Second International Summer School of the GDR PH-QCD

"Correlations between partons in nucleons"

Multidimensional pictures of the nucleon (2/3)

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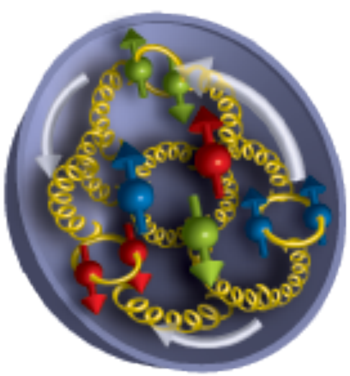
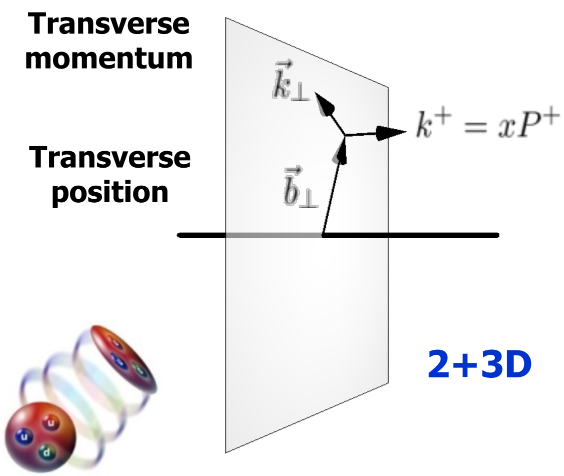
Lecture 1

- **Understanding nucleon internal structure is essential**
- **Concept of phase space can be generalized to QM and QFT**
- **Relativistic effects force us to abandon 1D in phase space**





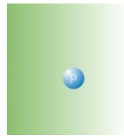
Outline

Lecture 1

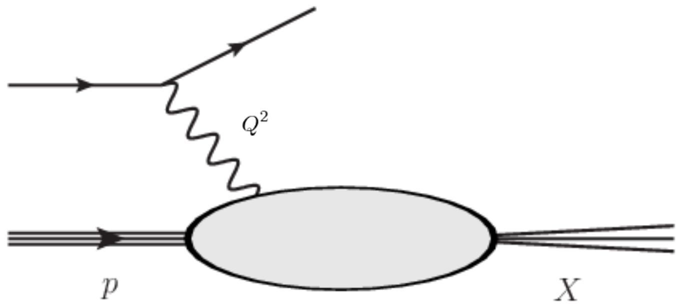
- **Introduction**
- **Tour in phase space**
- **Galileo *vs* Lorentz**
- **Photon point of view**

Lecture 2

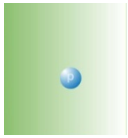
- **Nucleon 1D picture**
- **Nucleon 2D picture**
- **Nucleon 2+1D picture**
- **Energy-momentum tensor**



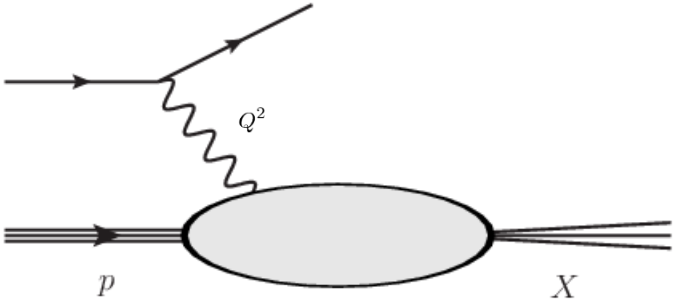
Coherent scattering

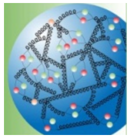


Deep inelastic scattering

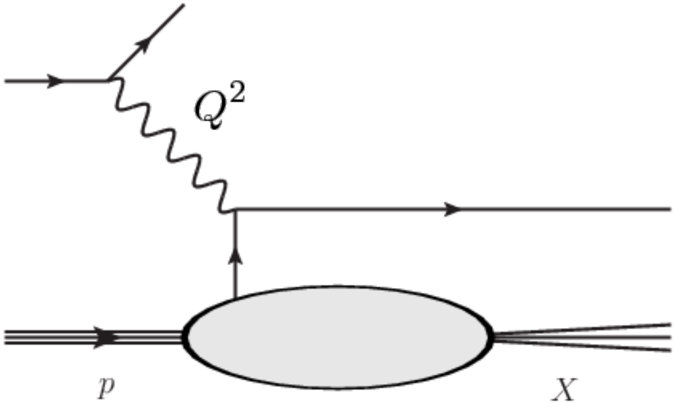


Coherent scattering

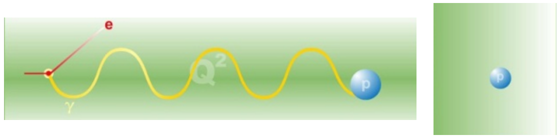




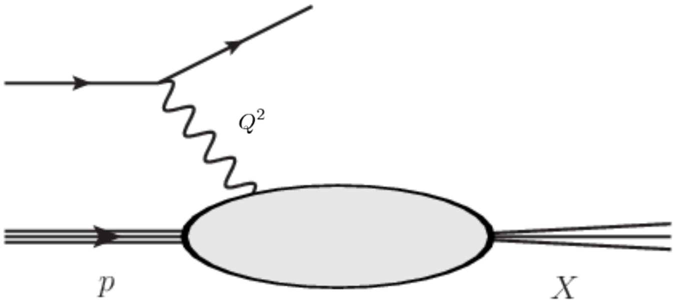
Incoherent scattering

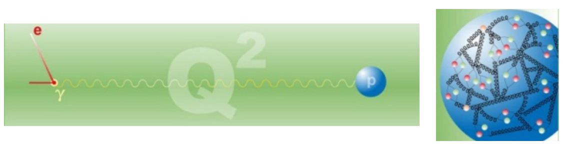


Deep inelastic scattering

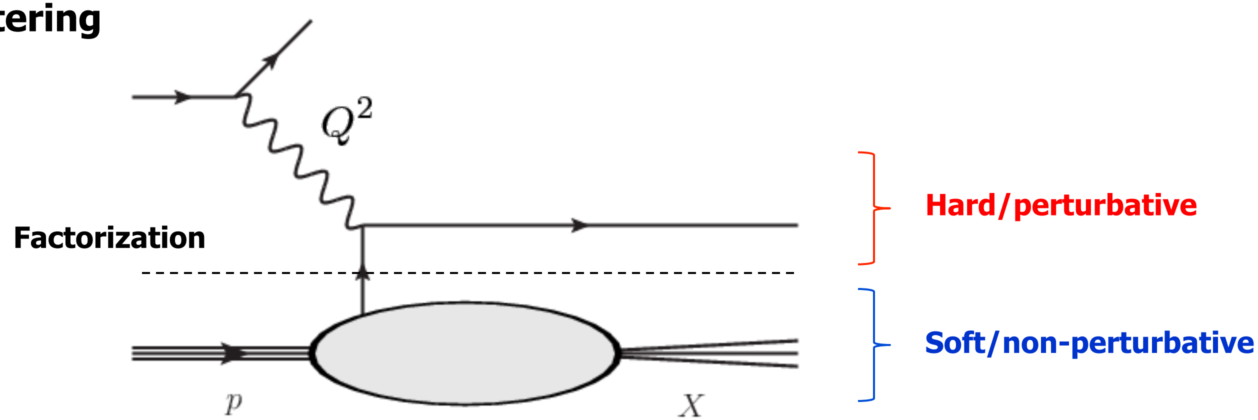


Coherent scattering



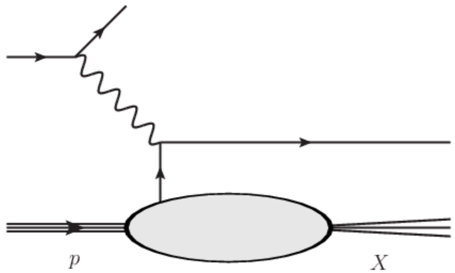


Incoherent scattering



Deep inelastic scattering

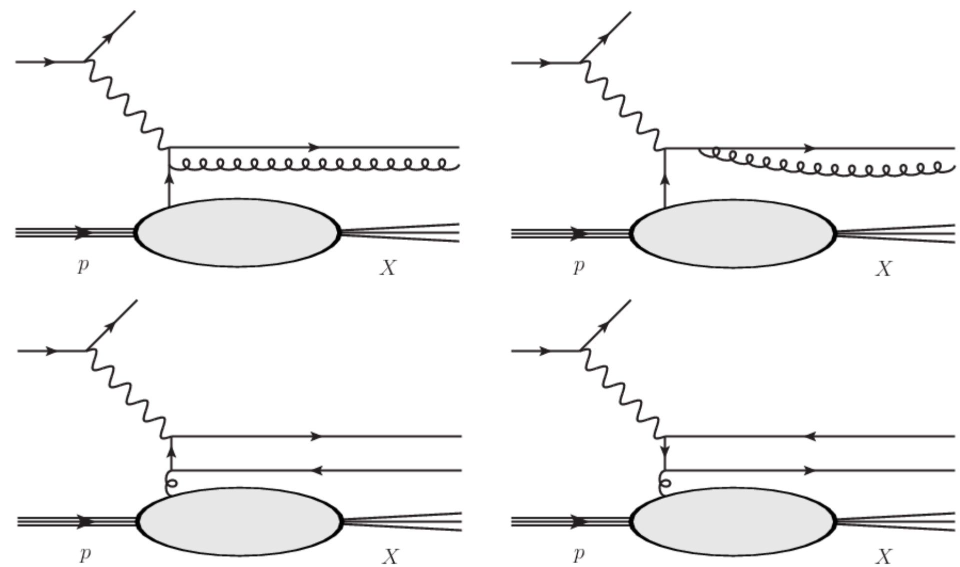
Parton model



$$d\sigma^{lp\to lX} \sim \sum_i f_i(x)$$

Probability to find parton i with momentum fraction x

QCD corrections



+ virtual diagrams

$$\boxed{d\sigma^{lp\to lX} \sim \sum_i \int_x^1 \frac{dy}{y}\, C_i\left(\frac{x}{y}, \frac{Q^2}{\mu_F^2}; \alpha_S(\mu_R)\right) f_i(y, \mu_F^2)}$$

Perturbative
Process-dependent

Non-perturbative
Process-independent

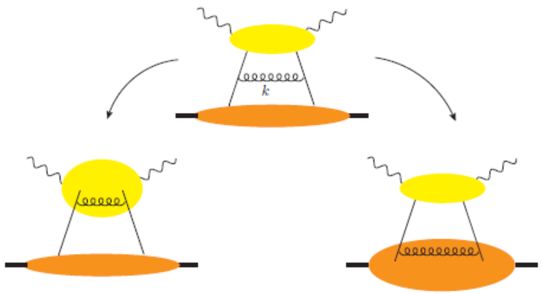
$i = q_f, \bar{q}_f, g$

$\mu_F \approx \mu_R$

Deep inelastic scattering

Cross-section must not depend on μ

$$\frac{\mathrm{d}\sigma^{lp\to lX}}{\mathrm{d}\ln\mu} = 0$$

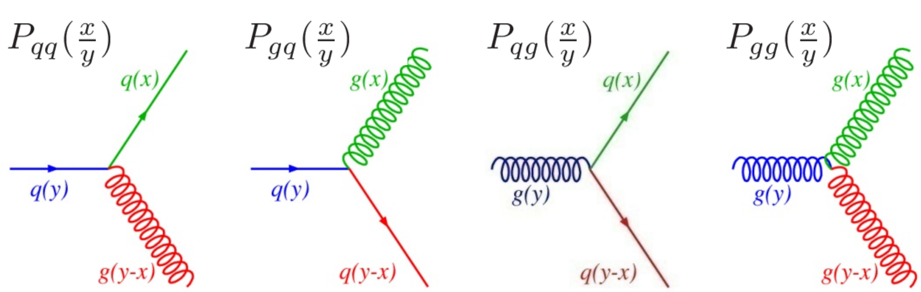


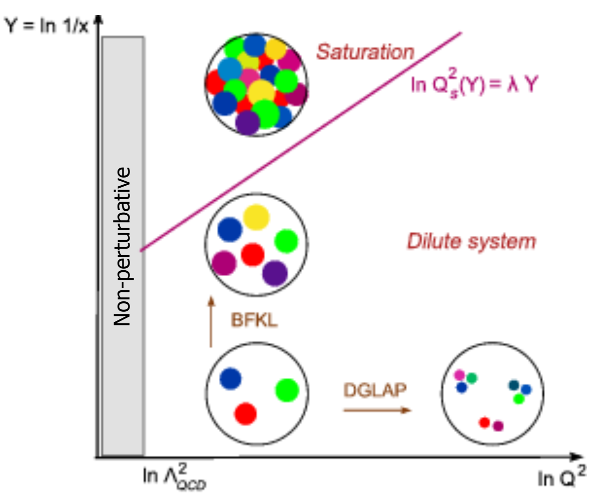
→ **DGLAP evolution equations**

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu} f_i(x,\mu^2) = \sum_j \int_x^1 \frac{\mathrm{d}y}{y} P_{ij}(\tfrac{x}{y})\, f_j(y,\mu^2)$$

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu} C_i(x,\tfrac{Q^2}{\mu^2}) = -\sum_j \int_x^1 \frac{\mathrm{d}y}{y} C_j(y,\tfrac{Q^2}{\mu^2})\, P_{ji}(\tfrac{x}{y})$$

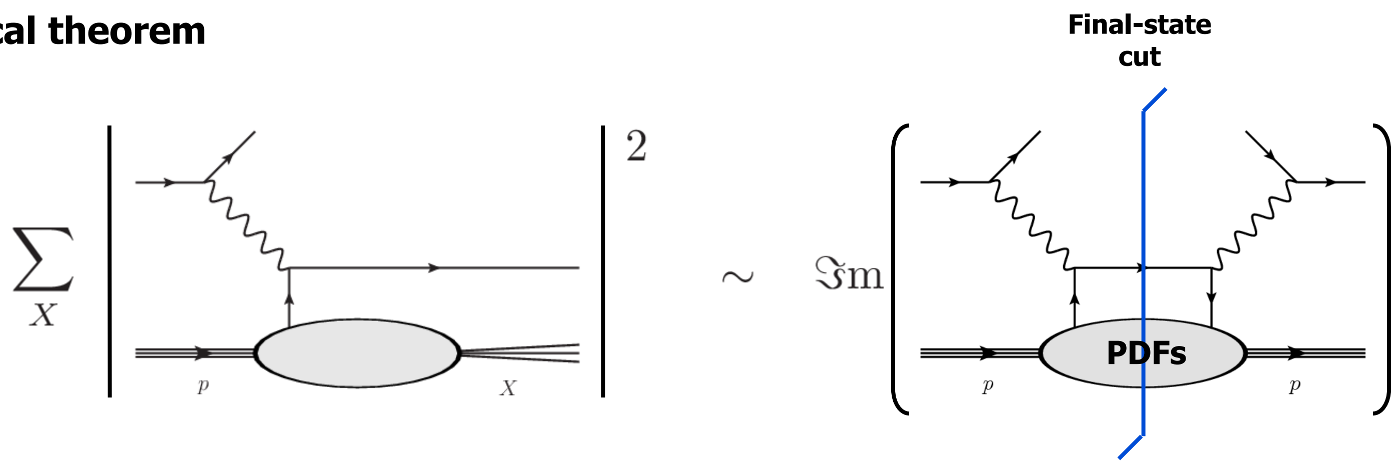
Splitting functions





Parton distribution functions (PDFs)

Optical theorem



PDF correlator

$$\mathcal{F}^{[\Gamma]}_{\Lambda'\Lambda}(x) = \frac{1}{2}\int \frac{\mathrm{d}z^-}{2\pi}\, e^{ixP^+z^-} \langle P,\Lambda'|\overline{\psi}(-\tfrac{z}{2})\,\Gamma\,\mathcal{W}\,\psi(\tfrac{z}{2})|P,\Lambda\rangle\big|_{z^+=z_\perp=0}$$

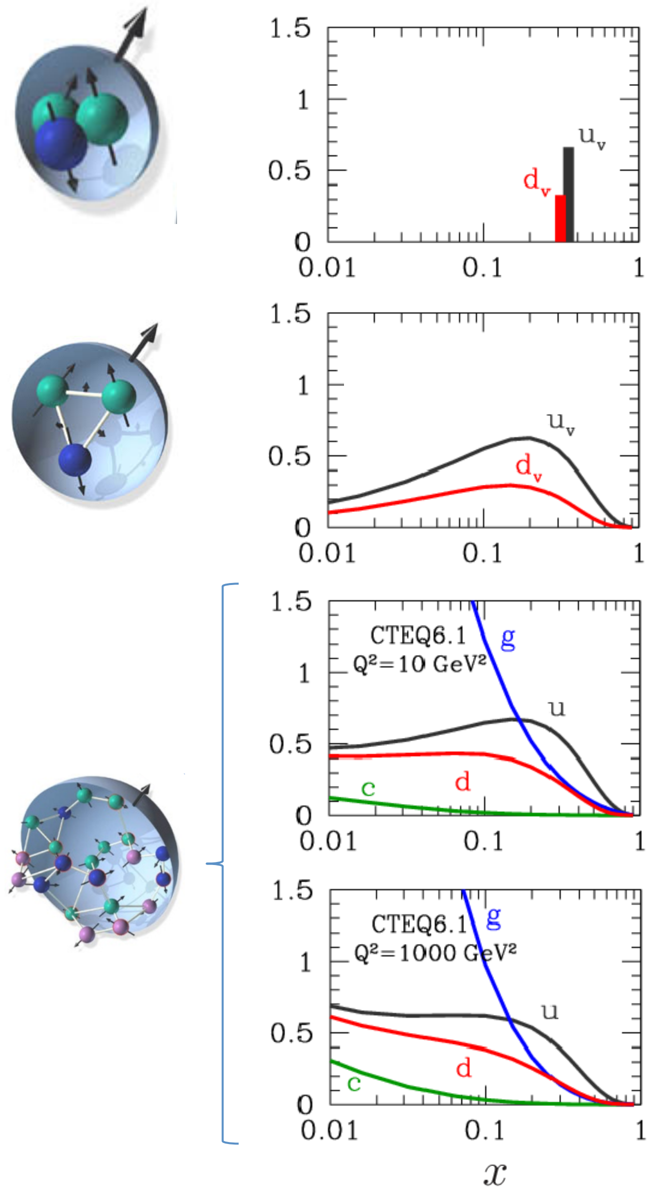
$$= \int \mathrm{d}^2k_\perp\, W^{[\Gamma]}_{\Lambda'\Lambda}(x,0,\vec{k}_\perp,\vec{0}_\perp)$$

Parametrization

$$\mathcal{F}^{[\Gamma]}_{\Lambda'\Lambda}(x) = \sum_i \left[\overline{u}(P,\Lambda')\,\Gamma_i\,u(P,\Lambda)\right]\,\mathrm{PDF}_i(x)$$

Constrained by Lorentz and discrete space-time symmetries

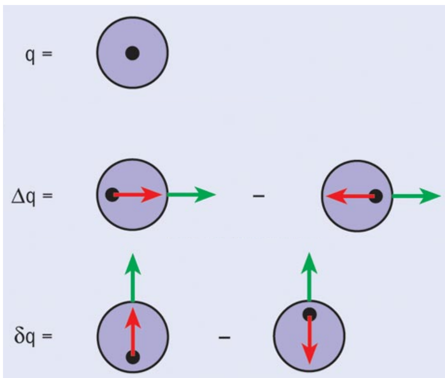
Parton distribution functions (PDFs)

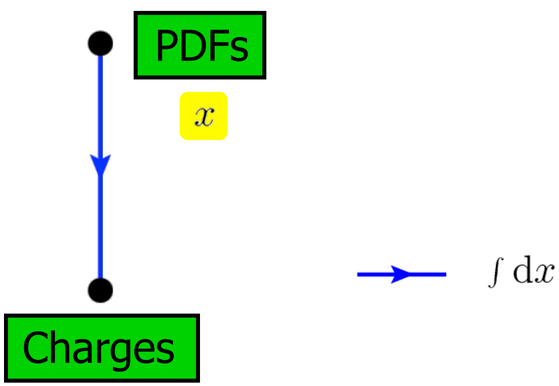


Vector
Quark number

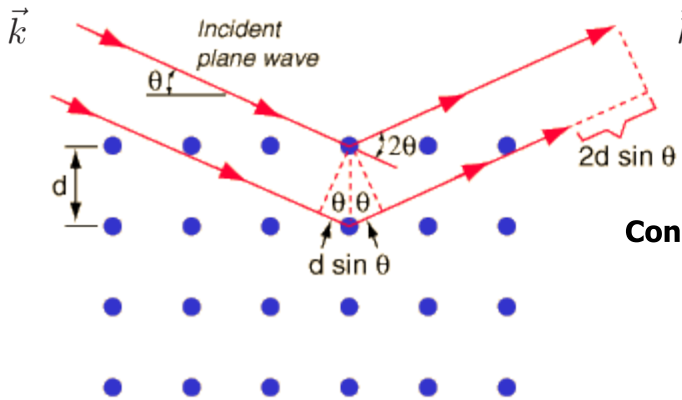
Axial
Quark helicity

Tensor
Quark transversity





Elastic scattering

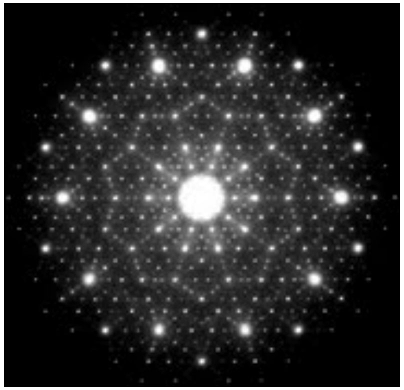


Crystal

Constructive interference (Bragg's law)

$$2d\,\sin\theta = n\lambda$$

Diffraction pattern



$$\propto |A_{\rm scatt}|^2$$

Scattered amplitude

$$A_{\rm scatt} \propto F(\vec{q}) = \int \mathrm{d}^3r\, e^{i\vec{q}\cdot\vec{r}}\, \rho(\vec{r}) \qquad \vec{q} = \vec{k} - \vec{k}'$$

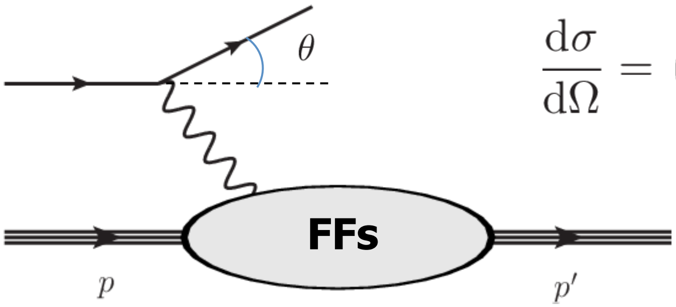
Form factor **Scatterer distribution**

Reconstructed charge distribution

$$\rho(\vec{r}) = \int \frac{\mathrm{d}^3q}{(2\pi)^3}\, e^{-i\vec{q}\cdot\vec{r}}\, F(\vec{q})$$

Let's replace the crystal by a nucleon !

Form factors (FFs)



$$\frac{d\sigma}{d\Omega} = \underbrace{\left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{Mott}}}_{\textbf{Pointlike}} \underbrace{\left[F_1^2(Q^2) + \tau F_2^2(Q^2) + 2\tau\, G_M^2(Q^2)\, \tan^2\frac{\theta}{2}\right]}_{\textbf{Spatial structure}}$$

$$Q^2 = -\Delta^2$$

$$\tau = \frac{Q^2}{4M^2}$$

FF correlator

$$\begin{aligned} A^{[\Gamma]}_{\Lambda'\Lambda}(\Delta) &= \frac{1}{2P^+} \langle p', \Lambda' | \overline{\psi}(0)\, \Gamma\, \psi(0) | p, \Lambda\rangle \\ &= \int dx\, d^2k_\perp\, W^{[\Gamma]}_{\Lambda'\Lambda}(x, \xi, \vec{k}_\perp, \vec{\Delta}_\perp) \end{aligned}$$

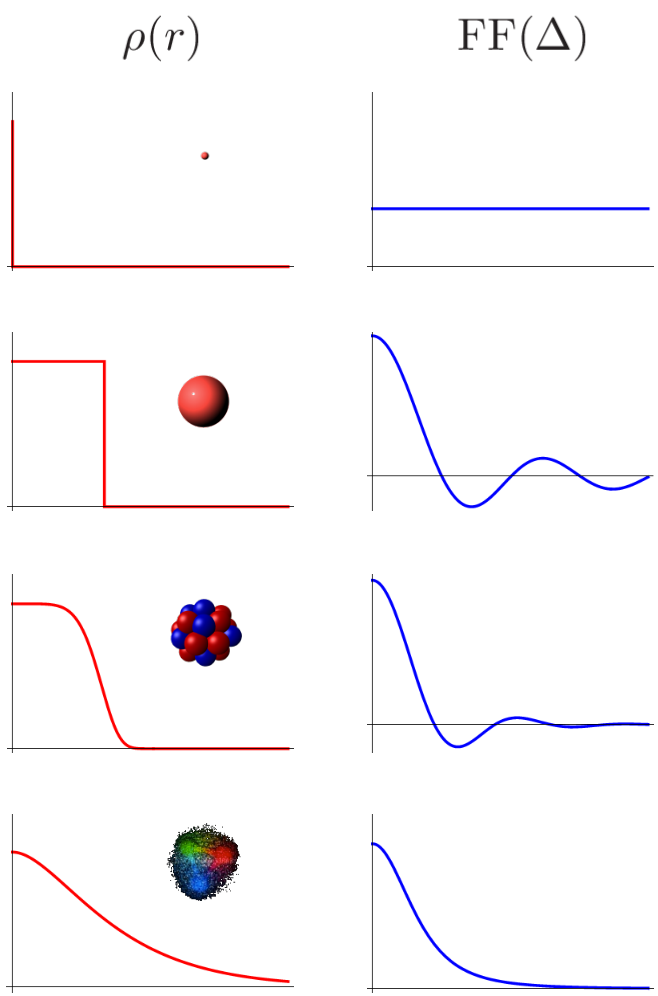
Parametrization (electromagnetic case)

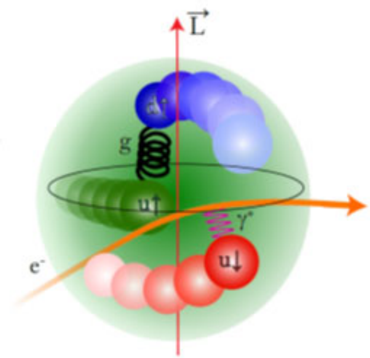
$$\langle p', \Lambda' | \overline{\psi}(0) \gamma^\mu \psi(0) | p, \Lambda\rangle = \overline{u}(p', \Lambda') \left[\gamma^\mu\, F_1(Q^2) + \frac{i\sigma^{\mu\nu}\Delta_\nu}{2M}\, F_2(Q^2)\right] u(p, \Lambda)$$

Sachs FFs

$$\begin{aligned} G_E(Q^2) &= F_1(Q^2) - \tau\, F_2(Q^2) \\ G_M(Q^2) &= F_1(Q^2) + F_2(Q^2) \end{aligned}$$

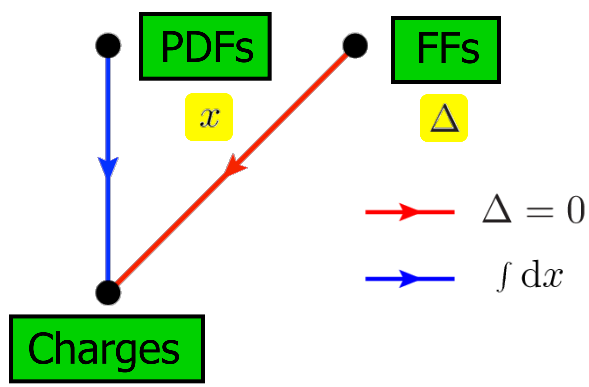
Form factors (FFs)





$F_1(0) = e$ **Electric charge**

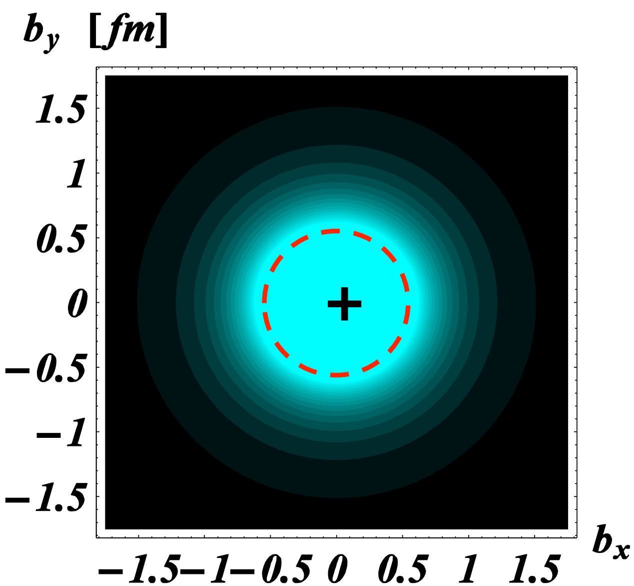
$F_2(0) = \kappa$ **Anomalous magnetic moment**



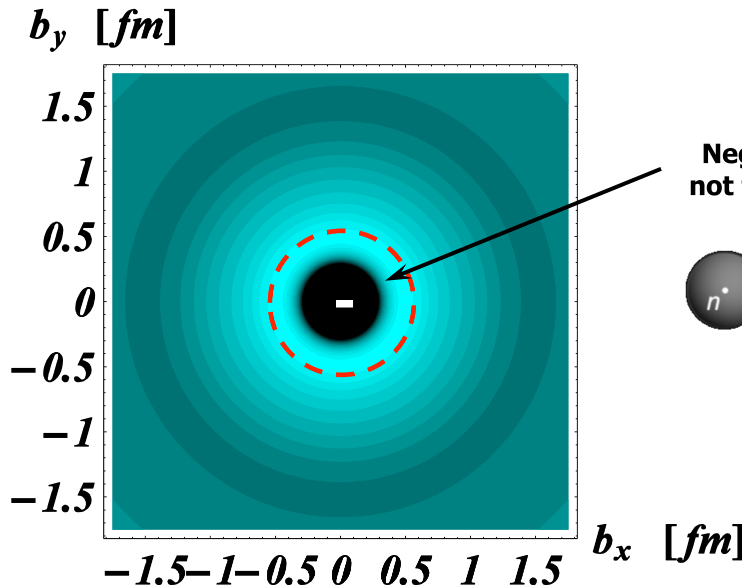
Longitudinally polarized nucleon

$$\rho(\vec{b}_\perp) = \underbrace{\int \frac{\mathrm{d}Q}{2\pi}\, Q\, J_0(Qb_\perp)\, F_1(Q^2)}_{\textbf{Monopole}}$$

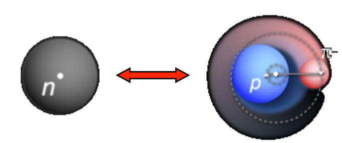
Proton



Neutron



Negative core does not fit naive picture !



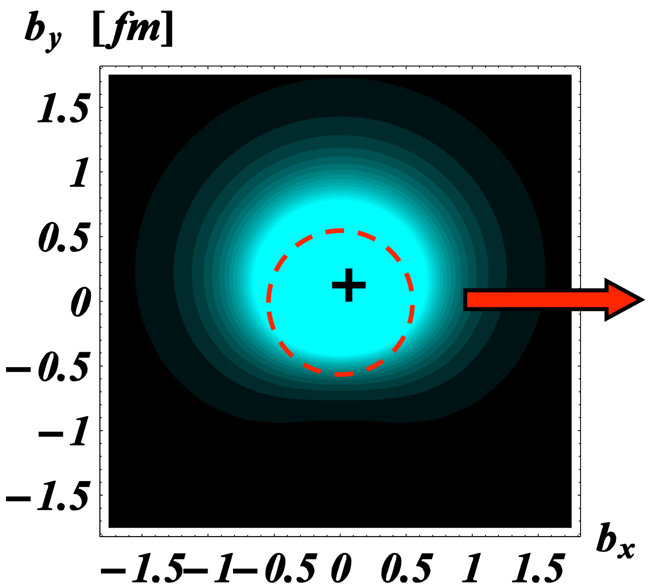
[Miller (2007)]

Form factors (FFs)

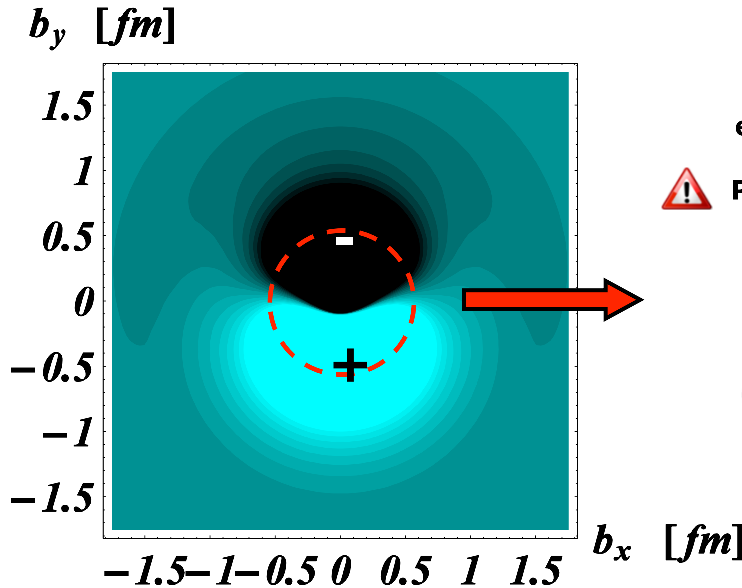
Transversely polarized nucleon

$$\rho_T(\vec{b}_\perp) = \underbrace{\rho(\vec{b}_\perp)}_{\text{Monopole}} + \underbrace{\sin(\phi_b - \phi_S) \int \frac{\mathrm{d}Q}{2\pi} \frac{Q^2}{2M} J_1(Q b_\perp) F_2(Q^2)}_{\text{Dipole}}$$

Proton

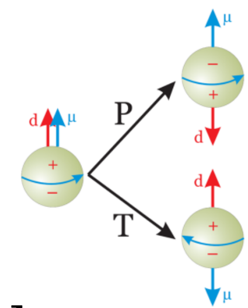


Neutron



Apparent electric dipole !

P & T violations ???

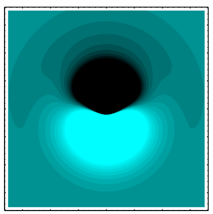


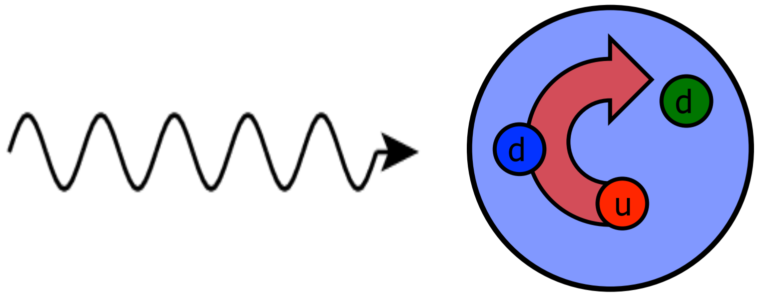
No !!!

[Carlson, Vanderhaeghen (2008)]

Transversely polarized nucleon

Origin of EDM : light-front artifact





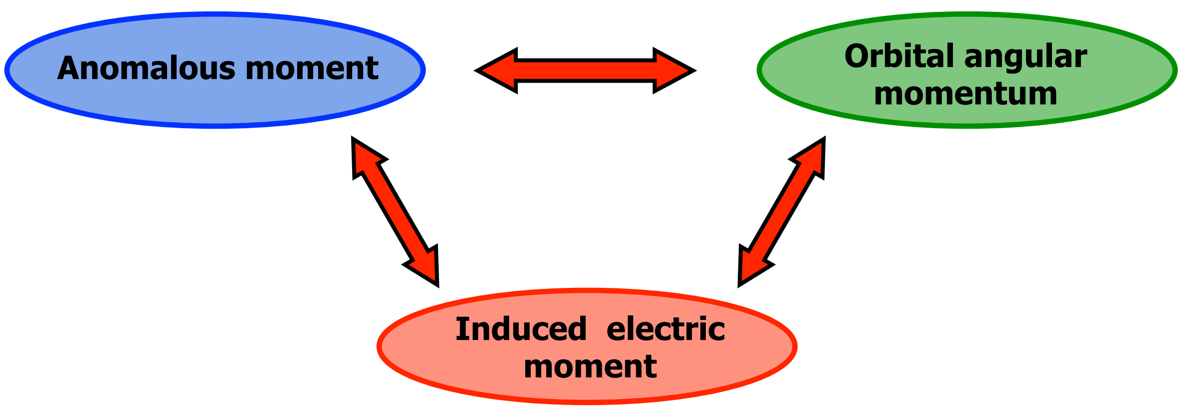
$j^3 < 0$

$j^3 > 0$

$$\boxed{j^+ = \frac{1}{\sqrt{2}}(j^0 + j^3)}$$

$\vec{S} \odot \quad \kappa_n = -1.91$

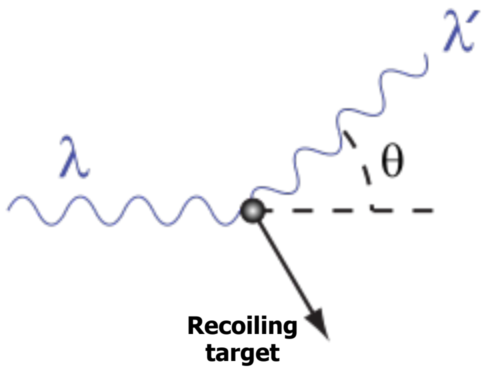
[Burkardt (2003)]



[Lorcé (2009)]

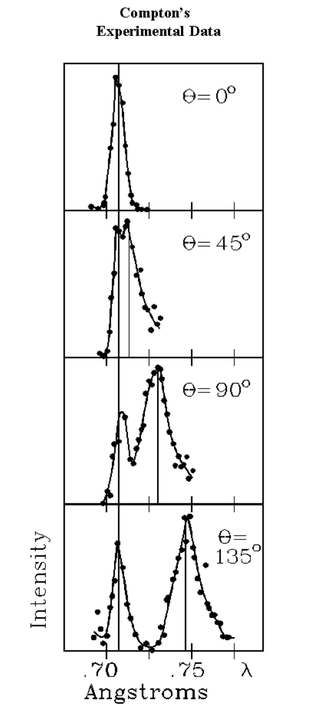
Compton scattering

Real Compton Scattering



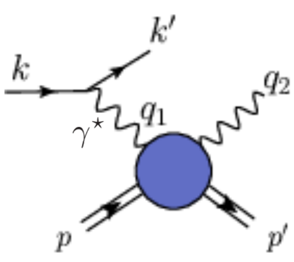
$$\Delta\lambda = \frac{2\pi}{m}(1-\cos\theta)$$

Compton wavelength



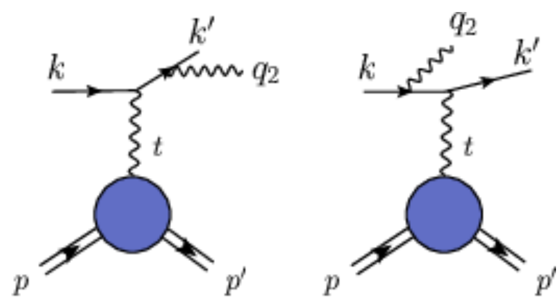
Question : why are there 2 peaks?

Virtual Compton scattering (VCS)



VCS

interferes with



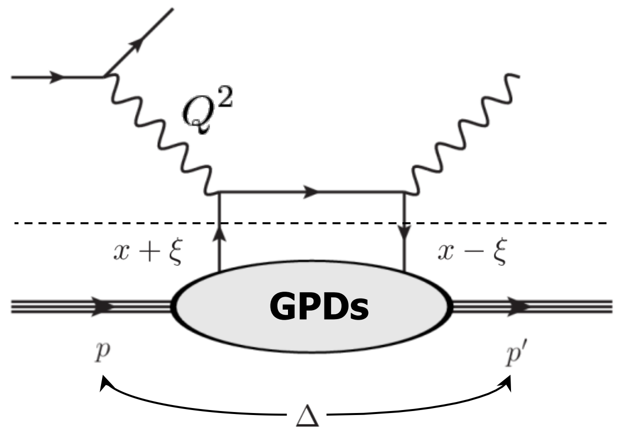
Bethe-Heitler

Deeply virtual Compton scattering (DVCS)

$P = \frac{p'+p}{2}$ $\quad x = \frac{k^+}{P^+}$

$\Delta = p' - p$ $\quad \xi = -\frac{\Delta^+}{2P^+}$

$t = \Delta^2$



Compton FF

$$T_{\rm DVCS} \sim \boxed{\int_{-1}^{1} \mathrm{d}x\, \frac{\mathrm{GPD}(x,\xi,t)}{x \pm \xi - i\epsilon}} + \mathcal{O}(\tfrac{t}{Q^2})$$

$$\sim \mathrm{PV} \int_{-1}^{1} \mathrm{d}x\, \frac{\mathrm{GPD}(x,\xi,t)}{x \pm \xi} \pm i\pi\, \mathrm{GPD}(\mp\xi,\xi,t) + \mathcal{O}(\tfrac{t}{Q^2})$$

Factorization
Evolution @ NLO

[Collins, Freund (1999)]
[Belitsky *et al.* (2000)]

GPD correlator

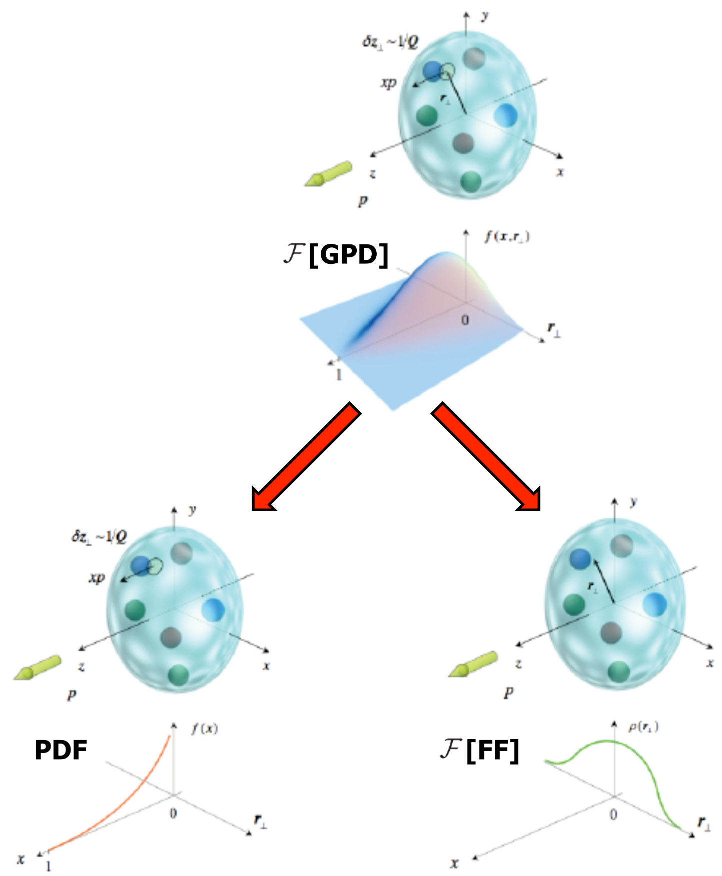
$$F^{[\Gamma]}_{\Lambda'\Lambda}(x,\Delta) = \frac{1}{2}\int \frac{\mathrm{d}z^-}{2\pi}\, e^{ixP^+z^-}\, \langle p',\Lambda'|\overline{\psi}(-\tfrac{z}{2})\,\Gamma\,\mathcal{W}\,\psi(\tfrac{z}{2})|p,\Lambda\rangle\big|_{z^+=z_\perp=0}$$

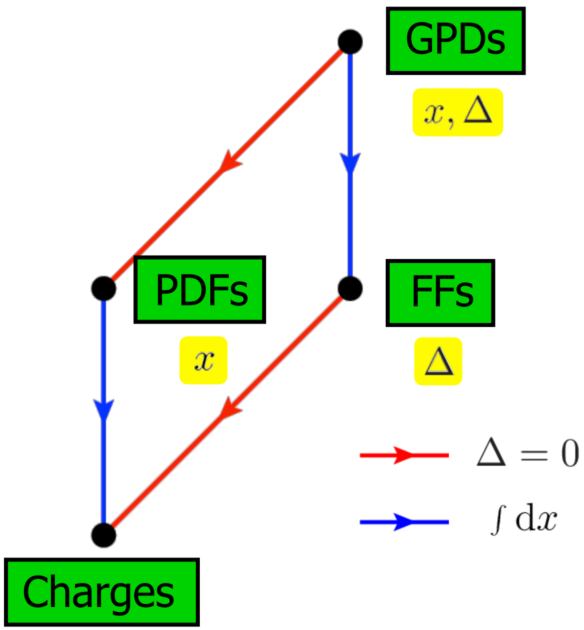
$$= \int \mathrm{d}^2k_\perp\, W^{[\Gamma]}_{\Lambda'\Lambda}(x,\xi,\vec{k}_\perp,\vec{\Delta}_\perp)$$

Reviews :

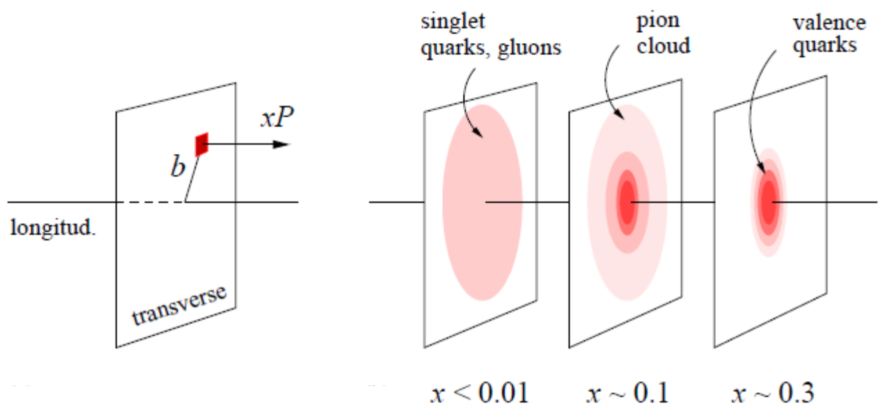
[Diehl (2003)]
[Belitsky, Radyushkin (2005)]
[Boffi, Pasquini (2008)]

Nucleon tomography/imaging

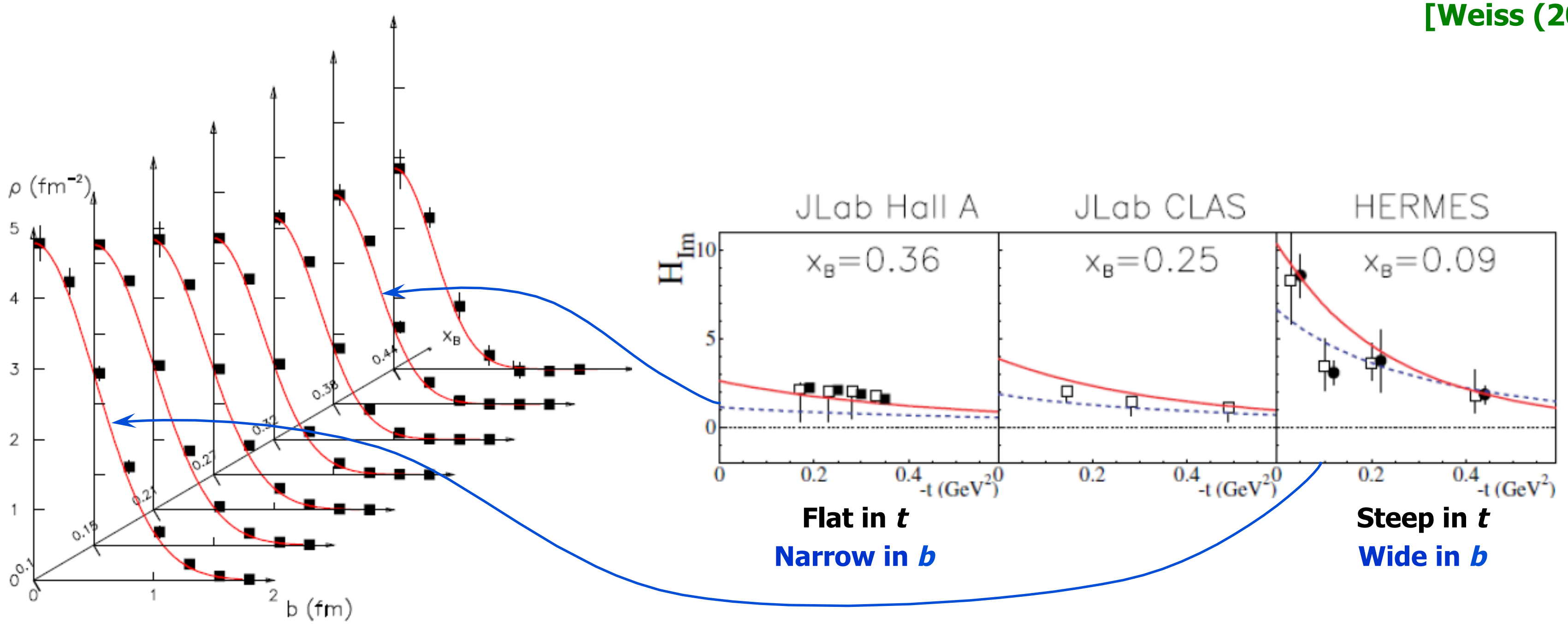




Generalized parton distributions (GPDs)



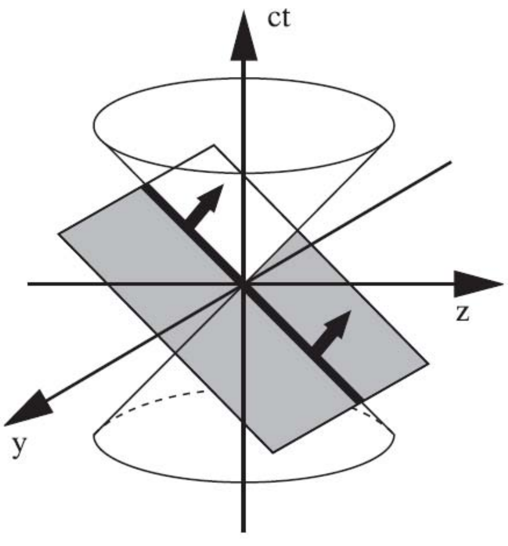
[Weiss (2009)]



[Guidal *et al.* (2013)]

Generalized parton distributions (GPDs)

Explicit Lorentz invariance is broken by the preferred light-front direction

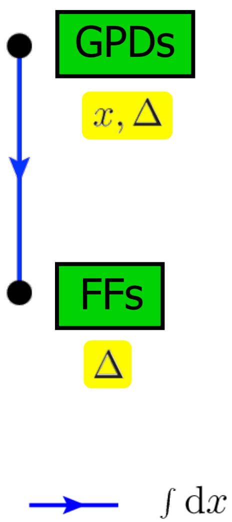


	Lorentz invariance
$t = \Delta^2$	✓
$x = \dfrac{k^+}{P^+}$	✗
$\xi = -\dfrac{\Delta^+}{2P^+}$	✗

→ x **and** ξ **dependences constrained by Lorentz invariance !**

Lowest moment

$$
\begin{aligned}
&P^+ \int \mathrm{d}x \int \frac{\mathrm{d}z^-}{2\pi}\, e^{ixP^+z^-}\, \overline{\psi}(-\tfrac{z^-}{2})\,\Gamma\,\mathcal{W}\,\psi(\tfrac{z^-}{2}) \\
&= \cancel{P}^+ \int \frac{\mathrm{d}z^-}{2\pi}\, 2\pi\,\delta(\cancel{P}^+ z^-)\,\overline{\psi}(-\tfrac{z^-}{2})\,\Gamma\,\mathcal{W}\,\psi(\tfrac{z^-}{2}) \\
&= \overline{\psi}(0)\,\Gamma\,\psi(0)
\end{aligned}
$$



Generic moment in LF gauge $A^+ = 0 \Longrightarrow \mathcal{W} = \mathbb{1}$ $\quad \mathcal{W}_{ba} = \mathcal{P}\left[e^{ig\int_a^b \mathrm{d}x^- A^+}\right]$

$$
\begin{aligned}
&(P^+)^{n+1}\int \mathrm{d}x\, x^n \int \frac{\mathrm{d}z^-}{2\pi}\, e^{ixP^+z^-}\,\overline{\psi}(-\tfrac{z^-}{2})\,\Gamma\,\psi(\tfrac{z^-}{2})\\
&= P^+\int \mathrm{d}x\int\frac{\mathrm{d}z^-}{2\pi}\left[\left(-i\tfrac{\mathrm{d}}{\mathrm{d}z^-}\right)^n e^{ixP^+z^-}\right]\overline{\psi}(-\tfrac{z^-}{2})\,\Gamma\,\psi(\tfrac{z^-}{2})\\
&= P^+\int \mathrm{d}x\int\frac{\mathrm{d}z^-}{2\pi}\, e^{ixP^+z^-}\left(i\tfrac{\mathrm{d}}{\mathrm{d}z^-}\right)^n\left[\overline{\psi}(-\tfrac{z^-}{2})\,\Gamma\,\psi(\tfrac{z^-}{2})\right]\\
&= \cancel{P}^+\int\frac{\mathrm{d}z^-}{2\pi}\,2\pi\,\delta(\cancel{P}^+z^-)\left[\overline{\psi}(-\tfrac{z^-}{2})\,\Gamma\,(\tfrac{i}{2}\overleftrightarrow{\partial}^+)^n\,\psi(\tfrac{z^-}{2})\right]\\
&= \overline{\psi}(0)\,\Gamma\,(\tfrac{i}{2}\overleftrightarrow{\partial}^+)^n\,\psi(0)
\end{aligned}
$$

Generalized parton distributions (GPDs)

Generic moment

$$
\begin{aligned}
&(P^+)^{n+1} \int \mathrm{d}x\, x^n \int \frac{\mathrm{d}z^-}{2\pi}\, e^{ixP^+z^-}\, \overline{\psi}(-\tfrac{z^-}{2})\, \Gamma\, \mathcal{W}\, \psi(\tfrac{z^-}{2}) \\
&= P^+ \int \mathrm{d}x \int \frac{\mathrm{d}z^-}{2\pi} \left[\left(-i\tfrac{\mathrm{d}}{\mathrm{d}z^-}\right)^n e^{ixP^+z^-} \right] \overline{\psi}(-\tfrac{z^-}{2})\, \Gamma\, \mathcal{W}\, \psi(\tfrac{z^-}{2}) \\
&= P^+ \int \mathrm{d}x \int \frac{\mathrm{d}z^-}{2\pi}\, e^{ixP^+z^-} \left(i\tfrac{\mathrm{d}}{\mathrm{d}z^-}\right)^n \left[\overline{\psi}(-\tfrac{z^-}{2})\, \Gamma\, \mathcal{W}\, \psi(\tfrac{z^-}{2}) \right] \\
&= \cancel{P}^+ \int \frac{\mathrm{d}z^-}{2\pi}\, 2\pi\, \delta(\cancel{P}^+ z^-) \left[\overline{\psi}(-\tfrac{z^-}{2})\, \Gamma \left(\mathcal{W}\, \tfrac{i}{2} \overrightarrow{D}^+ - \tfrac{i}{2} \overleftarrow{D}^+\, \mathcal{W} \right)^n \psi(\tfrac{z^-}{2}) \right] \\
&= \overline{\psi}(0)\, \Gamma\, (\tfrac{i}{2} \overleftrightarrow{D}^+)^n\, \psi(0)
\end{aligned}
$$

using

$$
\tfrac{\mathrm{d}}{\mathrm{d}z^-} \mathcal{W}_{-\frac{z^-}{2}\,\frac{z^-}{2}} = -\frac{ig}{2} \left[A^+(-\tfrac{z^-}{2})\, \mathcal{W}_{-\frac{z^-}{2}\,\frac{z^-}{2}} + \mathcal{W}_{-\frac{z^-}{2}\,\frac{z^-}{2}}\, A^+(\tfrac{z^-}{2}) \right]
$$

Derivative of Wilson line

$$
\begin{aligned}
\overrightarrow{D}^+ &= \overrightarrow{\partial}^+ - ig\, A^+(\tfrac{z^-}{2}) \\
\overleftarrow{D}^+ &= \overleftarrow{\partial}^+ + ig\, A^+(-\tfrac{z^-}{2})
\end{aligned}
$$

Covariant derivatives

Generalized parton distributions (GPDs)

Generic local operator

$$\hat{O}_{\Gamma}^{\mu_1\cdots\mu_n} = \mathbf{S}\left[\overline{\psi}(0)\,\Gamma\,\tfrac{i}{2}\overleftrightarrow{D}^{\mu_1}\cdots\tfrac{i}{2}\overleftrightarrow{D}^{\mu_n}\,\psi(0)\right]$$

Symmetrization
+ trace removal

Generic FFs

Constrained by Lorentz and
discrete space-time symmetries

$$\langle p'|\hat{O}_{\Gamma}^{\mu_1\cdots\mu_n}|p\rangle = \sum_{\Gamma'} \overline{u}(p')\,\Gamma'\,u(p)\sum_{i=0}^{n}\mathrm{FF}_{n+1,i}^{\Gamma\Gamma'}(t)\,\Delta^{\mu_1}\cdots\Delta^{\mu_i}\,P^{\mu_{i+1}}\cdots P^{\mu_n}$$

Polynomiality of GPDs

$$\boxed{\int_{-1}^{1}\mathrm{d}x\,x^n\,\mathrm{GPD}(x,\xi,t) = \sum_{i=1}^{n}(-2\xi)^{c_i}\,\mathrm{FF}_{n+1,i}(t)}$$

Generalized parton distributions (GPDs)

How to ensure polynomiality property?

$$\langle p'|\overline{\psi}(-\tfrac{z}{2})\,\gamma^+\,\mathcal{W}\,\psi(\tfrac{z}{2})|p\rangle\Big|_{z^+=z_\perp=0} \longrightarrow \langle p'|\overline{\psi}(-\tfrac{z}{2})\,\not{z}\,\mathcal{W}\,\psi(\tfrac{z}{2})|p\rangle\Big|_{z^2=0}$$

« Covariantization »

Lorentz-invariant variables

$$P^\mu \quad \Delta^\mu \quad z^\mu \longrightarrow \begin{array}{ccc} P\cdot\Delta = 0 & \boxed{P\cdot z} & \boxed{\Delta\cdot z} \\ P^2 = M^2 - \frac{t}{4} & \boxed{\Delta^2 = t} & z^2 = 0 \end{array}$$

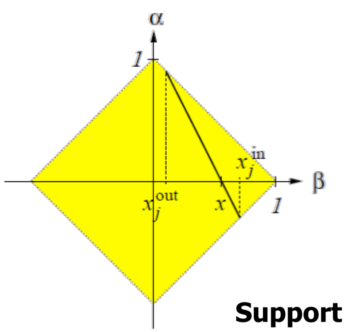
GPD parametrization

$$\langle p'|\overline{\psi}(-\tfrac{z}{2})\,\not{z}\,\mathcal{W}\,\psi(\tfrac{z}{2})|p\rangle\big|_{z^2=0} = \overline{u}(p')\,\not{z}\,u(p)\int \mathrm{d}x\, e^{-ixP\cdot z}\boxed{H^q(x,\xi,t)}$$
$$+\,\overline{u}(p')\,\frac{i\sigma^{\mu\nu}z_\mu\Delta_\nu}{2M}\,u(p)\int \mathrm{d}x\, e^{-ixP\cdot z}\boxed{E^q(x,\xi,t)}$$

Constraint : $\xi = -\frac{\Delta\cdot z}{2P\cdot z}$

Double distribution (DD) parametrization

$$\langle p'|\overline{\psi}(-\tfrac{z}{2})\,\not{z}\,\mathcal{W}\,\psi(\tfrac{z}{2})|p\rangle\big|_{z^2=0} = \overline{u}(p')\,\not{z}\,u(p)\int \mathrm{d}\beta\,\mathrm{d}\alpha\, e^{-i(\beta P-\alpha\frac{\Delta}{2})\cdot z}\boxed{f^q(\beta,\alpha,t)}$$
$$+\,\overline{u}(p')\,\frac{i\sigma^{\mu\nu}z_\mu\Delta_\nu}{2M}\,u(p)\int \mathrm{d}\beta\,\mathrm{d}\alpha\, e^{-i(\beta P-\alpha\frac{\Delta}{2})\cdot z}\boxed{k^q(\beta,\alpha,t)}$$
$$-\,\overline{u}(p')\,\frac{\Delta\cdot z}{2M}\,u(p)\int \mathrm{d}\alpha\, e^{i\alpha\frac{\Delta}{2}\cdot z}\boxed{D^q(\alpha,t)}$$



Support : $|\beta|+|\alpha|\leq 1$

[Müller *et al.* (1994)]
[Radyushkin (1999)]
[Polyakov, Weiss (1999)]

Generalized parton distributions (GPDs)

Relation between GPDs and DDs

$$H^q(x,\xi,t) = \int \mathrm{d}\beta\,\mathrm{d}\alpha\,\delta(x-\beta-\xi\alpha)\,f^q(\beta,\alpha,t) + \mathrm{sgn}(\xi)\,D^q(\tfrac{x}{\xi},t)$$

$$E^q(x,\xi,t) = \int \mathrm{d}\beta\,\mathrm{d}\alpha\,\delta(x-\beta-\xi\alpha)\,k^q(\beta,\alpha,t) - \mathrm{sgn}(\xi)\,D^q(\tfrac{x}{\xi},t)$$

Polynomiality property

$$\begin{aligned}\int \mathrm{d}x\,x^n\,H^q(x,\xi,t) &= \int \mathrm{d}x\,x^n\left[\int \mathrm{d}\beta\,\mathrm{d}\alpha\,\delta(x-\beta-\xi\alpha)\,f^q(\beta,\alpha,t) + \mathrm{sgn}(\xi)\,D^q(\tfrac{x}{\xi},t)\right]\\ &= \int \mathrm{d}\beta\,\mathrm{d}\alpha\,(\beta+\xi\alpha)^n\,f^q(\beta,\alpha,t) + \mathrm{sgn}(\xi)\,\xi^{n+1}\int \mathrm{d}y\,y^n\,D^q(y,t)\end{aligned}$$

$\Rightarrow$ α **and** β **dependences not constrained by Lorentz invariance !**

[Müller *et al.* (1994)]
[Radyushkin (1999)]
[Polyakov, Weiss (1999)]

A lot of interesting physics is contained in the EM tensor

[Polyakov, Shuvaev (2002)]
[Polyakov (2003)]
[Goeke *et al.* (2007)]
[Cebulla *et al.* (2007)]

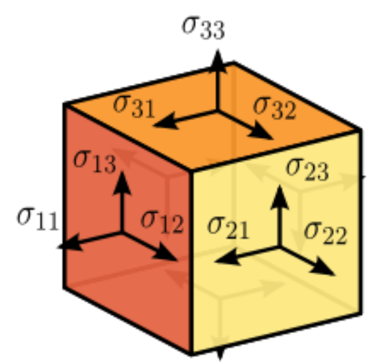
Energy density — Momentum density

$$T^{\mu\nu} = \begin{bmatrix} T^{00} & T^{01} & T^{02} & T^{03} \\ T^{10} & T^{11} & T^{12} & T^{13} \\ T^{20} & T^{21} & T^{22} & T^{23} \\ T^{30} & T^{31} & T^{32} & T^{33} \end{bmatrix}$$

Shear stress

Normal stress (pressure)

Energy flux — Momentum flux



In rest frame

$$M = \int \mathrm{d}^3r\, T^{00}(\vec{r})$$

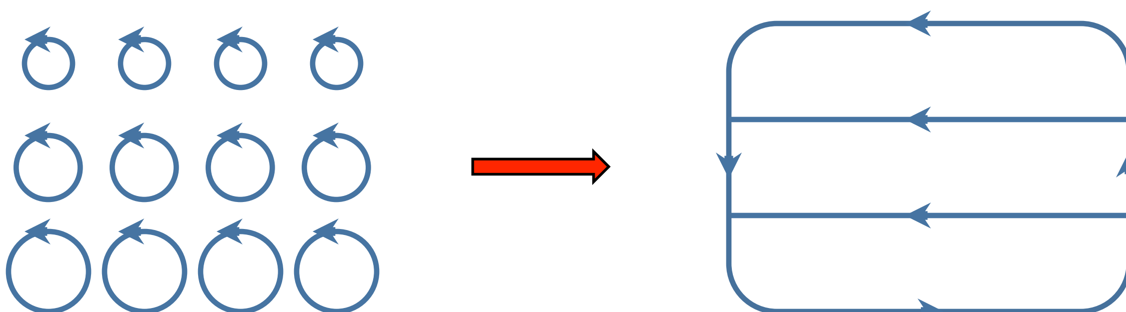
$$L^i = \int \mathrm{d}^3r\, \epsilon^{ijk} r^j T^{0k}(\vec{r})$$

Energy-momentum tensor

In presence of spin density $T^{0i} \neq T^{i0}$

Belinfante « improvement »

$$\begin{aligned} T_B^{\mu\nu} &\equiv T^{\mu\nu} + \tfrac{1}{2}\partial_\lambda[S^{\lambda\mu\nu} + S^{\mu\nu\lambda} + S^{\nu\mu\lambda}] \\ &= T_B^{\nu\mu} \end{aligned}$$



Spin density gradient **Four-momentum circulation**

In rest frame

$$M = \int \mathrm{d}^3r\, T_B^{00}(\vec{r})$$

$$J^i = \int \mathrm{d}^3r\, \epsilon^{ijk} r^j T_B^{0k}(\vec{r})$$

No « spin » contribution !

QCD Energy-momentum operator

$$T_B^{\mu\nu} = \tfrac{1}{2}\,\overline{\psi}\gamma^{\{\mu}iD^{\nu\}}\psi - 2\,\mathrm{Tr}[F^{\mu\alpha}F^{\nu}{}_{\alpha}] + \tfrac{1}{2}\,g^{\mu\nu}\,\mathrm{Tr}\big[F^{\alpha\beta}F_{\alpha\beta}\big]$$

$$a^{\{\mu}b^{\nu\}} = a^\mu b^\nu + a^\nu b^\mu$$

Matrix elements

Normalization

$$\langle p', \vec{s}|p, \vec{s}\rangle = 2P^0\,(2\pi)^2\,\delta^{(3)}(\vec{p}' - \vec{p})$$

$$\langle\langle \textstyle\int \mathrm{d}^3r\, O(\vec{r})\rangle\rangle \equiv \frac{\langle P, \vec{s}|\int \mathrm{d}^3r\, O(\vec{r})|P, \vec{s}\rangle}{\langle P, \vec{s}|P, \vec{s}\rangle} = \frac{1}{2P^0}\,\langle P, \vec{s}|O(\vec{0})|P, \vec{s}\rangle$$

$$\langle\langle \textstyle\int \mathrm{d}^3r\, \vec{r}\, O(\vec{r})\rangle\rangle \equiv \lim_{\vec{\Delta}\to\vec{0}} \frac{\langle p', \vec{s}|\int \mathrm{d}^3r\, \vec{r}\, O(\vec{r})|p, \vec{s}\rangle}{\langle p', \vec{s}|p, \vec{s}\rangle} = \frac{1}{2P^0}\left[-i\vec{\nabla}_\Delta\langle p', \vec{s}|O(\vec{0})|p, \vec{s}\rangle\right]_{\vec{\Delta}=\vec{0}}$$

$$\Delta = p' - p$$

Energy-momentum FFs

$$\langle p'|T_B^{\mu\nu}(0)|p\rangle = \overline{u}(p')\left[\frac{P^{\{\mu}\gamma^{\nu\}}}{2}\,A(t) + \frac{P^{\{\mu}i\sigma^{\nu\}\alpha}\Delta_\alpha}{4M}\,B(t) + \frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{M}\,C(t)\right]u(p)$$

Momentum sum rule

$$\langle\langle \int \mathrm{d}^3r\, T_B^{0\mu}(\vec{r})\rangle\rangle = A(0)\,P^\mu \quad \Rightarrow \quad \boxed{A(0) = 1}$$

Angular momentum sum rule

$$\langle\langle \int \mathrm{d}^3r\, \vec{J}_B(\vec{r})\rangle\rangle = \frac{\vec{s}}{2}\left[A(0) + \frac{P^0}{M}\,B(0)\right] - \frac{(\vec{P}\cdot\vec{s})\vec{P}}{2M(P^0+M)}\,B(0)$$

$P^\mu = (P^0, 0, 0, P_z)$

$\vec{s} = (0, 0, 1)$

$$\langle\langle \int \mathrm{d}^3r\, J_B^z(\vec{r})\rangle\rangle = \frac{1}{2}\left[A(0) + B(0)\right] \quad \Rightarrow \quad \boxed{A(0) + B(0) = 1}$$

$$\Rightarrow \quad \boxed{B(0) = 0}$$

Vanishing gravitomagnetic moment !

[Ji (1997)]

Energy-momentum FFs

$$\langle p'|T^{\mu\nu}_{Bq,G}(0)|p\rangle = \overline{u}(p')\left[\frac{P^{\{\mu}\gamma^{\nu\}}}{2}\,A_{q,G}(t) + \frac{P^{\{\mu}i\sigma^{\nu\}\alpha}\Delta_\alpha}{4M}\,B_{q,G}(t) + \frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{M}\,C_{q,G}(t) + \underbrace{M g^{\mu\nu}\,\overline{C}_{q,G}(t)}_{\text{Non-conserved current}}\right]u(p)$$

$$\boxed{\sum_{q,G}\overline{C}_{q,G}(t) = 0}$$

Momentum sum rule

$$\langle\langle\textstyle\int \mathrm{d}^3r\, T^{0\mu}_{Bq,G}(\vec{r})\rangle\rangle = A_{q,G}(0)\,P^\mu \quad\Longrightarrow\quad \boxed{\sum_{q,G} A_{q,G}(0) = 1}$$

Angular momentum sum rule

$$\langle\langle\textstyle\int \mathrm{d}^3r\, \vec{J}_{Bq,G}(\vec{r})\rangle\rangle = \frac{\vec{s}}{2}\left[A_{q,G}(0) + \frac{P^0}{M}\,B_{q,G}(0)\right] - \frac{(\vec{P}\cdot\vec{s})\vec{P}}{2M(P^0+M)}\,B_{q,G}(0)$$

$P^\mu = (P^0, 0, 0, P_z)$

$\vec{s} = (0, 0, 1)$

$$\langle\langle\textstyle\int \mathrm{d}^3r\, J^z_{Bq,G}(\vec{r})\rangle\rangle = \frac{1}{2}\left[A_{q,G}(0) + B_{q,G}(0)\right] \quad\Longrightarrow\quad \boxed{\sum_{q,G}\left[A_{q,G}(0) + B_{q,G}(0)\right] = 1}$$

$$\Longrightarrow\quad \boxed{\sum_{q,G} B_{q,G}(0) = 0}$$

Vanishing gravitomagnetic moment !

[Ji (1997)]

Leading-twist component of $T^{\mu\nu}_{Bq} = \frac{1}{2}\,\overline{\psi}\gamma^{\{\mu}iD^{\nu\}}\psi$ $\qquad a^+ = \frac{1}{\sqrt{2}}(a^0 + a^3)$

$$\langle p'|T^{++}_{Bq}(0)|p\rangle = \overline{u}(p')P^+\gamma^+u(p)\left[A_q(t) + 4\xi^2\,C_q(t)\right] + \overline{u}(p')P^+\frac{i\sigma^{+\alpha}\Delta_\alpha}{2M}\,u(p)\left[B_q(t) - 4\xi^2\,C_q(t)\right]$$

Link with GPDs

$$\begin{aligned}\langle p'|\overline{\psi}(0)\gamma^+iD^+\psi(0)|p\rangle &= 2(P^+)^2\int \mathrm{d}x\,x\left[\frac{1}{2}\int\frac{\mathrm{d}z^-}{2\pi}\,e^{ixP^+z^-}\,\langle p'|\overline{\psi}(-\tfrac{z^-}{2})\gamma^+\mathcal{W}\psi(\tfrac{z^-}{2})|p\rangle\right]\\ &= \overline{u}(p')P^+\gamma^+u(p)\int \mathrm{d}x\,x\,H_q(x,\xi,t)\\ &\quad + \overline{u}(p')P^+\frac{i\sigma^{+\alpha}\Delta_\alpha}{2M}\,u(p)\int \mathrm{d}x\,x\,E_q(x,\xi,t)\end{aligned}$$

$$\Longrightarrow\quad \boxed{\langle\langle \textstyle\int \mathrm{d}^3r\,J^z_{Bq}(\vec{r})\rangle\rangle = \frac{1}{2}\int \mathrm{d}x\,x\left[H_q(x,0,0) + E_q(x,0,0)\right]}$$

Accessible *e.g.* in DVCS !

[Ji (1997)]

Summary

Lecture 2

- **PDFs provide 1D pictures of the nucleon**
- **FFs provide 2D pictures of the nucleon**
- **GPDs generalize both PDFs and FFs and give access to the EMT**

$$\mathcal{F}[\mathrm{GPD}](x,\vec{b}_\perp) = \int \mathrm{d}^2k_\perp\, \rho_W(x,\vec{k}_\perp,\vec{b}_\perp)$$

$\mathcal{F}$[GPD]

$$T^{\mu\nu} = \begin{bmatrix} T^{00} & T^{01} & T^{02} & T^{03} \\ T^{10} & T^{11} & T^{12} & T^{13} \\ T^{20} & T^{21} & T^{22} & T^{23} \\ T^{30} & T^{31} & T^{32} & T^{33} \end{bmatrix}$$

Energy density (T^{00})

Momentum density (T^{01}, T^{02}, T^{03})

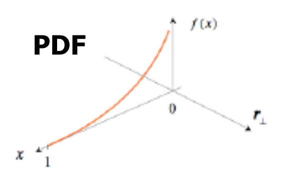
Energy flux (T^{10}, T^{20}, T^{30})

Momentum flux (T^{ij})

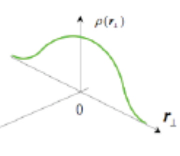
Shear stress

Normal stress (pressure)

PDF



$\mathcal{F}$[FF]

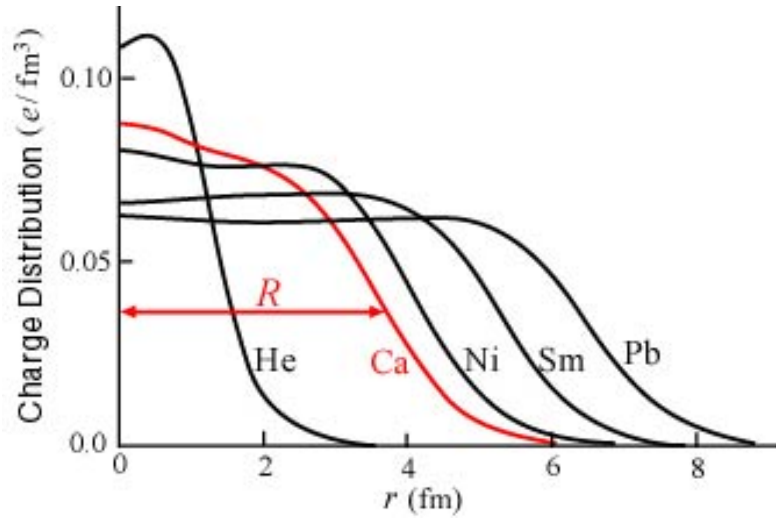


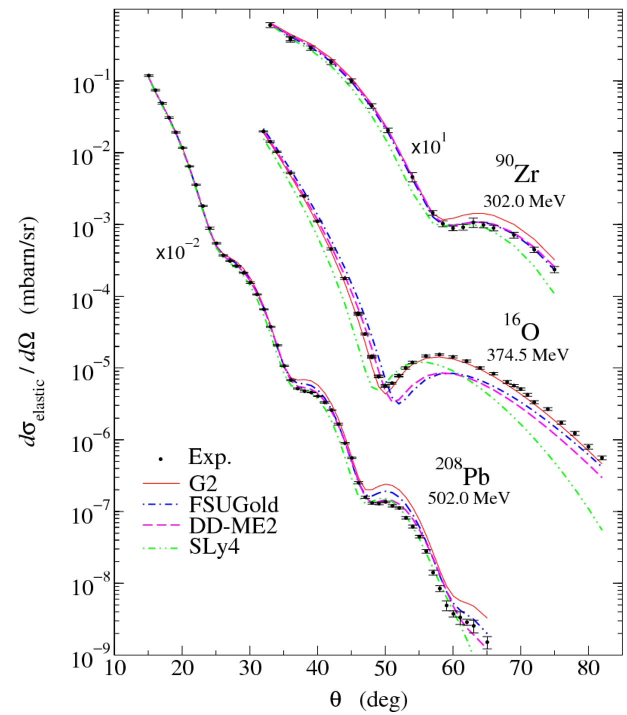
$$\mathrm{PDF}(x) = \int \mathrm{d}^2k_\perp\, \mathrm{d}^2b_\perp\, \rho_W(x,\vec{k}_\perp,\vec{b}_\perp)$$

$$\mathcal{F}[\mathrm{FF}](\vec{b}_\perp) = \int \mathrm{d}x\, \mathrm{d}^2k_\perp\, \rho_W(x,\vec{k}_\perp,\vec{b}_\perp)$$

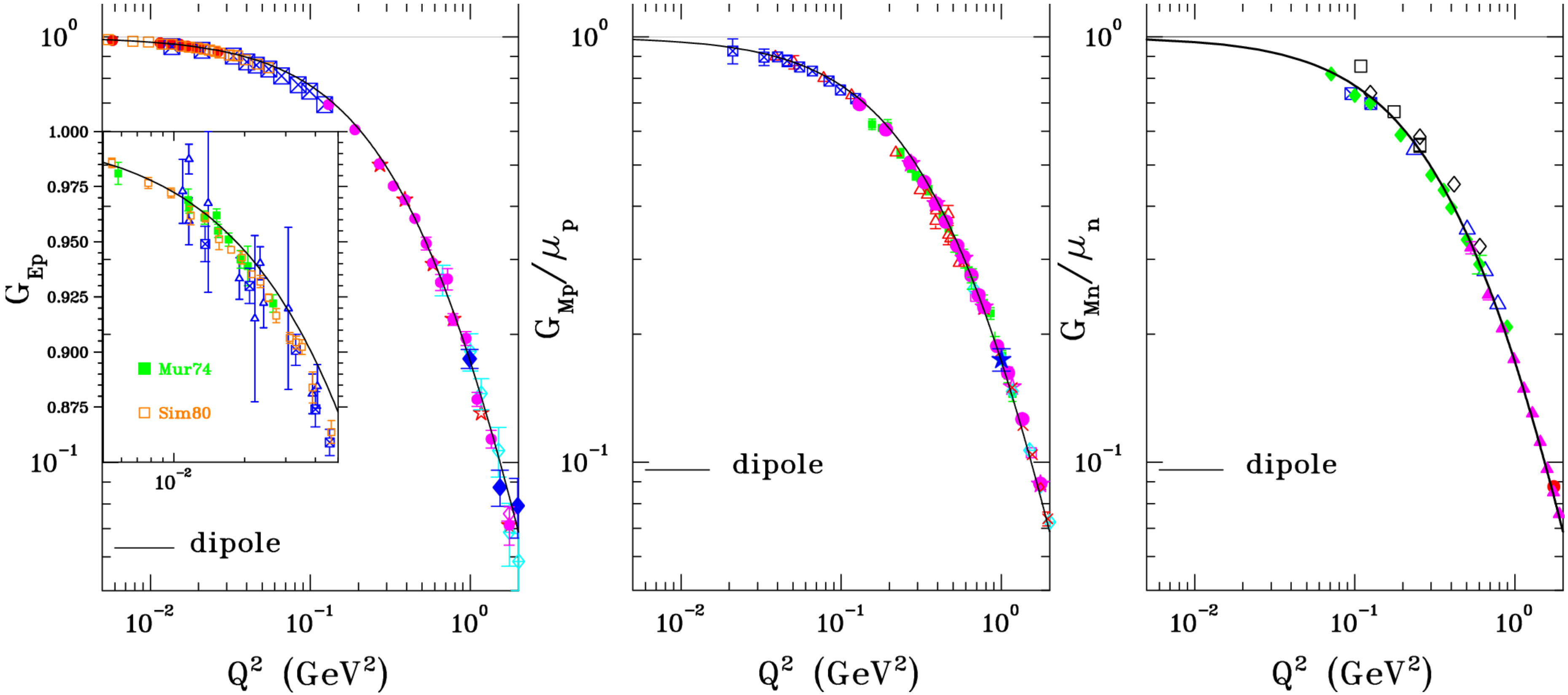
Backup slides

Nuclear charge densities and FFs





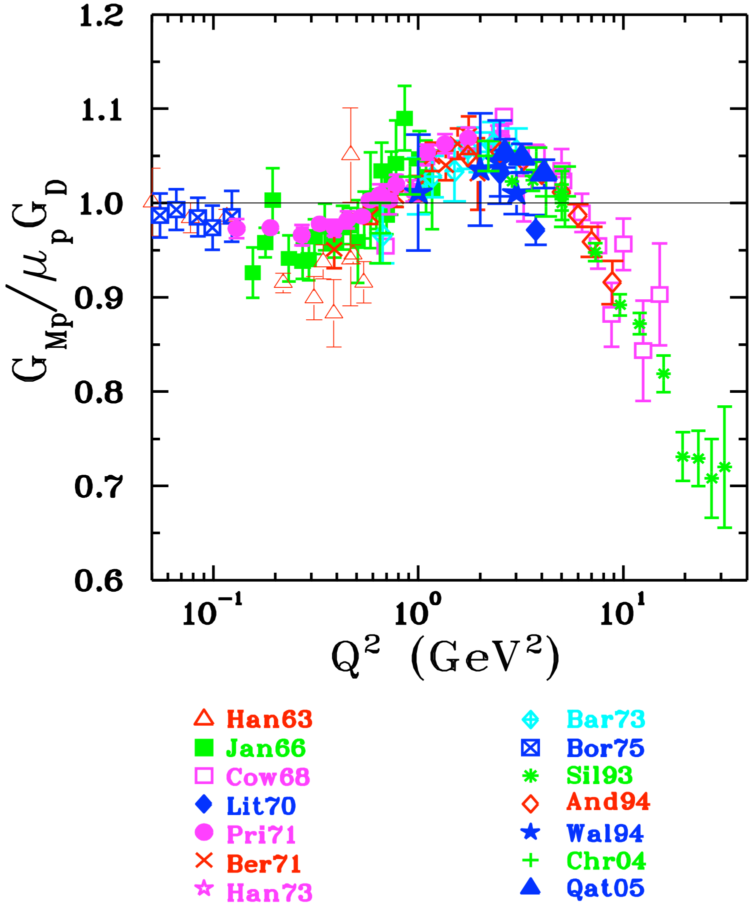
Nucleon FFs



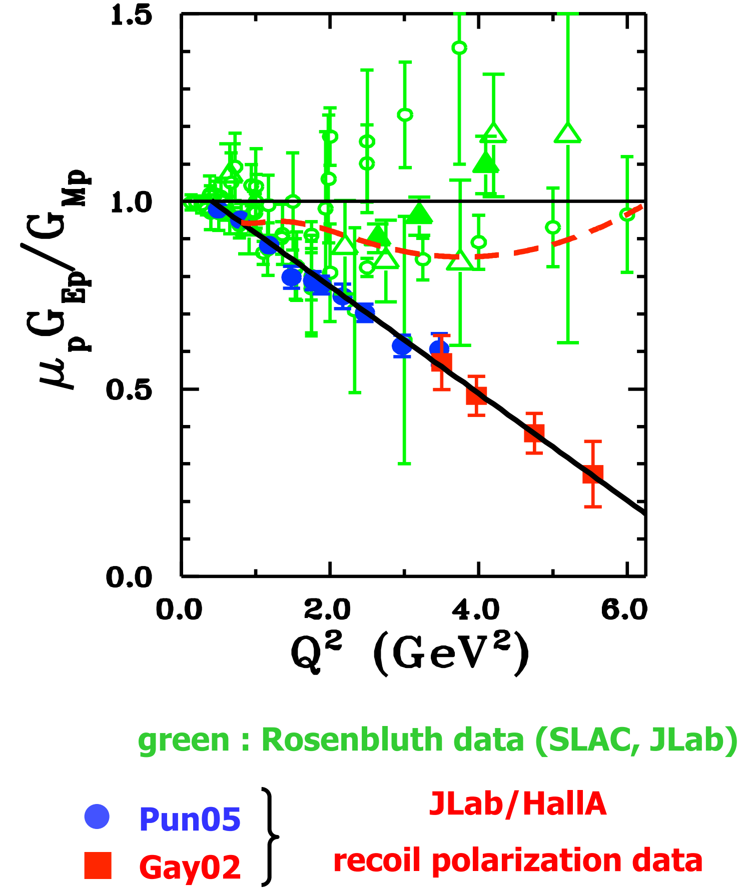
$$G_D(Q^2) = \frac{1}{(1+\frac{Q^2}{\Lambda_D^2})^2} \qquad \Lambda_D^2 = 0.71\ \mathrm{GeV}^2$$

[Perdrisat *et al.* (2006)]

Nucleon FFs



$G_{Mp}/\mu_p G_D$
Q^2 (GeV^2)
Han63
Jan66
Cow68
Lit70
Pri71
Ber71
Han73
Bar73
Bor75
Sil93
And94
Wal94
Chr04
Qat05



$\mu_p G_{Ep}/G_{Mp}$
Q^2 (GeV^2)
green : Rosenbluth data (SLAC, JLab)
Pun05
Gay02
JLab/HallA
recoil polarization data