Impact factor for high-energy two and three jets diffractive production

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[RB, A.Grabovsky, L.Szymanowski, S.Wallon (arXiv:1405.7676)]

Overview

1. Diffractive DIS
   - Rapidity gap events at HERA
   - Collinear factorization approach
   - k_T-factorization approach : two exchanged gluons
   - Confrontation of the two approaches with HERA data
2. Diffractive production of jets : our approach
   - $q\bar{q}$ production
   - $q\bar{q}g$ production
   - Linear approximation : 2 and 3 exchanged gluons
3. Conclusion

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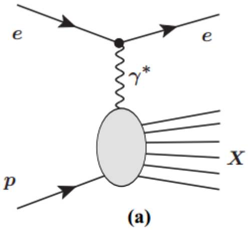
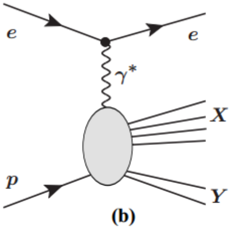
Diffractive DIS

Rapidity gap events at HERA

Experiments at HERA : about 10% of scattering events reveal a rapidity gap



DIS events



DDIS events

Rapidity gap events at HERA

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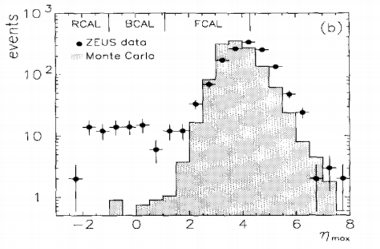
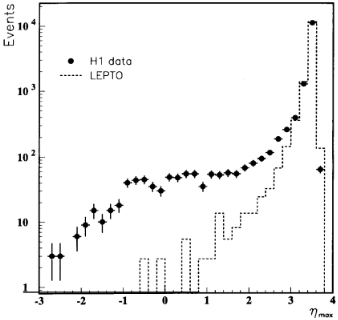


ZEUS, 1993



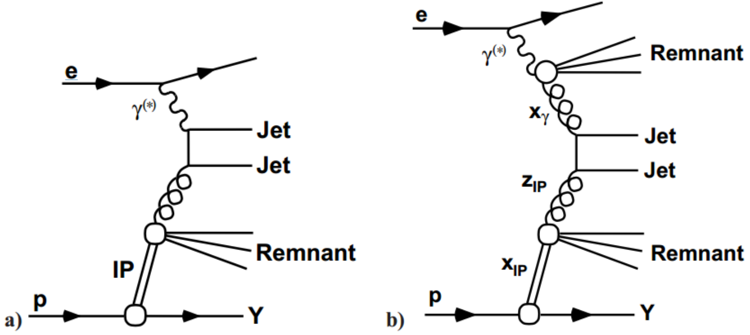
H1,1994

Diffractive DIS

Theoretical approaches for DDIS using pQCD

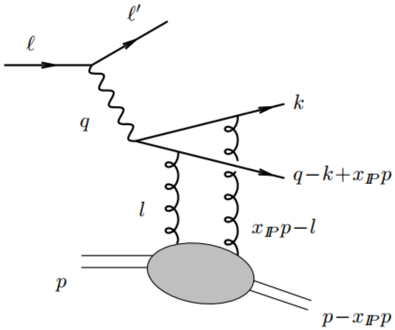
- Collinear factorization approach
  - Relies on QCD factorization theorem, using a hard scale such as the virtuality Q^2 of the incoming photon
  - One needs to introduce a diffractive distribution function for partons *within a pomeron*
- k_T factorization approach for two exchanged gluons
  - low-x QCD approach : $s \gg Q^2 \gg \Lambda_{QCD}$
  - The pomeron is described as a two-gluon color-singlet state

Collinear factorization approach



Theoretical approaches for DDIS using pQCD

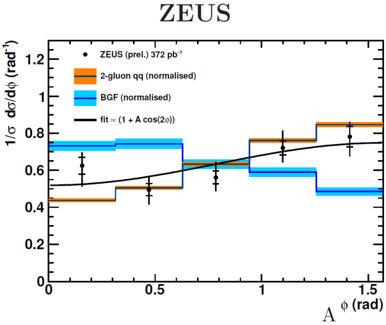
k_T-factorization approach : two gluon exchange



Theoretical approaches for DDIS using pQCD

Confrontation of the two approaches with HERA data

According to experimentalists, it seems that the k_T-factorization approach gives a better description of diffractive events at HERA :



Valkárová, low-x 2014, Kyoto

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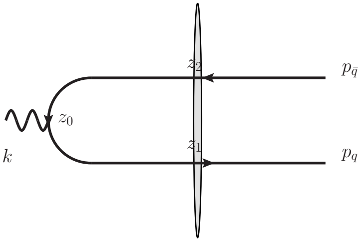
Assumptions

- Regge limit : $s \gg Q^2 \gg \Lambda_{QCD}$
- Pomeron $=$ Wilson lines
- No approximation for the outgoing gluon

Matrix element for EM current



$q\bar{q}$ production

$$M_0^{\alpha} = \delta_l^n \frac{\langle 0| b^l_{p_{\bar{q}}} (a_{p_q})_n \overline{\psi}(z_0)\, \gamma^{\alpha} \psi(z_0)\, e^{i\int \mathcal{L}_i(z)dz} |0\rangle}{\langle 0| e^{i\int \mathcal{L}(z)dz} |0\rangle}$$

$q\bar{q}$ production

We introduce the Wilson lines :

$$U_i = U_{\vec{z}_i} = U(\vec{z}_i, \eta) = P \exp\left[ig \int_{-\infty}^{+\infty} b_\eta^-(z_i^+, \vec{z}_i)\, dz_i^+\right]$$

Then we project the result on the color singlet state, eliminate the non-interacting term and get :

$$M_0^\alpha \quad = \quad \int d\vec{z}_1 d\vec{z}_2 F\left(p_q, p_{\bar{q}}, z_0, \vec{z}_1, \vec{z}_2\right)^\alpha \left[\mathrm{tr}\left(U_1 U_2^\dagger - N_c\right)\right]$$

$q\bar{q}$ production

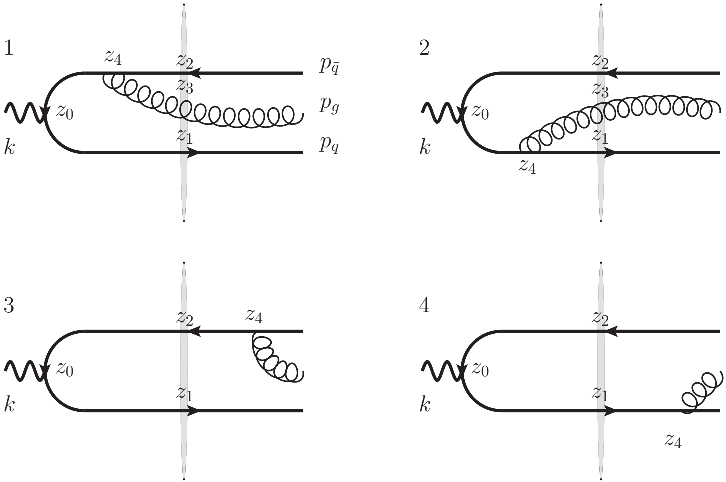
After some algebra, we recover the well-known results :

$$F\left(p_q, p_{\bar{q}}, k, \vec{z}_1, \vec{z}_2\right)^{\alpha} \varepsilon_{L\alpha} = \theta(p_q^+)\,\theta(p_{\bar{q}}^+)\frac{\delta\left(k^+ - p_q^+ - p_{\bar{q}}^+\right)}{(2\pi)^2} e^{-i\vec{p}_q\cdot\vec{z}_1 - i\vec{p}_{\bar{q}}\cdot\vec{z}_2}$$
$$\times(-2i)\delta_{\lambda_q,-\lambda_{\bar{q}}}\, x_q x_{\bar{q}}\, Q\, K_0\left(Q\sqrt{x_q x_{\bar{q}}\vec{z}_{12}^{\,2}}\right)$$

and

$$F\left(p_q, p_{\bar{q}}, k, \vec{z}_1, \vec{z}_2\right)^{j} \varepsilon_{Tj} = \theta(p_q^+)\,\theta(p_{\bar{q}}^+)\frac{\delta\left(k^+ - p_q^+ - p_{\bar{q}}^+\right)}{(2\pi)^2} e^{-i\vec{p}_q\cdot\vec{z}_1 - i\vec{p}_{\bar{q}}\cdot\vec{z}_2}$$
$$\times\delta_{\lambda_q,-\lambda_{\bar{q}}}\left(x_q - x_{\bar{q}} + s\lambda_q\right)\frac{\vec{z}_{12}\cdot\vec{\varepsilon}_T}{\vec{z}_{12}^{\,2}} Q\sqrt{x_q x_{\bar{q}}\vec{z}_{12}^{\,2}} K_1\left(Q\sqrt{x_q x_{\bar{q}}\vec{z}_{12}^{\,2}}\right).$$

$q\bar{q}g$ production



$q\bar{q}g$ production

As before :

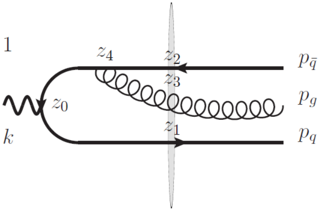
$$\tilde{M}^{\alpha} = (t^b)_l^n \frac{\langle 0| c_{p_g}^b \, b_{p_{\bar{q}}}^l (a_{p_q})_n \overline{\psi}(z_0)\, \gamma^{\alpha} \psi(z_0)\, e^{i\int \mathcal{L}_i(z) dz} |0\rangle}{\langle 0| e^{i\int \mathcal{L}(z) dz} |0\rangle}$$

Projection on color singlet and substraction of non-interacting terms give :

$$2M^{\alpha} = \int d\vec{z}_1 d\vec{z}_2 d\vec{z}_3 F_1 (p_q, p_{\bar{q}}, p_g, z_0, \vec{z}_1, \vec{z}_2, \vec{z}_3)^{\alpha} \left(\mathrm{tr}(U_1 U_3^{\dagger})\, \mathrm{tr}(U_3 U_2^{\dagger}) - N_c\, \mathrm{tr}(U_1 U_2^{\dagger}) \right)$$

$$+ \int d\vec{z}_1 d\vec{z}_2\, F_2 (p_q, p_{\bar{q}}, p_g, z_0, \vec{z}_1, \vec{z}_2)^{\alpha} \, \frac{N_c^2 - 1}{N_c} \left(\mathrm{tr}(U_1 U_2^{\dagger}) - N_c \right)$$

$q\bar{q}g$ production : first kind



$q\bar{q}g$ production : first kind

Result for a longitudinal photon

$$
\begin{aligned}
& F_{1L}\left(p_q, p_{\bar{q}}, p_g, k, \vec{z}_1, \vec{z}_2, \vec{z}_3\right) \\
= \quad & \delta(k^+ - p_g^+ - p_q^+ - p_{\bar{q}}^+)\theta(p_g^+ - \sigma)2Qg\frac{e^{-i\vec{p}_q\cdot\vec{z}_1 - i\vec{p}_{\bar{q}}\cdot\vec{z}_2 - i\vec{p}_g\cdot\vec{z}_3}}{\pi\sqrt{2p_g^+}}K_0(QZ_{123}) \\
\times \quad & \delta_{\lambda_q,-\lambda_{\bar{q}}}\left\{(x_{\bar{q}} + x_g\delta_{-s_g\lambda_q})x_q\frac{\vec{z}_{32}\cdot\vec{\varepsilon}_g^{\,*}}{\vec{z}_{32}^{\,2}} - (x_q + x_g\delta_{-s_g\lambda_{\bar{q}}})x_{\bar{q}}\frac{\vec{z}_{31}\cdot\vec{\varepsilon}_g^{\,*}}{\vec{z}_{31}^{\,2}}\right\}
\end{aligned}
$$

$q\bar{q}g$ production : first kind

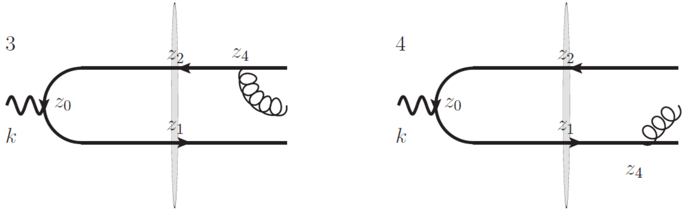
Result for a transverse photon

$$F_{1T}\left(p_q, p_{\bar{q}}, p_g, k, \vec{z}_1, \vec{z}_2, \vec{z}_3\right) = 2igQ\delta(k^+ - p_g^+ - p_q^+ - p_{\bar{q}}^+)\theta(p_g^+ - \sigma)\frac{e^{-i\vec{p}_q\cdot\vec{z}_1 - i\vec{p}_{\bar{q}}\cdot\vec{z}_2 - i\vec{p}_g\cdot\vec{z}_3}}{\pi Z_{123}\sqrt{2p_g^+}}$$

$$\times \quad \delta_{\lambda_q,-\lambda_{\bar{q}}} K_1(QZ_{123}) \left\{ -\frac{\left(\vec{z}_{23}\cdot\vec{\varepsilon}_g^{\,*}\right)\left(\vec{z}_{13}\cdot\vec{\varepsilon}_T\right)}{\vec{z}_{23}^{\,2}} x_q \left(x_q - \delta_{s\lambda_{\bar{q}}}\right)\left(x_{\bar{q}} + x_g\delta_{-s_g\lambda_q}\right) \right.$$

$$\left. -\frac{\left(\vec{z}_{23}\cdot\vec{\varepsilon}_g^{\,*}\right)\left(\vec{z}_{23}\cdot\vec{\varepsilon}_T\right)}{\vec{z}_{23}^{\,2}} x_q x_{\bar{q}} \left(x_{\bar{q}} + x_g\delta_{-s_g\lambda_q} - \delta_{s\lambda_q}\right) \right\} - (q \leftrightarrow \bar{q})$$

$q\bar{q}g$ production : second kind



$q\bar{q}$ production : second kind

$q\bar{q}g$ production : second kind

Result for a longitudinal photon

$$\tilde{F}_{2L}\left(p_q, p_{\bar{q}}, p_g, k, \vec{z}_1, \vec{z}_2\right) = 4ig\, Q\, \theta(p_g^+ - \sigma)\delta(k^+ - p_g^+ - p_q^+ - p_{\bar{q}}^+)\frac{e^{-i\vec{p}_q\cdot\vec{z}_1 - i\vec{p}_{\bar{q}}\cdot\vec{z}_2}}{\sqrt{2p_g^+}}$$

$$\times\delta_{\lambda_q,-\lambda_{\bar{q}}}\frac{x_q\left(x_g + x_{\bar{q}}\right)\left(\delta_{-s_g\lambda_q}x_g + x_{\bar{q}}\right)}{x_{\bar{q}}\, x_g}\frac{\vec{P}_{\bar{q}}\cdot\vec{\varepsilon}_g^{\,*}}{\vec{P}_{\bar{q}}^2}\, e^{-i\vec{p}_g\cdot\vec{z}_2}K_0(QZ_{122}) - (q \leftrightarrow \bar{q})$$

$q\bar{q}g$ production : second kind

Result for a transverse photon

$$\tilde{F}_{2T}\,(p_q, p_{\bar{q}}, p_g, k, \vec{z}_1, \vec{z}_2) = -4g\,\theta(p_g^+ - \sigma)\,\delta(k^+ - p_g^+ - p_q^+ - p_{\bar{q}}^+)\,\frac{e^{-i\vec{p}_q\cdot\vec{z}_1 - i\vec{p}_{\bar{q}}\cdot\vec{z}_2}}{\sqrt{2p_g^+}}\delta_{\lambda_q,-\lambda_{\bar{q}}}$$

$$\times\frac{\left(\delta_{\lambda_{\bar{q}}s} - x_q\right)\left(\delta_{-s_g\lambda_q}x_g + x_{\bar{q}}\right)}{x_{\bar{q}}\,x_g}\,\frac{\vec{P}_{\bar{q}}\cdot\vec{\varepsilon}_g^{\,*}}{\vec{P}_{\bar{q}}^2}\,\frac{\vec{z}_{12}\cdot\vec{\varepsilon}_T}{\vec{z}_{12}^2}\,Q\,Z_{122}K_1(QZ_{122})e^{-i\vec{p}_g\cdot\vec{z}_2} - (q\leftrightarrow\bar{q})$$

Linear approximation

Back to the general expression for the matrix element M^{α} :

$$2M^{\alpha} = \int d\vec{z}_1 d\vec{z}_2 d\vec{z}_3 F_1 \left(p_q, p_{\bar{q}}, p_g, z_0, \vec{z}_1, \vec{z}_2, \vec{z}_3\right)^{\alpha} \left(\mathrm{tr}(U_1 U_3^{\dagger})\,\mathrm{tr}(U_3 U_2^{\dagger}) - N_c\,\mathrm{tr}(U_1 U_2^{\dagger})\right)$$
$$+ \int d\vec{z}_1 d\vec{z}_2\, F_2 \left(p_q, p_{\bar{q}}, p_g, z_0, \vec{z}_1, \vec{z}_2\right)^{\alpha} \frac{N_c^2 - 1}{N_c} \left(\mathrm{tr}(U_1 U_2^{\dagger}) - N_c\right)$$

For 2 or 3 gluon exchange, one can linearize this expression :

$$\begin{aligned}\mathrm{tr}(U_1 U_3^{\dagger})\,\mathrm{tr}(U_3 U_2^{\dagger}) - N_c\,\mathrm{tr}(U_1 U_2^{\dagger}) &= N_c^2 \left(\mathbf{U}_{32} + \mathbf{U}_{13} - \mathbf{U}_{12} + \mathbf{U}_{32}\mathbf{U}_{13}\right) \\ &\simeq N_c^2 \left(\mathbf{U}_{32} + \mathbf{U}_{13} - \mathbf{U}_{12}\right)\end{aligned}$$

Linear approximation

After linearization one gets :

$$M^{\alpha} \overset{g^3}{=} \frac{1}{2}\int d\vec{z}_1 d\vec{z}_2 \mathbf{U}_{12}\left\{\tilde{F}_1\left(p_q, p_{\bar{q}}, p_g, z_0, \vec{z}_1, \vec{z}_2\right)^{\alpha}\right.$$
$$+ \left.\left(N_c^2-1\right)\tilde{F}_2\left(p_q, p_{\bar{q}}, p_g, z_0, \vec{z}_1, \vec{z}_2\right)^{\alpha}\right\}$$

$$\tilde{F}_1\left(p_q, p_{\bar{q}}, p_g, z_0, \vec{z}_1, \vec{z}_2\right)^{\alpha} = \int d\vec{z}_3 \left[N_c^2 F_1\left(p_q, p_{\bar{q}}, p_g, z_0, \vec{z}_1, \vec{z}_3, \vec{z}_2\right)^{\alpha}\right.$$
$$\left.+N_c^2 F_1\left(p_q, p_{\bar{q}}, p_g, z_0, \vec{z}_3, \vec{z}_2, \vec{z}_1\right)^{\alpha} - F_1\left(p_q, p_{\bar{q}}, p_g, z_0, \vec{z}_1, \vec{z}_2, \vec{z}_3\right)^{\alpha}\right]$$

$\Rightarrow$ One has to integrate the previously derived expressions

Linear approximation

No analytical expression for most of the integrals. BUT :

- For null transverse momenta $\vec{p}_q$, $\vec{p}_{\bar{q}}$ and $\vec{p}_g$ they can be performed
- In any case, they can be reduced to convergent integrals over a real parameter in [0,1] so a numerical calculation can be done.

For example :

$$\int d\vec{z}_3 e^{-i\vec{p}_g\cdot\vec{z}_3} \frac{\vec{z}_{32}}{\vec{z}_{32}^{\,2}} K_0\left(QZ_{123}\right)$$

$$= -\frac{\pi e^{-i\vec{p}_g\cdot\vec{z}_2}}{(1-x_g)x_g} \int_0^1 d\alpha e^{\alpha\frac{ix_q(\vec{z}_{21}\cdot\vec{p}_g)}{x_{\bar{q}}+x_q}} \left(\frac{i\vec{p}_g Z_{q\bar{q}g}}{Q_g(\alpha)} K_1\left(Q_g(\alpha) Z_{q\bar{q}g}\right) + x_g x_q \vec{z}_{21} K_0\left(Q_g(\alpha) Z_{q\bar{q}g}\right)\right)$$

$$\int d\vec{z}_3 \frac{\vec{z}_{32}}{\vec{z}_{32}^{\,2}} K_0\left(QZ_{123}\right) = -\frac{2\pi}{x_g x_q Q} \frac{\vec{z}_{21}}{\vec{z}_{21}^{\,2}} \left(Z_{q\bar{q}} K_1\left(QZ_{q\bar{q}}\right) - Z_{122} K_1\left(QZ_{122}\right)\right)$$

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Conclusion

Further analysis from our results

- A lot of phenomenology can be done from HERA data :
  - From $q\bar{q}$ production to jet production
  - Is our approach better than the 2-gluon approximation for H1 and ZEUS data?
- The same calculation can be done again for massive quarks
- The calculation of virtual correction for the cross section is still to be performed

Backup slides : Results in momentum space

Results in momentum space

First kind, longitudinal photon

$$F_{1L}\left(p_q, p_{\bar{q}}, p_g, z_0, \vec{p}_1, \vec{p}_2, \vec{p}_3\right) = \delta(k^+ - p_g^+ - p_q^+ - p_{\bar{q}}^+)\delta\left(\vec{p}_{1q} + \vec{p}_{2\bar{q}} + \vec{p}_{3g}\right)\theta(p_g^+ - \sigma)$$
$$\times \frac{\delta_{\lambda_q, -\lambda_{\bar{q}}}}{\sqrt{2p_g^+}} \frac{4iQ\, g\left(x_q + x_g \delta_{-s_g \lambda_{\bar{q}}}\right)\left((\vec{p}_{2\bar{q}} \cdot \vec{\varepsilon}_g^{\,*})x_q + (\vec{p}_{1q} \cdot \vec{\varepsilon}_g^{\,*})\left(1 - x_{\bar{q}}\right)\right)}{\left(1 - x_{\bar{q}}\right) x_g x_q \left(Q^2 + \frac{\vec{p}_{2\bar{q}}^{\,2}}{x_{\bar{q}}(1 - x_{\bar{q}})}\right)\left(Q^2 + \frac{\vec{p}_{1q}^{\,2}}{x_q} + \frac{\vec{p}_{2\bar{q}}^{\,2}}{x_{\bar{q}}} + \frac{\vec{p}_{3g}^{\,2}}{x_g}\right)} - (q \leftrightarrow \bar{q}),$$

Results in momentum space

Second kind, longitudinal photon

$$\tilde{F}_{2L}\left(p_q, p_{\bar{q}}, p_g, k, \vec{p}_1, \vec{p}_2\right) = \theta(p_g^+ - \sigma)\delta(k^+ - p_g^+ - p_q^+ - p_{\bar{q}}^+)\delta(\vec{p}_{1q} + \vec{p}_{2\bar{q}} - \vec{p}_g)$$
$$\times \frac{4igQ}{\sqrt{2p_g^+}}\delta_{\lambda_q, -\lambda_{\bar{q}}} \frac{x_{\bar{q}} + \delta_{-s_g \lambda_q} x_g}{x_{\bar{q}} x_g} \frac{2\pi}{Q^2 + \frac{\vec{p}_{1q}^{\,2}}{x_q\left(1-x_q\right)}} \frac{\vec{P}_{\bar{q}} \cdot \vec{\varepsilon}_g^{\,*}}{\vec{P}_{\bar{q}}^2} - (q \leftrightarrow \bar{q})\,.$$

Results in momentum space

First kind, transverse photon

$$F_{1T}\left(p_q, p_{\bar{q}}, p_g, z_0, \vec{p}_1, \vec{p}_2, \vec{p}_3\right) = \frac{2ig}{\sqrt{2p_g^+}} \frac{\delta(k^+ - p_g^+ - p_q^+ - p_{\bar{q}}^+)\delta\left(\vec{p}_{1q} + \vec{p}_{2\bar{q}} + \vec{p}_{3g}\right)\theta(p_g^+ - \sigma)\delta_{-\lambda_{\bar{q}}\lambda_q}}{Q^2\,(1-x_q)\left(\frac{\vec{p}_{1q}{}^2}{x_q} + \frac{\vec{p}_{2\bar{q}}{}^2}{x_{\bar{q}}} + \frac{\vec{p}_{3g}{}^2}{x_g} + Q^2\right)}\left\{\delta_{ss_g}\delta_{s\lambda_q}\right.$$

$$\left. + 2(\vec{p}_{1q}\cdot\vec{\varepsilon}_T)((\vec{p}_{2\bar{q}}\cdot\vec{\varepsilon}_g{}^*)\left(x_{\bar{q}} + x_g\right) + (\vec{p}_{1q}\cdot\vec{\varepsilon}_g{}^*)\,x_{\bar{q}})\frac{\left(x_q - \delta_{s\lambda_{\bar{q}}}\right)\left(x_g\,\delta_{-s_g\lambda_q} + x_{\bar{q}}\right)}{(1-x_q)\,x_q x_{\bar{q}} x_g\left(Q^2 + \frac{\vec{p}_{1q}{}^2}{(1-x_q)x_q}\right)}\right\} - (q \leftrightarrow \bar{q}).$$

Results in momentum space

Second kind, transverse photon

$$\tilde{F}_{2T}\left(p_q, p_{\bar{q}}, p_g, k, \vec{p}_1, \vec{p}_2\right) = -\,\theta(p_g^+ - \sigma)\,\delta(k^+ - p_g^+ - p_q^+ - p_{\bar{q}}^+)\delta(\vec{p}_{1q} + \vec{p}_{2\bar{q}} - \vec{p}_g)\frac{\delta_{\lambda_q,-\lambda_{\bar{q}}}}{\sqrt{2p_g^+}}$$

$$\times 4g\frac{\left(\delta_{\lambda_{\bar{q}}s} - x_q\right)\left(\delta_{-s_g\lambda_q}x_g + x_{\bar{q}}\right)}{x_{\bar{q}}x_g}\frac{2\pi i\left(\vec{p}_{1q}\cdot\vec{\varepsilon}_T\right)}{x_q\left(1-x_q\right)Q^2 + \vec{p}_{1q}^{\,2}}\frac{\vec{P}_{\bar{q}}\cdot\vec{\varepsilon}_g^{\,*}}{\vec{P}_{\bar{q}}^2} - (q \leftrightarrow \bar{q})$$