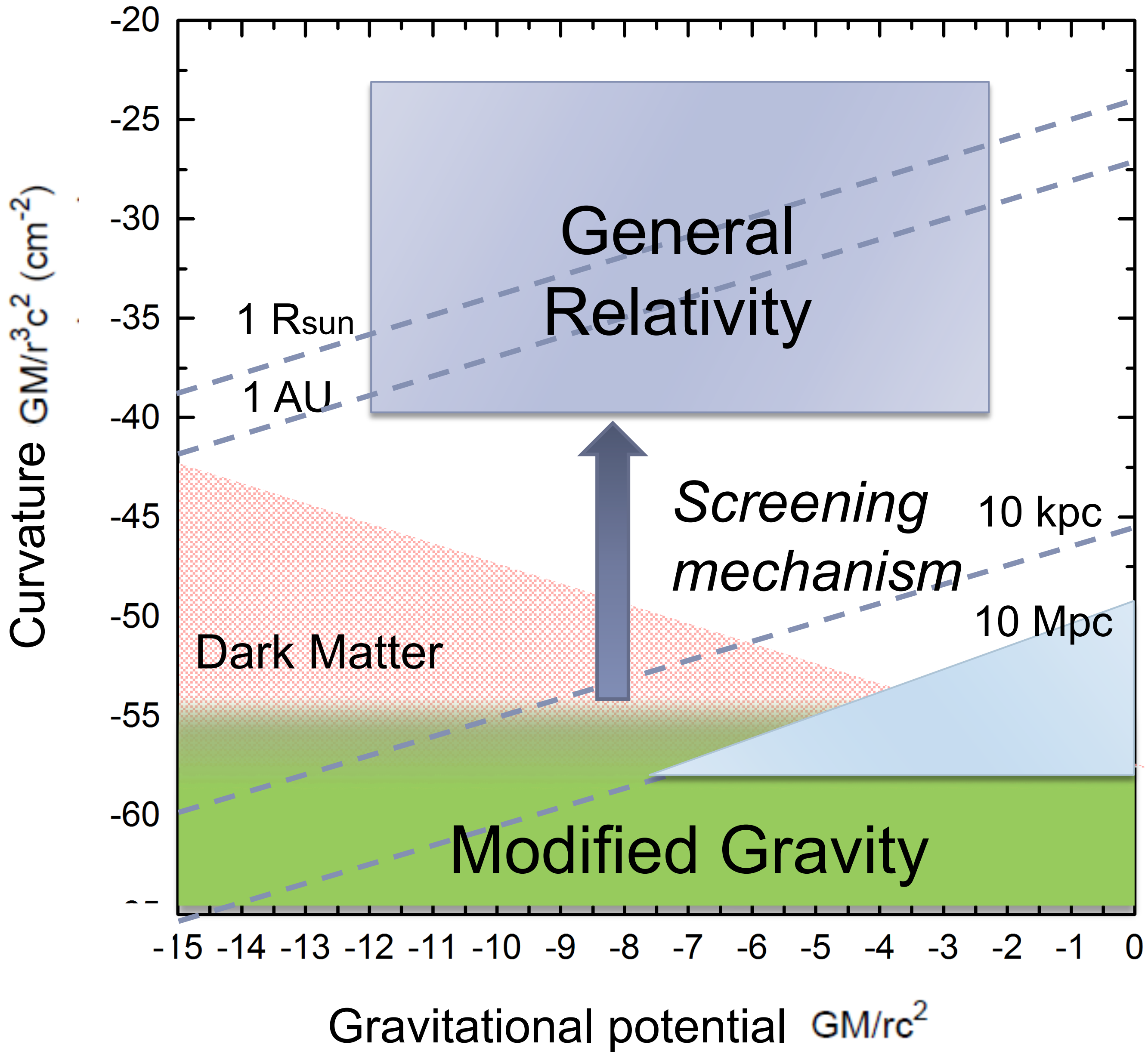


curvature

$$R = \frac{GM}{r^3 c^2}$$

potential

$$\Phi = \frac{GM}{rc^2}$$



Cosmological tests of GR

Taruya's talk

General picture

- Largest scales

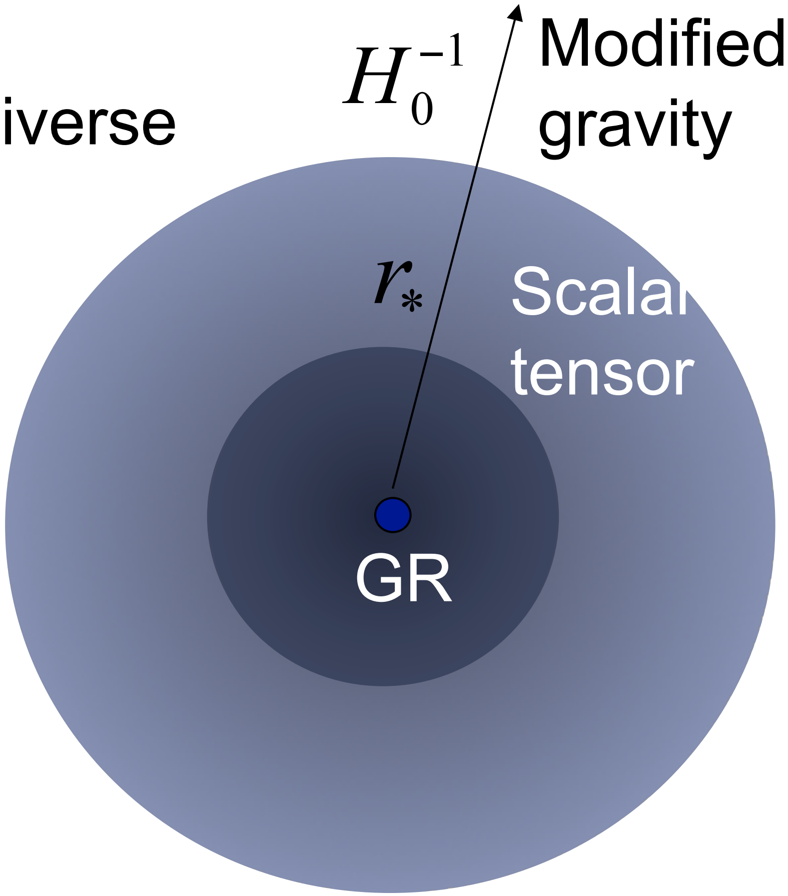
  gravity is modified so that the universe accelerates without dark energy

- Large scale structure scales

  gravity is still modified by a fifth force from scalar graviton

- Small scales (solar system)

  GR is recovered



Brans-Dicke theory

- Action

$$S = \int d^4x \left(\psi R - \frac{\omega_{BD}}{\psi} (\nabla \psi)^2 + L_m \right)$$

f(R) gravity: $\omega_{BD} = 0$

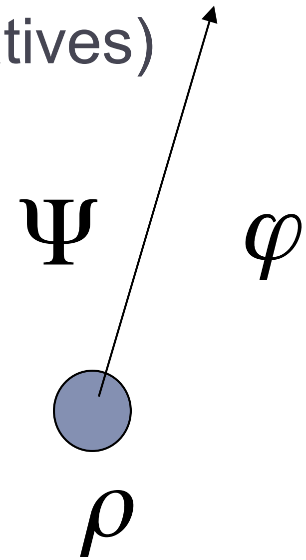
quasi-static approximations (neglecting time derivatives)

$$ds^2 = -(1 + 2\Psi) dt^2 + a(t)^2 (1 - 2\Phi) d\vec{x}^2 \quad \psi = \psi_0 + \varphi$$

$$(3 + 2\omega_{BD}) \nabla^2 \varphi = -8\pi G \rho$$

$$\nabla^2 \Psi = 4\pi G \rho - \frac{1}{2} \nabla^2 \varphi$$

$$\Phi - \Psi = -\varphi$$



Constraints on BD parameter

- Solutions

$$(3+2\omega_{BD})\nabla^2\varphi = -8\pi G\rho$$

$$\nabla^2\Psi = -4\pi G\left(\frac{4+2\omega_{BD}}{3+2\omega_{BD}}\right)\rho, \quad G_{eff} = \left(\frac{4+2\omega_{BD}}{3+2\omega_{BD}}\right)G$$

$$\Psi = \frac{2+\omega_{BD}}{1+\omega_{BD}}\Phi \equiv \gamma^{-1}\Phi$$

- PPN parameter

$$\gamma = \frac{1+\omega_{BD}}{2+\omega_{BD}}$$

$$\gamma - 1 = (2.1 \pm 2.3)\times 10^{-5} \qquad \omega_{BD} \geq 40{,}000$$

This constraint excludes any detectable modifications in cosmology

Screening mechanism

- Require screening mechanism to restore GR

$$S = \int d^4x \left(\psi R - \frac{\omega_{BD}(\psi)}{\psi} (\nabla \psi)^2 + V(\psi) + N(\nabla \psi, \nabla^2 \psi) \right)$$

recovery of GR must be environmental dependent

- make the scalar short-ranged using $V(\psi)$ (*chameleon*)
- make the kinetic term large to suppress coupling to matter using $\omega_{BD}(\psi)$ (*dilaton/symmetron*)
  or $N(\nabla \psi)$ (k-mouflage) $N(\nabla^2 \psi)$ (*Vainshtein*)

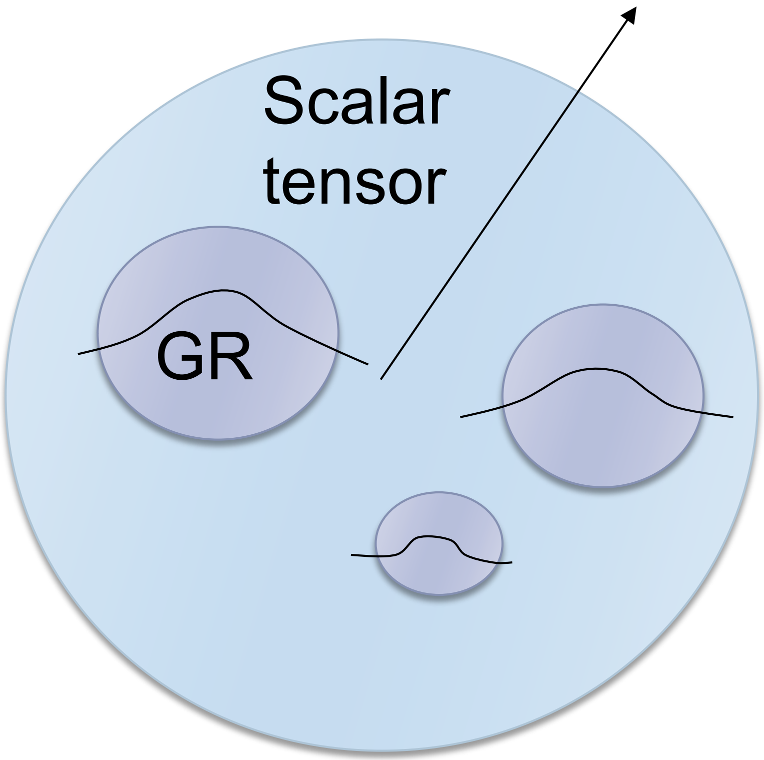
Break equivalence principle

$$\int d^4x \left(B(\psi) L_{baryon} + L_{CDM} \right)$$

remove the fifth force from baryons (*interacting DE models* in Einstein frame)

Behaviour of gravity

There regimes of gravity



In most models, the scalar mode obeys non-linear equations describing the transition from scalar tensor theory on large scales to GR on small scales

$$\rho_{crit} \approx 10^{-29} g / cm^3,$$

$$\rho_{galaxy} \approx 10^{-24} g / cm^3,$$

$$\rho_{solar} \approx 10 g / cm^3$$

Understandings of non-linear clustering require N-body simulations

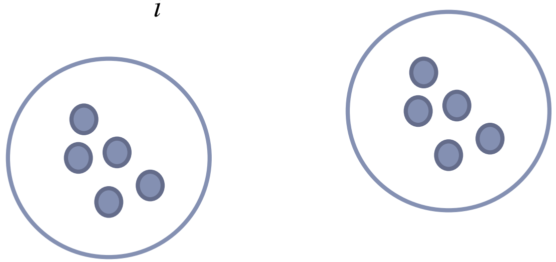
where the non-linear scalar equation needs to be solved

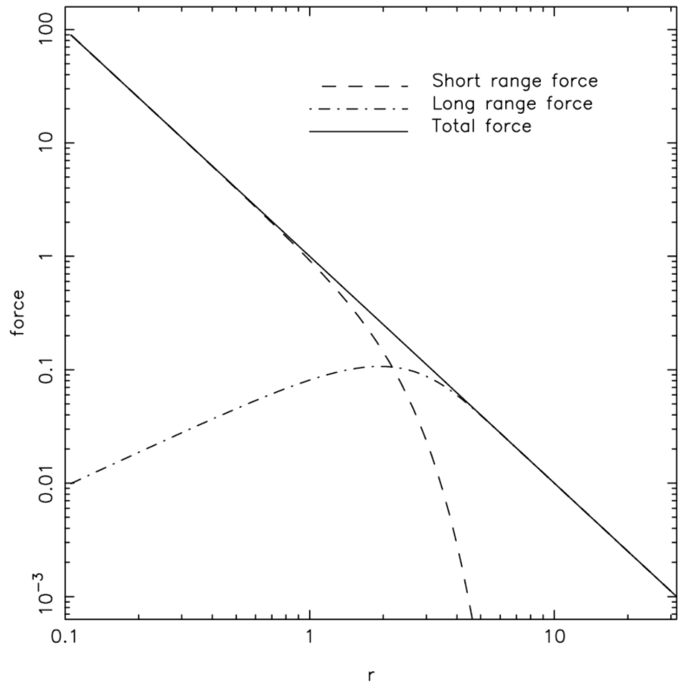
N-body simulations

- GR

  superposition of forces

$$\vec{F}(r) = \sum_i m_i \vec{f}_i(r - r_i)$$





- Modified gravity models

  the non-linear nature of the scalar field equation implies that the superposition rule does not hold

  - It is required to solve the non-linear scalar equation directly

    a computational challenge
  - The breakdown of the superposition rule has interesting consequences

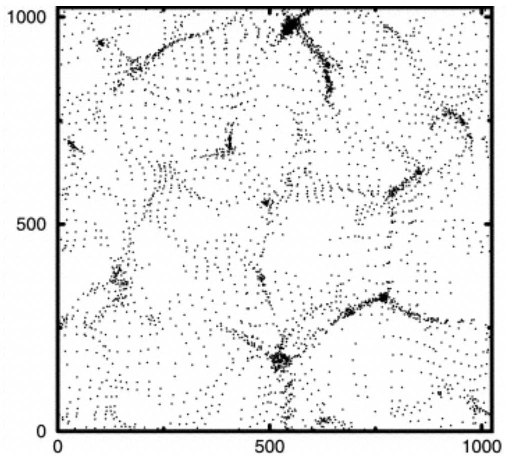
N-body Simulations

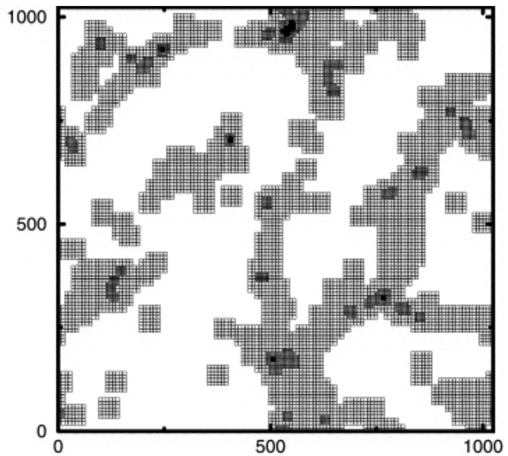
- Multi-level adaptive mesh refinement
- solve Poisson equation using a linear Gauss-Seidel relaxation
- add a scalar field solver using a non-linear Gauss Seidel relaxation

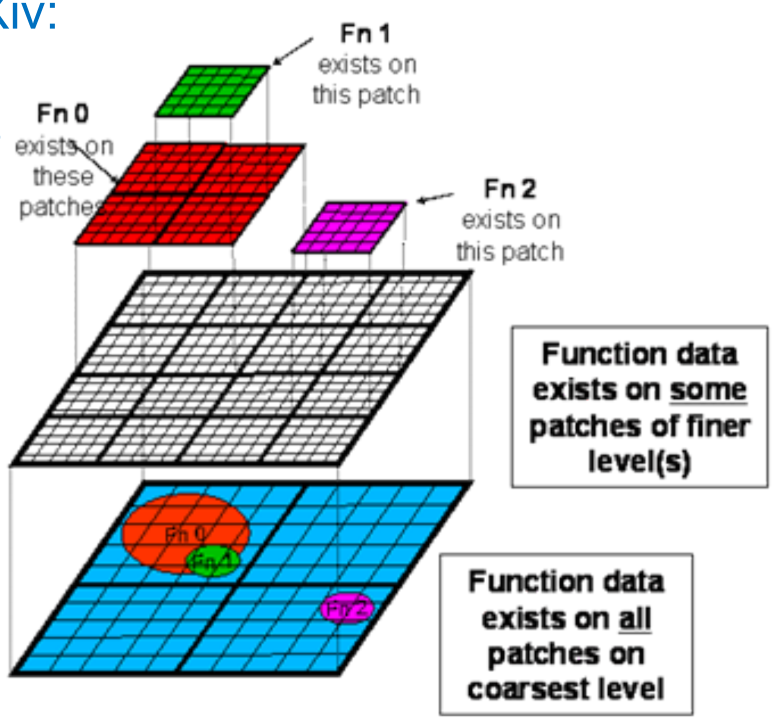
  ECOSMOG Li, Zhao, Teyssier, KK arXiv:1110.1379

  MG-GADGET Puchwein, Baldi, Springel arXiv: 1305.2418

  ISIS Llinares, Mota, Winther arXiv:13076748







Chameleon mechanism

- Scalar is coupled to matter

  Depending on density, the mass can change

  It is easier to understand the dynamics in Einstein frame

$$S = \int d^4x \left[\sqrt{-g} \left(\psi R - \frac{\omega_{BD}}{\psi} (\nabla \psi)^2 + V(\psi) \right) + L_m[g_{\mu\nu}] \right]$$

$$g_{\mu\nu} = \exp\left(-\frac{\alpha\phi}{M_{pl}} \right) \bar{g}_{\mu\nu},$$

$$\log\psi = 2\frac{\alpha\phi}{M_{pl}}$$

$$S_E = \int d^4x \left[\sqrt{-\bar{g}} \left(\bar{R} - \frac{1}{2} (\bar{\nabla}\phi)^2 + \bar{V}(\phi) \right) + L_m[e^{-\alpha\phi/M_{pl}} \bar{g}_{\mu\nu}] \right]$$

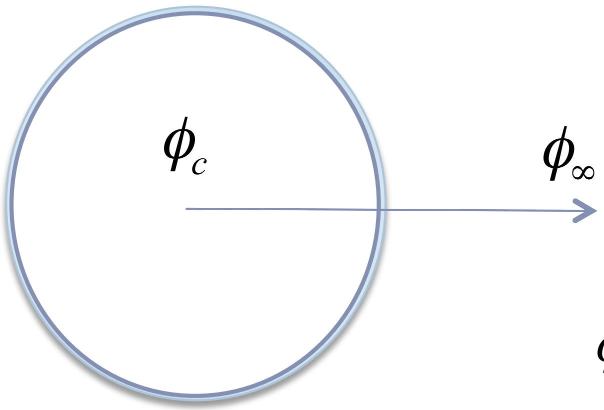
$$\alpha = \sqrt{\frac{1}{3 + 2\omega_{BD}}}$$

$$\nabla^2\phi = \partial_\phi V + \frac{\alpha}{M_{pl}} \rho e^{\alpha\phi/M_{pl}} \qquad V_{eff}(\phi) = V + \rho e^{\alpha\phi/M_{pl}}$$

Thin shell condition

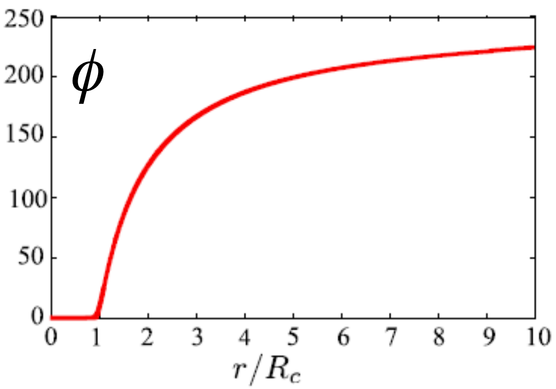
- If the thin shell condition is satisfied, only the shell of the size ΔR_c contributes to the fifth force

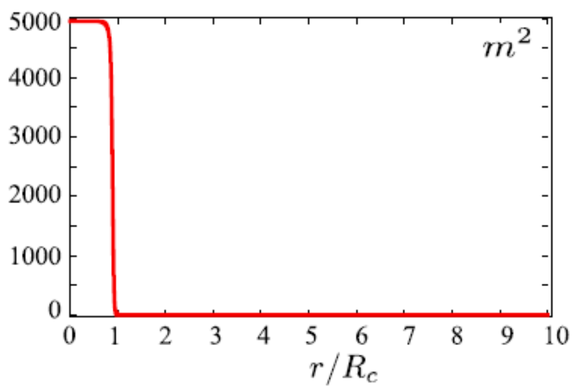
$$V_{eff}(\phi) = V + \rho e^{\alpha\phi/M_{pl}}$$



$$\frac{\Delta R_c}{R_c} = \frac{(\phi_\infty - \phi_c)/M_{pl}}{6\alpha\Psi_c} << 1 \qquad \Psi_c = \frac{GM}{R_c}$$

$$\phi(r) = -\left(\frac{\alpha}{4\pi M_{pl}}\right)\left(\frac{3\Delta R_c}{R_c}\right)\frac{M\exp\left(-m_\infty(r-R_c)\right)}{r} + \phi_\infty$$





Vainshtein mechanism

- Vainshtein mechanism

  originally discueed in massive gravity

  rediscovered in DGP brane world model

  linear theory $\omega_{BD} = 0$

$$3\nabla^2\varphi = -8\pi G\rho$$

$$\nabla^2\Psi = 4\pi G\rho - \frac{1}{2}\nabla^2\varphi$$

  even if gravity is weak, the scalar can be non-linear

$$3\nabla^2\varphi + r_c^2\left\{\left(\nabla^2\varphi\right)^2 - \partial_i\partial_j\varphi\,\partial^i\partial^j\varphi\right\} = 8\pi G a^2\rho \qquad r_c : m^{-1} : H_0^{-1}$$

Vainshtein radius

- Spherically symmetric solution for the scalar

$$\frac{d\varphi}{dr} = \frac{r_g}{r^2}\Delta(r), \quad \Delta(r) = \frac{2}{3}\left(\frac{r}{r_V}\right)^3\left(\sqrt{1+\left(\frac{r_V}{r}\right)^3}-1\right) \quad r_V = \left(\frac{8r_c^2 r_g}{9}\right)^{\frac{1}{3}}, \quad r_g = 2GM$$

r_g | r_V | r_c

4D Einstein	4D BD	
$\Phi = \frac{r_g}{2r} + \sqrt{\frac{r_g r}{2r_c^2}}$, $\Psi = -\frac{r_g}{2r} + \sqrt{\frac{r_g r}{2r_c^2}}$	$\Phi = \frac{r_g}{2r}\left(\frac{2}{3}\right)$, $\Psi = -\frac{r_g}{2r}\left(\frac{4}{3}\right)$	

2.95km | 0.1 kpc | 3000Mpc for the Sun

Chameleon v Vainshtein

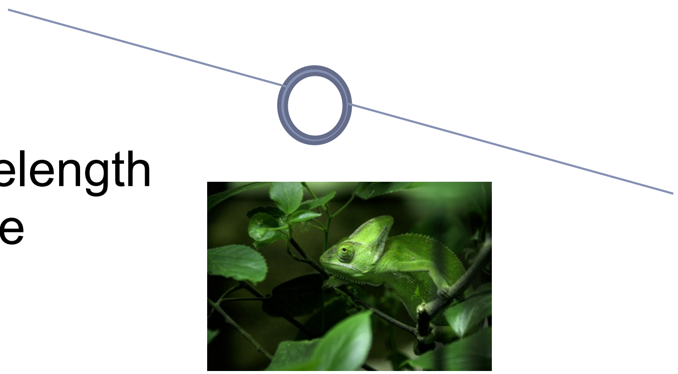
- Screening mechanism

  The self-field of the screened body and the external field do not in general superpose but rather interfere in a manner dependent on the non-linear interactions

$$\nabla\varphi_{\text{total}} = \nabla\varphi_{\text{external}} + \nabla\varphi_{\text{internal}} + \nabla\varphi_{\text{interference}}$$

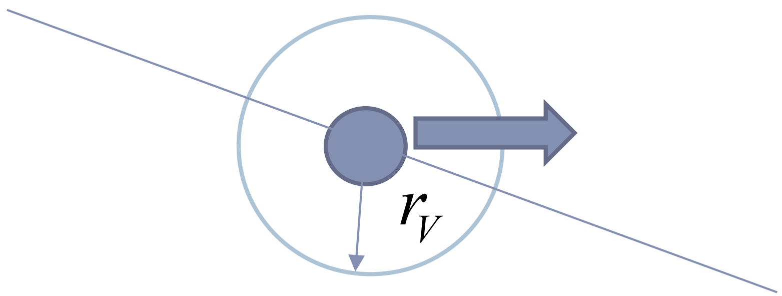
Chameleon

long wavelength mode



$$\nabla\varphi_{\text{total}} \sim 0$$

Vainshtein



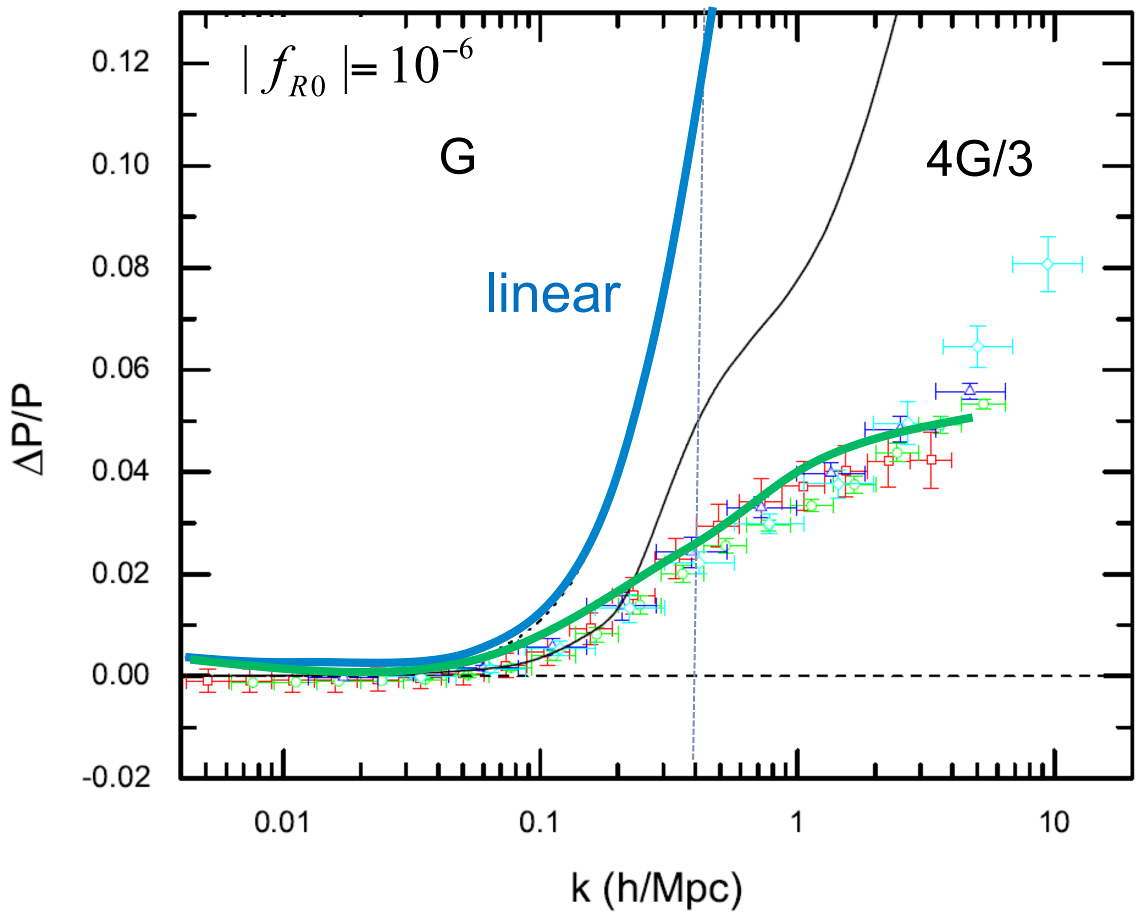
$$\nabla\varphi_{\text{total}} \sim \nabla\varphi_{\text{external}}$$

Chameleon models

Khoury & Weltman, Brax, Davis and Shaw

- Models: f(R) gravity $R - 2\Lambda + f_{R0}\frac{\bar{R}^2}{R}$ Hu & Sawicki

the background is indistinguishable from LCDM



Compton wavelength in the background

$$k_C : 0.4\left(\frac{10^{-6}}{|f_{R0}|}\right) h\,\mathrm{Mpc}^{-1}$$

enhanced gravity on small scales $k < k_C$

this enhancement is suppressed by the chameleon

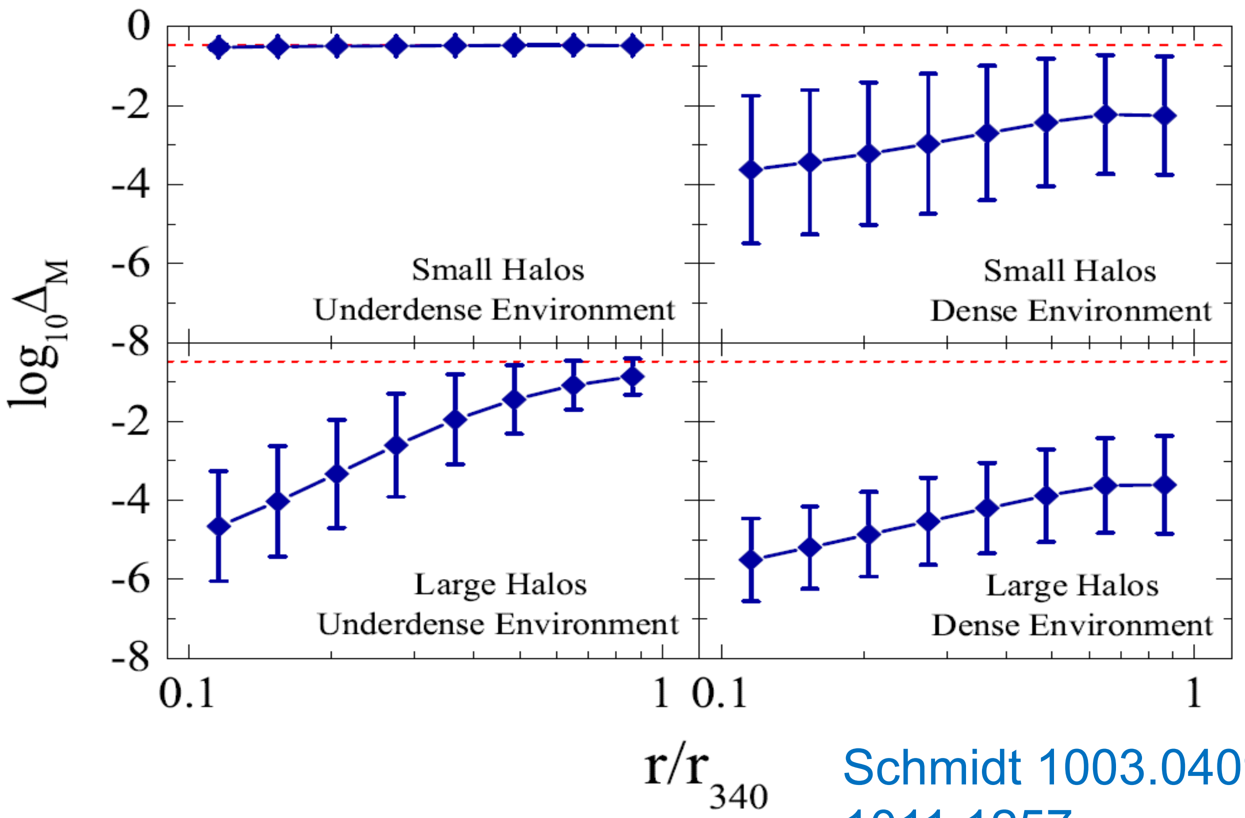
Li, Hellwing, Koyama, Zhao, Jennings, Baugh 1206.4317

Screening of dark matter halos

- Environmental dependence $\frac{\Delta R_c}{R_c} = \frac{(\phi_\infty - \phi_c)/M_{pl}}{6\alpha\Psi_c} << 1$

  dark matter halos

  screening depends on environment and halo mass



$$\Delta_M(r) = \frac{d\Phi(r)/dr}{d\Phi_+(r)/dr} - 1$$

$$\Phi_+ \equiv (\Phi + \Psi)/2$$

difference between lensing and dynamical mass cf. f(R) gravity

$$\Delta_M = [0:1/3]$$

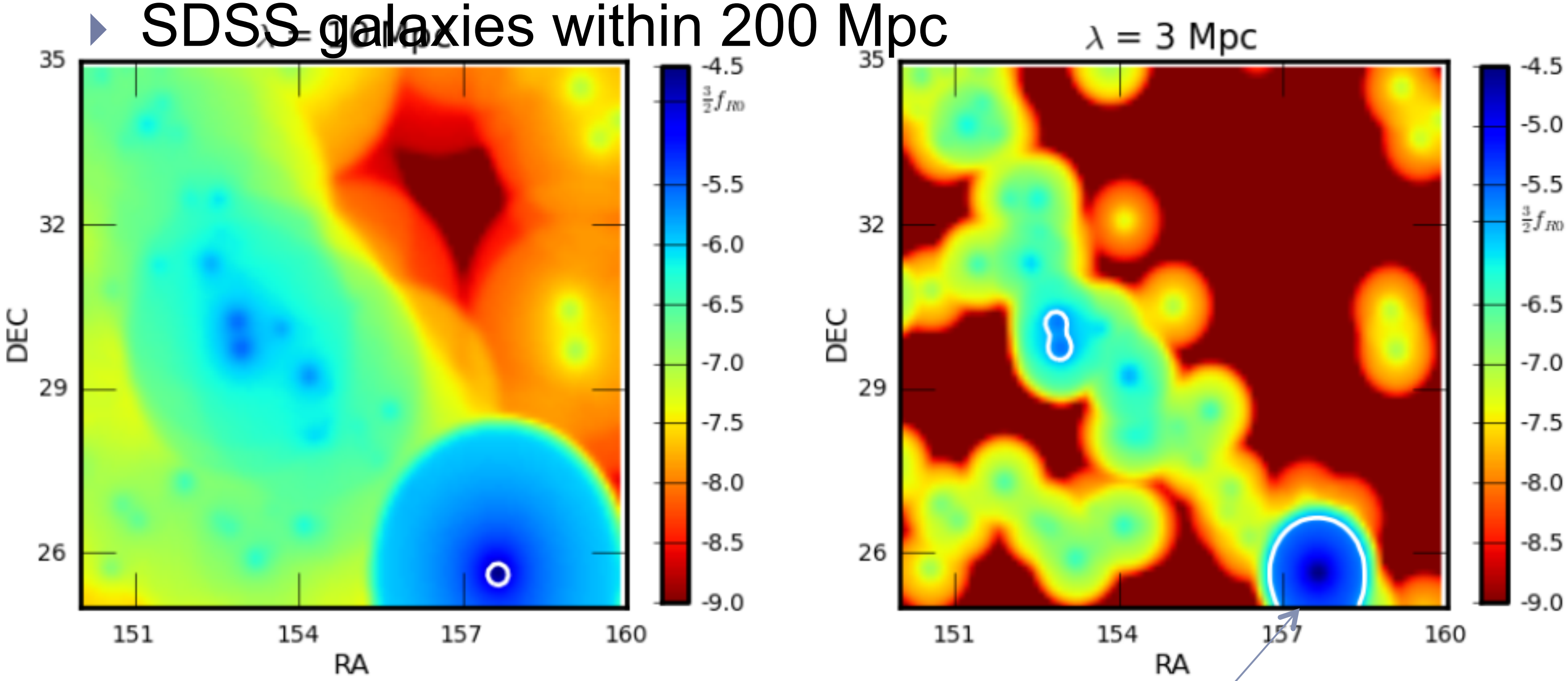
Schmidt 1003.0409, Zhao, Li Koyama 1011.1257

Li, Zhao, Koyama 1111.2602

Screening map

- It is essential to find places where GR is not recovered
  - Small galaxies in underdense regions
  - SDSS galaxies within 200 Mpc

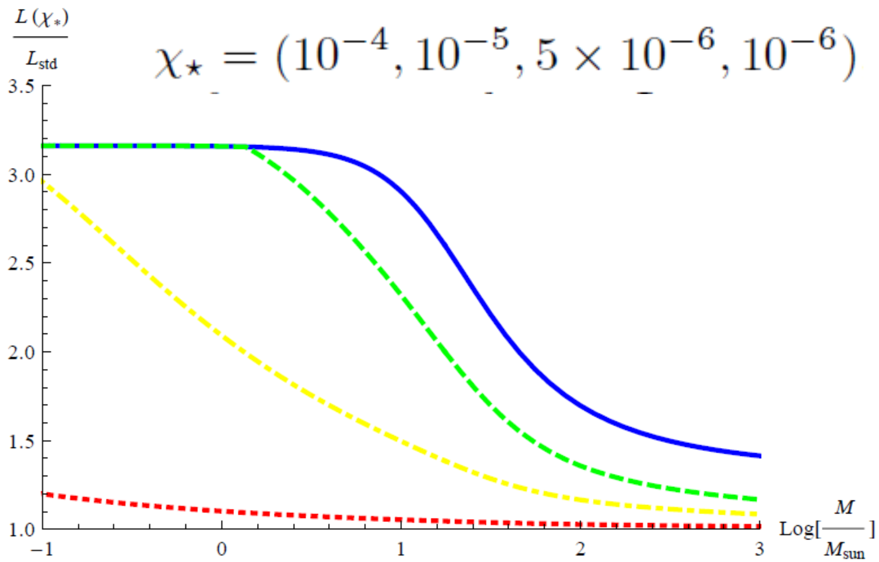
Cabre, Vikram, Zhao, Jain, KK 1204.6046



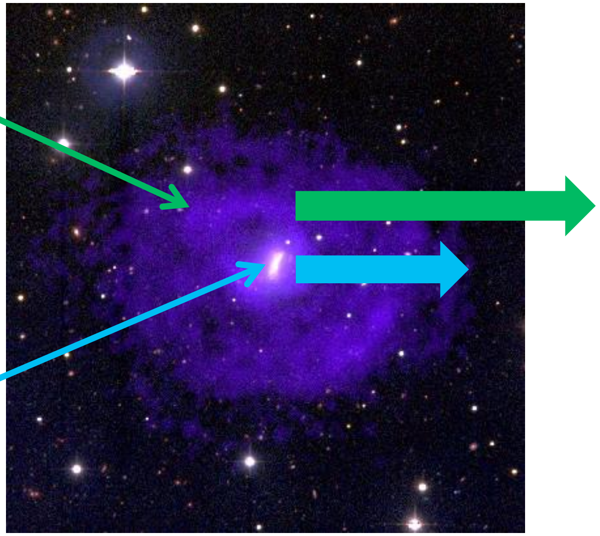
GR is recovered

Tests of gravity on small scales

- dwarf galaxies in voids
  - shallow potentials $\Psi \leq 10^{-7}$
  - strong modified gravity effects
    - Galaxies are brighter
    - A displacement of the stellar disks from HI gases

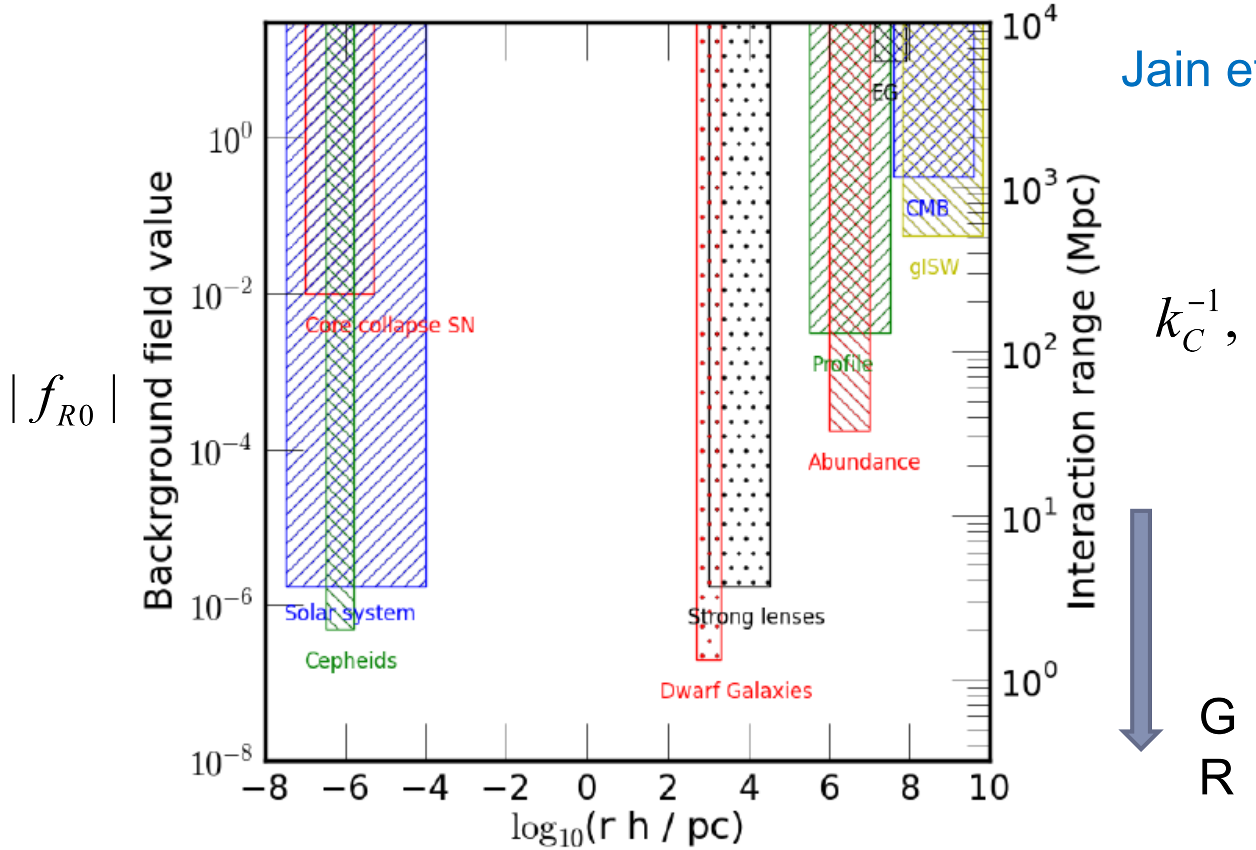


Davis et.al. 1102.5278



Jain & VanderPlas 1106.0065

Constraints on chameleon gravity



Jain et.al. 1309.5389

$$k_C^{-1}, \quad k_c \sim 0.4\left(\frac{10^{-6}}{|f_{R0}|}\right) h\,\mathrm{Mpc}^{-1}$$

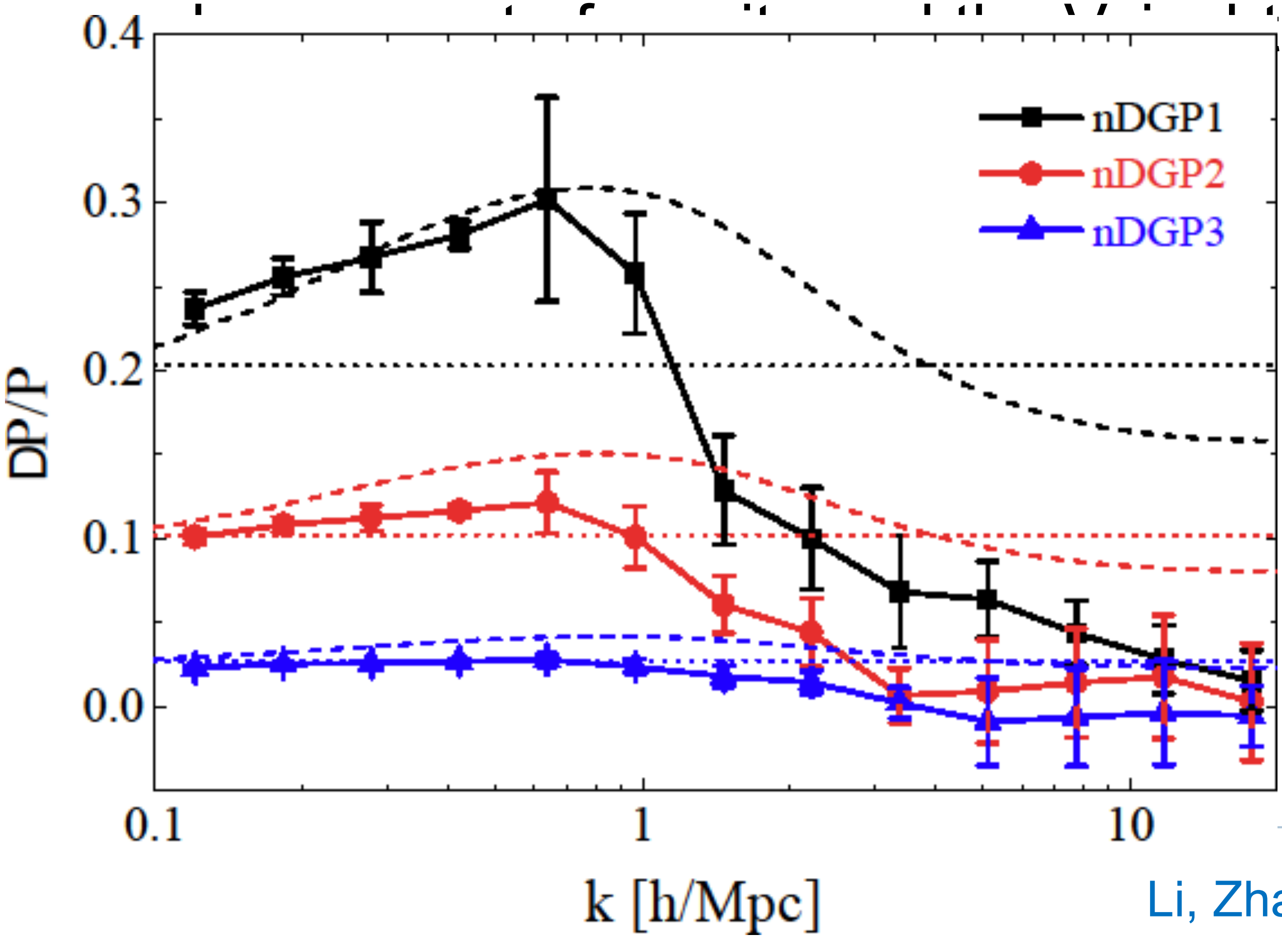
- Non-linear regime is powerful for constraining chameleon gravity
- Astrophysical tests could give better constraints than the solar system tests

Vainshtein mechanism

- Models: nDGP with the same expansion history as LCDM

  one model parameter r_c, which controls the linear

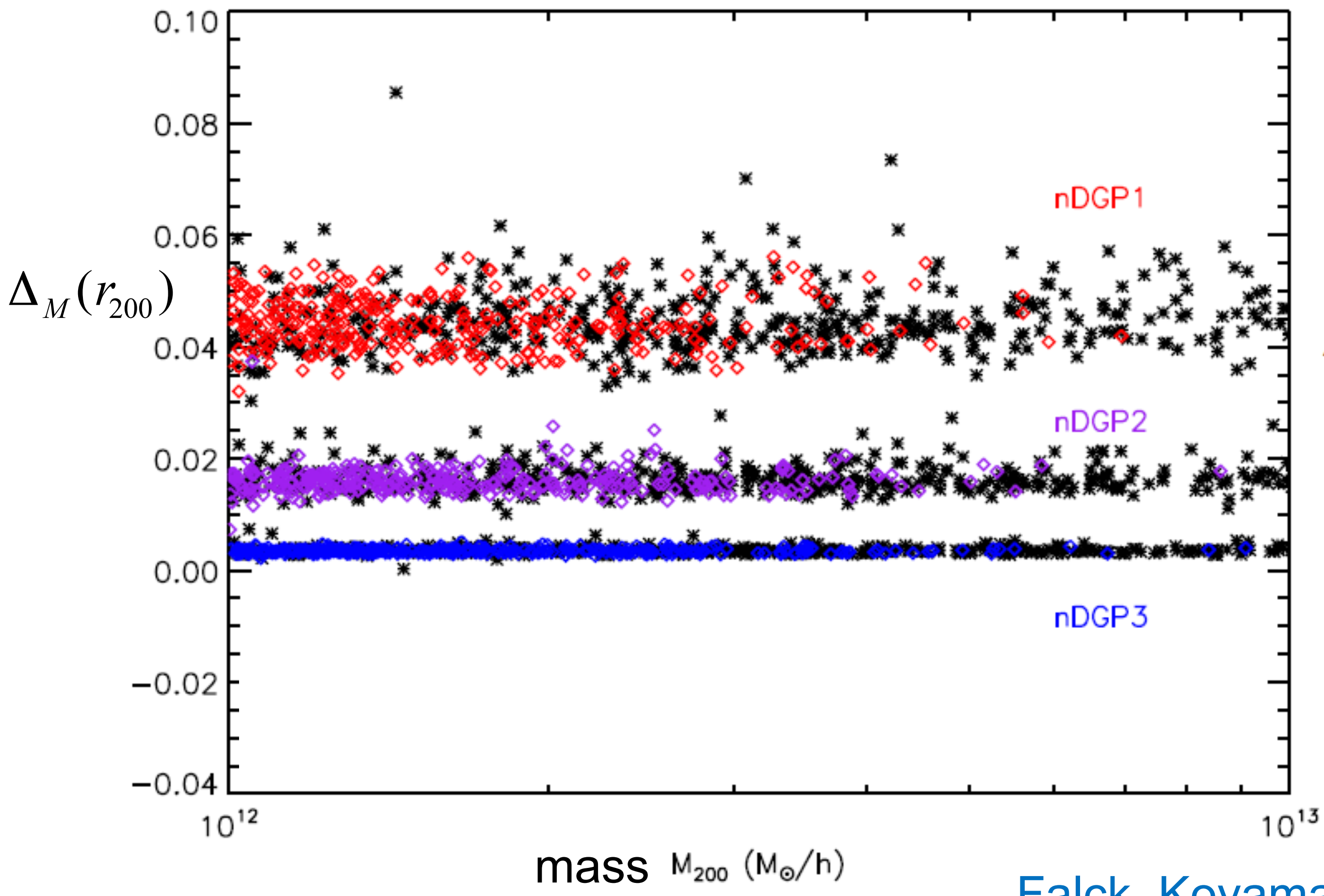
  ein suppression



Li, Zhao and Koyama 1303.0008

Screening of dark matter halos

- No mass and environmental dependence



difference between lensing and dynamical mass

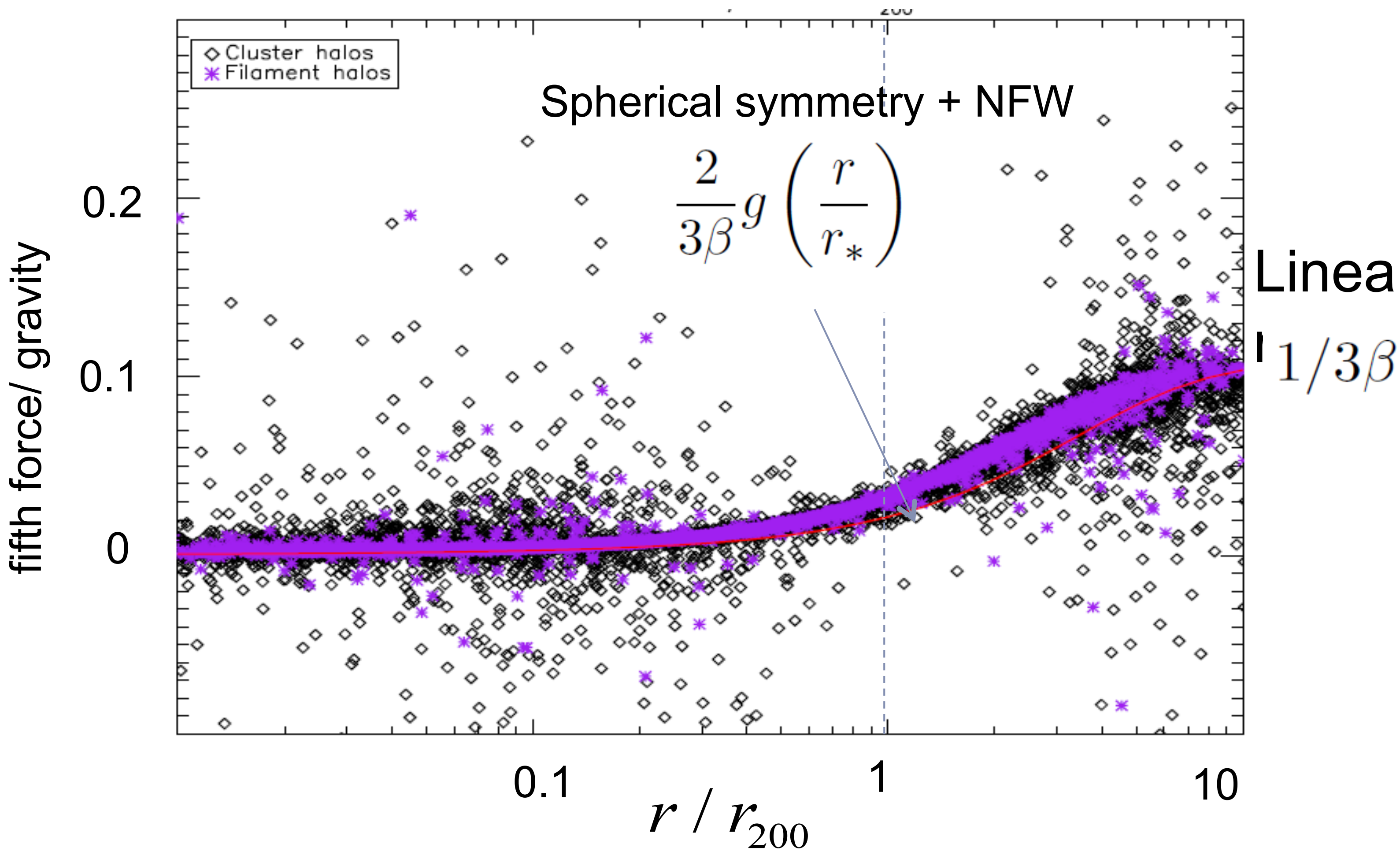
$$\Delta_M(r) = \frac{d\Phi(r)/dr}{d\Phi_+(r)/dr} - 1$$

$$\Phi_+ \equiv (\Phi + \Psi)/2$$

Falck, Koyama, Zhao and Li 14004.2206

Profiles

- The fifth force inside dark matter halos is suppressed



Two features

- "Morphology dependence"

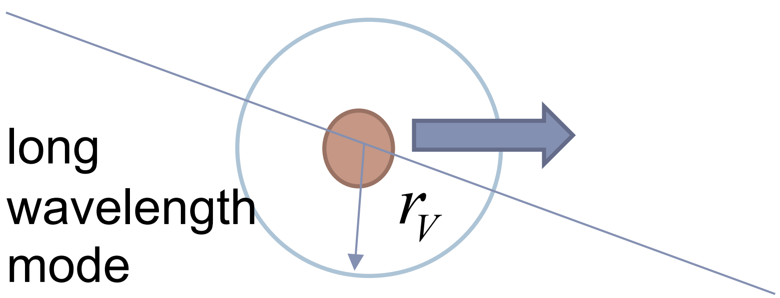
  Vainshtein mechanism does not work for one dimensional object

  $$\left(\partial^2\varphi\right)^2 - \left(\partial_i\partial_j\varphi\right)\left(\partial^i\partial^j\varphi\right) = 0$$

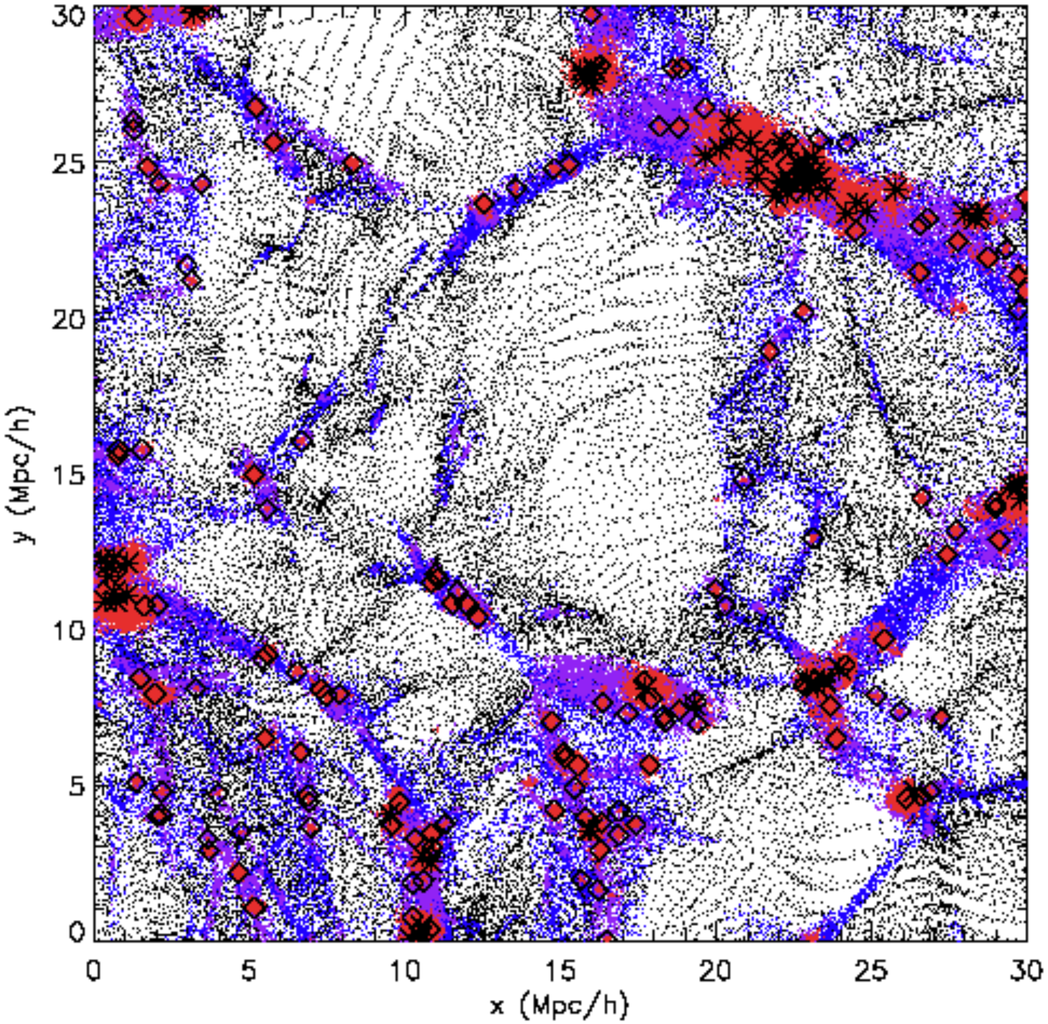
- Galileon symmetry

  we can add a constant gradient to the solution

  $$\nabla\varphi = \nabla\varphi_{galaxy} + \nabla\varphi_{LSS} \quad \nabla\varphi_{LSS} : \text{ const.}$$



Morphology



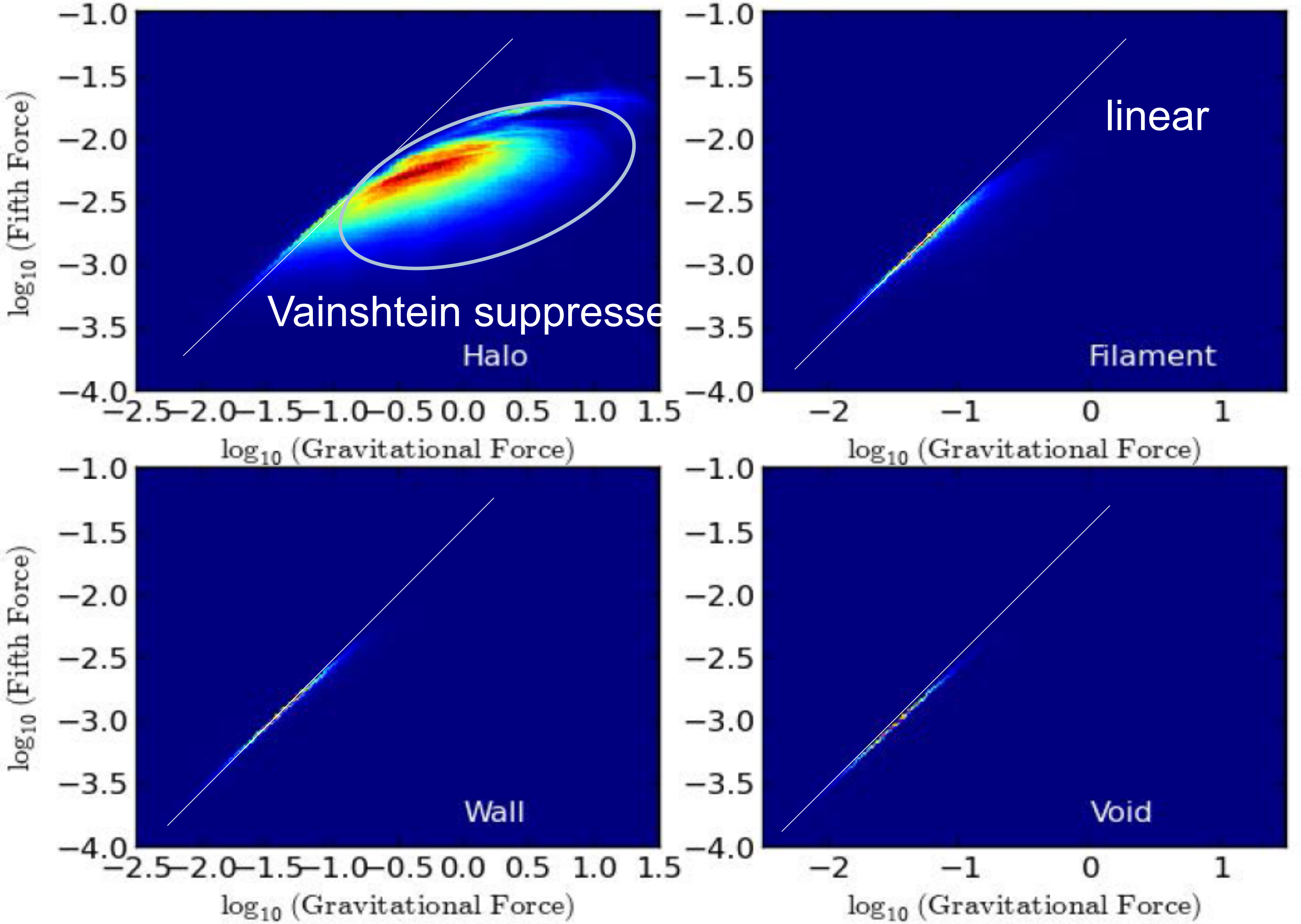
ORIGAMI finds shell-crossing by looking for particles out of order with respect to their original configuration

Halo particles have undergone shell-crossing along 3 orthogonal axes, filaments along 2, walls 1, and voids 0

Neyrinck, Falck & Szalay 1309.4787

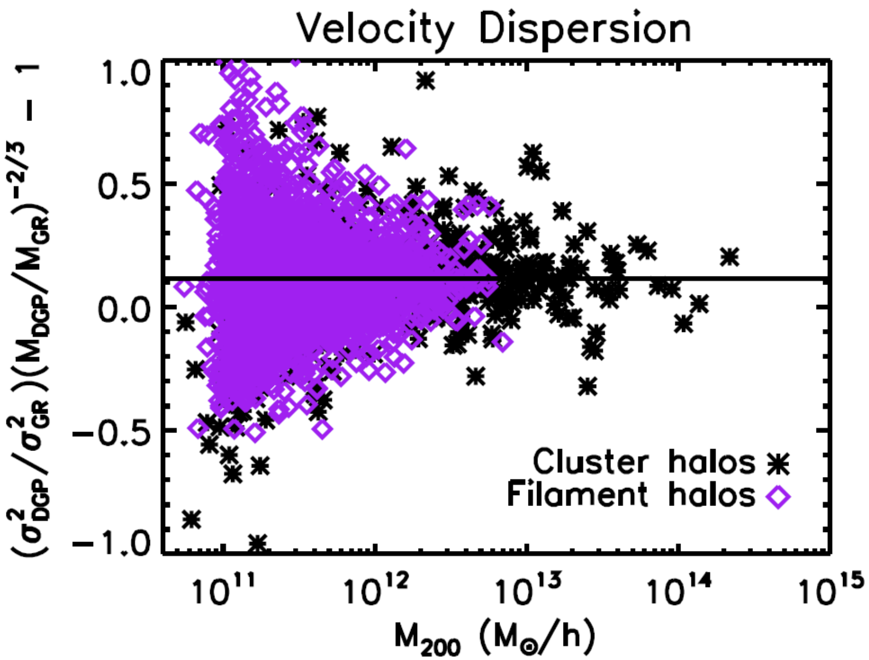
Dark matter particles

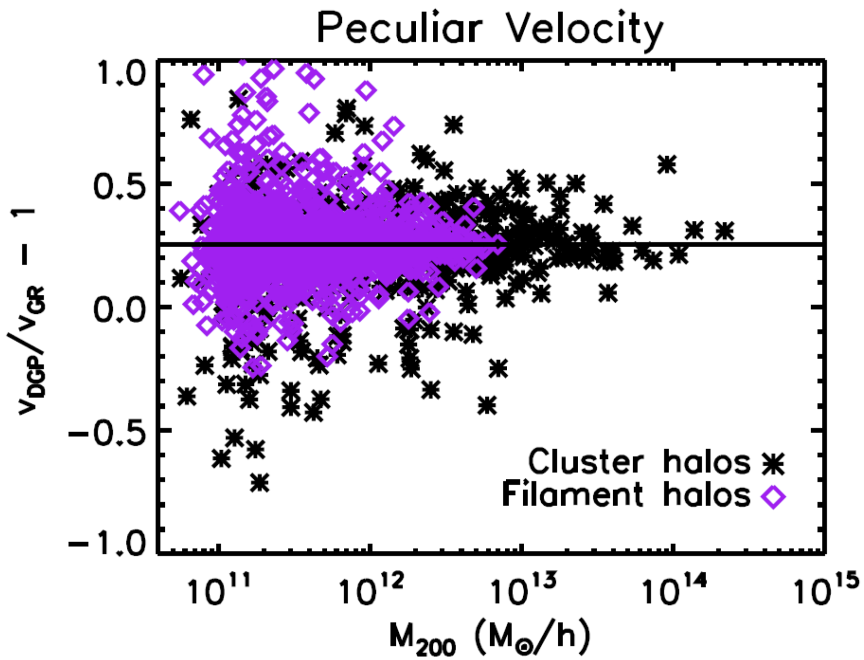
Falck, Koyama, Zhao and Li
14004.2206



Velocities

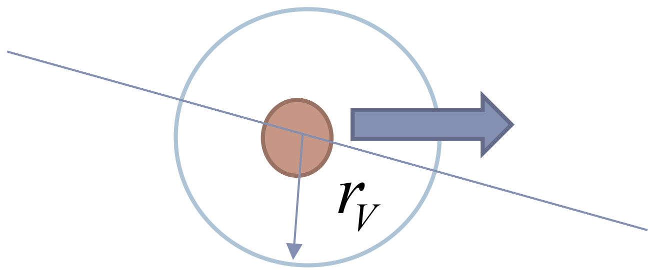
Falck, Koyama, Zhao and Li
14004.2206





Vainshtein suppression

$$\frac{\sigma_{DGP}}{\sigma_{GR}} : 1$$



Linear enhancement

$$\frac{V_{DGP}}{V_{GR}} = \left.\frac{V_{DGP}}{V_{GR}}\right|_{\text{linear}}$$

Summary

- Successful modified gravity models require a screening mechanism

Chameleon	Vainshtein
Environmentally dependent mass	Derivative self-interactions
Scale dependent growth on linear scale	Scale independent growth on linear scale
Screening of dark matter halos depends on mass and environment	Screening of dark matter halos does not depend on mass and environment
Strongest constraints come from objects with a shallow potential in low density environment	Strongest constraints come from linear scales
Screened objects do not feel external force (linear theory could not work)	Screened objects do feel external force (linear theory works well)