Second International Summer School of the GDR PH-QCD

"Correlations between partons in nucleons"

Multidimensional pictures of the nucleon (1/3)

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Outline

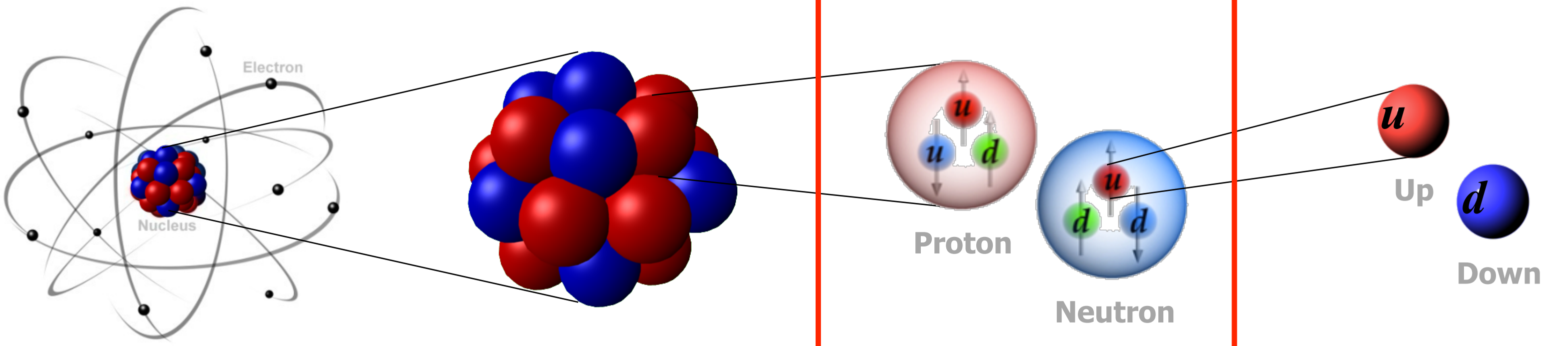
Lecture 1

- **Introduction**
- **Tour in phase space**
- **Galileo *vs* Lorentz**
- **Photon point of view**

Introduction

Structure of matter

Atom **Nucleus** **Nucleons** **Quarks**



10^{-10}m **10^{-14}m** **10^{-15}m** **10^{-18}m**

Atomic physics

Nuclear physics

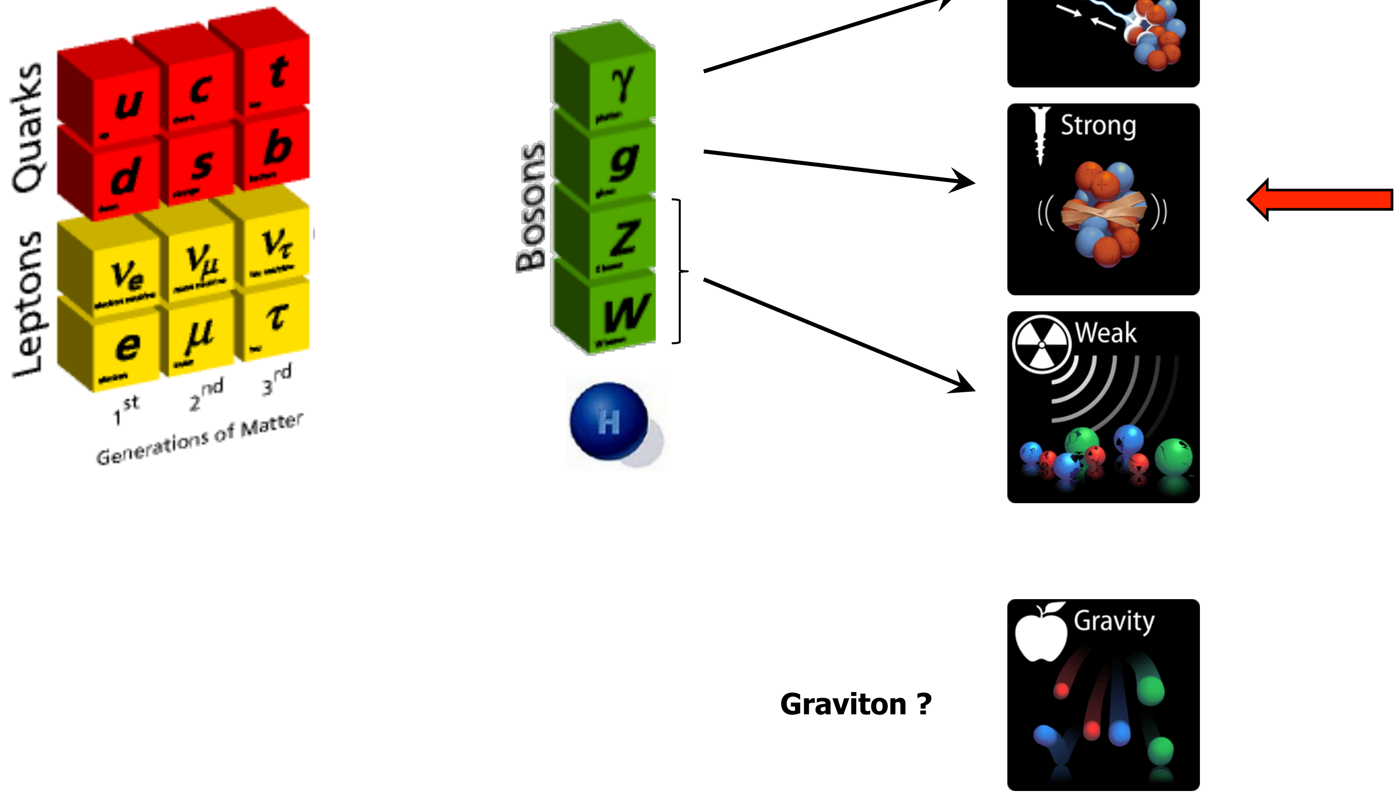
Hadronic physics

Particle physics

Elementary particles

Matter

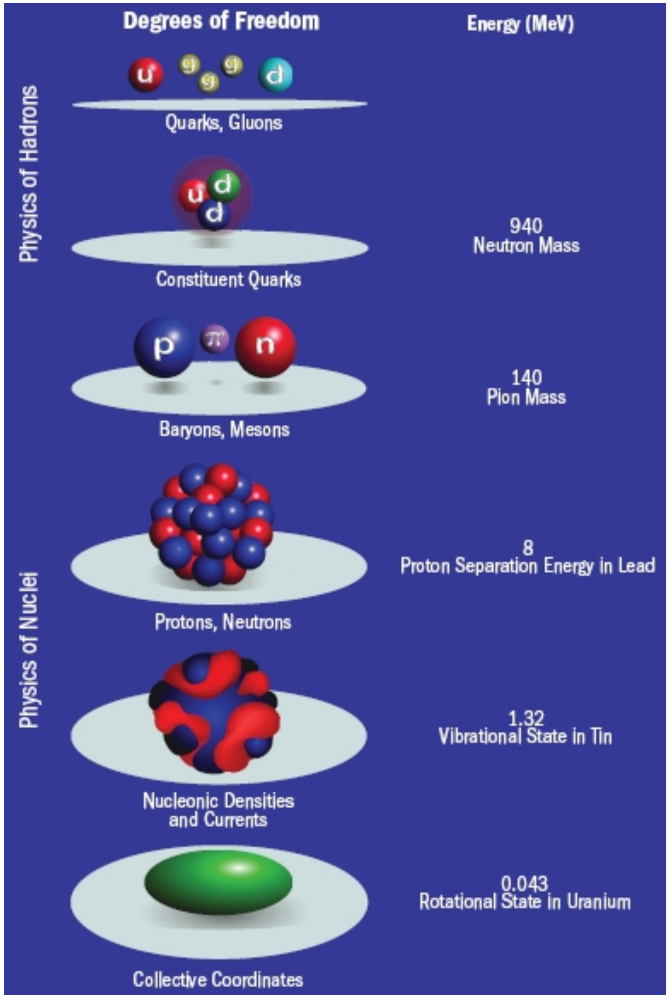
Interaction



Graviton ?

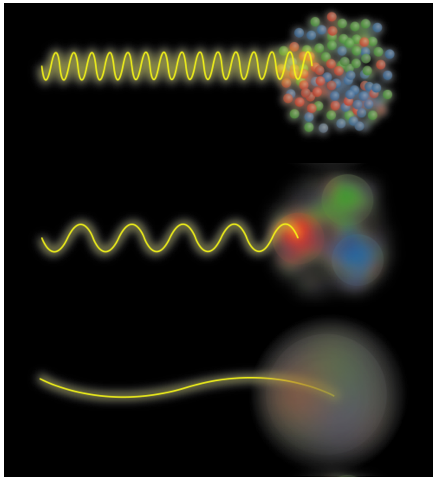
Introduction

Degrees of freedom



« Resolution »

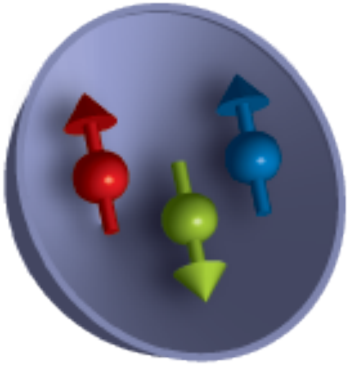
Relevant degrees of freedom depend on typical energy scale



Introduction

Nucleon pictures

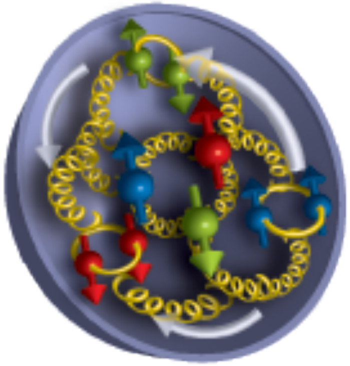
« Naive »



3 non-relativistic heavy quarks

$M_N \sim \Sigma\ m_{const}$

Realistic



Indefinite # of relativistic light quarks and gluons

$M_N = \Sigma\ m_{const}$ (~2%) + E_{int} (~98% !)

↑ **Brout-Englert-Higgs mechanism**

↑ **QCD**

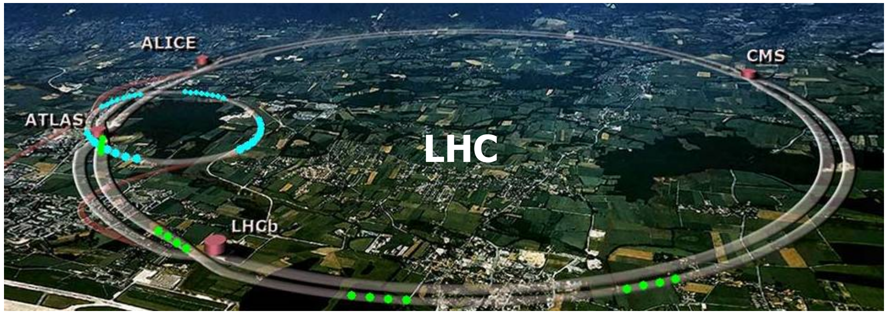
→ **Quantum Mechanics + Special Relativity = Quantum Field Theory**

Introduction

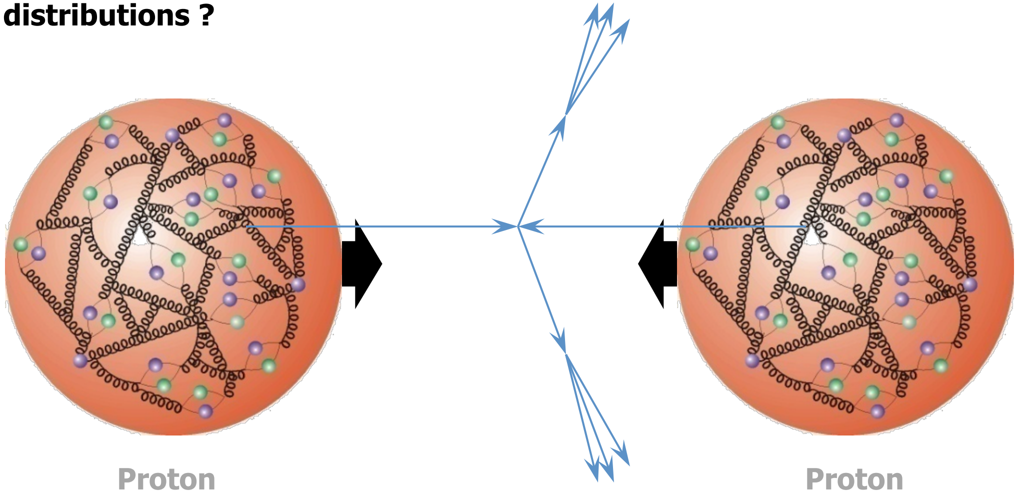
Goal : understanding the nucleon internal structure

Why ?

At the energy frontier



Quark & gluon distributions ?

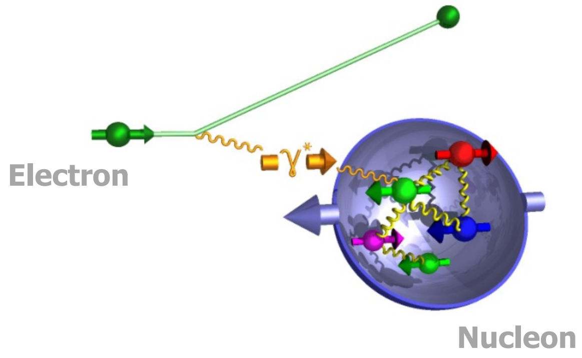


Introduction

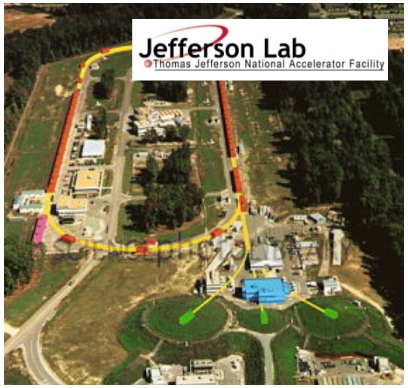
Goal : understanding the nucleon internal structure

Why ?

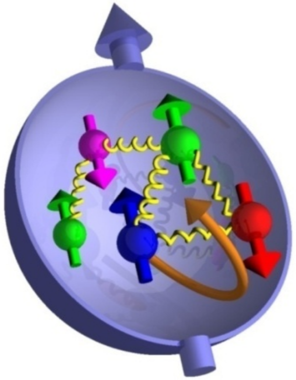
At the intensity frontier



E.g.



Nucleon spin structure ?



Nucleon shape & size ?

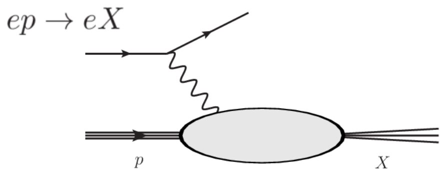
...

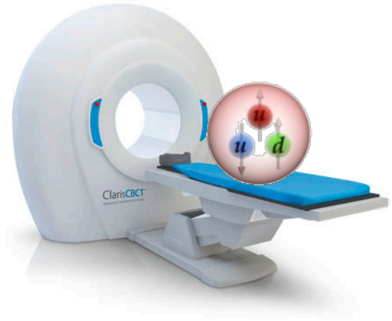
Introduction

Goal : understanding the nucleon internal structure

How ?

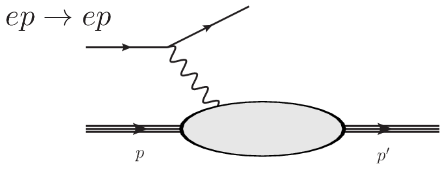
Deep Inelastic Scattering



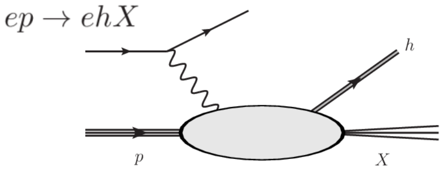


Proton
« tomography »

Elastic Scattering

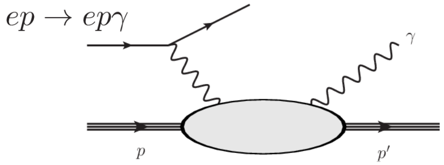


Semi-Inclusive DIS



Phase-space
distribution

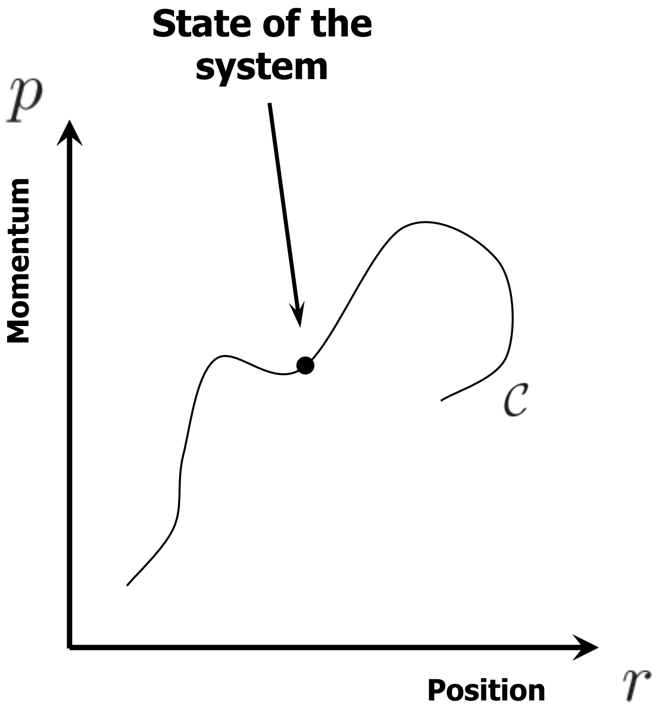
Deeply Virtual Compton Scattering



Phase space

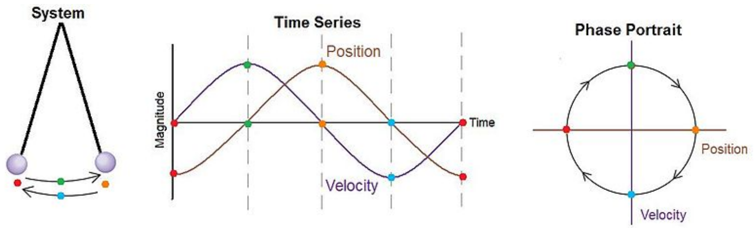
Classical Mechanics

$$p = \frac{\partial L}{\partial \dot{r}}$$



Particles follow well-defined trajectories

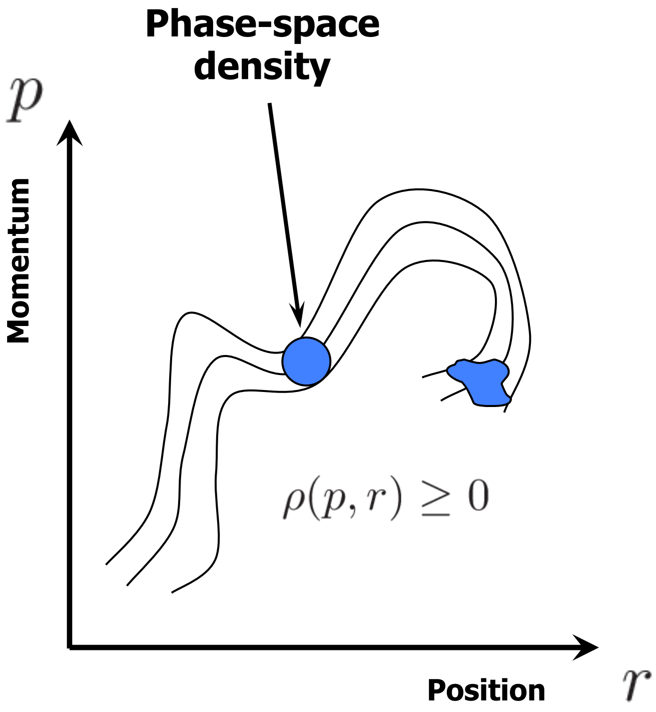
$$f[p(t), r(t), t] = 0$$



[Gibbs (1901)]

Phase space

Statistical Mechanics



Position-space density

$$\rho(r) = \int \mathrm{d}p \, \rho(p,r)$$

Momentum-space density

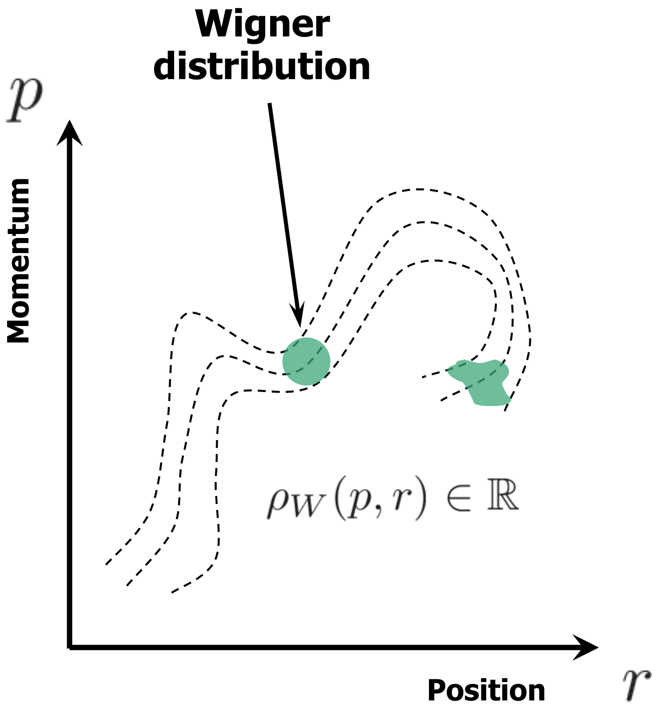
$$\rho(p) = \int \mathrm{d}r \, \rho(p,r)$$

Phase-space average

$$\overline{O} = \int \mathrm{d}p \, \mathrm{d}r \, O(p,r) \, \rho(p,r)$$

[Gibbs (1902)]

Quantum Mechanics



Position-space density

$$|\psi(r)|^2 = \int \mathrm{d}p\, \rho_W(p,r)$$

Momentum-space density

$$\tfrac{1}{2\pi}\,|\varphi(p)|^2 = \int \mathrm{d}r\, \rho_W(p,r)$$

Phase-space average

$$\begin{aligned}\langle\widehat{O}\rangle &= \int \mathrm{d}r\, \psi^*(r)\, \widehat{O}(-i\tfrac{\partial}{\partial r}, r)\, \psi(r) \\ &= \int \frac{\mathrm{d}p}{2\pi}\, \varphi^*(p)\, \widehat{O}(p, i\tfrac{\partial}{\partial p})\, \varphi(p) \\ &= \int \mathrm{d}p\, \mathrm{d}r\, O(p,r)\, \rho_W(p,r)\end{aligned}$$

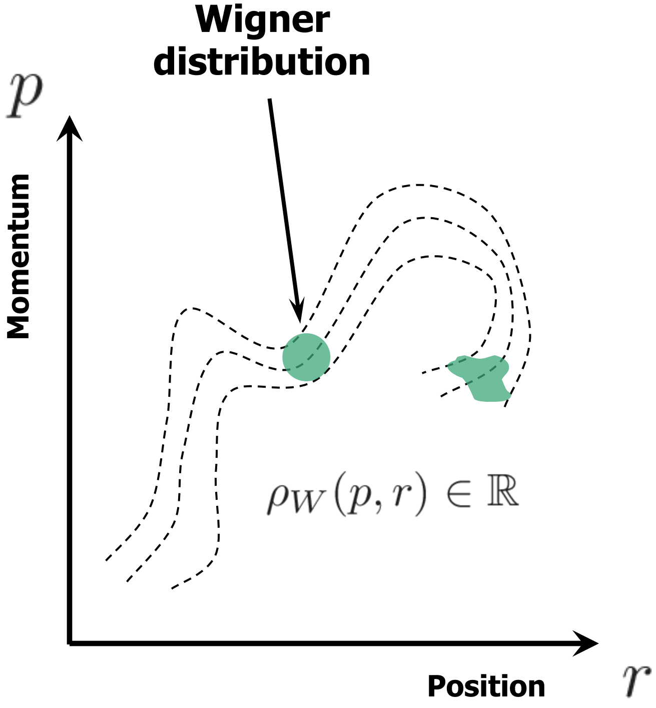
[Wigner (1932)]
[Moyal (1949)]

Phase space

Quantum Mechanics

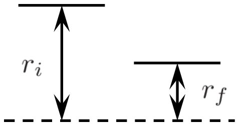
$$\psi(r) = \int \frac{\mathrm{d}p}{2\pi}\, e^{ipr}\, \varphi(p)$$

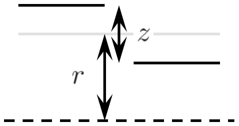
$$\boxed{\begin{aligned}\rho_W(p,r) &= \int \frac{\mathrm{d}z}{2\pi}\, e^{-ipz}\, \psi^*(r-\tfrac{z}{2})\, \psi(r+\tfrac{z}{2}) \\ &= \int \frac{\mathrm{d}\Delta}{(2\pi)^2}\, e^{-i\Delta r}\, \varphi^*(p+\tfrac{\Delta}{2})\, \varphi(p-\tfrac{\Delta}{2})\end{aligned}}$$



$$\begin{aligned}p\,\rho_W(p,r) &= \int \frac{\mathrm{d}z}{2\pi}\, p\, e^{-ipz}\, \psi^*(r-\tfrac{z}{2})\, \psi(r+\tfrac{z}{2}) \\ &= \int \frac{\mathrm{d}z}{2\pi}\, \big[(i\tfrac{\partial}{\partial z})e^{-ipz}\big]\, \psi^*(r-\tfrac{z}{2})\, \psi(r+\tfrac{z}{2}) \\ &= \int \frac{\mathrm{d}z}{2\pi}\, e^{-ipz}\big(-i\tfrac{\partial}{\partial z}\big)\big[\psi^*(r-\tfrac{z}{2})\, \psi(r+\tfrac{z}{2})\big] \\ &= \int \frac{\mathrm{d}z}{2\pi}\, e^{-ipz}\, \psi^*(r-\tfrac{z}{2})\underbrace{\big(-\tfrac{i}{2}\tfrac{\overleftrightarrow{\partial}}{\partial r}\big)}\psi(r+\tfrac{z}{2})\end{aligned}$$

Hermitian (symmetric) derivative $\equiv -\frac{i}{2}\left(\frac{\overrightarrow{\partial}}{\partial r} - \frac{\overleftarrow{\partial}}{\partial r}\right)$





Fourier conjugate variables

$p_f r_f - p_i r_i$

$\Delta r - pz$

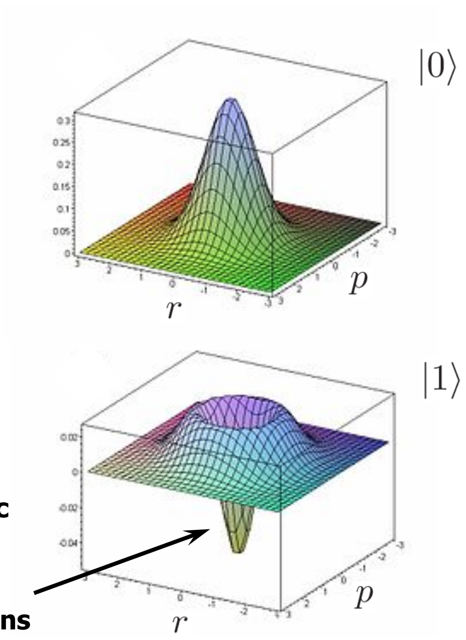
[Wigner (1932)]
[Moyal (1949)]

Phase space

Wigner distributions have applications in:

- **Nuclear physics**
- **Quantum chemistry**
- **Quantum molecular dynamics**
- **Quantum information**
- **Quantum optics**
- **Classical optics**
- **Signal analysis**
- **Image processing**
- **Quark-gluon plasma**
- **…**

Harmonic oscillator



Quasi-probabilistic

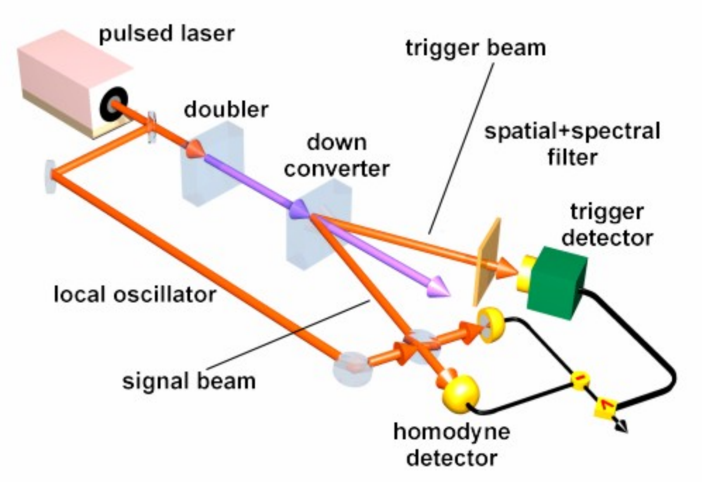
Heisenberg's uncertainty relations

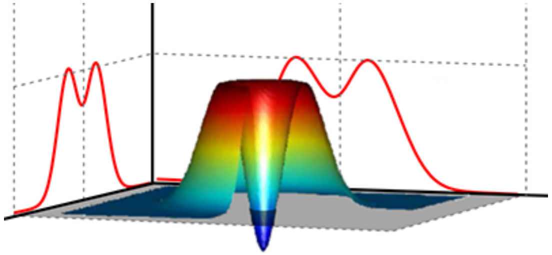
$$\Delta p \, \Delta r \geq \frac{\hbar}{2}$$

Phase space

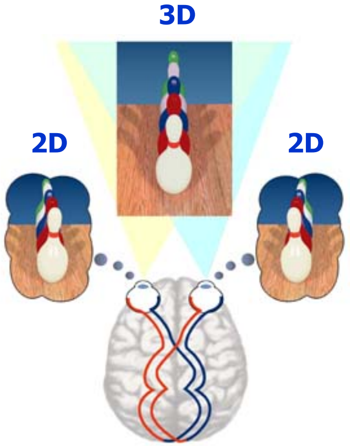
In quantum optics, Wigner distributions are « measured » using homodyne tomography

[Lvovski *et al.* (2001)]
[Bimbard *et al.* (2014)]



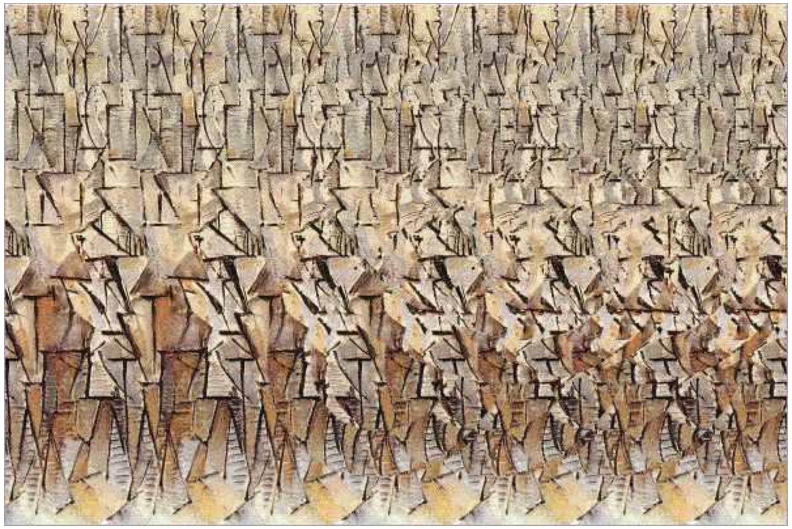


Idea : measuring projections of Wigner distributions from different directions



Binocular vision in phase space !

Find the hidden 3D picture



Phase space

Quantum Field Theory

Covariant Wigner operator

$$\widehat{W}(k,r) = \int \frac{\mathrm{d}^4 z}{(2\pi)^4}\, e^{ik\cdot z}\, \phi^*(r - \tfrac{z}{2})\, \phi(r + \tfrac{z}{2})$$

Time ordering ?

Scalar fields

Equal-time Wigner operator

$$\begin{aligned}\widehat{W}(\vec{k},\vec{r},t) &= \int \mathrm{d}k^0\, \widehat{W}(k,r) \\ &= \int \frac{\mathrm{d}^3 z}{(2\pi)^3}\, e^{-i\vec{k}\cdot\vec{z}}\, \phi^*(\vec{r} - \tfrac{\vec{z}}{2}, t)\, \phi(\vec{r} + \tfrac{\vec{z}}{2}, t)\end{aligned}$$

$$\boxed{\int \frac{\mathrm{d}k^0}{2\pi}\, e^{ik^0 z^0} = \delta(z^0)}$$

Phase-space/Wigner distribution

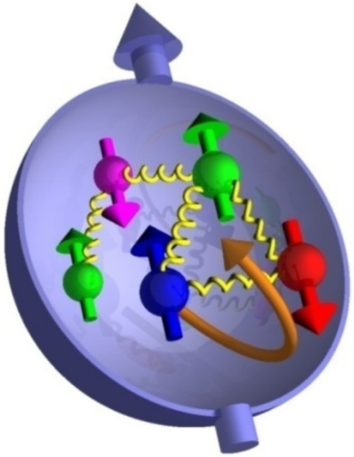
$$\rho_W(\vec{k},\vec{r},t;\Psi) = \langle\Psi|\widehat{W}(\vec{k},\vec{r},t)|\Psi\rangle$$

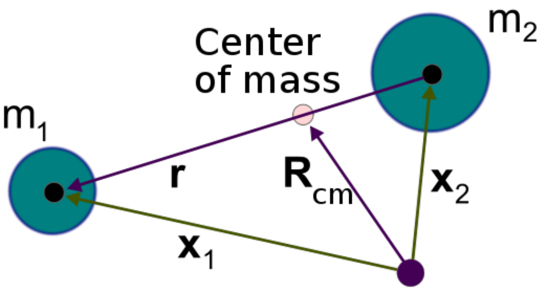
[Carruthers, Zachariasen (1976)]
[Ochs, Heinz (1997)]

Phase space

Interesting **Not so interesting**

Nucleons are composite systems ~ internal motion $\otimes$ center-of-mass motion





Phase space

Localized state in momentum space $|\Psi\rangle = |\vec{P}\rangle$

in position space $|\Psi\rangle = |\vec{R}\rangle$

$$= \int \frac{\mathrm{d}^3P}{(2\pi)^3}\, e^{-i\vec{P}\cdot\vec{R}}\, |\vec{P}\rangle$$

CoM momentum $\vec{P} = \sum_i \vec{k}_i$

CoM position $\vec{R} = \frac{\sum_i m_i \vec{r}_i}{\sum_i m_i}$

Phase-space compromise

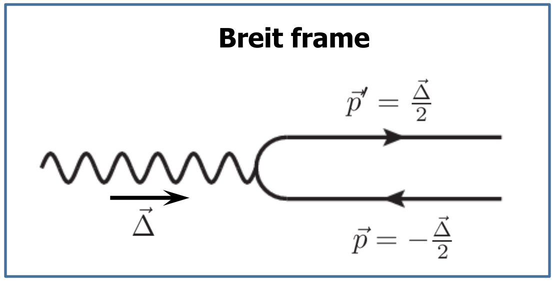
$$\rho_W(\vec{k},\vec{r},t;\vec{P},\vec{R}) = \int \frac{\mathrm{d}^3\Delta}{(2\pi)^3}\, e^{i\vec{\Delta}\cdot\vec{R}}\, \langle \vec{P}+\tfrac{\vec{\Delta}}{2}|\widehat{W}(\vec{k},\vec{r},t)|\vec{P}-\tfrac{\vec{\Delta}}{2}\rangle$$

$$= \int \mathrm{d}^3Z\, e^{i\vec{P}\cdot\vec{Z}}\, \langle \vec{R}-\tfrac{\vec{Z}}{2}|\widehat{W}(\vec{k},\vec{r},t)|\vec{R}+\tfrac{\vec{Z}}{2}\rangle$$

Intrinsic phase-space/Wigner distribution

$$\rho_W(\vec{k},\vec{r},t;\vec{0},\vec{0}) = \int \frac{\mathrm{d}^3\Delta}{(2\pi)^3}\, \langle \tfrac{\vec{\Delta}}{2}|\widehat{W}(\vec{k},\vec{r},t)|-\tfrac{\vec{\Delta}}{2}\rangle$$

Identified with intrinsic variables



Breit frame

[Ji (2003)]
[Belitsky, Ji, Yuan (2004)]

Phase space

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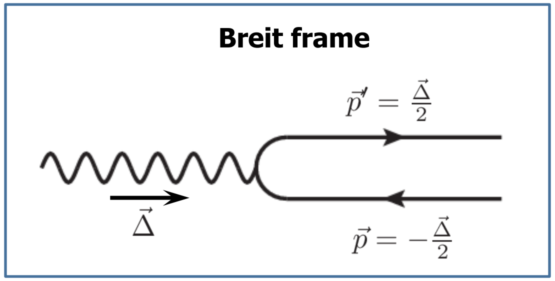
$$= \int \mathrm{d}^3 Z\, e^{i\vec{P}\cdot\vec{Z}}\, \langle \vec{R}-\tfrac{\vec{Z}}{2}|\widehat{W}(\vec{k},\vec{r},t)|\vec{R}+\tfrac{\vec{Z}}{2}\rangle$$

Intrinsic phase-space/Wigner distribution

$$\rho_W(\vec{k},\vec{r},\cancel{t};\vec{0},\vec{0}) = \int \frac{\mathrm{d}^3\Delta}{(2\pi)^3}\, \langle \tfrac{\vec{\Delta}}{2}|\widehat{W}(\vec{k},\vec{r},t)|-\tfrac{\vec{\Delta}}{2}\rangle$$

Identified with **intrinsic** **variables**

Same energy !



Breit frame

Time translation $\widehat{O}(t) = e^{i\widehat{H}t}\,\widehat{O}(0)\,e^{-i\widehat{H}t}$

[Ji (2003)]
[Belitsky, Ji, Yuan (2004)]

Galilean symmetry

All is fine as long as space-time symmetry is Galilean

$$[J^i, J^j] = i\epsilon^{ijk} J^k$$

$$[J^i, B^j] = i\epsilon^{ijk} B^k$$

$$[B^i, B^j] = 0$$

$$[B^i, H] = -iP^i \qquad E \mapsto E + \vec{P} \cdot \delta\vec{v}$$

$$[B^i, P^j] = -i\delta^{ij} M \qquad \vec{P} \mapsto \vec{P} + M\delta\vec{v}$$

$$[B^i, M] = [J^i, M] = 0$$

Position operator can be defined

$$\boxed{B^i = -MR^i} \quad \Longrightarrow$$

$$[R^i, P^j] = i\delta^{ij}\mathbb{1}$$

$$[R^i, R^j] = 0$$

$$[J^i, R^j] = i\epsilon^{ijk} R^k$$

Lorentz symmetry

But in relativity, space-time symmetry is Lorentzian

$$J^i = \tfrac{1}{2}\,\epsilon^{ijk} M^{jk}$$
$$K^i = M^{0i}$$

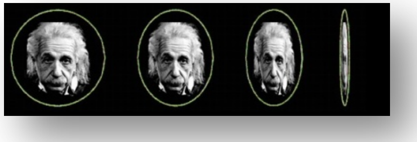
$$[J^i, J^j] = i\epsilon^{ijk} J^k$$
$$[J^i, K^j] = i\epsilon^{ijk} K^k$$
$$[K^i, K^j] = -i\epsilon^{ijk} J^k$$

$$[K^i, P^0] = -iP^i \qquad E \mapsto E + \vec{P}\cdot\delta\vec{v}$$
$$[K^i, P^j] = -i\delta^{ij} P^0 \qquad \vec{P} \mapsto \vec{P} + E\delta\vec{v}$$

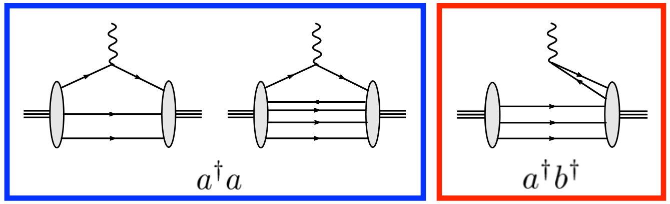
Position operator is ill-defined ! → **No separation of CoM and internal coordinates**

Further issues :

Lorentz contraction



Creation/annihilation of pairs



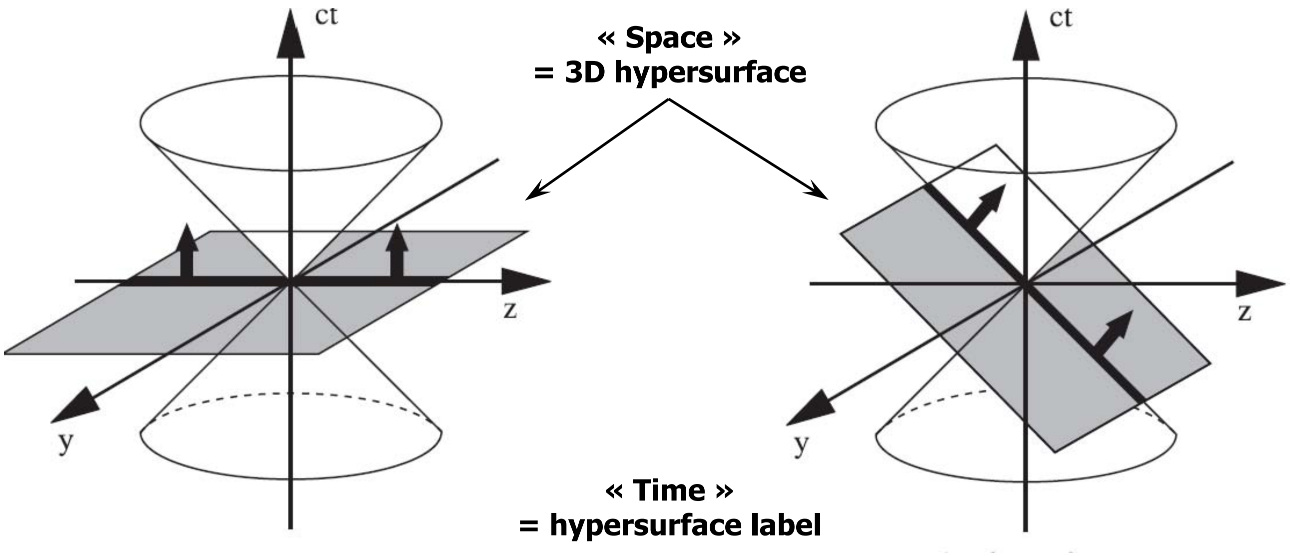
Spoils (quasi-) probabilistic interpretation

Forms of dynamics

Space-time foliation

Light-front components

$$\boxed{a^{\pm} = \frac{1}{\sqrt{2}}(a^0 \pm a^3)}$$



« Space »
= 3D hypersurface

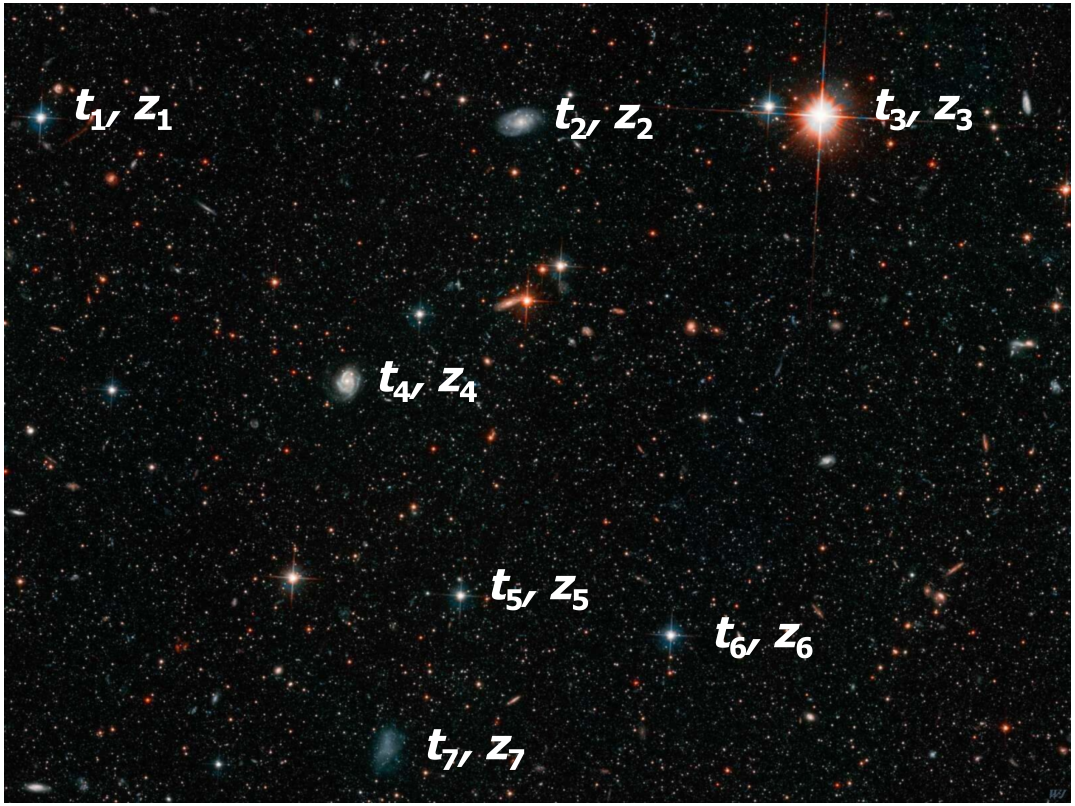
« Time »
= hypersurface label

	Instant-form dynamics	**Light-front form dynamics**
Time	x^0	x^+
Space	$\vec{x}$	$\vec{x}_\perp, x^-$
Energy	p^0	p^-
Momentum	$\vec{p}$	$\vec{p}_\perp, p^+$

[Dirac (1949)]

Instant form *vs* light-front form

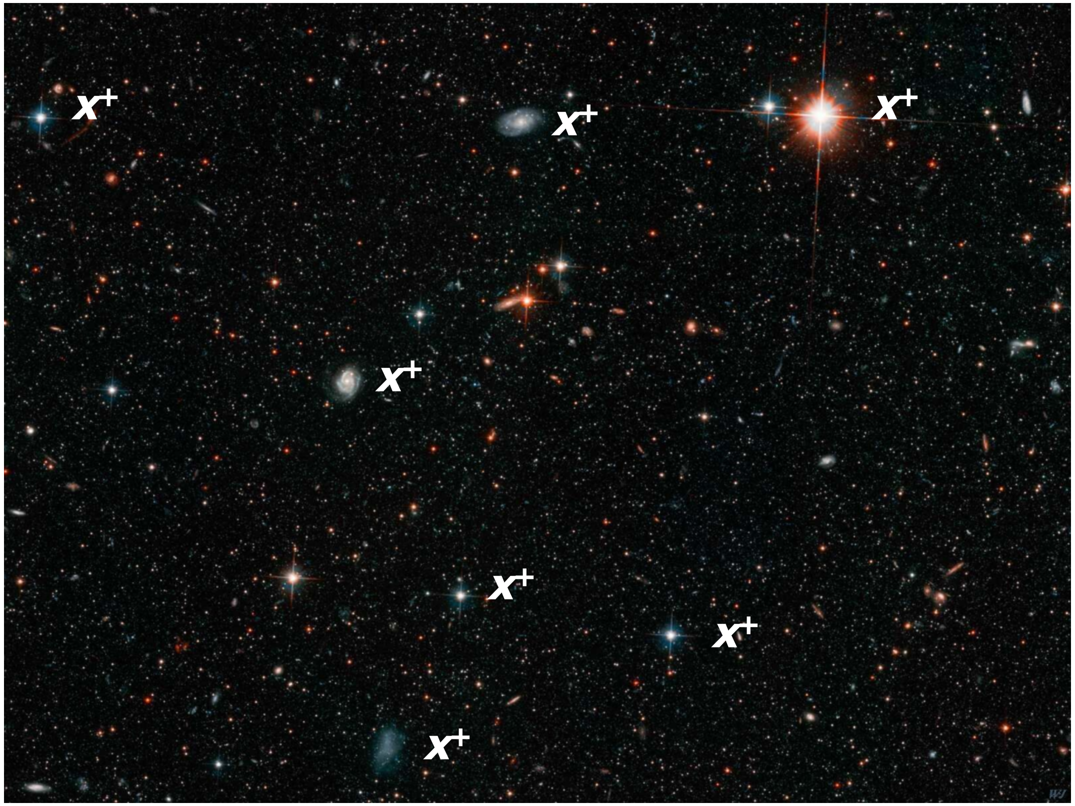
Ordinary point of view



Instant form *vs* light-front form

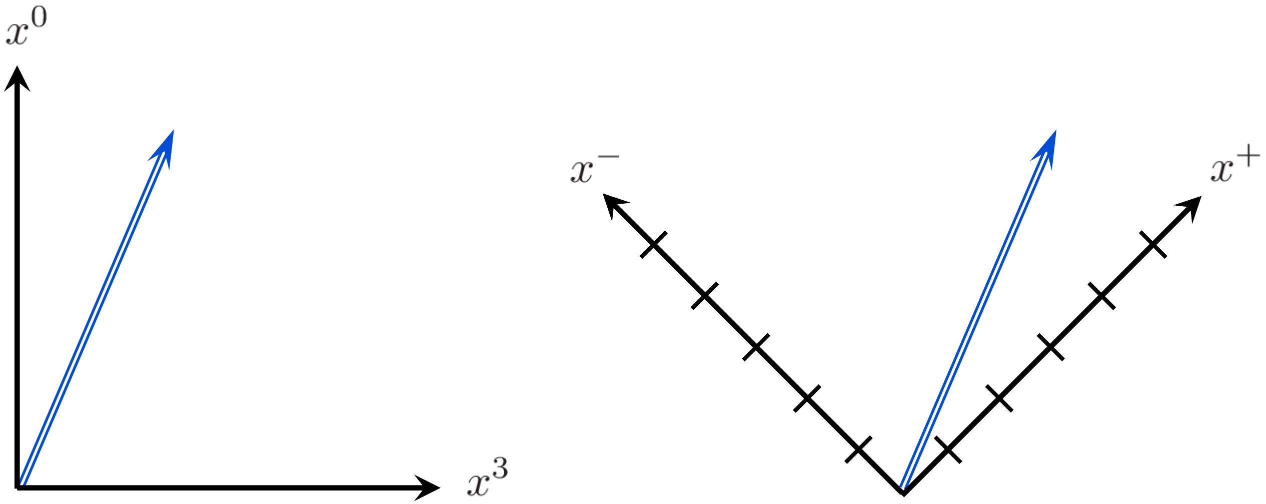
Photon point of view

$$\boxed{x^+ = \frac{1}{\sqrt{2}}(t + z)}$$



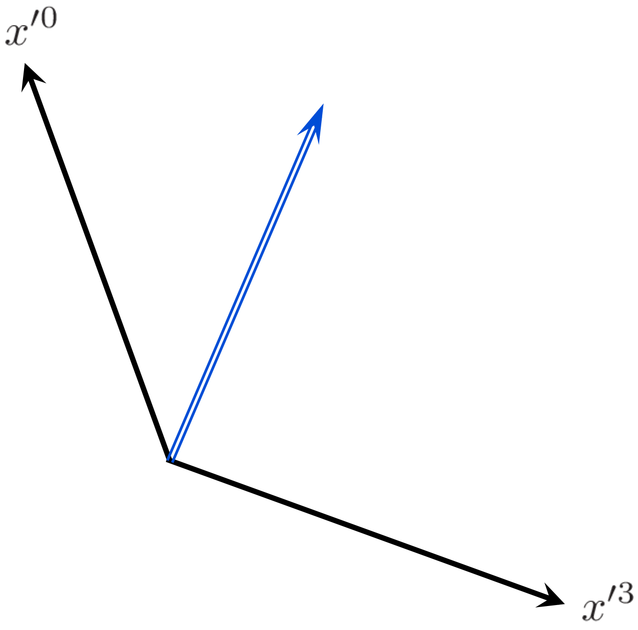
Instant form *vs* light-front form

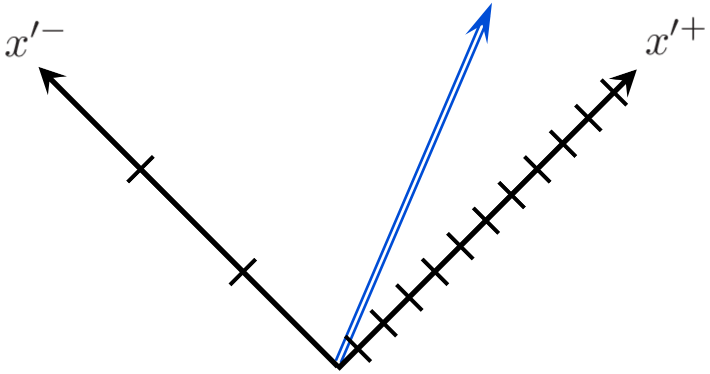
Initial frame



Instant form *vs* light-front form

Boosted frame





$$x'^0 = \frac{x^0 + vx^3}{\sqrt{1-v^2}}$$

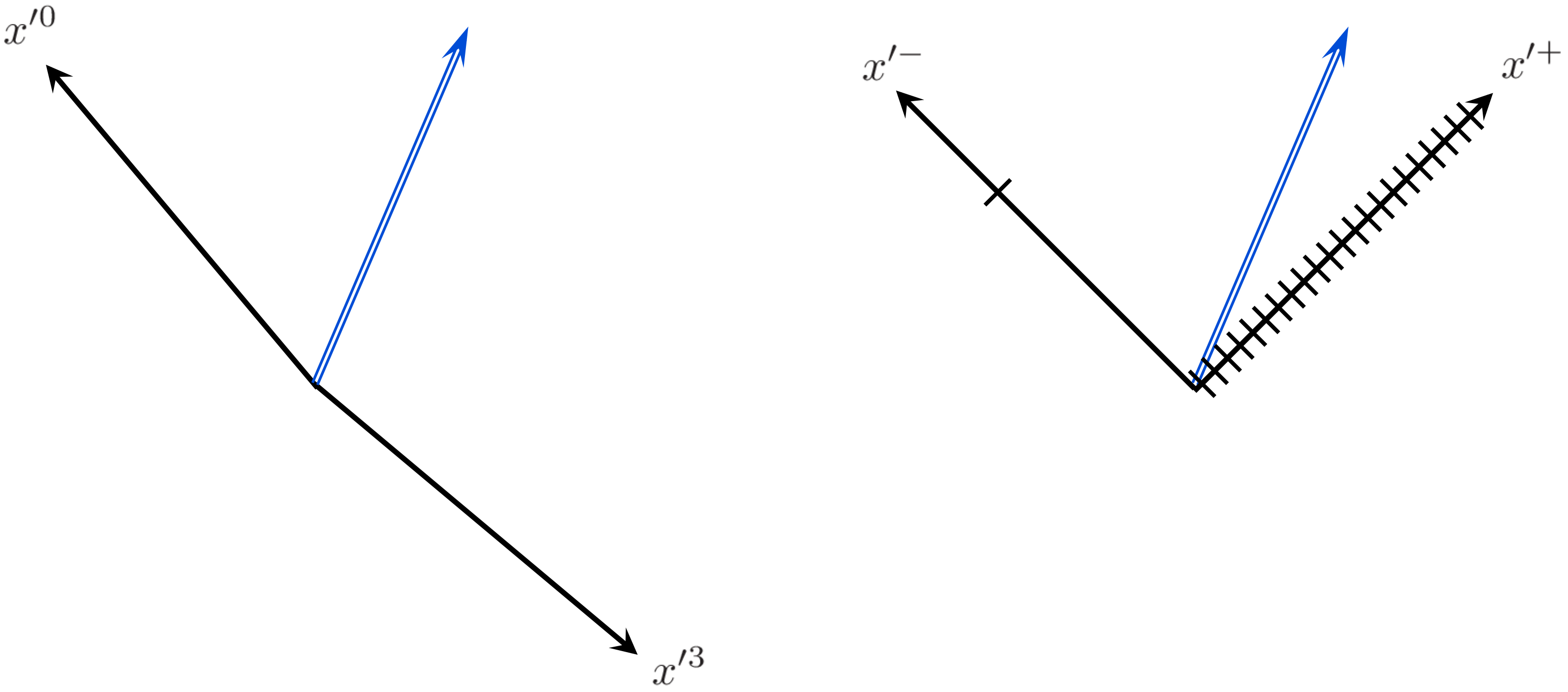
$$x'^3 = \frac{x^3 + vx^0}{\sqrt{1-v^2}}$$

$$x'^+ = \sqrt{\frac{1+v}{1-v}}\,x^+$$

$$x'^- = \sqrt{\frac{1-v}{1+v}}\,x^-$$

Instant form *vs* light-front form

(Quasi) infinite-momentum frame



$$x'^0 \approx x'^3 \approx \frac{x'^+}{\sqrt{2}} \approx \frac{x^+}{\sqrt{1-v}} \qquad x'^- \approx 0$$

Light-front operators

$$\begin{aligned} K^1_\perp &= M^{+1} & J^1_\perp &= M^{-1} \\ &= \tfrac{1}{\sqrt{2}}(K^1 + J^2) & &= \tfrac{1}{\sqrt{2}}(K^1 - J^2) \\ K^2_\perp &= M^{+2} & J^2_\perp &= M^{-2} \\ &= \tfrac{1}{\sqrt{2}}(K^2 - J^1) & &= \tfrac{1}{\sqrt{2}}(K^2 + J^1) \end{aligned}$$

Transverse space-time symmetry is Galilean

$$\begin{aligned} [J^3, J^i_\perp] &= i\epsilon^{3ij} J^j_\perp & [K^i_\perp, P^-] &= -iP^i_\perp \\ [J^3, K^i_\perp] &= i\epsilon^{3ij} K^j_\perp & [K^i_\perp, P^j_\perp] &= -i\delta^{ij}_\perp P^+ \\ [K^i_\perp, K^j_\perp] &= 0 & [K^i_\perp, P^+] &= [J^3, P^+] = 0 \end{aligned}$$

Transverse position operator can be defined !

$$\boxed{K^i_\perp = -P^+ R^i_\perp} \quad \Longrightarrow \quad \begin{aligned} [R^i_\perp, P^j_\perp] &= i\delta^{ij}_\perp \mathbb{1} \\ [R^i_\perp, R^j_\perp] &= 0 \\ [J^3, R^i_\perp] &= i\epsilon^{3ij} R^j_\perp \end{aligned}$$

Longitudinal momentum plays the role of mass in the transverse plane $P^+ \sim M$

Transverse boost $p'^+ = p^+$ $\vec{p}'_\perp = \vec{p}_\perp + p^+ \vec{v}_\perp$

[Kogut, Soper (1970)]

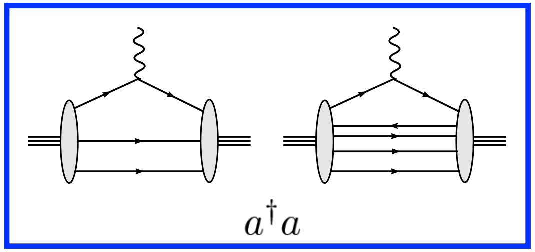
Quasi-probabilistic interpretation

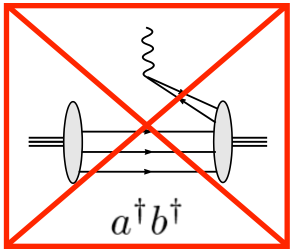
What about the further issues with Special Relativity ?

✓ **Transverse boosts are Galilean** → **No transverse Lorentz contraction !**

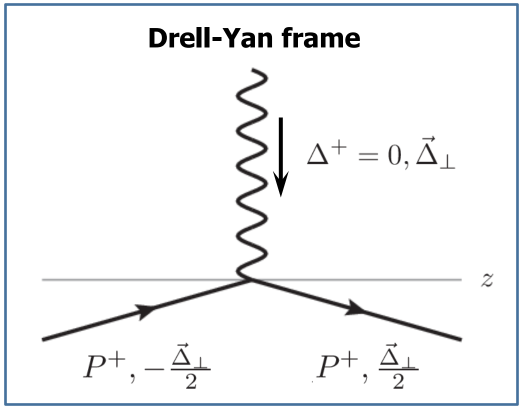
$\int \mathrm{d}r^-$ → **No sensitivity to longitudinal Lorentz contraction !**

✓ **Particle number is conserved in Drell-Yan frame**





k^+ **is conserved and positive**


Drell-Yan frame

→ **Conclusion : quasi-probabilistic interpretation is possible !** ←

Non-relativistic phase space

Localized state in momentum space $|\Psi\rangle = |\vec{P}\rangle$

in position space $|\Psi\rangle = |\vec{R}\rangle$

$$= \int \frac{\mathrm{d}^3 P}{(2\pi)^3}\, e^{-i\vec{P}\cdot\vec{R}}\, |\vec{P}\rangle$$

CoM momentum $\vec{P} = \sum_i \vec{k}_i$

CoM position $\vec{R} = \frac{\sum_i m_i \vec{r}_i}{\sum_i m_i}$

Equal-time Wigner operator

$$\widehat{W}(\vec{k},\vec{r},t) = \int \mathrm{d}k^0\, \widehat{W}(k,r)$$

Intrinsic phase-space/Wigner distribution

$$\rho_W(\vec{k},\vec{r},\cancel{t},\vec{0},\vec{0}) = \int \frac{\mathrm{d}^3\Delta}{(2\pi)^3}\, \langle \tfrac{\vec{\Delta}}{2} | \widehat{W}(\vec{k},\vec{r},t) | -\tfrac{\vec{\Delta}}{2} \rangle$$

Identified with
intrinsic variables

Time translation $\widehat{O}(t) = e^{i\widehat{H}t}\, \widehat{O}(0)\, e^{-i\widehat{H}t}$

[Ji (2003)]
[Belitsky, Ji, Yuan (2004)]

Relativistic phase space

Localized state in momentum space $|\Psi\rangle = |P^+, \vec{P}_\perp\rangle$

« CoM » momentum $\vec{P}_\perp = \sum_i \vec{k}_{i\perp}$

in position space

$$|\Psi\rangle = |P^+, \vec{R}_\perp\rangle = \int \frac{\mathrm{d}^2 P_\perp}{(2\pi)^2}\, e^{-i\vec{P}_\perp\cdot\vec{R}_\perp} |P^+, \vec{P}_\perp\rangle$$

« CoM » position $\vec{R}_\perp = \frac{\sum_i k_i^+ \vec{r}_{i\perp}}{\sum_i k_i^+}$

[Soper (1977)]
[Burkardt (2000)]
[Burkardt (2003)]

Equal light-front time Wigner operator

$$\widehat{W}(k^+, \vec{k}_\perp, r^-, \vec{r}_\perp, r^+) = \int \mathrm{d}k^-\, \widehat{W}(k,r)$$

[Ji (2003)]
[Belitsky, Ji, Yuan (2004)]

Intrinsic relativistic phase-space/Wigner distribution

$$\rho_W(x, \vec{k}_\perp, \vec{b}_\perp) = \frac{P^+}{\langle P^+|P^+\rangle} \int \mathrm{d}r^- \frac{\mathrm{d}^2\Delta_\perp}{(2\pi)^2} \langle P^+, \tfrac{\vec{\Delta}_\perp}{2}|\widehat{W}(xP^+, \vec{k}_\perp, r^-, \vec{b}_\perp, r^+)|P^+, -\tfrac{\vec{\Delta}_\perp}{2}\rangle$$

$$= \frac{1}{2}\int \frac{\mathrm{d}^2\Delta_\perp}{(2\pi)^2} \langle P^+, \tfrac{\vec{\Delta}_\perp}{2}|\widehat{W}(xP^+, \vec{k}_\perp, 0, \vec{b}_\perp, 0)|P^+, -\tfrac{\vec{\Delta}_\perp}{2}\rangle$$

$x = \frac{k^+}{P^+}$

Boost invariant !

$$\int \mathrm{d}k^+\, f(k^+) = \int \mathrm{d}x\, P^+ f(xP^+)$$

Space-time translation $\widehat{O}(r) = e^{i\widehat{P}\cdot r}\,\widehat{O}(0)\,e^{-i\widehat{P}\cdot r}$

Normalization $\langle P^+|P^+\rangle = 2P^+\, 2\pi\, \delta(0)$

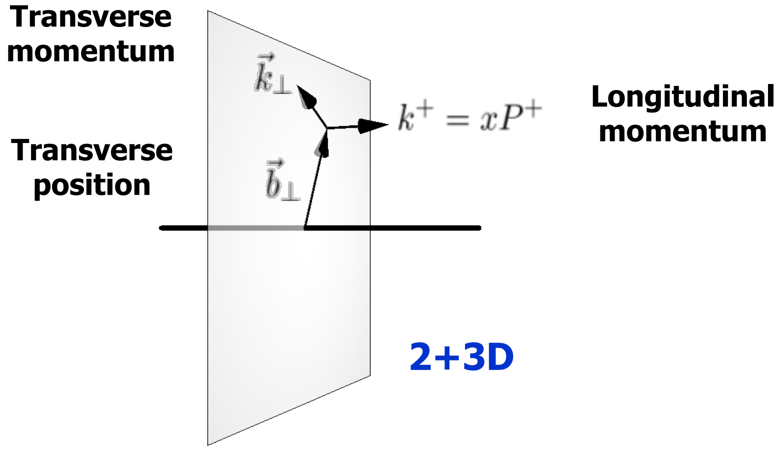
[C.L., Pasquini (2011)]

Relativistic phase space

Our intuition is instant form and not light-front form

NB : $\rho_W(x,\vec{k}_\perp,\vec{b}_\perp)$ **is invariant under light-front boosts**

→ **It can be thought as instant form phase-space/Wigner distribution in IMF !**



In IMF, the nucleon looks like a pancake

[C.L., Pasquini (2011)]

Link with parton correlators

Phase-space/Wigner distributions are Fourier transforms

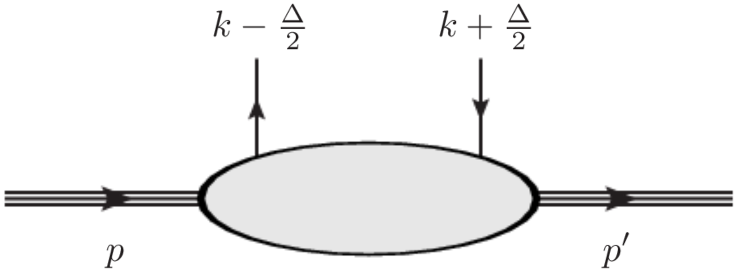
$$\rho_W(x,\vec{k}_\perp,\vec{b}_\perp) = \frac{1}{2}\int\frac{\mathrm{d}^2\Delta_\perp}{(2\pi)^2}\,\langle P^+,\tfrac{\vec{\Delta}_\perp}{2}|\widehat{W}(xP^+,\vec{k}_\perp,0,\vec{b}_\perp,0)|P^+,-\tfrac{\vec{\Delta}_\perp}{2}\rangle$$
$$= \int\frac{\mathrm{d}^2\Delta_\perp}{(2\pi)^2}\,e^{-i\vec{\Delta}_\perp\cdot\vec{b}_\perp}\,W(x,0,\vec{k}_\perp,\vec{\Delta}_\perp)$$

Space-time translation

$$\widehat{O}(r) = e^{i\widehat{P}\cdot r}\,\widehat{O}(0)\,e^{-i\widehat{P}\cdot r}$$

General parton correlator

$$W(x,\xi,\vec{k}_\perp,\vec{\Delta}_\perp) = \frac{1}{2}\int\frac{\mathrm{d}z^-\,\mathrm{d}^2z_\perp}{(2\pi)^3}\,e^{ik\cdot z}\,\langle p'|\phi^*(-\tfrac{z}{2})\,\phi(\tfrac{z}{2})|p\rangle\Big|_{z^+=0}$$



$P = \frac{1}{2}(p'+p)$ $\quad k^+ = xP^+$

$\Delta = p'-p$ $\quad \Delta^+ = -2\xi P^+$

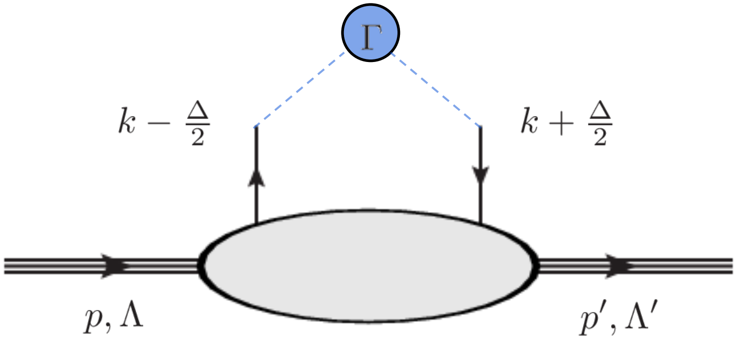
Link with parton correlators

Adding spin to the picture

$$W^{[\Gamma]}_{\Lambda'\Lambda}(x,\xi,\vec{k}_\perp,\vec{\Delta}_\perp) = \frac{1}{2}\int \frac{\mathrm{d}z^-\,\mathrm{d}^2z_\perp}{(2\pi)^3}\,e^{ik\cdot z}\,\langle p',\Lambda'|\overline{\psi}(-\tfrac{z}{2})\,\Gamma\,\psi(\tfrac{z}{2})|p,\Lambda\rangle\Big|_{z^+=0}$$

Dirac matrix

~ quark polarization



Leading twist

$$\gamma^+ \sim \delta_{\lambda'\lambda}$$
$$\gamma^+\gamma_5 \sim (\tau_3)_{\lambda'\lambda}$$
$$i\sigma^{j+}\gamma_5 \sim (\tau_j)_{\lambda'\lambda}$$

Link with parton correlators

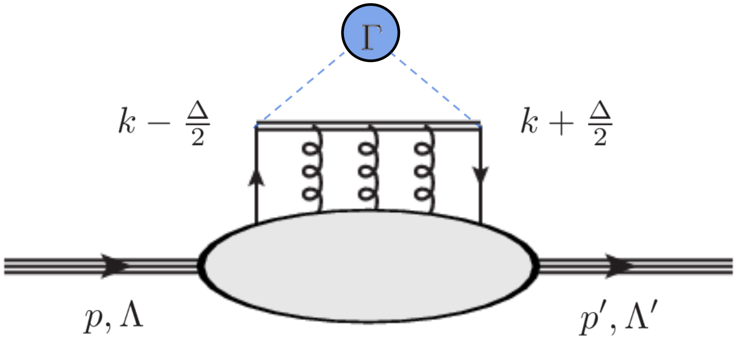
Adding spin and color to the picture

$$W^{[\Gamma]}_{\Lambda'\Lambda}(x,\xi,\vec{k}_\perp,\vec{\Delta}_\perp;\mathcal{W}) = \frac{1}{2}\int\frac{\mathrm{d}z^-\,\mathrm{d}^2z_\perp}{(2\pi)^3}\,e^{ik\cdot z}\,\langle p',\Lambda'|\overline{\psi}(-\tfrac{z}{2})\,\Gamma\,\mathcal{W}_{-\frac{z}{2}\,\frac{z}{2}}\,\psi(\tfrac{z}{2})|p,\Lambda\rangle\Big|_{z^+=0}$$

↑

Wilson line

$$\mathcal{W}_{ba} = \mathcal{P}\Big[e^{ig\int_a^b \mathrm{d}x^-\,A^+}\Big]$$



Gauge transformation

$$\psi(x) \mapsto U(x)\psi(x)$$
$$\mathcal{W}_{yx} \mapsto U(y)\mathcal{W}_{yx}U^{-1}(x)$$

Very very very complicated object not directly measurable

⇒ **Let's look for simpler measurable correlators**

Summary

Lecture 1

- **Understanding nucleon internal structure is essential**
- **Concept of phase space can be generalized to QM and QFT**
- **Relativistic effects force us to abandon 1D in phase space**