The Glasma: instabilities, turbulence, thermalization

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From the violence of a nuclear collision
…to the calm of a quark-gluon fluid

Initial state:
Far from equilibrium

Non-equilibrium dynamics

Final state:
Thermal equilibrium

How is thermal equilibrium achieved?

Approaches to thermalization

Two ``clean" theoretical limits:

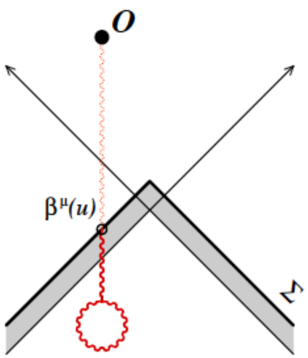
- **Holographic thermalization (based on duality of strongly coupled ($g^2 N_c \rightarrow \infty$; $N_c \rightarrow \infty$) N=4 SUSY YM to classical gravity in $AdS_5 \times S_5$)**

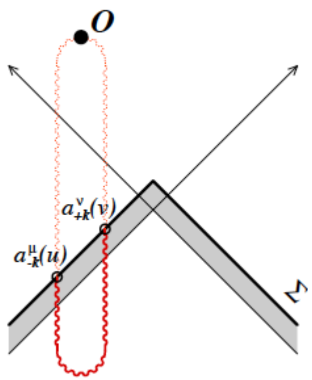
- **Highly occupied QCD at weak coupling ($g^2 \rightarrow 0$; $g^2 f \sim 1$)**

Our focus: non-equilibrium strongly correlated gluodynamics at weak coupling

RG evolution for 2 nuclei

Gelis,Lappi,RV (2008)



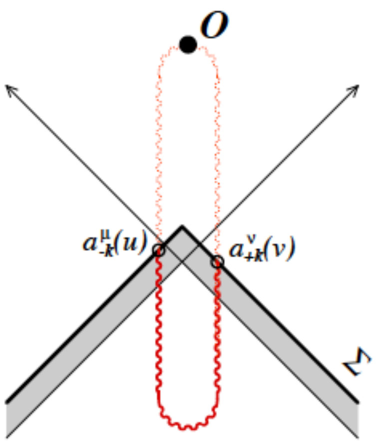


Log divergent contributions crossing nucleus 1 or 2:

$$\mathcal{O}_{\rm NLO} = \left[\frac{1}{2}\int_{\vec{u},\vec{v}} \mathcal{G}(\vec{u},\vec{v})\;\mathcal{T}_u\mathcal{T}_v + \int_{\vec{u}} \beta(\vec{u})\;\mathcal{T}_u\right]\mathcal{O}_{\rm LO}$$

$\mathcal{G}(\vec{u},\vec{v})$ **and** $\beta(\vec{u})$ **can be computed on the initial Cauchy surface**

$$\mathcal{T}_u = \frac{\delta}{\delta A(\vec{u})}$$ **linear operator on initial surface**



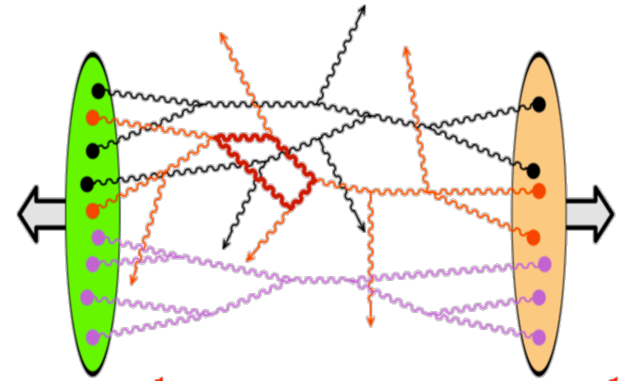
Contributions across both nuclei are finite-no log divergences => factorization

$$\mathcal{O}_{\rm NLO} = \left[\ln\left(\frac{\Lambda^+}{p^+}\right)\mathcal{H}_1 + \ln\left(\frac{\Lambda^-}{p^-}\right)\mathcal{H}_2\right]\mathcal{O}_{\rm LO}$$

Factorization + temporal evolution in the Glasma

$$T^{\mu\nu}_{\rm LO} = \frac{1}{4} g^{\mu\nu} F^{\lambda\delta} F_{\lambda\delta} - F^{\mu\lambda} F^{\nu}_{\lambda} \qquad O\left(\frac{Q_S^4}{g^2}\right)$$

ε=20-40 GeV/fm³ for τ=0.3 fm @ RHIC



NLO terms are as large as LO for $\alpha_s \ln(1/x)$:
small x (leading logs) and strong field (gρ) resummation

Gelis,Lappi,RV (2008)

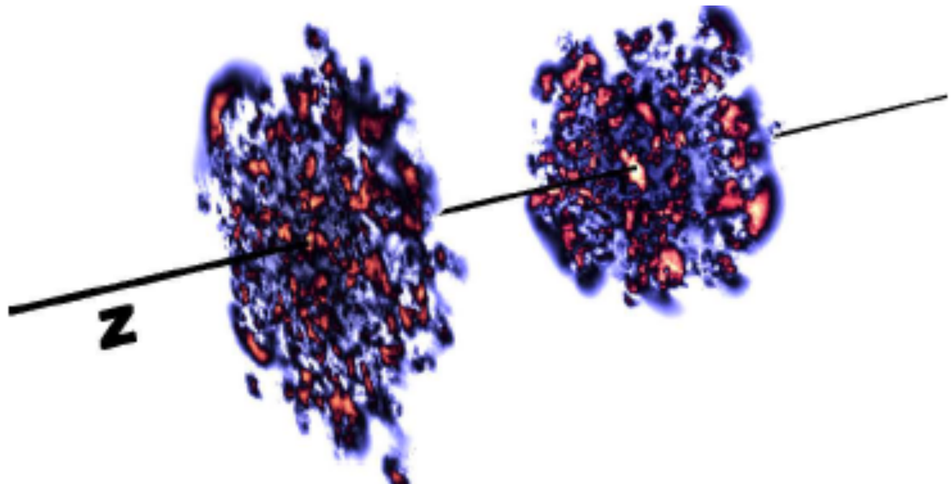
$$\langle T^{\mu\nu}(\tau, \underline{\eta}, x_\perp) \rangle_{\rm LLog} = \int [D\rho_1 d\rho_2]\, W_{Y_1}[\rho_1]\, W_{Y_2}[\rho_2]\, T^{\mu\nu}_{\rm LO}(\tau, x_\perp)$$

$$Y_1 = Y_{\rm beam} - \eta\, ;\ Y_2 = Y_{\rm beam} + \eta$$

Glasma factorization => universal "density matrices W" ⊗ "matrix element"

Heavy ion phenomenology in weak coupling

Collisions of lumpy gluon ``shock" waves



Leading order solution: Solution of QCD Yang-Mills eqns

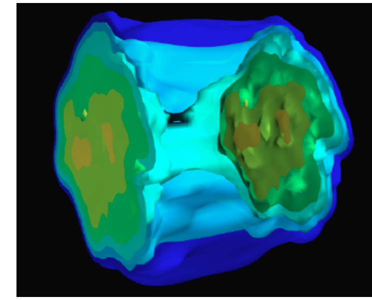
$$D_\mu F^{\mu\nu,a} = \delta^{\nu +}\rho^a_A(x_\perp)\delta(x^-) + \delta^{\nu -}\rho^a_B(x_\perp)\delta(x^+)$$

$$x^\pm = t \pm z$$

$$\langle \rho^a_{A(B)}(x_\perp)\rho^a_{A(B)}(y_\perp)\rangle = Q^2_{S,A(B)}\delta^{(2)}(x_\perp - y_\perp)$$

$$F^{\mu\nu,a} = \partial_\mu A^{\nu,a} - \partial_\nu A^{\nu,a} + g f^{abc} A^{\mu,b} A^{\nu,c}$$

Heavy ion phenomenology in weak coupling-II

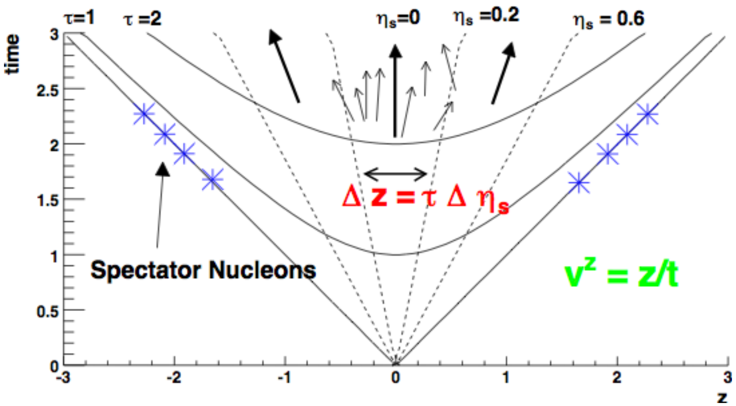


Leading order Yang-Mills solutions are boost invariant:

$$A^{\mu,a}(x_\perp,\tau,\eta) \equiv A^{\mu,a}(x_\perp,\tau)$$

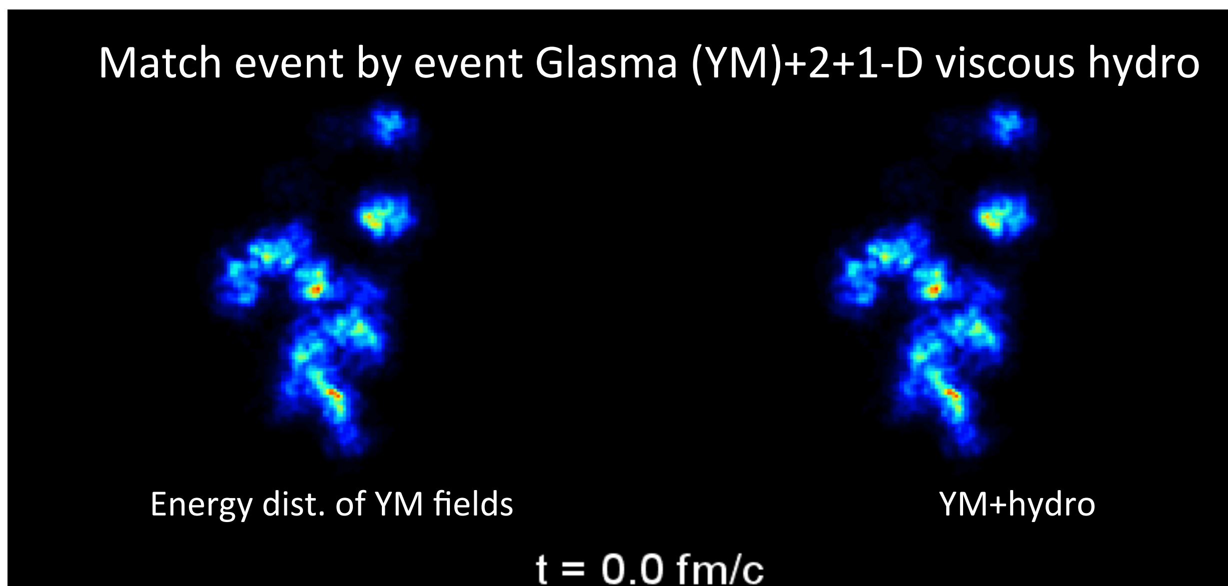
$$\tau = \sqrt{x^+x^-}$$

$$\eta = \frac{1}{2}\ln\left(\frac{x^+}{x^-}\right)$$



State of the art phenomenology:

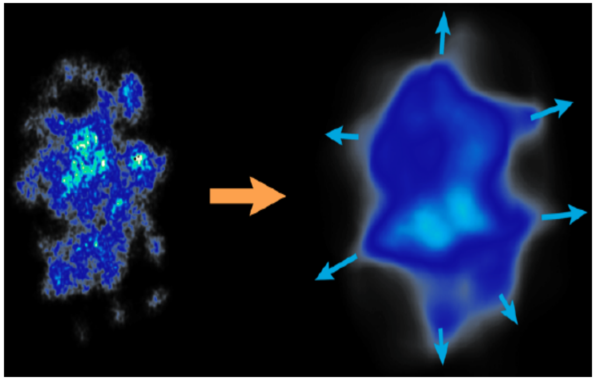
Solve viscous hydro equations with Glasma initial conditions

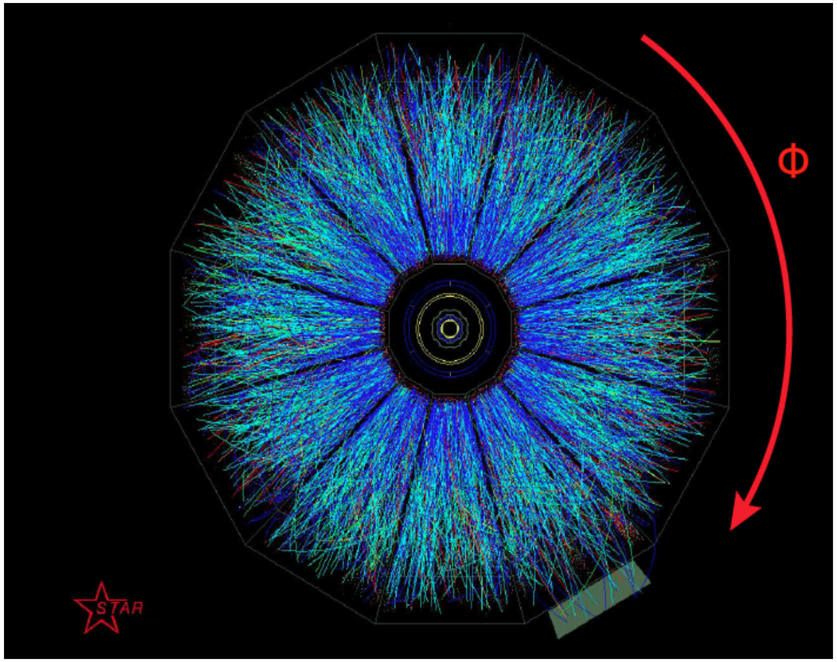


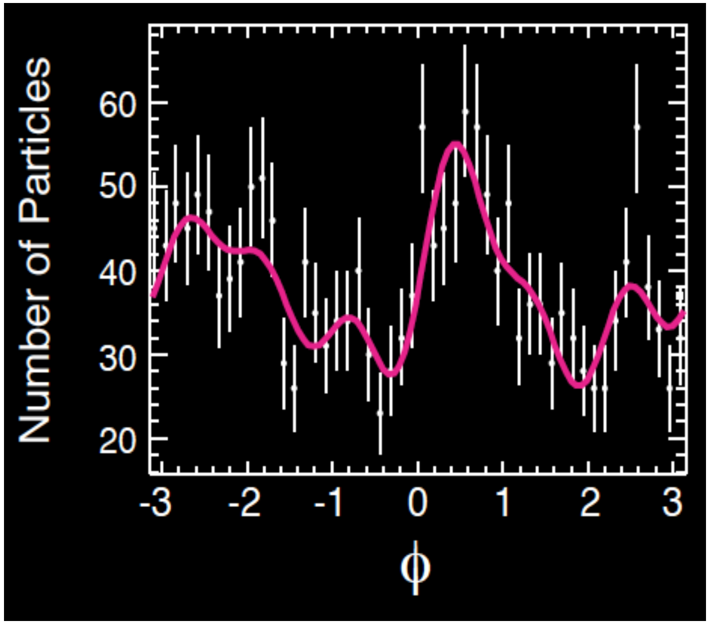
Heavy ion phenomenology in weak coupling

Hydrodynamics: efficient translation of spatial anisotropy into momentum anisotropy

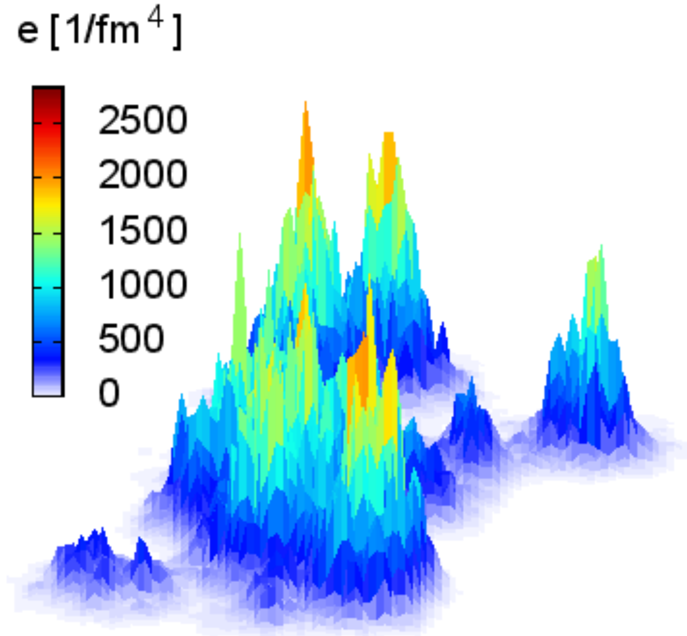
$$\frac{dN}{d\phi} = \frac{N}{2\pi}\left(1 + 2v_1\cos(\phi) + 2v_2\cos(2\phi) + 2v_3\cos(3\phi) + 2v_4\cos(4\phi) + \cdots\right)$$

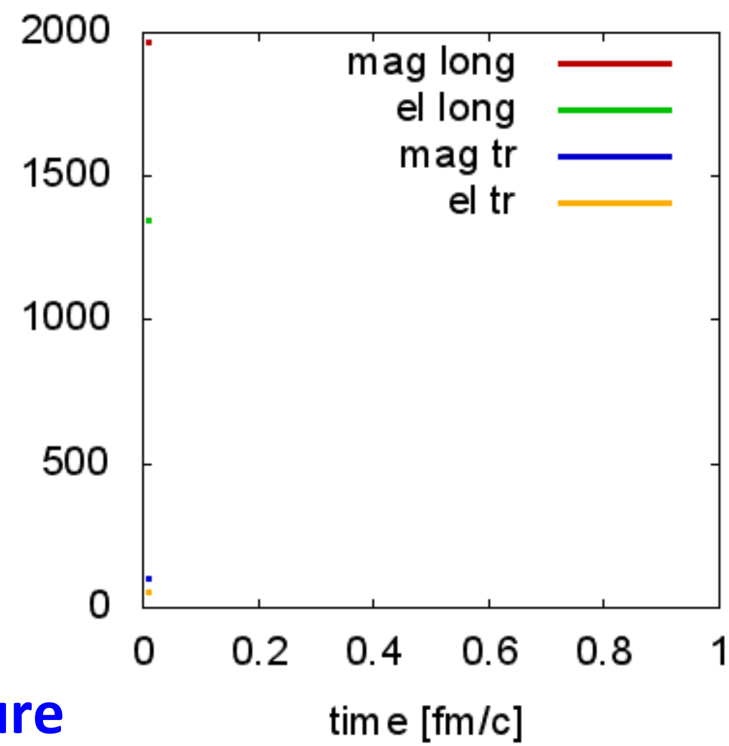






Matching boost invariant Yang-Mills to hydrodynamics



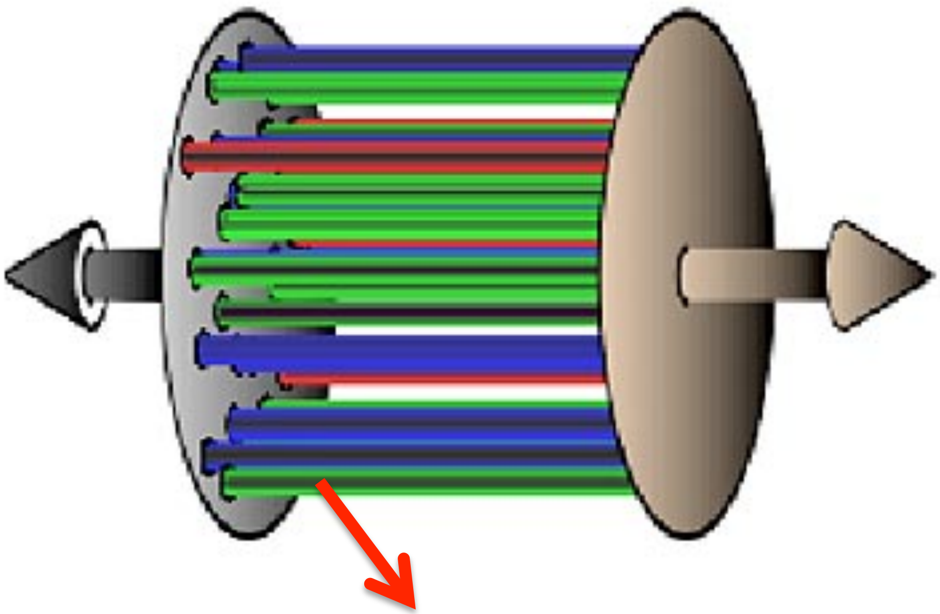


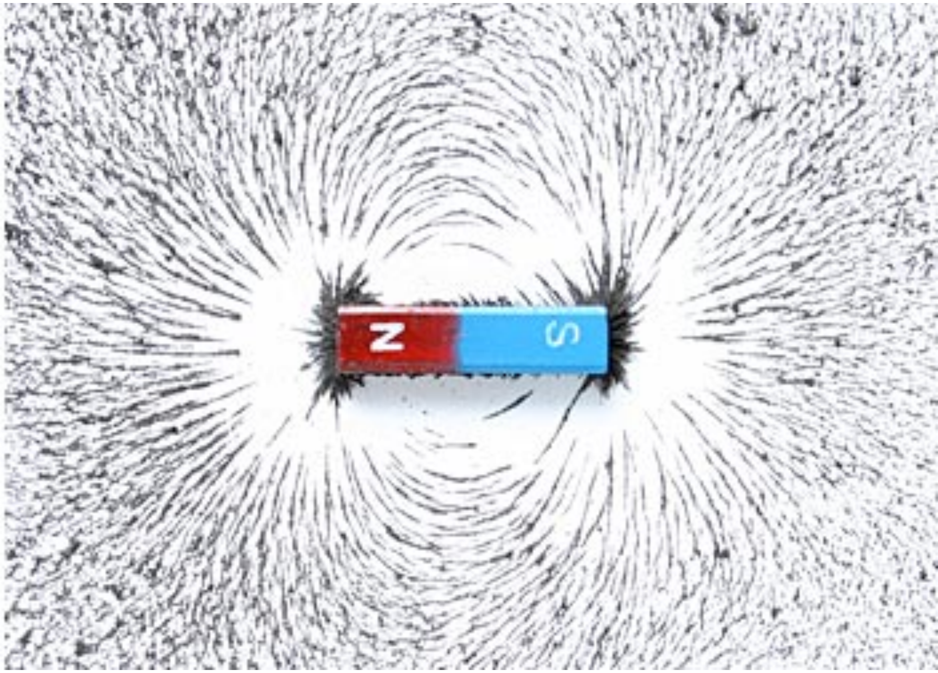
Glasma energy density and pressure

$$T_{\mu\nu}(\tau = 0) = \frac{1}{2}(B_z^2 + E_z^2) \times \mathrm{diag}(1, 1, 1, -1)$$

Initial longitudinal pressure is negative:
Goes to P_L =0 from below with time evolution

Imaging the force fields of QCD



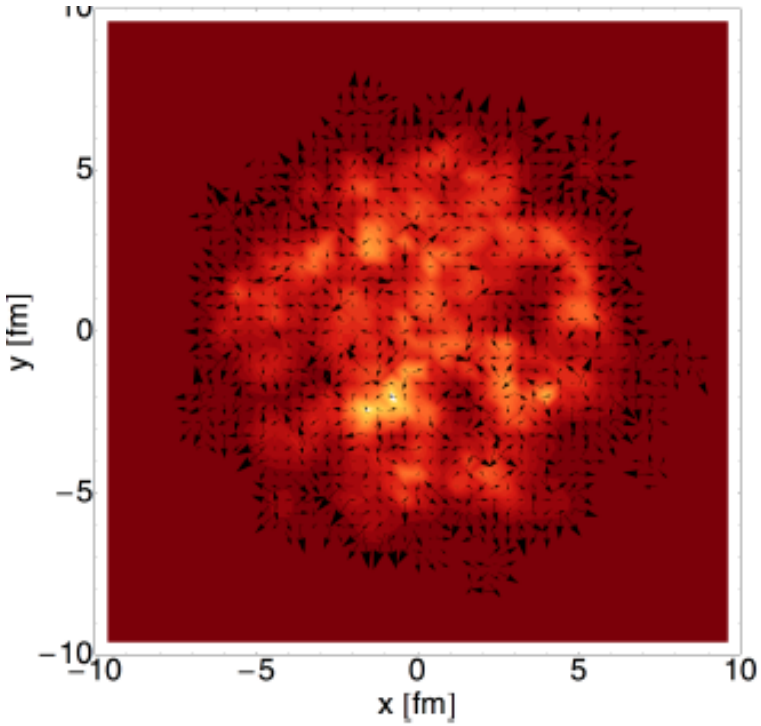


Solns. of QCD Yang-Mills eqns. demonstrate that each of these color "flux tubes" stretching out in rapidity is of transverse size $1/Q_S << 1$ fm

Multiparticle dynamics is controlled by sub-nucleon QCD scales

There are ~ $\pi R^2 Q_S^2$ flux tubes – multiplicity, $dn/d\eta \approx \pi R^2 Q_S^2/\alpha_S$

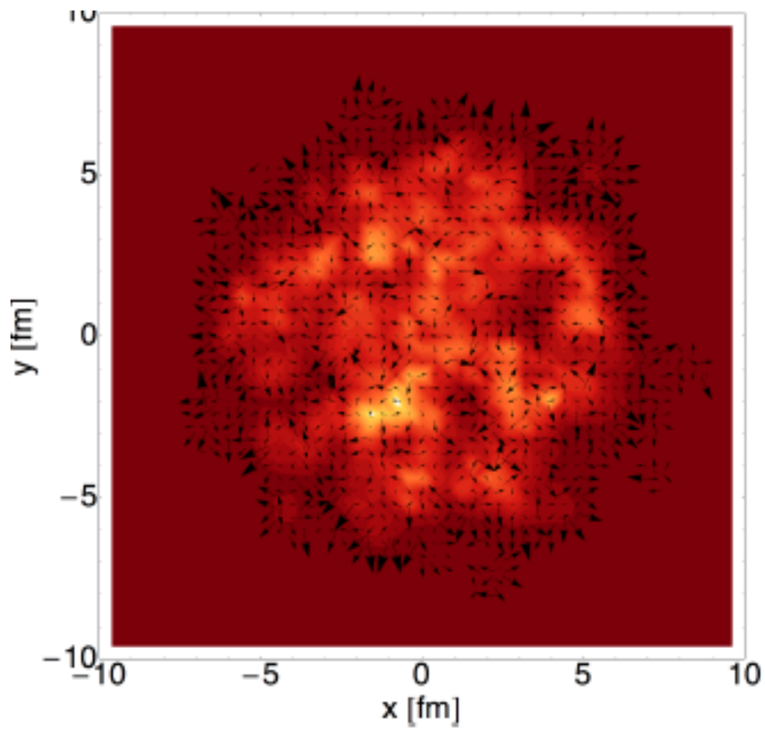
Matching boost invariant Yang-Mills to hydrodynamics



Energy density and (u_x, u_y) at $\tau = 0.4\,\mathrm{fm}/c$

Energy density and (u_x, u_y) from $u_\mu T^{\mu\nu} = \varepsilon u^\nu$

Matching boost invariant Yang-Mills to hydrodynamics



Energy density and (u_x, u_y) at $\tau = 0.4\,\mathrm{fm}/c$

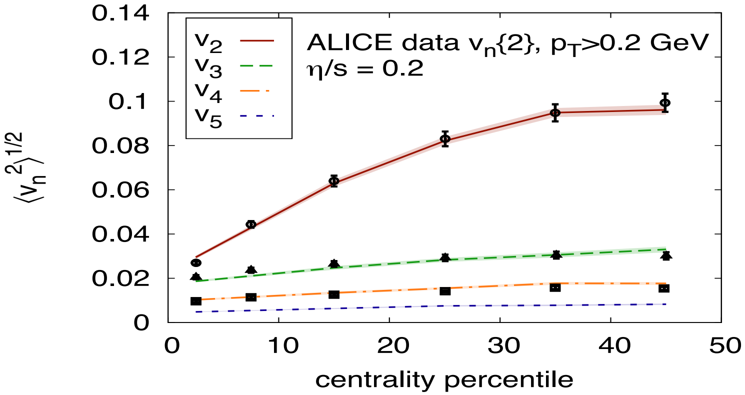
Matching to viscous hydro is "brutal" :
assume very rapid isotropization at initial hydro time

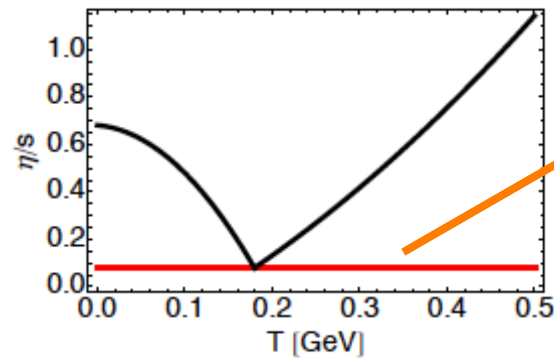
Large systematic uncertainty: how does isotropization/ thermalization occur on times < 1 fm/c ?

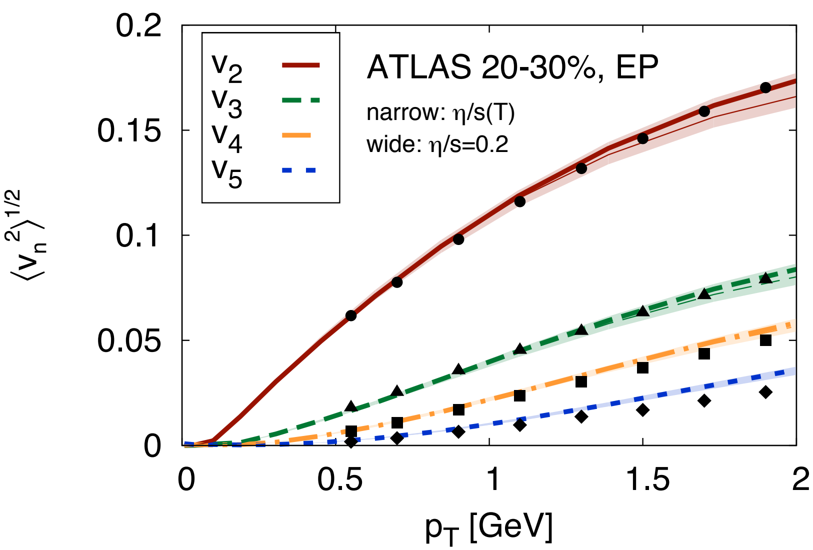
Heavy ion phenomenology in weak coupling

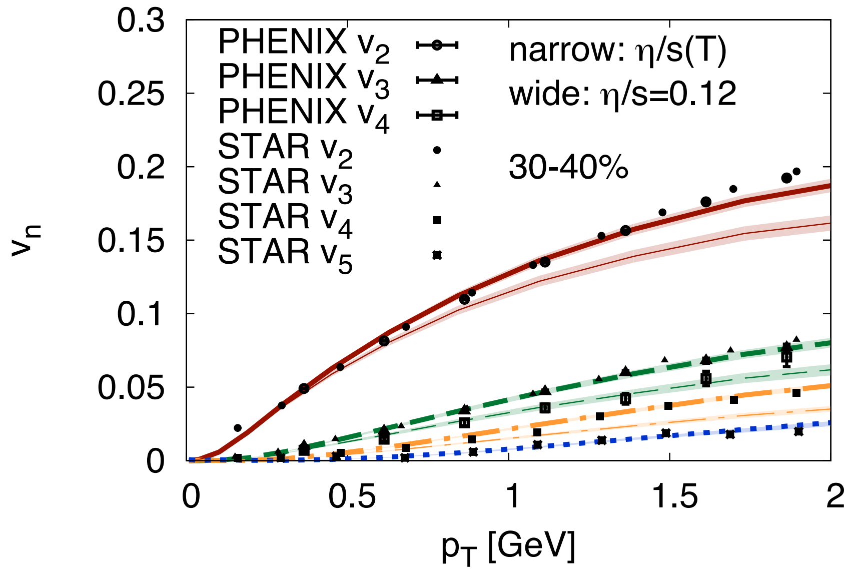
Gale,Jeon,Schenke,Tribedy,Venugopalan, PRL (2013) 012302

Results from the IP-Glasma +MUSIC model:



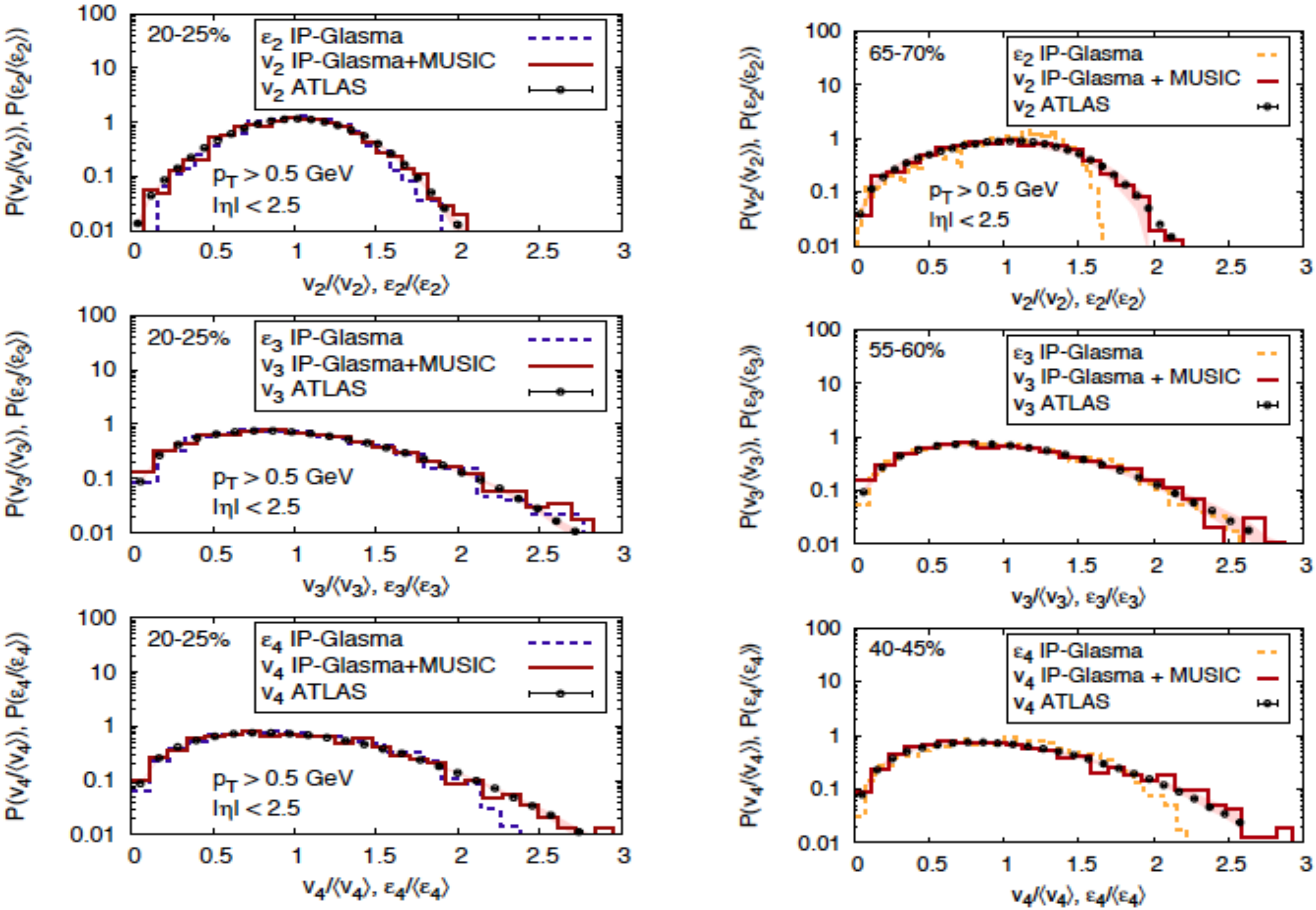






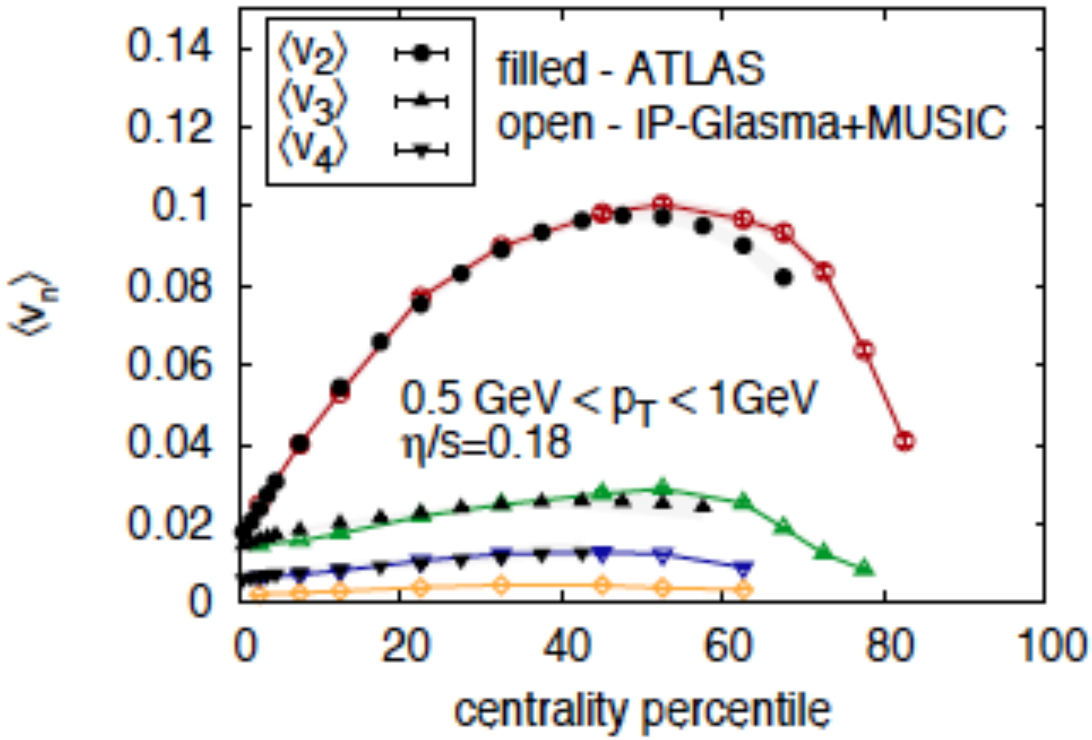
RHIC data require lower average value of η/s relative to LHC

Heavy ion phenomenology in weak coupling



Heavy ion phenomenology in weak coupling

Schenke,Venugopalan, arXiv:1405.3605



Remarkable agreement of IP-Glasma+MUSIC with data out to fairly peripheral overlap geometries…

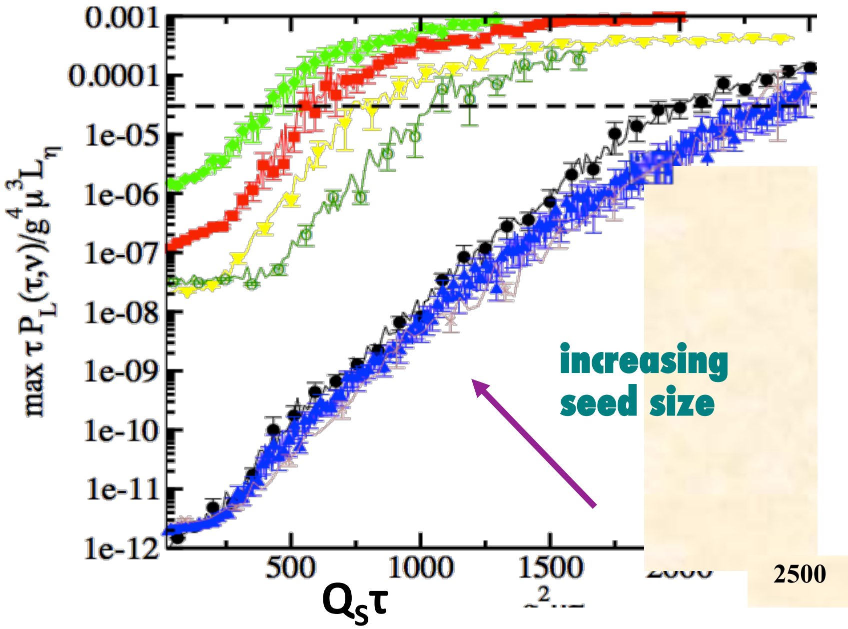
The Glasma at NLO: plasma instabilities

Romatschke,Venugopalan
Dusling,Gelis,Venugopalan
Gelis, Epelbaum

At LO: boost invariant gauge fields $A_{cl}^{\mu,a}(x_T,\tau) \sim 1/g$

NLO: $A^{\mu,a}(xT,\tau,\eta) = A_{cl}^{\mu,a}(x_T,\tau) + a^{\mu,a}(\eta)$

$a^{\mu,a}(\eta) = O(1)$



- Small fluctuations grow exponentially as ~ $e^{\sqrt{Q_S \tau}}$
- Same order of classical field at $\tau = \frac{1}{Q_S} \ln^2 \frac{1}{\alpha_S}$
- Resum such contributions to all orders $(g\, e^{\sqrt{Q_S \tau}})^n$

$$T^{\mu\nu}_{\text{resum}} = \int_{\tau=0^+} [da]\; F_{\text{init.}}[a]\; T_{\text{LO}}[A_{\text{cl}} + a]$$

◆ Systematic Schwinger-Keldysh approach allows one to separate divergences before collision (factorization) from those after…

Gelis,Lappi,RV
Jeon

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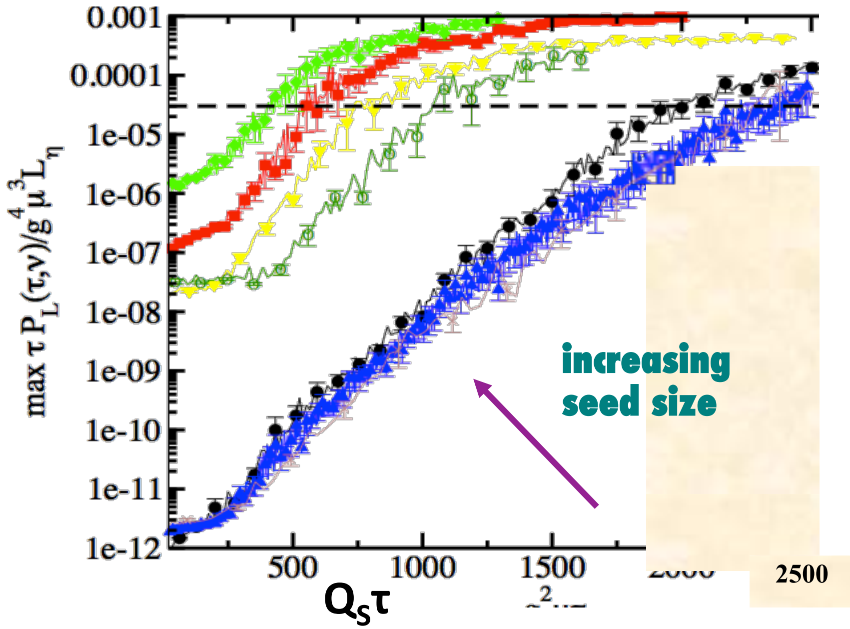

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- Same order of classical field at $\tau = \frac{1}{Q_S}\ln^2\frac{1}{\alpha_S}$
- Resum such contributions to all orders $(g\, e^{\sqrt{Q_S\tau}})^n$

◆ **Leading quantum corrections can be expressed as average over a classical-statistical ensemble of initial conditions**

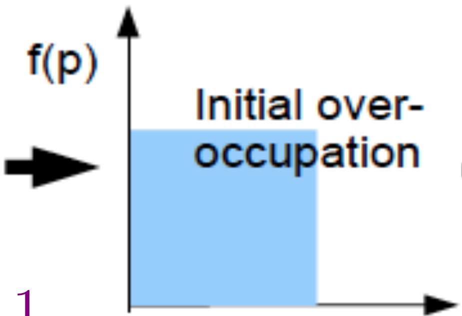
Initial conditions in the Glasma

Glasma + vacuum fluctuations

$$\tau = \frac{1}{Q_S}$$

Plasma Instabilities

$$\tau = \frac{1}{Q_S}\ln^2\frac{1}{\alpha_S}$$



???

$$\tau >> \frac{1}{Q_S}\ln^2\frac{1}{\alpha_S}$$

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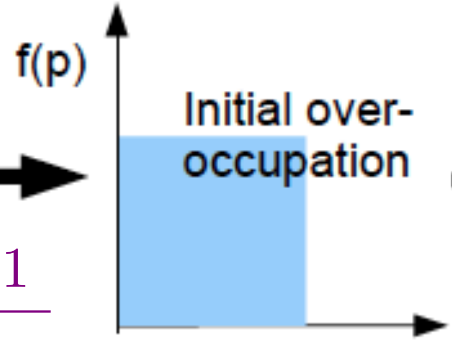
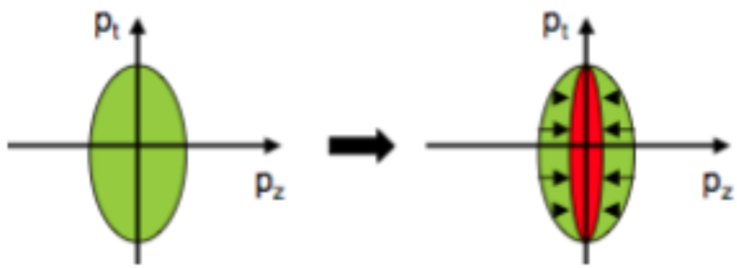


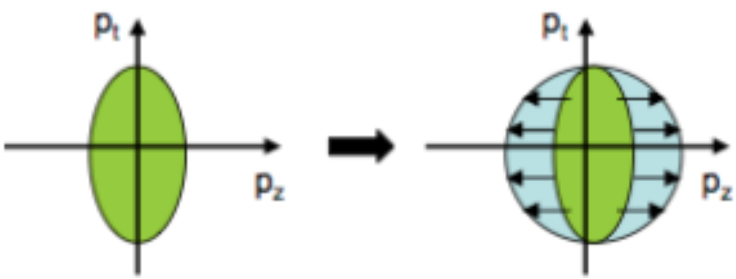
???

$$\tau >> \frac{1}{Q_S} \ln^2 \frac{1}{\alpha_S}$$

- There is a natural ***competition*** between ***interactions*** and the ***longitudinal expansion*** which renders the system ***anisotropic*** on large time scales



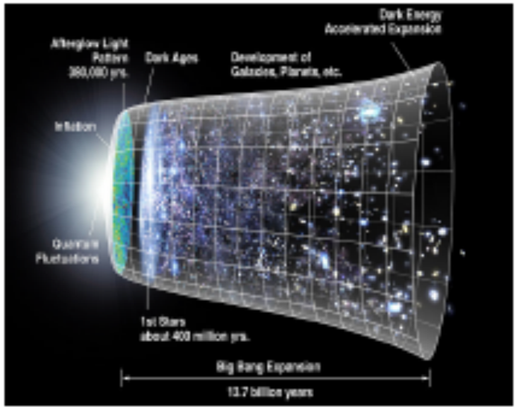
Longitudinal Expansion:
- Red-shift of longitudinal momenta p_z
  $\rightarrow$ increase of anisotropy
- Dilution of the system



Interactions:
- Isotropize the system

Early universe cosmology: turbulent thermalization

Micha,Tkachev, PRD 70 (2004) 043538



Model for early universe thermalization:

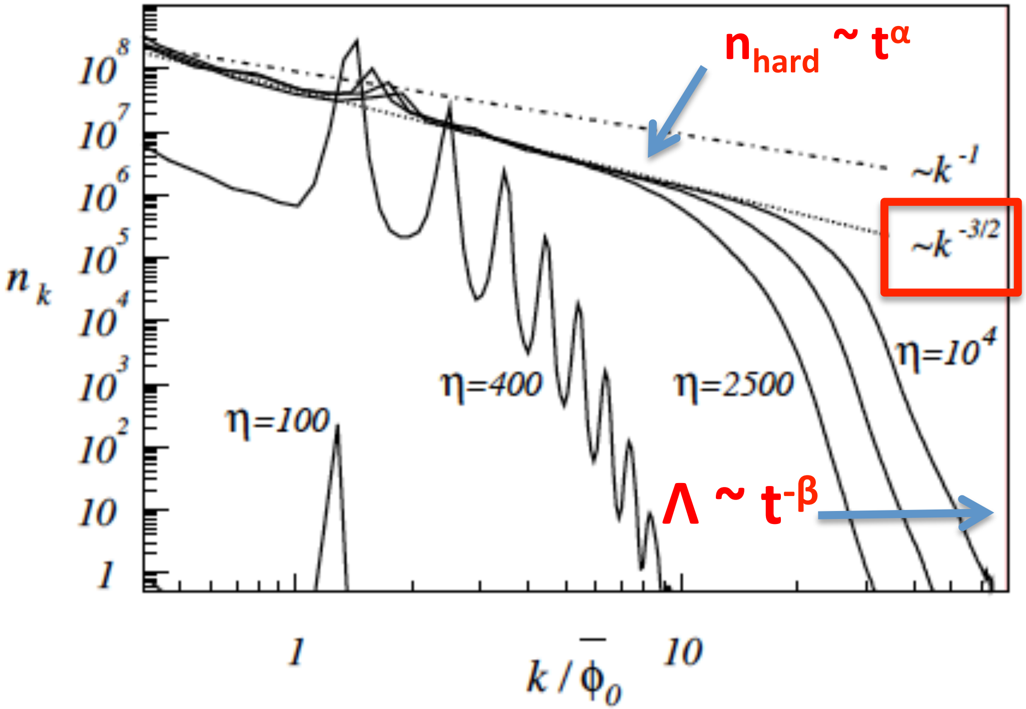
Weakly coupled scalar field theory ($\lambda\Phi^4$) (λ=10^{-8})

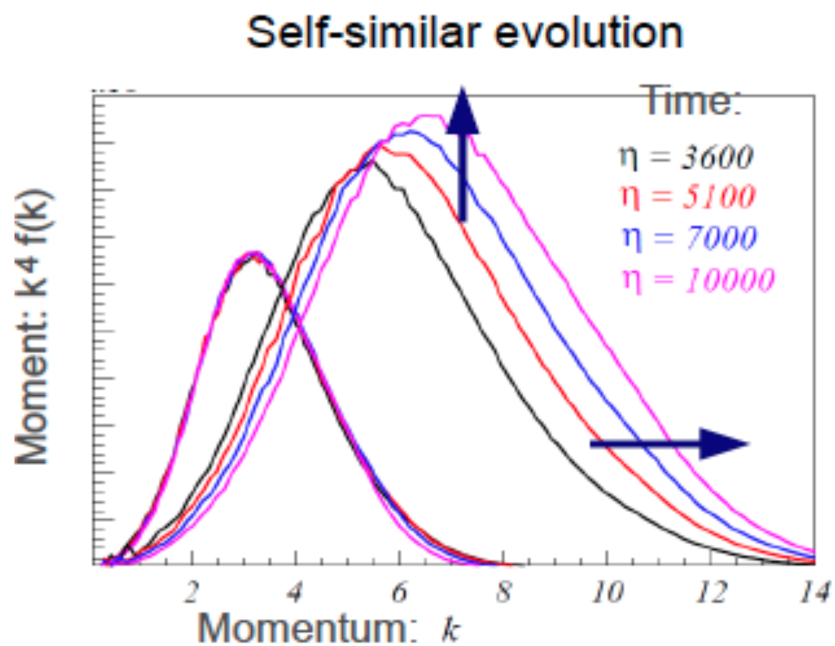
In homogeneous background field $\Phi_0 \sim \frac{1}{\sqrt{\lambda}}$
+ vacuum fluctuations

- **Growth of instabilities via parametric resonance of classical field and vacuum fluctuations**

Turbulent thermalization in Cosmology

Micha,Tkachev, PRD 70 (2004)043538



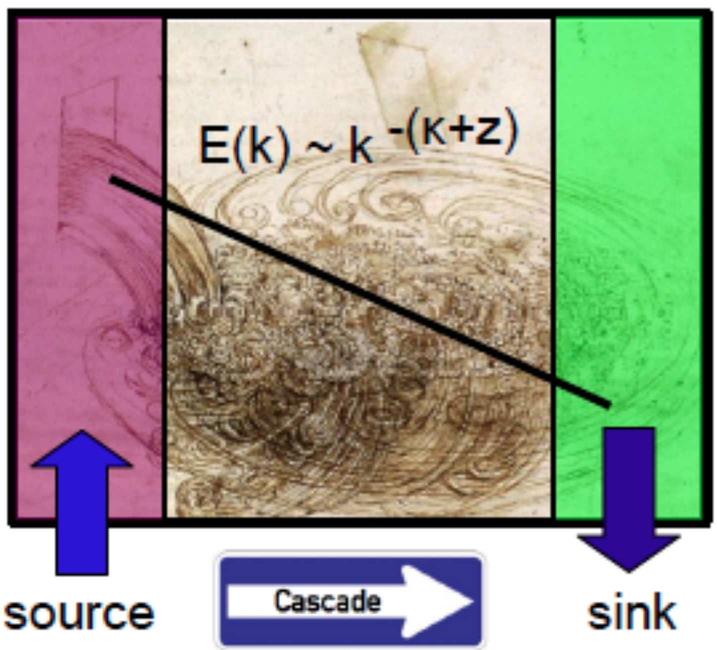


Thermalization process characterized by quasi-stationary evolution with scaling exponents.

Dynamic: α=-4/5, β=-1/5 ; Spectral: κ= -3/2

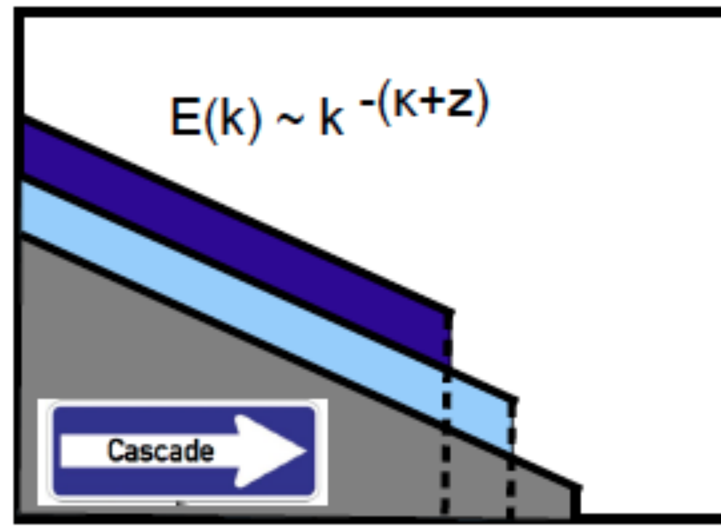
Turbulent thermalization in Cosmology

"Driven" Turbulence –
Kolmogorov wave turbulence



vs.

"Free" Turbulence –
Turbulent Thermalization



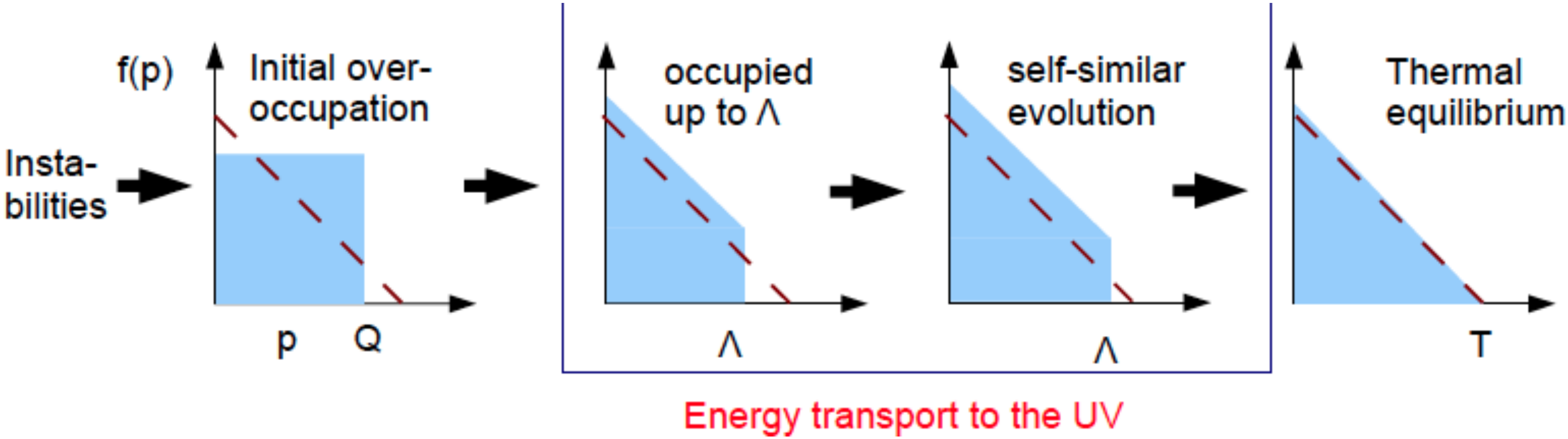
closed system

Free Turbulence:

Quasi-stationary solution: universal, non-thermal spectral exponents

Self-similar evolution with universal dynamical scaling exponents

Kinetic interpretation



Self consistent solutions:

$$f(p,t) = t^{\alpha} f_S(t^{\beta}, p) \qquad \partial_t f(p,t) = C[f](p,t)$$

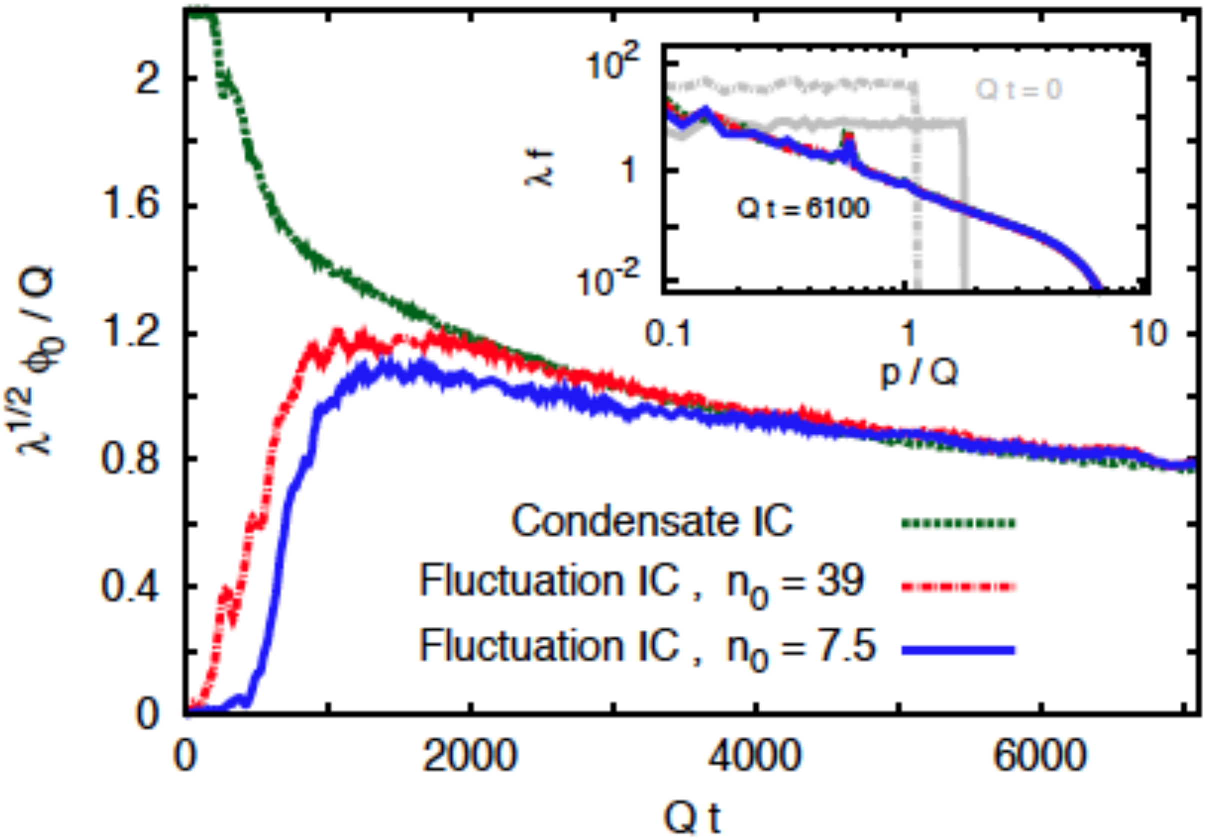
Stationary solution: $C[p_T, p_z, t : f] = t^{\mu}\, C[t^{\beta} p_T, t^{\gamma} p_z; f_S]$

Fixed point equation + scaling solution

$$\alpha f_S + \beta \partial_p f_S = C[f_S] \qquad \alpha - 1 = \mu(\alpha, \beta)$$

Dependence on initial conditions

Berges,Boguslavskii,Schlichting,Venugopalan, arXiv:1312.5216



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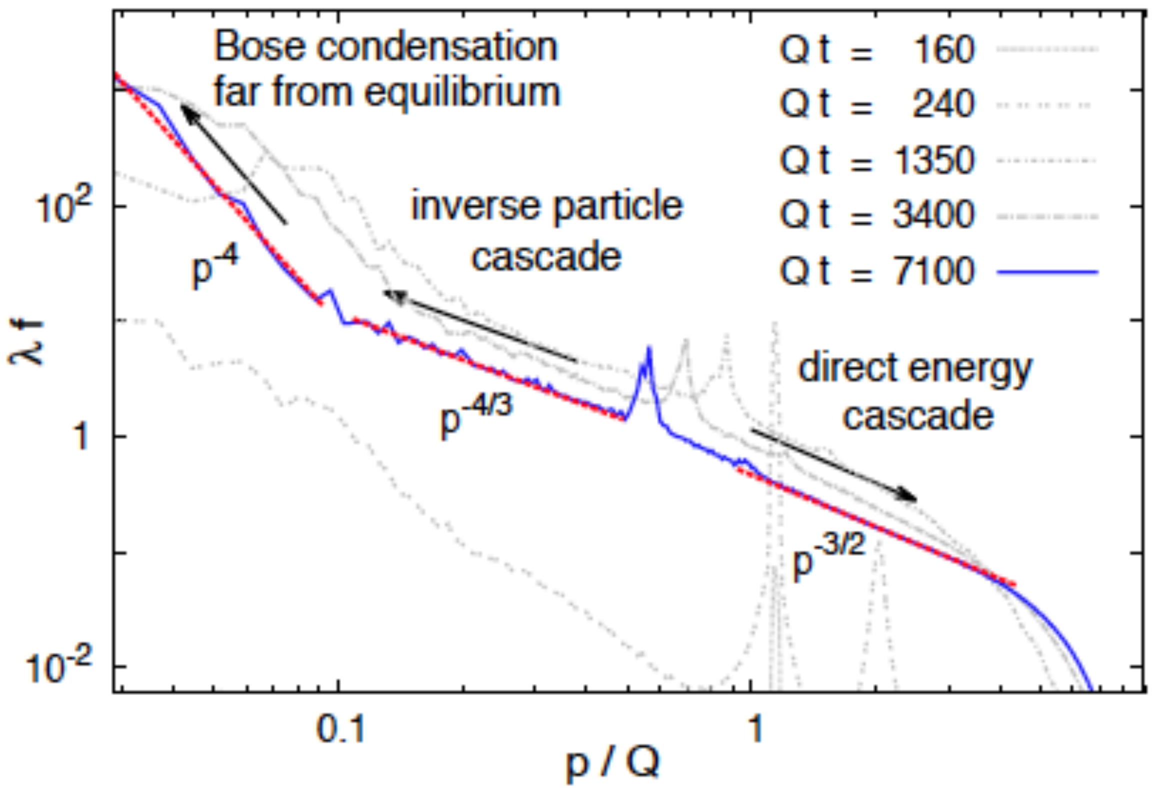


◆ **Non-trivial transport of quantities conserved in different inertial ranges of momenta**

Kinetic interpretation

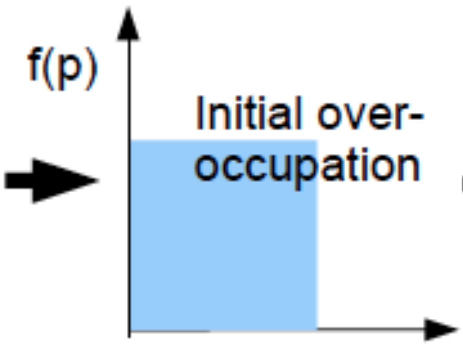
Interaction	Spectral Shape (Exponent κ)	Λ evolution (Exponent β)	Occupancy evolution (Exponent α)
2<->1+soft	3/2	-1/5	-4/5
2<->2	4/3	-1/7	-4/7
2<->3 (gauge theory)	??	-1/7	-4/7

- ***Scalar theory:*** turbulent cascade is driven by ***2<->(1+soft)*** interaction and leads to a ***transient condensate*** formation

(Cosmology: Micha, Tkachev PRD 70 (2004) 043538)

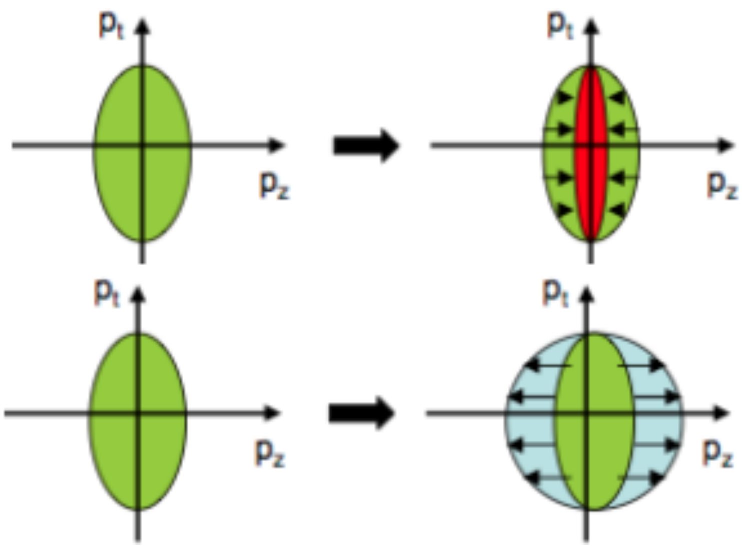
Back to the Glasma…

→ Glasma + vacuum fluctuations → Plasma Instabilities →



→ ???

- There is a natural ***competition*** between ***interactions*** and the ***longitudinal expansion*** which renders the system ***anisotropic*** on large time scales



Longitudinal Expansion:
- Red-shift of longitudinal momenta p_z → increase of anisotropy
- Dilution of the system

Interactions:
- Isotropize the system

Initial conditions in the overpopulated QGP

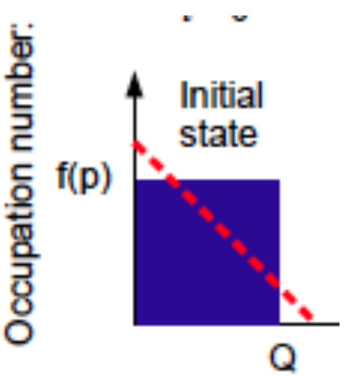
Choose for the initial classical-statistic ensemble of gauge fields

$$A_\nu(\tau,\eta,x_\perp) = \sum_\lambda \int \frac{d^2k_\perp}{(2\pi)^2}\frac{d\nu}{2\pi}\sqrt{f_{k_\perp\nu}+\frac{1}{2}}\left[c^{(\lambda)k_\perp\nu}\,\xi_\mu^{(\lambda)k_\perp\nu+}(\tau)\,e^{ik_\perp x_\perp}\,e^{i\nu\eta} + c^{*(\lambda)k_\perp\nu}\,\xi_\mu^{(\lambda)k_\perp\nu+*}(\tau)\,e^{-ik_\perp x_\perp}\,e^{-i\nu\eta}\right]$$

Stochastic random variables

$$\begin{aligned}
\langle c^{(\lambda)k_\perp\nu}c^{(\lambda')k'_\perp\nu'}\rangle &= 0\,,\\
\langle c^{(\lambda)k_\perp\nu}c^{*(\lambda')k'_\perp\nu'}\rangle &= (2\pi)^3\delta^{\lambda\lambda'}\delta(k-k')\delta(\nu-\nu')\\
\langle c^{*(\lambda)k_\perp\nu}c^{*(\lambda')k'_\perp\nu'}\rangle &= 0.
\end{aligned}$$

Polarization vectors ξ expressed in terms of Hankel functions in Fock-Schwinger gauge $A^\tau = 0$



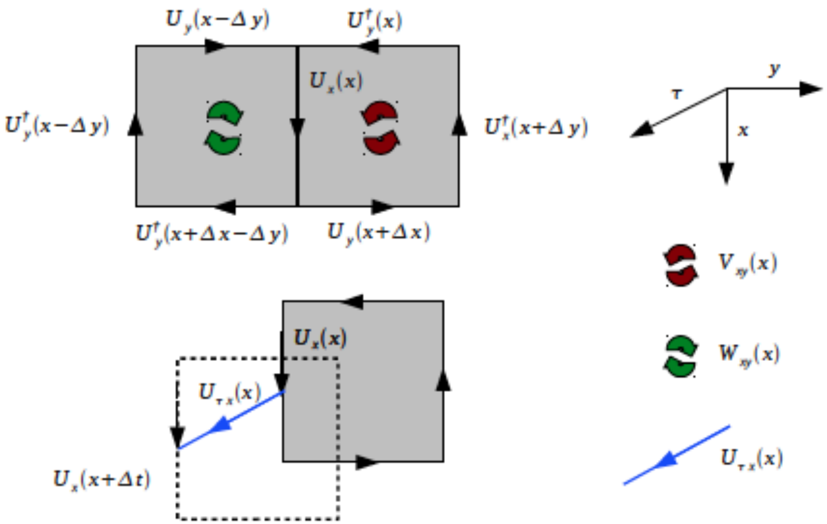
$$f(p_\perp, p_z, t_0) = \frac{n_0}{\alpha_S}\,\Theta\left(Q - \sqrt{p_\perp^2 + (\xi_0 p_z)^2}\right)$$

Controls "prolateness" or "oblateness" of initial momentum distribution

Temporal evolution in the overpopulated QGP

Berges,Boguslavski,Schlichting,Venugopalan
arXiv: 1303.5650, 1311.3005

Solve Hamilton's equation for 3+1-D SU(2) gauge theory in Fock-Schwinger gauge



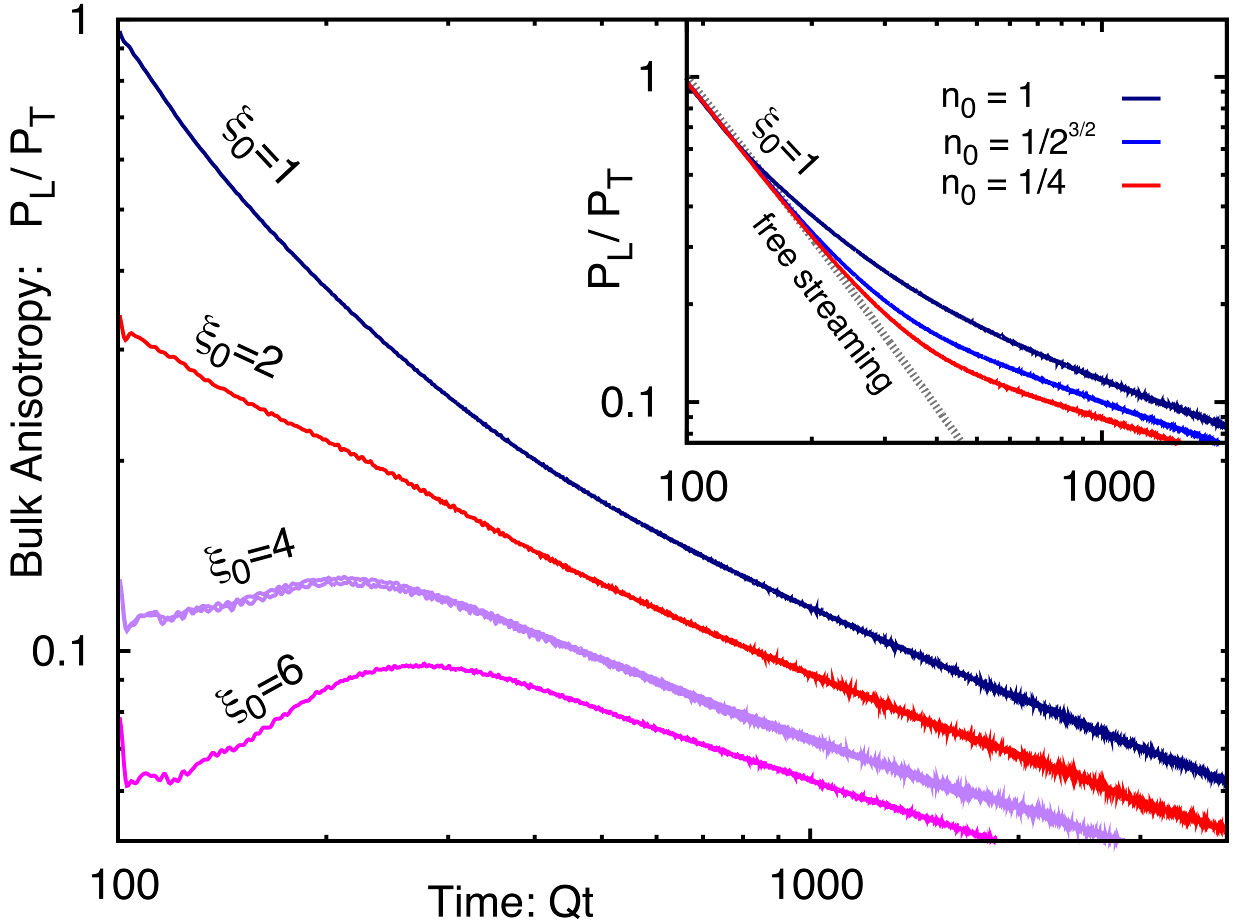
Fix residual gauge freedom imposing Coloumb gauge at each readout time

$$\partial_i A_i + t^{-2}\partial_\eta A_\eta = 0$$

- Largest classical-statistical numerical simulations of expanding Yang-Mills to date: $256^2 \times 4096$ lattices

- Classical-statistical computations performed at very weak coupling $\alpha_S = 10^{-5}$ for "classical dominance" at all times in simulation:
-- corresponds to $Q\tau_0 \approx \mathrm{Ln}^2(1/\alpha_S) \approx 100$

Result: pressure become increasingly anisotropic



P_L/P_T approaches universal $\tau^{-2/3}$ behavior

Hard scales show universal scaling

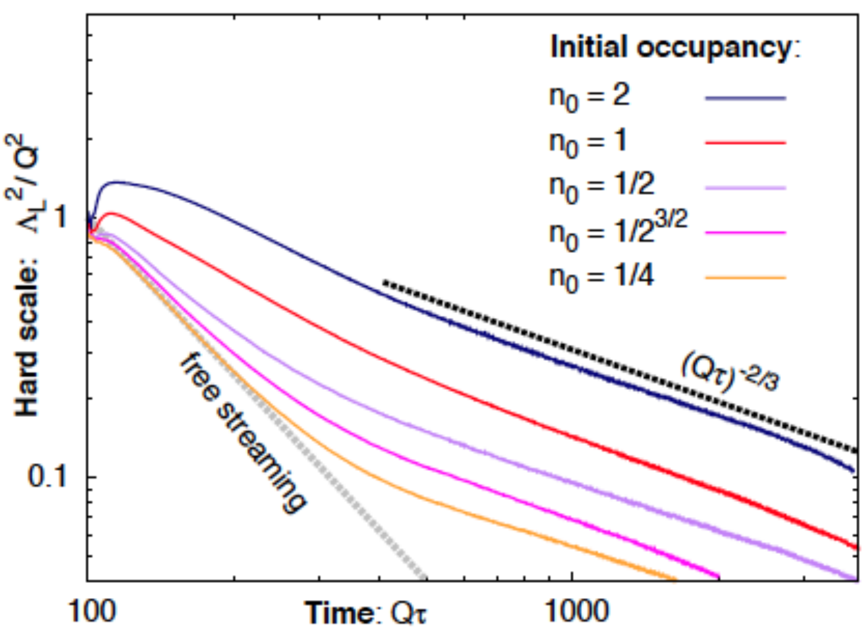
Gauge invariant quantities: simple quasi-particle interpretation for weak fields

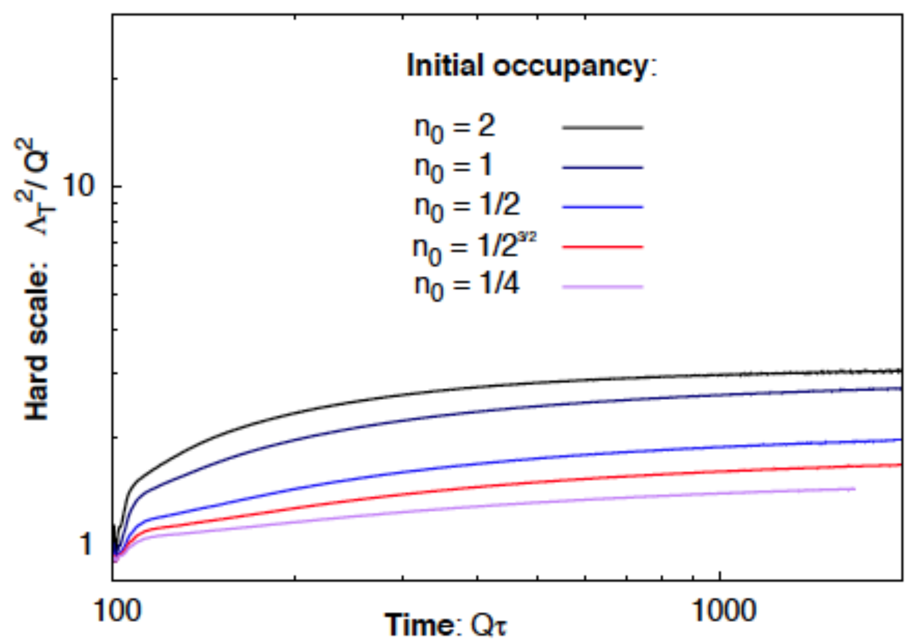
Kurkela,Moore

$$\Lambda_L^2(t) \sim (Qt)^{-2\gamma}$$

$$\Lambda_T^2(t) \sim (Qt)^{-2\beta}$$

$$\Lambda_{T,L}^2(t) \simeq \frac{\int d^2p_T \int dp_z \; p_{T,z}^2 \, \omega_p f(p_T, p_z, t)}{\int d^2p_T \int dp_z \; \omega_p f(p_T, p_z, t)}$$





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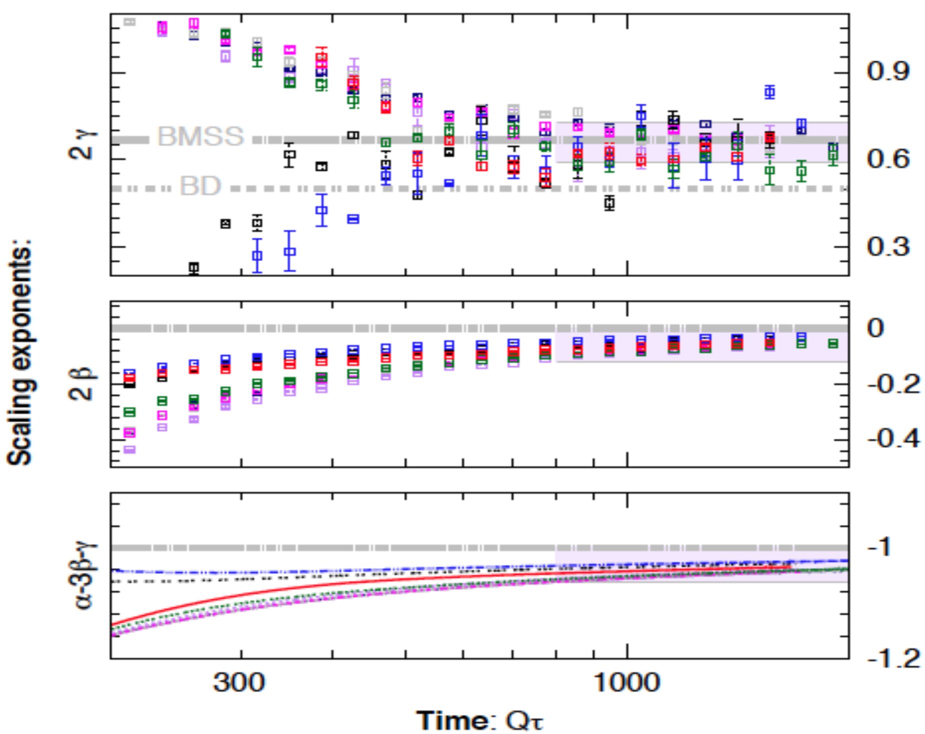

$$\alpha \sim -2/3$$

$$|\beta| < 0.06 \; ; \; 2\gamma = 0.67 \pm 0.07$$

β = 0 suggests no shift of hard transverse momentum scale in time

γ = 1/3 system longitudinal hard scale red-shifts but slower than free streaming value of γ=1

Kinetic theory in the overoccupied regime

For $1 < f < 1/\alpha_S$ a dual description is feasible either in terms of kinetic theory or classical-statistical dynamics …

Mueller,Son (2002)
Jeon (2005)

Different scenarios:

- ❑ Elastic multiple scattering dominates in the Glasma

  BMSS: Baier,Mueller,Schiff,Son

- ❑ Rescattering influenced by plasma (Weibel) instabilities

  KM: Kurkela, Moore

- ❑ Transient Bose condensation+multiple scattering

  BGLMV: Blaizot,Gelis,Liao,McLerran,Venugopalan

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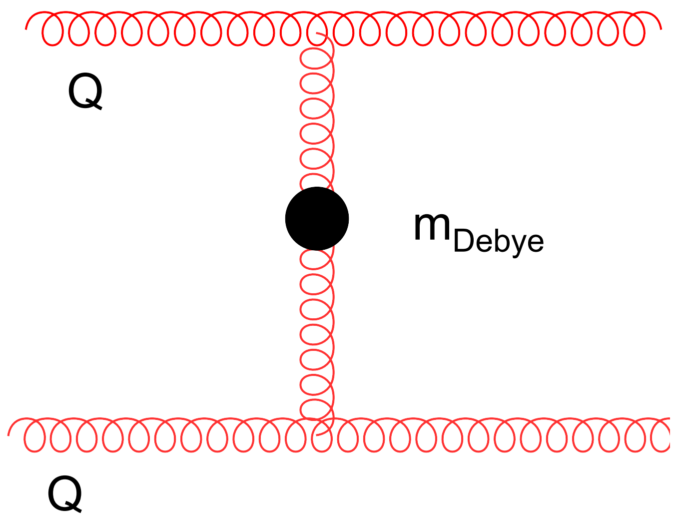
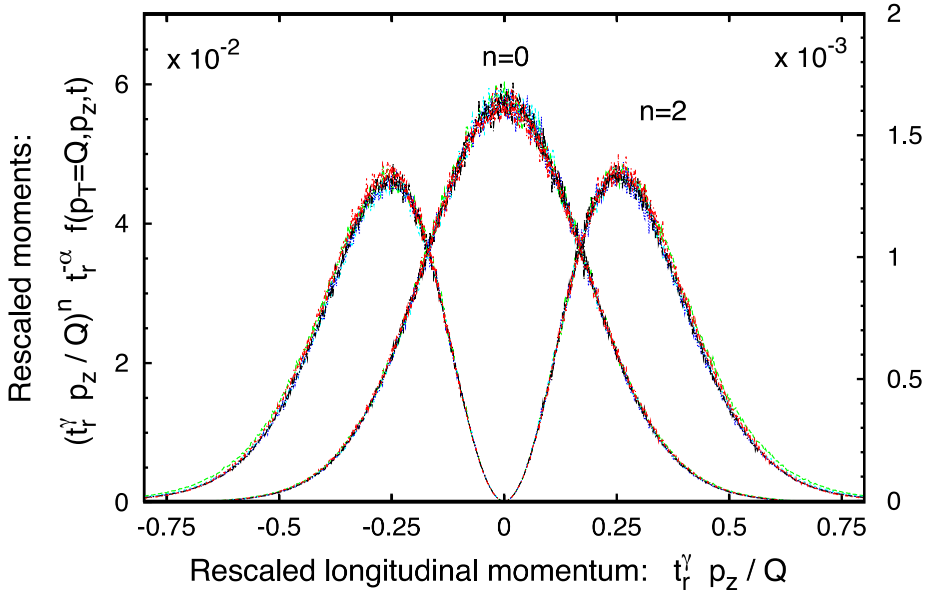
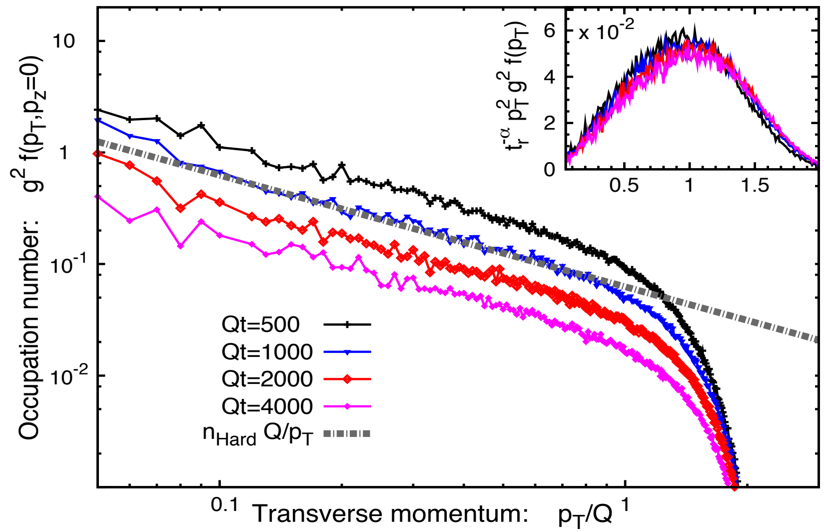


- **Differences arise due to assumptions of non-perturbative behavior at $p \approx m_{Debye}$**

Result: universal non-thermal fixed point

Conjecture: $f(p_\perp, p_z, t) = t^\alpha \; f_S(t^\beta p_T, t^\gamma p_z)$





Moments of distribution extracted over range of time slices lie on universal curves

Distribution as function of p_T displays 2-D thermal behavior

Kinetic interpretation of self-similar behavior

Follow wave turbulence kinetic picture of Zakharov, as developed by Micha & Tkachev in context of inflation

$$\left[\partial_t - \frac{p_z}{t}\partial_{p_z}\right] f(p_T, p_z, t) = C[p_T, p_z, t; f]$$

$$C[p_T, p_z, t; f] = t^{\mu}\, C[t^{\beta} p_T, t^{\gamma} p_z; f_S]$$

Fixed point solution satisfies

$$\alpha f_S(p_T, p_z) + \beta p_T \partial_{p_T} f_S(p_T, p_z) + (\gamma - 1)\, p_z \partial_{p_z} f_S(p_T, p_z) = C[p_T, p_z; f_S]$$

μ=α-1

If we assume that small angle elastic scattering dominates

$$C^{(\text{elast})}[p_T, p_z; f] = \hat{q}\ \partial^2_{p_z} f(p_T, p_z, t)$$

$$\hat{q} \sim \alpha_S^2 \int \frac{d^2 p_T}{(2\pi)^2} \int \frac{dp_z}{2\pi}\, f^2(p_T, p_z, t)$$

Kinetic interpretation of self-similar behavior

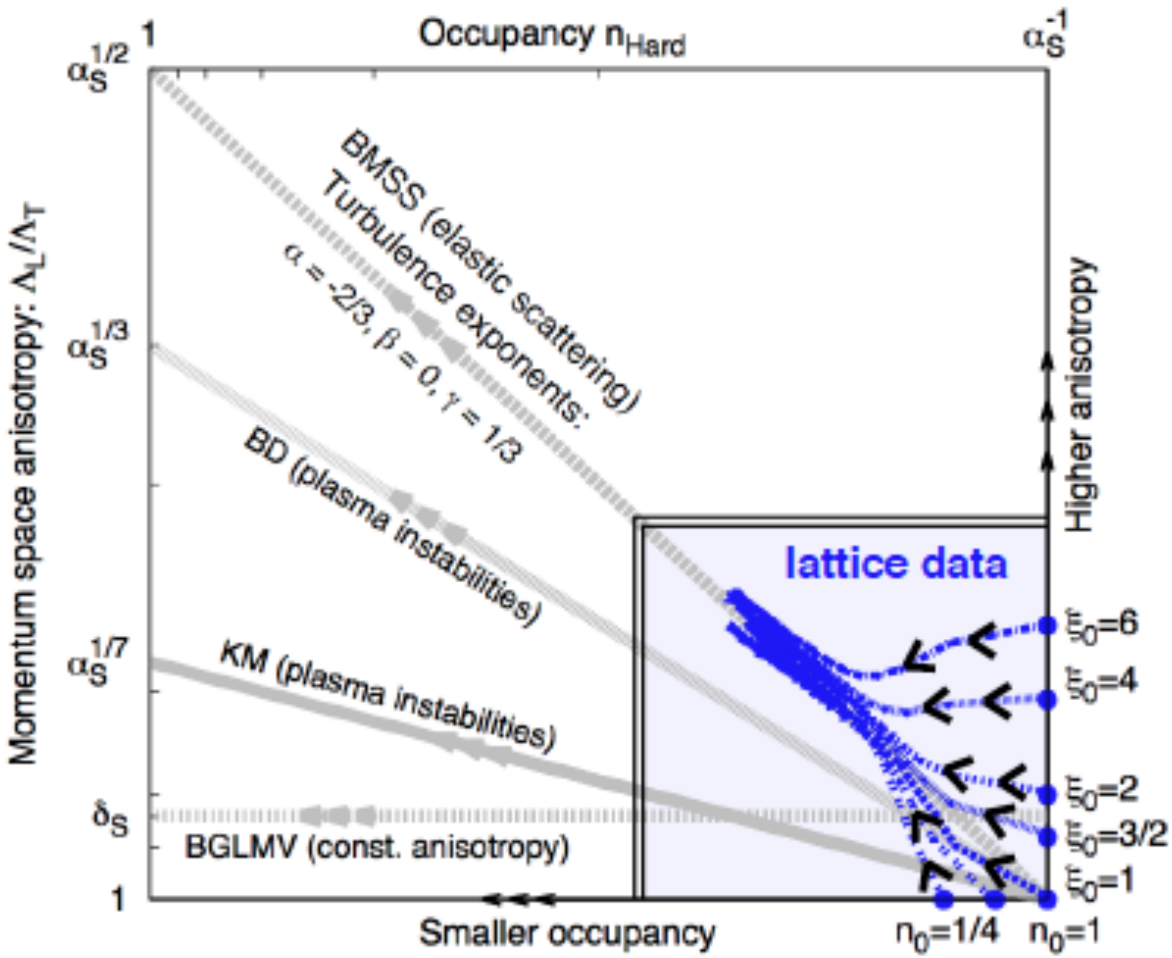
For self-similar scaling solution, a) small angle elastic scattering
b) energy conservation
c) number conservation

Give unique results: $\alpha = -2/3, \quad \beta = 0, \quad \gamma = 1/3$

- **These are the same exponents (within errors) extracted from our numerical simulations !**

- **The same exponents appear in the "bottom-up" thermalization scenario of Baier, Mueller, Schiff, Son (BMSS)**

Non-thermal fixed point in overpopulated QGP



KM: Kurkela, Moore
BGLMV: Blaizot,Gelis,Liao,McLerran,Venugopalan

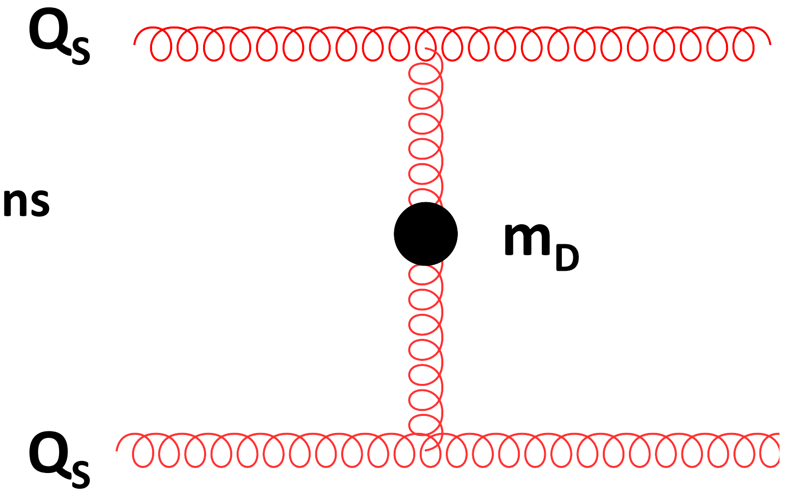
Quo vadis, thermal QGP?

The “bottom-up” scenario:

Baier,Mueller,Schiff,Son

Scale for scattering of produced gluons ($t > 1/Q_S$) set by

$$m_D^2 \propto g^2 \int_{\vec{p}} \frac{f_{\rm hard}}{p}$$



Build up p_z (which fights the red shift of $p_z \sim 1/t$) with mult. scattering

$$p_z^2 = N_{\rm coll.}\, m_D^2 \qquad \Rightarrow \qquad p_z \sim \frac{Q_s}{(Q_s \tau)^{1/3}}$$

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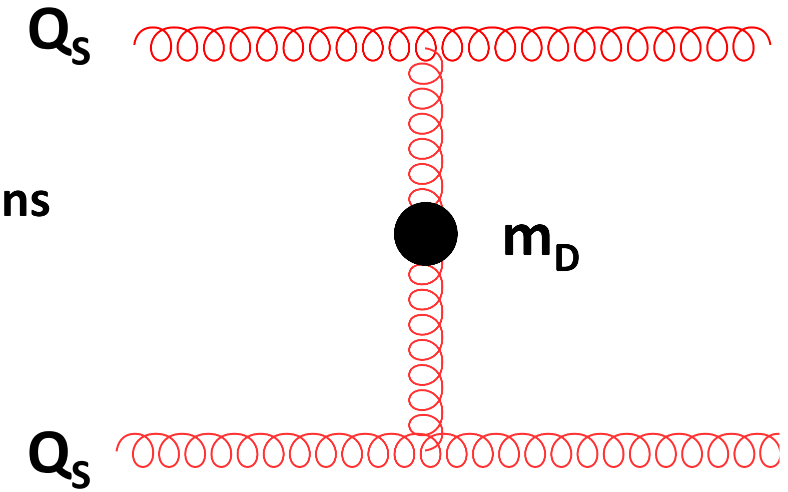

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Our simulations support this picture

– no significant role of instabilities -- imaginary component in m_D

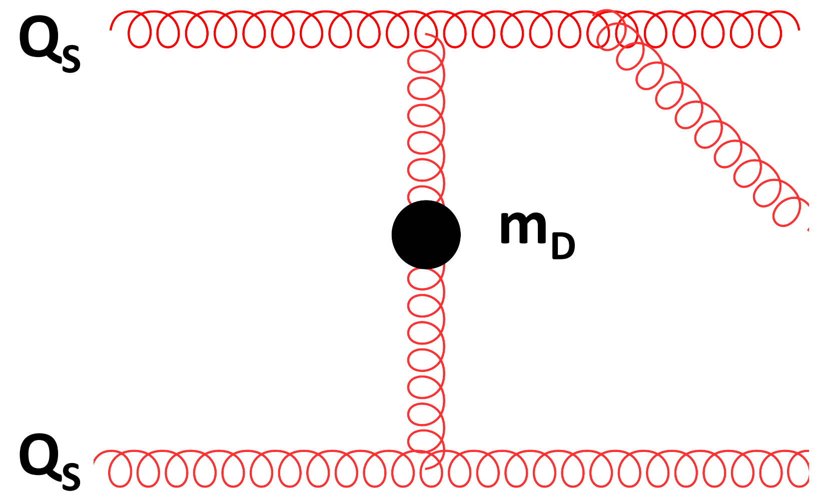
(conjectured by Arnold, Moore, Lenaghan and Yaffe to spoil the bottom up scenario)

Quo vadis, thermal QGP?

Occupation # $f = \frac{1}{p_z Q_s^2} \frac{Q_s^3}{\alpha_S(Q_s \tau)}$

$$f < 1 \text{ for } \tau > \frac{1}{\alpha_S^{3/2}} \frac{1}{Q_s}$$

Classical statistical simulations break down in this regime

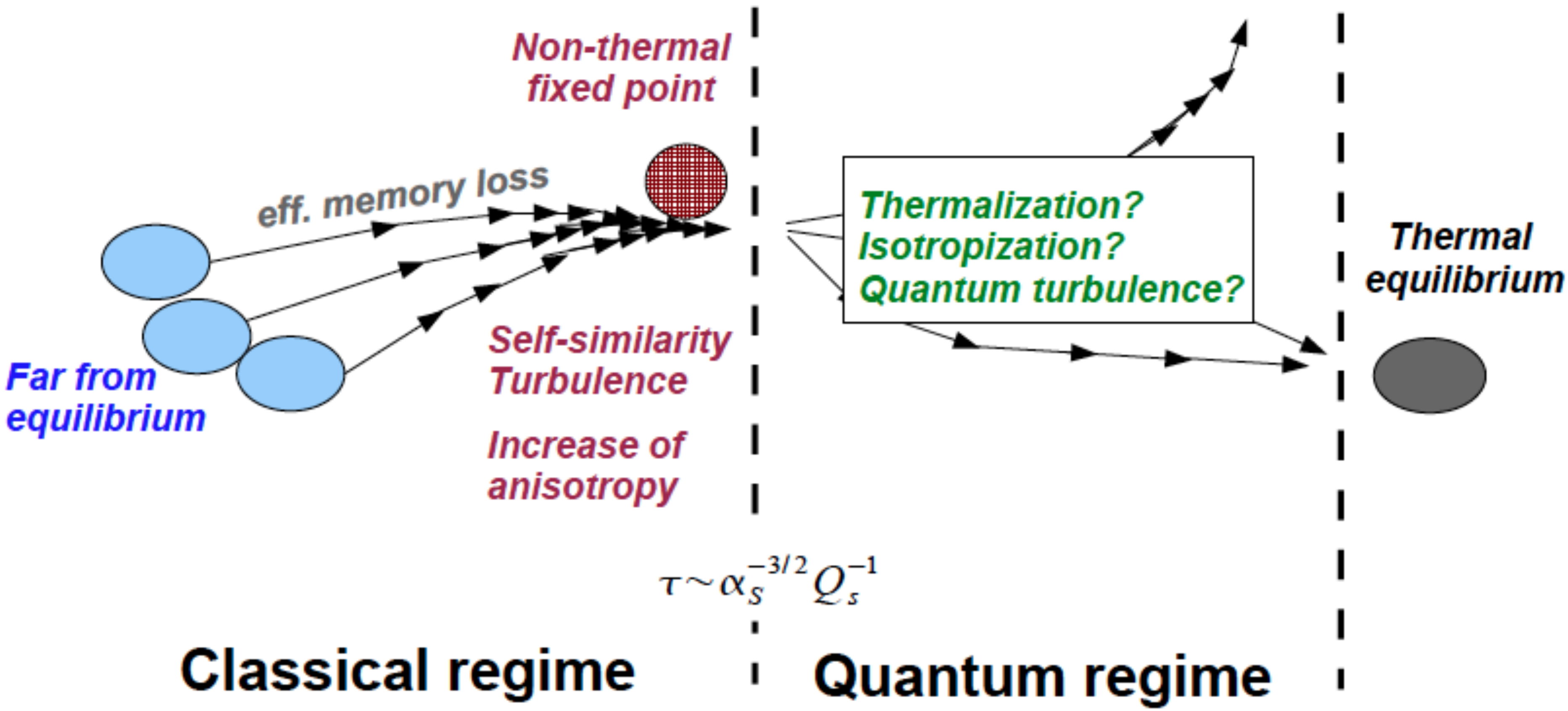


In quantum regime, thermalization proceeds through number changing inelastic processes

Thermalized soft bath of gluons for $\tau > \frac{1}{\alpha_S^{5/2}} \frac{1}{Q_S}$

Thermalization temperature of $T_i = \alpha_S^{2/5} Q_S$

Quo vadis, thermal QGP?



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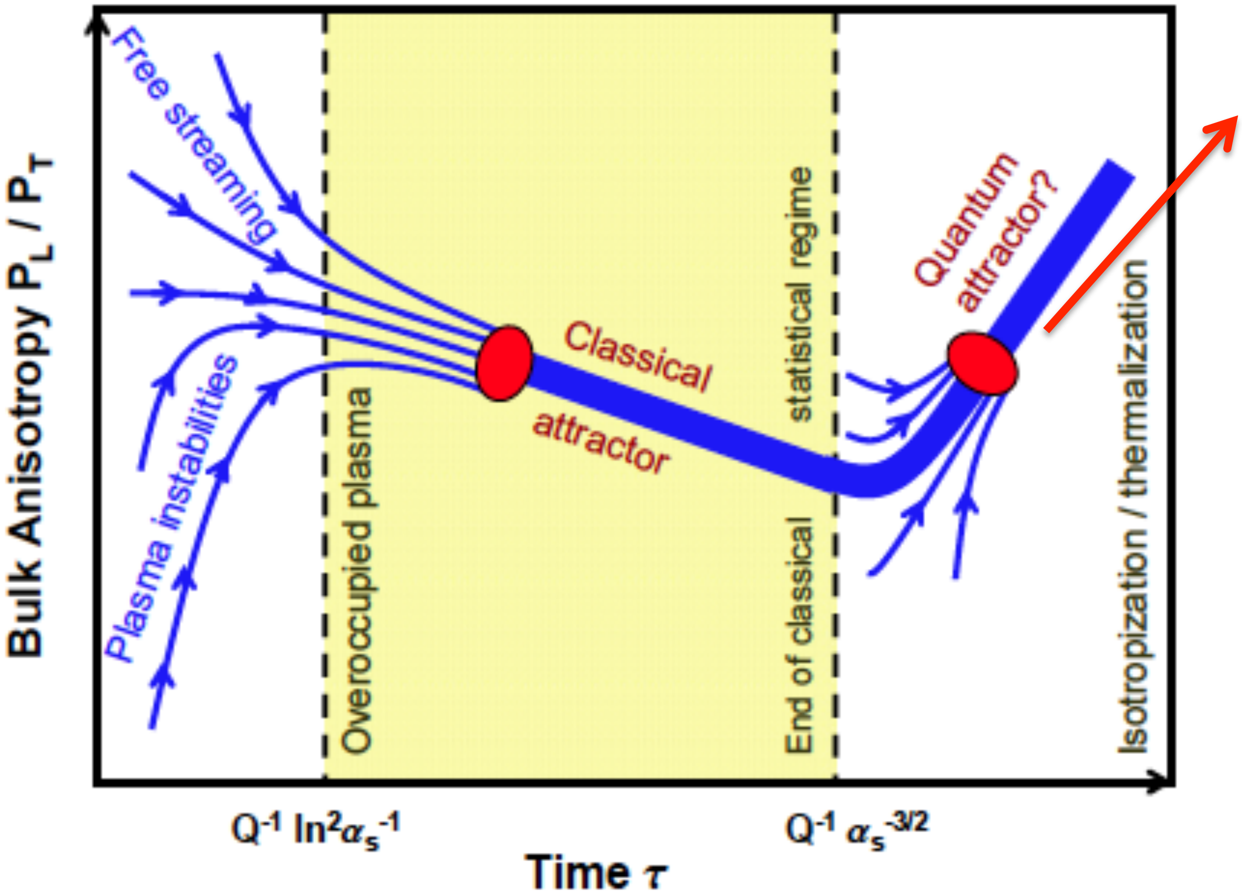


Blaizot,Dominguez, Iancu,Mehtar-Tani

Recent kinetic theory computations by Kurkela and Lu suggest that thermalization can occur between 0.2 and 1 fm for Q_S = 2 GeV and α_S between 0.3 and 0.2 respectively

Kurkela,Lu:1405.6318

Proof in principle of universality

Universality:

Dynamics in the vicinity of the attractor is independent of the details of the theory and is governed by universal numbers that depend on

a) dimensionality of the theory

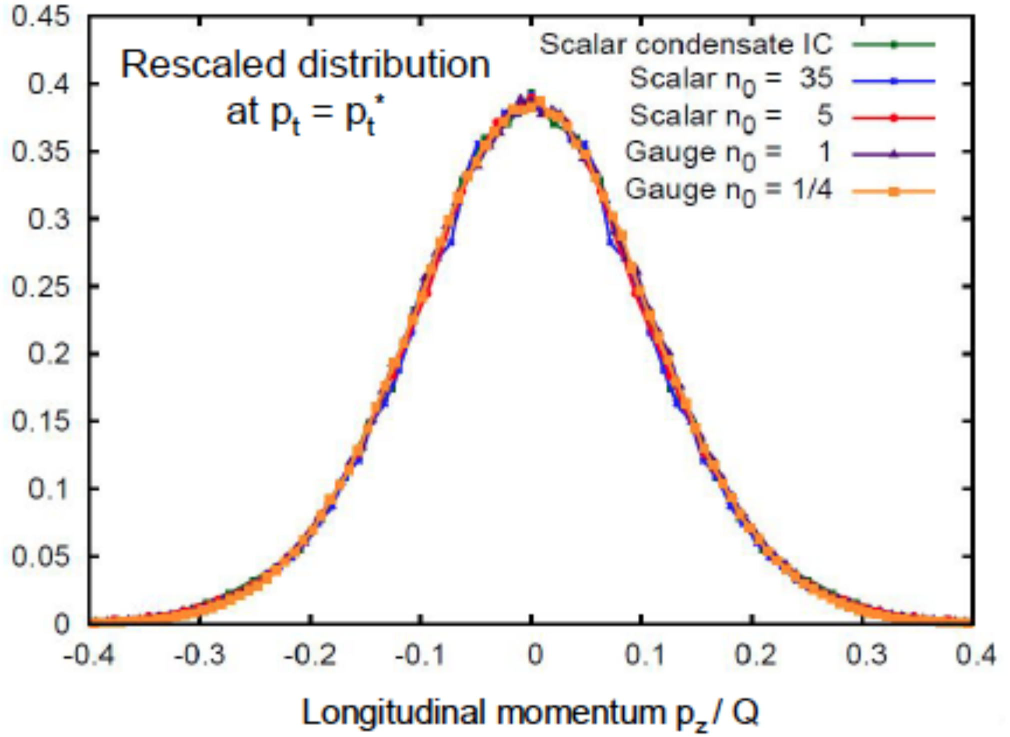
b) generic features of the scattering,

eg: 2 <-> 2 , 2 <-> 3, 2 <-> 1 + condensate

c) Boundary conditions

Proof in principle of universality

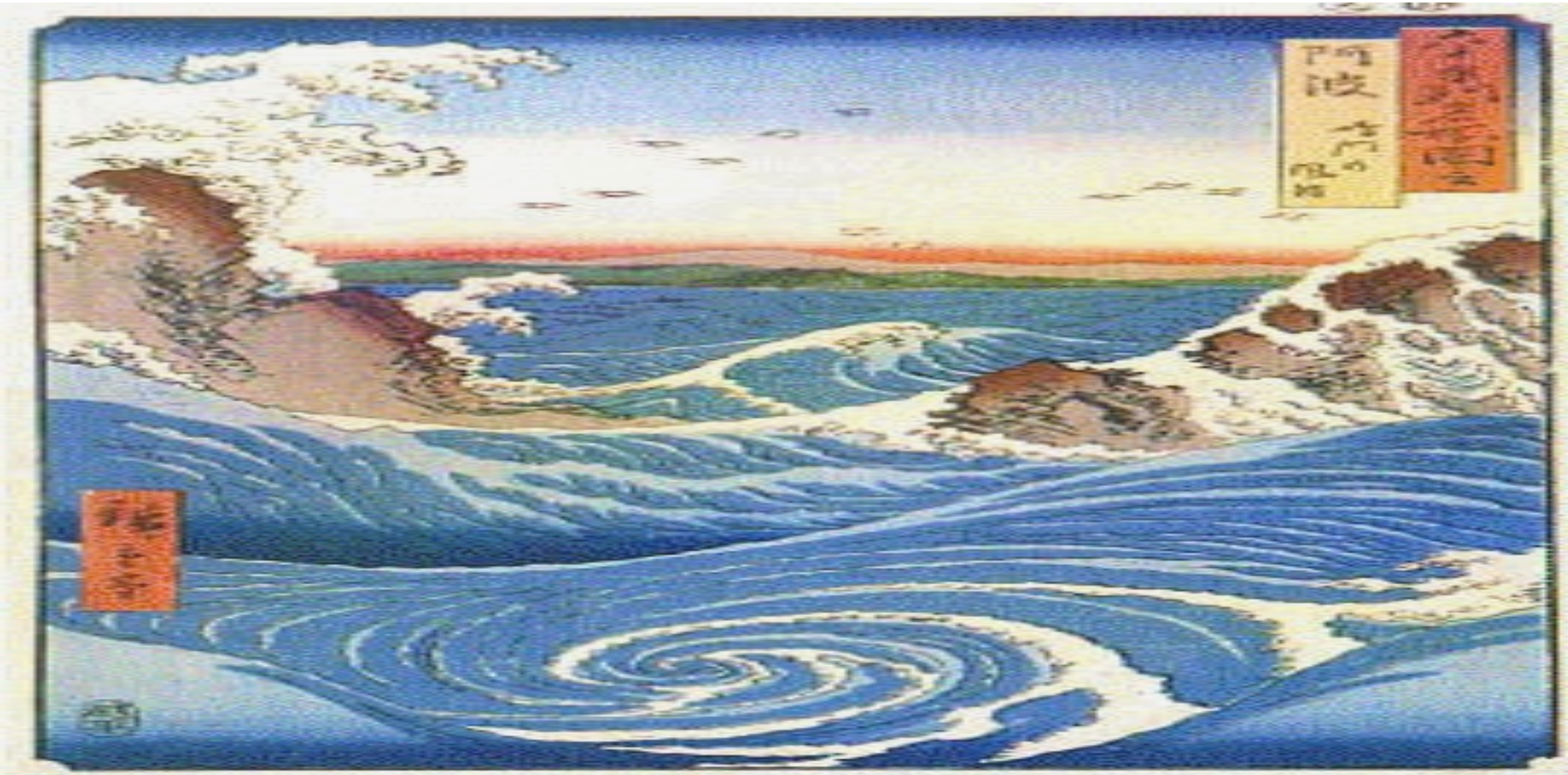
Berges,Boguslavski,Schlichting,Venugopalan,
In preparation



Rescaled distributions of expanding scalar and expanding gauge theory lie on top of each other !

– stationary distributions with identical scaling exponents…

Universal non-thermal attractor in QCD



“Big whorls have little whorls, which feed on their velocity,
And little whorls have lesser whorls, and so on to viscosity.”

Turbulence: a preamble

`I am an old man now, and when I die and go to heaven there are two matters on which I hope for enlightenment. One is quantum electrodynamics, and the other is the turbulent motion of fluids. And about the former I am rather optimistic.'
- Horace Lamb