Multiparton interactions
Part 4

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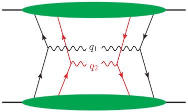
Correlations between Partons in Nucleons
Summer School, Orsay, June 30 to July 4, 2014

HELMHOLTZ
GEMEINSCHAFT

DESY

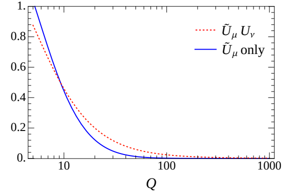
Color structure



- quark lines in amplitude and its conjugate can couple to color singlet or octet:

$$^1F \to (\bar{q}_2 \mathbb{1} q_2)\,(\bar{q}_1 \mathbb{1} q_1) \qquad\qquad ^8F \to (\bar{q}_2 t^a q_2)\,(\bar{q}_1 t^a q_1)$$

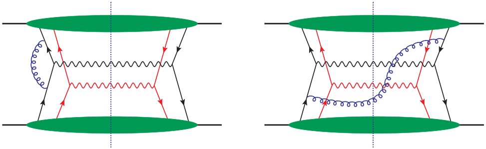
- ^8F describes color correlation between quarks 1 and 2 is essentially unknown (no probability interpretation as a guide)
- for two-gluon dist's more color structures: 1, 8_S, 8_A, 10, $\overline{10}$, 27
- for k_T integrated distributions:
  color correlations suppressed by Sudakov logarithms
  but not necessarily negligible for moderately hard scales



← Manohar, Waalewijn, arXiv:1202.3794

Sudakov factors

- origin: incomplete cancellation betwen real and virtual graphs with soft/collinear gluons



- large single and double logarithms $(\alpha_s \log^2$ and $\alpha_s \log)$
resummation to all orders $\rightsquigarrow \exp(-\dots)$
- calculation requires appropriate regulator ("rapidity regulator")
tools: Wilson line operators, Collins-Soper evolution equations, effective field theory (SCET)
  - collinear factorization with color correlations: $\log(Q/\Lambda)$
  - TMD factorization (single or double hard scattering):
$\log(Q/q_T)$ in all color channels
double logs same in DPS and single scattering, single logs differ

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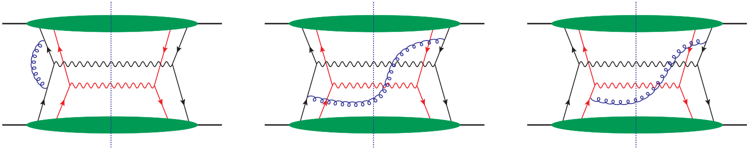

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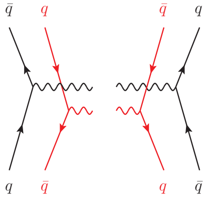
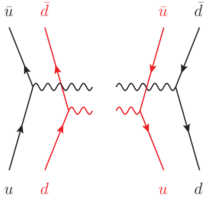
Interference effects

- so far: distributions with operators $\bar{q}_2\, q_2\ \bar{q}_1\, q_1$
  indices 1 and 2 refer to momentum fractions x_1, x_2
- but also have interference contributions (no probability interpretation)



operator: $\bar{q}_2\, \bar{q}_2\ q_1\, q_1$



operator: $\bar{u}_2\, d_2\ \bar{d}_1\, u_1$

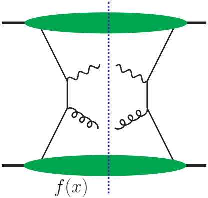
- can be included in cross section formula but involves further unknown DPDs
- expect to decrease for small x_1, x_2, since does not mix with gluons under evolution
- fermion number interference $\to$ color correlations $\to$ Sudakov suppression
  Manohar, Waalewijn 2012

Summary so far

- various parton correlations relevant in double parton scattering
- can include in cross sect. formula $\rightarrow$ new DPDs
  but are excluded in pocket formula estimate
- expect large correlations of several types at $x \gtrsim 0.1$
- large scale and/or small x suppress correlations
  Sudakov factors for color, evolution effects for spin

High q_T: general remarks

example: single Drell-Yan process with $\Lambda \ll q_T \ll Q$



- collinear factorization: compute emission of recoiling parton(s)
  - find large Sudakov logs in q_T/Q $\rightsquigarrow$ need to resum

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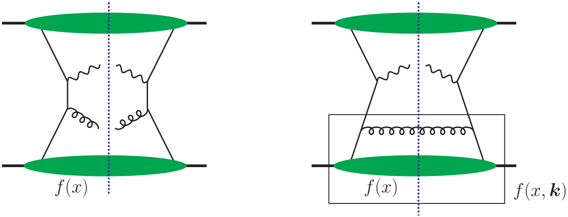

- collinear factorization: compute emission of recoiling parton(s)
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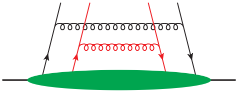
- k_T factorization: compute
  $f(x, \boldsymbol{k})$ = hard scattering $\otimes$ collinear dist'n
  - hard scattering closely related to DGLAP splitting functions
    k_T integrated distribution essentially is
    $$f(x; \mu^2) = \pi \int_0^{\mu^2} d\boldsymbol{k}^2 \, f(x, \boldsymbol{k})$$
  - Collins-Soper evolution equation $\rightsquigarrow$ resummation of Sudakov logs

High q_T in double hard scattering

- consider region $\Lambda \ll q_T \ll Q$, with $q_T \sim |\boldsymbol{q}_i|$ have $|\boldsymbol{k}_i| \sim q_T$
- adapt formalism for single hard scattering:
  k_T dependent dist'n = hard scattering $\otimes$ collinear dist'n
- ladder graphs: independent hard scatters for pair 1 and 2



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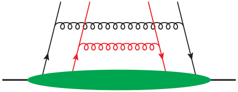

  - $|\boldsymbol{y}|$ of hadronic size
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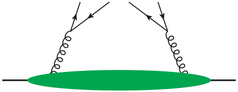
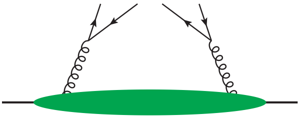
- splitting graphs



  - perturbatively small $|\boldsymbol{y}|$
  - longitudinal q and $\bar{q}$ spins fully correlated also large transv. spin correlation
- ladder graphs power suppressed by Λ^2/q_T^2 compared with splitting but have small-x enhancement

Behavior at small interparton distance

- splitting graphs also dominate collinear DPDs for $|\boldsymbol{y}| \ll 1/\Lambda$



- can compute short-distance behavior

$$F(x_1, x_2, \boldsymbol{y}) \sim \frac{1}{\boldsymbol{y}^2} \text{ splitting fct} \otimes \text{usual PDF}$$

nonzero results for all spin correlations not forbidden by symmetries

Scale evolution: a second look

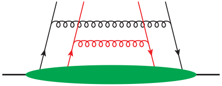
for collinear distributions without color correlation

- if define DPD from renormalized twist-two operators $\mathcal{O}$ in analogy with usual PDFs

$$F(x_1, x_2, \boldsymbol{y}; \mu) \sim \langle p | \mathcal{O}_1(\boldsymbol{0}; \mu)\, \mathcal{O}_2(\boldsymbol{y}; \mu) | p \rangle \qquad f(x; \mu) \sim \langle p | \mathcal{O}(\boldsymbol{0}; \mu) | p \rangle$$

$\Rightarrow$ for $F(x_i, \boldsymbol{y})$ at $\boldsymbol{y} \neq \boldsymbol{0}$ have

separate DGLAP evolution for partons 1 and 2



$$\frac{d}{d \log \mu} F(x_i, \boldsymbol{y}) = P \otimes_{x_1} F + P \otimes_{x_2} F$$

μ dep'ce of DPD $\leftrightarrow$ μ dep'ce of hard-scattering procs. at higher order

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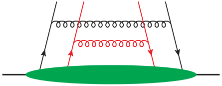

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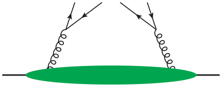
- $\int d^2\boldsymbol{y}\, F(x_i, \boldsymbol{y})$:

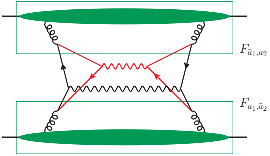
extra term from $2 \to 4$ parton transition

since $F(x_i, \boldsymbol{y}) \sim 1/\boldsymbol{y}^2$



Kirschner 1979; Shelest, Snigirev, Zinovev 1982
Gaunt, Stirling 2009; Ceccopieri 2011

Deeper problems with the splitting graphs



- contribution from splitting graphs in cross section
  gives divergent integrals $\int d^2\boldsymbol{y}\, F(x_1, x_2, \boldsymbol{y})\, F(\bar{x}_1, \bar{x}_2, \boldsymbol{y}) \sim \int d\boldsymbol{y}^2/\boldsymbol{y}^4$

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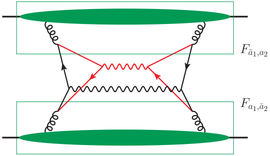

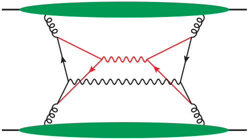

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- double counting problem between double scattering with splitting and single scattering at loop level

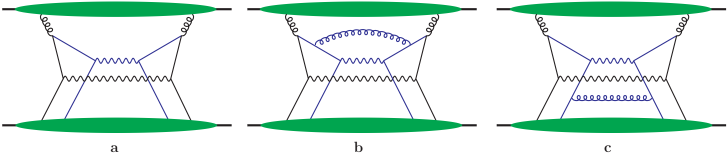
MD, Ostermeier, Schäfer 2011; Gaunt, Stirling 2011; Gaunt 2012
Blok, Dokshitzer, Frankfurt, Strikman 2011; Ryskin, Snigirev 2011, 2012
same problem for jets: Cacciari, Salam, Sapeta 2009

- need consistent separation of physics at small and large $\boldsymbol{y}$

"What is double parton scattering?"

solution will also determine evolution equations for DPDs

Deeper problems with the splitting graphs

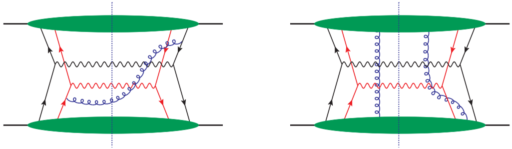


- also have graphs with short-distance splitting on one side but not on the other — B. Blok et al. 2011-13; J. Gaunt 2012
- nomenclature:

2 vs. 1	Gaunt, Stirling	
$3 \to 4$	Blok et al.	(s channel counting, for double dijets)
4×2	Diehl, Ostermeier, Schäfer	(t channel counting)

Double parton scattering: towards factorization

- no complete proof of factorization yet
  several elements worked out four double Drell-Yan process
  MD, Ostermeier, Schäfer 2011; Mahorhar, Waalewijn 2012
- major issue: soft gluon exchange between hard-scattering processes and between spectators



  - with soft-gluon approximation $\to$ Sudakov logarithms, describe using Wilson lines
  - in Glauber region $(l_T^2 \gg l^+ l^-)$: open problem
  - soft gluon exchange $\overset{?}{\leftrightarrow}$ color reconnection in Monte Carlos

Summary

- have many elements for rigirous theory of hard double parton scattering
  adapt technology and insight from single hard scattering
- but important open questions
  - soft gluon exchange (Glauber region, soft rescattering)
  - need a scheme for separating single and double hard scattering