Multiparton interactions
Part 2

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Correlations between Partons in Nucleons
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Recap: cross section formulae for double hard scattering

- transverse-momentum dependent:

$$\frac{d\sigma}{dx_1\, d\bar{x}_1\, d^2\boldsymbol{q}_1\; dx_2\, d\bar{x}_2\, d^2\boldsymbol{q}_2} = \frac{1}{C}\left[\prod_{i=1}^{2} \hat{\sigma}_i\big(q_i^2 = x_i\bar{x}_i s\big)\right]$$

$$\times \left[\prod_{i=1}^{2} \int d^2\boldsymbol{k}_i\, d^2\bar{\boldsymbol{k}}_i\, \delta^{(2)}(\boldsymbol{q}_i - \boldsymbol{k}_i - \bar{\boldsymbol{k}}_i)\right] \int d^2\boldsymbol{y}\, F(x_i, \boldsymbol{k}_i, \boldsymbol{y})\, F(\bar{x}_i, \bar{\boldsymbol{k}}_i, \boldsymbol{y})$$

- collinear

$$\frac{d\sigma}{dx_1\, d\bar{x}_1\; dx_2\, d\bar{x}_2} = \frac{1}{C}\left[\prod_{i=1}^{2} \hat{\sigma}_i\big(q_i^2 = x_i\bar{x}_i s\big)\right]$$

$$\times \int d^2\boldsymbol{y}\, F(x_i, \boldsymbol{y})\, F(\bar{x}_i, \boldsymbol{y})$$

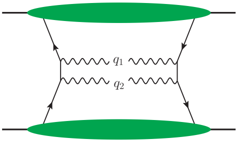
Power behavior: single versus double hard scattering

- example: ZZ production

$$\frac{d\sigma}{dx_1\, d\bar{x}_1\, d^2\boldsymbol{q}_1\; dx_2\, d\bar{x}_2\, d^2\boldsymbol{q}_2} \sim \frac{1}{Q^4 \Lambda^2}$$

$Q^2 = m_Z^2,\ \Lambda^2 \sim \mathrm{GeV}$

for both



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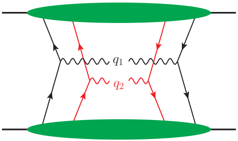

$\Rightarrow$ double scattering not power suppressed

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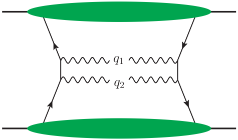

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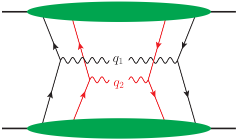

$\Rightarrow$ double scattering not power suppressed

- but if integrate over $\boldsymbol{q}_1$ and $\boldsymbol{q}_2$ then

single: $$\frac{d\sigma}{dx_1\, d\bar{x}_1\; dx_2\, d\bar{x}_2} \sim \frac{1}{Q^2}$$ since $\int d^2(\boldsymbol{q}_1 + \boldsymbol{q}_2) \sim \Lambda^2$ and $\int d^2(\boldsymbol{q}_1) \sim Q^2$

double: $$\frac{d\sigma}{dx_1\, d\bar{x}_1\; dx_2\, d\bar{x}_2} \sim \frac{\Lambda^2}{Q^4}$$ since $\int d^2\boldsymbol{q}_1 \int d^2\boldsymbol{q}_2 \sim \Lambda^4$

i.e. single hard scattering has larger phase space for transv. momenta

Power behavior: single versus double hard scattering

- example: ZZ production

$$\frac{d\sigma}{dx_1\, d\bar{x}_1\, d^2\boldsymbol{q}_1\; dx_2\, d\bar{x}_2\, d^2\boldsymbol{q}_2} \sim \frac{1}{Q^4 \Lambda^2}$$

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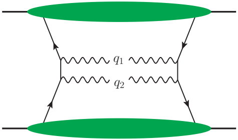

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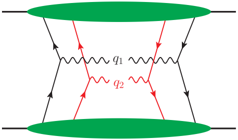
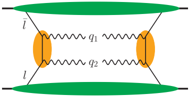
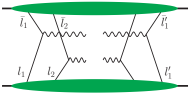
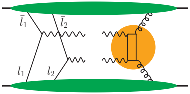


$\Rightarrow$ double scattering not power suppressed

- if integrate only over $\boldsymbol{q}_1 + \boldsymbol{q}_2$ then no power suppression yet

$$\frac{d\sigma}{dx_1\, d\bar{x}_1\, dx_2\, d\bar{x}_2\, d^2\boldsymbol{q}_1} \sim \frac{1}{Q^4}$$

Energy dependence

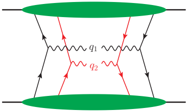
	$\dfrac{s\,d\sigma}{\prod_{i=1}^{2} dx_i\, d\bar{x}_i\, d^2\boldsymbol{q}_i}$	$\dfrac{s\,d\sigma}{\prod_{i=1}^{2} dx_i\, d\bar{x}_i}$
	$\dfrac{1}{\Lambda^2 Q^2}$	1
	$\dfrac{1}{\Lambda^2 Q^2}$	$\dfrac{\Lambda^2}{Q^2}$
	$\dfrac{1}{\Lambda^2 Q^2}$	$\dfrac{\Lambda^2}{Q^2}$

- interference between single and double scattering:
  - leading power when differential in $\boldsymbol{q}_i$
  - power suppressed when $\int d^2\boldsymbol{q}_i$, twist-three parton distributions
- at small $x_1 \sim x_2 \sim x$ expect
  - single scattering $\propto x^{-\lambda}$ with $x f(x) \sim x^{-\lambda}$
  - double scattering $\propto x^{-2\lambda}$
  - interference ? how do three-particle correlators behave for small x ?

Summary so far

- multiple hard scattering $\leftrightarrow$ high-multiplicity final states
- is not generically suppressed in sufficiently differential cross sections
- double hard scattering $\leftrightarrow$ double parton distributions
  depend on momentum fractions x_1, x_2 and on relative distance $\boldsymbol{y}$
  $\rightsquigarrow$ important aspect of hadron structure

Double parton scattering: pocket formula



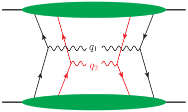
- if two-parton density factorizes as

$$F(x_1, x_2, \boldsymbol{y}) = f(x_1)\, f(x_2)\, G(\boldsymbol{y})$$

where $f(x_i)$ = usual PDF

- if assume same $G(\boldsymbol{y})$ for all parton types
then cross sect. formula turns from

$$\frac{d\sigma_{\text{double}}}{dx_1\, d\bar{x}_1\, dx_2\, d\bar{x}_2} = \frac{1}{C}\, \hat{\sigma}_1\, \hat{\sigma}_2 \int d^2\boldsymbol{y}\, F(x_i, \boldsymbol{y})\, F(\bar{x}_i, \boldsymbol{y})$$



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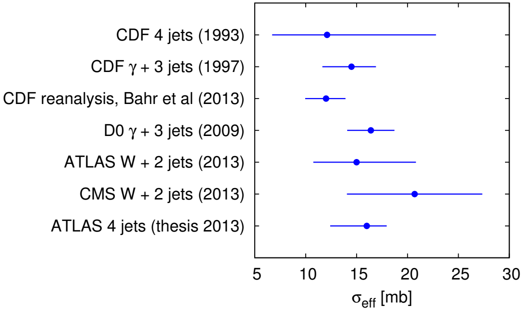
with $1/\sigma_{\text{eff}} = \int d^2\boldsymbol{y}\; G(\boldsymbol{y})^2$

$\rightsquigarrow$ scatters are completely independent

- derivation works including higher-order corrections to hard scattering
- pocket formula fails if any of the above assumptions is invalid
or if further terms must be added to original expression of cross sect.
(will encounter such terms later)

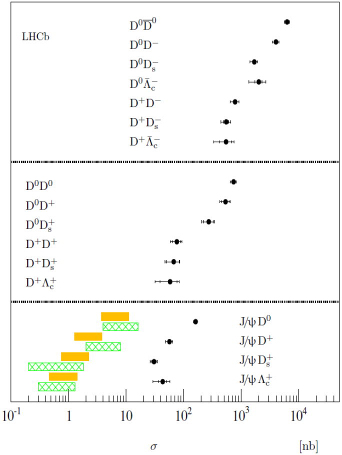
Experimental results: a sketch

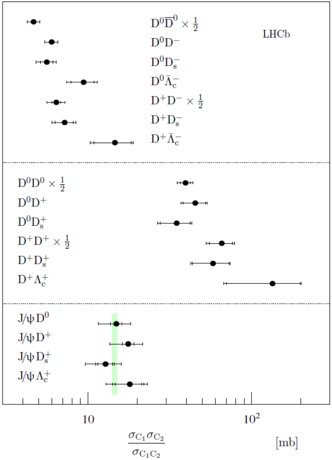


more detail in P. Bartalini's lecture

- double charm production ($c\bar{c}c\bar{c}$) at LHCb (2011, 2012): $J/\Psi + J/\Psi$, $J/\Psi + C$, $C + C$ with $C = D^0, D^+, D_s^+, \Lambda_c^+$

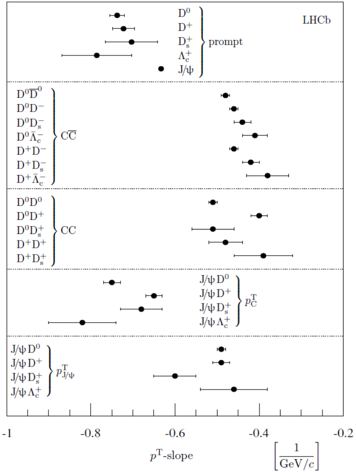
Double charm production: LHCb data





- $J/\Psi\, C$ channels:
  computed single-scattering contribution $\ll$ measured cross sect.
- ratio $\sigma_{C_1C_2}/(\sigma_{C_1}\sigma_{C_2})$ in ballpark of σ_{eff} of other proc's

Double charm production: LHCb data



- exponential p^T slope (fitted for $3\,\mathrm{GeV} < p^T < 12\,\mathrm{GeV}$)
- $J/\Psi\,C$ channels: slope for C $\sim$ slope in single C prod'n but does not hold for J/Ψ

Phenomenological estimates

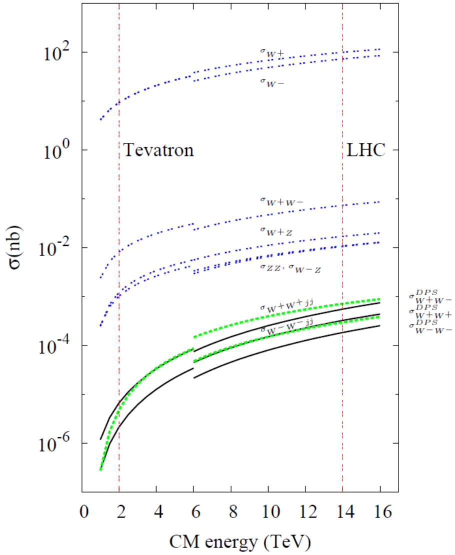
- pocket formula used in most estimate for DPS contribution to various processes
- some recent studies:
  - double dijets — Domdey, Pirner, Wiedemann 2009; Berger, Jackson, Shaughnessy 2009
  - W/Z + jets — Maina 2009, 2011
  - like-sign W pairs — Kulesza, Stirling 2009; Gaunt et al 2010; Berger et al 2011
  - double Drell-Yan — Kom, Kulesza, Stirling 2011
  - double charmonium — Kom, Kulesza, Stirling 2011; Baranov et al. 2011, 2012; Novoselov 2011
  - double charm — Berezhnoy et al 2012; Luszczak et al 2011; Maciula, Szczurek 2012, 2013; Cazaroto et al 2013

Phenomenological estimates

example: like-sign W pairs

single scattering:
$qq \to qq + W^+W^+$
suppressed by α_s^2

plot: Gaunt et al, arXiv:1003.3953



Approximating DPDs by single-parton distributions

$$F(x_i, \boldsymbol{k}_i, \boldsymbol{y}) = \underset{z_i \to (x_i, \boldsymbol{k}_i)}{\mathcal{FT}} \langle p| \bar{q}\big(-\tfrac{1}{2}z_2\big)\Gamma_2\, q\big(\tfrac{1}{2}z_2\big)\, \bar{q}\big(y - \tfrac{1}{2}z_1\big)\Gamma_1\, q\big(y + \tfrac{1}{2}z_1\big)|p\rangle$$

- between $\bar{q}_2\, q_2$ and $\bar{q}_1\, q_1$ insert complete set $\sum_X |X\rangle\langle X|$ of states
- if assume that single-proton states $|p\rangle\langle p|$ dominate in $\sum |X\rangle\langle X|$ then $F(x_i, \boldsymbol{k}_i, \boldsymbol{y}_1) \approx$ product of single-quark distributions

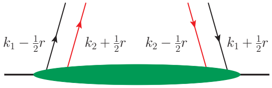
  $$\langle p|\bar{q}_2\, q_2\;\bar{q}_1\, q_1|p\rangle \approx \sum_{p'} \langle p|\bar{q}_2\, q_2|p'\rangle\, \langle p'|\bar{q}_1\, q_1|p\rangle$$

  in physical terms: neglect correlations between parton 1 and 2
- transv. momenta $\boldsymbol{p}$ and $\boldsymbol{p}'$ differ $\rightsquigarrow$ generalized parton distributions
  - appear in exclusive processes, e.g. $\gamma p \to J/\Psi\, p$ measured in ep and μp scattering
  - Fourier trf. from $\boldsymbol{p} - \boldsymbol{p}'$ to impact parameter $\rightsquigarrow$ joint dist'n of partons in long. mom. and transv. position

Approximating DPDs by single-parton distributions

$$\langle p|\bar{q}_2\, q_2\;\bar{q}_1\, q_1|p\rangle \approx \sum_{p'} \langle p|\bar{q}_2\, q_2|p'\rangle\,\langle p'|\bar{q}_1\, q_1|p\rangle$$

- relation is approximate but
  - $\sigma_{\text{DPD}} \propto \int d^2\boldsymbol{y}\, F(x_i, \boldsymbol{k}_i, \boldsymbol{y})\, F(\bar{x}_i, \bar{\boldsymbol{k}}_i, \boldsymbol{y}) \;\rightsquigarrow\; \boldsymbol{y}$ unobservable
  - in generalized parton dist's $\boldsymbol{p} - \boldsymbol{p}'$ is measurable
- especially simple for collinear distributions



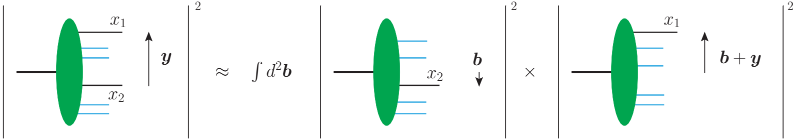
  momentum space:

$$F(x_1, x_2, \boldsymbol{r}) \;\approx\; f(x_1, \boldsymbol{r})\, f(x_2, -\boldsymbol{r})$$

  position space:

$$F(x_1, x_2, \boldsymbol{y}) \;\approx\; \int d^2\boldsymbol{b}\; f(x_2, \boldsymbol{b})\, f(x_1, \boldsymbol{b} + \boldsymbol{y})$$

with $f(x, \boldsymbol{b})$ = impact-parameter distribution



used for long time in literature, based on geometric intuition

Approximating DPDs by single-parton distributions

- if neglect correlations between two partons

$$F(x_1, x_2, \boldsymbol{y}) \;\approx\; \int d^2\boldsymbol{b}\, f(x_2, \boldsymbol{b})\, f(x_1, \boldsymbol{b}+\boldsymbol{y})$$

where $f(x_i, \boldsymbol{b})$ = impact parameter dependent single-parton density

- if neglect correlations between x and $\boldsymbol{b}$ of single parton

$$f(x_i, \boldsymbol{b}) = f(x_i) F(\boldsymbol{b})$$

with same $F(\boldsymbol{b})$ for all partons

- then $G(\boldsymbol{y}) = \int d^2\boldsymbol{b}\, F(\boldsymbol{b})\, F(\boldsymbol{b}+\boldsymbol{y})$

in pocket formula ansatz $F(x_1, x_2, \boldsymbol{y}) = f(x_1)\, f(x_2)\, G(\boldsymbol{y})$

recall: $1/\sigma_{\text{eff}} = \int d^2\boldsymbol{y}\, G(\boldsymbol{y})^2$

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- then $G(\boldsymbol{y}) = \int d^2\boldsymbol{b}\, F(\boldsymbol{b})\, F(\boldsymbol{b}+\boldsymbol{y})$
- for Gaussian $F(\boldsymbol{b})$ with average $\langle \boldsymbol{b}^2 \rangle$

$$\sigma_{\text{eff}} = 4\pi \langle \boldsymbol{b}^2 \rangle = 41\,\text{mb}\ \times \langle \boldsymbol{b}^2 \rangle / (0.57\,\text{fm})^2$$

determinations of $\langle \boldsymbol{b}^2 \rangle$ range from $\sim (0.57\,\text{fm} - 0.67\,\text{fm})^2$

is $\gg \sigma_{\text{eff}} \sim 10$ to $20\,\text{mb}$ from experimental extractions

if $F(\boldsymbol{b})$ is Fourier trf. of dipole then $41\,\text{mb} \to 36\,\text{mb}$

complete independence between two partons is disfavored
or something is systematically wrong with σ_{eff} extractions

cf. Calucci, Treleani 1999; Frankfurt, Strikman, Weiss 2003, 2004