Higher-Spin Resolution of a Toy Big-Bang
(+ where string theory might fit in)

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- CK, Shubho Roy. 1305.1277, 1311.7315.
- Ben Craps, CK, Ayush Saurabh. 1405.3935.
- CK, Joan Simon, ... To appear.

Background and Philosophy

General Relativity is expected to require modifications at short distances. The oft-stated reason for this expectation is the existence of an infinite number of perturbative UV divergent couplings when one quantizes metric fluctuations, and the accompanying loss of predictivity.

String theory solves this problem because it has an enormous gauge symmetry, called worldsheet conformal invariance. This gauge symmetry essentially uniquely fixes the infinite number of couplings arising in perturbation theory, even though the theory contains gravity.

Analogue: Fermi theory vs $SU(2) \times U(1)$ gauge theory.

Apart from the quantum problem of divergences, there is also a purely classical reason why we expect that gravity might require modifications at short distances.

This is because in gravity, spacetime singularities are ubiquitous.

[Penrose, Hawking]

Since string theory is expected to be perturbatively finite in the UV, it is natural to wonder whether it can also resolve spacetime singularities. Plausible, because string is a finite sized object.

Some mechanisms for handling singularities are known in string theory, eg: for resolving naked singularities [Strominger, APS, ...] or for cloaking singularities with back hole horizons [Dabholkar-Kallosh-Maloney, ...].

Cosmological singularities are even harder. Why?

Because Cosmology $\equiv$ Time Dependent Background, and we don't know how to quantize string theory in time-dependent backgrounds.

Typically we only understand how to quantize string theory in supersymmetric backgrounds, and supersymmetric backgrounds are automatically time independent.

Punchline: understanding singularities in cosmology from the context of string theory is doubly hard.

One way forward is to consider cosmological quotients of flat space as simple examples of time dependent singular backgrounds. The idea is that since the covering space is flat, we should be able to use the tools from flat space string theory, to explore these singular geometries [numerous papers].

Our mainexample for this talk: Milne orbifold, which is a time dependent orbifold of flat space.

However, it is known that 4-point tree level string scattering amplitudes on the Milne orbifold have divergences [Berkooz-Craps-Kutasov-Rajesh-Pioline-Nekrasov].

So some new ingredient is needed.

Enter Higher Spin Theories: Vasiliev has constructed interacting theories with gravity and higher spin fields.

There is evidence that higher spin theories are the $\alpha' \to \infty$ (tensionless) limit of string theory [Sundborg, Witten, Minwalla-Yin-...]

So one can think of higher spin theory as a Lagrangian theory in spacetime for the worldsheet spectrum of states of the string, in which all the massive modes have become massless (masses are inversely related to α').

But Vasiliev theory = complicated theory.

In any event, this offers another Paradigm in Which De-Singularization Might Happen in String Theory:

We mentioned string theory has an enormous gauge invariance. Much bigger than the usual diffeomoprhism invariance (freedom to change coordinates) of gravity. Working with stringy gauge invariance from a spacetime point of view is hard. But in the tensionless limit, higher spin theories capture these gauge invariances as bigger gauge symmetries than diffeomorphisms: higher spin gauge symmetries.

Diffeomorphisms cannot remove singularities, but these bigger gauge invariances might change that!

So: Maybe at least **some** of the singularities are just artifacts of a choice of gauge in string theory? Can a higher spin gauge transformation put Milne in a non-singular gauge?

We will embed Milne in three dimensions and work with flat space higher spin theory in 3 dimensions, because the theory has a Chern-Simons formulation, which makes things simpler.

Because the orbifolds we will consider are quotients of 2D flat space, we can always embed them in 3D.

This is good because higher dimensional Vasiliev theories are far more complicated.

In 3D, the higher spin situation is formally very similar to pure gravity. The idea is that Einstein-Hilbert action can be written in 3 dimensions as

$$I_{EH} \sim I[A, \tilde{A}] = I_{CS}[A] - I_{CS}[\tilde{A}] \quad (1)$$

where

$$I_{CS}[A] = \frac{k}{4\pi} \int_M Tr \left[A \wedge dA + \frac{2}{3} A \wedge A \wedge A \right]$$

with

$$A = (\omega + \epsilon e), \ \tilde{A} = (\omega - \epsilon e) \quad (2)$$

where

$$A = A^a_\mu T_a dx^\mu = (\omega^a_\mu + \epsilon e^a_\mu) T_a dx^\mu \equiv (\omega + \epsilon e) \quad (3)$$

similarly for $\tilde{A}$. Here T_a are the generators of $SL(2)$:

$$\left[T_a, T_b \right] = \epsilon_{abc} T^c. \quad (4)$$

Even thought the generators satisfy $SL(2)$ algebra, the theory is flat space gravity because we have taken ϵ to be a Grassmann parameter. [CK-S.Roy-A.Raju].

If we increase the rank of the gauge group the theory becomes a higher spin theory. A very simple set up.

Working with higher spin theories in higher dimensions provides the possibility of resolving curvature singularities.

Milne Geometry a quotient of flat space:

$$ds^2 = -dT^2 + r_C^2 dX^2 + \alpha^2 T^2 d\phi^2. \tag{5}$$

X is noncompact and $\phi \sim \phi + 2\pi$. The paramters can be set to 1.

Space time behaves like a double cone and there is a causal singularity at $T = 0$ where ϕ-circle crunches to a point before expanding in a big-bang. From (5), the triads and spin connection one forms for the Milne universe are

$$\begin{aligned} e^T = dT,\, e^X = r_C dX,\, e^\phi = \alpha T d\phi, \\ \omega^T = 0 = \omega^\phi,\, \omega^X = \alpha d\phi. \end{aligned} \tag{6}$$

The Chern-Simons connection for Milne is then

$$\begin{aligned} A^\pm &= (\omega^a \pm \epsilon e^a) T_a \\ &= \pm(\epsilon dT) T_T + (\alpha d\phi \pm r_C dX) T_X \pm (\epsilon \alpha T d\phi) T_\phi \end{aligned} \tag{7}$$

We want to do a gauge transformation that preserves the holonomy of the solution, since Einstein solutions correspond to flat connections.

The ϕ-circle holonomy matrix can be directly computed to be $\omega_\phi^\pm = 2\pi\alpha(T_X \pm \epsilon T T_\phi)$, it has the eigen values $(0, \pm 2\pi\alpha)$. The characteristic polynomials coefficients of these holonomy matrices are

$$\Theta_\phi^0 \equiv det(\omega_\phi) = 0, \quad \Theta_\phi^1 \equiv Tr(\omega_\phi^2) = 8\pi^2\alpha^2, \tag{8}$$

the $\pm$ superscript is dropped as the polynomials are identical for both.

The higher spin gauge transformed solution that we consider should also have same characteristic polynomial for it to describe the same physical configuration.

Adding the higher spin components, we get

$$A' = A + \sum_{n=-2}^{n=2} (C^n + \epsilon D^n) W_n \tag{9}$$

where C^n and D^n are frame fields and connection associated with new higher spin generators W_n. If work with a spin-3 theory $n \in -2, -1, 0, 1, 2$.

Our goal is to find the simplest resolution, so we try the coefficients to be constants, luckily this works.

We make two demands-
1. The holonomy is preserved for the new connection.
2. It is still a flat connection.

Turns out we can satisfy both these conditions if we set $D_{\phi}^{\pm 1} = D_{\phi}^{-2} = 0$ together with $D_{\phi}^{0} = 3D_{\phi}^{2}$ and all the C's set to zero. The resultant metric is identical to Milne except now,

$$g'_{\phi\phi} = g_{\phi\phi} + 12(D_{\phi}^{2})^{2},$$

Since $g_{\phi\phi} \sim r^2$, this means that the singularity is now resolved to a circle of minimum radius, there is a bounce. The curvature scalars are everywhere finite. The solution now contains non-trivial (but regular, upto a subtlety) higher spin fields which can be thought of as the matter supporting the throat.

Connection to string theory?

We expect morally that the tensionless limit of string theory should capture aspects of higher spin theories, even though precise proposals exist only in specific cases [Minwalla-et-al(AdS4), Gaberdiel-et-al(in AdS3, to appear?)].

If this is true, the singularities in the amplitudes that people have considered previously should not arise in the tensionless limit, if our resolutions are capturing something physical. (The scattering amplitude captures gauge-invariant information).

When we say tensionless (ie., $\alpha' \to \infty$) what we mean is the dimensionless α'.

Since we are working with quotients of flat space, the only available dimensionless α' has to constructed from the the momenta of the scattering states - there is no background scale.

The string scattering amplitudes are complicated beasts with integrals over numerous Gamma functions and such, so analyzing its divergence properties is messy. I will spare you the details: the equations are in our paper, the qualitative end result is [Craps-CK-Sourabh, CK-Simon-...]:

- All UV divergences that were previously identified as arising from the pathological singularity arise when $\alpha'(\mathbf{p}_1 - \mathbf{p}_3)^2 \leq 2$ or similar conditions hold. Roughly speaking the α' is being measured in units of momentum transfer and when it is large enough, there are no divergences as we wanted!
- We exhaustively scan for all divergences, and all the other divergences are sensible IR divergences that are unrelated to the singularity. Eg: poles giving rise to logarithmic divergences which have interpretation as the tower of intermediate string states going on-shell.

Questions Raised, Some Responses, Comments

- Quotients of de Sitter space using dS_3 higher spin theory have been resolved before [CK-Roy]. What is intersting about flat space quotients is that they allow connections with string theory.
- Similar story like Milne holds for the null orbifold cosmology of Liu-Moore-Seiberg: Higher spins can resolve it, string scattering amplitudes are better behaved at large α', etc.

- One interesting aspect of the LMS orbifold is that its C-S holonomy has trivial eigenvalues. There are some (implied) statements in the literature that demanding trivial CS holonomy eigenvalues can be used as an indicator of regularity of the geometry. This is a counter-example.
- The simplest resolution of the LMS orbifold has the interesting peoperty that all its curvature scalars vanish. The geometry is in fact a pp-wave/Kundt geometry and in 2+1 dimensions these geometries have the VSI property.

- The $\alpha' \to \infty$ limit is precisely the opposite limit of the GR limit in string theory: strings are long and floppy, not pointlike. But the message that singularities might be gauge artefacts and might be resolved via gauge transformations is perhaps a useful paradigm to keep in mind.
- Does the fact that we are in a gauge where the metric is singular mean that we have to worry about singular Jacobians etc in the path integral? Not likely....

However: One real question/problem in my view-

When we construct resolutions, we need to turn on higher spin fields. How do we know we haven't created some new kind of singularity in the higher spin field? They typically vanish somewhere in the bulk. Is this bad?

We don't have a real answer. Part of the reason to look at the string scattering amplitude was this question. Hopefully, the well-definedness of the string scattering amplitude at large α' might be some indication that things are fine. Clearly more work is needed. Perhaps to satisfyingly answer this question without resorting to string theory, we need a higher spin generalization of Riemannian geometry.