**Second International Summer School of the GDR PH-QCD**

**“Correlations between partons in nucleons”**

# Multidimensional pictures of the nucleon (3/3)

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IFPA Liège

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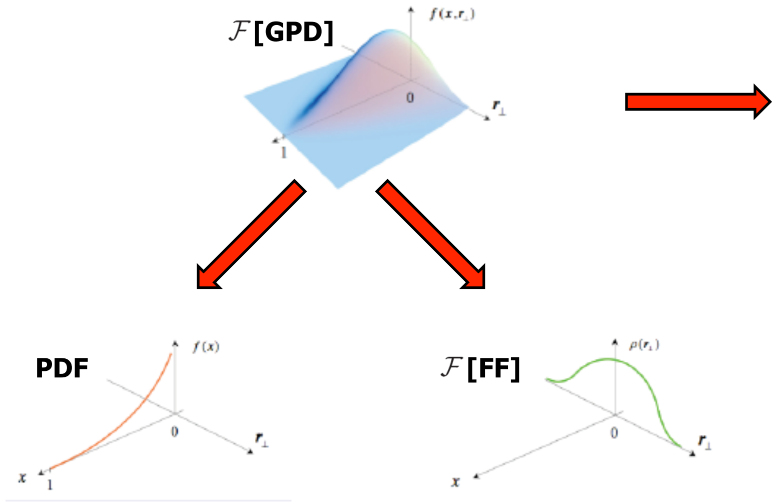
# Reminder

## Lecture 2

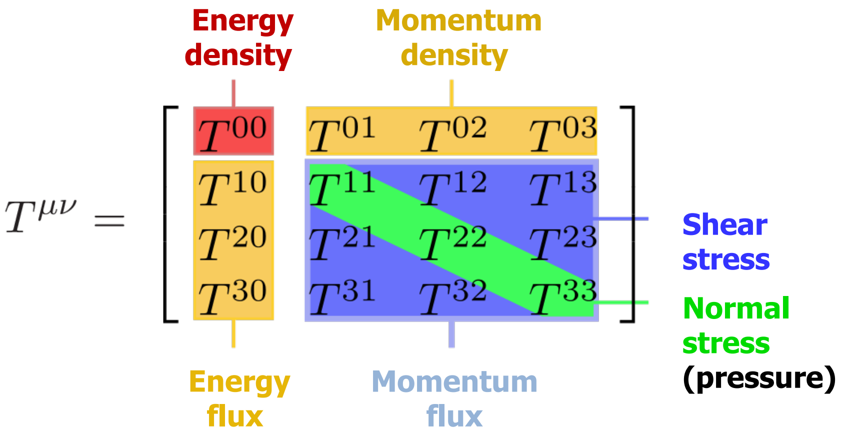
- **PDFs provide 1D pictures of the nucleon**
- **FFs provide 2D pictures of the nucleon**
- **GPDs generalize both PDFs and FFs and give access to the EMT**

$$\mathcal{F}[\text{GPD}](x,\vec{b}_\perp) = \int \mathrm{d}^2k_\perp\, \rho_W(x,\vec{k}_\perp,\vec{b}_\perp)$$



$$\text{PDF}(x) = \int \mathrm{d}^2k_\perp\, \mathrm{d}^2b_\perp\, \rho_W(x,\vec{k}_\perp,\vec{b}_\perp)$$

$$\mathcal{F}[\text{FF}](\vec{b}_\perp) = \int \mathrm{d}x\, \mathrm{d}^2k_\perp\, \rho_W(x,\vec{k}_\perp,\vec{b}_\perp)$$



**Energy density** **Momentum density**

$$T^{\mu\nu} = \begin{bmatrix} T^{00} & T^{01} & T^{02} & T^{03} \\ T^{10} & T^{11} & T^{12} & T^{13} \\ T^{20} & T^{21} & T^{22} & T^{23} \\ T^{30} & T^{31} & T^{32} & T^{33} \end{bmatrix}$$

**Shear stress**

**Normal stress** **(pressure)**

**Energy flux** **Momentum flux**

# Outline

## Lecture 1

- Introduction
- Tour in phase space
- Galileo *vs* Lorentz
- Photon point of view

## Lecture 2

- Nucleon 1D picture
- Nucleon 2D picture
- Nucleon 2+1D picture
- Energy-momentum tensor

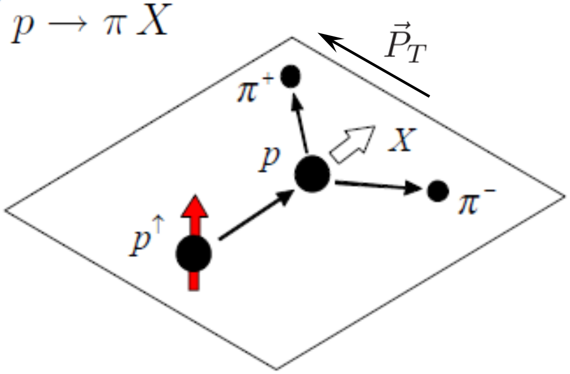
## Lecture 3

- Another nucleon 3D picture
- Tour in Fock space
- 3D+3D=… 5D !

# What about $k_T$ ?

**Large single-spin asymmetries have been observed at high energy !**

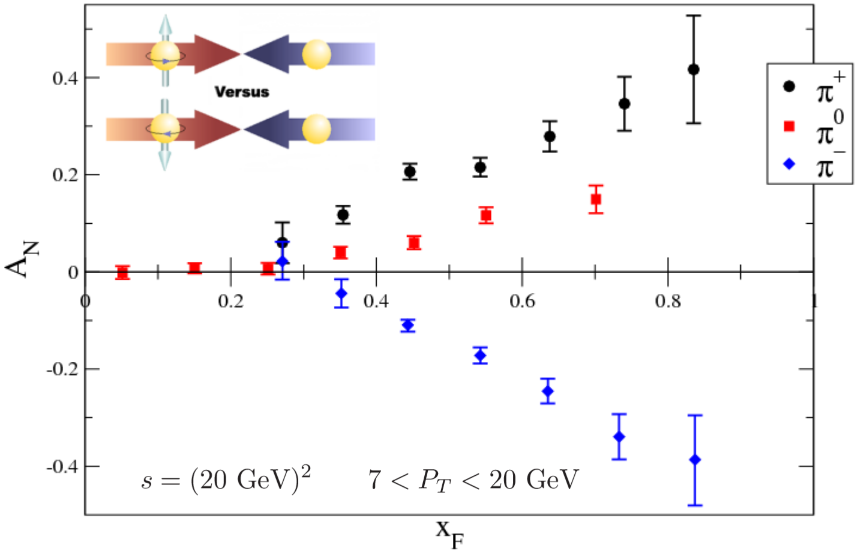
$$p^\uparrow p \to \pi X$$



$$A_N = \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow} \propto \vec{S}_N \cdot (\vec{p}_N \times \vec{P}_T)$$

$A_N$ vs $x_F$ in $\pi$ Production

(FNAL 1991)

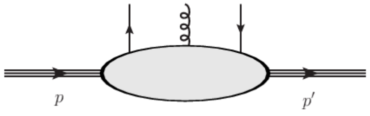


## Partonic origin ?

**Collinear twist-2** ✗ $A_N \propto \frac{m_q}{E_q}\alpha$ **Price for helicity flip** ➡ **Too small !** **[Kane, Pumplin, Repko (1978)]**

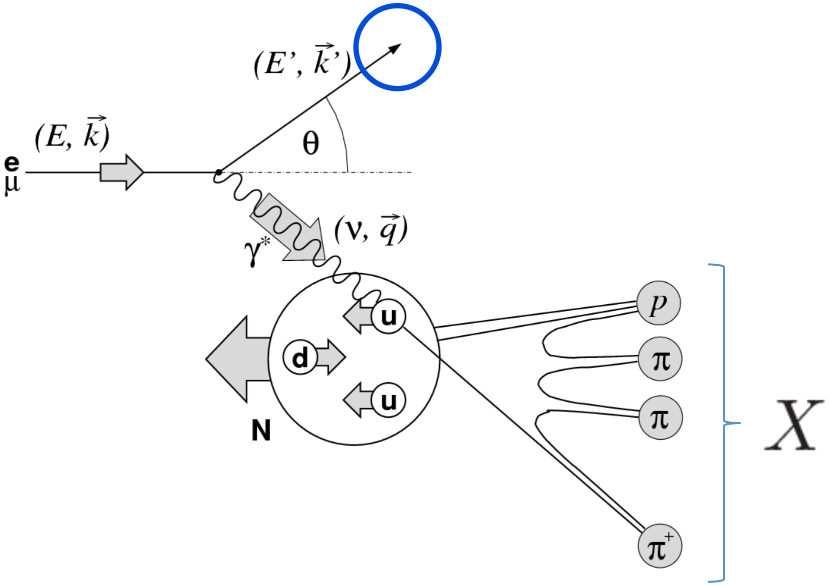
**Intrinsic $k_T$** ✓ $Q \gg P_T \sim \Lambda_{\mathrm{QCD}}$

**Collinear twist-3** ✓ $Q \sim P_T \gg \Lambda_{\mathrm{QCD}}$

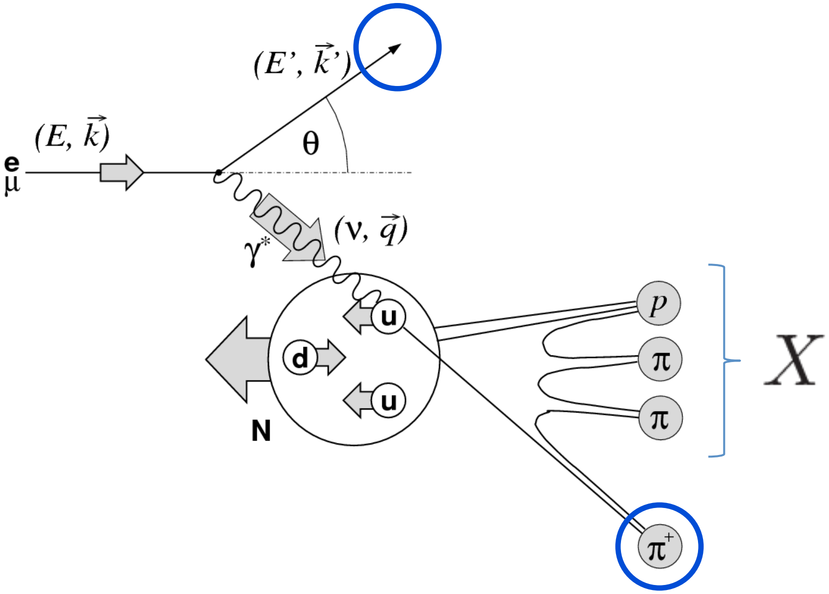


# Semi-inclusive DIS

**Inclusive DIS**



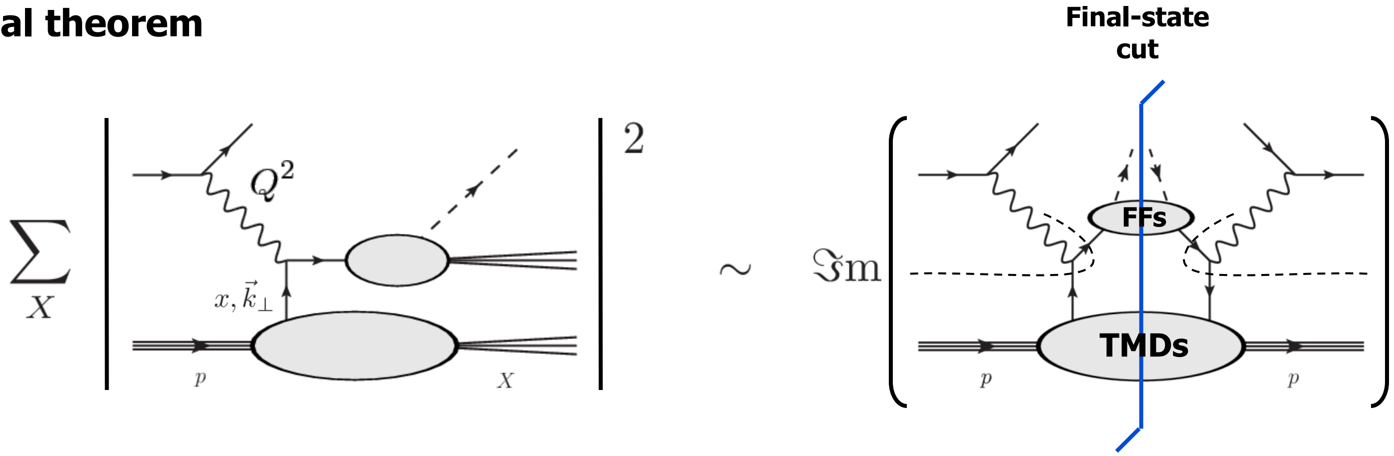
**Semi-inclusive DIS (SIDIS)**



○ **Identified particles in final state**

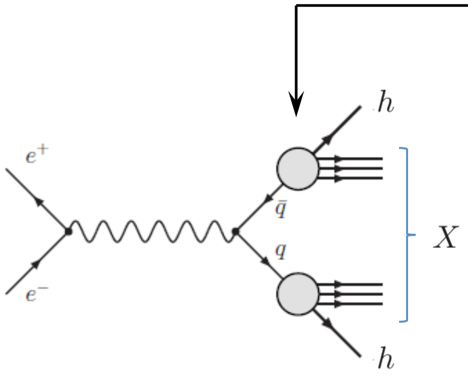
# Transverse-momentum distributions (TMDs)

**Optical theorem**



$$d\sigma \sim \sum \mathrm{TMD}(x,\vec{k}_\perp) \otimes d\hat{\sigma}_{hard} \otimes \underbrace{\mathrm{FF}(z,\vec{p}_\perp)}_{\text{Fragmentation Function}} + \mathcal{O}(\frac{P_T}{Q})$$

$e^+e^- \to hh'X$



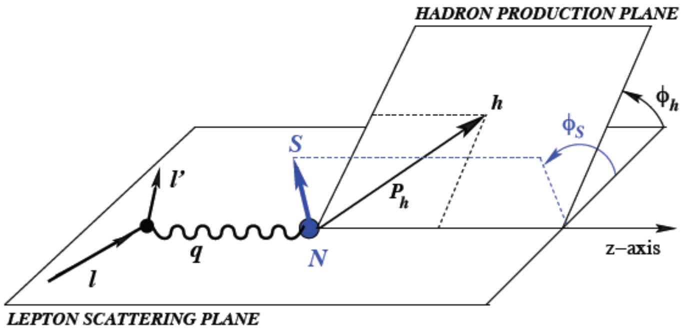
[Collins, Soper, Sterman (1985)]
[Ji, Ma, Yuan (2004)]
[Idilbi *et al.* (2004)]
[Cherednikov, Stefanis (2008)]
[Trentadue, Ceccopieri (2008)]
[Hautman (2008)]
[Echevarria, Idilbi, Scimemi (2011)]
[Collins (2011)]

# Transverse-momentum distributions (TMDs)

**TMD correlator**

$$\begin{aligned}\Phi^{[\Gamma]}_{\Lambda'\Lambda}(x,\vec{k}_\perp) &= \frac{1}{2}\int \frac{\mathrm{d}z^-\,\mathrm{d}^2 z_\perp}{(2\pi)^3}\, e^{ik\cdot z}\,\langle P,\Lambda'|\overline{\psi}(-\tfrac{z}{2})\,\Gamma\,\mathcal{W}\,\psi(\tfrac{z}{2})|P,\Lambda\rangle\Big|_{z^+=0} \\ &= W^{[\Gamma]}_{\Lambda'\Lambda}(x,0,\vec{k}_\perp,\vec{0}_\perp)\end{aligned}$$

**SIDIS modulations**



$$x = \frac{Q^2}{2\,P\cdot q} \qquad y = \frac{P\cdot q}{P\cdot l} \qquad z = \frac{P\cdot P_h}{P\cdot q}$$

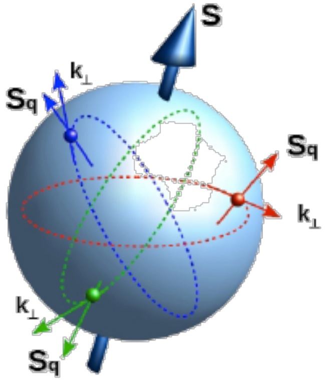
**[Mulders, Tangermann (1996)]**
**[Boer, Mulders (1998)]**
**[Bacchetta *et al.* (2004)]**
**[Bacchetta *et al.* (2007)]**
**[Anselmino *et al.* (2011)]**

$$\begin{aligned}\frac{\mathrm{d}^6\sigma}{\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z\,\mathrm{d}\phi_S\,\mathrm{d}^2P_T} &= F_{UU} + \cos(2\phi)\,F_{UU}^{\cos(2\phi)} + \frac{1}{Q}\cos\phi\,F_{UU}^{\cos\phi} + \lambda_e\frac{1}{Q}\sin\phi\,F_{UU}^{\sin\phi} \\ &+ S_L\left\{\sin(2\phi)\,F_{UL}^{\sin(2\phi)} + \frac{1}{Q}\sin\phi\,F_{UL}^{\sin\phi} + \lambda_e\left[F_{LL} + \frac{1}{Q}\cos\phi\,F_{LL}^{\cos\phi}\right]\right\} \\ &+ S_T\left\{\sin(\phi-\phi_S)\,F_{UT}^{\sin(\phi-\phi_S)} + \sin(\phi+\phi_S)\,F_{UT}^{\sin(\phi+\phi_S)} + \sin(3\phi-\phi_S)\,F_{UT}^{\sin(3\phi-\phi_S)}\right. \\ &\qquad + \frac{1}{Q}\left[\sin(2\phi-\phi_S)\,F_{UT}^{\sin(2\phi-\phi_S)} + \sin\phi_S\,F_{UT}^{\sin\phi_S}\right] \\ &\qquad \left. + \lambda_e\left[\cos(\phi-\phi_S)\,F_{LT}^{\cos(\phi-\phi_S)} + \frac{1}{Q}\left(\cos\phi_S\,F_{LT}^{\cos\phi_S} + \cos(2\phi-\phi_S)\,F_{LT}^{\cos(2\phi-\phi_S)}\right)\right]\right\}\end{aligned}$$

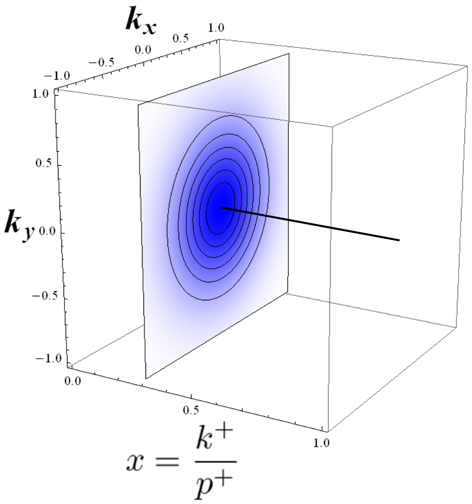
$$F_{S_q S} \propto \mathrm{TMD} \otimes \mathrm{FF}$$

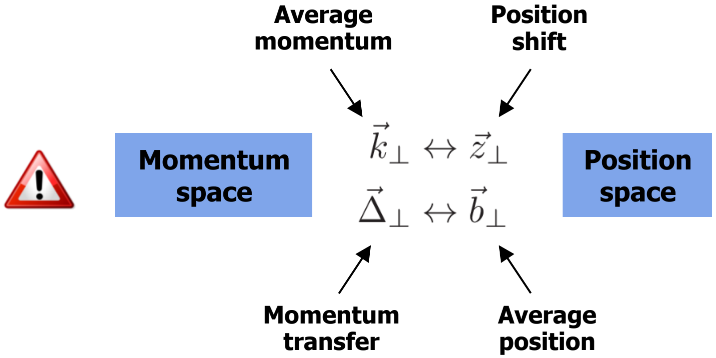
# Transverse-momentum distributions (TMDs)

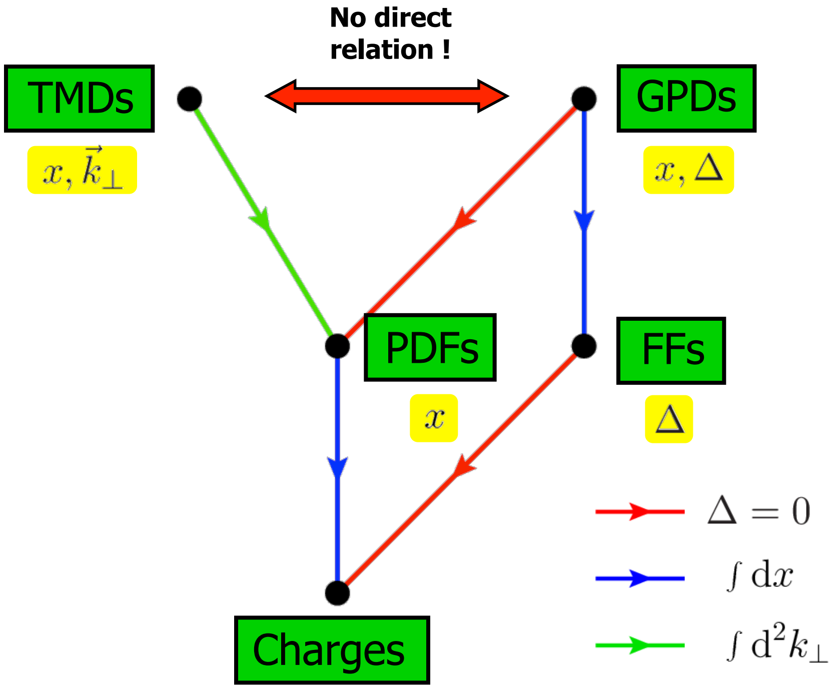
## Spin-orbit correlations



## Momentum-space imaging







**Complementary hadron 3D pictures !**

# Transverse-momentum distributions (TMDs)

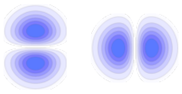
**Multipole structure**

**Quark polarization**

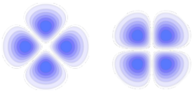
**Nucleon polarization**

| | $U$ | $T_x$ | $T_y$ | $L$ |
|---|---|---|---|---|
| $U$ | $f_1$ | $\frac{k_y}{M}\, h_1^\perp$ | $-\frac{k_x}{M}\, h_1^\perp$ | |
| $T_x$ | $\frac{k_y}{M}\, f_{1T}^\perp$ | $h_1 + \frac{k_x^2 - k_y^2}{2M^2}\, h_{1T}^\perp$ | $\frac{k_x k_y}{M^2}\, h_{1T}^\perp$ | $\frac{k_x}{M}\, g_{1T}$ |
| $T_y$ | $-\frac{k_x}{M}\, f_{1T}^\perp$ | $\frac{k_x k_y}{M^2}\, h_{1T}^\perp$ | $h_1 - \frac{k_x^2 - k_y^2}{2M^2}\, h_{1T}^\perp$ | $\frac{k_y}{M}\, g_{1T}$ |
| $L$ | | $\frac{k_x}{M}\, h_{1L}^\perp$ | $\frac{k_y}{M}\, h_{1L}^\perp$ | $g_{1L}$ |

**Monopole**



**Dipole**



**Quadrupole**

# Transverse-momentum distributions (TMDs)

## Multipole structure

Quark polarization

Nucleon polarization

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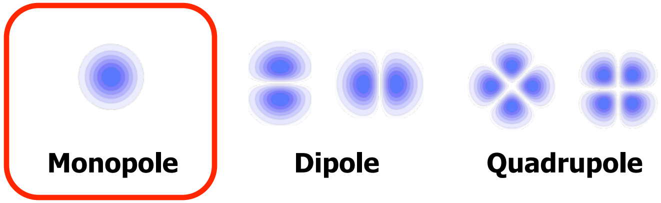

# Transverse-momentum distributions (TMDs)

**Multipole structure**

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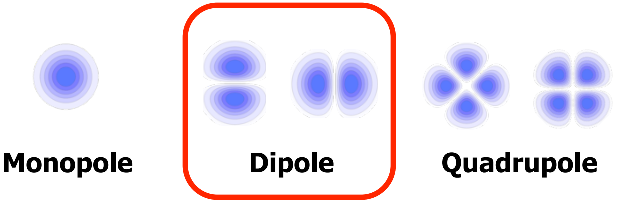

# Transverse-momentum distributions (TMDs)

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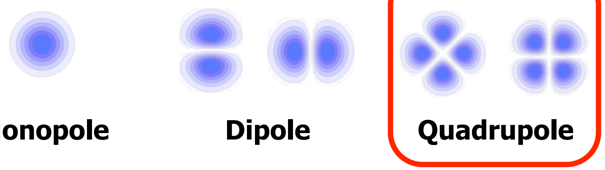

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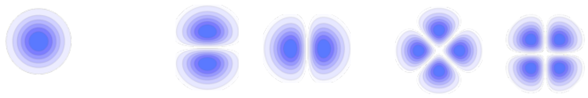


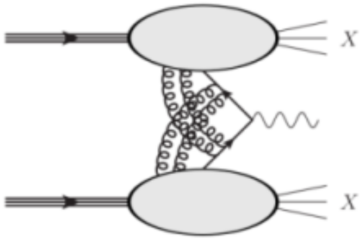
🟩 **Naive T-odd !**

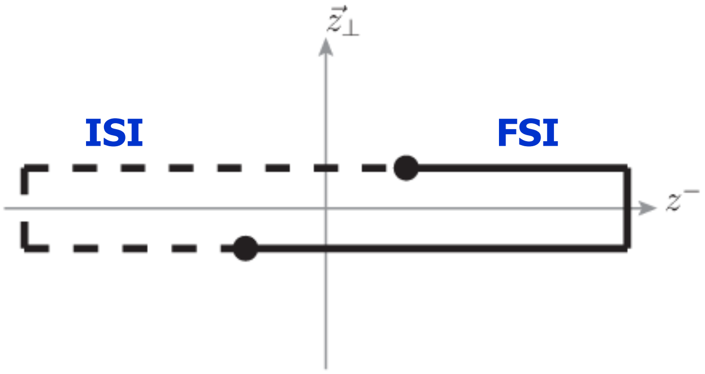
# Transverse-momentum distributions (TMDs)

**Non-trivial gauge link** $\mathcal{W}$

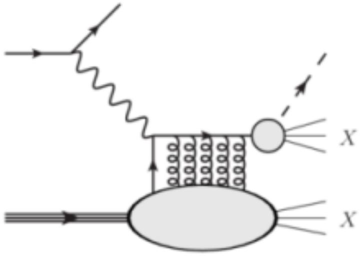
[Belitsky *et al.* (2003)]
[Boer *et al.* (2003)]

**Drell-Yan**





**SIDIS**



**CPT invariance** ⟶

$$\boxed{\begin{aligned} f_{1T}^{\perp}(x,\vec{k}_\perp)\big|_{\mathrm{DY}} &= -f_{1T}^{\perp}(x,\vec{k}_\perp)\big|_{\mathrm{SIDIS}} \\ h_{1}^{\perp}(x,\vec{k}_\perp)\big|_{\mathrm{DY}} &= -h_{1}^{\perp}(x,\vec{k}_\perp)\big|_{\mathrm{SIDIS}} \end{aligned}}$$

**Naive T-odd**

**Fundamental test !**

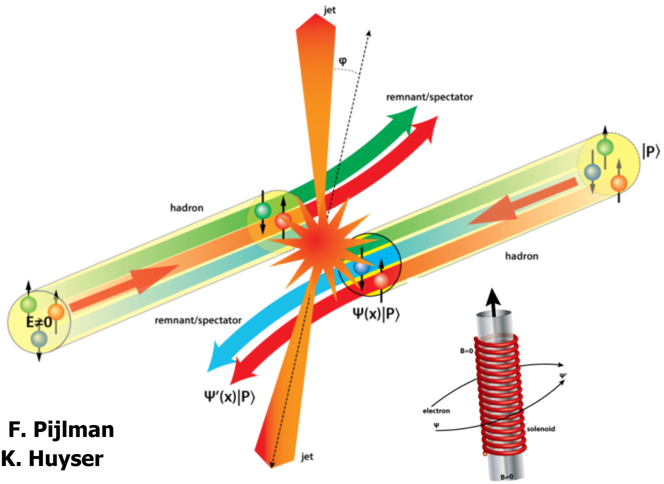
## Generalized universality

$$\underbrace{\mathrm{TMD}(x,\vec{k}_\perp;\mathcal{W})}_{\text{Process dependent}} = \sum_i \underbrace{C_i(\mathcal{W})}_{\text{Calculable}}\,\underbrace{\mathrm{TMD}_i(x,\vec{k}_\perp)}_{\text{Universal}}$$

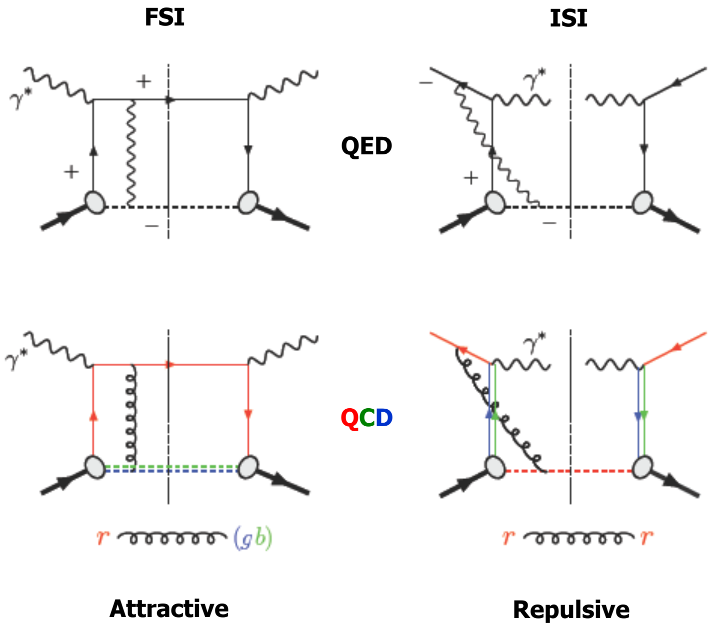
[Buffing *et al.* (2012)]
[Buffing *et al.* (2013)]

# Transverse-momentum distributions (TMDs)

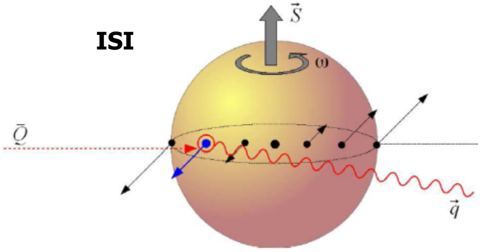
## Color-induced phase !

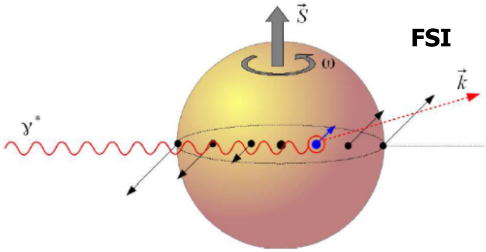


**Analogy: F. Pijlman**
**Figure: K. Huyser**



## Quasi-classical interpretation

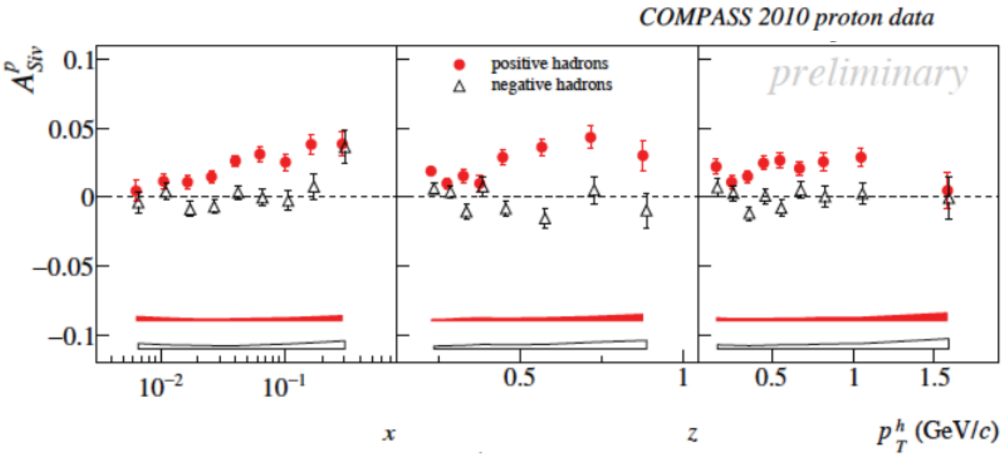




[Sivers (2006)]
[Sievert, Kovchegov (2014)]

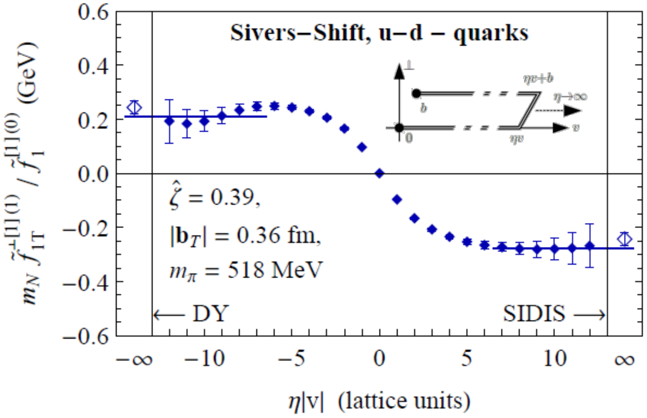
# Transverse-momentum distributions (TMDs)

## Clear experimental signal

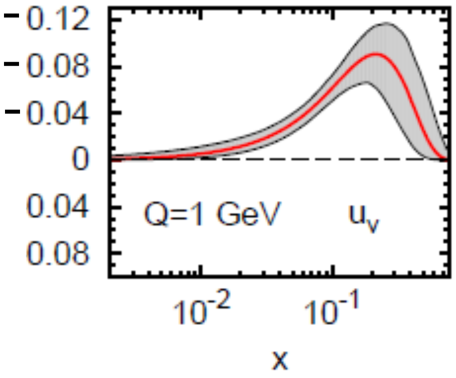


## Lattice QCD

**[Musch *et al.* (2012)]**



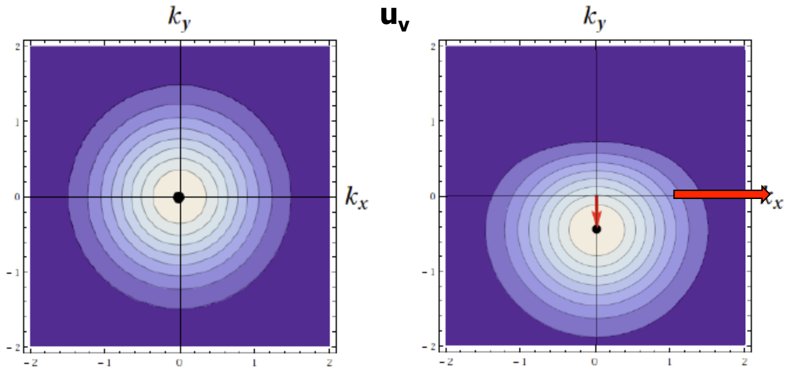
$$x f_{1T}^{\perp(1)}(x) = x \int \mathrm{d}^2 k_\perp \, \frac{\vec{k}_\perp^2}{M^2} \, f_{1T}^{\perp}(x, \vec{k}_\perp)$$



**[Anselmino *et al.* (2012)]**

**Dipole shift**

$$\boxed{\rho_X(x, \vec{k}_\perp) = f_1(x, \vec{k}_\perp) + \frac{k_y}{M} \, f_{1T}^{\perp}(x, \vec{k}_\perp)}$$

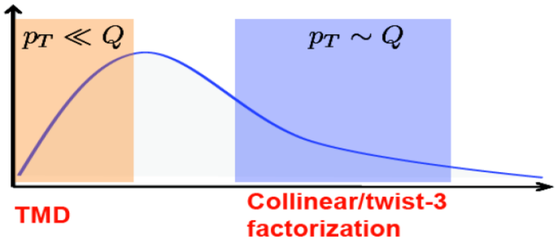


**Courtesy of Alexei Prokudin**

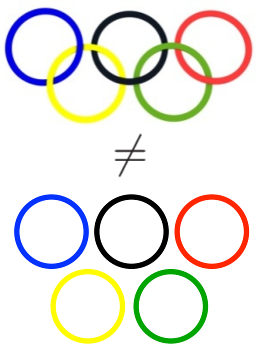
# Transverse-momentum distributions (TMDs)

**Open questions and problems**

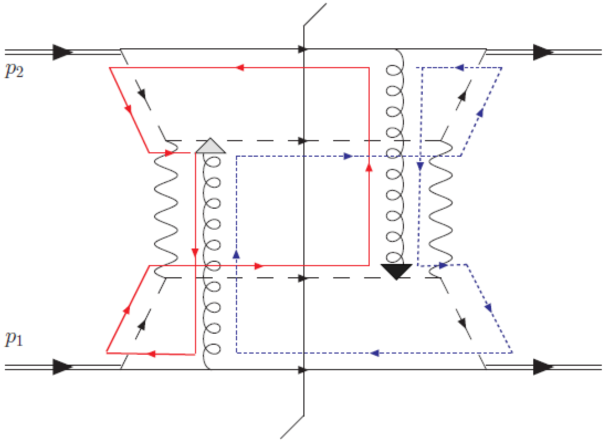
- Tests of universality (*e.g.* with DY) and evolution
- Model dependence and extrapolations
- Precise determination of polarized TMDs
- Extraction of gluon TMDs
- Accessing higher-twist distributions
- Link with low $x$, $k_T$ factorization
- Sign mismatch with collinear twist-3 approach
- Factorization breaks down in some pp scattering
- …



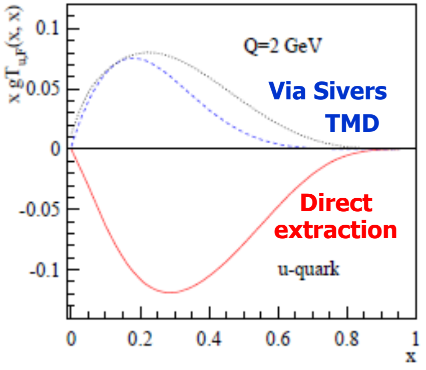
**Entangled color flow**



M. Buffing



[Rogers, Mulders (2011)]

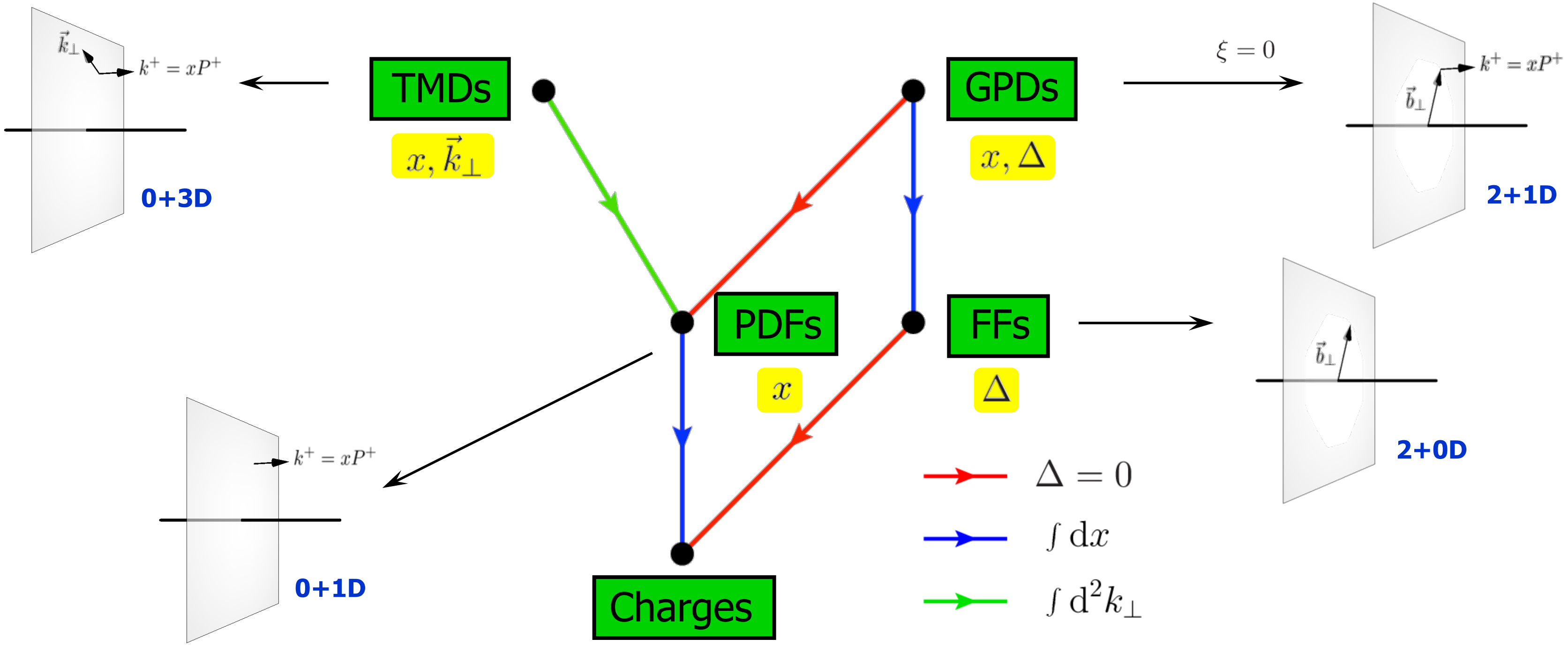


[Kang *et al.* (2011)]

# Parton distribution zoo

**Theoretical tools**

**« Physical » objects**



TMDs

$x, \vec{k}_\perp$

GPDs

$x, \Delta$

PDFs

$x$

FFs

$\Delta$

Charges

$\Delta = 0$

$\int \mathrm{d}x$

$\int \mathrm{d}^2k_\perp$

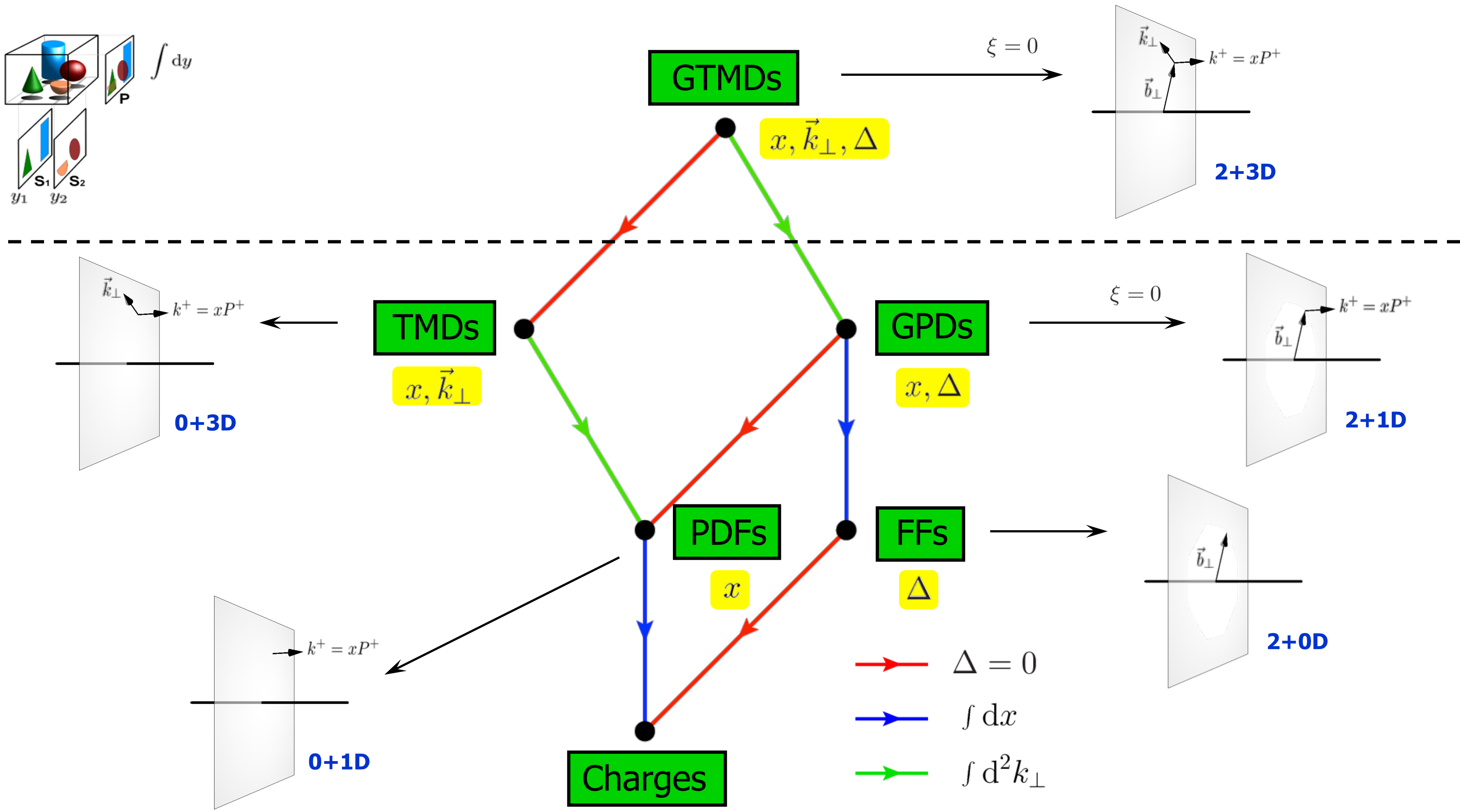
**[C.L., Pasquini, Vanderhaeghen (2011)]**

# Parton distribution zoo

**Theoretical tools**

**« Physical » objects**

**Phase-space (Wigner) distribution**



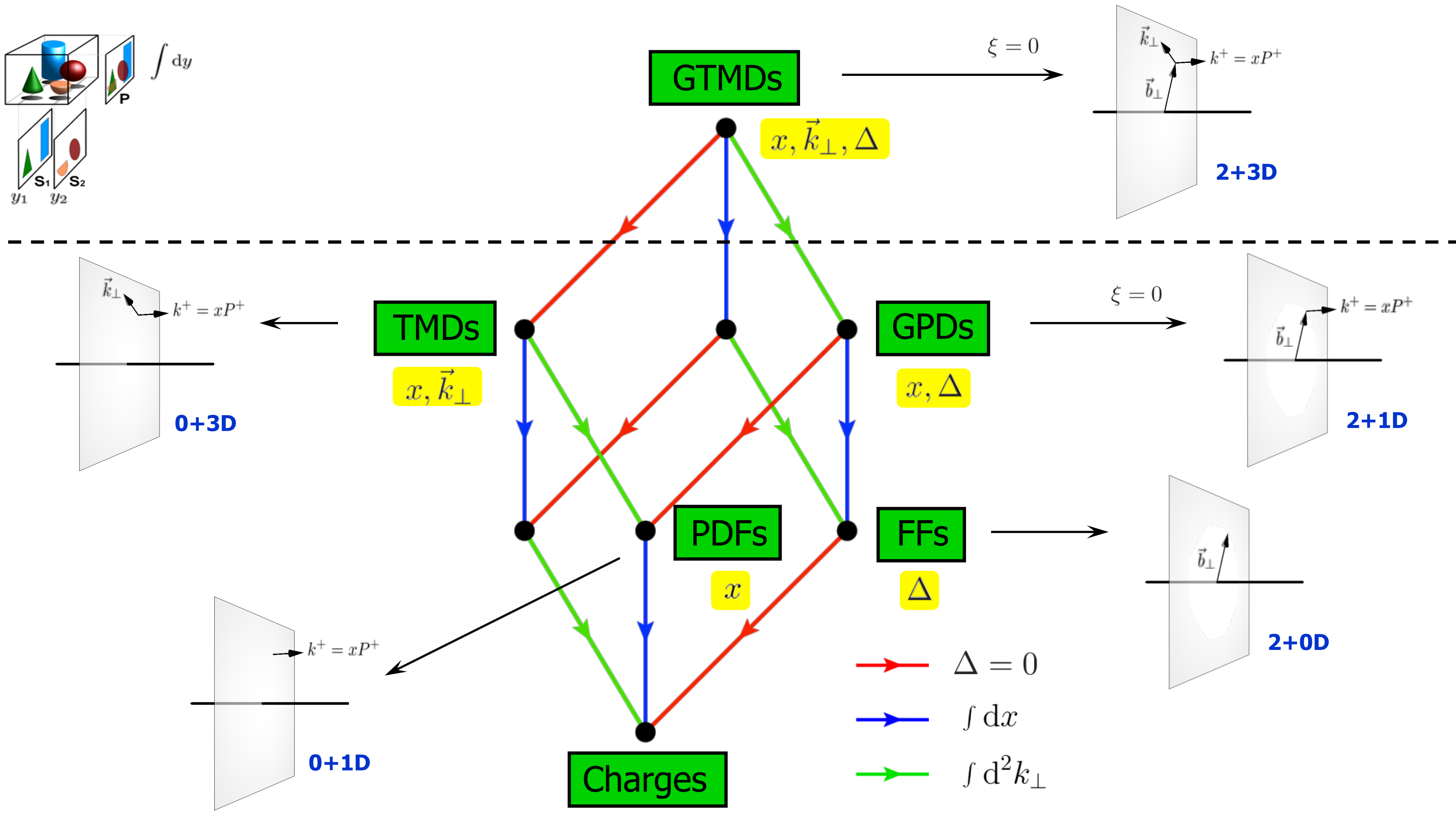
**[C.L., Pasquini, Vanderhaeghen (2011)]**

# Parton distribution zoo

**Theoretical tools**

**« Physical » objects**

**Phase-space (Wigner) distribution**



GTMDs

$x, \vec{k}_\perp, \Delta$

TMDs

$x, \vec{k}_\perp$

GPDs

$x, \Delta$

PDFs

$x$

FFs

$\Delta$

Charges

$\Delta = 0$

$\int \mathrm{d}x$

$\int \mathrm{d}^2 k_\perp$

**2+3D**

**0+3D**

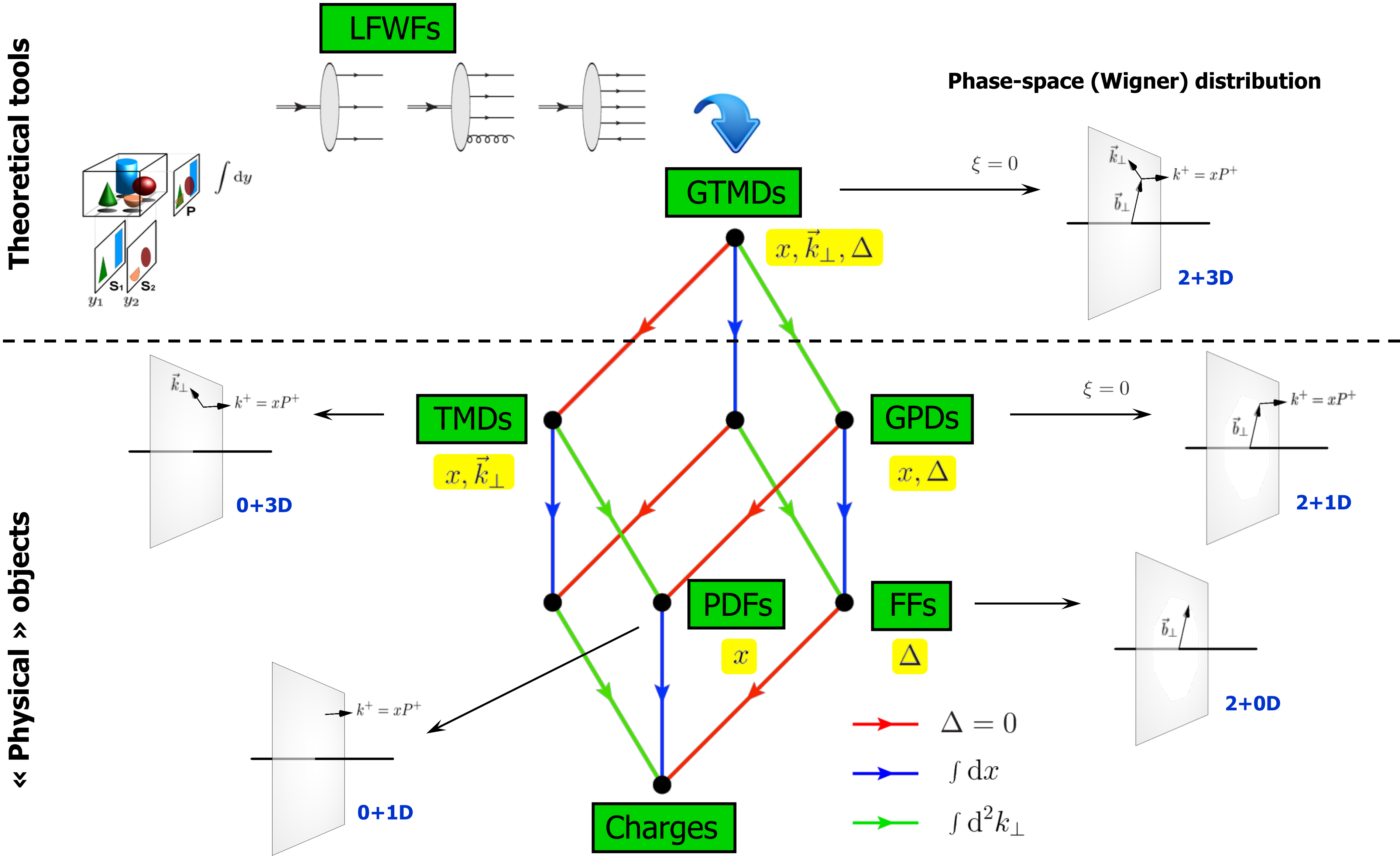
**2+1D**

**0+1D**

**2+0D**

**[C.L., Pasquini, Vanderhaeghen (2011)]**

# Parton distribution zoo



**Theoretical tools**

LFWFs

**Phase-space (Wigner) distribution**

GTMDs $x, \vec{k}_\perp, \Delta$

**2+3D**

**« Physical » objects**

TMDs $x, \vec{k}_\perp$

**0+3D**

GPDs $x, \Delta$

**2+1D**

PDFs $x$

**0+1D**

FFs $\Delta$

**2+0D**

Charges

🟥 $\Delta = 0$

🟦 $\int \mathrm{d}x$

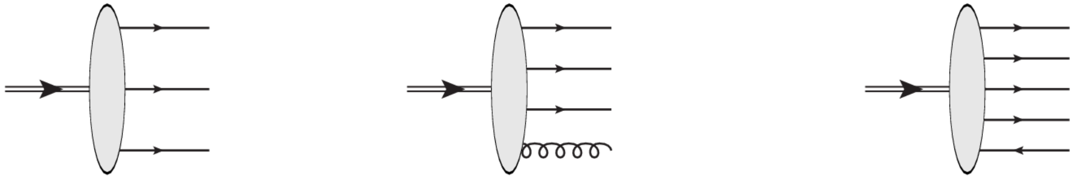
🟩 $\int \mathrm{d}^2 k_\perp$

**[C.L., Pasquini, Vanderhaeghen (2011)]**

# Light-front wave functions (LFWFs)

**Fock expansion of the nucleon state**

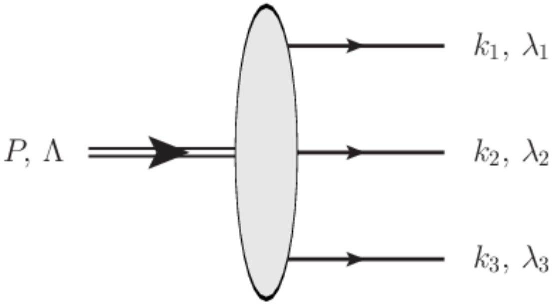
$$|p\rangle \quad = \quad \Psi_{qqq} \quad |qqq\rangle \quad + \quad \Psi_{qqqg} \quad |qqqg\rangle \quad + \quad \Psi_{qqqq\bar{q}} \quad |qqqq\bar{q}\rangle \quad + \quad \cdots$$



**Probability associated with the Fock states**

$$\rho^{\Lambda}_{N,\beta} = \int [\mathrm{d}x]_N \left[\mathrm{d}^2k_\perp\right]_N \left|\Psi^{\Lambda}_{\lambda_1\cdots\lambda_N}\right|^2$$

$$\sum_{N,\beta} \rho^{\Lambda}_{N,\beta} = 1$$



**Momentum and angular momentum conservation**

$$P^+ = \sum_{i=1}^{N} k_i^+$$

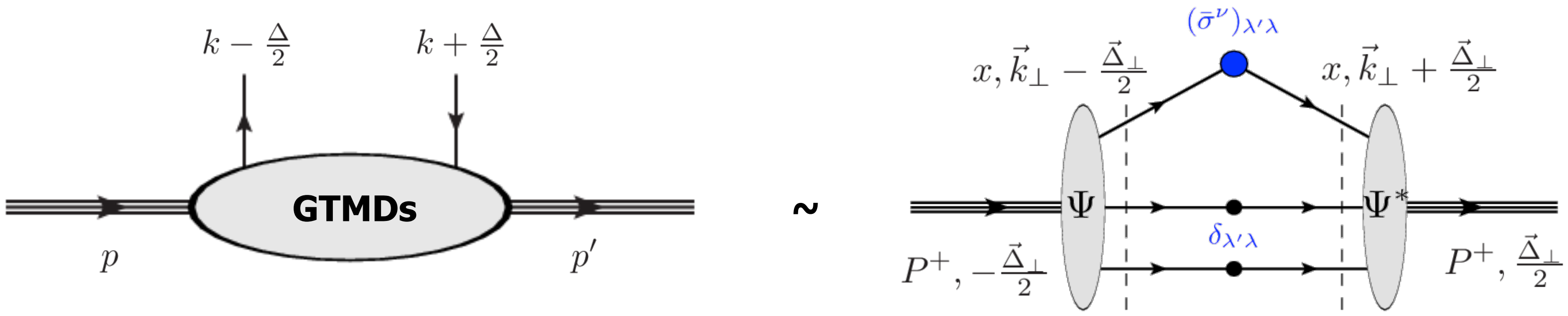
$$\vec{0}_\perp = \vec{P}_\perp = \sum_{i=1}^{N} \vec{k}_{i\perp}$$

$$\boxed{\Lambda = \sum_{i=1}^{N} \lambda_i + l_z}$$

⚠ $A^+ = 0$ **gauge**

# Light-front wave functions (LFWFs)

**Overlap representation**



$$\boxed{W^{[\Gamma]}_{\Lambda'\Lambda}(x,\xi,\vec{k}_\perp,\vec{\Delta}_\perp) = \frac{1}{\sqrt{1-\xi^2}} \sum_{\beta',\beta} \int [\mathrm{d}x]_3 [\mathrm{d}^2k_\perp]_3 \, \bar{\delta}(\tilde{k}) \, \psi^*_{\Lambda'\beta'}(r') \, \psi_{\Lambda\beta}(r) \, M^{[\Gamma]\beta'\beta}}$$

**Momentum**

**Polarization**

$$[\mathrm{d}x]_3 \equiv \left[\prod_{i=1}^{3} \mathrm{d}x_i\right] \delta\left(1 - \sum_{i=1}^{3} x_i\right)$$

$$[\mathrm{d}^2k_\perp]_3 \equiv \left[\prod_{i=1}^{3} \frac{\mathrm{d}^2 k_{i\perp}}{2(2\pi)^3}\right] 2(2\pi)^3 \, \delta^{(2)}\left(\sum_{i=1}^{3} \vec{k}_{i\perp}\right)$$

$$\bar{\delta}(\tilde{k}) \equiv \sum_{i=1}^{3} \Theta(x) \, \delta(x - x_i) \, \delta^{(2)}(\vec{k}_\perp - \vec{k}_{i\perp})$$

$$M^{[\Gamma]\beta'\beta} = M^{[\Gamma]\lambda'_1\lambda_1} \, \delta^{\lambda'_2\lambda_2} \, \delta^{\lambda'_3\lambda_3}$$

$$M^{[\Gamma]\lambda'\lambda} \equiv \frac{\overline{u}(p',\lambda')\Gamma u(p,\lambda)}{2P^+\sqrt{1-\xi^2}}$$

**[C.L., Pasquini, Vanderhaeghen (2011)]**

# Light-front wave functions (LFWFs)

**Light-front quark models**

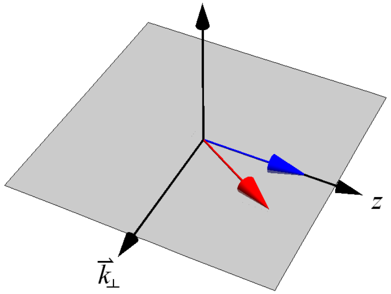
$$\psi_{\Lambda\beta}(r) = \mathcal{N}\,\Psi(r)\sum_{\sigma_i}\underset{\uparrow}{\Phi_\Lambda^{\sigma_1\sigma_2\sigma_3}}\prod_{i=1}^{3} D_{\lambda_i\sigma_i}(\tilde{k}_i)$$

**SU(6) spin-flavor wave function**

**Wigner rotation**

$$\underset{\uparrow}{q_\lambda^{LC}(k)} = \sum_s D_{\lambda s}^{(1/2)*}(k)\,\underset{\uparrow}{q_s^{C}(k)}$$

**Light-front helicity** **Canonical spin**



$$D(\tilde{k}) = \frac{1}{|\vec{K}|}\begin{pmatrix} K_z & K_L \\ -K_R & K_z \end{pmatrix}$$

$$K_{R,L} = K_x \pm iK_y$$

| Model | $\Psi(r)$ | $K_z$ | $\vec{K}_\perp$ | $\kappa_z$ |
|---|---|---|---|---|
| LFCQM | $\tilde{\psi}(r)$ | $m + y\mathcal{M}_0$ | $\vec{\kappa}_\perp$ | $y\mathcal{M}_0 - \omega$ |
| LF$\chi$QSM | $\prod_{i=1}^{3}\lvert\vec{K}_i\rvert$ | $f_{/\!/}(y,\kappa_\perp)$ | $\vec{\kappa}_\perp\, f_\perp(y,\kappa_\perp)$ | $y\mathcal{M}_N - E_{\text{lev}}$ |

[C.L., Pasquini, Vanderhaeghen (2011)]

# Parametrization

## Twist-2

**GTMDs**

Quark polarization

Nucleon polarization

| | $U$ | $T_x$ | $T_y$ | $L$ |
|---|---|---|---|---|
| $U$ | $F_{11}$ | $\frac{i}{M}\,(k_y H_{11} + \Delta_y H_{12})$ | $-\frac{i}{M}\,(k_x H_{11} + \Delta_x H_{12})$ | $\frac{i(\vec{\Delta}_\perp \times \vec{k}_\perp)_z}{M^2}\,G_{11}$ |
| $T_x$ | $\frac{i}{M}\,\left(k_y F_{12} + \Delta_y(F_{13} - \frac{1}{2}F_{11})\right)$ | $\dots$ | $\dots$ | $\dots$ |
| $T_y$ | $-\frac{i}{M}\,\left(k_x F_{12} + \Delta_x(F_{13} - \frac{1}{2}F_{11})\right)$ | $\dots$ | $\dots$ | $\dots$ |
| $L$ | $-\frac{i(\vec{\Delta}_\perp \times \vec{k}_\perp)_z}{M^2}\,F_{14}$ | $\frac{1}{M}\,(k_x H_{17} + \Delta_x H_{18})$ | $\frac{1}{M}\,(k_y H_{17} + \Delta_y H_{18})$ | $G_{14}$ |

$\vec{\Delta}_\perp = \vec{0}_\perp$

$\int \mathrm{d}^2 k_\perp$

$\vec{k}_\perp \leftrightarrow i\vec{\Delta}_\perp$

**TMDs**

| | $U$ | $T_x$ | $T_y$ | $L$ |
|---|---|---|---|---|
| $U$ | $f_1$ | $\frac{k_y}{M}\,h_1^\perp$ | $-\frac{k_x}{M}\,h_1^\perp$ | |
| $T_x$ | $\frac{k_y}{M}\,f_{1T}^\perp$ | $h_1 + \frac{k_x^2 - k_y^2}{2M^2}\,h_{1T}^\perp$ | $\frac{k_x k_y}{M^2}\,h_{1T}^\perp$ | $\frac{k_x}{M}\,g_{1T}$ |
| $T_y$ | $-\frac{k_x}{M}\,f_{1T}^\perp$ | $\frac{k_x k_y}{M^2}\,h_{1T}^\perp$ | $h_1 - \frac{k_x^2 - k_y^2}{2M^2}\,h_{1T}^\perp$ | $\frac{k_y}{M}\,g_{1T}$ |
| $L$ | | $\frac{k_x}{M}\,h_{1L}^\perp$ | $\frac{k_y}{M}\,h_{1L}^\perp$ | $g_{1L}$ |

**GPDs**

$\xi = 0$

| | $U$ | $T_x$ | $T_y$ | $L$ |
|---|---|---|---|---|
| $U$ | $H$ | $i\,\frac{\Delta_y}{2M}\,(2\tilde{H}_T + E_T)$ | $-i\,\frac{\Delta_x}{2M}\,(2\tilde{H}_T + E_T)$ | |
| $T_x$ | $i\,\frac{\Delta_y}{2M}\,E$ | $H_T - \frac{\Delta_x^2 - \Delta_y^2}{4M^2}\,\tilde{H}_T$ | $-\frac{\Delta_x \Delta_y}{2M^2}\,\tilde{H}_T$ | |
| $T_y$ | $-i\,\frac{\Delta_x}{2M}\,E$ | $-\frac{\Delta_x \Delta_y}{2M^2}\,\tilde{H}_T$ | $H_T + \frac{\Delta_x^2 - \Delta_y^2}{4M^2}\,\tilde{H}_T$ | |
| $L$ | | | | $\tilde{H}$ |

**Complete parametrizations :** **Quarks** **[Meissner, Metz, Schlegel (2009)]**

**Quarks & gluons** **[C.L., Pasquini (2013)]**

# Parametrization

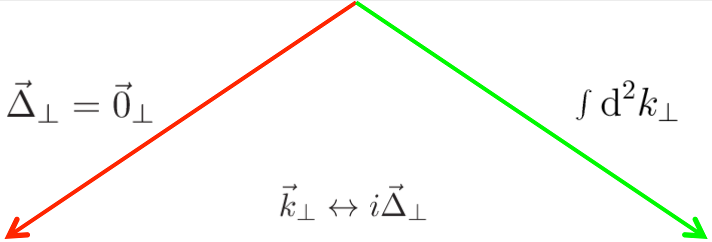
**Twist-2**

**GTMDs**

Quark polarization

Nucleon polarization

| | $U$ | $T_x$ | $T_y$ | $L$ |
|---|---|---|---|---|
| $U$ | $F_{11}$ | $\frac{i}{M}\left(k_y H_{11}+\Delta_y H_{12}\right)$ | $-\frac{i}{M}\left(k_x H_{11}+\Delta_x H_{12}\right)$ | $\frac{i(\vec{\Delta}_\perp\times\vec{k}_\perp)_z}{M^2}\,G_{11}$ |
| $T_x$ | $\frac{i}{M}\left(k_y F_{12}+\Delta_y(F_{13}-\frac{1}{2}F_{11})\right)$ | $\cdots$ | $\cdots$ | $\cdots$ |
| $T_y$ | $-\frac{i}{M}\left(k_x F_{12}+\Delta_x(F_{13}-\frac{1}{2}F_{11})\right)$ | $\cdots$ | $\cdots$ | $\cdots$ |
| $L$ | $-\frac{i(\vec{\Delta}_\perp\times\vec{k}_\perp)_z}{M^2}\,F_{14}$ | $\frac{1}{M}\left(k_x H_{17}+\Delta_x H_{18}\right)$ | $\frac{1}{M}\left(k_y H_{17}+\Delta_y H_{18}\right)$ | $G_{14}$ |



$\vec{\Delta}_\perp=\vec{0}_\perp$

$\int \mathrm{d}^2k_\perp$

$\vec{k}_\perp \leftrightarrow i\vec{\Delta}_\perp$

**TMDs**

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| $U$ | $f_1$ | $\frac{k_y}{M}\,h_1^\perp$ | $-\frac{k_x}{M}\,h_1^\perp$ | |
| $T_x$ | $\frac{k_y}{M}\,f_{1T}^\perp$ | $h_1+\frac{k_x^2-k_y^2}{2M^2}\,h_{1T}^\perp$ | $\frac{k_xk_y}{M^2}\,h_{1T}^\perp$ | $\frac{k_x}{M}\,g_{1T}$ |
| $T_y$ | $-\frac{k_x}{M}\,f_{1T}^\perp$ | $\frac{k_xk_y}{M^2}\,h_{1T}^\perp$ | $h_1-\frac{k_x^2-k_y^2}{2M^2}\,h_{1T}^\perp$ | $\frac{k_y}{M}\,g_{1T}$ |
| $L$ | | $\frac{k_x}{M}\,h_{1L}^\perp$ | $\frac{k_y}{M}\,h_{1L}^\perp$ | $g_{1L}$ |

**GPDs** $\xi=0$

| | $U$ | $T_x$ | $T_y$ | $L$ |
|---|---|---|---|---|
| $U$ | $H$ | $i\,\frac{\Delta_y}{2M}\,(2\tilde{H}_T+E_T)$ | $-i\,\frac{\Delta_x}{2M}\,(2\tilde{H}_T+E_T)$ | |
| $T_x$ | $i\,\frac{\Delta_y}{2M}\,E$ | $H_T-\frac{\Delta_x^2-\Delta_y^2}{4M^2}\,\tilde{H}_T$ | $-\frac{\Delta_x\Delta_y}{2M^2}\,\tilde{H}_T$ | |
| $T_y$ | $-i\,\frac{\Delta_x}{2M}\,E$ | $-\frac{\Delta_x\Delta_y}{2M^2}\,\tilde{H}_T$ | $H_T+\frac{\Delta_x^2-\Delta_y^2}{4M^2}\,\tilde{H}_T$ | |
| $L$ | | | | $\tilde{H}$ |

**Complete parametrizations :** **Quarks** **[Meissner, Metz, Schlegel (2009)]**

**Quarks & gluons** **[C.L., Pasquini (2013)]**

# Parametrization

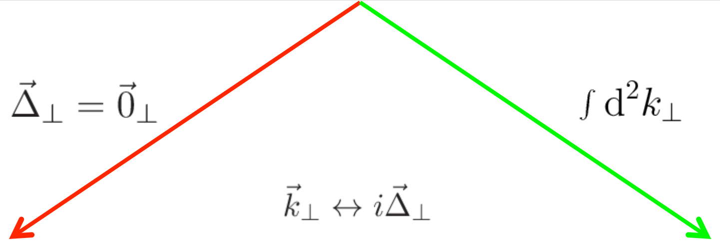
**Twist-2**

**GTMDs**

Quark polarization

Nucleon polarization

|  | $U$ | $T_x$ | $T_y$ | $L$ |
|---|---|---|---|---|
| $U$ | $F_{11}$ | $\frac{i}{M}\left(k_y H_{11}+\Delta_y H_{12}\right)$ | $-\frac{i}{M}\left(k_x H_{11}+\Delta_x H_{12}\right)$ | $\frac{i(\vec{\Delta}_\perp\times\vec{k}_\perp)_z}{M^2}G_{11}$ |
| $T_x$ | $\frac{i}{M}\left(k_y F_{12}+\Delta_y(F_{13}-\frac{1}{2}F_{11})\right)$ | $\cdots$ | $\cdots$ | $\cdots$ |
| $T_y$ | $-\frac{i}{M}\left(k_x F_{12}+\Delta_x(F_{13}-\frac{1}{2}F_{11})\right)$ | $\cdots$ | $\cdots$ | $\cdots$ |
| $L$ | $-\frac{i(\vec{\Delta}_\perp\times\vec{k}_\perp)_z}{M^2}F_{14}$ | $\frac{1}{M}\left(k_x H_{17}+\Delta_x H_{18}\right)$ | $\frac{1}{M}\left(k_y H_{17}+\Delta_y H_{18}\right)$ | $G_{14}$ |



$\vec{\Delta}_\perp=\vec{0}_\perp$

$\int \mathrm{d}^2k_\perp$

$\vec{k}_\perp \leftrightarrow i\vec{\Delta}_\perp$

**TMDs**

|  | $U$ | $T_x$ | $T_y$ | $L$ |
|---|---|---|---|---|
| $U$ | $f_1$ | $\frac{k_y}{M}h_1^\perp$ | $-\frac{k_x}{M}h_1^\perp$ |  |
| $T_x$ | $\frac{k_y}{M}f_{1T}^\perp$ | $h_1+\frac{k_x^2-k_y^2}{2M^2}h_{1T}^\perp$ | $\frac{k_xk_y}{M^2}h_{1T}^\perp$ | $\frac{k_x}{M}g_{1T}$ |
| $T_y$ | $-\frac{k_x}{M}f_{1T}^\perp$ | $\frac{k_xk_y}{M^2}h_{1T}^\perp$ | $h_1-\frac{k_x^2-k_y^2}{2M^2}h_{1T}^\perp$ | $\frac{k_y}{M}g_{1T}$ |
| $L$ |  | $\frac{k_x}{M}h_{1L}^\perp$ | $\frac{k_y}{M}h_{1L}^\perp$ | $g_{1L}$ |

**GPDs** $\xi=0$

|  | $U$ | $T_x$ | $T_y$ | $L$ |
|---|---|---|---|---|
| $U$ | $H$ | $i\frac{\Delta_y}{2M}\left(2\tilde{H}_T+E_T\right)$ | $-i\frac{\Delta_x}{2M}\left(2\tilde{H}_T+E_T\right)$ |  |
| $T_x$ | $i\frac{\Delta_y}{2M}E$ | $H_T-\frac{\Delta_x^2-\Delta_y^2}{4M^2}\tilde{H}_T$ | $-\frac{\Delta_x\Delta_y}{2M^2}\tilde{H}_T$ |  |
| $T_y$ | $-i\frac{\Delta_x}{2M}E$ | $-\frac{\Delta_x\Delta_y}{2M^2}\tilde{H}_T$ | $H_T+\frac{\Delta_x^2-\Delta_y^2}{4M^2}\tilde{H}_T$ |  |
| $L$ |  |  |  | $\tilde{H}$ |

**Complete parametrizations :** **Quarks** **[Meissner, Metz, Schlegel (2009)]**
**Quarks & gluons** **[C.L., Pasquini (2013)]**

# Parametrization

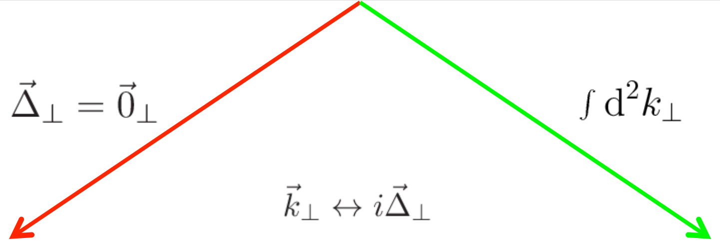
**Twist-2**

**GTMDs**

Quark polarization

Nucleon polarization

| | $U$ | $T_x$ | $T_y$ | $L$ |
|---|---|---|---|---|
| $U$ | $F_{11}$ | $\frac{i}{M}\left(k_y H_{11} + \Delta_y H_{12}\right)$ | $-\frac{i}{M}\left(k_x H_{11} + \Delta_x H_{12}\right)$ | $\frac{i(\vec{\Delta}_\perp \times \vec{k}_\perp)_z}{M^2} G_{11}$ |
| $T_x$ | $\frac{i}{M}\left(k_y F_{12} + \Delta_y (F_{13} - \frac{1}{2}F_{11})\right)$ | $\cdots$ | $\cdots$ | $\cdots$ |
| $T_y$ | $-\frac{i}{M}\left(k_x F_{12} + \Delta_x (F_{13} - \frac{1}{2}F_{11})\right)$ | $\cdots$ | $\cdots$ | $\cdots$ |
| $L$ | $-\frac{i(\vec{\Delta}_\perp \times \vec{k}_\perp)_z}{M^2} F_{14}$ | $\frac{1}{M}\left(k_x H_{17} + \Delta_x H_{18}\right)$ | $\frac{1}{M}\left(k_y H_{17} + \Delta_y H_{18}\right)$ | $G_{14}$ |



$\vec{\Delta}_\perp = \vec{0}_\perp$

$\int \mathrm{d}^2 k_\perp$

$\vec{k}_\perp \leftrightarrow i\vec{\Delta}_\perp$

**TMDs**

| | $U$ | $T_x$ | $T_y$ | $L$ |
|---|---|---|---|---|
| $U$ | $f_1$ | $\frac{k_y}{M} h_1^\perp$ | $-\frac{k_x}{M} h_1^\perp$ | |
| $T_x$ | $\frac{k_y}{M} f_{1T}^\perp$ | $h_1 + \frac{k_x^2 - k_y^2}{2M^2} h_{1T}^\perp$ | $\frac{k_x k_y}{M^2} h_{1T}^\perp$ | $\frac{k_x}{M} g_{1T}$ |
| $T_y$ | $-\frac{k_x}{M} f_{1T}^\perp$ | $\frac{k_x k_y}{M^2} h_{1T}^\perp$ | $h_1 - \frac{k_x^2 - k_y^2}{2M^2} h_{1T}^\perp$ | $\frac{k_y}{M} g_{1T}$ |
| $L$ | ✗ | $\frac{k_x}{M} h_{1L}^\perp$ | $\frac{k_y}{M} h_{1L}^\perp$ | $g_{1L}$ |

**GPDs**

$\xi = 0$

| | $U$ | $T_x$ | $T_y$ | $L$ |
|---|---|---|---|---|
| $U$ | $H$ | $i\frac{\Delta_y}{2M}\left(2\tilde{H}_T + E_T\right)$ | $-i\frac{\Delta_x}{2M}\left(2\tilde{H}_T + E_T\right)$ | |
| $T_x$ | $i\frac{\Delta_y}{2M} E$ | $H_T - \frac{\Delta_x^2 - \Delta_y^2}{4M^2}\tilde{H}_T$ | $-\frac{\Delta_x \Delta_y}{2M^2}\tilde{H}_T$ | |
| $T_y$ | $-i\frac{\Delta_x}{2M} E$ | $-\frac{\Delta_x \Delta_y}{2M^2}\tilde{H}_T$ | $H_T + \frac{\Delta_x^2 - \Delta_y^2}{4M^2}\tilde{H}_T$ | |
| $L$ | ✗ | | | $\tilde{H}$ |

**Complete parametrizations :** **Quarks** **[Meissner, Metz, Schlegel (2009)]**

**Quarks & gluons** **[C.L., Pasquini (2013)]**

# Parametrization

**Twist-2**

**GTMDs**

Quark polarization

Nucleon polarization

| | $U$ | $T_x$ | $T_y$ | $L$ |
|---|---|---|---|---|
| $U$ | $F_{11}$ | $\frac{i}{M}\left(k_y H_{11} + \Delta_y H_{12}\right)$ | $-\frac{i}{M}\left(k_x H_{11} + \Delta_x H_{12}\right)$ | $\frac{i(\vec{\Delta}_\perp \times \vec{k}_\perp)_z}{M^2} G_{11}$ |
| $T_x$ | $\frac{i}{M}\left(k_y F_{12} + \Delta_y (F_{13} - \frac{1}{2} F_{11})\right)$ | $\cdots$ | $\cdots$ | $\cdots$ |
| $T_y$ | $-\frac{i}{M}\left(k_x F_{12} + \Delta_x (F_{13} - \frac{1}{2} F_{11})\right)$ | $\cdots$ | $\cdots$ | $\cdots$ |
| $L$ | $-\frac{i(\vec{\Delta}_\perp \times \vec{k}_\perp)_z}{M^2} F_{14}$ | $\frac{1}{M}\left(k_x H_{17} + \Delta_x H_{18}\right)$ | $\frac{1}{M}\left(k_y H_{17} + \Delta_y H_{18}\right)$ | $G_{14}$ |

$\vec{\Delta}_\perp = \vec{0}_\perp$

$\int d^2 k_\perp$

$\vec{k}_\perp \leftrightarrow i\vec{\Delta}_\perp$

**TMDs**

| | $U$ | $T_x$ | $T_y$ | $L$ |
|---|---|---|---|---|
| $U$ | $f_1$ | $\frac{k_y}{M} h_1^\perp$ | $-\frac{k_x}{M} h_1^\perp$ | ✗ |
| $T_x$ | $\frac{k_y}{M} f_{1T}^\perp$ | $h_1 + \frac{k_x^2 - k_y^2}{2M^2} h_{1T}^\perp$ | $\frac{k_x k_y}{M^2} h_{1T}^\perp$ | $\frac{k_x}{M} g_{1T}$ |
| $T_y$ | $-\frac{k_x}{M} f_{1T}^\perp$ | $\frac{k_x k_y}{M^2} h_{1T}^\perp$ | $h_1 - \frac{k_x^2 - k_y^2}{2M^2} h_{1T}^\perp$ | $\frac{k_y}{M} g_{1T}$ |
| $L$ | | $\frac{k_x}{M} h_{1L}^\perp$ | $\frac{k_y}{M} h_{1L}^\perp$ | $g_{1L}$ |

**GPDs** $\xi = 0$

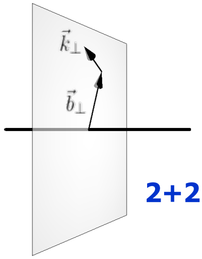
| | $U$ | $T_x$ | $T_y$ | $L$ |
|---|---|---|---|---|
| $U$ | $H$ | $i\,\frac{\Delta_y}{2M}\,(2\tilde{H}_T + E_T)$ | $-i\,\frac{\Delta_x}{2M}\,(2\tilde{H}_T + E_T)$ | ✗ |
| $T_x$ | $i\,\frac{\Delta_y}{2M}\,E$ | $H_T - \frac{\Delta_x^2 - \Delta_y^2}{4M^2}\,\tilde{H}_T$ | $-\frac{\Delta_x \Delta_y}{2M^2}\,\tilde{H}_T$ | |
| $T_y$ | $-i\,\frac{\Delta_x}{2M}\,E$ | $-\frac{\Delta_x \Delta_y}{2M^2}\,\tilde{H}_T$ | $H_T + \frac{\Delta_x^2 - \Delta_y^2}{4M^2}\,\tilde{H}_T$ | |
| $L$ | | | | $\tilde{H}$ |

**Complete parametrizations :** **Quarks** **[Meissner, Metz, Schlegel (2009)]**

**Quarks & gluons** **[C.L., Pasquini (2013)]**

# Model results

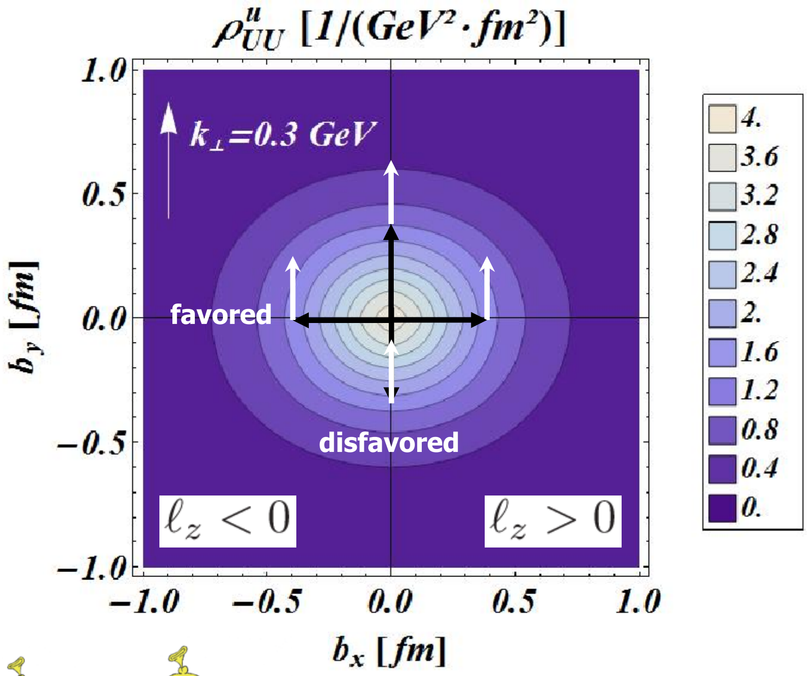
$$\int \mathrm{d}x\, \rho_W(x, \vec{k}_\perp, \vec{b}_\perp)$$

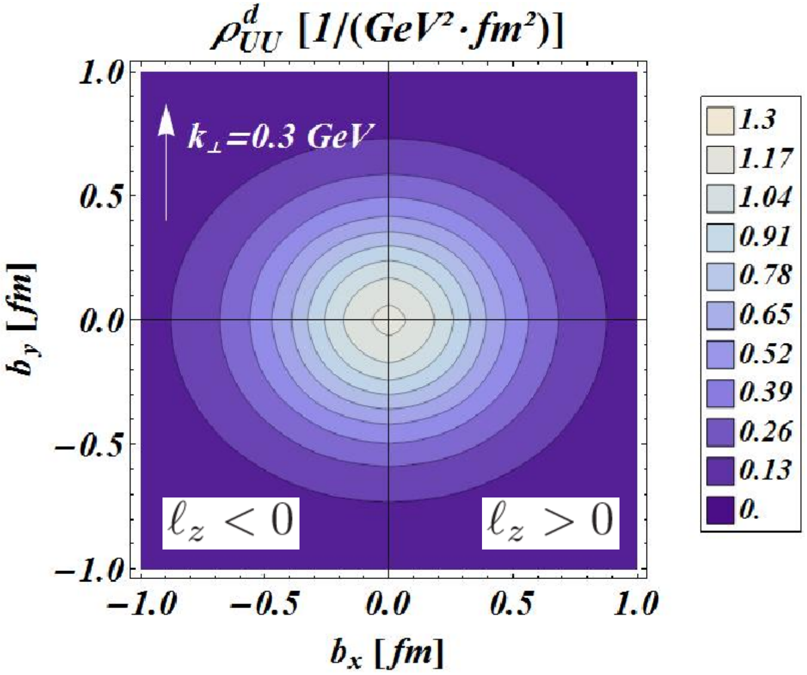


**2+2D**

**Wigner distribution of unpolarized quark in unpolarized nucleon**

$$\boxed{\rho_{UU} = \frac{1}{2}\left(\rho^{[\gamma^+]}_{++} + \rho^{[\gamma^+]}_{--}\right) \propto F_{11}}$$



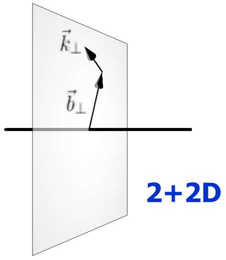


**Left-right symmetry** → $\boxed{\ell_z^{\text{tot}} = 0}$

**[C.L., Pasquini (2011)]**

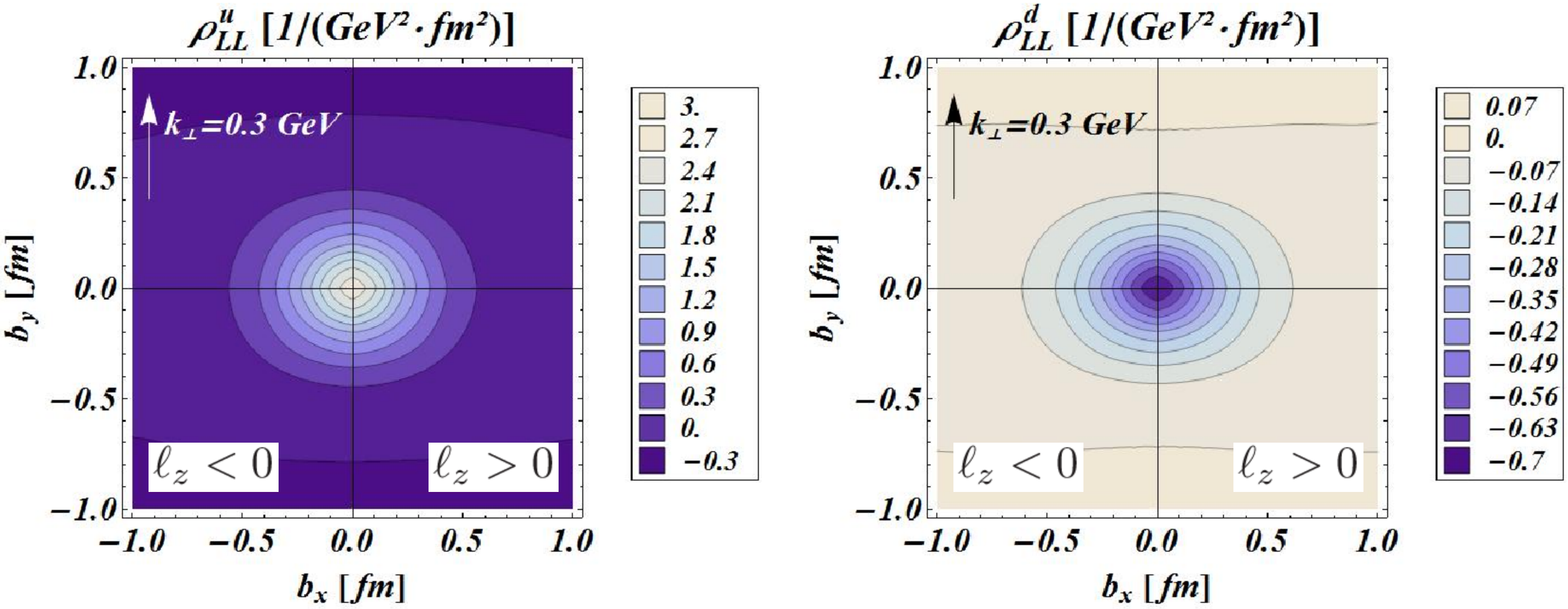
# Model results

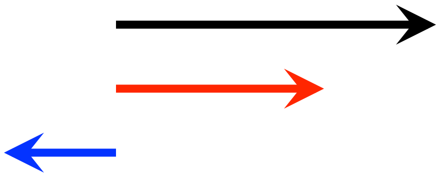
$\int \mathrm{d}x\, \rho_W(x, \vec{k}_\perp, \vec{b}_\perp)$



**Quark spin-nucleon spin correlation**

$$\boxed{\rho_{LL} = \frac{1}{2}\left(\rho^{[\gamma^+\gamma_5]}_{++} - \rho^{[\gamma^+\gamma_5]}_{--}\right) \propto G_{14}}$$





**Proton spin**

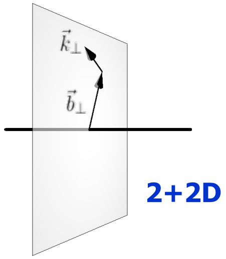
***u*-quark spin**

***d*-quark spin**

[C.L., Pasquini (2011)]

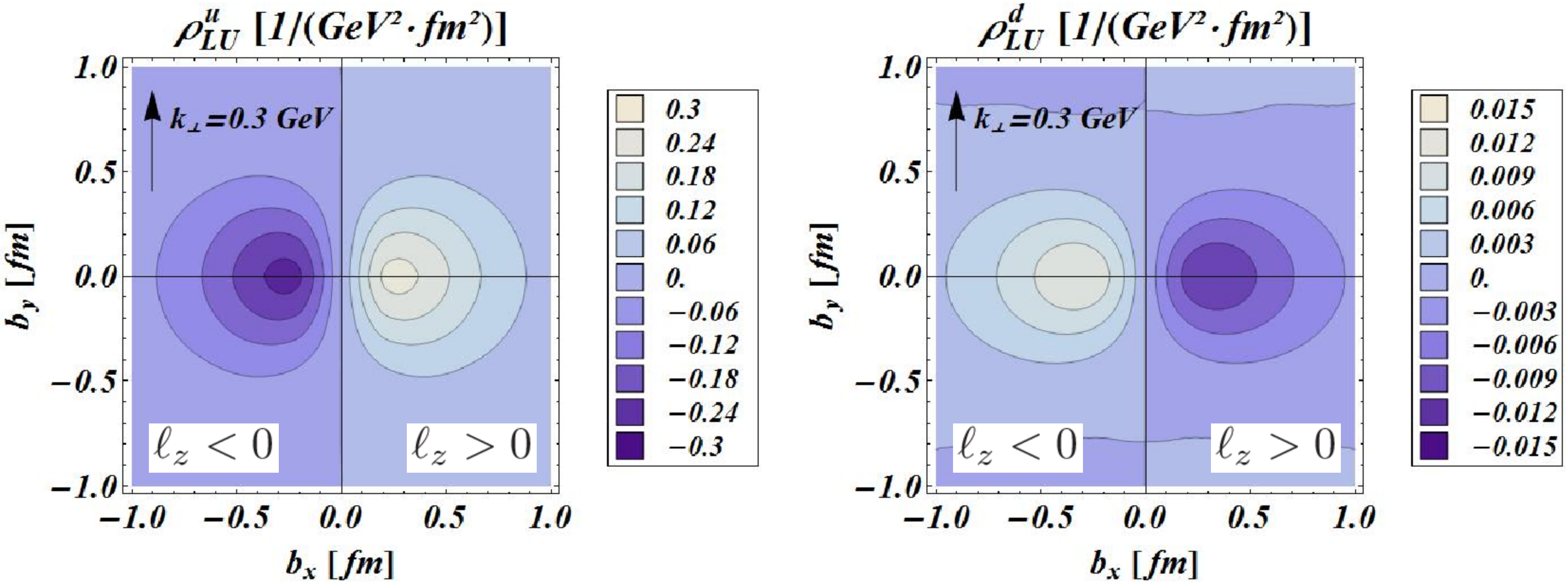
# Model results

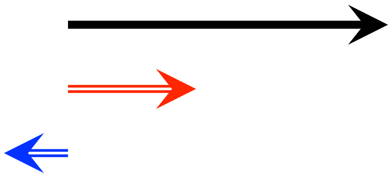
$$\int \mathrm{d}x\, \rho_W(x, \vec{k}_\perp, \vec{b}_\perp)$$



**Distortion correlated to nucleon spin**

$$\boxed{\rho_{LU} = \frac{1}{2}\left(\rho^{[\gamma^+]}_{++} - \rho^{[\gamma^+]}_{--}\right) \propto F_{14}}$$





**Proton spin**

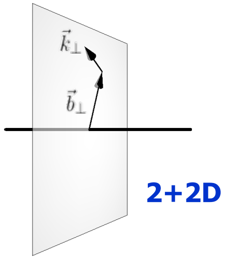
***u*-quark OAM**

***d*-quark OAM**

**[C.L., Pasquini (2011)]**

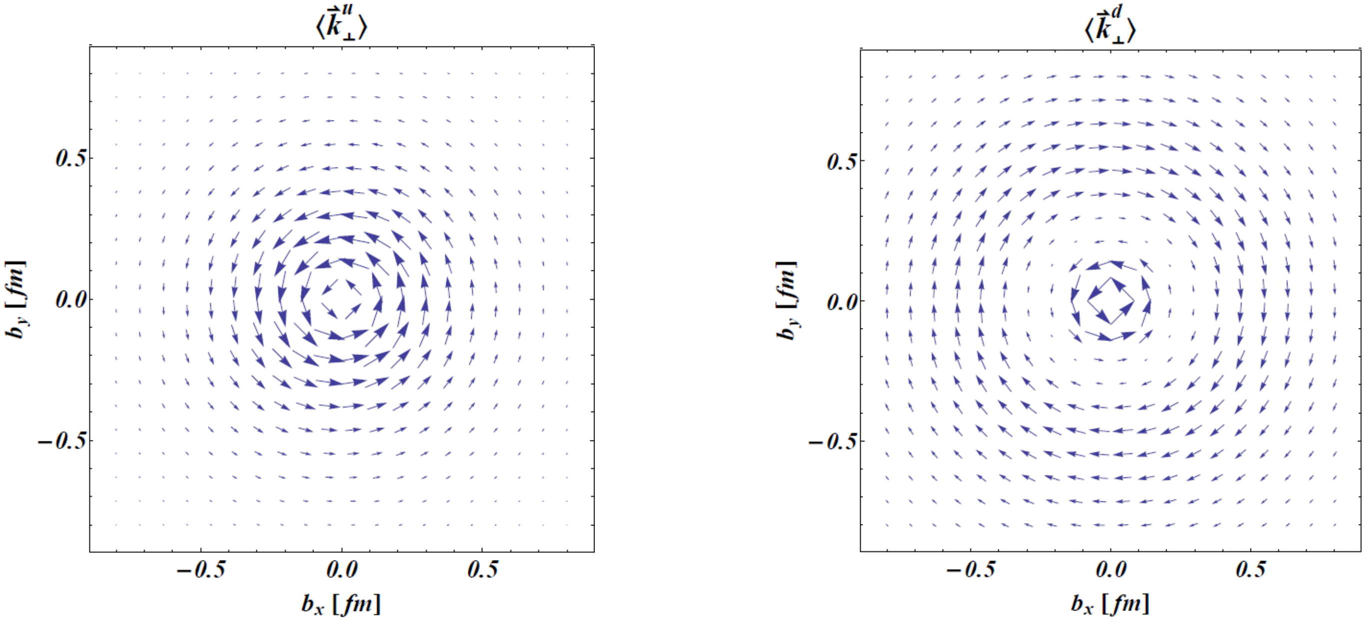
# Model results

$$\int \mathrm{d}x\, \rho_W(x, \vec{k}_\perp, \vec{b}_\perp)$$



**Average transverse quark momentum correlated to nucleon spin**

$$\boxed{\langle \vec{k}_\perp \rangle(\vec{b}_\perp) = \int \mathrm{d}x\, \mathrm{d}^2k_\perp\, \vec{k}_\perp\, \rho_{LU}(x, \vec{k}_\perp, \vec{b}_\perp)}$$



$$\boxed{\begin{aligned} \ell_z &= \int \mathrm{d}^2b_\perp\, \vec{b}_\perp \times \langle \vec{k}_\perp \rangle(\vec{b}_\perp) \\ &= -\int \mathrm{d}x\, \mathrm{d}^2k_\perp\, \frac{\vec{k}_\perp^2}{M^2}\, F_{14}(x, 0, \vec{k}_\perp, \vec{0}_\perp) \end{aligned}}$$

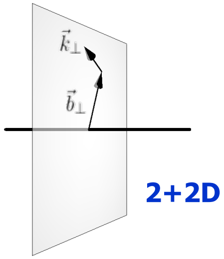
$F_{14}$

**« Vorticity »**

**[C.L., Pasquini, Xiong, Yuan (2012)]**

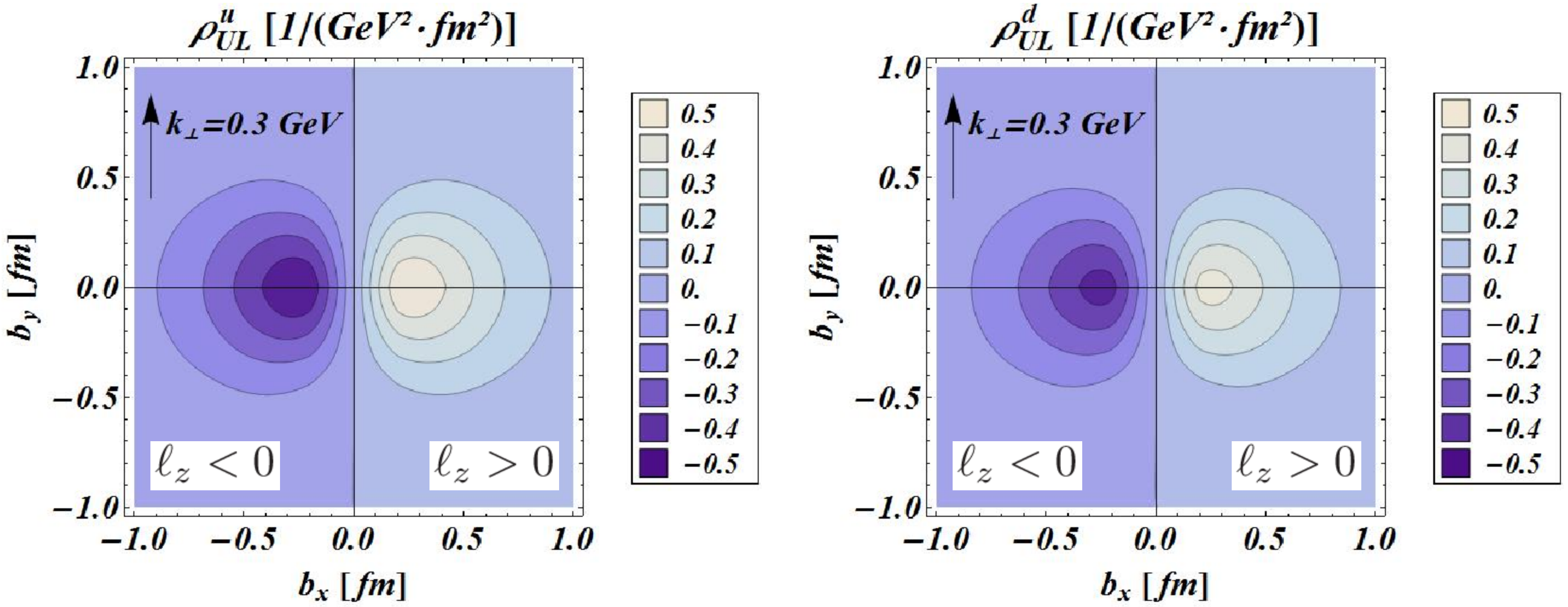
# Model results

$$\int \mathrm{d}x\, \rho_W(x, \vec{k}_\perp, \vec{b}_\perp)$$



**Distortion correlated to quark spin**

$$\boxed{\rho_{UL} = \frac{1}{2}\left(\rho_{++}^{[\gamma^+\gamma_5]} + \rho_{--}^{[\gamma^+\gamma_5]}\right) \propto G_{11}}$$



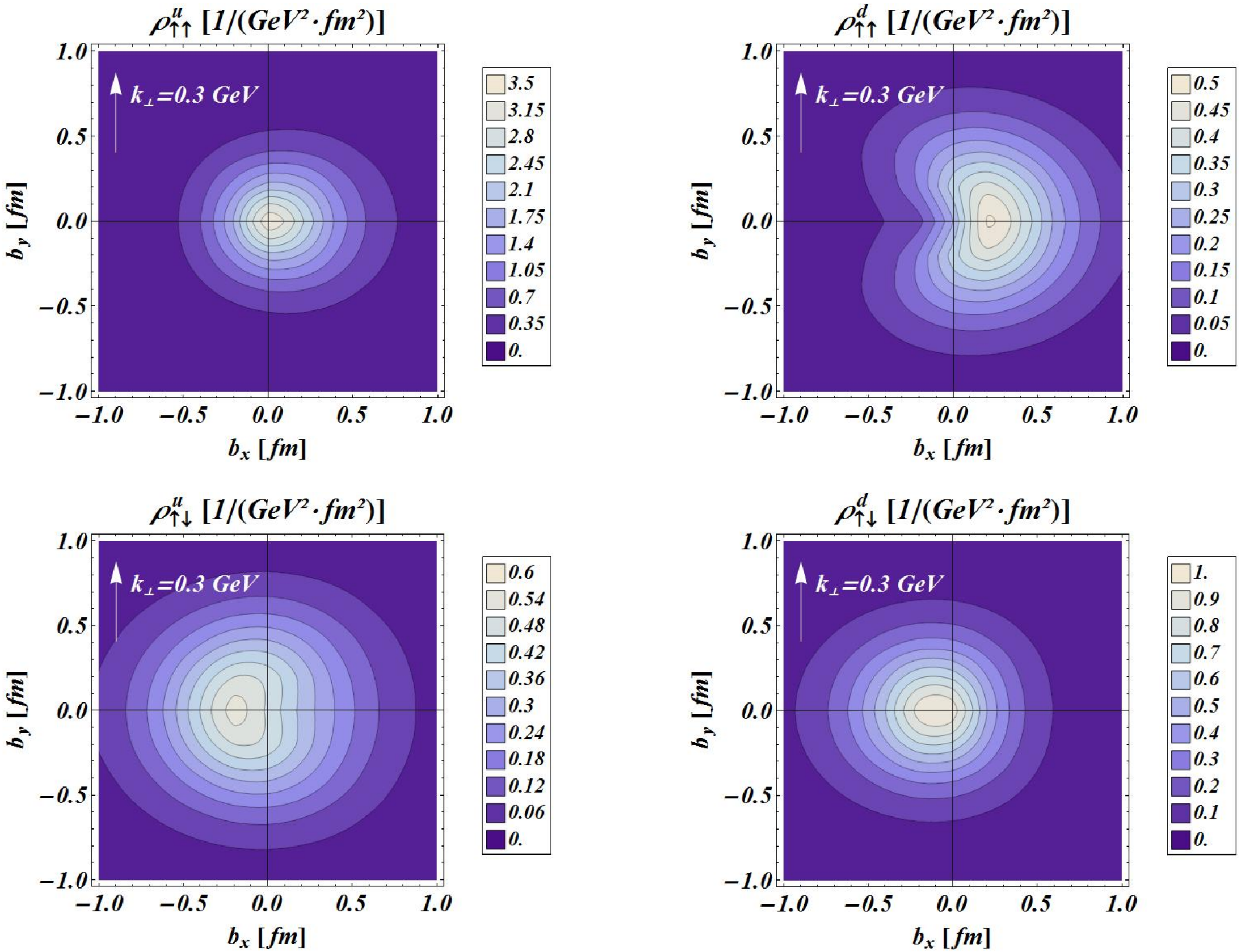
Quark spin

*u*-quark OAM

*d*-quark OAM

[C.L., Pasquini (2011)]

# Model results



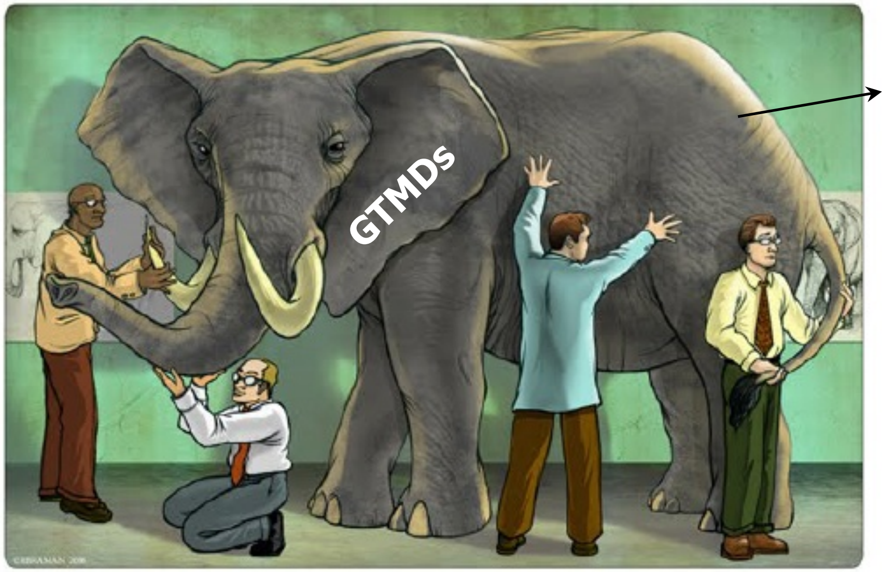
**[C.L., Pasquini (2011)]**

# Summary

## Lecture 3

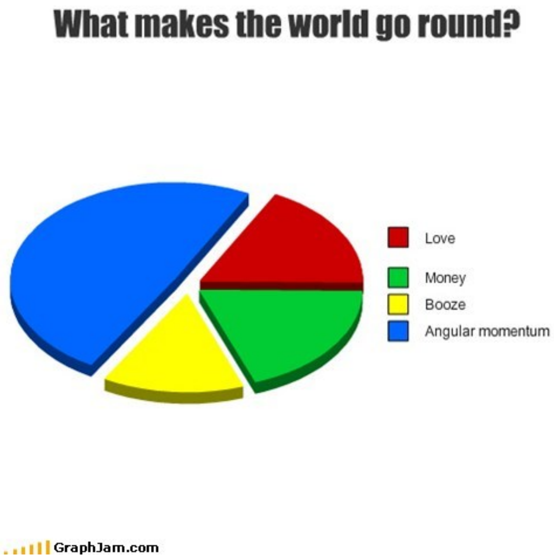
- TMDs provide complementary 3D pictures of the nucleon
- All distributions can be seen as overlaps of light-front wave functions
- Models constrained by data give access to Wigner distributions

**Nucleon**



**GPDs** **TMDs** **FFs** **PDFs**

**LFWFs**



# Backup slides

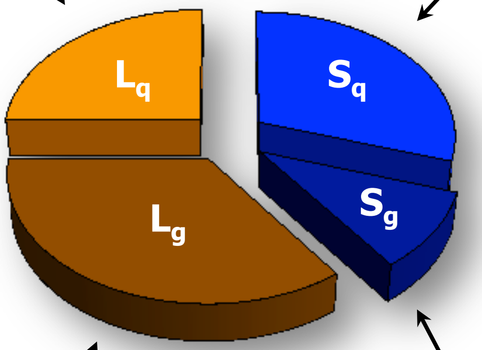
# Spin decompositions in a nutshell

## Canonical

$$\vec{p} = \frac{\partial L}{\partial \vec{v}}$$

$$\vec{\mathcal{L}}_q = \int \mathrm{d}^3r\, \psi^\dagger \vec{r} \times (-i\vec{\nabla})\psi$$

$$\vec{S}_q = \frac{1}{2} \int \mathrm{d}^3r\, \psi^\dagger \vec{\Sigma} \psi$$



$$\vec{\mathcal{L}}_g = \int \mathrm{d}^3r\, E^{ai} \vec{r} \times \vec{\nabla} A^{ai}$$

$$\vec{S}_g = \int \mathrm{d}^3r\, \vec{E}^a \times \vec{A}^a$$

**Gauge non-invariant !**

**[Jaffe, Manohar (1990)]**

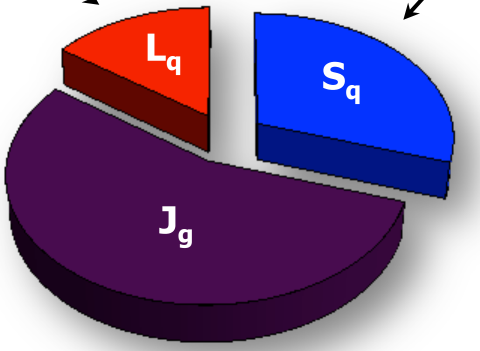
## Kinetic

$$\vec{\pi} = m\vec{v} = \vec{p} + g\vec{A}$$

$$\boxed{\vec{D} = -\vec{\nabla} - ig\vec{A}}$$

$$\vec{L}_q = \int \mathrm{d}^3r\, \psi^\dagger \vec{r} \times (i\vec{D})\psi$$

$$\vec{S}_q = \frac{1}{2} \int \mathrm{d}^3r\, \psi^\dagger \vec{\Sigma} \psi$$



$$\vec{J}_g = \int \mathrm{d}^3r\, \vec{r} \times (\vec{E}^a \times \vec{B}^a)$$

**« Incomplete »**

**[Ji (1997)]**

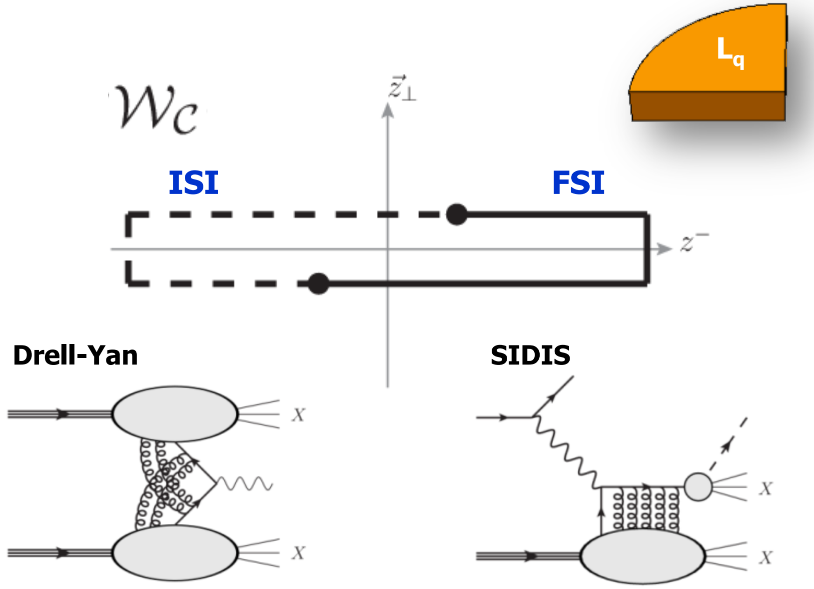
# OAM and path dependence

[Ji, Xiong, Yuan (2012)]
[Hatta (2012)]
[C.L. (2013)]

**Quark generalized OAM operator**

$$\boxed{L^{\mu\nu\rho}_{q,\mathcal{C}}(x)=\overline{\psi}(x)\gamma^\mu\left(x^\nu\,\tfrac{i}{2}\overleftrightarrow{D}^\rho_{\text{pure}}-x^\rho\,\tfrac{i}{2}\overleftrightarrow{D}^\nu_{\text{pure}}\right)\psi(x)}$$

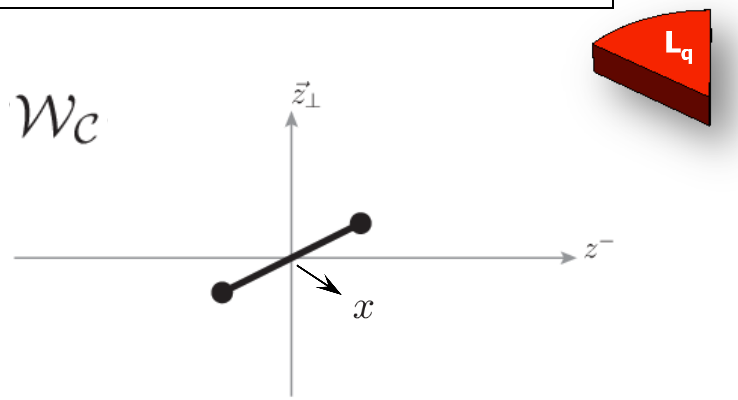
## Light-front



$$D^{\text{pure}}_\mu \overset{A^+=0}{=} \partial_\mu \qquad \boxed{\ell^{\text{DY}}_z=\ell^{\text{SIDIS}}_z}$$

**Naive T-even**

## *x*-based Fock-Schwinger



**Coincides *locally* with kinetic quark OAM**

$$A_\mu(x)=A^{\text{pure}}_\mu(x)$$

$$A_\mu(y)\neq A^{\text{pure}}_\mu(y)\qquad y\neq x$$

$$L^{\mu\nu\rho}_q(x)=\overline{\psi}(x)\gamma^\mu(x^\nu iD^\rho-x^\rho iD^\nu)\psi(x)$$

# Kinetic *vs* canonical OAM

## Kinetic OAM (Ji)

$$L_z = \underbrace{\frac{1}{2}\int \mathrm{d}x\, x[H(x,0,0)+E(x,0,0)]}_{J_z} - \underbrace{\frac{1}{2}\int \mathrm{d}x \tilde{H}(x,0,0)}_{S_z}$$

$$= -\underbrace{\int \mathrm{d}x\, x\, G_2(x,0,0)}_{\textbf{Pure twist-3}} = \int \mathrm{d}x\, x\,[H(x,0,0)+E(x,0,0)+\tilde{E}_{2T}(x,0,0)]$$

## Quark *naive* canonical OAM (Jaffe-Manohar)

**[C.L., Pasquini, PLB710 (2012) 486]**

$$\mathcal{L}_z = -\int \mathrm{d}x\, \mathrm{d}^2k_\perp \frac{\vec{k}_\perp^2}{2M^2}\, h_{1T}^\perp(x,\vec{k}_\perp)$$

⚠ **Model-dependent !**

## Canonical OAM (Jaffe-Manohar)

**[C.L., Pasquini, PRD84 (2011) 014015]**
**[C.L., Pasquini, Xiong, Yuan, PRD85 (2012) 114006]**
**[Kanazawa, C.L., Metz, Pasquini, Schlegel, PRD (2014)]**

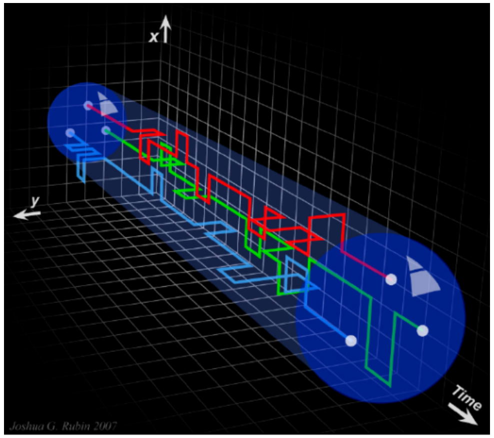
$$\ell_z = -\int \mathrm{d}x\, \mathrm{d}^2k_\perp \frac{\vec{k}_\perp^2}{M^2}\, F_{14}(x,0,\vec{k}_\perp,\vec{0}_\perp)$$

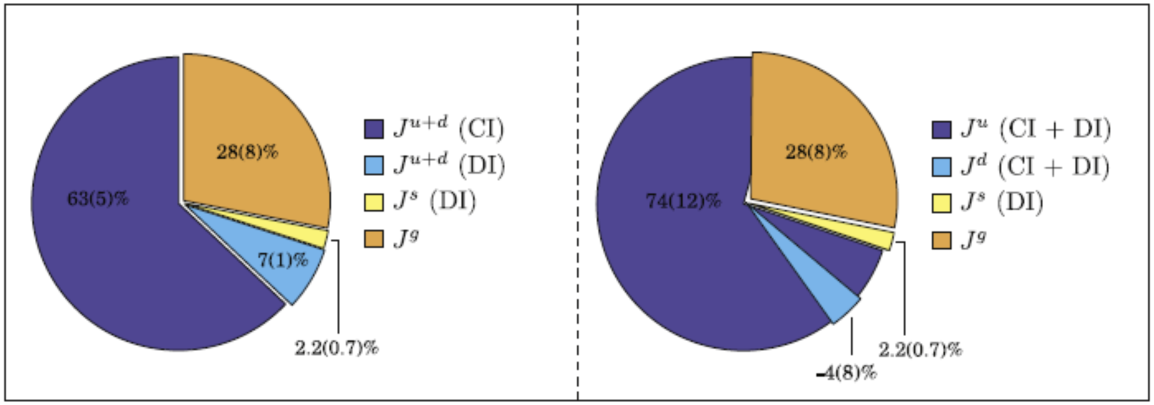
| Model | LCCQM | | | $\chi$QSM | | |
|---|---|---|---|---|---|---|
| $q$ | $u$ | $d$ | Total | $u$ | $d$ | Total |
| $\ell_z^q$ | 0.131 | −0.005 | 0.126 | 0.073 | −0.004 | 0.069 |
| $L_z^q$ | 0.071 | 0.055 | 0.126 | −0.008 | 0.077 | 0.069 |
| $\mathcal{L}_z^q$ | 0.169 | −0.042 | 0.126 | 0.093 | −0.023 | 0.069 |

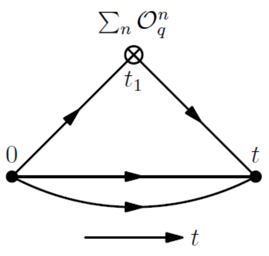
⚠ **No gluons and not QCD EOM !**

$\ell_z = L_z$ **but** $\ell_z^q \neq L_z^q$

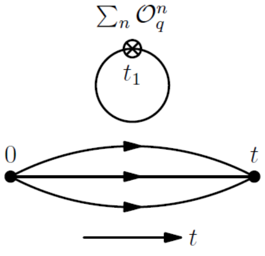
# Lattice results



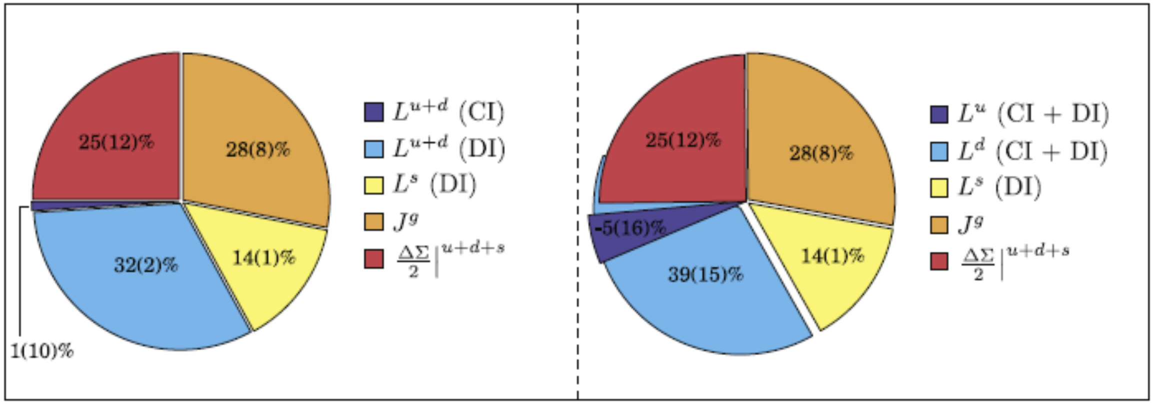




**CI**



**DI**



**[Deka *et al.* (2013)]**