Holographic inflation and conservation of ζ

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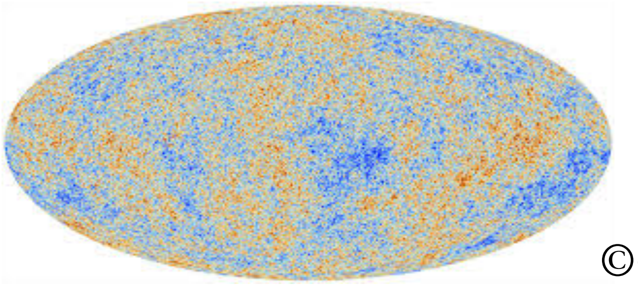
arXiv:1303.5997, JCAP 1307,033
arXiv:1403.5497, JHEP in press.
+ in progress (with K. Skenderis)

Observational evidence for inflation is very compelling, and the possible detection of primordial tensor modes can only make things better.



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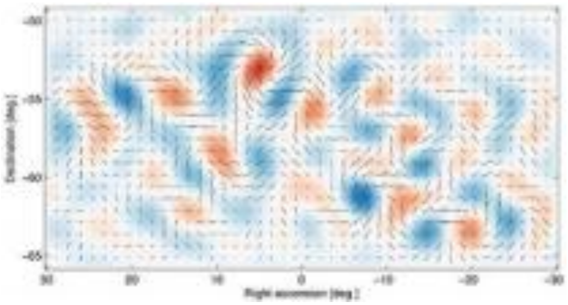

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Predictions follow from perturbative calculations, involving a weakly coupled inflaton which drives a phase of accelerated expansion.

Linearized perturbations

$$S=-\frac{1}{2}\int \mathrm{d}^4x\sqrt{-g}[g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi+2V(\phi)]+\text{gravity}$$

$\delta\phi=0$ gauge

spatial metric $h_{ij}=e^{2(\rho+\zeta)}(\delta_{ij}+\delta\gamma_{ij})$ $a:=e^{\rho}$

$$\delta^{ij}\delta\gamma_{ij}=\partial_i\delta\gamma_{ij}=0$$

ζ : Curvature perturbation ${}^{s}R\simeq-4e^{-2\rho}\partial^2\zeta$

$\delta\gamma_{ij}$: Tensor modes

$$\zeta''+2\frac{z'}{z}\zeta'-\cancel{\partial^2\zeta}=0$$

$$\delta\gamma''_{ij}+2\frac{a'}{a}\delta\gamma'_{ij}-\cancel{\partial^2\delta\gamma_{ij}}=0$$

$\partial^2=\partial_i\partial^i$

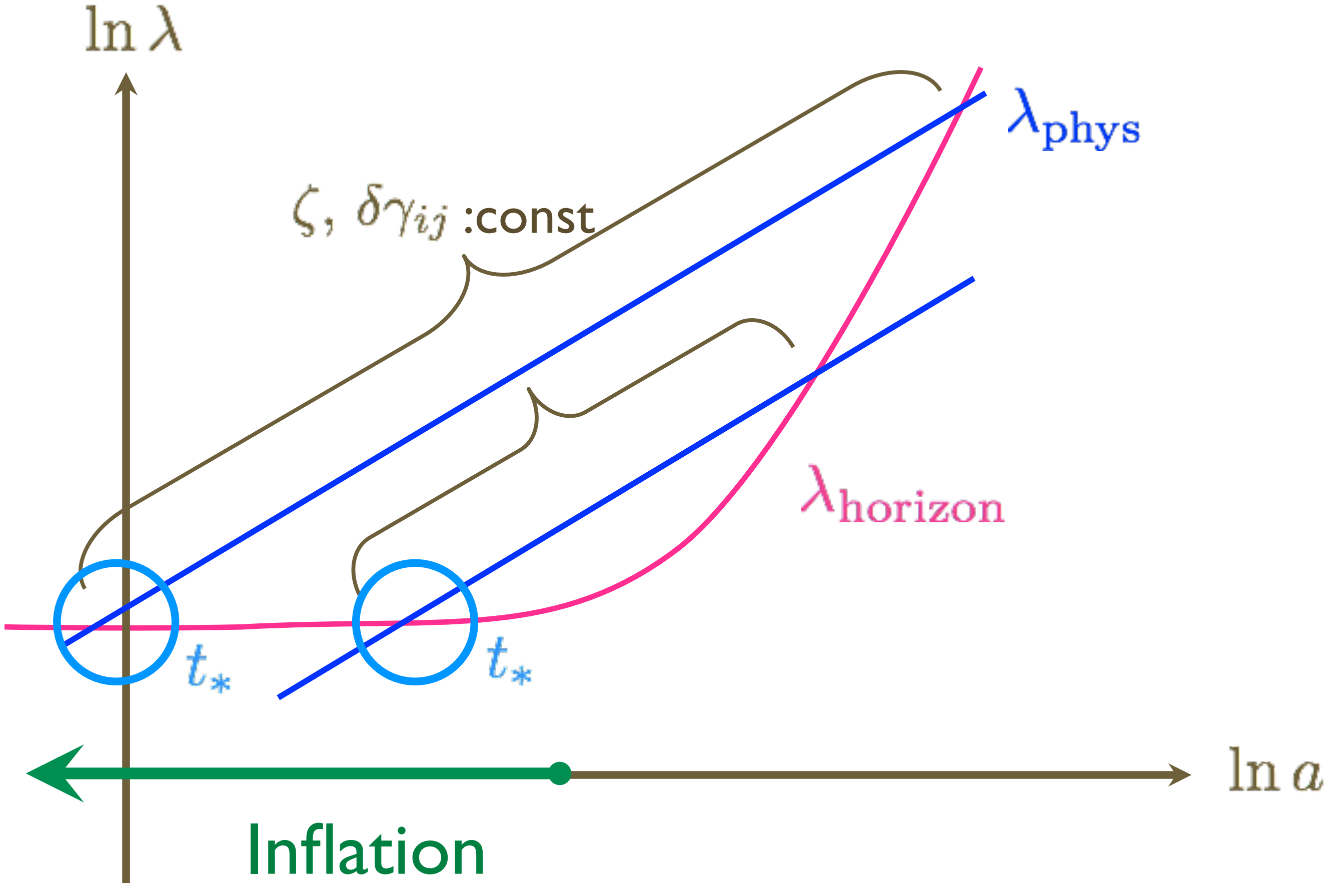
$z:=\dfrac{\phi'}{H}$

At large scales, ζ, $\delta\gamma_{ij}$ freeze.

Evolution of perturbations

Physical scale $\lambda_{\rm phys} \propto a$

Horizon scale $\lambda_{\rm horizon} \propto H^{-1}$

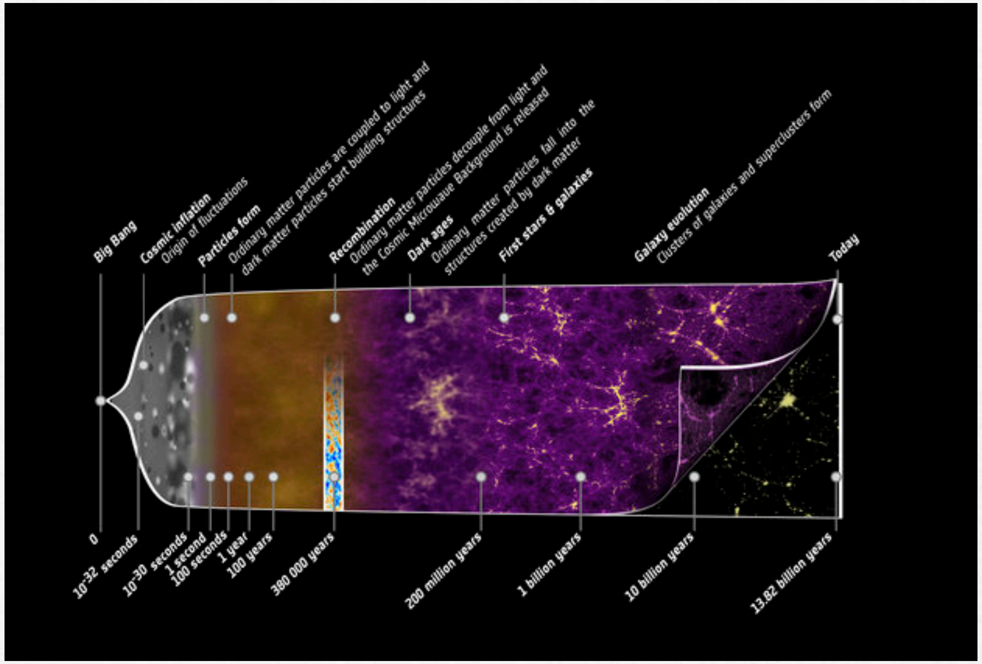


$$\Delta_k^2 = \frac{k^3}{2\pi^2} P_k$$

$$\Delta_\zeta^2 = \frac{1}{2M_{\rm pl}^2}\frac{1}{\varepsilon_*}\left(\frac{H_*}{2\pi}\right)^2$$

$$\Delta_{\rm GW}^2 = \frac{8}{M_{\rm pl}^2}\left(\frac{H_*}{2\pi}\right)^2$$

UV sensitivity of inflation



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time

energy scale

- Cosmological perturbation theory
- Quantum field theory

$$P(k) \propto \frac{1}{\varepsilon}\left(\frac{H}{M_{\rm pl}}\right)^2 \frac{1}{k^3}$$

UV sensitivity of inflation

Scale of inflation may be close to the Planck or string scale.

Super Planckian field excursions require careful embedding in a UV complete setup.

GR coupled to an inflaton is not expected to be a complete theory.

Eternal inflation leads to measure issues which may be UV sensitive.

Outline

1. dS/CFT
2. Inflation/QFT
3. Boundary QFT
4. ζ correlators from boundary

AdS/CFT correspondence

Maldacena (97)

SU(N) super Yang-Mills theory in 4D

Duality ↕ $(N \gg 1, \quad \lambda \equiv N g_{YM}^2 \gg 1)$

Classical SUGRA on $AdS_5 \times S^5$

Boundary CFT		Bulk gravity
‘tHooft coupling λ	$\lambda = (l_0/l_s)^4$	Curvature scale l_0
Strong coupling	$\lambda \gg 1,\ l_0 \gg l_s$	Weak coupling
Weak coupling	$\lambda \ll 1,\ l_0 \ll l_s$	Strong coupling
Central charge	$N^2 = (l_0/l_p)^3$	Planck scale l_p

Gauge/Gravity correspondence

d-dim gauge theory ⟷ (d+1)-dim gravity theory
+ RG flow

- Correlation functions in strongly coupled CFT from classical gravity

$$Z_{\text{bulk}}\left[\Phi(z,\mathbf{x})|_{z=0}\right] = \left\langle e^{-\int d^4\mathbf{x}\,\Phi(\mathbf{x})O(\mathbf{x})}\right\rangle_{\text{CFT}} \equiv Z_{\text{CFT}}$$

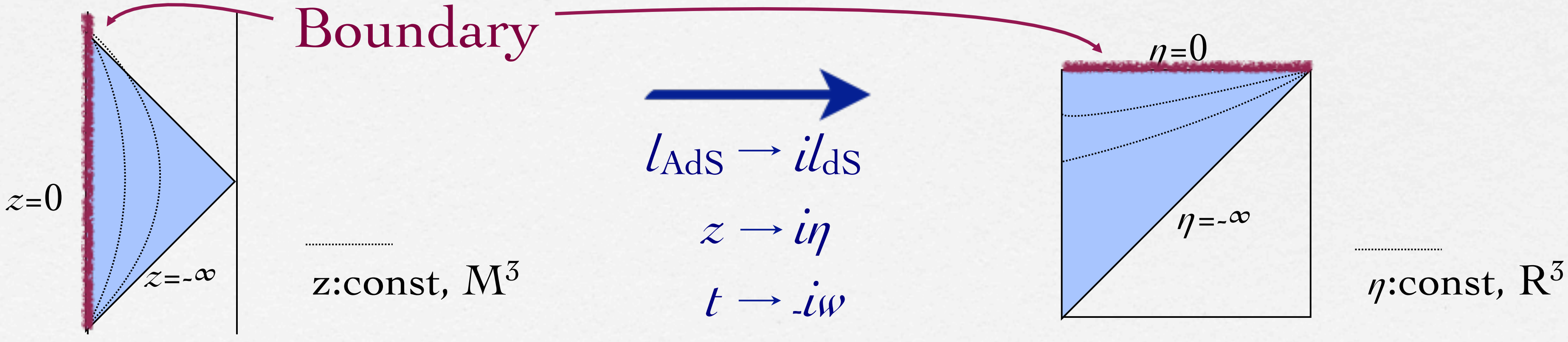
Gubser, Klebanov, Polyakov (98), Witten(98)

AdS and dS

Anti de Sitter (AdS)	de Sitter (dS)
Vacuum with $\Lambda < 0$	Vacuum with $\Lambda > 0$

$$ds^2 = l_{\rm AdS}^2 \left(\frac{-dt^2 + dx^2 + dy^2 + dz^2}{z^2} \right)$$

$$ds^2 = l_{\rm dS}^2 \left(\frac{-d\eta^2 + dx^2 + dy^2 + dw^2}{\eta^2} \right)$$



$$l_{\rm AdS} \to i l_{\rm dS}$$
$$z \to i\eta$$
$$t \to -iw$$

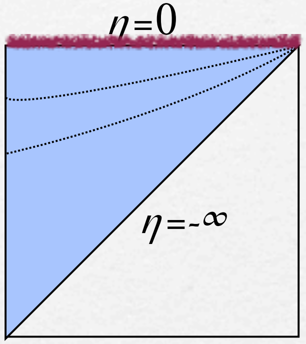
z:const, M^3

η:const, R^3

dS/CFT

Strominger(01), Witten(01)

- CFT lives on the spacelike boundary at the future infinity of dS.



Maldacena(02)

- Wave function for bulk gravity
  Probability distribution

$\Psi_{HH}[g]=Z_{CFT}$

$P[g]=|Z_{CFT}|^2$

- Time evolution in bulk can be described by RG flow of the boundary CFT.

Renormalization scale $\mu \propto$ Scale factor a

Central charge in dS/CFT

Strominger and Larsen '14

In CFT

$$T_{ij} = -\frac{2}{\sqrt{-g}}\frac{\delta S_3}{\delta g_{ij}}$$

$$\boxed{\langle T_{ij}(\vec{x}) T_{kl}(0)\rangle = \frac{C_T}{|\vec{x}|^6} I_{ij,kl}(\vec{x}) \ ,}$$

$$I_{ij,kl} = \frac{1}{2}\left(I_{ik}I_{jl} + I_{il}I_{jk}\right) - \frac{1}{3}\delta_{ij}\delta_{kl} \ . \qquad I_{ij}(\vec{x}) = \delta_{ij} - 2\frac{x_i x_j}{x^2}$$

Through the correspondence: for linearized gravitons

$$\Psi_{HH}[\psi] \sim e^{-\frac{1}{2}\int \frac{d^3k}{(2\pi)^3 P_t(k)}\psi(\vec{k})\bar{\psi}(\vec{k})} \sim \ \left\langle e^{-\frac{1}{2}\int d^3x \psi_{ij} T^{ij}} \right\rangle$$

$$\langle T(\vec{k})\bar{T}(\vec{k}')\rangle = \frac{4(2\pi)^6}{Z}\frac{\delta^2 Z[\psi]}{\delta\bar{\psi}(\vec{k})\delta\psi(\vec{k}')} = (2\pi)^3\delta(\vec{k}-\vec{k}')\frac{2}{P_t(k)}$$

$$C_T = \frac{12}{\pi^4 A_t} = 3\times 10^8 \ .$$

During inflation (if BICEP2 signal is of primordial origin)

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Breaking conformal symmetry

de Sitter space ⟷ SO(1,4) ⟷ CFT on R^3

+ inflaton φ

Breaking dS sym.

Inflation

+ φO (ex)mass

Breaking conf. sym.

QFT where couplings flow

Standard picture of inflation

4D bulk

Inflation
= dS + modulation

Holographic inflation

4D bulk

Inflation

= dS + modulation

←

3D boundary

QFT

CFT+1 deformation

$$\boxed{\Psi_{\text{bulk}}[\varphi] \;=\; Z_{\text{QFT}}\,[g]}$$

$$Z_{\text{QFT}} = \int D\chi \exp\left[-S_{\text{CFT}} - \underline{\int g\mathcal{O}[\chi]}\right]$$

deformation

Building blocks

- φ& g relation? we assume a local relation
- t & μ relation? what is time?

Conservation of ζ

From cosmological perturbation theory

Single clock $\partial_t \zeta = O((k/aH)^2)$

Wands et al.(00), Weinberg (03), Lyth et al(04), Langlois&Vernizzi(05), Naruko & Sasaki (11)

- Energy conservation $\nabla^{\mu} T^{0}{}_{\mu} = 0$
- Conservation holds at full non-linear order

Holographic inflation

4D bulk

Inflation
= dS + modulation

←

3D boundary

QFT
CFT+1 deformation

Conservation
of P_ζ

$$\boxed{\Psi_{\text{bulk}}[\varphi] \;=\; Z_{\text{QFT}}\,[g]}$$

$$Z_{\text{QFT}} = \int D\chi \exp\left[-S_{\text{CFT}} - \underline{\int g\mathcal{O}[\chi]}\right]$$

- φ& g relation? $\quad g(\mu, \mathbf{x}) = \kappa\,\phi(t(\mu), \mathbf{x})$
- t & μ relation? $\quad a(t) \propto \mu^p \quad p$: const J.G.&Y.U.(14)

N.B. Subtle issue occurs for bi-spectrum in higher order of SR

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Boundary QFT

Conformal perturbation theory (~ Slow-roll expansion)

$S_{\mathrm{QFT}} = S_{\mathrm{CFT}} + \delta S$ $$\delta S = \int \mathrm{d}^3x\, g\mathcal{O}[\chi]$$ $(0 \leqq g << 1)$

O : Boundary operator consists of χ

g : Dimensionless coupling

μ : Renormalization scale

- Correlators for CFT

$$\langle O(\boldsymbol{x})O(\boldsymbol{y})\rangle_{\mathrm{CFT}} = \frac{c}{|\boldsymbol{x}-\boldsymbol{y}|^{2\Delta}}$$

$$\langle O(\boldsymbol{x})O(\boldsymbol{y})O(\boldsymbol{z})\rangle_{\mathrm{CFT}} = \frac{C}{|\boldsymbol{x}-\boldsymbol{y}|^{\Delta}|\boldsymbol{y}-\boldsymbol{z}|^{\Delta}|\boldsymbol{z}-\boldsymbol{x}|^{\Delta}}$$

Beta function & FP

ß function $\beta(\mu) \equiv \frac{\mathrm{d}g(\mu)}{\mathrm{d}\ln\mu}$

Klebanov et al.(11)

$$\tilde{C} \sim \frac{C}{c}$$

$$\beta(\mu) = \underbrace{\lambda\, g(\mu)}_{\text{Classical scaling}} + \underbrace{\frac{\tilde{C}}{2} g^2(\mu) + \mathcal{O}(g^3)}_{\text{Quantum corrections}}.$$

$\lambda = \Delta - 3$

- Fixed point (FP) ß=0

For $\tilde{C}/\lambda < 0$ Two FPs $g=0, \quad -2\lambda/\tilde{C}$

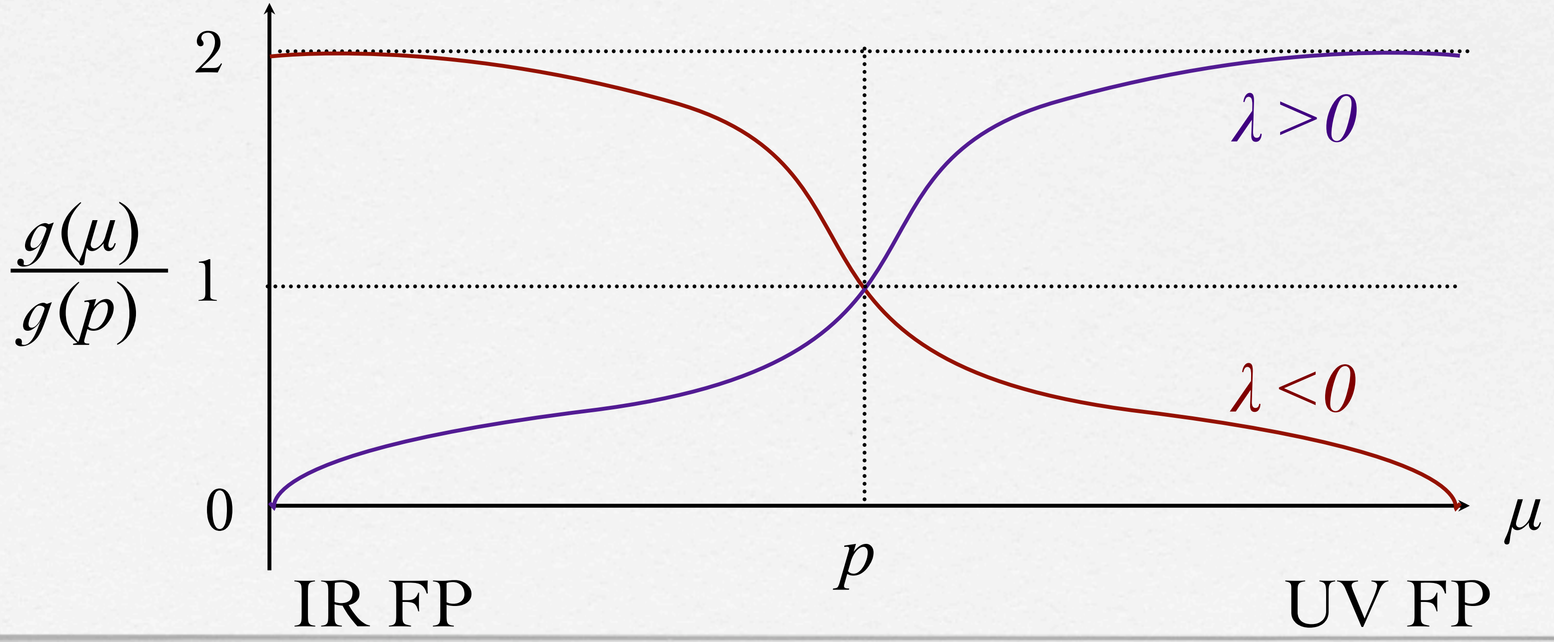
For $\tilde{C}/\lambda \geqq 0$ One FP $g=0,$

Solving RG flow

$$\beta(\mu) = \lambda\, g(\mu) + \frac{\tilde{C}}{2} g^2(\mu) + \mathcal{O}(g^3)$$

for $\tilde{C}/\lambda < 0$

$$g(\mu) = \frac{2}{1+\left(\frac{\mu}{p}\right)^{\lambda}} \left(\frac{\mu}{p}\right)^{\lambda} g(p) \qquad g(p) \equiv -\frac{\lambda}{\tilde{C}}$$



Reconstruction of potential

KG equation

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V(\phi)}{\partial \phi} = 0$$

$$\downarrow$$

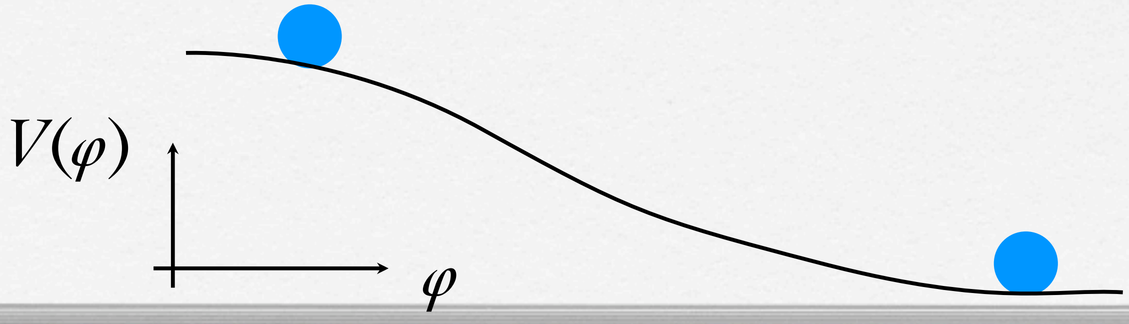
$$\frac{d\phi}{d\ln a} = -\frac{2}{\kappa^2}\frac{1}{W(\phi)}\frac{\partial W(\phi)}{\partial \phi}$$

$$V(\phi) = \frac{8}{\kappa^2}\left[\frac{3}{2}W^2(\phi) - \frac{1}{\kappa^2}\left(\frac{\partial W(\phi)}{\partial \phi}\right)^2\right]$$

RG equation

$$\frac{dg}{d\ln \mu} = \lambda g + \frac{\tilde{C}}{2}g^2 + O(g^3)$$

$$\longleftrightarrow \quad g(\mu, \mathbf{x}) = \kappa\phi(t(\mu), \mathbf{x})$$



Correlators of O

Expanding by correlators for CFT with cutoff

Bzowski, McFadden and Skenderis(12)

$$\langle \mathcal{O}(x_1)\cdots\mathcal{O}(x_n)\rangle_\mu$$

$$= \langle \mathcal{O}(x_1)\cdots\mathcal{O}(x_n)e^{-\int \mathrm{d}^3x\, g\mathcal{O}}\rangle_{\mu,\mathrm{CFT}}$$

$\downarrow$ integrating out $k>\mu$, changing μ, using OPE

$$Z^{-n/2}(\mu)\langle \mathcal{O}(\boldsymbol{x}_1)\cdots\mathcal{O}(\boldsymbol{x}_n)\rangle_\mu = Z^{-n/2}(\mu_0)\langle \mathcal{O}(\boldsymbol{x}_1)\cdots\mathcal{O}(\boldsymbol{x}_n)\rangle_{\mu_0}$$

$k<\mu$ $\qquad\qquad$ $k<\mu_0$

Wave fn. renormalization

$$\sqrt{Z(\mu)} = \mu^{-\lambda}\left[1+\left(\frac{\mu}{p}\right)^\lambda\right]^2 = 4p^{-\lambda}\frac{\beta(p)}{\beta(\mu)}$$

J.G.&Y.U.(14)

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Correlators

From boundary QFT to bulk gravity

- Distribution function

$$P[\zeta] = |A|^2 e^{-\delta W[\zeta]} \,,$$

where we defined

$$\delta W[\zeta] \equiv 2\mathrm{Re}\left[W_{\mathrm{QFT}}[\zeta] - W_{\mathrm{QFT}}[\zeta = 0]\right] \,.$$

We expand $\delta W[\zeta]$ as

$$\delta W[\zeta] = \sum_{n=1}^{n} \frac{1}{n!} \int \mathrm{d}^d \boldsymbol{x}_1 \cdots \int \mathrm{d}^d \boldsymbol{x}_n W^{(n)}(\boldsymbol{x}_1, \cdots, \boldsymbol{x}_n) \zeta(\boldsymbol{x}_1) \cdots \zeta(\boldsymbol{x}_n) \,,$$

where

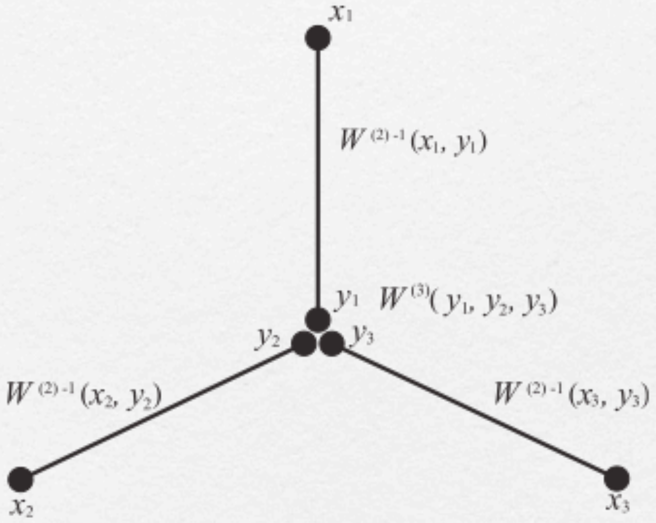
$$W^{(n)}(\boldsymbol{x}_1, \cdots, \boldsymbol{x}_n) \equiv 2\mathrm{Re}\left[\left. \frac{\delta^n W_{\mathrm{QFT}}[\zeta]}{\delta\zeta(\boldsymbol{x}_1) \cdots \delta\zeta(\boldsymbol{x}_n)} \right|_{\zeta=0} \right] \,.$$

Correlators and diagrams

- 2 point function

$$\langle \zeta(\boldsymbol{x}_1)\zeta(\boldsymbol{x}_2)\rangle_{\rm conn} = W^{(2)\,-1}(\boldsymbol{x}_1,\,\boldsymbol{x}_2)$$

- 3 point function



J.G.&Y.U.(13)

$$\langle \zeta(\boldsymbol{x}_1)\zeta(\boldsymbol{x}_2)\zeta(\boldsymbol{x}_3)\rangle_{\rm conn} = -\int \prod_{i=1}^{3} {\rm d}^d\boldsymbol{y}_i\, W^{(2)\,-1}(\boldsymbol{x}_i\,,\boldsymbol{y}_i)\, W^{(3)}(\boldsymbol{y}_1,\,\boldsymbol{y}_2,\,\boldsymbol{y}_3)\,.$$

ζ Correlators

in cosmology

$$\langle\zeta(x_1)\zeta(x_2)\cdots\zeta(x_n)\rangle$$

in boundary QFT

$$\langle\mathcal{O}(x_1)\mathcal{O}(x_2)\cdots\mathcal{O}(x_n)\rangle$$

$\delta\phi(t, \mathbf{x}) = 0$

flat space R=0

at large scales

$$\zeta = -\frac{H}{\dot\phi}\delta\phi + \frac{\varepsilon_2}{4}\left(\frac{H}{\dot\phi}\right)^2\delta\phi^2 + \cdots$$

$$\varepsilon_1 \equiv \frac{1}{2}\frac{\dot\phi^2}{H^2}$$

$$\varepsilon_2 \equiv \frac{\mathrm{d}\ln\varepsilon_2}{\mathrm{d}\ln a}$$

Vertex function

$$W^{(n)}(\boldsymbol{x}_1, \cdots, \boldsymbol{x}_n) \equiv 2\mathrm{Re}\left[\left.\frac{\delta^n W_{\mathrm{QFT}}[\zeta]}{\delta\zeta(\boldsymbol{x}_1)\cdots\delta\zeta(\boldsymbol{x}_n)}\right|_{\zeta=0}\right] \longleftarrow \langle\mathcal{O}(x_1)\cdots\mathcal{O}(x_m)\rangle_\mu$$

$$\frac{\delta}{\delta\zeta} = \frac{\delta\phi}{\delta\zeta}\frac{\delta}{\delta\phi} \sim \frac{\delta\phi}{\delta\zeta}\mathcal{O}$$

$$B_n(\mu) \equiv -\left.\frac{\partial^n \delta g_f(\mu,\, \boldsymbol{x})}{\partial \zeta^n(\mu,\, \boldsymbol{x})}\right|_{\zeta=0},$$

$$B_1 = \frac{\dot{\phi}}{H} = \frac{\mathrm{d}\phi}{\mathrm{d}\ln a}\,,$$

$$B_2 = -\frac{\dot{\phi}}{H}\frac{\varepsilon_2}{2} = -\frac{\mathrm{d}B_1}{\mathrm{d}\ln a} = -\frac{\mathrm{d}^2\phi}{\mathrm{d}\ln a^2}\,.$$

$$\boxed{W^{(2)}(\boldsymbol{x}_1,\, \boldsymbol{x}_2) = -2\mathrm{Re}\left[B_1^2(\mu)\langle\mathcal{O}(\boldsymbol{x}_1)\mathcal{O}(\boldsymbol{x}_2)\rangle_\mu\right]}$$

\mu independent correlator for \zeta implies

$$\frac{\mathrm{d}}{\mathrm{d}\mu}\left[B_1(\mu)\sqrt{Z(\mu)}\right] = 0\,.$$

where $$\sqrt{Z(\mu)} = \mu^{-\lambda}\left[1 + \left(\frac{\mu}{p}\right)^\lambda\right]^2 = 4p^{-\lambda}\frac{\beta(p)}{\beta(\mu)}\,.$$

$$\Rightarrow \quad B_1(\mu) = \mathcal{C}\beta(\mu) \quad \Rightarrow \quad \ln(\mu/\mu_0) = \mathcal{C}\ln(a/a_0)$$

So RG flow seems to be related to scale factor time

Power spectrum

$$P(k) = -\frac{3}{8\pi}\frac{1}{c\beta^2(p)}\frac{1}{k^3}\left(\frac{k}{p}\right)^{-2\lambda}\left[1+\left(\frac{k}{fp}\right)^{\lambda}\right]^4$$

cf Agrees with the result of Bzowski+(12) in μ → ∞

Remarks

1.Amplitude

$$\beta = \frac{dg}{d\ln\mu} \sim \frac{d(\phi/M_{pl})}{d\ln a} = \sqrt{2\varepsilon}$$

$$c \simeq (M_{\rm pl}/H_{\rm dS})^2$$ Strominger(01)

$$\longrightarrow \quad \frac{1}{c\beta^2} \sim \frac{1}{\varepsilon}\left(\frac{H}{M_{pl}}\right)^2$$

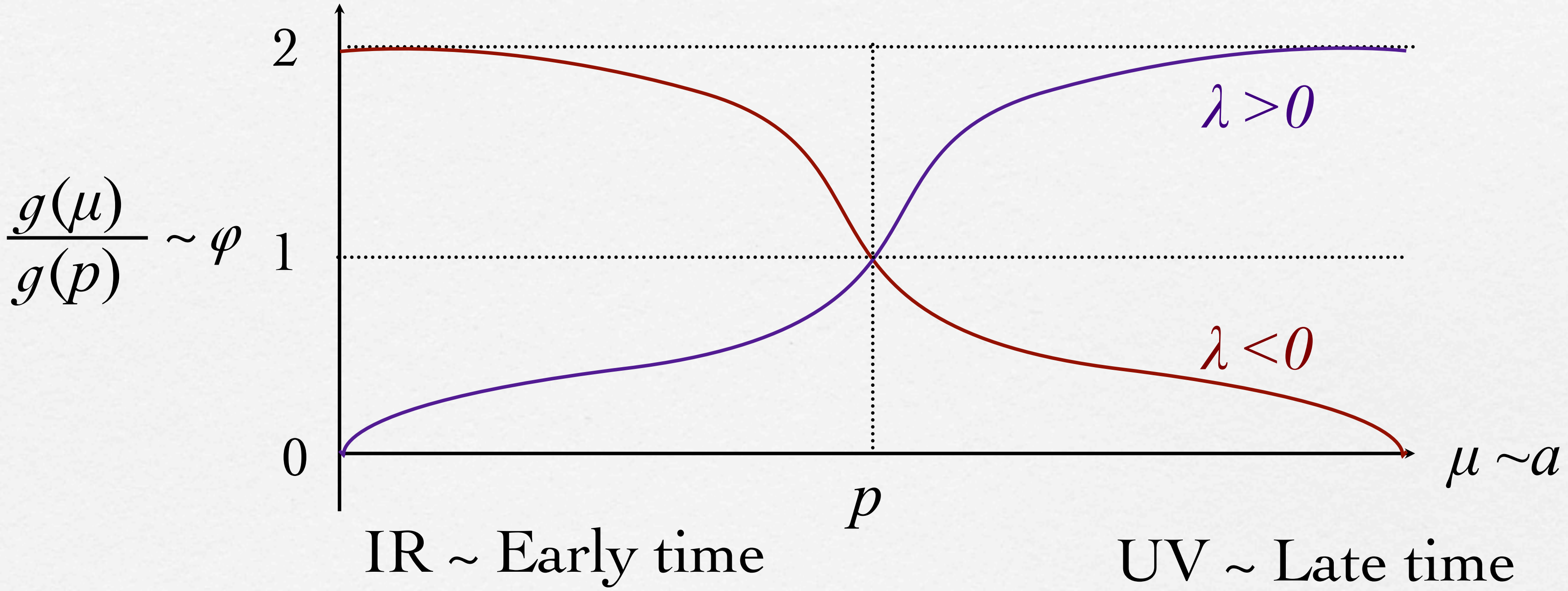
Maldacena(02)

2. Spectral index

For k>> fp $n_s-1=2|\lambda|$ Blue-tilted

For k<< fp $n_s-1=-2|\lambda|$ Red-tilted

Evolution of “inflaton”



N.B. n_s - 1=-6ε+2η

Puzzling issue

Conservation of bi-spectrum?

J.G.&Y.U.(14)

$$\langle\zeta(\boldsymbol{x}_1)\zeta(\boldsymbol{x}_2)\zeta(\boldsymbol{x}_3)\rangle_{\rm conn} = -\int\prod_{i=1}^{3}{\rm d}^d\boldsymbol{y}_i\,W^{(2)\,-1}(\boldsymbol{x}_i\,,\boldsymbol{y}_i)\,W^{(3)}(\boldsymbol{y}_1,\,\boldsymbol{y}_2,\,\boldsymbol{y}_3)\,.$$

$$\begin{aligned} &W^{(3)}(\boldsymbol{x}_1,\,\boldsymbol{x}_2,\,\boldsymbol{x}_3) \\ &= -2{\rm Re}\Big[B_1^3(\mu)\langle\mathcal{O}(\boldsymbol{x}_1)\mathcal{O}(\boldsymbol{x}_2)\mathcal{O}(\boldsymbol{x}_3)\rangle_\mu \\ &\qquad + B_2(\mu)B_1(\mu)\left\{\delta(\boldsymbol{x}_1-\boldsymbol{x}_2)\langle\mathcal{O}(\boldsymbol{x}_2)\mathcal{O}(\boldsymbol{x}_3)\rangle_\mu + (\text{cyclic perms})\right\}\Big] \end{aligned}$$

Conservation seems to require $B_2(\mu) = s_2B_1(\mu)$

with constant $s_2 = -\frac{\rm d}{{\rm d}\ln a}\ln B_1$

Puzzling issue

Conservation of bi-spectrum

J.G.&Y.U.(14)

$$\boxed{\beta(\mu) = \beta_0 \left(\frac{\mu}{\mu_0}\right)^{\lambda} \qquad \lambda = -\frac{s_2}{\mathcal{C}}}$$

- Restricted RG, at most 1 FP
- RG w/2 FPs, Break of conservation away from FPs
- Restricted bulk evolution $\varepsilon_1 \propto \beta^2 \propto a^{-2s_2}$

$$P_\zeta(k) = -\frac{6}{\pi^2}\frac{1}{\mathcal{C}^2\beta_0^2 c_0}\frac{1}{k^{3+2\lambda}} \qquad n_s - 1 = -2\lambda$$

Multi-field extension

J.G.&Y.U.(in preparation)

$S_{\mathrm{dCFT}} = S_{\mathrm{CFT}} + \delta S$

$$\delta S = \sum_{a=1}^{N} \int d\Omega \; g_a \mathcal{O}_a$$

μ : Renormalization scale

O_a : Boundary operator consists of χ

g_a : Dimensionless coupling

Gauge condition $\delta g_1(\mu, x)=0$

$g^{(np)}{}_1(\mu, x)= g_1(\mu)$

$g^{(np)}{}_a(\mu, x)= g_a(\mu)+ s_a(\mu, x)$ (a=2, ..., N)

Conclusion

Holographic description of inflation scenario

- primordial spectrum can be computed holographically, (the result may apply to strong/weak gravity regimes)

- The conservation of ζ power spectrum determines t & μ relation as $a(t) \propto \mu^p$. However, there is a puzzle for higher order correlators

- Holographic inflation (w/2 FPs) predicts broken power law spectrum.

Backup

Correlators

From boundary QFT to bulk gravity $\delta g(x)$

$\Psi[\delta g] = e^{\text{-WQFT}[\delta g]}$ $P[\delta g] = \left|\Psi_{\mathrm{qdS}}[\delta g]\right|^2$

$$\langle \delta g(x_1) \cdots \delta g(x_n) \rangle = \int D\delta g P[\delta g] \delta g(x_1) \cdots \delta g(x_n)$$

* Distribution function $P[\delta g] = e^{-\delta W[\delta g]}$

$$\delta W[\delta g] = \sum_{n=1}^{\infty} \int \mathrm{d}^d \mathbf{x}_1 \cdots \int \mathrm{d}^d \mathbf{x}_n \, W^{(n)}(\mathbf{x}_1, \cdots, \mathbf{x}_n) \, \delta g(\mathbf{x}_1) \cdots \delta g(\mathbf{x}_n)$$

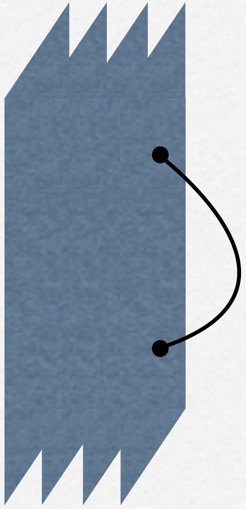
$$W^{(n)}(\mathbf{x}_1, \cdots, \mathbf{x}_n) \equiv 2\mathrm{Re} \left[\left. \frac{\delta^n W_{\mathrm{WFT}}[\delta g]}{\delta g(\mathbf{x}_1) \cdots \delta g(\mathbf{x}_n)} \right|_{\delta g = 0} \right]$$

AdS/CFT

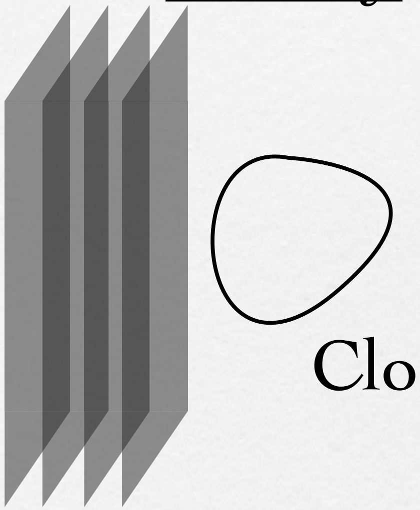
Maldacone(97)

Equivalence of 2 ways to describe D-branes

Gauge theory



Gravity



Low energy effective action of
open strings on N D2 branes

↓

gauge theory in 4D

Closed string in blackbrane
background of N D3 branes

↓

10D SUGRA in curved sp.

Correspondence

gauge theory in 4D ↔ 10D SUGRA in curved sp.

Closed string side

string theory in curved specetime → SUGRA

Horowitz&Strominger(91)

$$\mathrm{d}s^2 = f(r)^{-1/2}\underline{(\mathrm{d}x^\mu)^2} + f(r)^{1/2}(\mathrm{d}r^2 + r^2\mathrm{d}\Omega_5^2)$$

D3-brane

$$f(r) = 1 + (r_0/r)^4 \qquad r_0 := Q^{1/4} l_s$$

RR charge $Q = 4\pi g_s N$

Near horizon limit $r \ll r_0$

$$\mathrm{d}s^2 = \underline{\frac{r^2}{r_0^2}(\mathrm{d}x^\mu)^2 + \frac{r_0^2}{r^2}\mathrm{d}r^2} + \underline{r_0^2\mathrm{d}\Omega_5^2}$$

AdS$_5$: SO(4, 2) S^5: SO(6)

For $r_0 \gg l_s$, stringy corrections are negligible.

Open string side

In low energy limit $l_s \to 0,\ g_s$: fixed

10D SUGRA in flat spacetime

+ gauge theory on N D3 branes $U(N) = SU(N) \times U(1)$

Near horizon limit $r \ll r_0$

$\mathcal{N}=4$ SU(N) super YM theory in 4D ...U(1) is decoupled

$$\text{CFT with} \begin{cases} \text{gauge fields} & A_\mu \quad (\mu = 0, \cdots 3) \\ \text{scalar fields} & \phi^I \quad (I = 1, \cdots, 6) \\ \text{gaugino} & \lambda^i_\alpha \quad (i = 1, \cdots, 4 \quad \alpha = 1, 2) \end{cases}$$

- CFT in 4D Conformal group SO(4,2)
- Rotation of 6 adjoint scalars SO(6)

J. Maldacena(97)

Maldacena's limit

1) Quantum gravity corrections can be ignored

$r_0 \gg l_p \sim g_s^{1/4} l_s \quad \xrightarrow[r_0 = (4\pi g_s N)^{1/4} l_s]{} \quad N \gg 1$

2) Stringy corrections can be ignored $\lambda = g_{\rm YM}^2 N = 2\pi g_s N$

(= Massive modes of string excitation can be ignored)

$r_0 \gg l_s \quad \longrightarrow \quad N g_s \gg 1$

Keeping 'tHooft coupling $\lambda = g^2_{\rm YM} N = 4\pi g_s N$:const.

$$\boxed{g_s N \ : \text{fixed} \qquad N \to \infty}$$

Non-trivial duality

$r_0 \gg l_s \quad (\lambda \gg 1) \quad \longleftrightarrow \quad \lambda \ll 1 \quad (r_0 \ll l_s)$

Holography for dS

- dS/dS

Alishalia, Karch, Silverstein,&Tong(04)
Alishalia, Karch, &Silverstein(05)

dS_{p} with radius R can be foliated by $\mathrm{dS}_{\mathrm{p-1}}$

$$\mathrm{d}s^2 = \sin^2(\omega/R)\mathrm{d}s^2_{\mathrm{dS}_{d-1}} + \mathrm{d}\omega^2$$

near horizon $\omega \simeq 0, \pi R$ isomorphic to $\mathrm{AdS}_{\mathrm{p-1}}$

⟶ CFT on timelike boundary

- FRW from uplifted CFT

Dong et al.(11)

uplift AdS/CFT solutions by magnetic flavor branes

Progresses so far

From bulk gravity to boundary QFT

Weak gravity

- Power spectrum

  Maldacena (02), Larsen et al. (02)van der Schaar(03),...

- Beta fn. & Consistency relation

  Maldacena (02), Larsen & McNees(03),...

- Bi-spectrum

  Lidsey & Seery (06)

From boundary QFT to bulk gravity

???

CFT on R^d

SO(d,2) symmetry

$\#=(d+1)(d+2)/2$

- Poincare T.
  - Rotations $x^\mu \to \Lambda^\mu{}_\nu x^\nu$ $\#=d(d-1)/2$
  - Translations $x^\mu \to x^\mu + a^\mu$ $\#=d$
- Dilatation $x^\mu \to \lambda x^\mu$ $\#=1$
- Special C.T. $x^\mu \to \dfrac{x^\mu + a^\mu x^2}{1+2a^\mu x_\mu + a^2 x^2}$ $\#=d$

Correlators fixed by the symmetry

$$\langle O_I(\mathbf{x}_1) O_J(\mathbf{x}_2)\rangle = \frac{c_{IJ}}{|\mathbf{x}_1-\mathbf{x}_2|^{2\Delta_I}} \qquad \text{for} \quad \Delta_I = \Delta_J$$

$$\langle O_I(\mathbf{x}_1) O_J(\mathbf{x}_2) O_K(\mathbf{x}_3)\rangle = \frac{c_{IJK}}{|\mathbf{x}_1-\mathbf{x}_2|^{\Delta_I+\Delta_J-\Delta_K}|\mathbf{x}_2-\mathbf{x}_3|^{\Delta_J+\Delta_K-\Delta_I}|\mathbf{x}_3-\mathbf{x}_1|^{\Delta_K+\Delta_I-\Delta_J}}$$