Basics of Event Generators II

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and Theoretical Physics
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Correlations between partons in nucleons
Orsay 2014.06.30

Outline of Lectures

- Lecture I: Basics of Monte Carlo methods, the event generator strategy, matrix elements, LO/NLO, . . .
- Lecture II: Parton showers, initial/final state, matching/merging, . . .
- Lecture III: Underlying events, multiple interactions, minimum bias, pile-up, hadronization, decays, . . .
- Lecture IV: Correlations between partons in nucleons, summary, . . .

Buckley et al. (MCnet collaboration), *Phys. Rep.* **504** (2011) 145.

Outline

Final-State Showers
- Angular Ordering
- Evolution Variables
- The Sudakov Veto Algorithm

Initial-State Showers
- Backwards Evolution
- $k_\perp$-Factorization

Matching and Merging
- The Basic Idea
- Tree-level matching
- NLO Matching
- Multi-leg NLO Matching

The purpose of parton showers is to generate real exclusive events on parton level down to a very low (almost non-perutbative) jet resolution scale μ.

Starting from an initial hard scattering eg. $\mathrm{e}^+\mathrm{e}^- \to \mathrm{q}\bar{\mathrm{q}}$ or $\mathrm{q}\bar{\mathrm{q}} \to Z^0$, we basically need

$$\begin{aligned}
\sigma_{+0} &= \sigma_0(1 + C_{01}\alpha_\mathrm{s} + C_{02}\alpha_\mathrm{s}^2 + C_{03}\alpha_\mathrm{s}^3 + \ldots) \\
\sigma_{+1} &= \sigma_0(C_{11}\alpha_\mathrm{s} + C_{12}\alpha_\mathrm{s}^2 + C_{13}\alpha_\mathrm{s}^3 + \ldots) \\
\sigma_{+2} &= \sigma_0(C_{22}\alpha_\mathrm{s}^2 + C_{23}\alpha_\mathrm{s}^3 + C_{24}\alpha_\mathrm{s}^4 + \ldots) \\
&\vdots
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NLO generators only gives us one extra parton.

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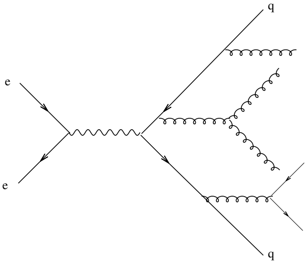
Final-State Showers

The tree-level matrix element for an *n*-parton state can be approximated by a product of splitting functions corresponding to a sequence of one-parton emissions from the zeroth order state.



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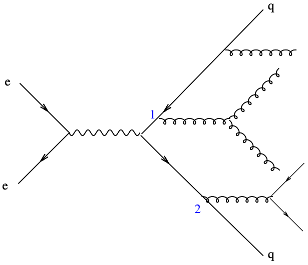

We can then order the emissions acording to some resolution scale, ρ, so that $\rho_1 \gg \rho_2 \gg \rho_3 \gg \ldots$

We have the standard DGLAP splitting kernels

$$
\begin{aligned}
P_{\mathrm{q}\to\mathrm{qg}}(\rho,z)d\rho dz &= \frac{\alpha_{\mathrm{s}}}{2\pi}\, dz\frac{d\rho}{\rho}\, C_F\, \frac{1+z^2}{1-z} \\
P_{\mathrm{g}\to\mathrm{gg}}(\rho,z)d\rho dz &= \frac{\alpha_{\mathrm{s}}}{2\pi}\, dz\frac{d\rho}{\rho}\, N_C\, \frac{(1-z(1-z))^2}{z(1-z)} \\
P_{\mathrm{g}\to\mathrm{q\bar{q}}}(\rho,z)d\rho dz &= \frac{\alpha_{\mathrm{s}}}{2\pi}\, dz\frac{d\rho}{\rho}\, T_R\,(z^2+(1-z)^2)
\end{aligned}
$$

where ρ is the squared invariant mass or transverse momentum, and z is the energy (or light-cone) fraction taken by one of the daugthers. (We ignore the ϕ-dependence here).

We now to make the events exclusive. This is done by saying that the first emission at some ρ_1 is given by the splitting kernel multiplied by the probability that there has been no emission above that scale.

In a given interval $d\rho$ we have the no-emission probability

$$1 - d\rho \sum_{bc} \int dz\, P_{a\to bc}(z,\rho)$$

Integrating from ρ_1 up to some maximum scale, ρ_0 we get

$$\Delta(\rho_0,\rho_1) = \exp\left(- \sum_{bc} \int_{\rho_1}^{\rho_0} d\rho \int dz\, P_{a\to bc}(z,\rho) \right)$$

In the same way we get the probability to have the *n*th emission at some scale ρ_n

$$
\begin{aligned}
P(\rho_n) &= \sum_{abc} \int dz\, P_{a\to bc}(\rho_n, z) \times \\
& \exp\left(-\sum_{abc} \int_{\rho_n}^{\rho_{n-1}} d\rho' \int dz'\, P_{a\to bc}(z', \rho') \right)
\end{aligned}
$$

Integrating we get schematically

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We still need a cutoff, ρ_{cut}, and the coefficients C_{nn}^{PS} diverges as $\log^{2n} \rho_{\max}/\rho_{\mathrm{cut}}$

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The divergencies comes from the soft and collinear poles in the splitting kernels, eg.

$$\int_{\rho_c}^{\rho_0} d\rho \int dz\, P_{q\to qg}(\rho, z) \sim \int_{\rho_c}^{\rho_0} \frac{\alpha_s d\rho}{\rho} \ln(\rho_0/\rho) \sim \alpha_s \ln^2(\rho_0/\rho_c)$$

Parton showers systematically resums all orders of $\alpha_s^n \ln^{2n}(\rho_0/\rho_c)$ which is the main part of the higher order corrections.
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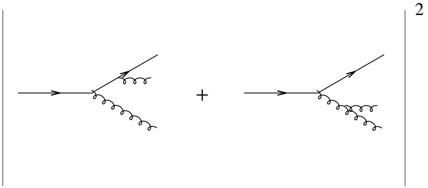
(Also important terms $\sim \alpha_s^n \ln^{2n-1}(\rho_0/\rho_c)$ are resummed.)

However if there is no strong ordering, $\rho_1 \gg \rho_2 \gg \rho_3 \gg \ldots$, the PS approximation breaks down

Parton showers cannot model several hard jets very well. Especially the correlations between hard jets are poorly described.

Angular Ordering

The splitting probabilities means that coherence effects are not taken into account



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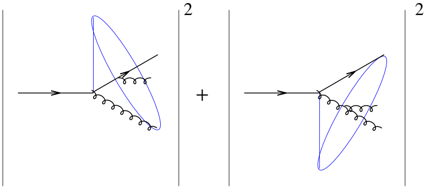

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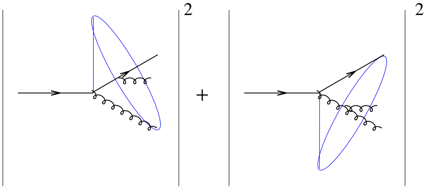

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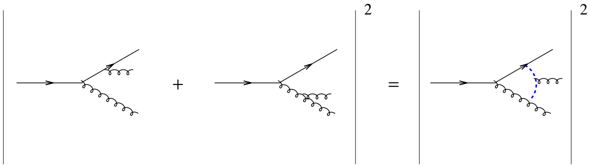
Some angular correlations can also be taken into account by adjusting the azimuthal angles after a shower is generated.

Coherence effects can be included directly, by considering gluon radiation from colour dipoles between colour-connected partons.



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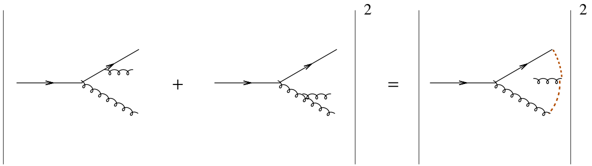


Rather than iterating $1 \to 2$ parton splitting we iterate $2 \to 3$ splittings. Each emission from a dipole will create two new dipoles which can continue radiating.

This was first implemented in the ARIADNE generator. Recently similar schemes have been implemented in PYTHIA, HERWIG++, SHERPA and VINCIA.

Evolution Variables

How do we choose the evolution variable, ρ?

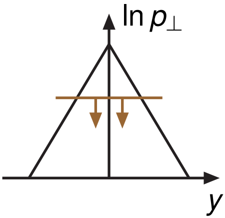
The most natural choice is to choose a variable which isolates both the soft and collinear poles in the splitting kernel. This is the case for $\rho = p_\perp^2$ as used in eg. ARIADNE.

In old versions of PYTHIA and SHERPA the evolution variable is the virtuality Q^2 which in principle is fine except that $\alpha_s(p_\perp^2)$ may diverge for any given Q^2. Also angular ordering needs to be imposed in separately.

In HERWIG the ordering is in angle, which ensures angular ordering, but does not isolate the soft pole, and an additional cutoff is needed.

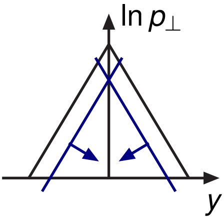
Transverse momentum

$\rho = p_\perp^2$



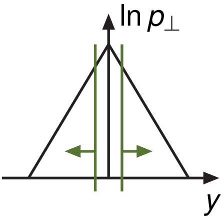
Virtuality

$\rho = Q^2 \sim \frac{p_\perp^2}{z(1-z)}$

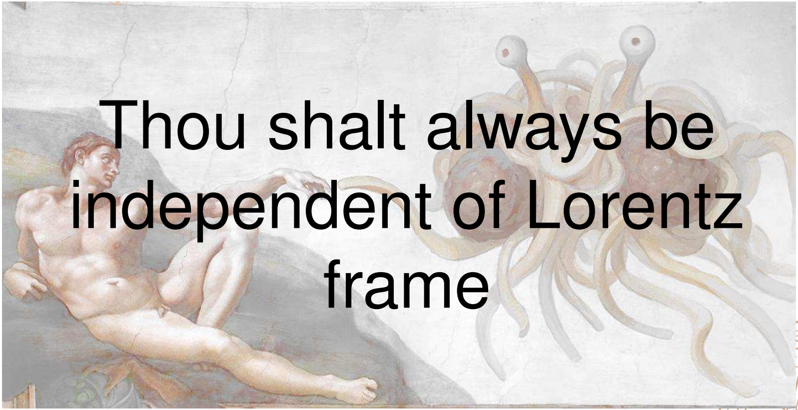


Angle

$\rho \sim E^2\theta^2 \sim \frac{p_\perp^2}{z^2(1-z)^2}$

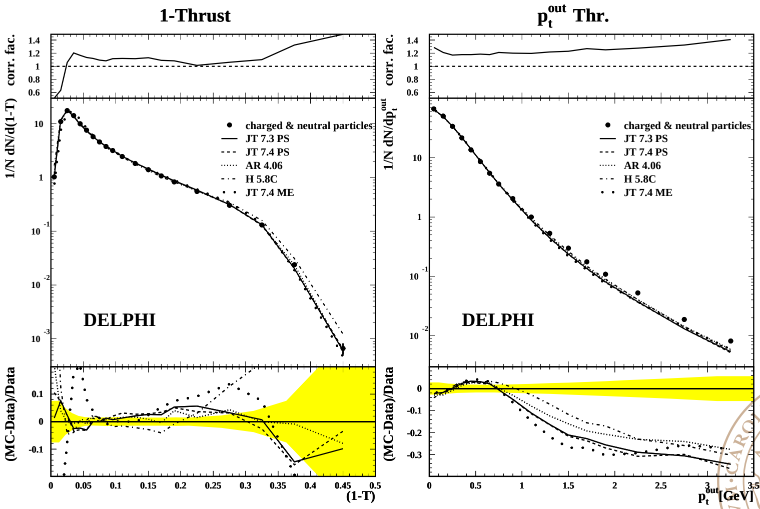


The Sixth Commandment of Event Generation



Thou shalt always be independent of Lorentz frame

Final-state parton showers did really well at LEP



How do we generate a parton shower emission?

$$\mathcal{P}(t) = P(t) \exp\left(-\int_t^{t_{\max}} dt' P(t')\right)$$

$\mathcal{P}(t)$ is a probability distribiution, so we can do the standard transformation method

$$1 - r = \int_r^1 dt\, p_R(t) = \int_t^{t_{\max}} dt \mathcal{P}(t) = 1 - \exp\left(-\int_t^{t_{\max}} dt' P(t')\right)$$

So if P has a simple primitive function F we get

$$t = F^{-1}(F(t_{\max}) - \ln r)$$

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Assume g is a simple function with a simple primitive function G such that $g(t) \geq P(t), \forall t$. Then we can use the following algorithm

- start with $t_0 = t_{\max}$;
- select $t_i = G^{-1}(G(t_{i-1}) - \ln R)$,
- compare a (new) R with the ratio $\mathcal{P}(t_i)/g(t_i)$; if $\mathcal{P}(t_i)/g(t_i) \leq R$, then return to point 2 for a new try, $i \to i+1$;
- otherwise t_i is retained as final answer.

If $t_i < t_{\rm cut}$, there is no emission and the shower is done.

Consider the various ways in which one can select a specific scale t. The probability that the first try works, $t = t_1$, i.e. that no intermediate t values need be rejected, is given by

$$p_0(t) = e^{-\int_t^{t_{\max}} g(t')\,dt'} g(t) \frac{P(t)}{g(t)} = P(t) e^{-\int_t^{t_{\max}} g(t')\,dt'}$$

The probability that we have thrown away one intermediate value t_1

$$p_1(t) = \int_t^{t_{\max}} dt_1\, e^{-\int_{t_1}^{t_{\max}} g(t')\,dt'} g(t_1) \left[1 - \frac{P(t_1)}{g(t_1)}\right] \times \\ \times e^{-\int_t^{t_1} g(t')\,dt'} g(t) \frac{P(t)}{g(t)}$$

$$p_1(t) = p_0(t) \int_t^{t_{\max}} dt_1 \, [g(t_1) - P(t_1)]$$

Similarly we get

$$\begin{aligned} p_2(t) &= p_0(t) \int_t^{t_{\max}} dt_1 \, [g(t_1) - P(t_1)] \int_t^{t_1} dt_2 \, [g(t_2) - P(t_2)] \\ &= p_0(t) \frac{1}{2} \left(\int_t^{t_{\max}} [g(t') - P(t')] \, dt' \right)^2 \end{aligned}$$

$$\begin{aligned} p_{tot}(t) &= \sum_{n=0}^{\infty} p_n(t) = p_0(t) \sum_{n=0}^{\infty} \frac{1}{n!} \left(\int_t^{t_{\max}} [g(t') - P(t')] \, dt' \right)^n \\ &= P(t) e^{-\int_t^{t_{\max}} g(t') \, dt'} e^{\int_t^{t_{\max}} [g(t') - P(t')] dt'} \\ &= P(t) e^{-\int_t^{t_{\max}} P(t') \, dt'} \end{aligned}$$

Also if several things may happen, $P_1(t)$, $P_2(t)$, $P_3(t)$, ... the probability of i happening first is

$$P_i(t) \times \prod_j e^{-\int_t^{t_{\max}} P_j(t')\,dt'}$$

Simply generate a scale for each i according to

$$P_i(t) \times e^{-\int_t^{t_{\max}} P_i(t')\,dt'}$$

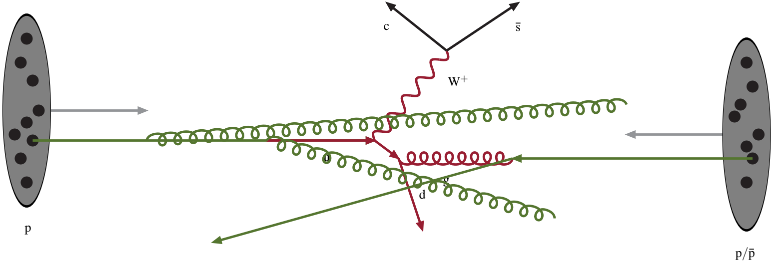
and pick the process with the largest scale.

Initial-State Showers

For incoming hadrons, we need to consider the evolution of the parton densities. Using collinear factorization and DGLAP evolution we have (with $t = \log k_\perp^2/\Lambda^2$)

$$\frac{df_b(x,t)}{dt} = \sum_a \int \frac{dx'}{x'} f_a(x',t) \frac{\alpha_s}{2\pi} P_{a\to b}\left(\frac{x}{x'}\right)$$

We can interpret this as during a small increase dt there is a probability for parton a with momentum fraction x' to become resolved into parton b at $x = zx'$ and another parton c at $x' - x = (1-z)x'$.


c
s̄
W+
d
p
p/p̄

In a backward evolution scenario we start out with the hard sub-process at some scale $t_{\max}$

$$\sigma_0 \propto \hat{\sigma}_{ab\to X} f_a(x_a, t_{\max}) f_b(x_b, t_{\max})$$

and we get the relative probability for the parton a to be *unresolved* into parton c during a decrease in scale dt

$$d\mathcal{P}_a = \frac{df_a(x_a,t)}{f_a(x_a,t)} = |dt| \sum_c \int \frac{dx'}{x'} \frac{f_c(x',t)}{f_a(x_a,t)} \frac{\alpha_s}{2\pi} P_{c\to a}\left(\frac{x_a}{x'}\right)$$

Summing up the cumulative effect of many small changes dt, the probability for no radiation exponentiates and we get a Sudakov

$$\Delta_{S_{+a}}(x_a, t_{\max}, t) = \exp\left\{ -\int_t^{t_{\max}} dt' \sum_c \int \frac{dx'}{x'} \frac{f_c(x',t')}{f_a(x_a,t')} \frac{\alpha_s(t')}{2\pi} P_{c\to a}\left(\frac{x_a}{x'}\right) \right\}$$

This now gives us the probability for the first backwards initial-state splitting

$$d\mathcal{P}_{ca} = \frac{\alpha_s}{2\pi} P_{ac}(z) \frac{f_c(x_a/z, t)}{f_a(x_a, t)} dt \frac{dz}{z} \times \Delta_{S_{+a}}(x_a, t_{\max}, t)$$

In a hadronic collision we first generate the hard scattering, then evolve the incoming partons backward to lower scales, and then alow for a final-state shower from all partons from the hard scattering and the initial-state shower.

This is like undoing the evolution of the PDFs

The small-x problem

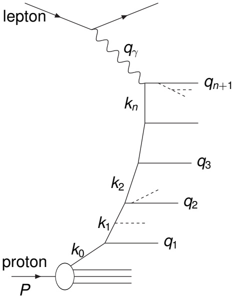
DGLAP evolution is not applicable if the hard scale is much smaller than the total energy and the virtuality of the incoming partons are not much smaller than the hard scale. (small x)

Collinear factorization $\Longrightarrow$ $k_\perp$-factorization

$$\int dx_a dx_b \hat{\sigma}_{ab\to X} f_a(x_a, Q^2) f_b(x_b, Q^2) \Longrightarrow$$

$$\int dx_a dx_b dk_{\perp a} dk_{\perp b} \hat{\sigma}^\star_{ab\to X} \mathcal{F}_a(x_a, k_{\perp a}, Q^2) \mathcal{F}_b(x_b, k_{\perp b}, Q^2)$$

$\mathcal{F}$ an unintegrated parton density.
$\hat{\sigma}^\star$ is the off-shell matrix element

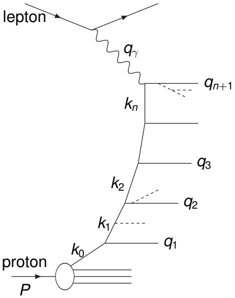


In DIS, the cross section is dominated by events with small $Q^2 = -q_\gamma^2$ and small x.

The available phase space for emitting partons is not limited by Q^2, but rather by the total hadronic energy, $W^2 \approx Q^2/x$.

The $1/z$ pole in the gluon splitting function makes it possible to emit many initial-state gluons even for small Q^2.

We need to take into account unordered evolution.



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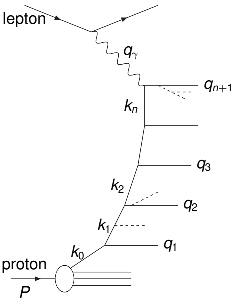

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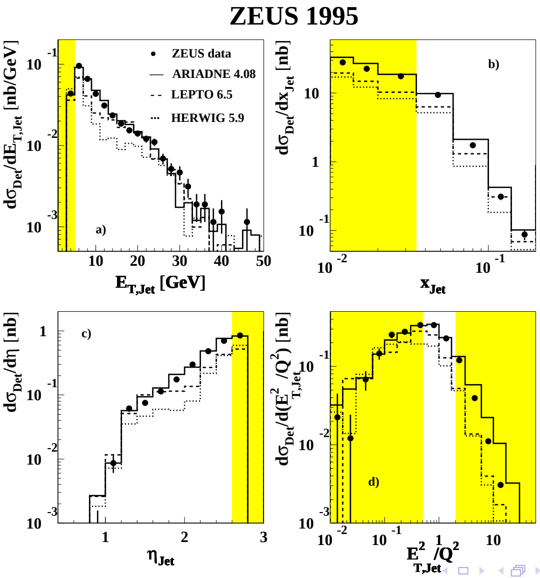
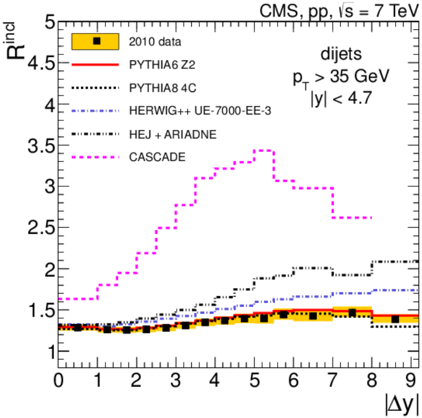
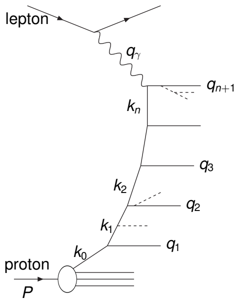

ZEUS 1995
ZEUS data
ARIADNE 4.08
LEPTO 6.5
HERWIG 5.9
a)
b)
c)
d)
$d\sigma_{Det}/dE_{T,Jet}$ [nb/GeV]
$E_{T,Jet}$ [GeV]
$d\sigma_{Det}/dx_{Jet}$ [nb]
x_{Jet}
$d\sigma_{Det}/d\eta$ [nb]
η_{Jet}
$d\sigma_{Det}/d(E^2_{T,Jet}/Q^2)$ [nb]
$E^2_{T,Jet}/Q^2$

However, at the LHC . . .





The power of $k_\perp$-factorization is the resummation of un-ordered emissions, which cannot be done by DGLAP-based showers.

BFKL- or CCFM-based showers do not yet describe data.

But we now have matrix element generators which can produce many un-ordered emissions.

But to get the complete picture we need to combine many multiplicities, and we still need parton showers.

Tree-level matrix element generators and are good for a handful hard, well separated partons, but bad for many soft and collinear partons.

Parton shower generators are not good for a handful hard, well separated partons, but good for many soft and collinear partons.

Let's try to get the best of both.

Fixed-order expansion of a parton shower

(using $\mathcal{P}_i = \frac{F_i}{F_{i-1}} P_i$)

$$\frac{d\sigma_0^{ex}}{d\phi_0} = F_0 \left|\mathcal{M}_0\right|^2 \left[1 - \alpha_S \int_{\rho_{MS}}^{\rho_0} d\rho dz\, \mathcal{P}_1 + \frac{\alpha_S^2}{2}\left(\int_{\rho_{MS}}^{\rho_0} d\rho dz\, \mathcal{P}_1\right)^2\right]$$

$$\frac{d\sigma_1^{ex}}{d\phi_0} = F_0 \left|\mathcal{M}_0\right|^2 \alpha_S \mathcal{P}_1 d\rho_1 dz_1$$

$$\times \left[1 - \alpha_S \int_{\rho_1}^{\rho_0} d\rho dz\, \mathcal{P}_1 - \alpha_S \int_{\rho_{MS}}^{\rho_1} d\rho dz\, \mathcal{P}_2\right]$$

$$\frac{d\sigma_2}{d\phi_0} = F_0 \left|\mathcal{M}_0\right|^2 \alpha_S^2 \mathcal{P}_1 d\rho_1 dz_1 \mathcal{P}_2 d\rho_2 dz_2 \Theta(\rho_1 - \rho_2)$$

Unitary to all orders in α_S — total cross section is $F_0 \left|\mathcal{M}_0\right|^2$.
1-jet cross section will not even be correct to LO.

ME reweighting

We really want to improve our parton shower.

The easiest thing is

$$P_i \to P_i^{ME} \equiv \frac{|\mathcal{M}_i|^2 \, d\phi_i}{|\mathcal{M}_{i-1}|^2 \, d\phi_{i-1} d\rho dz}$$

This has been around quite a while in PYTHIA for the first splitting in some processes. Preserves the unitarity of the shower

$$\frac{d\sigma_0^{ex}}{d\phi_0} = F_0\,|\mathcal{M}_0|^2\left[1-\alpha_S\int_{\rho_{\rm MS}}^{\rho_0} d\rho dz\,\mathcal{P}_1^{ME} + \frac{\alpha_S^2}{2}\left(\int_{\rho_{\rm MS}}^{\rho_0} d\rho dz\,\mathcal{P}_1^{ME}\right)^2\right]$$

$$\frac{d\sigma_1^{ex}}{d\phi_0} = F_0\,|\mathcal{M}_0|^2\,\alpha_S\mathcal{P}_1^{ME}d\rho_1 dz_1$$

$$\times\left[1-\alpha_S\int_{\rho_1}^{\rho_0} d\rho dz\,\mathcal{P}_1^{ME} - \alpha_S\int_{\rho_{\rm MS}}^{\rho_1} d\rho dz\,\mathcal{P}_2\right]$$

$$\frac{d\sigma_2}{d\phi_0} = F_0\,|\mathcal{M}_0|^2\,\alpha_S^2\mathcal{P}_1^{ME}d\rho_1 dz_1\mathcal{P}_2 d\rho_2 dz_2\Theta(\rho_1-\rho_2)$$

Still unitary to all orders of α_S. We can decrease $\rho_{\rm MS}$ to the non-perturbative boundary ρ_{cut}.

Going to higher multiplicities turns out to be difficult.

Matching: The Basic Idea

A fixed-order ME-generator gives the first few orders in α_s exactly.

The parton shower gives approximate (N)LL terms to all orders in α_s through the Sudakov form factors.

- Take a parton shower and correct the first few terms in the resummation with (N)LO ME.
- Take events generated with (N)LO ME with subtracted Parton Shower terms. Add parton shower.
- Take events samples generated with (N)LO ME, reweight and combine with Parton showers:

Tree-level Merging

Has been around the whole millennium: CKKW(-L), MLM, ...

Combines samples of tree-level (LO) ME-generated events for different jet multiplicities. Reweight with proper Sudakov form factors (or approximations thereof).

Needs a merging scales to separate ME and shower region and avoid double counting. Only observables involving jets above that scale will be correct to LO.

Typically the merging scale dependence is beyond the precision of the shower: $\sim \mathcal{O}(L^3\alpha_s^2)\frac{1}{N_c^2} + \mathcal{O}(L^2\alpha_s^2)$.

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CKKW(-L)

Generate inclusive few-jet samples according to exact tree-level $|\mathcal{M}_n|^2$ using some merging scale ρ_{MS}.

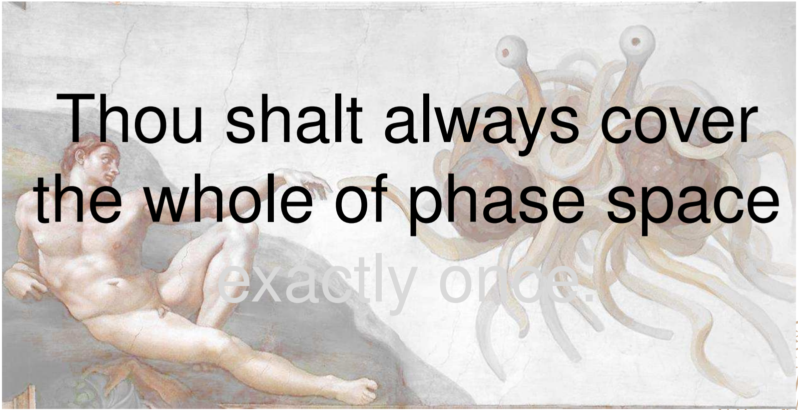
These are then made exclusive by reweighting no-emission probabilities (in CKKW-L generated by the shower itself)

Add normal shower emissions below ρ_{MS}.

Add all samples together.

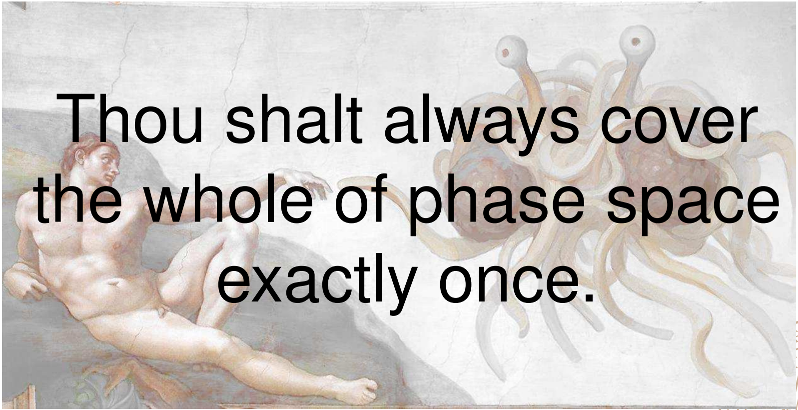
- Dependence on the merging scale cancels to the precision of the shower.
- If the merging scale is not defined in terms of the shower ordering variable, we need vetoed and truncated showers.
- Breaks the unitarity of the shower.

The Second Commandment of Event Generation



Thou shalt always cover the whole of phase space exactly once.

The Second Commandment of Event Generation



Thou shalt always cover the whole of phase space exactly once.

Multi-jet tree-level matching

$$\frac{d\sigma_0^{ex}}{d\phi_0} = F_0 \left|\mathcal{M}_0\right|^2 \left[1 - \alpha_S \int_{\rho_{\mathrm{MS}}}^{\rho_0} d\rho dz\, \mathcal{P}_1 + \frac{\alpha_S^2}{2}\left(\int_{\rho_{\mathrm{MS}}}^{\rho_0} d\rho dz\, \mathcal{P}_1\right)^2\right]$$

$$\frac{d\sigma_1^{ex}}{d\phi_0} = F_0 \left|\mathcal{M}_0\right|^2 \alpha_S \mathcal{P}_1^{ME} d\rho_1 dz_1 \times \left[1 - \alpha_S \int_{\rho_1}^{\rho_0} d\rho dz\, \mathcal{P}_1 - \alpha_S \int_{\rho_{\mathrm{MS}}}^{\rho_1} d\rho dz\, \mathcal{P}_2\right]$$

$$\frac{d\sigma_2}{d\phi_0} = F_0 \left|\mathcal{M}_0\right|^2 \alpha_S^2 \mathcal{P}_1^{ME} d\rho_1 dz_1 \mathcal{P}_2^{ME} d\rho_2 dz_2 \Theta(\rho_1 - \rho_2)$$

NOT unitary. Gives artificial dependence of ρ_{MS}.
e.g. extra contribution to $\int \alpha_S \mathcal{P}_1^{ME}$ is $\sim \alpha_S^2 L^3$.

Mature procedure. Available in HERWIG++, SHERPA, PYTHIA8.

The MLM-procedure (ALPGEN + HERWIG/PYTHIA) is similar, but even less control over the perturbative expansion.

There are recent procedures to restore unitarity:

- Vincia exponentiates the full *n*-parton matrix elements.
- UMEPS uses a add/subtract procedure combined with a re-clustering algorithm.

UMEPS – Restoring unitarity

$$\frac{d\sigma_0^{ex}}{d\phi_0} = F_0 \left|\mathcal{M}_0\right|^2 \left[1 - \alpha_S \int_{\rho_{MS}}^{\rho_0} d\rho dz\, \mathcal{P}_1 + \frac{\alpha_S^2}{2}\left(\int_{\rho_{MS}}^{\rho_0} d\rho dz\, \mathcal{P}_1\right)^2\right]$$

$$\frac{d\sigma_1^{ex}}{d\phi_0} = F_0 \left|\mathcal{M}_0\right|^2 \alpha_S \mathcal{P}_1^{ME} d\rho_1 dz_1 \left[1 - \alpha_S \int_{\rho_1}^{\rho_0} d\rho dz\, \mathcal{P}_1 - \alpha_S \int_{\rho_{MS}}^{\rho_1} d\rho dz\, \mathcal{P}_2\right]$$

$$\frac{d\sigma_2}{d\phi_0} = F_0 \left|\mathcal{M}_0\right|^2 \alpha_S^2 \mathcal{P}_1^{ME} d\rho_1 dz_1 \mathcal{P}_2^{ME} d\rho_2 dz_2 \Theta(\rho_1 - \rho_2)$$

UMEPS – Restoring unitarity

$$\frac{d\sigma_0^{fx}}{d\phi_0} = F_0 \left|\mathcal{M}_0\right|^2 \left[1 - \alpha_S \int_{\rho_{MS}}^{\rho_0} d\rho dz \, \mathcal{P}_1 + \frac{\alpha_S^2}{2}\left(\int_{\rho_{MS}}^{\rho_0} d\rho dz \, \mathcal{P}_1\right)^2\right]$$

$$\frac{d\sigma_1^{fx}}{d\phi_0} = F_0 \left|\mathcal{M}_0\right|^2 \alpha_S \mathcal{P}_1^{ME} d\rho_1 dz_1 \left[1 - \alpha_S \int_{\rho_1}^{\rho_0} d\rho dz \, \mathcal{P}_1 - \alpha_S \int_{\rho_{MS}}^{\rho_1} d\rho dz \, \mathcal{P}_2\right]$$

$$\frac{d\sigma_2}{d\phi_0} = F_0 \left|\mathcal{M}_0\right|^2 \alpha_S^2 \mathcal{P}_1^{ME} d\rho_1 dz_1 \mathcal{P}_2^{ME} d\rho_2 dz_2 \Theta(\rho_1 - \rho_2)$$

UMEPS – Restoring unitarity

$$\frac{d\sigma_0^{fx}}{d\phi_0} = F_0 \left|\mathcal{M}_0\right|^2 \left[1 - \alpha_S \int_{\rho_{MS}}^{\rho_0} d\rho dz \, \mathcal{P}_1 + \frac{\alpha_S^2}{2} \left(\int_{\rho_{MS}}^{\rho_0} d\rho dz \, \mathcal{P}_1 \right)^2 \right]$$

$$- \int d\rho_1 dz_1 \frac{d\sigma_1^{fx}}{d\phi_0 d\rho_1 dz_1}$$

$$\frac{d\sigma_1^{fx}}{d\phi_0} = F_0 \left|\mathcal{M}_0\right|^2 \alpha_S \mathcal{P}_1^{ME} d\rho_1 dz_1 \left[1 - \alpha_S \int_{\rho_1}^{\rho_0} d\rho dz \, \mathcal{P}_1 - \alpha_S \int_{\rho_{MS}}^{\rho_1} d\rho dz \, \mathcal{P}_2 \right]$$

$$- \int d\rho_2 dz_2 \frac{d\sigma_2^{fx}}{d\phi_0 d\rho_1 dz_1 d\rho_2 dz_2}$$

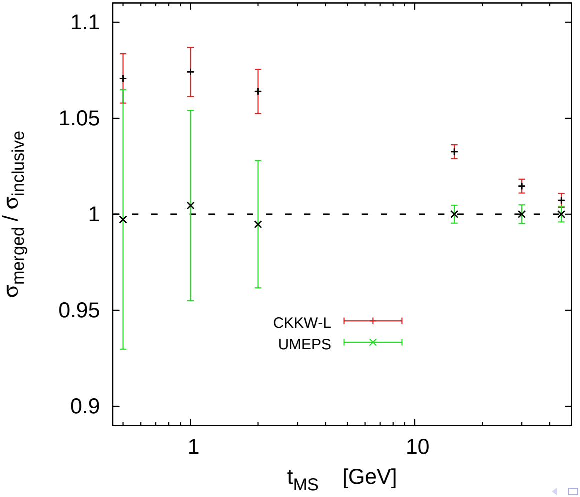
$$\frac{d\sigma_2}{d\phi_0} = F_0 \left|\mathcal{M}_0\right|^2 \alpha_S^2 \mathcal{P}_1^{ME} d\rho_1 dz_1 \mathcal{P}_2^{ME} d\rho_2 dz_2 \Theta(\rho_1 - \rho_2)$$

In CCKW we need to recreate the sequence of emissions.

In CKKW-L this is done by selecting a full parton shower history of an *n*-parton state.

In UMEPS performing the integration is simply to replace the *n*-jet the state with the one with one jet less in the history.

But why worry about unitarity, the cross sections are never better than LO anyway, so scale uncertainties are huge.



NLO

The anatomy of NLO calculations.

$$\langle\mathcal{O}\rangle = \int d\phi_n \left(B_n + V_n\right) \mathcal{O}_n(\phi_n) + \int d\phi_{n+1} B_{n+1} \mathcal{O}_{n+1}(\phi_{n+1}).$$

Not practical, since V_n and B_{n+1} are separately divergent, although their sum is finite.

The standard subtraction method:

$$\langle\mathcal{O}\rangle = \int d\phi_n \left(B_n + V_n + \sum_p \int d\psi^{(a)}_{n,p} S^{(a)}_{n,p}\right) \mathcal{O}_n(\phi_n) + \int d\phi_{n+1} \left(B_{n+1} \mathcal{O}_{n+1}(\phi_{n+1}) - \sum_p S^{(a)}_{n,p} \mathcal{O}_n\left(\frac{\phi_{n+1}}{\psi^{(a)}_{n,p}}\right)\right),$$

MC@NLO

(Frixione et al.)

The subtraction terms must contain all divergencies of the real-emission matrix element. A parton shower splitting kernel does exactly that.

Generating two samples, one according to $B_n + V_n + \int S_n^{\mathrm{PS}}$, and one according to $B_{n+1} - S_n^{\mathrm{PS}}$, and just add the parton shower from which S_n is calculated.

POWHEG

(Nason et al.)

Calculate $\overline{B}_n = B_n + V_n + \int B_{n+1}$ and generate n-parton states according to that.

Generate a first emission according to B_{n+1}/B_n, and then add any[1] parton shower for subsequent emissions.

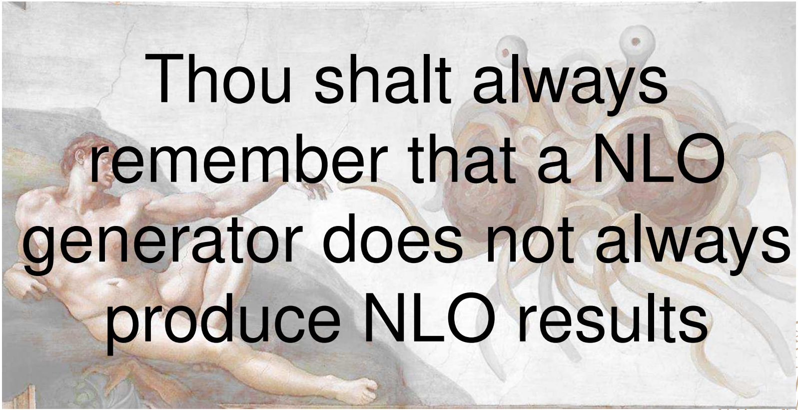
[1] As long as it is transverse-momentum ordered in the same way as in POWHEG or properly truncated

POWHEG and MC@NLO are very similar. They are both correct to NLO, but differ at higher orders

- POWHEG exponentiates also non singular pieces of the $n+1$ parton cross section
- POWHEG multiplies the $n+1$ parton cross section with $\overline{B}_n/B_n$ (the phase-space dependent K-factor).

POWHEG may also resum $k_\perp > \mu_R$, and will then generate additional logarithms, $log(S/\mu_R) \sim \log(1/x)$.

The Fifth Commandment of Event Generation



Thou shalt always remember that a NLO generator does not always produce NLO results

Really NLO?

Do NLO-generators always give NLO-predictions?

For simple Born-level processes such as Z^0-production, all inclusive Z^0 observables will be correct to NLO.

- y_Z
- y_e
- $p_{\perp e}$

But note that for $p_{\perp e} > m_Z/2$ the prediction is only leading order!

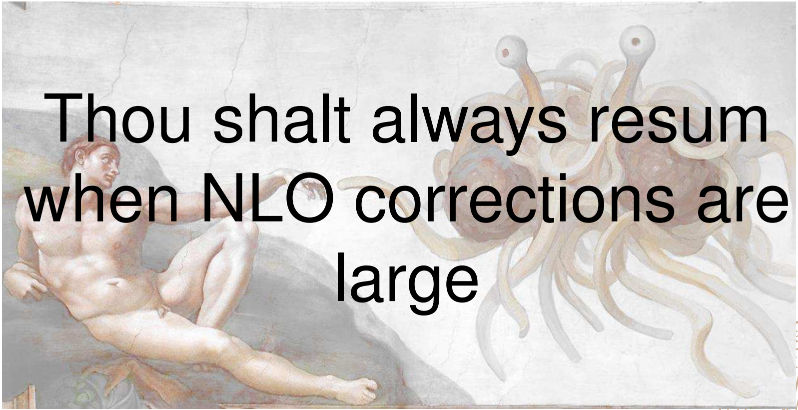
Also $p_{\perp Z}$ is LO. To get NLO we need to start with Z+jet at Born-level and calculate full α_S^2.

But for small $p_{\perp Z}$ the NLO cross section diverges due to $L^{2n}\alpha_s^n$, $L = \log(p_{\perp Z}/\mu_R)$.

If $L^2\alpha_s \sim 1$, the α_s^2 corrections are parametrically as large as the NLO corrections.

Can be alleviated by clever choices for μ_R, but in general you need to resum.

The Seventh Commandment of Event Generation



Thou shalt always resum when NLO corrections are large

Multi-leg Matching

We need to be able to combine several NLO calculations and add (parton shower) resummation in order to get reliable predictions.

- No double (under) counting.
  - No parton shower emissions which are already included in (tree-level) ME states.
  - No terms in the PS no-emission resummation which are already in the NLO
- Dependence of any merging scale must not destroy NLO accuracy.
  - The NLO 0-jet cross section must not change too much when adding NLO 1-jet.
  - Dependence on logarithms of the merging scale should be less than $L^3\alpha_s^2$ in order for predictions to be stable for small scales.

SHERPA

First *working* solution for hadronic collisions.

CKKW-like combining of (MC@)NLO-generated events, fixing up double counting of NLO real and virtual terms.

Any jet multiplicity possible.

Dependence on merging scale canceled at NLO and parton-shower precision.

Residual dependence: $L^3\alpha_s^2/N_C^2$ — can't take merging scale too low.

MINLO

No merging scale!

- Take e.g. POWHEG Higgs+1-jet calculation down to *very* low $p_\perp$.
- Use clever (nodal) renormalization scales
- Multiply with (properly subtracted) Sudakov form factor
- Add non-leading terms to Sudakov form factor to get correct NLO 0-jet cross section.

Possible to go to NNLO!

Not clear how to go to higher jet multiplicities.

UNLOPS

Start from UMEPS (unitary version of CKKW-L).
Add (and subtract) *n*-jet NLO samples, fixing up double counting of NLO real and virtual terms.

$$\frac{d\sigma_1^{sub}}{d\phi_0} = \alpha_S \mathcal{P}_1^{ME} d\rho_1 dz_1 \left[\Pi_0(\rho_0, \rho_1) - 1 + \alpha_S \int_{\rho_1}^{\rho_0} d\rho dz \, \mathcal{P}_1 \right]$$

Note that PS uses $\alpha_S(\rho)$ and $f(x, \rho)$
rather than $\alpha_S(\mu_R)$ and $f(x, \mu_F)$

Any jet multiplicity possible.

Although there is a merging scale, the dependence of an *n*-jet cross section due to addition of higher multiplicities drops out completely. Merging scale can be taken arbitrarily small.

— Lots of negative weights.

Possible to go to NNLO?

Available in PYTHIA8
(and HERWIG++ in Simon Plätzer's incarnation)

GENEVA

- Analytic (SCET) resummation of NLO cross section to NLL (or even NNLL!) in the merging scale variable.
- Only $\mathrm{e}^+\mathrm{e}^-$ so far (W-production in *pp* on its way).

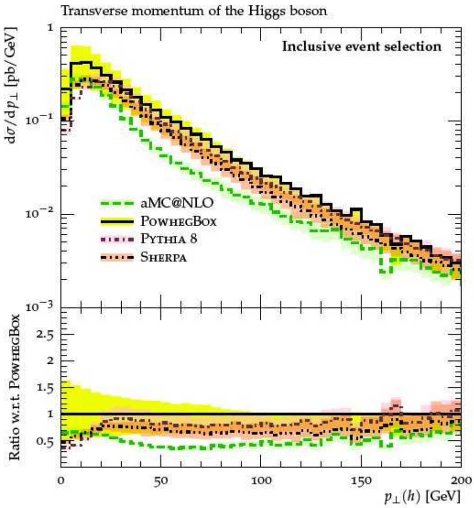
VINCIA

- Exponentiate NLO Matrix Elements in no-emission probability — no merging scale.
- Only $\mathrm{e}^+\mathrm{e}^-$ so far

FxFx

- MLM-like merging of different MC@NLO calculations.
- Difficult to understand merging scale dependence

Les Houches comparison



Outline of Lectures

- Lecture I: Basics of Monte Carlo methods, the event generator strategy, matrix elements, LO/NLO, . . .
- Lecture II: Parton showers, initial/final state, matching/merging, . . .
- Lecture III: Underlying events, multiple interactions, minimum bias, pile-up, hadronization, decays, . . .
- Lecture IV: Correlations between partons in nucleons, summary, . . .

Buckley et al. (MCnet collaboration), *Phys. Rep.* **504** (2011) 145.