Multiparton interactions
Part 1

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Correlations between Partons in Nucleons
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Some literature an entirely incomplete list

- two "classics"

  A. Del Fabbro and D. Treleani, A Double parton scattering background to Higgs boson production at the LHC, Phys. Rev. D **61** (2000) 077502 [hep-ph/9911358]

  T. Sjöstrand and M. van Zijl, Multiple Parton-parton Interactions in an Impact Parameter Picture, Phys. Lett. B **188** (1987) 149

- recent overviews (proceedings):

  P. Bartalini et al., Multi-Parton Interactions at the LHC, arXiv:1111.0469

  S. Plätzer and M. D., Proceedings of MPI@LHC 2011, DESY-PROC-2012-03

  phenomenological mini-review in:
  T. Sjöstrand and P. Z. Skands, Multiple interactions and the structure of beam remnants, JHEP **0403** (2004) 053 [hep-ph/0402078].

- theoretical approach followed in these lectures (short):
  M. D. and A. Schäfer, Theoretical considerations on multiparton interactions in QCD, Phys. Lett. B **698** (2011) 389 [arXiv:1102.3081]

  (not so short):
  M. D., D. Ostermeier and A. Schäfer, Elements of a theory for multiparton interactions in QCD, JHEP **1203** (2012) 089 [arXiv:1111.0910].

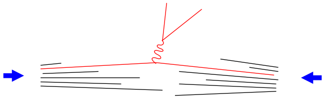
Starting from basics: single and multiple hard scattering

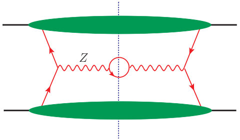
Factorization formulae: single hard scattering

- standard description for hard processes in pp collisions
  example: Z production (followed by decay $Z \to \ell^+ \ell^-$)

$$\frac{d\sigma(pp \to Z + X)}{dx\, d\bar{x}} = f_q(x)\, f_{\bar{q}}(\bar{x})\, \hat{\sigma}(q\bar{q} \to Z)$$

x and $\bar{x}$ measurable, related to Z rapidity, $\hat{\sigma}$ includes $\delta(sx\bar{x} - m_Z^2)$





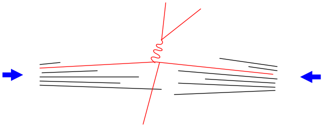
- factorization formulae are for inclusive cross sections $pp \to Y + X$
  where $Y =$ produced in parton-level scattering, can specify in detail
  $X =$ summed over, no details
- above formula is just tree level, usually not precise enough

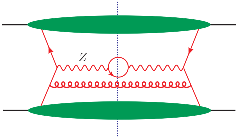
Factorization formulae: single hard scattering

- standard description for hard processes in pp collisions
  include radiation:

$$\frac{d\sigma(pp \to Z + X)}{dx\, d\bar{x}} = f_q(x)\, f_{\bar{q}}(\bar{x})\, \hat{\sigma}(q\bar{q} \to Z)$$
$$+ \int_x^1 dz \int_{\bar{x}}^1 d\bar{z}\, f_q(z)\, f_{\bar{q}}(\bar{z})\, \hat{\sigma}(q\bar{q} \to Z + g) + \text{further terms}$$

$\hat{\sigma}(q\bar{q} \to Z)$ now includes one-loop corrections



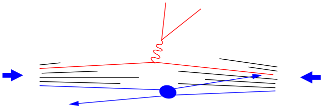


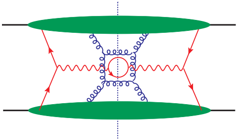
- extra radiation part of unobserved system X
- but this is still oversimplified: “spectator partons” interact as well

Factorization formulae: single hard scattering

- standard description for hard processes in pp collisions
  inclusive cross section:

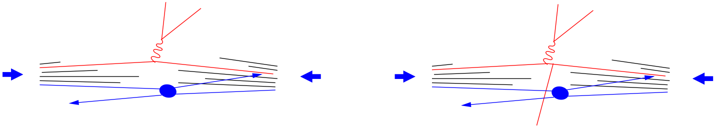
$$\frac{d\sigma(pp \to Z + X)}{dx\, d\bar{x}} = f_q(x)\, f_{\bar{q}}(\bar{x})\, \hat{\sigma}(q\bar{q} \to Z) + \int_x^1 dz \int_{\bar{x}}^1 d\bar{z}\, f_q(z)\, f_{\bar{q}}(\bar{z})\, \hat{\sigma}(q\bar{q} \to Z + g) + \text{further terms}$$





- "spectator" interactions produce additional particles
  which are also part of unobserved system X ("underlying event")
- need not calculate this thanks to unitarity
  as long as cross section/observable sufficiently inclusive

Multiparton interactions (MPI)



- generically take place in hadron-hadron collisions
- prodominantly low-p_T scattering $\rightsquigarrow$ underlying event (UE)
- at high c.m. energy several interactions can be hard
  $\rightsquigarrow$ multiple hard scattering

  extra interactions enhanced because

  (density of two small-x partons) $\gg$ (density of one small-x parton)
- effects cancel or are suppressed in sufficiently inclusive quantities
  but do affect final state properties
- these lectures: work within hard-scattering factorization
  focus on double parton scattering (DPS)

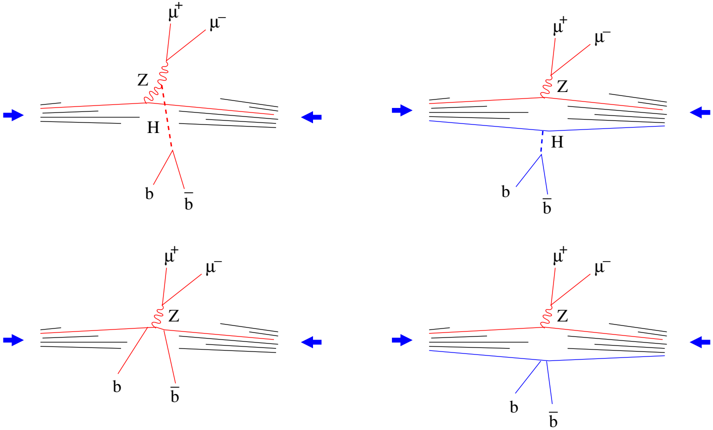
  alternative approach: small-x factorization ("BFKL ladders")

  $\rightarrow$ lectures by Raju Venugopalan

Relevance for LHC

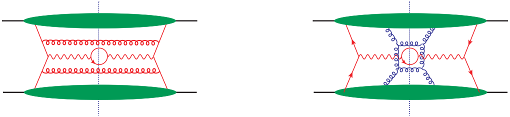
example: $pp \to H + Z \to b\bar{b} + Z$ Del Fabbro, Treleani 1999

- multiple interactions contribute to signal and background



same for $pp \to H + W \to b\bar{b} + W$ study for Tevatron: Bandurin et al, 2010

Double parton scattering



- assumed factorization formula, example: production of $Z+$ 2 jets

$$\frac{d\sigma(pp \to Z + 2\ \text{jets} + X)}{dx_1\, d\bar{x}_1\, dx_2\, d\bar{x}_2} = \text{single hard scattering}$$
$$+ \frac{1}{C} \int d^2\boldsymbol{y}\, F_{qg}(x_1, x_2, \boldsymbol{y})\, F_{\bar{q}g}(\bar{x}_1, \bar{x}_2, \boldsymbol{y})\, \hat{\sigma}(q\bar{q} \to Z)\, \hat{\sigma}(gg \to 2\ \text{jets})$$
$$+ \text{other subprocesses} + \text{higher orders} + \ldots$$

F_{qg} = double parton distribution (DPD)
$\boldsymbol{y}$ = transverse distance between two partons
C = combinatorial factor, here $C = 1$

- X includes further radiation from each hard scattering at higher orders and particles from further "spectator" interactions
- also have contribution from triple hard scattering
e.g. $q\bar{q} \to Z$, $gg \to$ jet $+ X$, $gg \to$ jet $+ X$

Inclusive and "exclusive" cross sections

- standard factorization formulae are for inclusive cross sections
- requires proper counting: an event with "3 jets" counts several times in $\sigma(pp \to Z + 2 \text{ jets} + X)$ → M. Seymour, A. Siodmok 2013
- computation of "exclusive" cross sections in general more complicated

  example: $pp \to Z + \text{exactly } 2 \text{ jets} + X$ with no further jet above $p_{T\text{cut}}$

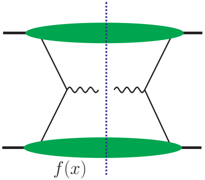
$$\begin{aligned}\sigma &= \sigma_{\text{single hard}}(pp \to Z + 2 \text{ jets} + X) - \sigma_{\text{single hard}}(pp \to Z + 3 \text{ jets} + X) \\ &\quad + \sigma_{\text{double hard}}(pp \to Z + 2 \text{ jets} + X) - \sigma_{\text{double hard}}(pp \to Z + 3 \text{ jets} + X) \\ &\quad - \sigma_{\text{triple hard}}(pp \to Z + 2 \text{ jets} + 2 \text{ jets} + X) - \ldots\end{aligned}$$

  where "jets" are required to have $p_T > p_{T\text{cut}}$

- if $p_{T\text{cut}} \ll$ other hard scales (e.g. m_Z) then
  - hardest scale for approximations is $p_{T\text{cut}}$, not m_Z
  - must resum Sudakov logarithms $\ln(p_{T\text{cut}}/m_Z)$ to all orders

Cross sections for definite transverse momenta

- standard factorization formulae have $\int$ over total transv. momentum produced in hard scattering
- example: Z production



- two possibilities to compute for measured q_T of Z
  both can be extended to double hard scattering
  needed for $d\sigma/dq_T$ and for $\sigma(q_T > q_{T\text{cut}})$

Cross sections for definite transverse momenta

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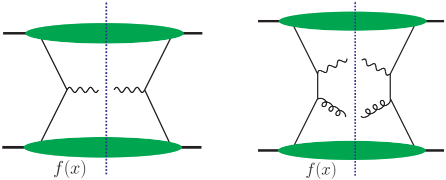
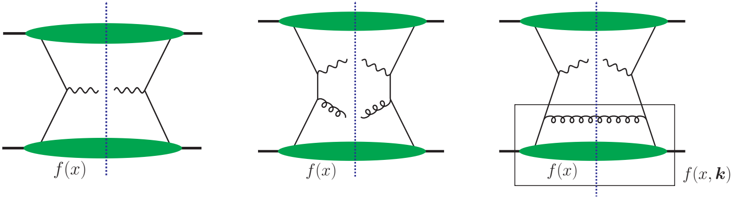
- example: Z production



- collinear factorization: compute emission of recoiling parton(s)
  - need $q_T \gg \Lambda$ since q_T is now a hard scale
  - for $q_T \ll m_Z$ find large Sudakov logs in q_T/m_Z $\rightsquigarrow$ must resum

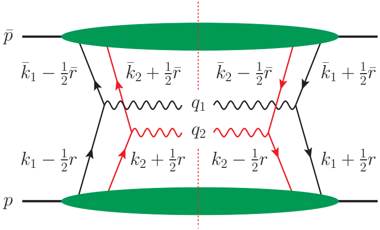
Cross sections for definite transverse momenta

- standard factorization formulae have $\int$ over total transv. momentum produced in hard scattering
- example: Z production



- for $q_T \ll m_Z$ have TMD factorization
  (only worked out for prod'n of color singlet particles: $Z, W, \gamma\gamma, H, \ldots$)
  - use TMD $f(x, \boldsymbol{k})$ and $q\bar{q} \to Z$ without parton emission
  - for $q_T \gg \Lambda$ compute
    $$f(x, \boldsymbol{k}) = \text{hard scattering} \otimes \text{collinear dist'n}$$
  - resummation of Sudakov logs with Collins-Soper evolution equation

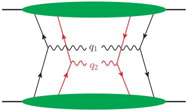
DPS: momentum and space-time structure



momentum conservation $\Rightarrow r + \bar{r} = 0$

- large (plus or minus) momenta of partons $x_i p$, $\bar{x}_i \bar{p}$ fixed by final state exactly as for single hard scattering
- transverse parton momenta not the same in amplitude $\mathcal{A}$ and in $\mathcal{A}^*$
  cross section $\propto \int d^2\boldsymbol{r}\, F(x_i, \boldsymbol{k}_i, \boldsymbol{r})\, F(\bar{x}_i, \bar{\boldsymbol{k}}_i, -\boldsymbol{r})$
- Fourier trf to impact parameter: $F(x_i, \boldsymbol{k}_i, \boldsymbol{r}) \to F(x_i, \boldsymbol{k}_i, \boldsymbol{y})$
  cross section $\propto \int d^2\boldsymbol{y}\, F(x_i, \boldsymbol{k}_i, \boldsymbol{y})\, F(\bar{x}_i, \bar{\boldsymbol{k}}_i, \boldsymbol{y})$
- interpretation: $\boldsymbol{y}$ = transv. dist. between two scattering partons
  = equal in both colliding protons

DPS cross section



- get cross section formula

$$\frac{d\sigma}{dx_1\, d\bar{x}_1\, d^2\boldsymbol{q}_1\; dx_2\, d\bar{x}_2\, d^2\boldsymbol{q}_2} = \frac{1}{C} \left[\prod_{i=1}^{2} \hat{\sigma}_i\big(q_i^2 = x_i \bar{x}_i s\big) \right]$$

$$\times \left[\prod_{i=1}^{2} \int d^2\boldsymbol{k}_i\, d^2\bar{\boldsymbol{k}}_i\, \delta^{(2)}(\boldsymbol{q}_i - \boldsymbol{k}_i - \bar{\boldsymbol{k}}_i) \right] \int d^2\boldsymbol{y}\, F(x_i, \boldsymbol{k}_i, \boldsymbol{y})\, F(\bar{x}_i, \bar{\boldsymbol{k}}_i, \boldsymbol{y})$$

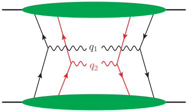
$\hat{\sigma}_i =$ parton-level cross section
$F(x_i, \boldsymbol{k}_i, \boldsymbol{y}) = k_T$ dependent double parton distribution

- result follows from Feynman graphs and hard-scattering approximation
  no semi-classical approximation required
- $\int d^2\boldsymbol{q}_1 \int d^2\boldsymbol{q}_2$ in cross sect. $\to k_T$ integrated (= collinear) distributions

$$F(x_i, \boldsymbol{y}) = \int d^2\boldsymbol{k}_1 \int d^2\boldsymbol{k}_2\, F(x_i, \boldsymbol{k}_i, \boldsymbol{y})$$

recover usual cross section formula

DPS cross section



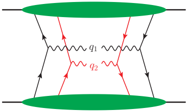
- get cross section formula

$$\frac{d\sigma}{dx_1\, d\bar{x}_1\; dx_2\, d\bar{x}_2} = \frac{1}{C} \left[\prod_{i=1}^{2} \hat{\sigma}_i\big(q_i^2 = x_i \bar{x}_i s\big) \right] \times \int d^2\boldsymbol{y}\, F(x_i, \boldsymbol{y})\, F(\bar{x}_i, \boldsymbol{y})$$

$\hat{\sigma}_i =$ parton-level cross section

$F(x_i, \boldsymbol{y}) = k_T$ integrated double parton distribution

Double parton distributions

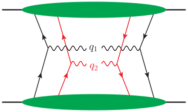


- define as operator matrix element (like for TMDs)

$$F(x_i, \boldsymbol{k}_i, \boldsymbol{y}) = \underset{z_i \to (x_i, \boldsymbol{k}_i)}{\mathcal{FT}} \langle p | \bar{q}\big(-\tfrac{1}{2} z_2\big) \Gamma_2\, q\big(\tfrac{1}{2} z_2\big)\; \bar{q}\big(y - \tfrac{1}{2} z_1\big) \Gamma_1\, q\big(y + \tfrac{1}{2} z_1\big) | p \rangle$$

- essential for studying factorization, scale evolution, etc.
- possibility for lattice calculations
- in $F(x_i, \boldsymbol{y})$: bilinear op's $\bar{q}\,\Gamma_i\, q$ at different transv. positions
  $\Rightarrow$ not a twist-four operator
  but product of two twist-two operators

Double parton distributions



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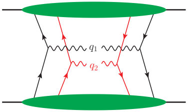
  - in $F(x_i, \boldsymbol{y})$: bilinear op's $\bar{q}\,\Gamma_i\, q$ at different transv. positions
    $\Rightarrow$ not a twist-four operator
    but product of two twist-two operators

- interpretation of $F(x_i, \boldsymbol{k}_i, \boldsymbol{y})$ as Wigner function:
  $\boldsymbol{k}_1, \boldsymbol{k}_2 =$ transv. parton momenta averaged over $\mathcal{A}$ and $\mathcal{A}^*$
  $\boldsymbol{y} =$ transv. distance between partons averaged over $\mathcal{A}$ and $\mathcal{A}^*$

  can introduce full 2 dim. Wigner function $F(x_i, \boldsymbol{k}_i, \boldsymbol{b}_i)$ with
  $\boldsymbol{b}_1, \boldsymbol{b}_2 =$ transv. parton positons averaged over $\mathcal{A}$ and $\mathcal{A}^*$

$$F(x_i, \boldsymbol{k}_i, \boldsymbol{y}) = \int d^2\boldsymbol{b}_1\; F(x_i, \boldsymbol{k}_i, \boldsymbol{b}_i)\Big|_{\boldsymbol{y}=\boldsymbol{b}_1-\boldsymbol{b}_2}$$

Double parton distributions



- define as operator matrix element (like for TMDs)

$$F(x_i, \boldsymbol{k}_i, \boldsymbol{y}) = \underset{z_i \to (x_i, \boldsymbol{k}_i)}{\mathcal{FT}} \langle p | \bar{q}\big(-\tfrac{1}{2} z_2\big) \Gamma_2\, q\big(\tfrac{1}{2} z_2\big)\, \bar{q}\big(y - \tfrac{1}{2} z_1\big) \Gamma_1\, q\big(y + \tfrac{1}{2} z_1\big) | p \rangle$$

  - essential for studying factorization, scale evolution, etc.
  - possibility for lattice calculations
  - in $F(x_i, \boldsymbol{y})$: bilinear op's $\bar{q}\,\Gamma_i\, q$ at different transv. positions
    $\Rightarrow$ not a twist-four operator
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- interpretation of $F(x_i, \boldsymbol{k}_i, \boldsymbol{y})$ as Wigner function:
  $\boldsymbol{k}_1, \boldsymbol{k}_2 =$ transv. parton momenta averaged over $\mathcal{A}$ and $\mathcal{A}^*$
  $\boldsymbol{y} =$ transv. distance between partons averaged over $\mathcal{A}$ and $\mathcal{A}^*$
  $\rightsquigarrow$ simple interpretation of cross section formula

$$\frac{d\sigma}{d\ldots} = \Bigg[\prod_{i=1}^{2} \hat{\sigma}_i \int d^2\boldsymbol{k}_i\, d^2\bar{\boldsymbol{k}}_i\, \delta(\boldsymbol{q}_i - \boldsymbol{k}_i - \bar{\boldsymbol{k}}_i) \Bigg] \int d^2\boldsymbol{y}\, F(x_i, \boldsymbol{k}_i, \boldsymbol{y})\, F(\bar{x}_i, \bar{\boldsymbol{k}}_i, \boldsymbol{y})$$

apart from "average over $\mathcal{A}$ and $\mathcal{A}^*$"

Aside: transverse momentum vs. position

- variables related by 2d Fourier transforms, e.g.
  - quark fields $\tilde{q}(\boldsymbol{k}, z^-, z^+) = \int d^2\boldsymbol{z}\, e^{-i\boldsymbol{z}\boldsymbol{k}}\, q(\boldsymbol{z}, z^-, z^+)$
  - proton states $|p^+, \boldsymbol{b}\rangle = \int d^2\boldsymbol{p}\, e^{-i\boldsymbol{b}\boldsymbol{p}}\, |p^+, \boldsymbol{p}\rangle$
- in bilinear operators

$$\bar{\tilde{q}}(\boldsymbol{k})\,\tilde{q}(\boldsymbol{k}') = \int d^2\boldsymbol{z}\, d^2\boldsymbol{z}'\, e^{i(\boldsymbol{z}\boldsymbol{k}-\boldsymbol{z}'\boldsymbol{k}')}\, \bar{q}(\boldsymbol{z})\, q(\boldsymbol{z}')$$

$$\boldsymbol{z}\boldsymbol{k} - \boldsymbol{z}'\boldsymbol{k}' = \tfrac{1}{2}(\boldsymbol{z}+\boldsymbol{z}')(\boldsymbol{k}-\boldsymbol{k}') + \tfrac{1}{2}(\boldsymbol{z}-\boldsymbol{z}')(\boldsymbol{k}+\boldsymbol{k}')$$

'average' transv. momentum $\leftrightarrow$ position difference
transv. momentum transfer $\leftrightarrow$ 'average' position

- 'average' transv. mom. and position not Fourier conjugate
- density interpretation:
  - it $\int d^2(\boldsymbol{k}+\boldsymbol{k}') \ldots$ then $\boldsymbol{z} = \boldsymbol{z}' =$ position
  - if $\int d^2(\boldsymbol{z}+\boldsymbol{z}') \ldots$ then $\boldsymbol{k} = \boldsymbol{k}' =$ momentum