Questions in Hadron Physics

Deep Inelastic Scattering with & without spin

Lecture 1 B
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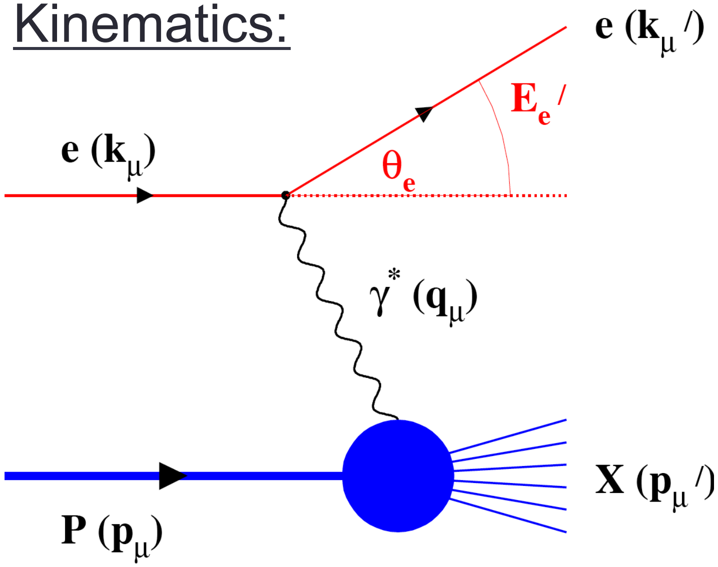
Deep Inelastic Scattering

Kinematics:



$$Q^2 = -q^2 = -(k_\mu - k'_\mu)^2$$

Measure of resolution power

$$Q^2 = 2E_e E'_e(1-\cos\Theta_{e'})$$

$$y = \frac{pq}{pk} = 1 - \frac{E'_e}{E_e}\cos^2\left(\frac{\theta'_e}{2}\right)$$

Measure of inelasticity

$$x = \frac{Q^2}{2pq} = \frac{Q^2}{sy}$$

Measure of momentum fraction of struck quark

***Hadron*:**

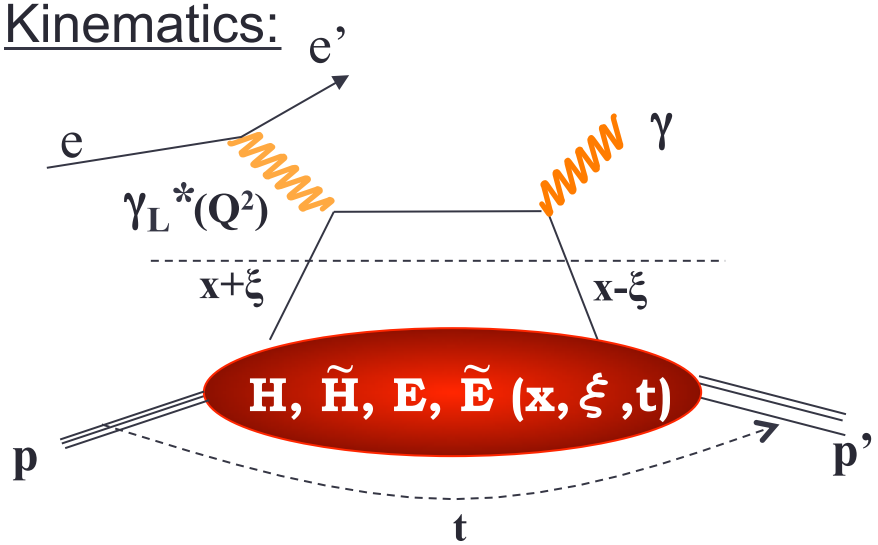
$$z = \frac{E_h}{\nu}; p_t^{\text{with respect to } \gamma}$$

Inclusive events:
e+p/A → e'+X
detect only the scattered lepton in the detector

Semi-inclusive events:
e+p/A → e'+h(π,K,p,jet)+X
detect the scattered lepton in coincidence with identified hadrons/jets in the detector

Deep Inelastic Scattering

Kinematics:



$$Q^2 = -q^2 = -(k_\mu - k'_\mu)^2$$

Measure of resolution power

$$Q^2 = 2E_e E'_e(1-\cos\Theta_{e'})$$

$$y = \frac{pq}{pk} = 1 - \frac{E'_e}{E_e}\cos^2\left(\frac{\theta'_e}{2}\right)$$

Measure of inelasticity

$$x_B = \frac{Q^2}{2pq} = \frac{Q^2}{sy}$$

Measure of momentum fraction of struck quark

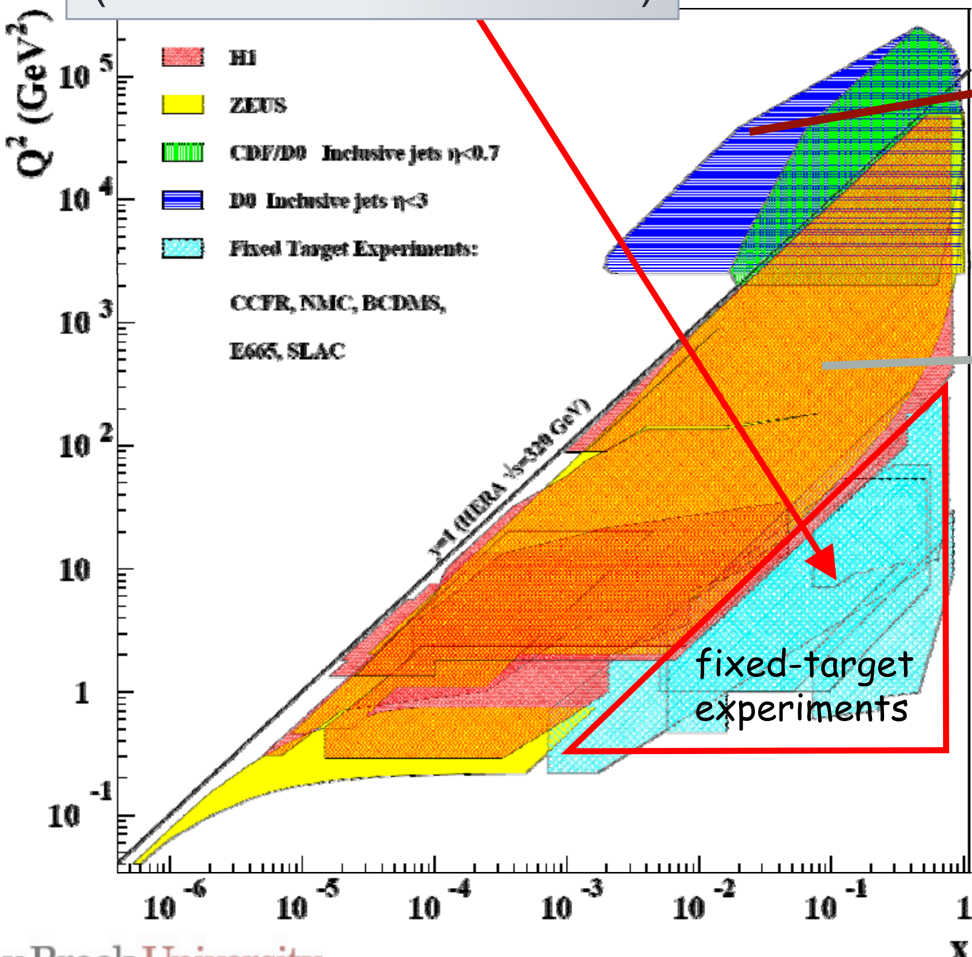
$$t = (p-p')^2, \xi = \frac{x_B}{2-x_B}$$

Exclusive events:
e + **(p/A)** → e'+ **(p'/A')**+ γ / J/ψ / ρ / ϕ
detect **all** event products in the detector

Special sub-event category rapidity gap events
e + **(p/A)** → e' + γ / J/ψ / ρ / ϕ / jet
Don't detect **(p'/A') in final state**

Perspective on x,Q^2, Center of Mass

Fixed target e-N experiments
(center of mass < 30 GeV)

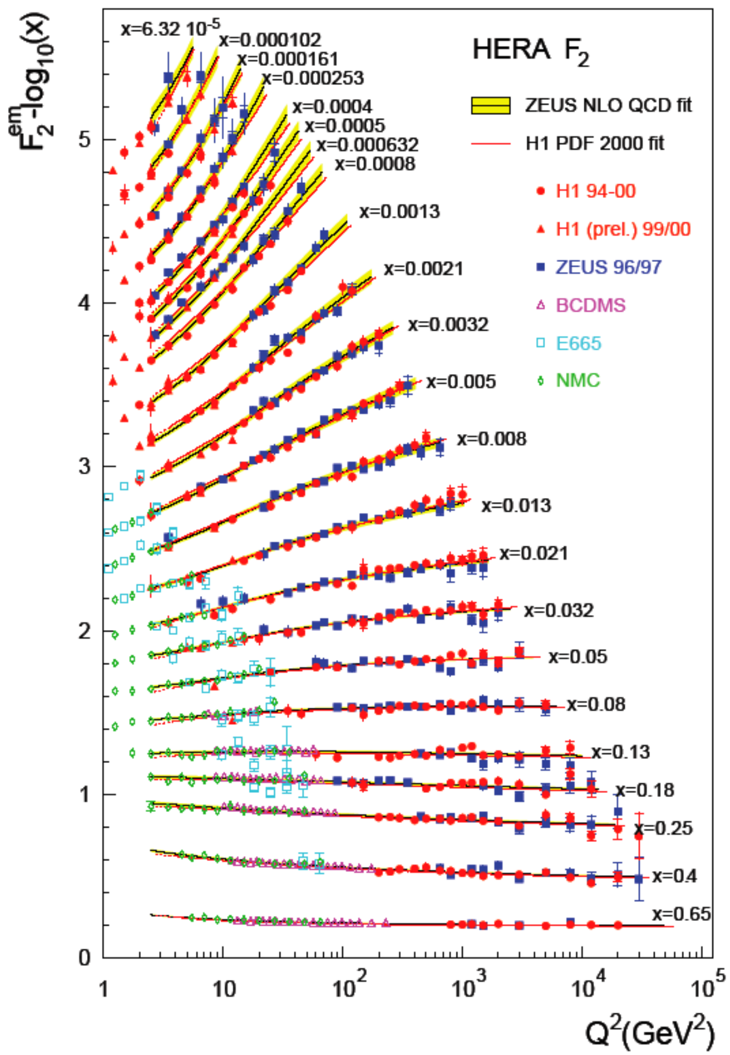


Hadron-Hadron
Collider: CM ~2 TeV

Typically accessible
By e-N collider experiments
CM ~ 300 GeV

$Q^2 = S\ x\ y$

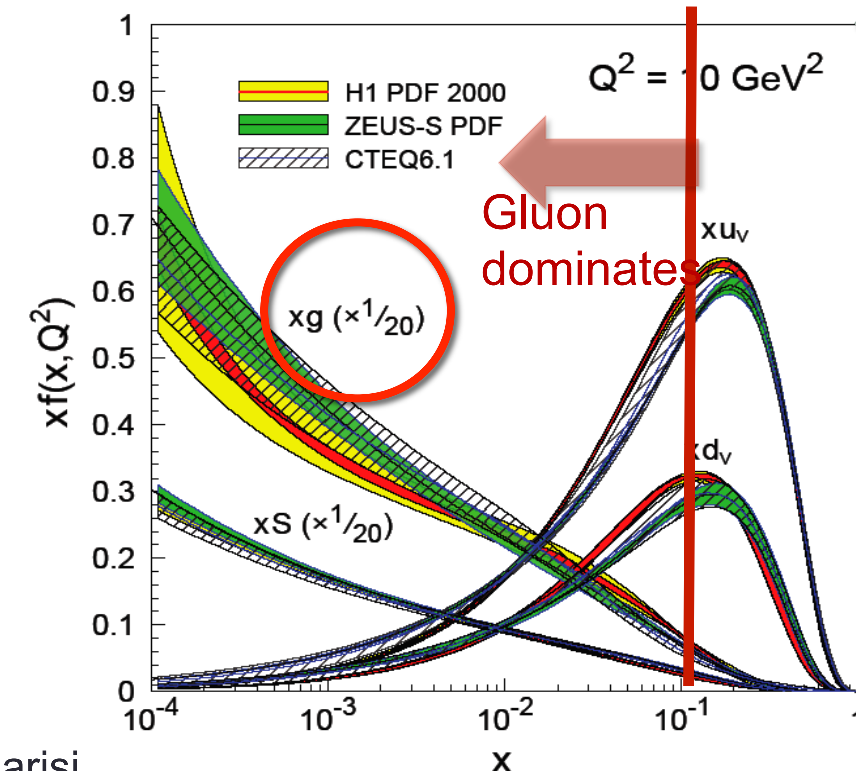
Measurement of Glue at HERA



- Scaling violations of $F_2(x,Q^2)$

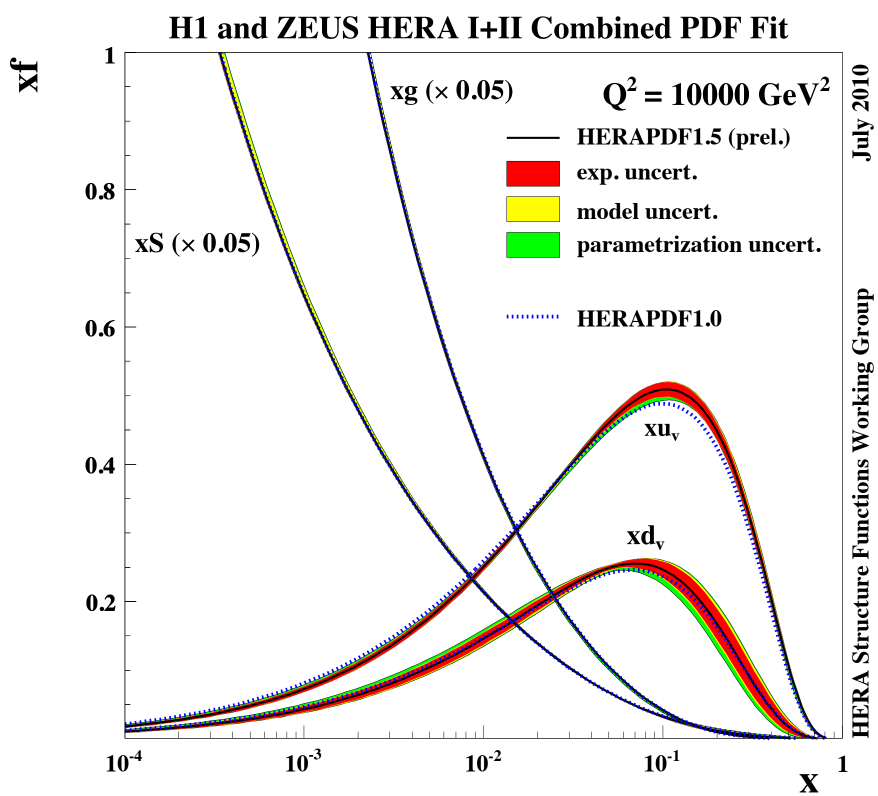
$$\frac{\partial F_2(x,Q^2)}{\partial \ln Q^2} \propto G(x,Q^2)$$

- NLO pQCD analyses: fits with **linear** DGLAP* equations



*Dokshitzer, Gribov, Lipatov, Altarelli, Parisi

Measurement of Gluons at HERA



- Inclusive e-p scattering at 300 GeV center of mass
- Low x gluon distribution measured up to x = 10^{-4}
- Evolved to a high value of Q^2 using the Altarelli Parisi equation
- Gluon distribution keeps on rising: “the Low x singularity”

Method of extraction covered by M. Stratmann? -- If not, I will briefly Visit this later in my lectures.

What this means to our understanding of QCD: R. Venugopalan’s lectures

Some equations…

Assume only γ^* exchange

- Lepton Nucleon Cross Section

Nucleon spin

$$\frac{d^3\sigma}{dx\,dy\,d\phi} = \frac{\alpha^2 y}{2Q^4} L_{\mu\nu}(k, q, s,) W^{\mu\nu}(P, q, S)$$

Lepton spin

- Lepton tensor $L_{\mu\nu}$ affects the kinematics (QED)
- Hadronic tensor $W^{\mu\nu}$ has information about the hadron structure

$$W^{\mu\nu}(P, q, S) = -(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}) \underline{F_1(x, Q^2)} + (p^\mu - \frac{P\cdot q}{q^2} q^\mu)(p^\nu - \frac{P\cdot q}{q^2} q^\nu)\frac{1}{P\cdot q}\underline{F_2(x, Q^2)}$$

$$-i\epsilon^{\mu\nu\lambda\sigma} q_\lambda \left[\frac{M S_\sigma}{P\cdot q}(g_1(x, Q^2) + g_2(x, Q^2)) - \frac{M(S\cdot q)P_\sigma}{P\cdot q} g_2(x, Q^2) \right]$$

Lepton-nucleon Cross Section

$$\sigma = \bar{\sigma} - \tfrac{1}{2} h_l \delta\sigma .$$

$$\gamma = \frac{2Mx}{\sqrt{Q^2}} = \frac{\sqrt{Q^2}}{\nu} .$$

$$\frac{d^2\bar{\sigma}}{dx\,dQ^2} = \frac{4\pi\alpha^2}{Q^4 x} \left[xy^2 \left(1 - \frac{2m_l^2}{Q^2} \right) F_1(x,Q^2) + \left(1 - y - \frac{\gamma^2 y^2}{4} \right) F_2(x,Q^2) \right],$$

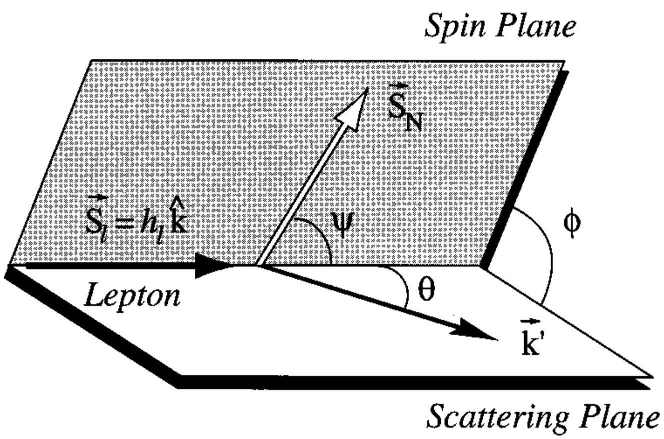
lepton helicity $h_l = \pm 1$

unpolarized structure functions $F_{1,2}(x, Q^2)$

scaling variable $x = Q^2/2M\nu$

exchanged virtual photon energy $= \nu$

Polarized lepton-nucleon cross section…



$$\Delta\sigma = \cos\psi \Delta\sigma_{\parallel} + \sin\psi \cos\phi \Delta\sigma_{\perp}$$

$$\gamma = \frac{2Mx}{\sqrt{Q^2}} = \frac{\sqrt{Q^2}}{\nu}.$$

For high energy γ is small

$$\frac{d^2\Delta\sigma_{\parallel}}{dx\,dQ^2} = \frac{16\pi\alpha^2 y}{Q^4}\left[\left(1 - \frac{y}{2} - \frac{\gamma^2 y^2}{4}\right) g_1 - \frac{\gamma^2 y}{2} g_2\right]$$

$$\frac{d^3\Delta\sigma_T}{dx\,dQ^2\,d\phi} = -\cos\phi \frac{8\alpha^2 y}{Q^4}\gamma\sqrt{1 - y - \frac{\gamma^2 y^2}{4}}\left(\frac{y}{2} g_1 + g_2\right)$$

Cross section asymmetries….

- $\Delta\sigma_{||}$ = anti-parallel – parallel spin cross sections
- $\Delta\sigma_{perp}$= lepton-nucleon spins orthogonal
- Instead of measuring cross sections, it is prudent to measure the differences: Asymmetries in which many measurement imperfections might cancel:

$$A_{||} = \frac{\Delta\sigma_{||}}{2\overline{\sigma}}, \quad A_{\perp} = \frac{\Delta\sigma_{\perp}}{2\overline{\sigma}},$$

which are related to virtual photon-proton asymmetries A_1,A_2:

$$A_{||} = D(A_1 + \eta A_2), \quad A_{\perp} = d(A_2 - \xi A_1)$$

$$\boxed{A_1 = \frac{\sigma_{1/2} - \sigma_{3/2}}{\sigma_{1/2} + \sigma_{3/2}} = \frac{g_1 - \gamma^2 g_2}{F_1}}$$

$$A_2 = \frac{2\sigma^{TL}}{\sigma_{1/2} + \sigma_{3/2}} = \gamma\frac{g_1 + g_2}{F_1}$$

$$d = \frac{\sqrt{1 - y - \gamma^2 y^2/4}}{1 - y/2} D,$$

$$\eta = \frac{\gamma(1 - y - \gamma^2 y^2/4)}{(1 - y/2)(1 + \gamma^2 y/2)},$$

$$\xi = \frac{\gamma(1 - y/2)}{1 + \gamma^2 y/2}.$$

d, η, ξ are kinematic factors

D = Depolarization factor: how much polarization of the incoming electron is taken by the virtual photon, calculable in QED

$$D = \frac{y(2 - y)(1 + \gamma^2 y/2)}{y^2(1 + \gamma^2)(1 - 2m_l^2/Q^2) + 2(1 - y - \gamma^2 y^2/4)(1 + R)}$$

- $A_{||}$ could be written down in terms of spin structure function g_1, and A_2 along with kinematic factors:

$$\frac{A_{\|}}{D} = (1+\gamma^2)\,\frac{g_1}{F_1} + (\eta - \gamma)A_2$$

Where A_1 is bounded by 1, and A_2 by sqrt($R=\sigma_T/\sigma_L$), when terms related A_2 can be neglected, and γ is small,

$$A_1 \simeq \frac{A_{\|}}{D}, \quad \frac{g_1}{F_1} \simeq \frac{1}{1+\gamma^2}\,\frac{A_{\|}}{D}$$

- Where: $F_1 = \frac{1+\gamma^2}{2x(1+R)} F_2$ and $A_2 = \frac{1}{1+\eta\xi}\left(\frac{A_{\perp}}{d} + \xi\,\frac{A_{\|}}{D}\right)$

Relation to spin structure function g_1

$$g_1(x) = \frac{1}{2}\sum_{i=1}^{n_f} e_i^2 \Delta q_i(x)$$

$$\Delta q_i(x) = q_i^+(x) - q_i^-(x) + \overline{q}_i^+(x) - \overline{q}_i^-(x)$$

$$q_i^+ \ (\overline{q}_i^+) \ \text{ and } \ q_i^- (\overline{q}_i^-)$$

Quark and anti-quark with spin orientation along and against the proton spin.

- In QCD quarks interact with each other through gluons, which gives rise to a weak Q^2 dependence of structure functions
- At any given Q^2 the spin structure function is related to polarized quark & gluon distributions by coefficients C_q and C_g

Composition & Q^2 or t dependence of Structure Functions

$$g_1(x,t) = \frac{1}{2}\sum_{k=1}^{n_f}\frac{e_k^2}{n_f}\int_x^1\frac{dy}{y}\left[C_q^{\mathrm{S}}\left(\frac{x}{y},\alpha_s(t)\right)\Delta\Sigma(y,t)\right.$$
$$+2n_f C_g\left(\frac{x}{y},\alpha_s(t)\right)\Delta g(y,t)$$
$$\left.+C_q^{\mathrm{NS}}\left(\frac{x}{y},\alpha_s(t)\right)\Delta q^{\mathrm{NS}}(y,t)\right].$$

In this equation:
t = ln(Q^2/Λ^2)
α_S = strong interaction constant
S & NS stand for flavor singlet & flavor non-singlet

$$\Delta\Sigma(x,t) = \sum_{i=1}^{n_f}\Delta q_i(x,t),$$

$$\Delta q^{\mathrm{NS}}(x,t) = \left[\sum_{i=1}^{n_f}\left(e_i^2 - \frac{1}{n_f}\sum_{k=1}^{n_f}e_k^2\right) \Big/ \frac{1}{n_f}\sum_{k=1}^{n_f}e_k^2\right]\Delta q_i(x,t).$$

Composition & Q² or t dependence of Structure Functions

$$\frac{d}{dt}\Delta\Sigma(x,t)=\frac{\alpha_s(t)}{2\pi}\int_x^1\frac{dy}{y}\left[P_{qq}^{S}\left(\frac{x}{y},\alpha_s(t)\right)\Delta\Sigma(y,t)\right.$$
$$\left.+2n_f P_{qg}\left(\frac{x}{y},\alpha_s(t)\right)\Delta g(y,t)\right],$$

Singlet quark distribution
And its t dependence

$$\frac{d}{dt}\Delta g(x,t)=\frac{\alpha_s(t)}{2\pi}\int_x^1\frac{dy}{y}\left[P_{gq}\left(\frac{x}{y},\alpha_s(t)\right)\Delta\Sigma(y,t)\right.$$
$$\left.+P_{gg}\left(\frac{x}{y},\alpha_s(t)\right)\Delta g(y,t)\right],$$

(Singlet) Gluon distribution
And its t dependence

Non-Singlet quark distribution
And its t dependence

$$\frac{d}{dt}\Delta q^{\mathrm{NS}}(x,t)=\frac{\alpha_s(t)}{2\pi}\int_x^1\frac{dy}{y}P_{qq}^{\mathrm{NS}}\left(\frac{x}{y},\alpha_s(t)\right)\Delta q^{\mathrm{NS}}(y,t).$$

At leading order g_1 decouples with ΔG

$$C_q^{0,\mathrm{S}}\left(\frac{x}{y},\alpha_s\right) = C_q^{0,\mathrm{NS}}\left(\frac{x}{y},\alpha_s\right) = \delta\left(1-\frac{x}{y}\right),$$

$$C_g^{0}\left(\frac{x}{y},\alpha_s\right) = 0.$$

Whenever you hear Analysis done at "Leading order"
--This means quark-gluon interactions are **dropped** from consideration

Beyond the leading order coefficient & splitting functions are not uniquely defined: There are some favorite schemes of theorists, each with distinct calculation advantage.

- Most are now available at α_S^2
- More comments on this in various theory talks

Life was easy in the Quark Parton Model (QPM)

Until first spin experiments were done!

"If I had known this earlier,
I would have done Biology"
--W. Pauli

@1960's # of hadrons > # of chemical elements

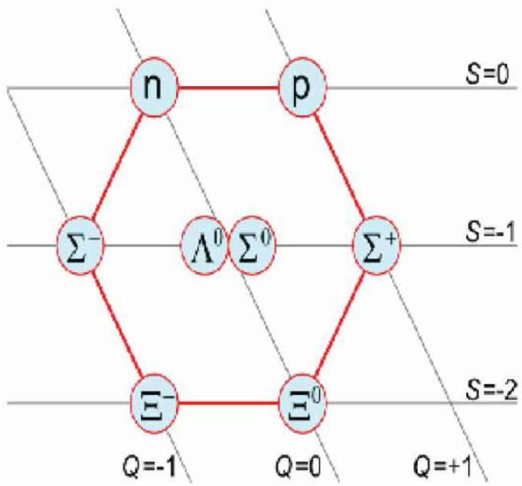
- baryons ($J = n/2$)$_{n=1,2,...}$
- mesons: ($J = n$)$_{n=0,1,...}$

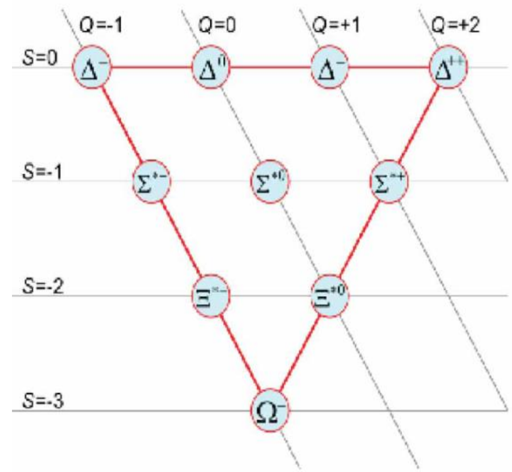
1961 'The Eightfold way' : all baryons and mesons grouped in multiplets definded by SU(3) symmetry

e.g., baryons:

$J^p = 1/2^+$

(same spin and parity)





$J^p = 3/2^+$

isolines of same charge and strangeness

Understanding the proton structure:

Friedman, Kendall, Taylor: 1960's SLAC Experiment

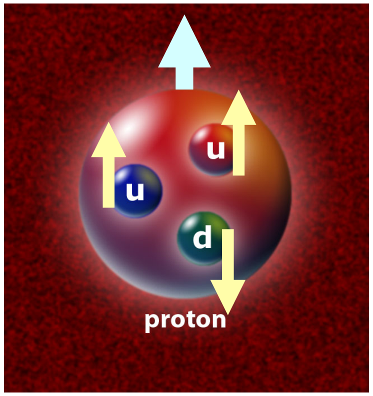
1990 Nobel Prize: "*for their pioneering investigations concerning **deep inelastic scattering of electrons on protons and bound neutrons**, which have been of essential importance for the development of the **quark model** in particle physics*".

Obvious next Question:

Could we understand other properties of proton,

e.g. SPIN, in the quark-parton model?

Proton Spin = ½, each quark is a spin ½ particle…



Structure Functions & PDFs

- The F_1 and F_2 are unpolarized structure functions or momentum distributions
- The g_1 and g_2 are polarized structure functions or spin distributions
- In QPM
  - $F_2(x) = 2xF_1$ (Calan Gross relation)
  - $g_2 = 0$ (Twist 3 quark gluon correlations)

$$F_1(x) = \frac{1}{2}\Sigma_f e_f^2 \{q_f^+(x) + q_f^-(x)\} = \frac{1}{2}\Sigma_f e_f^2 q_f(x)$$

$$g_1(x) = \frac{1}{2}\Sigma_f e_f^2 \{q_f^+(x) - q_f^-(x)\} = \frac{1}{2}\Sigma_f e_f^2 \Delta q_f(x)$$

Nucleon spin & Quark Probabilities

- Define

$$\Delta q = q^{+} - q^{-}$$

  - With q^{+} and q^{-} probabilities of quark & anti-quark with spin parallel and anti-parallel to the nucleon spin

- Total quark contribution then can be written as:

$$\Delta\Sigma = \Delta u + \Delta d + \Delta s$$

- The nucleon spin composition

New: we know only now

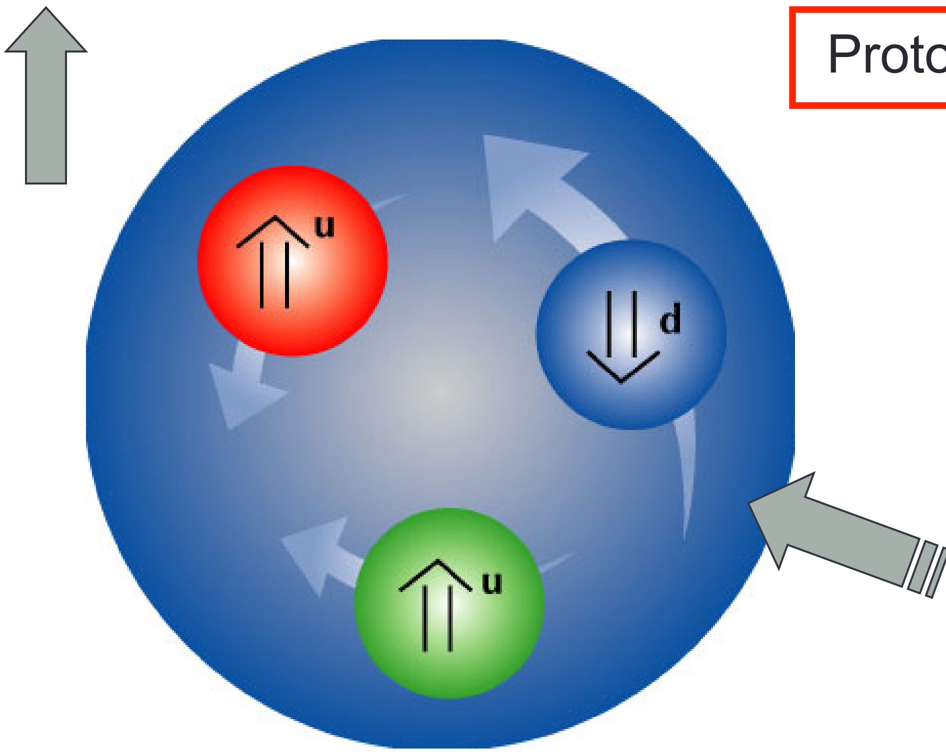
$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma$$

Nucleon's Spin: Naïve Quark Parton Model

- Protons and Neutrons are spin 1/2 particles
- Quarks that constitute them are also spin 1/2 particles
  - And there are three of them in the

Proton: u u d

Neutron: u d d



S_{proton} = Sum of all quark spins!

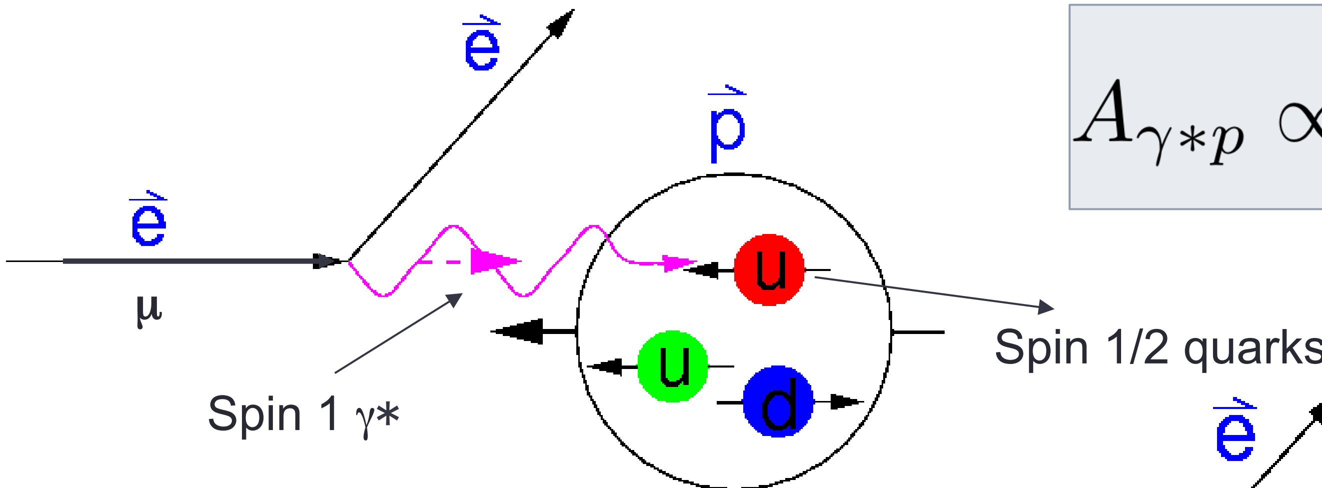
$$1/2 \overset{?}{=} 1/2 + 1/2 + 1/2$$

$$\boxed{1/2 = 1/2 - 1/2 + 1/2}$$

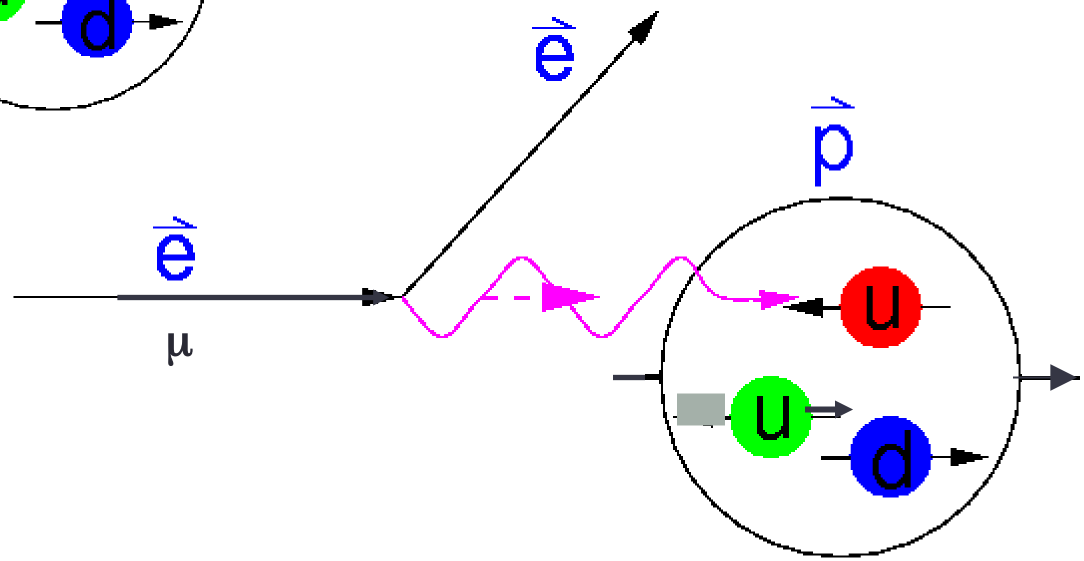
How was the Quark Spin measured?

- Deep Inelastic polarized electron or muon scattering

$$A_{\gamma * p} \propto \frac{N^{\uparrow\downarrow} - N^{\uparrow\uparrow}}{N^{\uparrow\downarrow} + N^{\uparrow\uparrow}}$$



$$\lambda = \frac{\text{Constant}}{p}$$



Measurements of spin structure functions:
What issues we need to worry about?

1) Design of experiments, operational issues
2) Calculations of spin structure functions

Experimental Needs in DIS

Polarized target, polarized beam

- Polarized targets: hydrogen (p), deuteron (pn), helium (^{3}He: 2p+n)
- Polarized beams: electron,muon used in DIS experiments

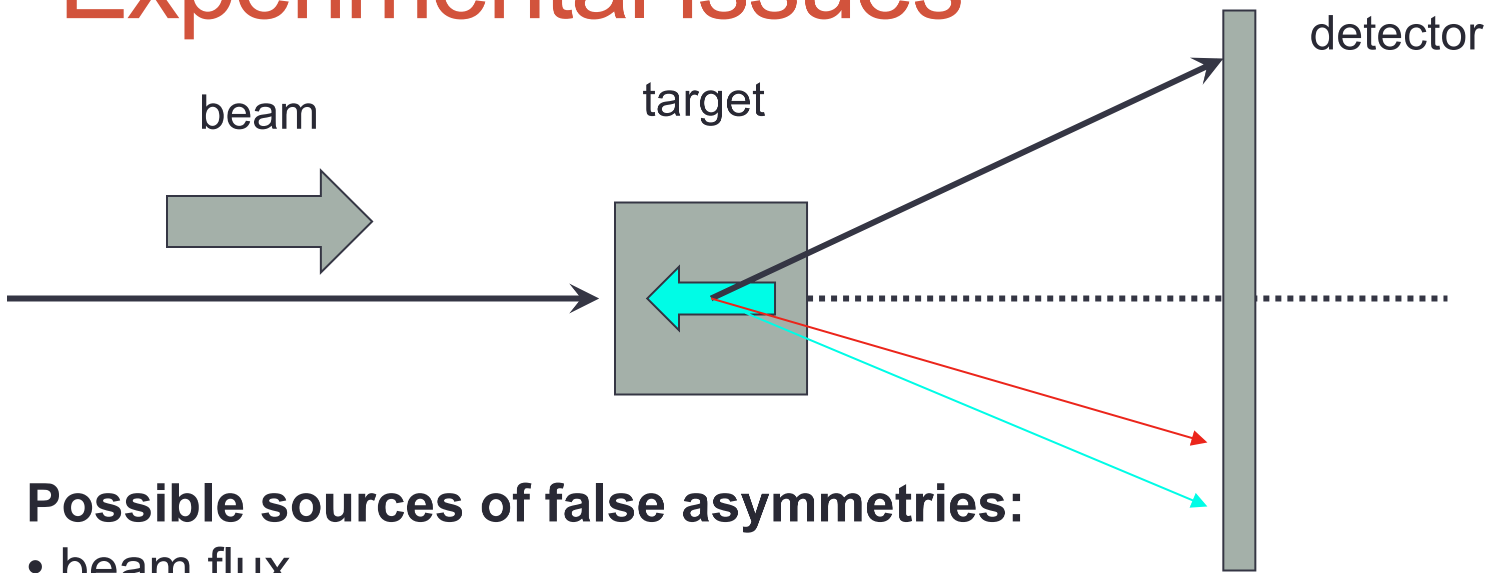
Determine the kinematics: measure with high accuracy:

- Energy of **incoming lepton**
- Energy, direction of **scattered lepton**: energy, direction
- Good identification of **scattered lepton**

Control of false asymmetries:

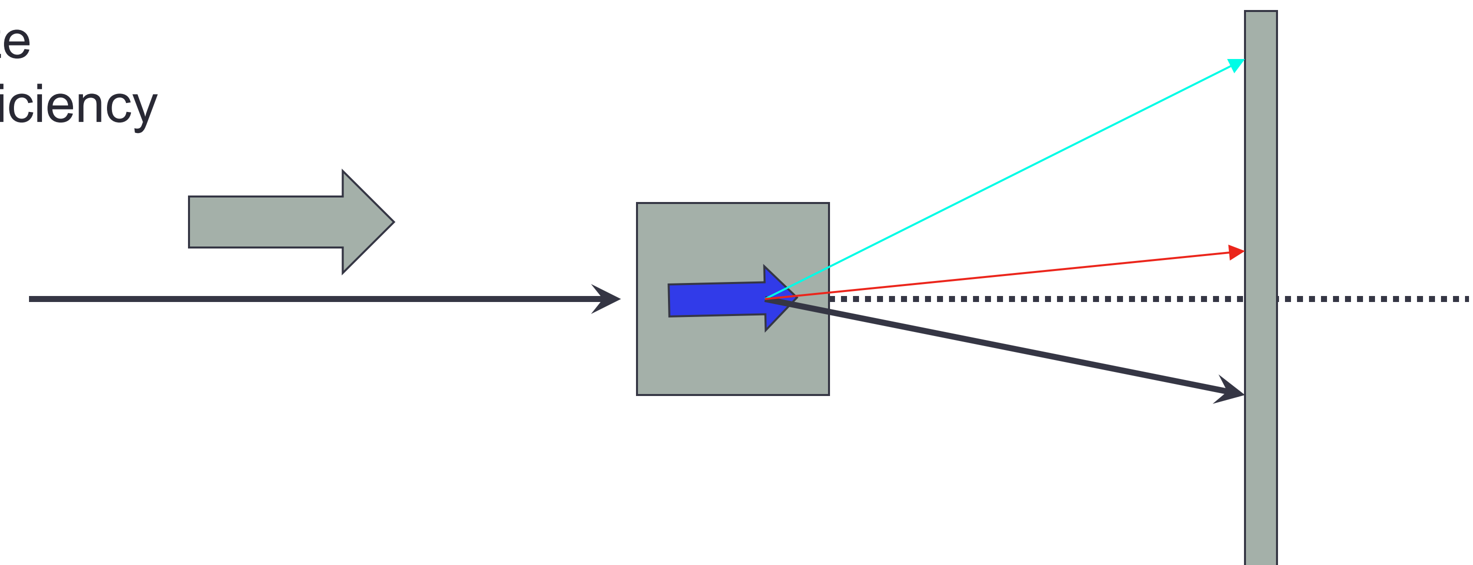
- Need excellent understanding and control of **false asymmetries** (time variation of the detector efficiency etc.)

Experimental issues



Possible sources of false asymmetries:

- beam flux
- target size
- detector size
- detector efficiency



An Ideal Situation

$$A_{measured} = \frac{N^{\rightarrow\leftarrow} - N^{\rightarrow\rightarrow}}{N^{\rightarrow\leftarrow} + N^{\rightarrow\rightarrow}}$$

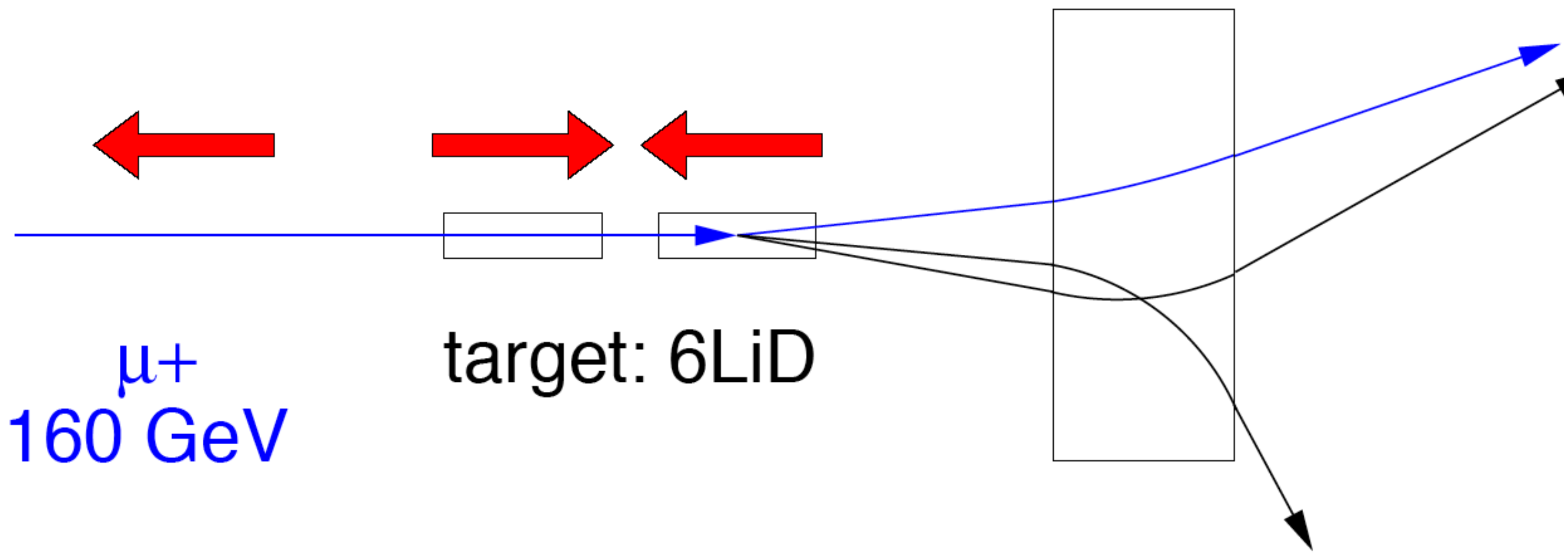
$$N^{\leftarrow\rightarrow} = N_b \cdot N_t \cdot \sigma^{\leftarrow\rightarrow} \cdot D_{acc} \cdot D_{eff}$$

$$N^{\rightarrow\rightarrow} = N_b \cdot N_t \cdot \sigma^{\rightarrow\rightarrow} \cdot D_{acc} \cdot D_{eff}$$

If all other things are equal, they cancel in the ratio and….

$$A_{measured} = \frac{\sigma^{\rightarrow\leftarrow} - \sigma^{\rightarrow\rightarrow}}{\sigma^{\rightarrow\leftarrow} + \sigma^{\rightarrow\rightarrow}}$$

A Typical Setup



- Target polarization direction reversed every 6-8 hrs
- Typically experiments try to limit false asymmetries to be about 10 times smaller than the physics asymmetry of interest

Asymmetry Measurement

$$\frac{N^{\uparrow\downarrow} - N^{\uparrow\uparrow}}{N^{\uparrow\downarrow} + N^{\uparrow\uparrow}} = A_{measured} = P_{beam} \cdot P_{target} \cdot f \cdot A_{||}$$

- f = dilution factor proportional to the polarizable nucleons of interest in the target "material" used, for example for NH_3, f=3/17

$$g_1 \approx \frac{A_{||}}{D} \cdot F_1 \approx \frac{A_{||}}{D} \frac{F_2}{2 \cdot x} \qquad \int_0^1 g_1^p(x, Q_0^2)dx = \Gamma_1^p(Q_0^2)$$

- D is the depolarization factor, kinematics, polarization transfer from polarized lepton to photon, D ~ y^2

First Moments of SPIN SFs

- With $$\Delta q = \int \Delta q(x)dx$$

$$g_1(x) = \frac{1}{2}\Sigma_f e_f^2\{q_f^+(x) - q_f^-(x)\} = \frac{1}{2}\Sigma_f e_f^2 \Delta q_f(x)$$

$$\Gamma_1^p = \frac{1}{2}\left[\frac{4}{9}\Delta u + \frac{1}{9}\Delta d + \frac{1}{9}\Delta s\right]$$

$$= \frac{1}{12}\underbrace{(\Delta u - \Delta d)}_{a_3=g_a} + \frac{1}{36}\underbrace{(\Delta u + \Delta d - 2\Delta s)}_{a_8} + \frac{1}{9}\underbrace{(\Delta u + \Delta d + \Delta s)}_{a_0}$$

Neutron decay

(3F-D)/3
Hyperon Decay

$\Delta\Sigma$

$$\Gamma_1^{p,n} = \frac{1}{12}\left[\pm a_3 + \frac{1}{\sqrt{3}}a_8\right] + \frac{1}{9}a_0$$

First moment of $g_1^p(x)$: Ellis-Jaffe SR

$$\Gamma_1^{p,n} = \frac{1}{12}\left[\pm a_3 + \frac{1}{\sqrt{3}} a_8\right] + \frac{1}{9} a_0$$

$$a_3 = \frac{g_A}{g_V} = F + D = 1.2601 \pm 0.0025$$

$$a_8 = 3F - D \Longrightarrow F/D = 0.575 \pm 0.016$$

Assuming $SU(3)_f$ & $\Delta s = 0$, Ellis & Jaffe: $\Gamma_1^p = 0.170 \pm 0.004$

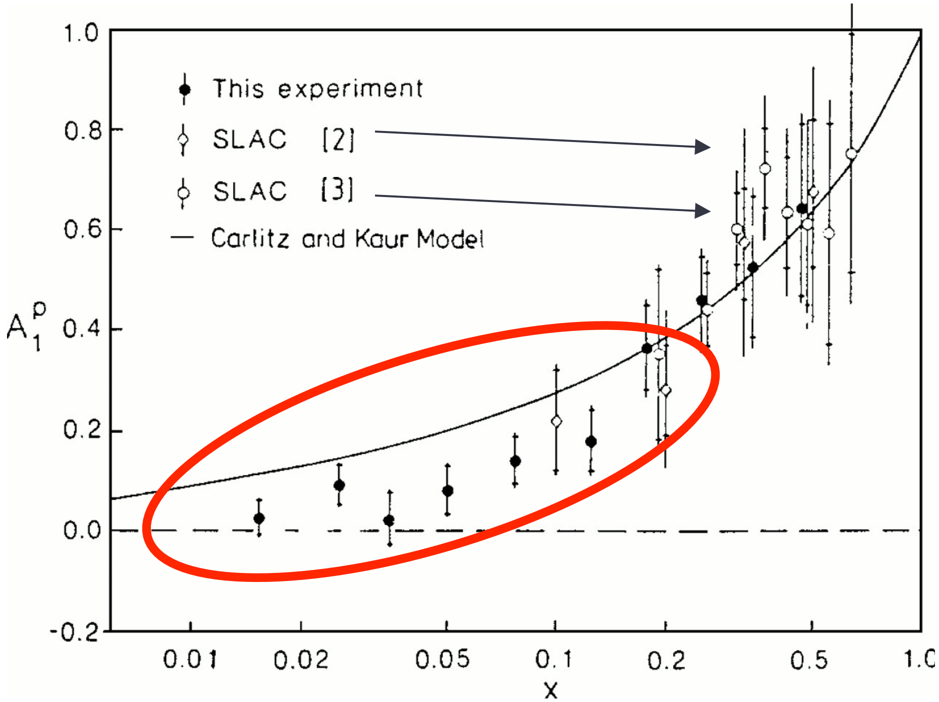
Measurements were done at SLAC (E80, E130) Experiments:

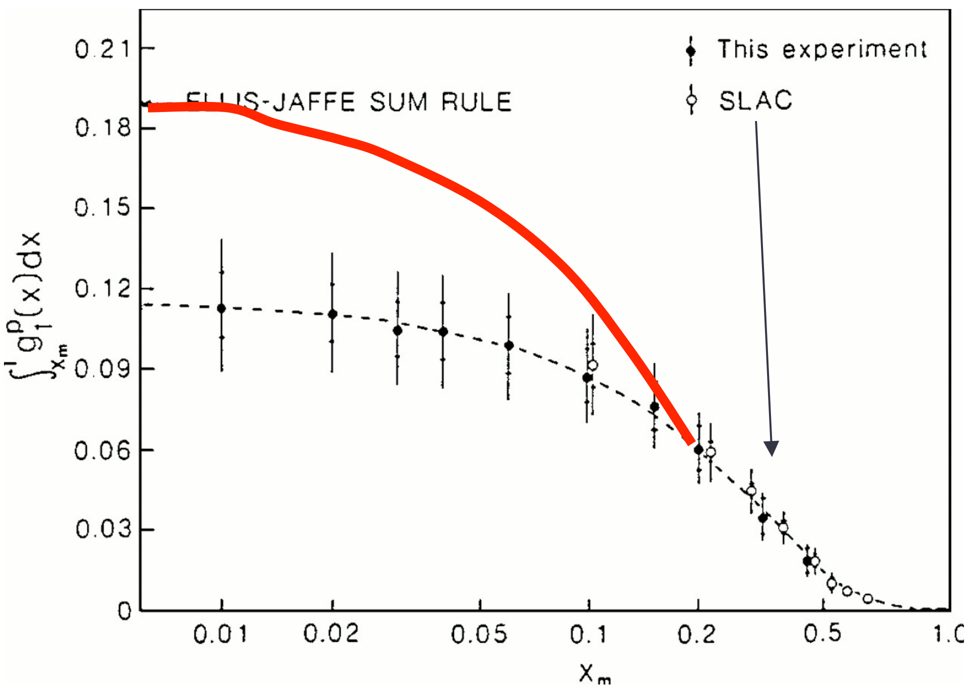
- Low 8-20 GeV electron beam on fixed target
- Did not reach low enough x → $x_{min} \sim 10^{-2}$
- Found consistency of data and E-J sum rule above

European Muon Collaboration at CERN

- 160 GeV muon beam (lower intensity), but significantly higher energy
- Significantly LOWER X reach → $x_{min} \sim 10^{-3}$
- Polarized target
- Repeated experiment for A_1 and measured g_1 of the proton!

Proton Spin Crisis (1989)!





$\Delta\Sigma$ = (0.12) +/- (0.17) (EMC, 1989)

$\Delta\Sigma$ = 0.58 expected from E-J sum rule….

Extrapolations!

The most simplistic but intuitive theoretical predictions for the polarized deep inelastic scattering are the **sum rules** for the nucleon structure function g_1.

$$\Gamma_1(Q^2) = \int_0^1 g_1(x,Q^2)\,dx$$

Due to experimental limitations, accessibility of x range is limited, and extrapolations to x= 0 and x = 1 are **unavoidable**.

Extrapolations to x = 1, are *somewhat* less problematic:

$$|A_1| \leq 1$$

- Small contribution to the integral
- Future precisions studies at JLab 12GeV of great interest

Low x behavior of $g_1(x)$ is theoretically not well established hence of significant debate and excitement in the community

Low x behavior of g_1

- Regge models (mostly used until mid 1990s):

$$Q^2 << 2M\nu, \text{ i.e. }, x \to 0, \; g_1^p \pm g_1^n \to x^{-\alpha} \qquad \boxed{-0.5 < \alpha < 0.5}$$

Where α is the intercept of the lowest contributing Regge trajectories

- Other model dependent expectations (non-QCD based):

$$\boxed{g_1(x) \quad \propto \quad [2\ \ln(1/x) - 1]} \qquad \boxed{g_1(x) \propto (x\ \ln^2 x)^{-1}}$$

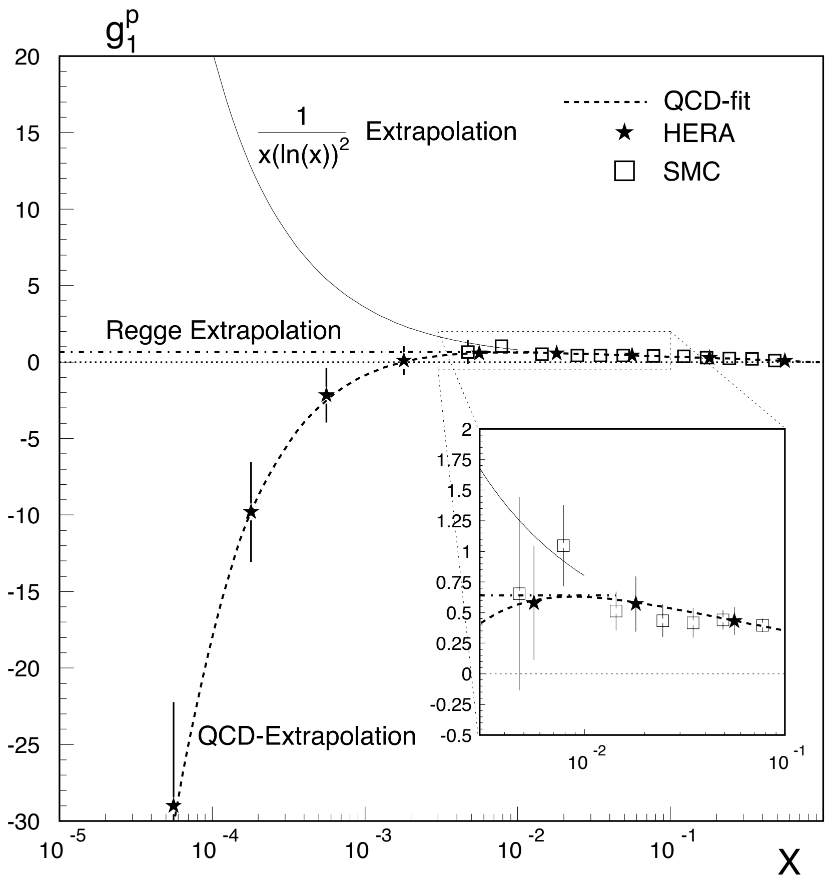
- QCD based calculations:

Resummation of AP: $\boxed{g_1(x,Q^2) \sim \exp A\sqrt{\ln[\alpha_x(Q_0^2)/\alpha_s(Q^2)]\ln(1/x)}}$

Resum of leading power of ln(1/x) gives:

$$\boxed{g_1^{NS}(x,Q^2) \sim x^{-w_{NS}}, \quad w_{NS} \sim 0.4} \qquad \boxed{g_1^{S}(x,Q^2) \sim x^{-w_S}, \quad w_S \sim 3w_{NS}}$$

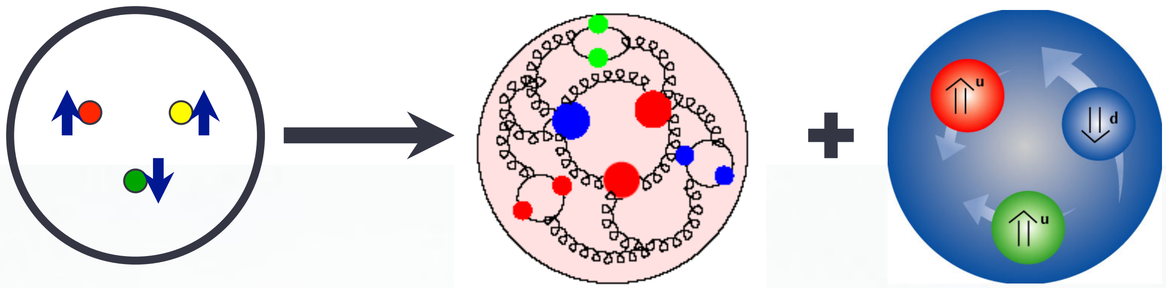
A collection of low x behaviors:



Deshpande, Hughes, Lichtenstadt, HERA low x WS (1999) Simulated data for polarized e-p scattering shown in the figure. Polarized HERA was not realize!

- Low x behavior all over the place
- No theoretical guidance for which one is correct
- Only logical path is though measurements.
  - Not easy
  - But planned in future
  - See lectures on EIC later in the week.

Evolution: Our Understanding of Nucleon Spin



1980s

1990/2000s

$\Delta\Sigma$ = 0.12 +/- 0.17

We have come a long way, but do we understand nucleon spin?

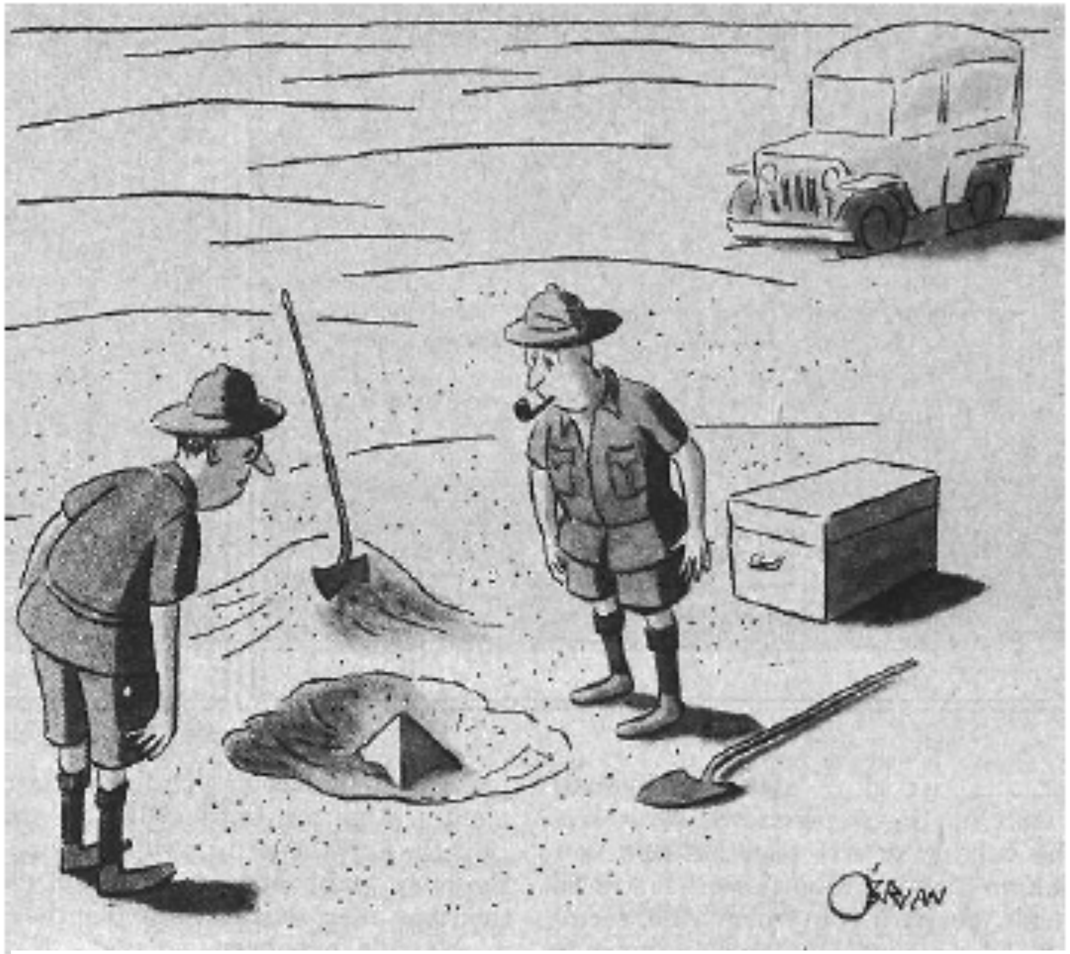
Lesson:

- Every time we explored a physical observable with “spin” as one of the experimental variable, we have learnt something new about nature….

- **But was this really a “ spin crisis”?**
  - Experimental uncertainties too large
  - The assumptions: naïve (constituent) quark model
  - ***We needed to examine and improve on both fronts!***

This is precisely what was done in the following decade….

How significant is this?



of course, on how far down it goes.

"It could the discovery of the century. Depending, of course on how far below it goes…"