Correlations between Partons in Nucleons
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QCD High Energy Evolution: from Basics to NLO

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How scattering amplitudes and cross sections depend on the collision energy $\sqrt{s}$?

At low energy hadrons consists of relatively small number of partons (Quarks + Gluons). As the collision energy (rapidity) increases, new partons are emitted (*Weizsacker Williams radiation*) due to color charge:

As long as the density of partons remains small, new particles are created *linearly*: the number of new partons due to the increase in collision energy is proportional to the number of emitting partons (so the density grows exponentially).

This regime is described the BFKL equation:

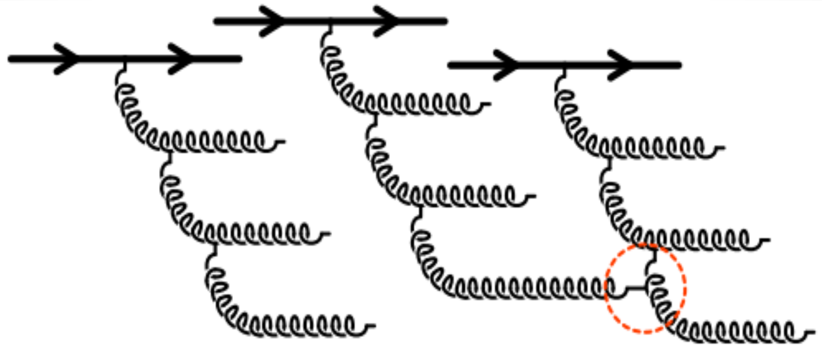
$$\frac{d\phi(p_T)}{dY} = \int_{k_T} K_{BFKL}(p_T, k_T)\phi(k_T)$$

Unfortunately, BFKL is not the whole story. BFKL solutions suffer from ***Unitarity Violation*** - for example the proton-proton cross section expected to grow asymptotically as a power of s, which violates the ***Froissart bound:***

$$\sigma_{\rm tot} \leq C \ln^2 s \;, \quad {\rm as}\; s \to \infty$$

What happens if we push to higher collision energy?

Eventually gluons start overlapping with each other and new parton emission become a *collective process*.
Emission process becomes *non-linear* and leads to *gluon saturation* phenomena (also known as *Color Glass Condensate (CGC), Cold Gluon Cloud, or JIMWLK*).
The gluon density grows logarithmically instead of exponentially (as in the case of BFKL).



The JIMWLK equation, $\frac{d}{dY}\mathcal{O} = -H^{JIMWLK}\mathcal{O}$, describes the rapidity (denoted by Y) evolution of observables $\mathcal{O}$ in scattering process (approximation of dilute probe on dense target).

The JIMWLK equation is a *non-linear functional* equation which is obtained by computing the expectation value of the S-matrix operator (expanded to first order in longitudinal phase space):

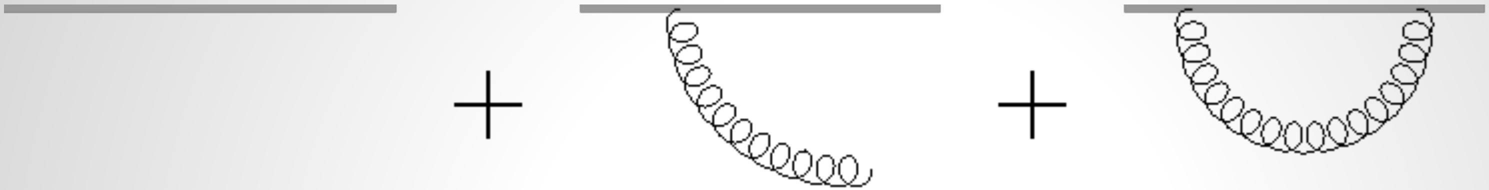
$$H^{JIMWLK} = \langle\psi|\,\hat{S} - 1\,|\psi\rangle$$

It takes into account both linear growth as well as saturation effects.

Leading-Order JIMWLK

J.Jalilian-Marian,
E.Iancu,
L. McLerran,
H. Weigert,
A. Leonidov,
A. Kovner/ 97’
-99’

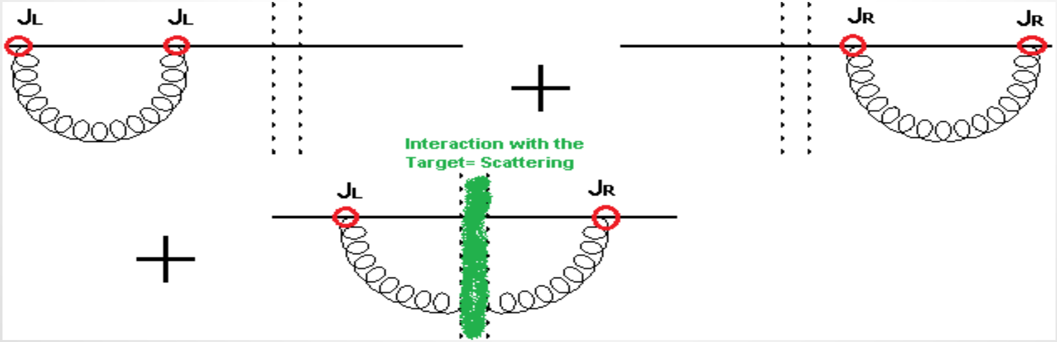
The leading order wave function composed of three terms:



It corresponds to the following light cone wave function:

$$|\psi\rangle = (1 - g_s^2\,\kappa_0\,JJ)\,|\,no\,soft\,gluons\rangle + g_s\kappa_1\,J\,|\,one\,soft\,gluon\rangle$$

If we insert the LO wave to the JIMWLK definition we find three contributions:



The LO JIMWLK Hamiltonian

The JIMWLK Hamiltonian reads:

$$\mathcal{H}_{LO\,JIMWLK} =$$
$$-\frac{\alpha_s}{2\pi}\int d^2x\, d^2y\, d^2z\, \frac{(x-z)(y-z)}{(x-z)^2(y-z)^2}\left[J_L^a(x)J_L^a(y) + J_R^a(x)J_R^a(y) - 2J_L^a(x)S^{ab}(z)J_R^b(y)\right]$$

This Hamiltonian describes the evolution of scattering amplitudes even *for large values of rapidity*. The target is modeled by a fixed background field A. The S-matrix of a fast particle interacting with a gluonic field A is the Wilson line (*Eikonal Approximation*):

$$|x,a\rangle \to S^{ab}(x)|x,b\rangle \qquad S^{ab}(x) = \mathcal{P}\exp\left\{i\int dx^-\, T^a\,\alpha_T^a(x,x^-)\right\}^{ab}$$

JL and JR are operators acting on Wilson lines as rotations:

$$J_L^a(x)S^{ij}(w) = (t^a S(w))^{ij}\delta(w-x) \qquad J_R^a(x)S^{ij}(w) = (S(w)t^a)^{ij}\delta(w-x)$$

Motivations for NLO JIMWLK Equation

The LO JIMWLK is only a first term in an infinite perturbative series:

$$H_{JIMWLK} = H_{LO}(\alpha_s) + \boxed{H_{NLO}(\alpha_s^2)} + H_{NNLO}(\alpha_s^3) + \ldots$$

The NLO term is necessary because:

a. NLO Corrections are *known to be large*.

b. Built-in information on the *running coupling* - better phenomenology. The running is known to slow down the evolution.

c. To determine *region of applicability* of the leading order equation (note that we expect a dependence on the scattering process).

d. Important step towards *all order resummation*.

As in LO case, NLO JIMWLK reduces to NLO BFKL in linear approximation.

Towards NLO JIMWLK

General structure of the wave-function up to g^3 (normalization up to g^4), which has the following structure:

$$
\begin{aligned}
|\psi\rangle \;=\; & (1 - g_s^2\,\kappa_0\, JJ - g_s^4(\delta_1\, JJ + \delta_2\, JJJ + \delta_3\, JJJJ)\,|\,no\,soft\,gluons\rangle + \\
& + (g_s\kappa_1\, J + g_s^3\epsilon_1\, J + g_s^3\,\epsilon_2\, J\, J)\,|\,one\,soft\,gluon\rangle + g_s^2(\epsilon_3\, J \;+ \epsilon_4\, JJ)\,|\,two\,soft\,gluons\rangle + g_s^2\,\epsilon_5\, J\,|\,q\,\bar{q}\rangle
\end{aligned}
$$

Combining this wave function with:

a. *Expected symmetries* of the NLO kernel, SU(N)xSU(N).

b. *Unitarity*, while S=1 we should get no evolution.

We can find the general form of the NLO JIMWLK Hamiltonian.

The General Form of NLO JIMWLK Hamiltonian

$$
\begin{aligned}
H^{NLO\ JIMWLK} = &\int_{x,y,z} K_{JSJ}(x,y;z)\, \left[J_L^a(x)\, J_L^a(y) + J_R^a(x)\, J_R^a(y) - 2\, J_L^a(x)\, S_A^{ab}(z)\, J_R^b(y)\right] + \\
&+ \int_{x\,y\,z\,z'} K_{JSSJ}(x,y;z,z')\, \left[f^{abc}\, f^{def}\, J_L^a(x)\, S_A^{be}(z)\, S_A^{cf}(z')\, J_R^d(y) - N_c\, J_L^a(x)\, S_A^{ab}(z)\, J_R^b(y)\right] + \\
&+ \int_{x,y,z,z'} K_{q\bar{q}}(x,y;z,z')\, \left[2\, J_L^a(x)\, tr[S^\dagger(z)\, T^a\, S(z') T^b]\, J_R^b(y) - J_L^a(x)\, S_A^{ab}(z)\, J_R^b(y)\right] + \\
&+ \int_{w,x,y,z,z'} K_{JJSSJ}(w;x,y;z,z') f^{acb} \Big[J_L^d(x)\, J_L^e(y)\, S_A^{dc}(z)\, S_A^{eb}(z')\, J_R^a(w) - J_L^a(w)\, S_A^{cd}(z)\, S_A^{be}(z')\, J_R^d(x)\, J_R^e(y) + \\
&+ \frac{1}{3}\left[\, J_L^c(x)\, J_L^b(y)\, J_L^a(w) - J_R^c(x)\, J_R^b(y)\, J_R^a(w)\right]\Big] + \\
&+ \int_{w,x,y,z} K_{JJSJ}(w;x,y;z)\, f^{bde} \Big[J_L^d(x)\, J_L^e(y)\, S_A^{ba}(z)\, J_R^a(w) - J_L^a(w)\, S_A^{ab}(z)\, J_R^d(x)\, J_R^e(y) + \\
&+ \frac{1}{3}\left[\, J_L^d(x)\, J_L^e(y)\, J_L^b(w) - J_R^d(x)\, J_R^e(y)\, J_R^b(w)\right]\Big]
\end{aligned}
$$

Shortcut to the Kernels

While we obtained the general form of the Hamiltonian the 5 kernels still have to be determined. How did we manage to find them?

Smart trick:

Evolution equation of ***quark dipole*** - $s(x,y) = \frac{1}{N}\mathrm{Tr}(S^{\dagger}(x)S(y))$

$$\frac{d}{dY}s(x,y) = -H_{NLO\,JIMWLK}\,s(x,y)$$

The NLO BK which was computed by I. Balitsky and G. A. Chirilli in hep-ph/**0710.4330 (PRD).**

Evolution equation of ***SU(3) Baryon*** - $B(x,y,z) = \epsilon^{ijk}\epsilon^{lmn}S^{il}(x)S^{jm}(y)S^{kn}(z)$

$$\frac{d}{dY}B(x,y,z) = -H_{NLO\,JIMWLK}\,B(x,y,z)$$

Connected part was computed by V. A. Grabovsky in **hep-ph/1307.5414**.

The Kernels for Gauge Invariant Operators (color singlet amplitudes)

$$K_{JSJ}(x,y;z) = -\frac{\alpha_s^2}{16\pi^3}\frac{(x-y)^2}{X^2Y^2}\Big[b\ln(x-y)^2\mu^2 - b\frac{X^2-Y^2}{(x-y)^2}\ln\frac{X^2}{Y^2} + \Big(\frac{67}{9}-\frac{\pi^2}{3}\Big)N_c - \frac{10}{9}n_f\Big] - \frac{N_c}{2}\int_{z'}\tilde{K}(x,y,z,z')$$

Here μ is the normalization point in the $\overline{MS}$ scheme and $b = \frac{11}{3}N_c - \frac{2}{3}n_f$ is the first coefficient of the β-function.

$$K_{JSSJ}(x,y;z,z') = \frac{\alpha_s^2}{16\,\pi^4}\Big[-\frac{4}{(z-z')^4} + \Big\{2\frac{X^2Y'^2 + X'^2Y^2 - 4(x-y)^2(z-z')^2}{(z-z')^4[X^2Y'^2 - X'^2Y^2]} + \frac{(x-y)^4}{X^2Y'^2 - X'^2Y^2}\Big[\frac{1}{X^2Y'^2} + \frac{1}{Y^2X'^2}\Big] + \frac{(x-y)^2}{(z-z')^2}\Big[\frac{1}{X^2Y'^2} - \frac{1}{X'^2Y^2}\Big]\Big\}\ln\frac{X^2Y'^2}{X'^2Y^2}\Big] + \tilde{K}(x,y,z,z')$$

$$\tilde{K}(x,y,z,z') = \frac{i}{2}\,[K_{JJSSJ}(x;x,y;z,z') - K_{JJSSJ}(y;x,y;z,z') - K_{JJSSJ}(x;y,x;z,z') + K_{JJSSJ}(y;y,x;z,z')]$$

$$K_{q\bar{q}}(x,y;z,z') = -\frac{\alpha_s^2\,n_f}{8\,\pi^4}\Big\{\frac{X'^2Y^2 + Y'^2X^2 - (x-y)^2(z-z')^2}{(z-z')^4(X^2Y'^2 - X'^2Y^2)}\ln\frac{X^2Y'^2}{X'^2Y^2} - \frac{2}{(z-z')^4}\Big\}$$

$$K_{JJSJ}(w;x,y;z) = -i\,\frac{\alpha_s^2}{4\,\pi^3}\Big[\frac{X\cdot W}{X^2\,W^2} - \frac{Y\cdot W}{Y^2\,W^2}\Big]\ln\frac{Y^2}{(x-y)^2}\ln\frac{X^2}{(x-y)^2}$$

$$K_{JJSSJ}(w;x,y;z,z') = -i\,\frac{\alpha_s^2}{2\,\pi^4}\Big(\frac{X_iY'_j}{X^2Y'^2}\Big)\Big(\frac{\delta_{ij}}{2(z-z')^2} + \frac{(z'-z)_iW'_j}{(z'-z)^2W'^2} + \frac{(z-z')_jW_i}{(z-z')^2W^2} - \frac{W_iW'_j}{W^2W'^2}\Big)\ln\frac{W^2}{W'^2}$$

Kernels for Color Non-singlets Operators

Most of our interest is in gauge invariant amplitudes. For the sake of completeness it is interesting to find the kernels applicable for action on non-gauge invariant structures. This has been done by comparison of the evolution equation of one, two and three Wilson lines $S^{ij}(x)$ $S^{ij}(x)S^{kl}(y)$ and with $S^{ij}(x)S^{kl}(y)S^{mn}(z)$ of ***I. I. Balitsky*** and ***G. A. Chirilli*** (hep-ph/1309.7644).

$$K_{JSJ}(x,y,z) \to \bar{K}_{JSJ}(x,y,z) \equiv K_{JSJ}(x,y,z)+$$
$$+\frac{\alpha_s^2}{16\pi^3}\left\{\left[\frac{1}{X^2}+\frac{1}{Y^2}\right]\left[\left(\frac{67}{9}-\frac{\pi^2}{3}\right)N_c-\frac{10}{9}n_f\right]+\frac{b}{X^2}\ln X^2\mu^2+\frac{b}{Y^2}\ln Y^2\mu^2\right\}$$

$$K_{JSSJ}(x,y;z,z') \to \bar{K}_{JSSJ}(x,y;z,z') \equiv K_{JSSJ}(x,y;z,z')$$
$$+\frac{\alpha_s^2}{8\pi^4}\left[\frac{4}{(z-z')^4}-\frac{I(x,z,z')}{(z-z')^2}-\frac{I(y,z,z')}{(z-z')^2}\right]$$

$$K_{qq}(x,y;z,z') \to \bar{K}_{qq}(x,y;z,z') \equiv K_{qq}(x,y;z,z')-\frac{\alpha_s^2 n_f}{8\pi^4}\left[\frac{I_f(x,z,z')}{(z-z')^2}+\frac{I_f(y,z,z')}{(z-z')^2}\right]$$

$$I(x,z,z')=\frac{1}{X^2-(X')^2}\ln\frac{X^2}{(X')^2}\left[\frac{X^2+(X')^2}{(z-z')^2}-\frac{X\cdot X'}{X^2}-\frac{X\cdot X'}{(X')^2}-2\right]$$
$$I_f(x,z,z')=\frac{2}{(z-z')^2}-\frac{2X\cdot X'}{(z-z')^2(X^2-(X')^2)}\ln\frac{X^2}{(X')^2}$$

These modifications are vanishing while acting on gauge invariant structures.

Conformal Properties of the NLO Hamiltonian hep-th/1401.0374

The leading order Hamiltonian is invariant under the following transformations (together known as conformal symmetry):

1. ***Scaling Symmetry:*** $$x \to \alpha x$$

2. ***Inversion:*** $$x \to \frac{1}{x}$$

We addressed the question whether these symmetries are still preserved at NLO.

QCD is NOT a conformal theory. Therefore, we looked for NLO JIMWLK Hamiltonian for $\mathcal{N} = 4$ theory, which is similar to QCD but with vanishing beta function. For this case we showed conformal symmetry still exist albit in a modified way.

Summary

1) We have constructed the NLO JIMWLK (suitable for acting on color singlet operators) based on symmetries and known results on dipole and baryon evolutions.

2) Generalization of the kernels applicable for non-color singlets has been done (to appear soon).

3) Full consistency with the Balitsky's hierarchy at NLO (I. Balitsky and A. Chirill, hep-ph/**1309.7644**) has been shown.

4) The NLO Hamiltonian (in N=4) was proven to preserve the conformal symmetry (which exists also in the leading-order Hamiltonian).