Towards a Pion Generalized Parton Distribution Model from Dyson-Schwinger Equations

C. Mezrag

CEA Saclay - IRFU/SPhN

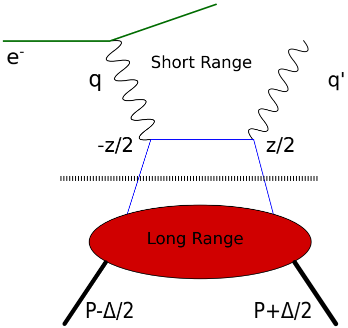
3rd July 2014

In collaboration with
H. Moutarde, C. Roberts, J. Rodriguez-Quintero, F. Sabatié

Based on : *Mezrag et al., arXiv 1406.7425*.

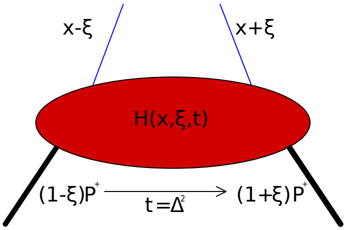
GPDs and Observables

Deep-Virtual Compton Scattering (DVCS)



- Fourier transform of a matrix element,
- contains non pertubative information on the 3D hadron structure,
- are *universal*, *i.e.* independent of the considered process.

GPDs and Observables



- Fourier transform of a matrix element,
- contains non pertubative information on the 3D hadron structure,
- are *universal*, *i.e.* independent of the considered process.
- GPD will be denoted by H,
- and depends on 3 variables : x, ξ, t.

Spinless meson GPD

$$H(x,\xi,t) = \frac{1}{2}\int \frac{\mathrm{d}z^-}{2\pi}\, e^{ixP^+z^-} \langle P+\frac{\Delta}{2}|\bar{q}\left(-\frac{z}{2}\right)\gamma^+\left[-\frac{z}{2};\frac{z}{2}\right] q\left(\frac{z}{2}\right)|P-\frac{\Delta}{2}\rangle_{z^+=0,z_\perp=0}$$

X. Ji,1997
D. Müller et al.,1994
A. Radyushkin,1997

Spinless meson GPD

$$H(x,\xi,t) = \frac{1}{2}\int \frac{\mathrm{d}z^-}{2\pi} e^{ixP^+z^-} \langle P+\frac{\Delta}{2}|\bar{q}\left(-\frac{z}{2}\right)\gamma^+\left[-\frac{z}{2};\frac{z}{2}\right]q\left(\frac{z}{2}\right)|P-\frac{\Delta}{2}\rangle_{z^+=0,z_\perp=0}$$

X. Ji,1997
D. Müller et al.,1994
A. Radyushkin,1997

Problem

Modeling such a matrix element is still an open problem today.

Current GPDs models

- The most popular models of GPDs are based on the **Double Distributions** (DD) F and G (see Cédric's lectures):

$$H(x,\xi,t) = \int_{|\alpha|+|\beta|\leq 1} \mathrm{d}\alpha\ \mathrm{d}\beta (F(\beta,\alpha,t) + \xi G(\beta,\alpha,t))\delta(x-\beta-\xi\alpha)$$

Current GPDs models

- The most popular models of GPDs are based on the **Double Distributions** (DD) F and G (see Cédric's lectures):

$$H(x,\xi,t) = \int_{|\alpha|+|\beta|\leq 1} \mathrm{d}\alpha\ \mathrm{d}\beta(F(\beta,\alpha,t)+\xi G(\beta,\alpha,t))\delta(x-\beta-\xi\alpha)$$

- F and G are based on the Radyushkin Double Distribution Ansatz.

Current GPDs models

- The most popular models of GPDs are based on the **Double Distributions** (DD) F and G (see Cédric's lectures):

$$H(x,\xi,t) = \int_{|\alpha|+|\beta|\leq 1} \mathrm{d}\alpha \, \mathrm{d}\beta (F(\beta,\alpha,t) + \xi G(\beta,\alpha,t))\delta(x-\beta-\xi\alpha)$$

- F and G are based on the Radyushkin Double Distribution Ansatz.
- Advantage : simple.

Current GPDs models

- The most popular models of GPDs are based on the **Double Distributions** (DD) F and G (see Cédric's lectures):

$$H(x,\xi,t) = \int_{|\alpha|+|\beta|\leq 1} \mathrm{d}\alpha\ \mathrm{d}\beta (F(\beta,\alpha,t) + \xi G(\beta,\alpha,t))\delta(x-\beta-\xi\alpha)$$

- F and G are based on the Radyushkin Double Distribution Ansatz.
- Advantage : simple.
- Drawback : rigid.
  Mezrag et al, Phys. Rev. D88

Current GPDs models

- The most popular models of GPDs are based on the **Double Distributions** (DD) F and G (see Cédric's lectures):

$$H(x,\xi,t) = \int_{|\alpha|+|\beta|\leq 1} \mathrm{d}\alpha\ \mathrm{d}\beta (F(\beta,\alpha,t) + \xi G(\beta,\alpha,t))\delta(x-\beta-\xi\alpha)$$

- F and G are based on the Radyushkin Double Distribution Ansatz.
- Advantage : simple.
- Drawback : rigid.
  Mezrag et al, Phys. Rev. D88

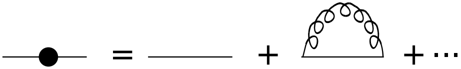
How to go beyond the Radyushkin Ansatz in the DD formalism?

Dyson-Schwinger Equations

- Equations between the different non pertubative Green functions, *i.e.* vertices and propagators.
- Knowing all the non pertubative Green functions means that you have solved your problem entirely.
- Infinite number of coupled equations. Until know, no one have solved that!
- $\rightarrow$ different approximation schemes.

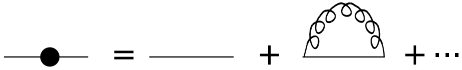
Example : the quark propagator

Pertubative case :



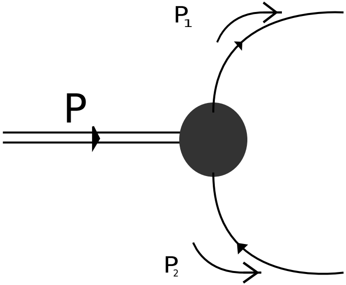
Example : the quark propagator

Pertubative case :



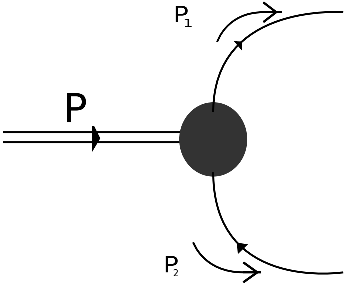
Schwinger-Dyson case :

The pion? But why?



- Advantages:
  - Two body system.
  - Pseudo-scalar meson.
  - Valence quarks u and d.
  - Isospin symmetry.
- Drawbacks:
  - Few experimental data available.
  - No data at $\xi \neq 0$
    $\rightarrow$ One can only compare the model to data at $\xi = 0$, *i.e.* to the Parton Distribution Function (PDF) and to the form factor.
- But :
  Amrath et al., Eur. Phys. J. C58 179

The pion? But why?



Good starting point before dealing with more complex objects.

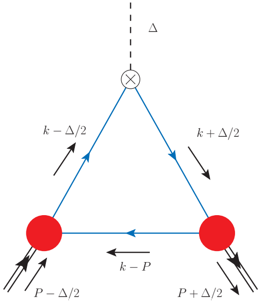
- Advantages:
  - Two body system.
  - Pseudo-scalar meson.
  - Valence quarks u and d.
  - Isospin symmetry.
- Drawbacks:
  - Few experimental data available.
  - No data at $\xi \neq 0$
    $\rightarrow$ One can only compare the model to data at $\xi = 0$, *i.e.* to the Parton Distribution Function (PDF) and to the form factor.
- But :
  Amrath et al., Eur. Phys. J. C58 179

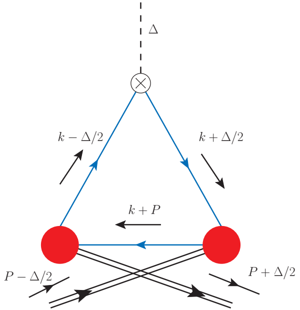
Mellin moments

$$\begin{aligned}\mathcal{M}_m(\xi,t) &= \int_{-1}^{1} \mathrm{d}x\ x^m\ H(x,\xi,t) \\ &= \frac{1}{2(P\cdot n)^{m+1}} \left\langle \pi, P+\frac{\Delta}{2} \left| \bar{\psi}(0)\gamma\cdot n(i\overleftrightarrow{D}\cdot n)^m\psi(0) \right| \pi, P-\frac{\Delta}{2} \right\rangle .\end{aligned}$$

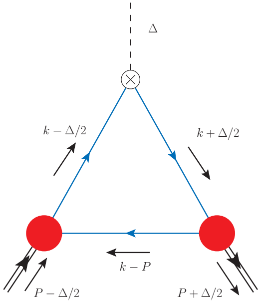
Mellin moments

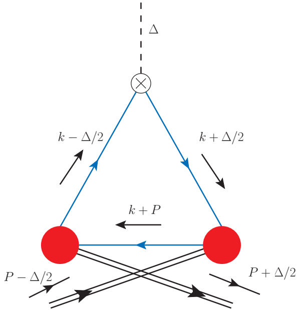
$$\mathcal{M}_m(\xi,t) = \int_{-1}^{1} \mathrm{d}x\ x^m\ H(x,\xi,t)$$
$$= \frac{1}{2(P\cdot n)^{m+1}} \left\langle \pi, P+\frac{\Delta}{2} \left| \bar{\psi}(0)\gamma\cdot n(i\overleftrightarrow{D}\cdot n)^m \psi(0) \right| \pi, P-\frac{\Delta}{2} \right\rangle .$$





Mellin moments





$$\begin{aligned} 2(P\cdot n)^{m+1}\mathcal{M}_m(\xi,t) &= \mathrm{tr}_{CFD}\int \frac{\mathrm{d}^4k}{(2\pi)^4}\,(k\cdot n)^m\ i\Gamma_\pi(k-\frac{\Delta}{2},P-\frac{\Delta}{2})\ S(k-\frac{\Delta}{2}) \\ & i\gamma\cdot n\ S(k+\frac{\Delta}{2})\ i\bar{\Gamma}_\pi(k+\frac{\Delta}{2},P+\frac{\Delta}{2})\ S(k-P) \end{aligned}$$

Analytic model

The propagators:

$$S(p^2) = \frac{-ip\cdot\gamma + M}{p^2 + M^2}$$

- p is quark momentum,
- M is a parameter that can be seen as the quark effective mass.

The vertex :

$$\Gamma_\pi \propto i\gamma_5 \int \frac{\mathrm{d}z \;\; M^2 \rho_\nu(z)}{(q(k,\Delta,P)^2 + M^2)^\nu}$$

- $\rho_\nu(z) \propto (1-z^2)^\nu$ is the z distribution.
- $q(k,\Delta,P) = k - \frac{1-z}{2}\left(P \pm \frac{\Delta}{2}\right)$.

Analytic model

The propagators:

$$S(p^2) = \frac{-ip\cdot\gamma + M}{p^2 + M^2}$$

- p is quark momentum,
- M is a parameter that can be seen as the quark effective mass.

The vertex :

$$\Gamma_\pi \propto i\gamma_5 \int \frac{\mathrm{d}z\ M^2 \rho_\nu(z)}{(q(k,\Delta,P)^2 + M^2)^\nu}$$

- $\rho_\nu(z) \propto (1-z^2)^\nu$ is the z distribution.
- $q(k,\Delta,P) = k - \frac{1-z}{2}\left(P \pm \frac{\Delta}{2}\right)$.

Those functional forms are inspired by numerical results coming from Dyson-Schwinger equations (*L. Chang and al.*,2013)

Theoretical constraint on Mellin moments

- $\int \mathrm{d}x \; x^m \; H(x, \xi, t) = \mathcal{M}_m(\xi, t)$

Theoretical constraint on Mellin moments

- $\int \mathrm{d}x \; x^m \; H(x,\xi,t) = \mathcal{M}_m(\xi,t)$
- Lorentz invariance $\Rightarrow \mathcal{M}_m(\xi,t)$ in a polynomial in ξ.
  our model fulfills this property!

Theoretical constraint on Mellin moments

- $\int \mathrm{d}x \; x^m \; H(x, \xi, t) = \mathcal{M}_m(\xi, t)$
- Lorentz invariance $\Rightarrow \mathcal{M}_m(\xi, t)$ in a polynomial in ξ.
  our model fulfills this property!
- Time reversal invariance $\Rightarrow \mathcal{M}_m(\xi, t) = \mathcal{M}_m(-\xi, t)$. This symmetry is also fulfilled.

Theoretical constraint on Mellin moments

- $\int \mathrm{d}x \; x^m \; H(x,\xi,t) = \mathcal{M}_m(\xi,t)$
- Lorentz invariance $\Rightarrow \mathcal{M}_m(\xi,t)$ in a polynomial in ξ.
  our model fulfills this property!
- Time reversal invariance $\Rightarrow \mathcal{M}_m(\xi,t) = \mathcal{M}_m(-\xi,t)$. This symmetry is also fulfilled.
- Charge conjugation demands $H^q + H^{\bar{q}}$ to be odd in x, $H^q - H^{\bar{q}}$ to be even in x.
  This is also fulfilled.

Theoretical constraint on Mellin moments

- $\int \mathrm{d}x \; x^m \; H(x, \xi, t) = \mathcal{M}_m(\xi, t)$
- Lorentz invariance $\Rightarrow \mathcal{M}_m(\xi, t)$ in a polynomial in ξ.
  our model fulfills this property!
- Time reversal invariance $\Rightarrow \mathcal{M}_m(\xi, t) = \mathcal{M}_m(-\xi, t)$. This symmetry is also fulfilled.
- Charge conjugation demands $H^q + H^{\bar{q}}$ to be odd in x, $H^q - H^{\bar{q}}$ to be even in x.
  This is also fulfilled.

Our model fulfills the symmetry properties coming from quantum field theory.

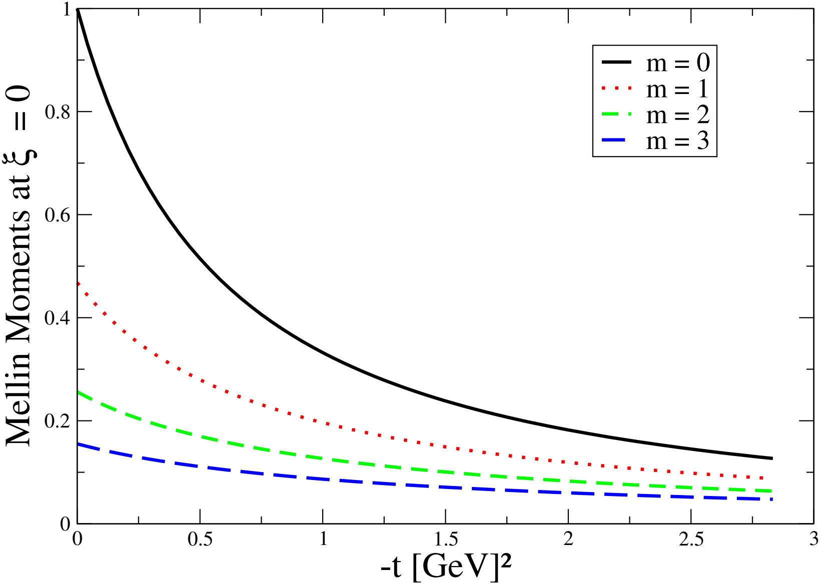
Theoretical constraint on Mellin moments

- $\int \mathrm{d}x \; x^m \; H(x, \xi, t) = \mathcal{M}_m(\xi, t)$
- Lorentz invariance $\Rightarrow \mathcal{M}_m(\xi, t)$ in a polynomial in ξ.
  our model fulfills this property!
- Time reversal invariance $\Rightarrow \mathcal{M}_m(\xi, t) = \mathcal{M}_m(-\xi, t)$. This symmetry is also fulfilled.
- Charge conjugation demands $H^q + H^{\bar{q}}$ to be odd in x, $H^q - H^{\bar{q}}$ to be even in x.
  This is also fulfilled.

Our model fulfills the symmetry properties coming from quantum field theory.

This is not necessarily the case of quark models.

Mellin moments depending on t



Analytic results

$$
\begin{aligned}
F^u(\beta,\alpha,t) \quad = \quad & \frac{48}{5}\Bigg\{-\frac{18M^4 t(\beta-1)(\alpha-\beta+1)(\alpha+\beta-1)\left(\left(\alpha^2-(\beta-1)^2\right)\tanh^{-1}\left(\frac{2\beta}{-\alpha^2+\beta^2+1}\right)+2\beta\right)}{\left(4M^2+t\left((\beta-1)^2-\alpha^2\right)\right)^3} \\
&+\frac{9M^4(\alpha-\beta+1)\left(-4\beta\left(-\alpha^2+\beta^2+1\right)+2\tanh^{-1}\left(\frac{2\beta}{-\alpha^2+\beta^2+1}\right)\right)}{4(\alpha-\beta-1)\left(4M^2+t\left((\beta-1)^2-\alpha^2\right)\right)^2} \\
&+\frac{9M^4(\alpha-\beta+1)\left(\left(\alpha^4-2\alpha^2\left(\beta^2+1\right)+\beta^2\left(\beta^2-2\right)\right)\log\left(\frac{(\alpha-\beta-1)(\alpha+\beta+1)}{\alpha^2-(\beta-1)^2}\right)\right)}{4(\alpha-\beta-1)\left(4M^2+t\left((\beta-1)^2-\alpha^2\right)\right)^2} \\
&+\frac{9M^4(\alpha+\beta-1)\left(-4\beta\left(-\alpha^2+\beta^2+1\right)+2\tanh^{-1}\left(\frac{2\beta}{-\alpha^2+\beta^2+1}\right)\right)}{4(\alpha+\beta+1)\left(4M^2+t\left((\beta-1)^2-\alpha^2\right)\right)^2} \\
&+\frac{9M^4(\alpha+\beta-1)\left(\left(\alpha^4-2\alpha^2\left(\beta^2+1\right)+\beta^4-2\beta^2\right)\log\left(\frac{(\alpha-\beta-1)(\alpha+\beta+1)}{\alpha^2-(\beta-1)^2}\right)\right)}{4(\alpha+\beta+1)\left(4M^2+t\left((\beta-1)^2-\alpha^2\right)\right)^2} \\
&+\frac{9M^4\beta(\alpha-\beta+1)^2(\alpha+\beta-1)^2\left(\frac{2\left(\alpha^2\beta-\beta^3+\beta\right)}{\alpha^4-2\alpha^2(\beta^2+1)+(\beta^2-1)^2}\right)}{\left(4M^2+t\left((\beta-1)^2-\alpha^2\right)\right)^2} \\
&+\frac{9M^4\beta(\alpha-\beta+1)^2(\alpha+\beta-1)^2\left(-\tanh^{-1}(\alpha-\beta)+\tanh^{-1}(\alpha+\beta)\right)}{\left(4M^2+t\left((\beta-1)^2-\alpha^2\right)\right)^2}\Bigg\},
\end{aligned}
$$

Analytic results

$$H^{u}_{x\geq\xi}(x,\xi,0) = \frac{48}{5}\Bigg\{ \frac{3\left(-2(x-1)^4\left(2x^2-5\xi^2+3\right)\log(1-x)\right)}{20\left(\xi^2-1\right)^3}$$

$$\frac{3\left(+4\xi\left(15x^2(x+3)+(19x+29)\xi^4+5(x(x(x+11)+21)+3)\xi^2\right)\tanh^{-1}\left(\frac{(x-1)\xi}{x-\xi^2}\right)\right)}{20\left(\xi^2-1\right)^3}$$

$$+\frac{3\left(x^3(x(2(x-4)x+15)-30)-15(2x(x+5)+5)\xi^4\right)\log\left(x^2-\xi^2\right)}{20\left(\xi^2-1\right)^3}$$

$$+\frac{3\left(-5x(x(x(x+2)+36)+18)\xi^2-15\xi^6\right)\log\left(x^2-\xi^2\right)}{20\left(\xi^2-1\right)^3}$$

$$+\frac{3\left(2(x-1)\left((23x+58)\xi^4+(x(x(x+67)+112)+6)\xi^2+x(x((5-2x)x+15)+3)\right)\right)}{20\left(\xi^2-1\right)^3}$$

$$+\frac{3\left(\left(15(2x(x+5)+5)\xi^4+10x(3x(x+5)+11)\xi^2\right)\log\left(1-\xi^2\right)\right)}{20\left(\xi^2-1\right)^3}$$

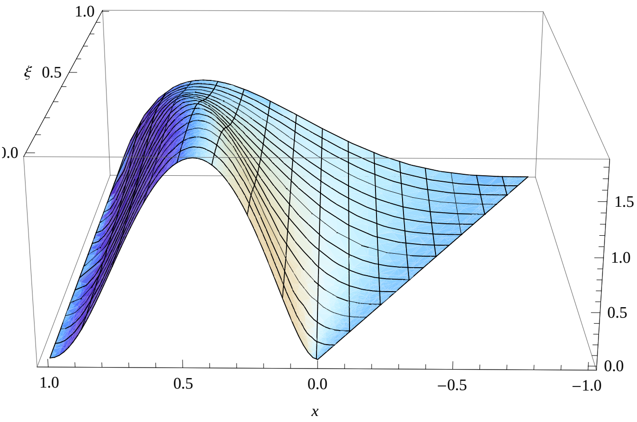
$$+\frac{3\left(2x(5x(x+2)-6)+15\xi^6-5\xi^2+3\right)\log\left(1-\xi^2\right)}{20\left(\xi^2-1\right)^3}\Bigg\},$$

Analytic results

$$
\begin{aligned}
H^{u}_{|x|\leq\xi}(x,\xi,0) \quad = \quad & \frac{48}{5}\left\{ \frac{6\xi(x-1)^4\left(-\left(2x^2-5\xi^2+3\right)\right)\log(1-x)}{40\xi\left(\xi^2-1\right)^3} \right. \\
& + \frac{6\xi\left(-4\xi\left(15x^2(x+3)+(19x+29)\xi^4+5(x(x(x+11)+21)+3)\xi^2\right)\log(2\xi)\right)}{40\xi\left(\xi^2-1\right)^3} \\
& + \frac{6\xi(\xi+1)^3\left((38x+13)\xi^2+6x(5x+6)\xi+2x(5x(x+2)-6)+15\xi^3-9\xi+3\right)\log(\xi+1)}{40\xi\left(\xi^2-1\right)^3} \\
& + \frac{6\xi(x-\xi)^3\left((7x-58)\xi^2+6(x-4)x\xi+x(2(x-4)x+15)+15\xi^3+75\xi-30\right)\log(\xi-x)}{40\xi\left(\xi^2-1\right)^3} \\
& + \frac{3(\xi-1)(x+\xi)\left(4x^4\xi-2x^3\xi(\xi+7)+x^2(\xi((119-25\xi)\xi-5)+15)\right)}{40\xi\left(\xi^2-1\right)^3} \\
& \left. + \frac{3(\xi-1)(x+\xi)\left(x\xi(\xi(\xi(71\xi+5)+219)+9)+2\xi(\xi(2\xi(34\xi+5)+9)+3)\right)}{40\xi\left(\xi^2-1\right)^3} \right\}.
\end{aligned}
$$

Those horrible formulas allow us to do nice pictures!

Reconstruction ($t = 0$)



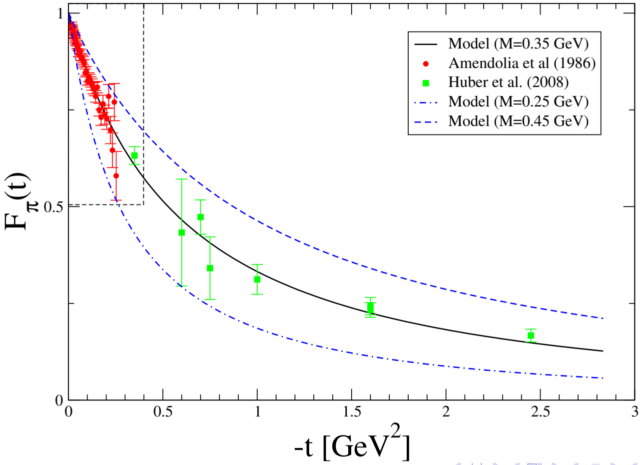
We get also the good support and continuity properties.

Form factor

$$F_{\pi}^{q}(t) = \mathcal{M}_0(t) = \int_{-1}^{1} \mathrm{d}x \; H^q(x, \xi, t)$$

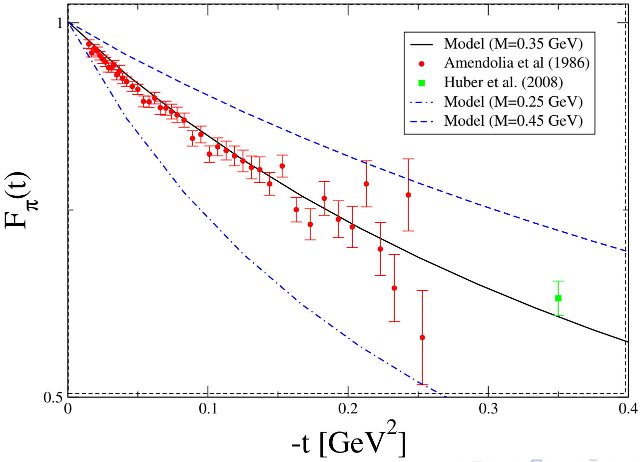
Form factor

$$F_\pi^q(t) = \mathcal{M}_0(t) = \int_{-1}^{1} \mathrm{d}x \; H^q(x, \xi, t)$$

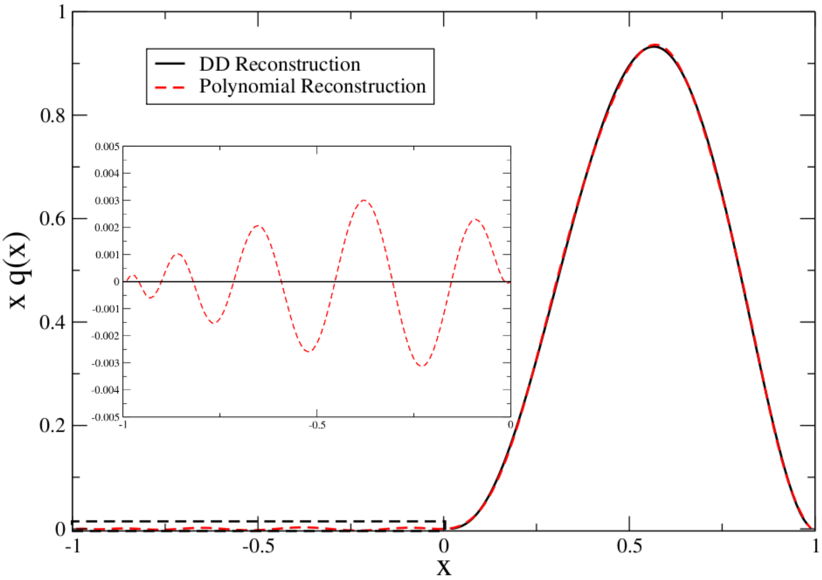


Form factor

$$F_\pi^q(t) = \mathcal{M}_0(t) = \int_{-1}^{1} \mathrm{d}x \; H^q(x, \xi, t)$$



The PDF case



DD Reconstruction
Polynomial Reconstruction
x q(x)
x
1
0.8
0.6
0.4
0.2
0
-1
-0.5
0
0.5
1
0.005
0.004
0.003
0.002
0.001
0
-0.001
-0.002
-0.003
-0.004
-0.005
-1
-0.5
0

3D imaging

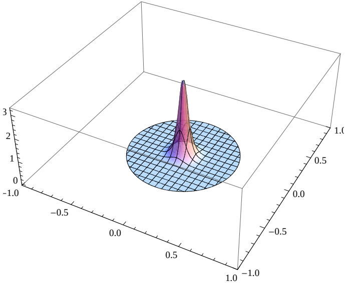
$$\rho^q(x, b_\perp) = \int_0^\infty \frac{\mathrm{d}|\Delta_\perp|}{2\pi} |\Delta_\perp| J_0\left(|b_\perp| \cdot |\Delta_\perp|\right) H^q(x, 0, -\Delta_\perp^2),$$

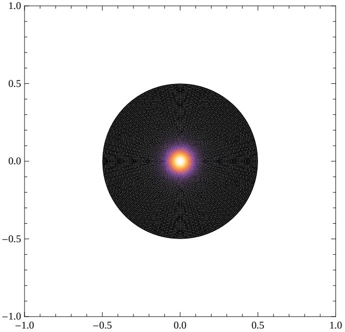
- $b_\perp$ is the Fourier conjugated variable to $\Delta_\perp$.
- $b_\perp$ stands for the position of a quark in the plane transverse to the hadron direction.
- J_0 is Bessel function of the first kind.
- $\rho^q(x, b_\perp)$ is here the probability density to find a quark q at a given position $b_\perp$ in the transverse plane and at a given longitudinal momentum fraction x.

3D imaging

$$\rho^q(x, b_\perp) = \int_0^\infty \frac{\mathrm{d}|\Delta_\perp|}{2\pi} |\Delta_\perp| J_0\big(|b_\perp| \cdot |\Delta_\perp|\big) H^q(x, 0, -\Delta_\perp^2),$$

Case $x = 0.95$

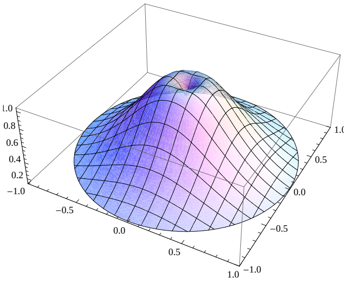


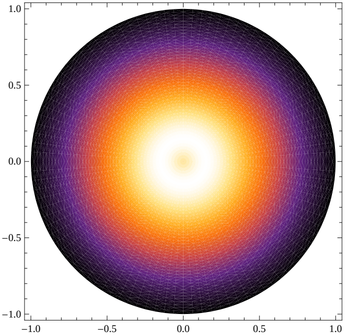


3D imaging

$$\rho^q(x, b_\perp) = \int_0^\infty \frac{\mathrm{d}|\Delta_\perp|}{2\pi} |\Delta_\perp| J_0\left(|b_\perp| \cdot |\Delta_\perp|\right) H^q(x, 0, -\Delta_\perp^2),$$

Case $x = 0.5$

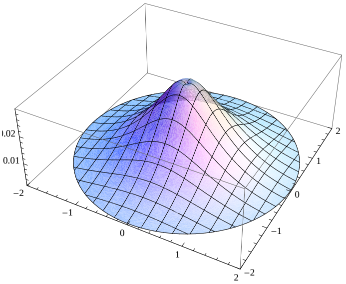


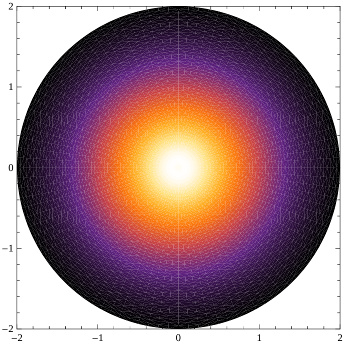


3D imaging

$$\rho^q(x, b_\perp) = \int_0^\infty \frac{\mathrm{d}|\Delta_\perp|}{2\pi} |\Delta_\perp| J_0\left(|b_\perp| \cdot |\Delta_\perp|\right) H^q(x, 0, -\Delta_\perp^2),$$

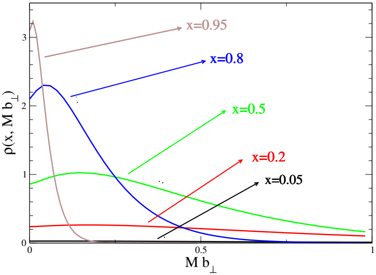
Case $x = 0.05$



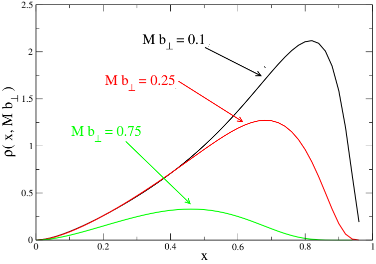


Graph summary

Comparison at given x.



Comparison at given $b_\perp$.



Summary and conclusion

- Summary: we want to build a new GPD model.
  - Dyson-Schwinger equations.
  - Pion case.
  - Mellin moments.
  - Triangle diagrams, and valence quarks.
  - Analytic model.
  - 3D imaging.
- Conclusion
  - New modeling tools.
  - Analytic model $\rightarrow$ agreement model/data.
  - Encouraging results.

Outlooks 1

- Transversity GPDs:
  $\rightarrow$ Work done by Pierre Fromholtz during his internship.
- Beyond the analytic model:
  - Use results coming directly from Dyson-Schwinger equations.
  - Build numerical model.
  - Understand the 3D structure of the pion in terms of chiral symmetry breaking?
  - Link between spectroscopy and partonic structure?

Outlook 2 : the proton case

- Proton : many more data!
- Difficulties,
  - 3 body system,
  - spin,
  - ...

Thank you!

Current GPD models

- Double Distribution Models : based on Radyushkin Ansatz.
  A.Radyushkin, Phys. Rev. D59 014030

Current GPD models

- Double Distribution Models : based on Radyushkin Ansatz.
  A.Radyushkin, Phys. Rev. D59 014030
- Mellin-Barnes formalism: fit of Mellin moments on data then resumming thanks to Inverse Mellin transform.
  D. Müler and A.Schäfer, Nucl. Phys. B 739 1

Current GPD models

- Double Distribution Models : based on Radyushkin Ansatz.
  A.Radyushkin, Phys. Rev. D59 014030
- Mellin-Barnes formalism: fit of Mellin moments on data then resumming thanks to Inverse Mellin transform.
  D. Müler and A.Schäfer, Nucl. Phys. B 739 1
- Dual model: fits of conformal moments on data, then partial resummation on polynomials.
  M. V. Polyakov, Nucl. Phys. B 555 231

Current GPD models

- Double Distribution Models : based on Radyushkin Ansatz.
  A.Radyushkin, Phys. Rev. D59 014030
- Mellin-Barnes formalism: fit of Mellin moments on data then resumming thanks to Inverse Mellin transform.
  D. Müler and A.Schäfer, Nucl. Phys. B 739 1
- Dual model: fits of conformal moments on data, then partial resummation on polynomials.
  M. V. Polyakov, Nucl. Phys. B 555 231

All those parameterisations are *phenomenological*.

Locality

GPD	Moments de Mellin
$H(x,\xi,t)$	$\int \mathrm{d}x \; x^m \; H(x,\xi,t)$
Non local object.	Local objects.
Wilson line.	Covariant derivatives.
Different kinematics interpretations.	Difficulty : resumming.

Locality

GPD

- $H(x, \xi, t)$
- Non local object.
- Wilson line.
- Different kinematics interpretations.

Moments de Mellin

- $\int \mathrm{d}x \ x^m \ H(x, \xi, t)$
- Local objects.
- Covariant derivatives.
- Difficulty : resumming.

We choose to model Mellin moments instead of the GPD itself.

Mellin Moments

$$
\begin{aligned}
\langle x^m \rangle^q \quad = \quad & \lambda(\nu) \int \mathrm{d}x\, \mathrm{d}y\, \mathrm{d}u\, \mathrm{d}v\, \mathrm{d}w\, \mathrm{d}z\, \mathrm{d}z' \, \left(\frac{M^2}{M'^2}\right)^{2\nu} \\
& \delta(1-x-y-u-v-w) x^{\nu-1} y^{\nu-1} \rho_\nu(z) \rho_\nu(z') \\
& \Big[(g-2\xi\ f)^m (g+1-2\xi f) \\
& + \frac{1}{2}(-2\xi f + g - 1)(g - 2\xi\ f)^m \\
& + \frac{m}{2}\left((g-2\xi f)^{m-1}((g-2\xi f)^2 - \xi^2)\right) \\
& + \frac{\Gamma(2\nu+1)}{2M'^2\Gamma(2\nu)}(g-2\xi f)^m \\
& \times \left((g-2\xi f)(tf^2 + P^2(g^2-2g) + \frac{t}{4} + M^2)\right]
\end{aligned}
$$

Mellin Moments

$$f = \frac{1}{2}\left(-\frac{1+z'}{2}y + \frac{1+z}{2}x + v - w\right)$$

$$g = \frac{1-z'}{2}y + x\frac{1-z}{2} + u$$

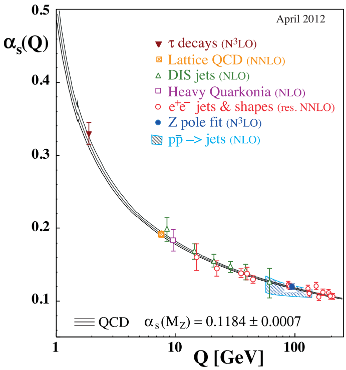
$$M'^2 = M^2 + \frac{t}{4}\left(-4f^2 + y\left(\frac{1+z'}{2}\right)^2 + x\left(\frac{1+z}{2}\right)^2 + v + w\right)$$
$$+P^2\left(-g^2 + \left(\frac{1-z'}{2}\right)^2 y + \left(\frac{1-z}{2}\right)^2 x + u\right).$$

The parton distribution function

- Form factor
  - Very good agreement with data.
  - Strong confidence in the t-dependence of our model.
  - Dependence in x and ξ?
- PDF:
  - Explicit dependence in x.
  - $\xi = 0$.
  - Moment to moment comparison.

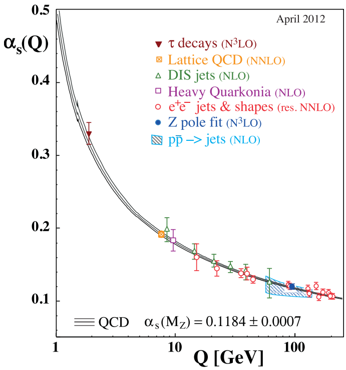
The parton distribution function

- Form factor
  - Very good agreement with data.
  - Strong confidence in the t-dependence of our model.
  - Dependence in x and ξ?
- PDF:
  - Explicit dependence in x.
  - $\xi = 0$.
  - Moment to moment comparison.



The parton distribution function

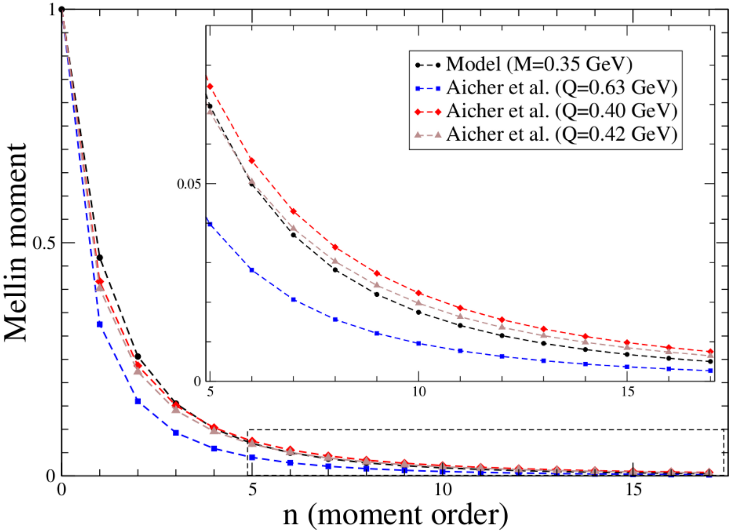
- Form factor
  - Very good agreement with data.
  - Strong confidence in the t-dependence of our model.
  - Dependence in x and ξ?
- PDF:
  - Explicit dependence in x.
  - $\xi = 0$.
  - Moment to moment comparison.



Scale troubles

The PDF depend on factorization and renormalization scales. In our model, those scales are hidden in M and ν. One do not know *a priori* the scale of the model.

Moments de Mellin de la PDF



Properties of Mellin moments

Polynomiality:

$$\left\langle \pi, P+\frac{\Delta}{2}\left|\bar{\psi}(0)\gamma\cdot n(i\overleftrightarrow{D}\cdot n)^m\psi(0)\right|\pi, P-\frac{\Delta}{2}\right\rangle$$

$$= n_\mu n_{\mu_1}\ldots n_{\mu_m} P^{\{\mu}\sum_{j=0}^{m}\binom{m}{j} F_{m,j}(t) P^{\mu_1}\ldots P^{\mu_j}\left(-\frac{\Delta}{2}\right)^{\mu_{j+1}}\ldots\left(-\frac{\Delta}{2}\right)^{\mu_m\}}$$

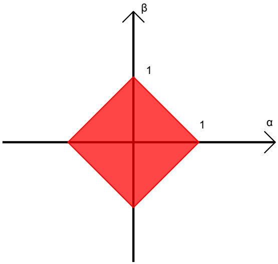
$$-n_\mu n_{\mu_1}\ldots n_{\mu_m}\frac{\Delta}{2}^{\{\mu}\sum_{j=0}^{m}\binom{m}{j} G_{m,j}(t) P^{\mu_1}\ldots P^{\mu_j}\left(-\frac{\Delta}{2}\right)^{\mu_{j+1}}\ldots\left(-\frac{\Delta}{2}\right)^{\mu_m\}}$$

$\xi = -\frac{\Delta\cdot n}{2P\cdot n} \Rightarrow \mathcal{M}_m(\xi,t)$ is a polynomial in ξ of order $m+1$.

Properties of Mellin moments

Double distributions:

$$
\begin{aligned}
F_{m,j}(t) &= \int_{-1}^{1} \mathrm{d}\beta \int_{-1+|\beta|}^{1-|\beta|} \mathrm{d}\alpha \; \beta^{m-j} \alpha^{j} \; F(\beta, \alpha, t) \\
G_{m,j}(t) &= \int_{-1}^{1} \mathrm{d}\beta \int_{-1+|\beta|}^{1-|\beta|} \mathrm{d}\alpha \; \beta^{m-j} \alpha^{j} \; G(\beta, \alpha, t)
\end{aligned}
$$



Properties of Mellin moments

$$\begin{aligned}
\mathcal{M}_m(\xi, t) \quad = \quad & n_\mu n_{\mu_1}...n_{\mu_m} \sum_{j=0}^{m} \binom{m}{j} \int_{-1}^{1} \mathrm{d}\beta \int_{-1+|\beta|}^{1-|\beta|} \mathrm{d}\alpha \; \beta^{m-j} \alpha^j \\
& F(\beta, \alpha, t) P^{\{\mu} P^{\mu_1} ... P^{\mu_j} \left(-\frac{\Delta}{2}\right)^{\mu_{j+1}} ... \left(-\frac{\Delta}{2}\right)^{\mu_m\}} \\
& - G(\beta, \alpha, t) \frac{\Delta}{2}^{\{\mu} P^{\mu_1} ... P^{\mu_j} \left(-\frac{\Delta}{2}\right)^{\mu_{j+1}} ... \left(-\frac{\Delta}{2}\right)^{\mu_m\}}
\end{aligned}$$

$$\mathcal{M}_m(\xi,t) = n_\mu n_{\mu_1}...n_{\mu_m}\sum_{j=0}^{m}\binom{m}{j}\int_{-1}^{1}\mathrm{d}\beta \int_{-1+|\beta|}^{1-|\beta|}\mathrm{d}\alpha\ \beta^{m-j}\alpha^{j}$$
$$F(\beta,\alpha,t)P^{\{\mu}P^{\mu_1}...P^{\mu_j}\left(-\frac{\Delta}{2}\right)^{\mu_{j+1}}...\left(-\frac{\Delta}{2}\right)^{\mu_m\}}$$
$$-G(\beta,\alpha,t)\frac{\Delta}{2}^{\{\mu}P^{\mu_1}...P^{\mu_j}\left(-\frac{\Delta}{2}\right)^{\mu_{j+1}}...\left(-\frac{\Delta}{2}\right)^{\mu_m\}}$$

Time Rerversal Invariance

$$\Delta \to -\Delta$$

$\mathcal{M}_m(\xi,t)$ is an *even* polynomial in ξ of order $m+1$.

$F(\beta,\alpha)$ is **even** in α.
$G(\beta,\alpha)$ is **odd** in α.