Shockwave approach to low-x evolution equations

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Correlations between partons in nucleons, LPT Orsay
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Outline

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- Introduction
- Shockwave formallism
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Definitions

Introduce the light cone vectors n_1 and n_2

$$n_1 = (1, 0, 0, 1)\,, \quad n_2 = \frac{1}{2}(1, 0, 0, -1)\,, \quad n_1^+ = n_2^- = n_1 n_2 = 1$$

For any p define $p^\pm$

$$p^+ = pn_2 = \frac{1}{2}\left(p^0 + p^3\right), \qquad p^- = pn_1 = p^0 - p^3,$$

$$p^2 = 2p^+p^- - \vec{p}^{\,\mathbf{2}};$$

The scalar products:

$$p = p^+ n_1 + p^- n_2 + p_\perp, \quad (p\,k) = p^\mu k_\mu = p^+ k^- + p^- k^+ - \vec{p}\vec{k}.$$

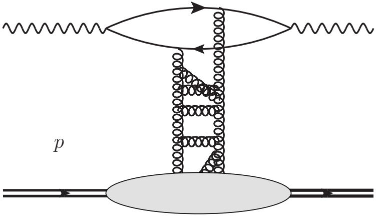
Definitions

Wilson line describing interaction with external field b_η^- made of slow gluons with $p^+ < e^\eta$

$$U_{\vec{z}}^\eta = Pe^{ig\int_{-\infty}^{+\infty} dz^+ b_\eta^-(z^+,\vec{z})}, \quad b_\eta^- = \int \frac{d^4p}{(2\pi)^4} e^{-ipz} b^-(p)\,\theta(e^\eta - p^+).$$

Introduction

Dipole picture $s \gg Q^2 \gg \Lambda^2_{QCD}$



$$\sigma_{\gamma*}(s, Q^2) = \int d^2\mathbf{r} |\Psi_{\gamma*}(\mathbf{r}, Q^2)|^2 \, \sigma_{dip}(\mathbf{r}, s), \quad \sigma_{dip}(\mathbf{r}, s) = 2 \int d\mathbf{b}\left(1 - \frac{1}{N_c} F(\mathbf{b}, \mathbf{r}, s)\right)$$

$\mathbf{r} = \mathbf{r_1} - \mathbf{r_2}$ — dipole size, $\mathbf{b} = \frac{1}{2}(\mathbf{r_1} + \mathbf{r_2})$ — impact parameter, $F = tr(U_1 U_2^\dagger)$, — dipole Green function, $U_i = U_i^\eta$ — Wilson lines, describing fast moving quarks interacting with the target.

η — rapidity divide, gluons with $p^+ > e^\eta$ belong to photon wavefunction, gluons with $p^+ < e^\eta$ belong to Wilson lines, describing the field of the target.

Introduction

$tr(U_1 U_2^\dagger)$ obeys the LO Balitsky-Kovchegov evolution equation

$$\frac{\partial tr(U_1 U_2^\dagger)}{\partial \eta} = \frac{\alpha_s}{2\pi^2} \int d\vec{r}_4 \frac{\vec{r}_{12}^{\,2}}{\vec{r}_{14}^{\,2}\vec{r}_{42}^{\,2}} \left[tr(U_1 U_4^\dagger) tr(U_4 U_2^\dagger) - N_c tr(U_1 U_2^\dagger) \right].$$

LO equation was obtained in 1996-99, NLO — in 2007-2010 (Balitsky and Chirilli).

Shock wave

For a fast moving particle with the velocity $-\beta$ and the field strength tensor $\mathbb{F}(x^+, x^-, \vec{x})$ in its rest frame, in the observer's frame the field will look like

$$\mathfrak{F}^{-i}\left(y^+, y^-, \vec{y}\right) = \lambda \mathbb{F}^{-i}(\lambda y^+, \frac{1}{\lambda} y^-, \vec{y}) \to \delta\left(y^+\right) \mathfrak{F}^{i}\left(\vec{y}\right),$$

$$\mathfrak{F}^{-i} \gg \mathfrak{F}^{\cdots}$$

in the Regge limit $\lambda \to +\infty$, $\lambda = \sqrt{\frac{1+\beta}{1-\beta}}$.

Therefore the natural choice for the gauge is $b^{i,+} = 0$,

b^- is the solution of the equations

$$\frac{\partial b^-}{\partial y^i} = \delta(y^+)\mathfrak{F}^{i}\left(\vec{y}\right), i.e.$$

$$b^{\mu}\left(y\right) = \delta(y^+)B\left(\vec{y}\right) n_2^{\mu}$$

It is the shock-wave field.

Propagator in the shock wave background

Choose the gluon field $\mathcal{A}$ in the gauge $\mathcal{A}n_2 = 0$ as a sum of external classical b and quantum A.

$$\mathcal{A} = A + b, \quad b^{\mu}(x) = \delta(x^+) B(\vec{x})\, n_2^{\mu}.$$

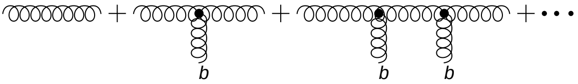
The A-b interaction lagrangian has only one vertex

$$\mathcal{L}_i = \frac{g}{2} f^{acb} (b^-)^c g_{\perp}^{\alpha\beta} \left[A_{\alpha}^a \frac{\overleftrightarrow{\partial}}{\partial x^-} A_{\beta}^b \right].$$

The free propagator $G_0^{\mu\nu}(x^+, p^+, \vec{p}) =$

$$= \frac{-d_0^{\mu\nu}(p^+, p_{\perp})}{2p^+} e^{-i\frac{\vec{p}^{\,2} x^+}{2p^+}} \left(\theta(x^+)\theta(p^+) - \theta(-x^+)\theta(-p^+)\right) + n_2^{\mu} n_2^{\nu} \ldots,$$

$$d_0^{\mu\nu}(p) = g_{\perp}^{\mu\nu} - \frac{p_{\perp}^{\mu} n_2^{\nu} + p_{\perp}^{\nu} n_2^{\mu}}{p^+} - \frac{n_2^{\mu} n_2^{\nu} \vec{p}^{\,2}}{(p^+)^2}.$$



Sum the diagrams

- b does not depend on x^-, hence the conservation of p^+,
- $b \sim \delta(x^+)$, hence $e^{-i\frac{\vec{p}^{\,2}(x_1^+ - x_2^+)}{2p^+}} \to 1$ in every internal vertex,
- $g_\perp^{\mu\nu} d_{0\nu\rho} g_\perp^{\rho\sigma} = g_\perp^{\mu\sigma}$, hence no dependence on $\vec{p} \Longrightarrow$ conservation of $\vec{x}$ in every internal vertex

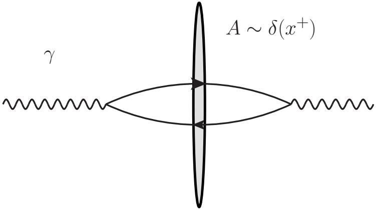
Propagator in the shock-wave background:

$$G_{\mu\nu}(x,y)|_{x^+>0>y^+} = 2iA^\mu(x)\int d^4z\,\delta(z^+)F^{+i}(z)\underset{\frac{\partial}{\partial z^-}}{\xrightarrow{U_{\vec{z}}}}F^{+i}(z)A^\nu(y)\,.$$

where the interaction with b is through Wilson line

$$U_{\vec{z}} = Pe^{ig\int_{y^+}^{x^+} dz^+ b^-\left(z^+,\vec{z}\right)}.$$

Dipole picture



Color field of a fast moving particle $A^- \sim \delta(z^+)A^\eta(z_\perp)$
$A^\eta(z_\perp)$ contains slow components with rapidities $< \eta$

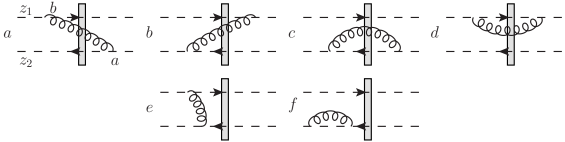
Quark propagator in such an external field $G(x,y) \sim U^\eta(z_\perp)$

DIS matrix element contains a Wilson loop = color dipole operator $U^\eta_{12} = tr(U^\eta(z_{1\perp})U^{\eta\dagger}(z_{2\perp}))$. Balitsky 1996

Balitsky derivation of the BK equation

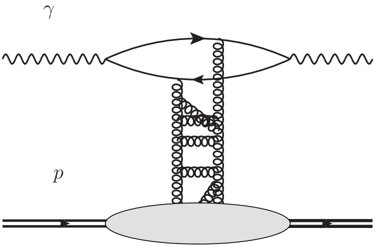
To derive the evolution equation we have to change $\eta \to \eta + \Delta\eta$ and integrate over the fields with the rapidities in the strip $\Delta\eta$

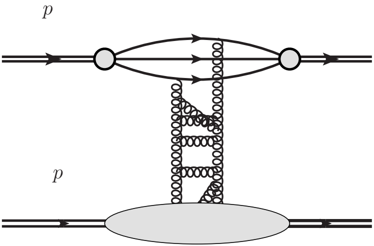
$$U_{12}^{\eta+\Delta\eta} = U_{12}^{\eta} + \frac{\langle 0|T(U_{12}^{\Delta\eta} e^{i\int \mathcal{L}(z)dz})|0\rangle}{\langle 0|T(e^{i\int \mathcal{L}(z)dz})|0\rangle}.$$



$$\frac{\partial U_{12}^{\eta}}{\partial\eta} = \frac{\alpha_s}{2\pi^2}\int d\vec{z}_4 \frac{\vec{z}_{12}^{\,2}}{\vec{z}_{14}^{\,2}\vec{z}_{42}^{\,2}}\left[U_{14}^{\eta}U_{42}^{\eta} - N_c U_{12}^{\eta}\right].$$

Motivation



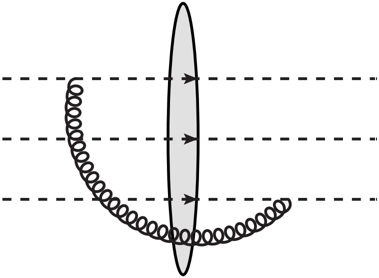


Dipole picture,
BK equation

?

Evolution equation for a 3-quark Wilson line

$$B_{123}^{\eta} = \varepsilon^{i'j'h'}\varepsilon_{ijh}U^{\eta}(\vec{z}_1)^{i}_{i'}\,U^{\eta}(\vec{z}_2)^{j}_{j'}\,U^{\eta}(\vec{z}_3)^{h}_{h'}$$

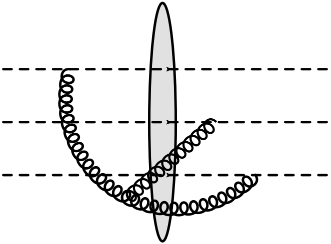


$$\frac{\partial B_{123}^{\eta}}{\partial\eta} = \frac{\alpha_s 3}{4\pi^2}\int d\vec{z}_4\left[\frac{\vec{z}_{12}^{\,2}}{\vec{z}_{41}^{\,2}\vec{z}_{42}^{\,2}}\left(-B_{123}^{\eta}+\frac{1}{6}\left(B_{144}^{\eta}B_{324}^{\eta}+B_{244}^{\eta}B_{314}^{\eta}-B_{344}^{\eta}B_{214}^{\eta}\right)\right)\right.$$

$$\left.+(1\leftrightarrow 3)+(2\leftrightarrow 3)\right].$$

NLO corrections: under check now

NLO evolution of 2 Wilson lines with open indices from Balitsky and Chirilli 2013



$$
\begin{aligned}
\frac{\partial B_{123}}{\partial \eta} = \frac{\alpha_s(\mu^2)}{8\pi^2} \int d\vec{r}_0 \Big[& (B_{100}B_{320} + B_{200}B_{310} - B_{300}B_{210} - 6B_{123}) \\
& \times \left\{ \frac{\vec{r}_{12}^{\,2}}{\vec{r}_{01}^{\,2}\vec{r}_{02}^{\,2}} - \frac{3\alpha_s}{4\pi}\frac{11}{3}\left[\ln\left(\frac{\vec{r}_{01}^{\,2}}{\vec{r}_{02}^{\,2}}\right)\left(\frac{1}{\vec{r}_{02}^{\,2}} - \frac{1}{\vec{r}_{01}^{\,2}}\right) - \frac{\vec{r}_{12}^{\,2}}{\vec{r}_{01}^{\,2}\vec{r}_{02}^{\,2}} \ln\left(\frac{\vec{r}_{12}^{\,2}}{\tilde{\mu}^2}\right) \right] \right\} \\
& - \frac{\alpha_s}{\pi} \ln\frac{\vec{r}_{20}^{\,2}}{\vec{r}_{21}^{\,2}} \ln\frac{\vec{r}_{10}^{\,2}}{\vec{r}_{21}^{\,2}} \left\{ \frac{1}{2}\left[\frac{\vec{r}_{13}^{\,2}}{\vec{r}_{10}^{\,2}\vec{r}_{30}^{\,2}} - \frac{\vec{r}_{32}^{2}}{\vec{r}_{30}^{2}\vec{r}_{20}^{\,2}} \right] (B_{100}B_{320} - B_{200}B_{310}) \right. \\
& \left. - \frac{\vec{r}_{12}^{2}}{\vec{r}_{10}^{2}\vec{r}_{20}^{\,2}} \left(9B_{123} - \frac{1}{2}\left[2\left(B_{100}B_{320} + B_{200}B_{130}\right) - B_{300}B_{120} \right] \right) \right\} + (1 \leftrightarrow 3) + (2 \leftrightarrow 3) \Big]
\end{aligned}
$$

NLO corrections

Work with I. Balitsky

$$-\frac{\alpha_s^2}{8\pi^4}\int d\vec{r}_0 d\vec{r}_4\ \left[\{\tilde{L}_{12}\left(U_0 U_4{}^\dagger U_2\right)\cdot\left(U_1 U_0{}^\dagger U_4\right)\cdot U_3\right.$$

$$+L_{12}\left[\left(U_0 U_4{}^\dagger U_2\right)\cdot\left(U_1 U_0{}^\dagger U_4\right)\cdot U_3+tr\left(U_0 U_4{}^\dagger\right)\left(U_1 U_0{}^\dagger U_2\right)\cdot U_3\cdot U_4\right.$$

$$\left.-\frac{3}{4}[B_{144}B_{234}+B_{244}B_{134}-B_{344}B_{124}]+\frac{1}{2}B_{123}\right]$$

$$+(M_{13}-M_{12}-M_{23}+M_2)\left[\left(U_0 U_4{}^\dagger U_3\right)\cdot\left(U_2 U_0{}^\dagger U_1\right)\cdot U_4\right.$$

$$\left.\left.+\left(U_1 U_0{}^\dagger U_2\right)\cdot\left(U_3 U_4{}^\dagger U_0\right)\cdot U_4\right]+(\text{all 5 permutations } 1\leftrightarrow 2\leftrightarrow 3)\}+\ (0\leftrightarrow 4)\right].$$

$$\frac{11}{3}\ln\frac{1}{\tilde{\mu}^2}=\frac{11}{3}\ln\left(\frac{\mu^2}{4e^{2\psi(1)}}\right)+\frac{67}{9}-\frac{\pi^2}{3}.$$

This equation has correct dipole limit.

NLO corrections

Pomeron contribution $L_{12}(0 \leftrightarrow 4) = L_{12}$

$$L_{12} = \left[\frac{1}{\vec{r}_{01}{}^2\vec{r}_{24}{}^2 - \vec{r}_{02}{}^2\vec{r}_{14}{}^2}\left(-\frac{\vec{r}_{12}{}^4}{8}\left(\frac{1}{\vec{r}_{01}{}^2\vec{r}_{24}{}^2} + \frac{1}{\vec{r}_{02}{}^2\vec{r}_{14}{}^2}\right) + \frac{\vec{r}_{12}{}^2}{\vec{r}_{04}{}^2} - \frac{\vec{r}_{02}{}^2\vec{r}_{14}{}^2 + \vec{r}_{01}{}^2\vec{r}_{24}{}^2}{4\vec{r}_{04}{}^4}\right)\right.$$
$$\left. + \frac{\vec{r}_{12}{}^2}{8\vec{r}_{04}{}^2}\left(\frac{1}{\vec{r}_{02}{}^2\vec{r}_{14}{}^2} - \frac{1}{\vec{r}_{01}{}^2\vec{r}_{24}{}^2}\right)\right] \ln\left(\frac{\vec{r}_{01}{}^2\vec{r}_{24}{}^2}{\vec{r}_{14}{}^2\vec{r}_{02}{}^2}\right) + \frac{1}{2\vec{r}_{04}{}^4}.$$

2-point contribution to odderon $\tilde{L}_{12}(0 \leftrightarrow 4) = -\tilde{L}_{12}$

$$\tilde{L}_{12} = \frac{\vec{r}_{12}{}^2}{8}\left[\frac{\vec{r}_{12}{}^2}{\vec{r}_{01}{}^2\vec{r}_{02}{}^2\vec{r}_{14}{}^2\vec{r}_{24}{}^2} - \frac{1}{\vec{r}_{01}{}^2\vec{r}_{04}{}^2\vec{r}_{24}{}^2} - \frac{1}{\vec{r}_{02}{}^2\vec{r}_{04}{}^2\vec{r}_{14}{}^2}\right] \ln\left(\frac{\vec{r}_{01}{}^2\vec{r}_{24}{}^2}{\vec{r}_{14}{}^2\vec{r}_{02}{}^2}\right).$$

New structures

$$M_{12} = \frac{\vec{r}_{12}{}^2}{16}\left[\frac{\vec{r}_{12}{}^2}{\vec{r}_{01}{}^2\vec{r}_{02}{}^2\vec{r}_{14}{}^2\vec{r}_{24}{}^2} - \frac{1}{\vec{r}_{01}{}^2\vec{r}_{04}{}^2\vec{r}_{24}{}^2} - \frac{1}{\vec{r}_{02}{}^2\vec{r}_{04}{}^2\vec{r}_{14}{}^2}\right] \ln\left(\frac{\vec{r}_{01}{}^2\vec{r}_{02}{}^2}{\vec{r}_{14}{}^2\vec{r}_{24}{}^2}\right).$$

$$M_2 = \left(\frac{\vec{r}_{12}{}^2\vec{r}_{23}{}^2}{\vec{r}_{01}{}^2\vec{r}_{02}{}^2\vec{r}_{24}{}^2\vec{r}_{34}{}^2} - \frac{\vec{r}_{14}{}^2\vec{r}_{23}{}^2}{\vec{r}_{01}{}^2\vec{r}_{04}{}^2\vec{r}_{24}{}^2\vec{r}_{34}{}^2} - \frac{\vec{r}_{03}{}^2\vec{r}_{12}{}^2}{\vec{r}_{01}{}^2\vec{r}_{02}{}^2\vec{r}_{04}{}^2\vec{r}_{34}{}^2} + \frac{\vec{r}_{13}{}^2}{\vec{r}_{01}{}^2\vec{r}_{04}{}^2\vec{r}_{34}{}^2}\right)$$
$$\times \frac{1}{4}\ln\left(\frac{\vec{r}_{02}{}^2}{\vec{r}_{24}{}^2}\right).$$

Summary

- The nonlinear LO low-x evolution equation (closed in color space) for a Baryon Green function.
- NLO evolution equation.
- Transformation of the NLO equation to the quasi-conformal form.

Work in progress and plans

- Solution of the equation.
- Comparison with the results from JIMWLK (Kovner Lublinsky Mulian 2014, S. Caron-Huot 2014).
- Phenomenology

Thank you for your attention