Testing the Standard Model with radiative B decays

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Some results in this talk are not yet approved by Babar; these results represent my personal work and are marked 'not yet approved'

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Les deux infinis

CENTRE DE PHYSIQUE DES PARTICULES DE MARSEILLE
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PRESENTATION OVERVIEW

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The Flavor Sector:

Complementarity to Direct Searches for New Physics

The Standard Model and the Flavor Sector

$$\mathcal{L}_{\rm SM} = \mathcal{L}_{\rm gauge}(A_{\rm a}, \psi_{\rm i}) + \mathcal{L}_{\rm Higgs}(\phi, A_{\rm a}, \psi_{\rm i})$$

- Experimentally tested with high accuracy
- Global flavor symmetry
- Stable with respect to quantum corrections

- Weakly tested in its dynamical form
- Origin of the flavor structure of the model
- Not stable with respect to quantum corrections

- Within the SM the flavor-degeneracy is broken only by the Yukawa interaction
- Quarks from different flavors are coupled through the complex matrix V_{CKM} (detailed in a few slides)

The Standard Model as an Effective Theory

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm gauge}(A_{\rm a}, \psi_{\rm i}) + \mathcal{L}_{\rm Higgs}(\phi, A_{\rm a}, \psi_{\rm i}) + \sum_{\rm d \geq 5} \frac{c_{\rm n}}{\Lambda^{d-4}} \mathcal{O}_{\rm n}^{\rm (d)}(\phi, A_{\rm a}, \psi_{\rm i})$$

Arguments from:

- **theory:**
  - inclusion of gravity
  - instability of the Higgs potential
  - neutrino masses
  - origin of flavor
  - ...
- **cosmological observations:**
  - dark matter
  - inflation
  - cosmological constant
  - ...
- **hierarchical structure of quark and lepton masses**

⇓

- **The current point of view on the SM Lagrangian: a low-energy limit of a more complete theory (effective theory)**
- **New degrees of freedom are expected at a scale Λ above the electroweak scale**

The Standard Model as an Effective Theory

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm gauge}(A_{\rm a}, \psi_{\rm i}) + \mathcal{L}_{\rm Higgs}(\phi, A_{\rm a}, \psi_{\rm i}) + \sum_{\rm d \geq 5} \frac{c_{\rm n}}{\Lambda^{d-4}} \mathcal{O}_{\rm n}^{\rm (d)}(\phi, A_{\rm a}, \psi_{\rm i})$$

$\mathcal{L}_{\rm SM}$: **renormalizable part of** $\mathcal{L}_{\rm eff}$

(i.e. all possible operators with d ≤ 4 compatible with the gauge symmetry)

Operators of d ≥ 5 containing SM fields only and compatible with the SM gauge symmetry

Two major questions of particle physics today:

- **Which is the energy scale of New Physics** (or the value of Λ) ⟺ **High-energy experiments** [the high-energy frontier]
- **Which is the symmetry structure of the new degrees of freedom** (or the structure of the c_n) ⟺ **High-precision low-energy exp.** [the high-intensity frontier]

General "Recipe" to Probe the Flavor Structure of Physics Beyond the SM

1 **From theoretically clean and non-suppressed tree-level processes**: Determine the CKM elements where the SM is likely to be largely dominant ⇔

- exclusive and inclusive semi-leptonic b→u decays

2 **Identify as many as possible processes** where the SM is calculable with good accuracy using the tree-level inputs, or suppressed for null tests. ⇔

- Neutral meson mixing (K, $B_{(d,s)}$, D)
- Penguin modes
- CP-violating observables
- Forbidden processes
- Helicity-suppressed observables
- …

3 Measure with good accuracy these rare processes and determine the allowed room for new physics

eventual extra step… →

…which is likely to be attributed to the theorist

General "Recipe" to Probe the Flavor Structure of Physics Beyond the SM

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1 From theoretically clean and non- ... d inclusive
sup ... c b→u
Det
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2

3

Here we will focus on radiative B decays and the photon polarization as a probe for potential NP contributions

Many experimental methods to probe the photon polarization exist:

- **CP-violation parameters in $B^0_{(s)} \to f_{CP}\,\gamma$:**
  - $B^0 \to K_S\,\pi^0\,\gamma$
  - $B^0 \to K_S\,\rho^0\,\gamma$ *Presented in detail today*
  - $B^0_s \to \phi\gamma$
- **Angular analyses:**
  - $B^+ \to K^+_{res}\,\gamma \to K^+\pi\pi\gamma$
  - $B^0 \to K^{*0}(\to K^-\pi^+)\ell^+\ell^-$

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Phenomenological Context and Experimental Status

(1) CP violation in the Standard Model
(2) The time-dependent CP asymmetry
(3) Radiative B decays and the photon polarization
(4) Physical observables and experimental status

CP Violation
Origin in the SM

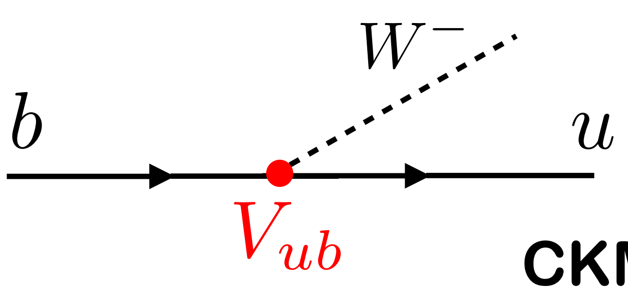
- **The CKM Formalism and CP Violation:**
  - In the Quark sector: Weak interaction eigenstates ≠ Flavor eigenstates
  - Existence of 3X3 unitary matrix describing the mixing of quarks: the CKM Matrix

$$V_{\mathrm{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \simeq \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & \lambda^3 A\left(\rho - i\eta\right) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & \lambda^2 A \\ \lambda^3 A\left(1-\rho-i\eta\right) & -\lambda^2 A & 1 \end{pmatrix} + \mathcal{O}\left(\lambda^4\right)$$

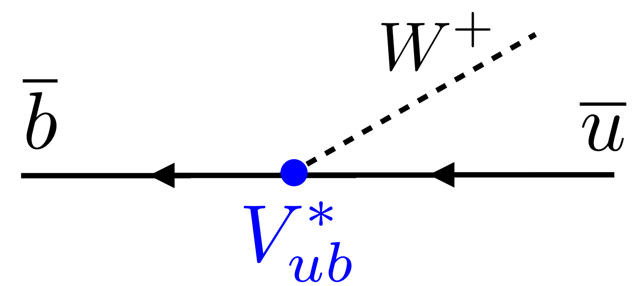
In the SM, CP violation originates from the presence an irreducible phase

(Only possible in the case of at least 3 generations and η ≠ 0)

Transition amplitude between b and u quarks



Transition amplitude between anti-b and anti-u quarks



CKM phase ⇒ $V^*_{ub} \neq V_{ub}$

Different behavior of matter and anti-matter

- Actually this single amplitude cannot give observable CP violation
- Must have a sum of amplitudes ⇒ contribution from a few processes

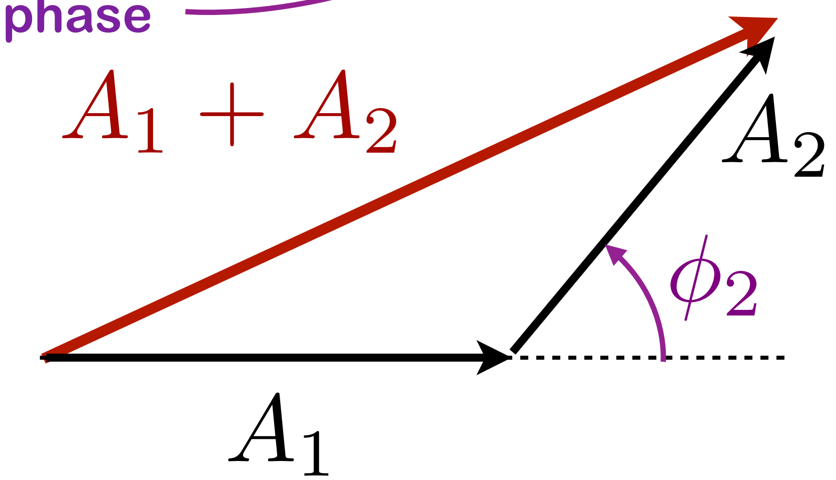
- **Two amplitudes with a relative phase changing under CP:**

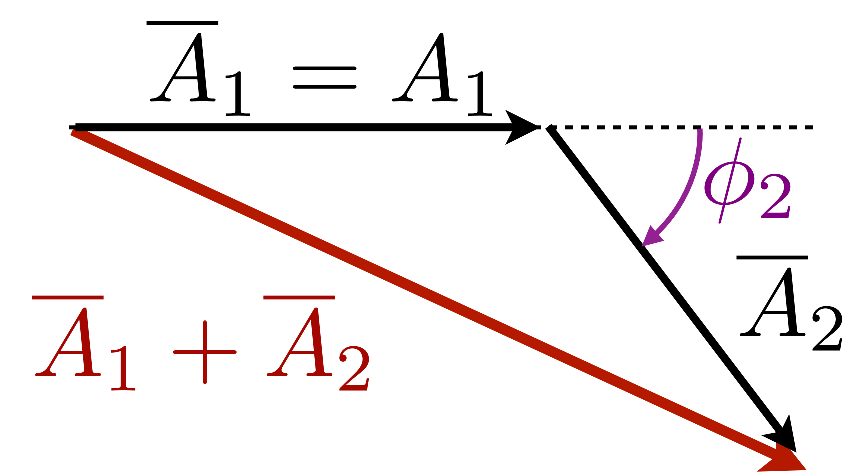
$$A = \langle \text{final} | \text{H} | \text{initial} \rangle$$
$$= A_1 + a_2 e^{+i\phi_2}$$

CP-violating, "weak" or CP-odd phase

CP

$$\overline{A} = \overline{A}_1 + a_2 e^{-i\phi_2}$$





$$|A_1 + A_2|^2 = |\overline{A}_1 + \overline{A}_2|^2 \Rightarrow \Gamma(i \to f) = \Gamma(\bar{i} \to \bar{f})$$

No CP asymmetry!!

CP Violation
Amplitudes and Observables

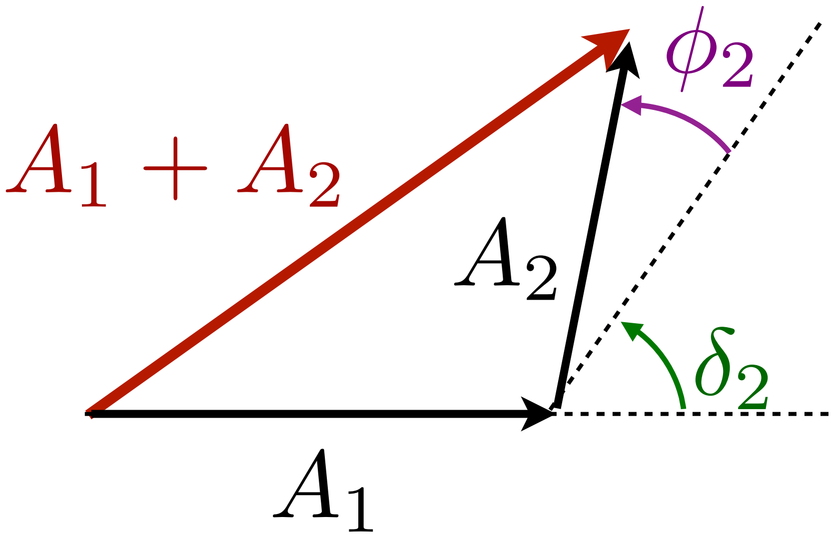
- **Two amplitudes with a phase changing under CP and a CP-conserving phase:**

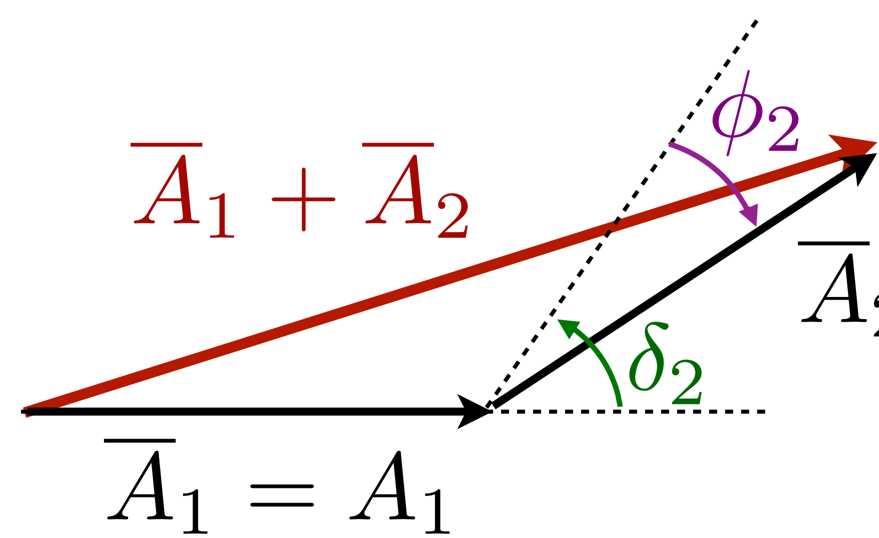
$$A = A_1 + a_2 e^{i(\phi_2 + \delta_2)}$$

CP-conserving, "strong" or CP-even phase

CP

$$\overline{A} = \overline{A}_1 + a_2 e^{i(-\phi_2 + \delta_2)}$$





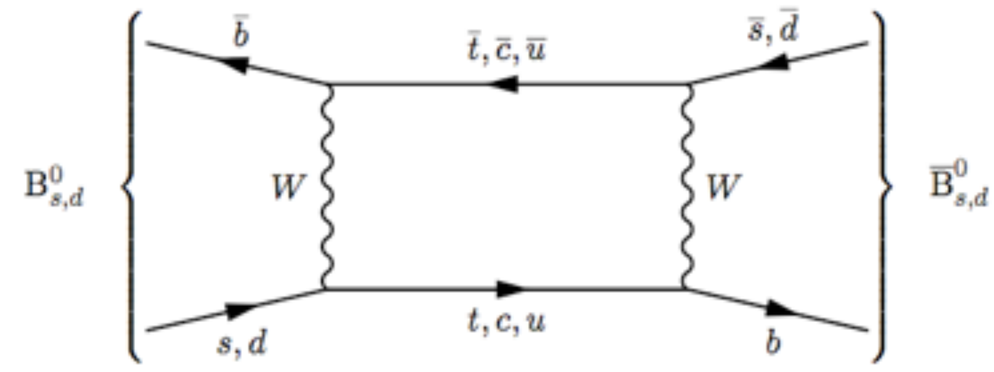
$$|A|^2 \neq |\overline{A}|^2 \Rightarrow \Gamma(i \to f) \neq \Gamma(\bar{i} \to \bar{f})$$

$$\mathcal{A}_{CP} = \frac{\Gamma(\bar{i} \to \bar{f}) - \Gamma(i \to f)}{\Gamma(\bar{i} \to \bar{f}) + \Gamma(i \to f)} \neq 0$$

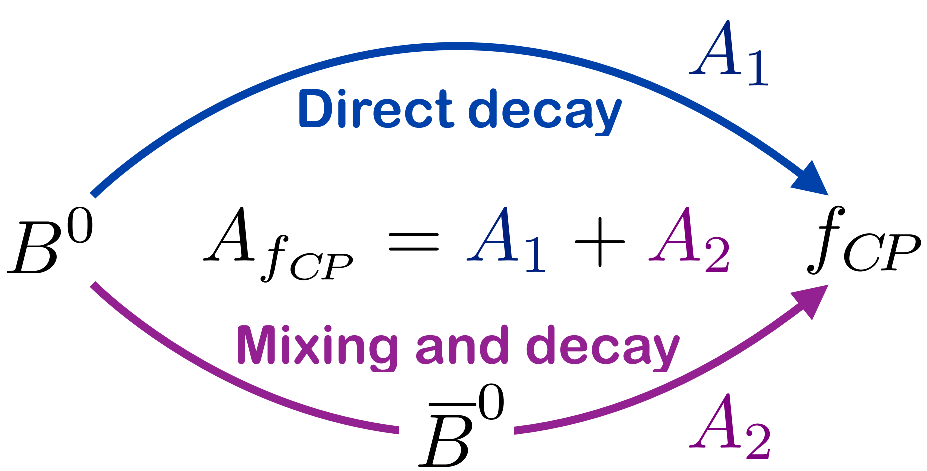
Now have a CP asymmetry!!

TIME-DEPENDENT CP ASYMMETRY

- **CP violation in the interference between decay and mixing:**
  - Need a final state accessible to both B^0 and $\overline{B}^0$: satisfied if a CP eigenstate

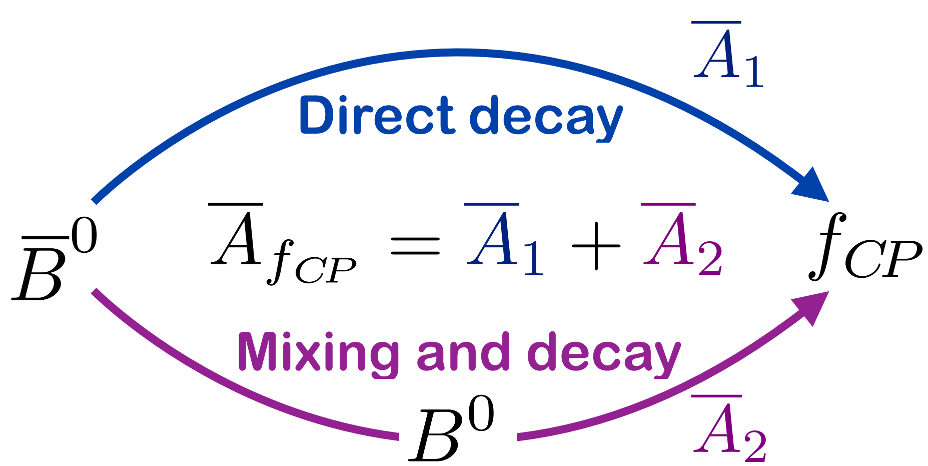


$$\Gamma(B^0 \to f) \neq \Gamma(\overline{B}^0 \to f)$$



$$\mathcal{A}_{CP}(\Delta t) \equiv \frac{\Gamma(\overline{B}^0(\Delta t) \to f_{CP}) - \Gamma(B^0(\Delta t) \to f_{CP})}{\Gamma(\overline{B}^0(\Delta t) \to f_{CP}) + \Gamma(B^0(\Delta t) \to f_{CP})}$$
$$= \mathcal{S} \sin(\Delta m_d \Delta t) - \mathcal{C} \cos(\Delta m_d \Delta t)$$

B_d states mass difference, oscillation frequency



$$\mathcal{S} = \frac{2\Im(\lambda_{f_{CP}})}{1 + |\lambda_{f_{CP}}|^2} \longrightarrow$$ CP in interference between decay and mixing

$$\mathcal{C} = \frac{1 - |\lambda_{f_{CP}}|^2}{1 + |\lambda_{f_{CP}}|^2} \longrightarrow$$ CP in decay, or direct CP

$$\lambda_{f_{CP}} = \frac{q}{p} \frac{\overline{A}_{f_{CP}}}{A_{f_{CP}}}$$

Radiative B Decays & The Photon Polarization

- **Radiative decays b → sγ (FCNC):**

  In SM interaction between left-handed quarks or right-handed antiquarks

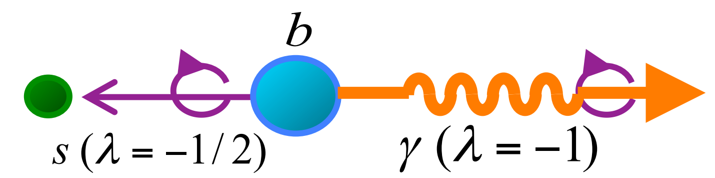
- **Effective Hamiltonian of the process:**

$$\mathrm{H}_{\mathrm{eff}} = -\frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{i=1}^{8} C_i(\mu)\mathcal{O}_i(\mu) + C_i'(\mu)\mathcal{O}_i'(\mu)$$

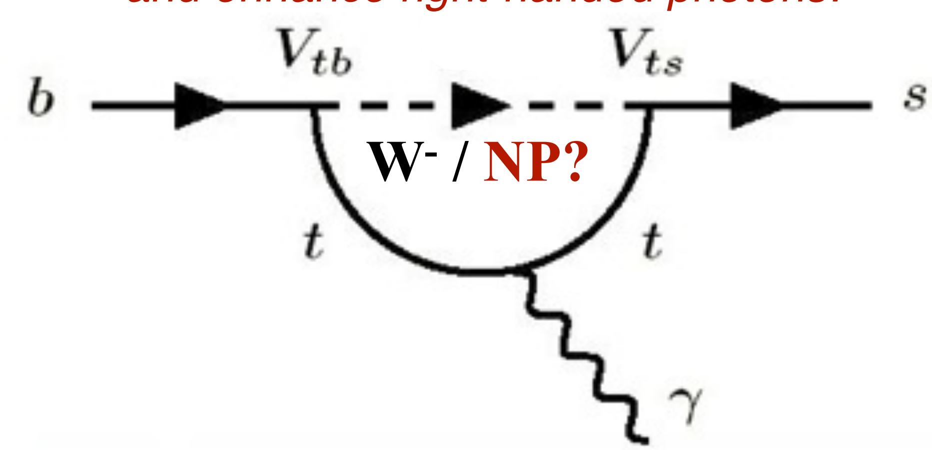
$\mathcal{O}_{7\gamma}$: dominant operator related to photon penguin diagram

In SM: primed operators related to $b \rightarrow s\gamma_R$ transitions with $i \leq 6$ are predicted to have no contribution

Helicity: spin projection on the momentum of a particle $\lambda = \frac{\vec{S}\cdot\vec{p}}{|\vec{p}|}$



B meson radiative decay,
NP particle may be present in the loop, and enhance right-handed photons:



- **Transition amplitude of b → sγ process:**

$$\mathcal{M}(b \rightarrow s\gamma)_{\mathrm{LO}} = \langle f|\mathrm{H}_{\mathrm{eff}}|i\rangle = -\frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \left(C_{7\gamma}^{(0)\mathrm{eff}}(\mu)\,\langle f|\mathcal{O}_{7\gamma}|i\rangle + C_{7\gamma}'^{(0)\mathrm{eff}}(\mu)\,\langle f|\mathcal{O}_{7\gamma}'|i\rangle \right)$$

$$\left|C_{7\gamma}^{(0)\mathrm{eff}}(\mu = m_b)\right| \approx 0.33\ , \qquad \frac{C_{7\gamma}'}{C_{7\gamma}} = \frac{m_s}{m_b} \sim 0.02$$

- **Mixing-induced CP asymmetry:**

$\xi \equiv CP(f_{CP}) = \pm 1$

$\phi_M = 2\beta$ for B_d^0

$\phi_M \simeq 0$ for B_s^0

$\phi_{L,R}$: *relative CP-odd weak phases in the $b \to s\gamma$ process. (~ 0 in SM)*

- $$\mathcal{A}_{CP}(t) = \frac{\Gamma(\overline{B}^0(t) \to f_{CP}\gamma) - \Gamma(B^0(t) \to f_{CP}\gamma)}{\Gamma(\overline{B}^0(t) \to f_{CP}\gamma) + \Gamma(B^0(t) \to f_{CP}\gamma)}$$

$\mathcal{C}_{f_{CP}} \simeq 0 \rightarrow$ $$\simeq -\xi \sin(2\psi) \sin(\phi_M - \phi_L - \phi_R) \sin(\Delta M t)$$
$$= \mathcal{S}_{f_{CP}} \sin(\Delta M t)$$

- $$\Gamma(B^0(t) \to f_{CP}\gamma) = |\mathcal{M}_L(t)|^2 + |\mathcal{M}_R(t)|^2 \,,$$
$$\Gamma(\overline{B}^0(t) \to f_{CP}\gamma) = |\overline{\mathcal{M}}_L(t)|^2 + |\overline{\mathcal{M}}_R(t)|^2$$

No interference between amplitudes of left- and right-handed transitions

- $$\sin(2\psi) \equiv \frac{2|\mathcal{M}_L \mathcal{M}_R|}{|\mathcal{M}_L|^2 + |\mathcal{M}_R|^2}$$

Parametrizes the relative amount of left- and right-handed photons $\rightarrow$ $\dfrac{\mathcal{M}_R}{\mathcal{M}_L} \propto \dfrac{|C'^{\text{eff}}_{7\gamma}|}{|C^{\text{eff}}_{7\gamma}|}$

$$\mathcal{M}_R^{\text{SM}} \simeq 0 \quad \rightarrow \quad \mathcal{A}_{CP}^{\text{SM}}(t) \sim 0$$

$\mathcal{A}_{CP}(t) \neq 0$:

- 😃 **Immediately indicate the existence of NP contributions**
- 😕 **Difficult to disentangle contributions from $\mathcal{M}_R/\mathcal{M}_L$ and the CP violating phases**

Physical Observables and Experimental Status

- **Mixing-induced CP asymmetry:**

Non exhaustive lists of references!

- D. Atwood, M. Gronau, and A. Soni, Mixing induced CP asymmetries in radiative B decays in and beyond the standard model, **Phys.Rev.Lett. 79 (1997) 185–188**, **arXiv:9704272 [hep-ph]**.
- D. Atwood, T. Gershon, M. Hazumi, and A. Soni, Mixing-induced CP violation in $B \to P(1)P(2)\gamma$ in search of clean new physics signals, **Phys.Rev. D71 (2005) 076003**, **arXiv:0410036 [hep-ph]**.

- **Belle Collaboration**, Y. Ushiroda et al., Time-Dependent CP Asymmetries in $\mathbf{B^0 \to K_S{}^0\pi^0\gamma}$ **transitions**, **Phys.Rev. D74 (2006) 111104**, **arXiv:0608017 [hep-ex]**.
- **BABAR Collaboration**, B. Aubert et al., Measurement of Time-Dependent CP Asymmetry in $\mathbf{B^0 \to K_S{}^0\pi^0\gamma}$ **Decays**, **Phys.Rev. D78 (2008) 071102**, **arXiv:0807.3103 [hep-ex]**.
- **Belle Collaboration**, J. Li et al., Time-dependent CP Asymmetries in $\mathbf{B^0 \to K_S{}^0\rho^0\gamma}$ Decays, **Phys.Rev.Lett. 101 (2008) 251601**, **arXiv:0806.1980 [hep-ex]**.
- **LHCb Collaboration**, R. Aaij et al., Measurement of the ratio of branching fractions $B(B^0 \to K^{*0}\gamma)/B(B_S{}^0 \to \varphi\gamma)$ and the direct CP asymmetry in $\mathbf{B^0 \to K^{*0}\gamma}$, **Nucl.Phys. B867 (2013) 1–18**, **arXiv:1209.0313 [hep-ex]**.

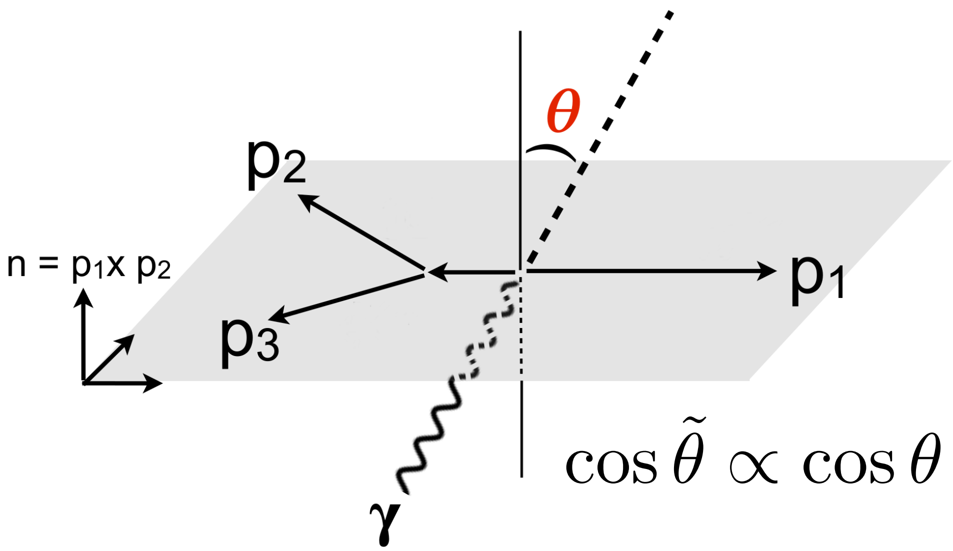
Physical Observables and Experimental Status

- **$B^+ \to K^+_{res}\gamma \to K^+\pi\pi\gamma$ angular analysis:**

- $$\left|A(\overline{B} \to \overline{K}_{\rm res}\gamma \to K\pi\pi\gamma)\right|^2 = |c_L|^2|\mathcal{M}_L|^2 + |c_R|^2|\mathcal{M}_R|^2$$

- $$\lambda_\gamma = \frac{|c_R|^2 - |c_L|^2}{|c_R|^2 + |c_L|^2} \qquad c_R \propto C_{7\gamma}^{\prime(0)\rm eff}, \quad c_L \propto C_{7\gamma}^{(0)\rm eff}$$

- $$\mathcal{A}_{\rm ud} \equiv \frac{\int_0^1 {\rm dcos}\,\tilde\theta \frac{{\rm d}\Gamma}{{\rm dcos}\,\tilde\theta} - \int_{-1}^0 {\rm dcos}\,\tilde\theta \frac{{\rm d}\Gamma}{{\rm dcos}\,\tilde\theta}}{\int_{-1}^1 {\rm dcos}\,\tilde\theta \frac{{\rm d}\Gamma}{{\rm dcos}\,\tilde\theta}} \propto \lambda_\gamma$$



The asymmetry corresponds to the number of photons in the up and down hemispheres wrt the plane defined by the hadronic system

$\mathcal{A}_{\rm ud} \neq 0$:

- 😃 **Immediately indicate if the photon is polarized**
- 😕 **Currently not possible to link directly A_{ud} to λ_γ due to theoretical limitations!** **Not possible to tell what is the photon polarization…**

Physical Observables and Experimental Status

- **$B^+ \to K^+_{res}\,\gamma \to K^+\pi\,\pi\,\gamma$ angular analysis:**

Non exhaustive lists of references!

◉ M. Gronau, Y. Grossman, D. Pirjol, and A. Ryd, Measuring the photon polarization in $B \to K\pi\pi\gamma$, **Phys.Rev.Lett. 88 (2002) 051802, arXiv:0107254 [hep-ph].**

◉ M. Gronau and D. Pirjol, Photon polarization in radiative B decays, **Phys.Rev. D66 (2002) 054008, arXiv:0205065 [hep-ph].**

✦ **LHCb Collaboration**, R. Aaij et al., CP and up-down asymmetries in $B^\pm \to K^\pm\pi^\mp\pi^\pm\gamma$ decays, **LHCb-CONF-2013-009 (Jun, 2013).**

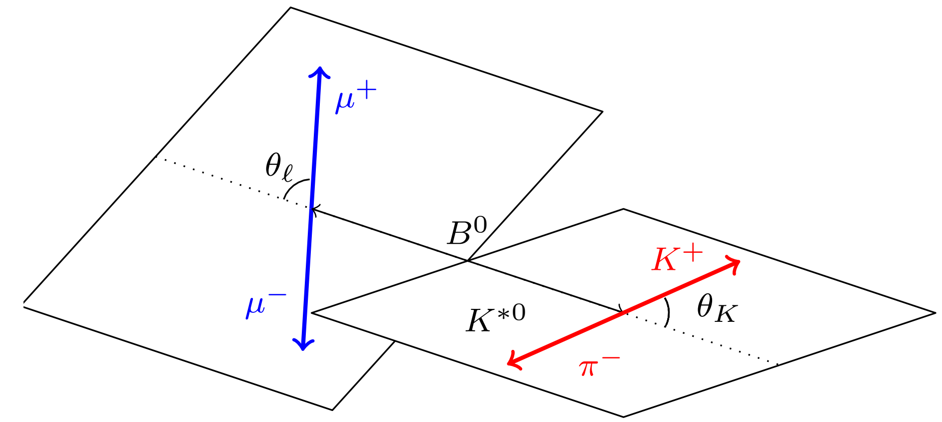
An LHCb paper should soon be published!!!

Physical Observables and Experimental Status

- **$B^0 \to K^{*0}(\to K^-\pi^+)\ell^+\ell^-$ angular analysis:**

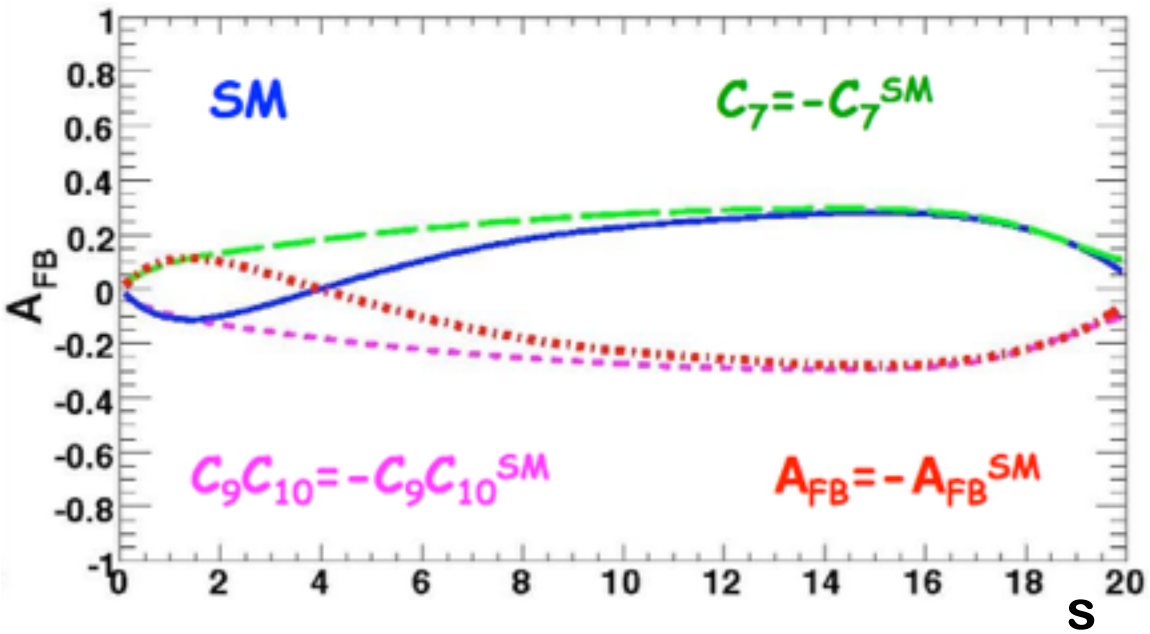
$$\frac{1}{\mathrm{d}\Gamma/\mathrm{d}q^2}\frac{\mathrm{d}^4\Gamma}{\mathrm{d}q^2\,\mathrm{d}\cos\theta_\ell\,\mathrm{d}\cos\theta_K\,\mathrm{d}\hat{\phi}} = \frac{9}{16\pi}\Big[F_L\cos^2\theta_K + \frac{3}{4}(1-F_L)(1-\cos^2\theta_K) - F_L\cos^2\theta_K(2\cos^2\theta_\ell - 1) + \frac{1}{4}(1-F_L)(1-\cos^2\theta_K)(2\cos^2\theta_\ell-1) + S_3(1-\cos^2\theta_K)(1-\cos^2\theta_\ell)\cos 2\hat{\phi} + \frac{4}{3}A_{FB}(1-\cos^2\theta_K)\cos\theta_\ell + A_9(1-\cos^2\theta_K)(1-\cos^2\theta_\ell)\sin 2\hat{\phi}\Big]$$

S_3: related to the photon polarization →



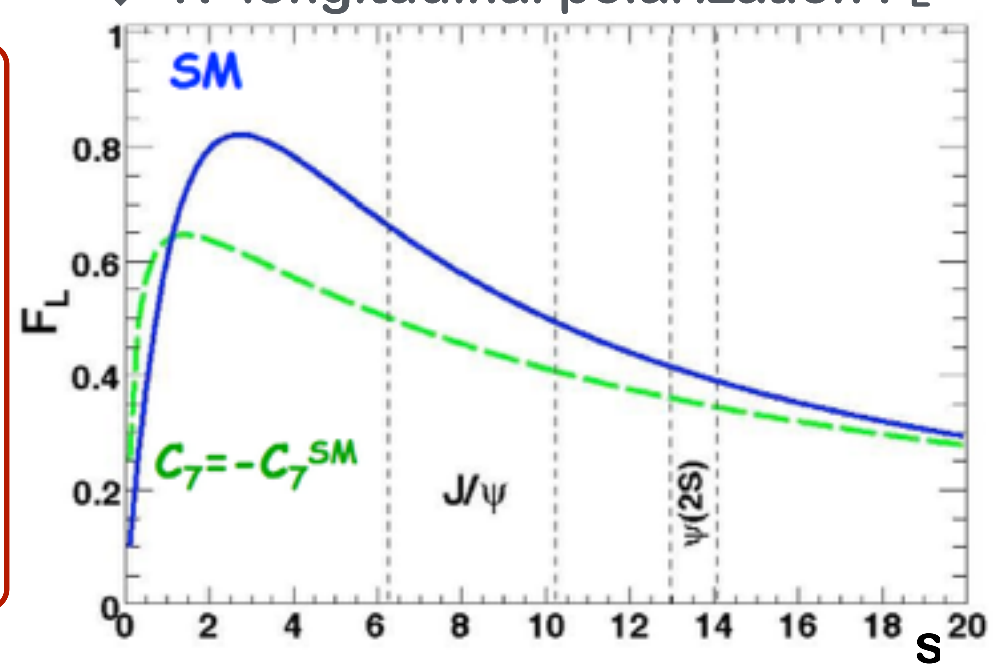
(a) θ_K and θ_ℓ definitions for the B^0 decay

❖ Lepton forward-backward asymmetry A_{FB}



Many additional observables sensitive to the presence of NP contributions 😃

❖ K^* longitudinal polarization F_L



- **$B^0 \to K^{*0}(\to K^-\pi^+)\ell^+\ell^-$ angular analysis:**

Non exhaustive lists of references! (in fact a huge list of phenomenological papers on this subject…)

- F. Kruger and J. Matias, Probing new physics via the transverse amplitudes of B0 → K∗0(→ K−π+)l+l− at large recoil, **Phys.Rev. D71 (2005) 094009, arXiv:0502060 [hep-ph].**

- **LHCb Collaboration**, R. Aaij et al., Measurement of the B0 → K^{*0}e+e− branching fraction at low dilepton mass, **JHEP 1305 (2013) 159, arXiv:1304.3035 [hep-ex].**
- **LHCb Collaboration**, R. Aaij et al., Differential branching fraction and angular analysis of the decay B0 → K∗0μ+μ− **JHEP 1308 (2013) 131, arXiv:1304.6325 [hep-ex].**
- **BABAR Collaboration**, B. Aubert et al., Measurements of branching fractions, rate asymmetries, and angular distributions in the rare decays B→Kℓ+ℓ− and B→K∗ℓ+ℓ− **Phys.Rev. D73 (2006) 092001**, **arXiv:0604007 [hep-ex]**.
- **Belle Collaboration**, J.-T. Wei et al., Measurement of the Differential Branching Fraction and Forward-Backward Asymmetry for B ---> K(*)l+l- **Phys.Rev.Lett. 103 (2009) 171801**, **arXiv:0904.0770 [hep-ex]**.
- **CDF Collaboration**, T. Aaltonen et al., Angular analysis and branching fraction measurement of the decay B0 to K*0 mu+ mu-, **Phys. Lett. B 727 (2013) 77**, **arXiv:1308.3409 [hep-ex]**.

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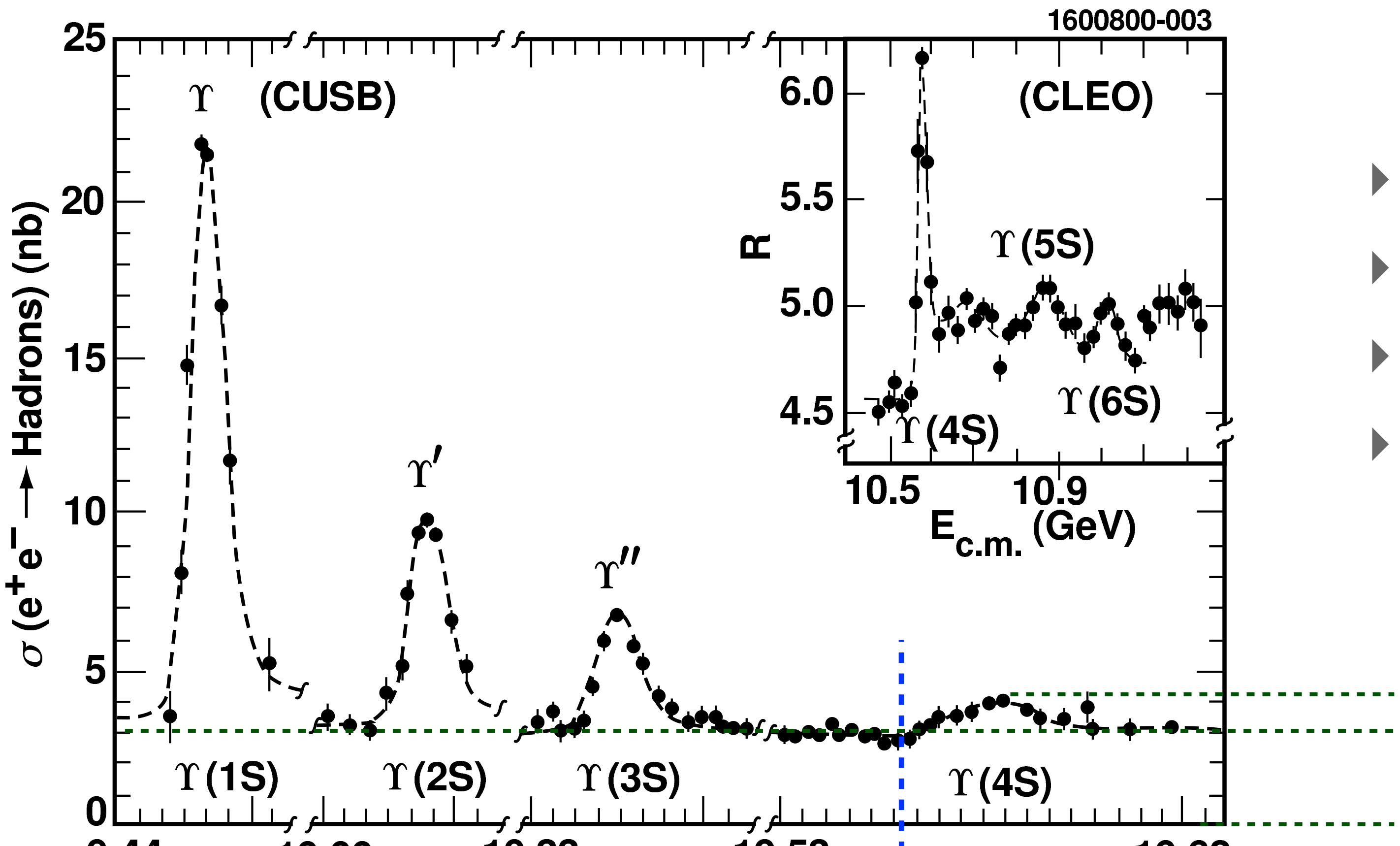
Study of $B \to K \pi \pi \gamma$ decays with the BaBar detector:

Overview of $B^0 \to K_S \rho^0 \gamma$ Analysis

(1) The BaBar detector: ingredients of the measurements
(2) Analysis goal
(3) Analysis strategy
(4) The dilution factor: analytical expression

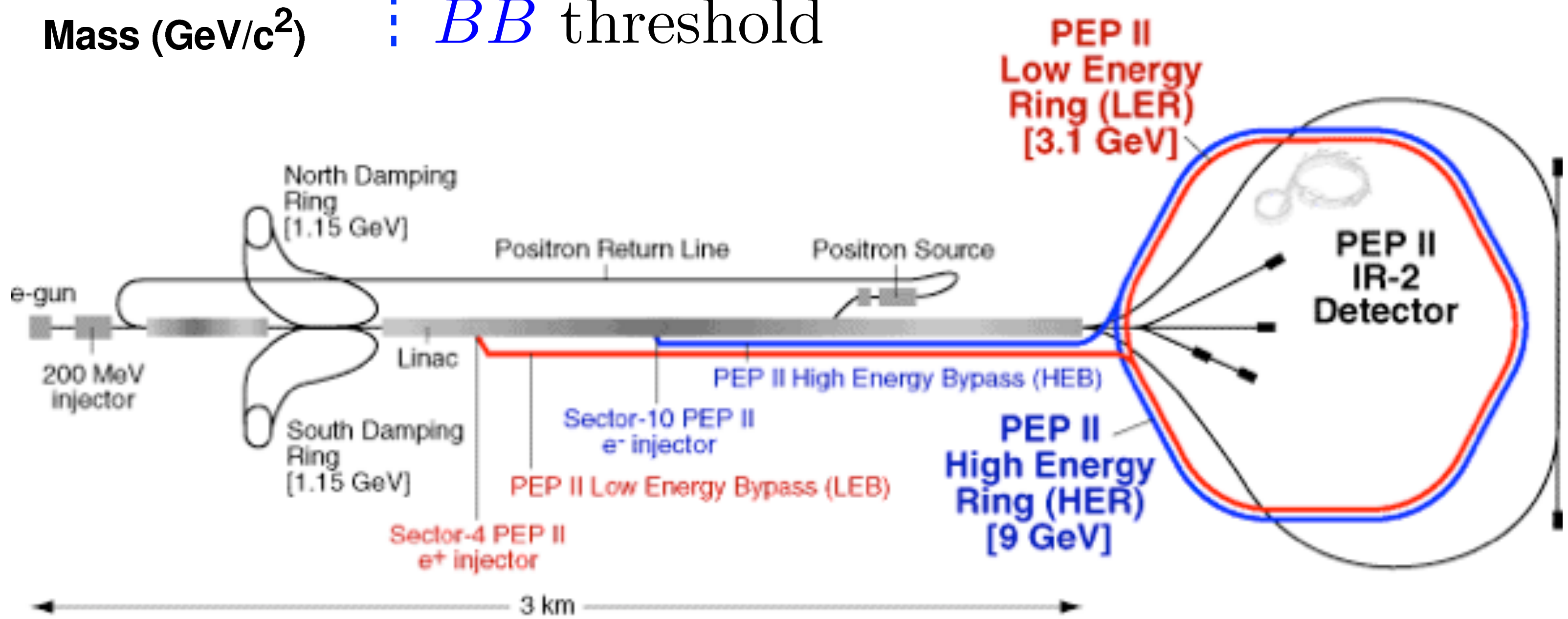
AN ASYMMETRIC e^+e^- ACCELERATOR: PEP-II



$B\overline{B}$ threshold

$$\frac{\sigma(b\bar{b})}{\sigma(\text{hadrons})} = 0.28$$

- **BaBar at SLAC**
- **Running with PEP-II accelerator**
- **Clean environment**
- **Data taking stopped in 2008**



The BaBar Detector & The Data Sample

$e^+e^- \rightarrow \Upsilon(4S) \rightarrow B\bar{B}$; $B^0\bar{B}^0$ (coherent state) or B^+B^-

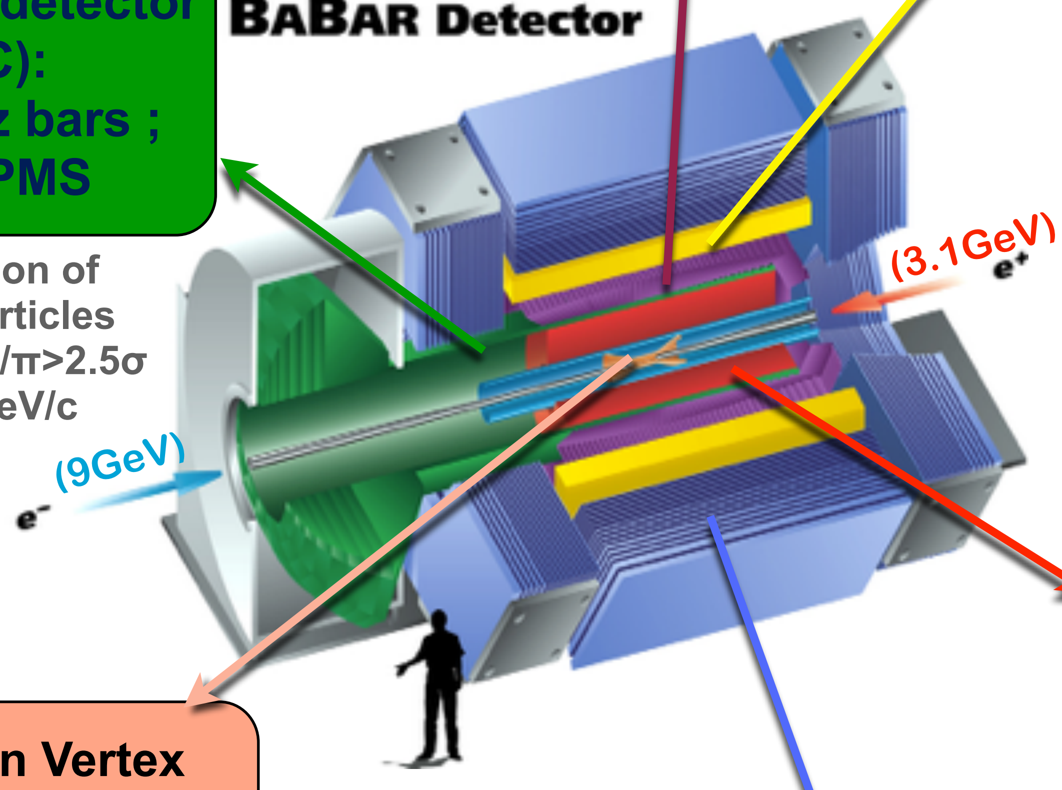
Electromagnetic calorimeter (EMC): 6580 CsI(Tl) crystals

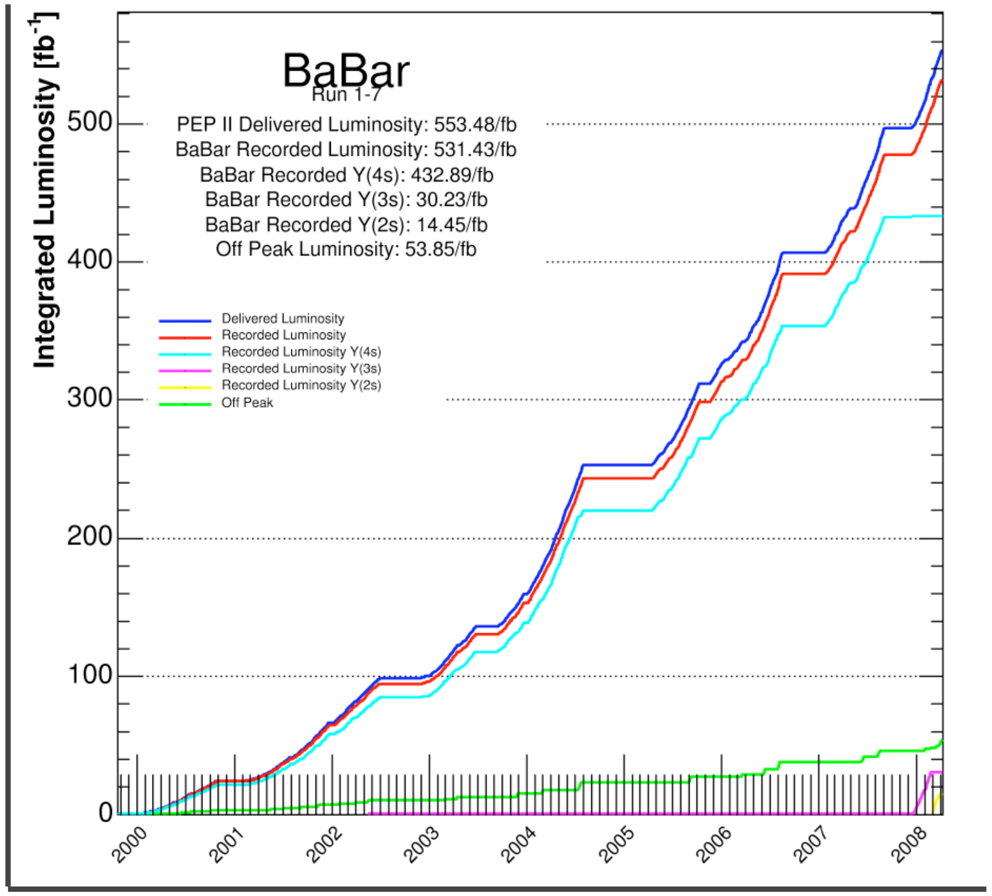
Particle identification (e^-, gamma)
Measures the energy of charged and neutral particles

1.5T solenoid

Cherenkov detector (DIRC): 144 quartz bars ; 11000 PMS

Identification of charged particles
Separation K/π>2.5σ up to 4 GeV/c





Silicon Vertex Tracker (SVT): 5 layers

Reconstruction of decay vertex and tracks close to the IP

Instrumented Flux Return (IFR)

Muon identification

Drift Chamber: 40 stereo layers

Reconstruction of charged particle tracks: momentum and angles

Full dataset:

$\int \mathscr{L}dt \sim 433\ \mathrm{fb^{-1}}$ @$\Upsilon(4S)$

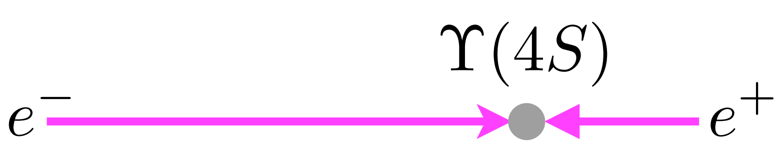
$470 \times 10^6\ B\bar{B}$

$\int \mathscr{L}dt \sim 550\ \mathrm{fb^{-1}}$ total
(Off resonance, $\Upsilon(nS)$)

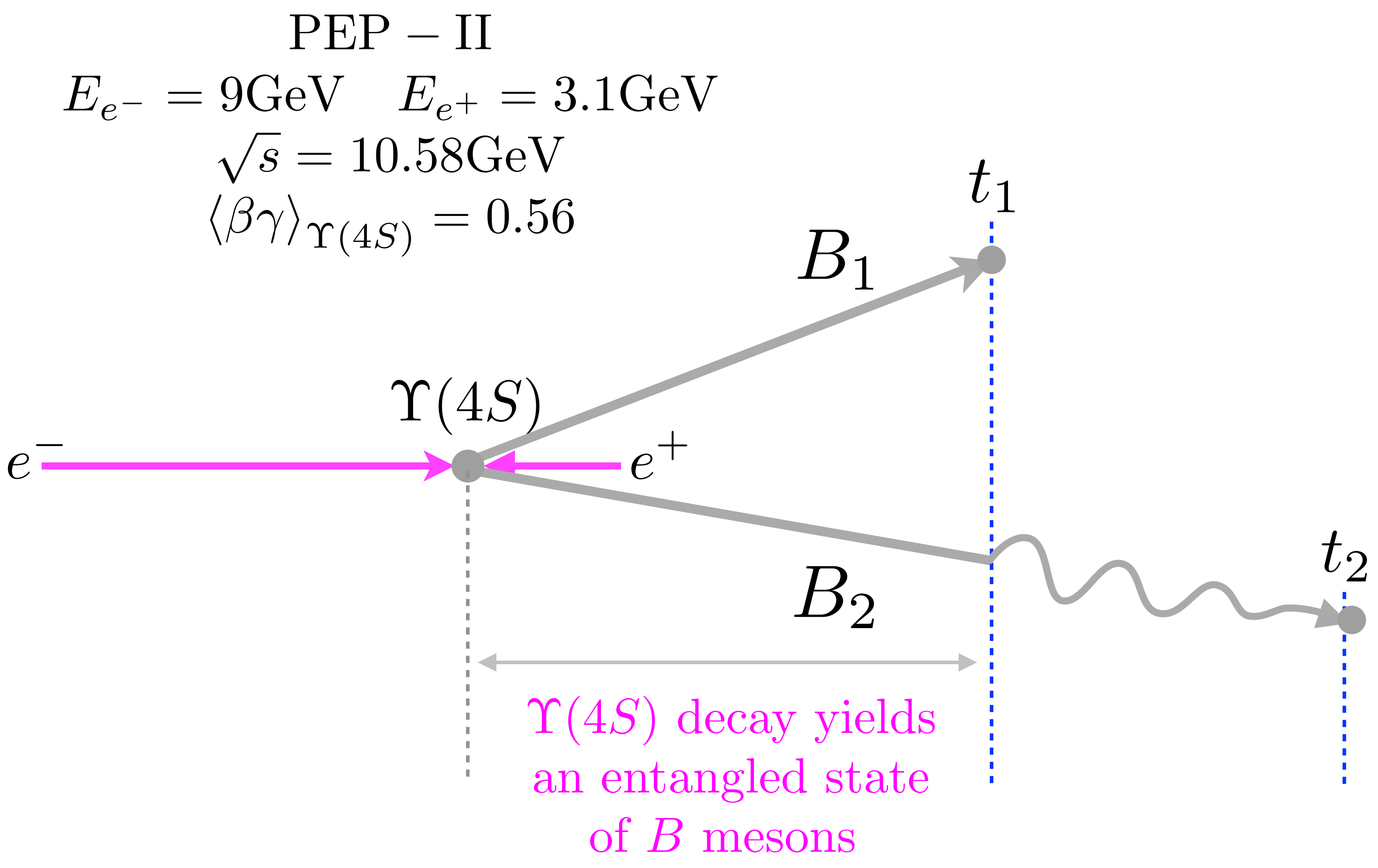
Time-Dependent CP Asymmetry
Ingredients of the Measurement

- **Flavor tagging and time measurement:**
  the "golden channel" $B^0 \to J/\psi\, K_S^0$ as an example

$$\mathrm{PEP-II}$$
$$E_{e^-} = 9\mathrm{GeV} \quad E_{e^+} = 3.1\mathrm{GeV}$$
$$\sqrt{s} = 10.58\mathrm{GeV}$$
$$\langle \beta\gamma \rangle_{\Upsilon(4S)} = 0.56$$



- **Flavor tagging and time measurement:**
  the "golden channel" $B^0 \to J/\psi\, K^0_S$ as an example



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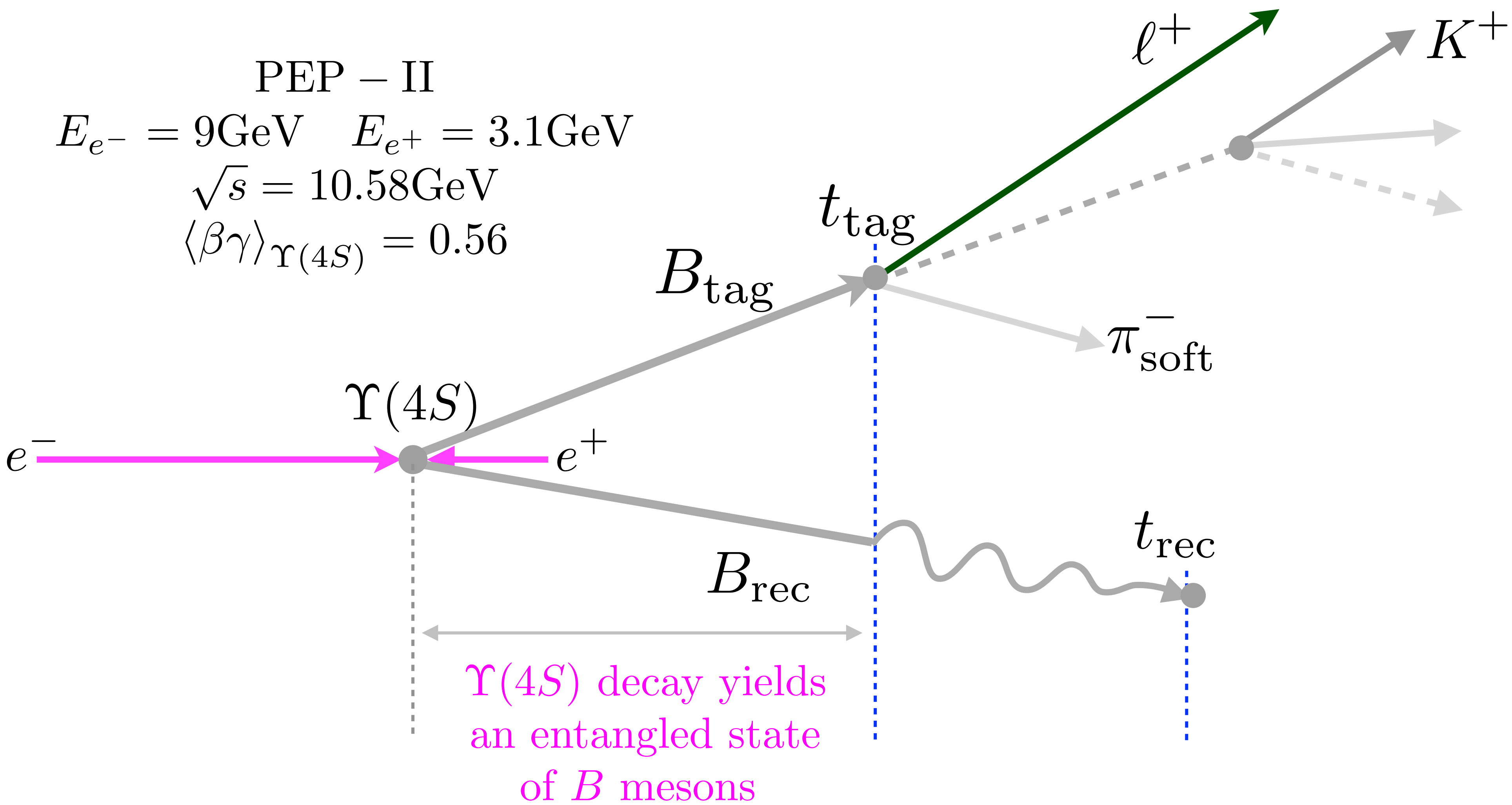
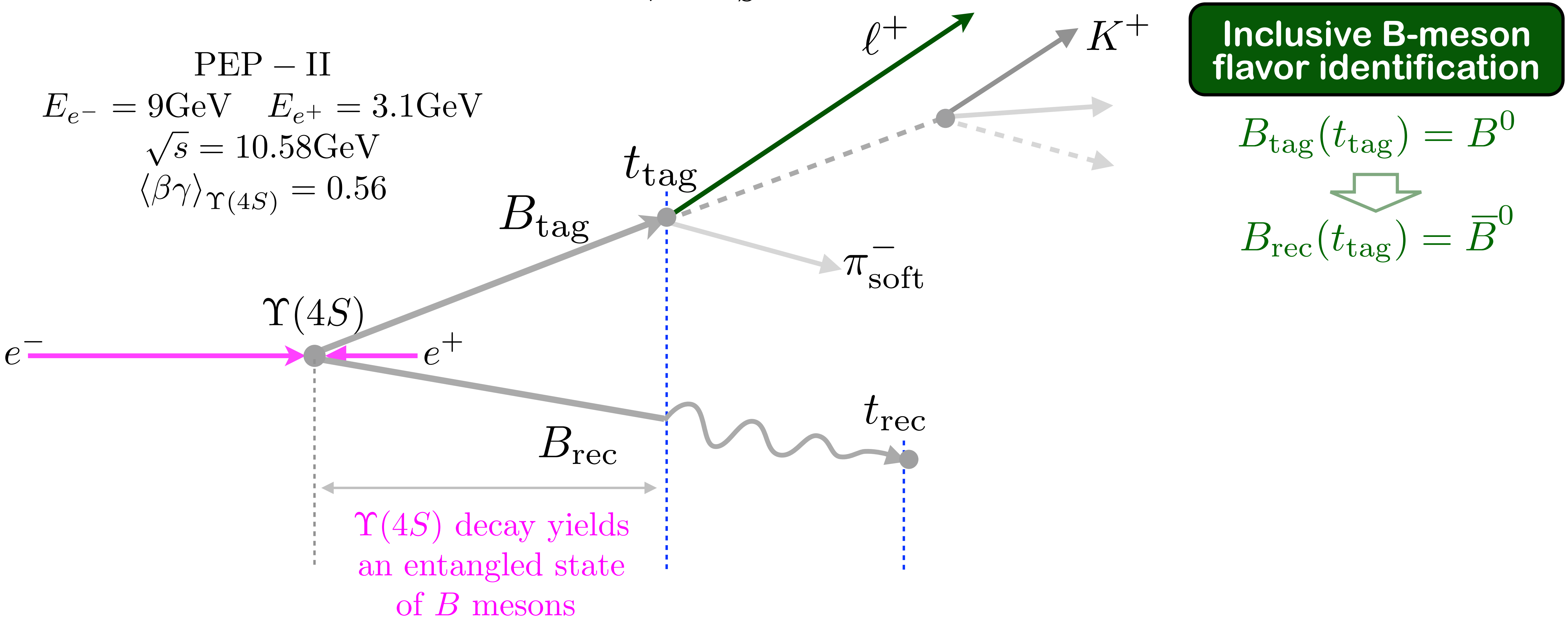


Inclusive B-meson flavor identification

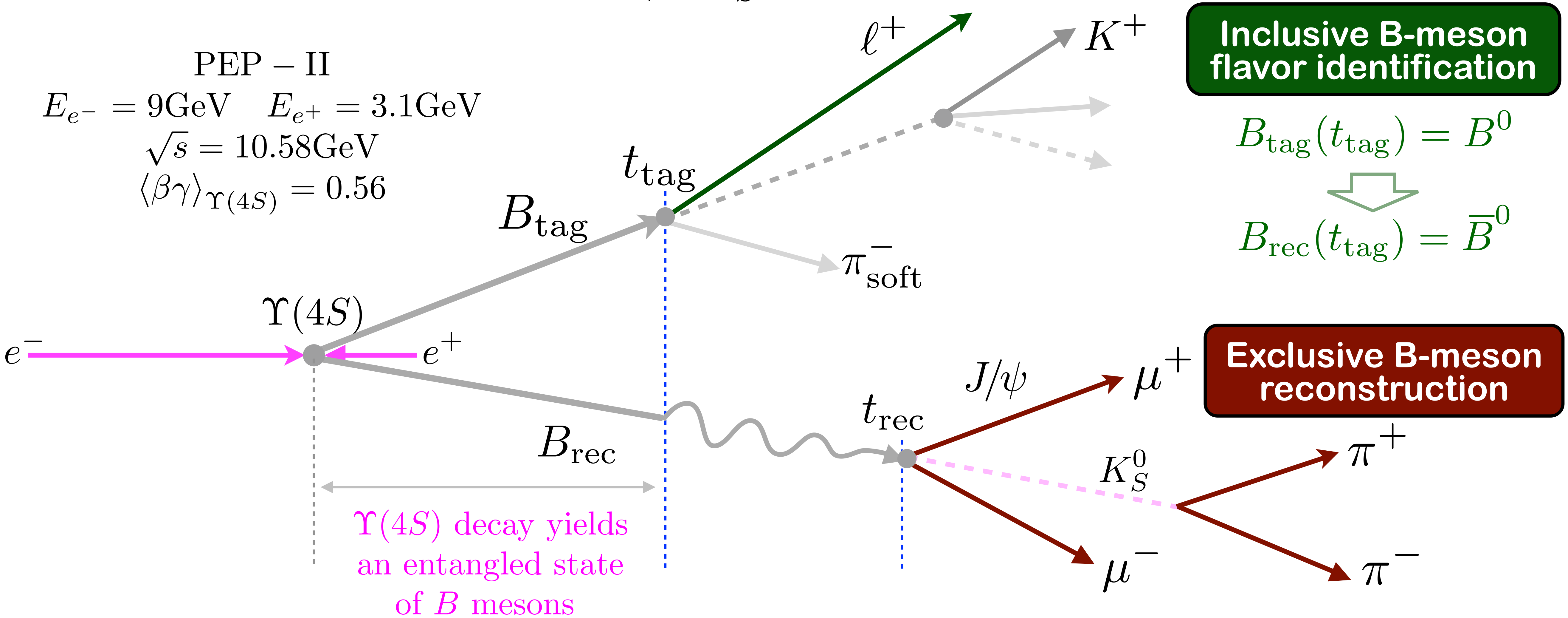
$B_{\mathrm{tag}}(t_{\mathrm{tag}}) = B^0$

- **Flavor tagging and time measurement:**
  the "golden channel" $B^0 \to J/\psi\, K_S^0$ as an example

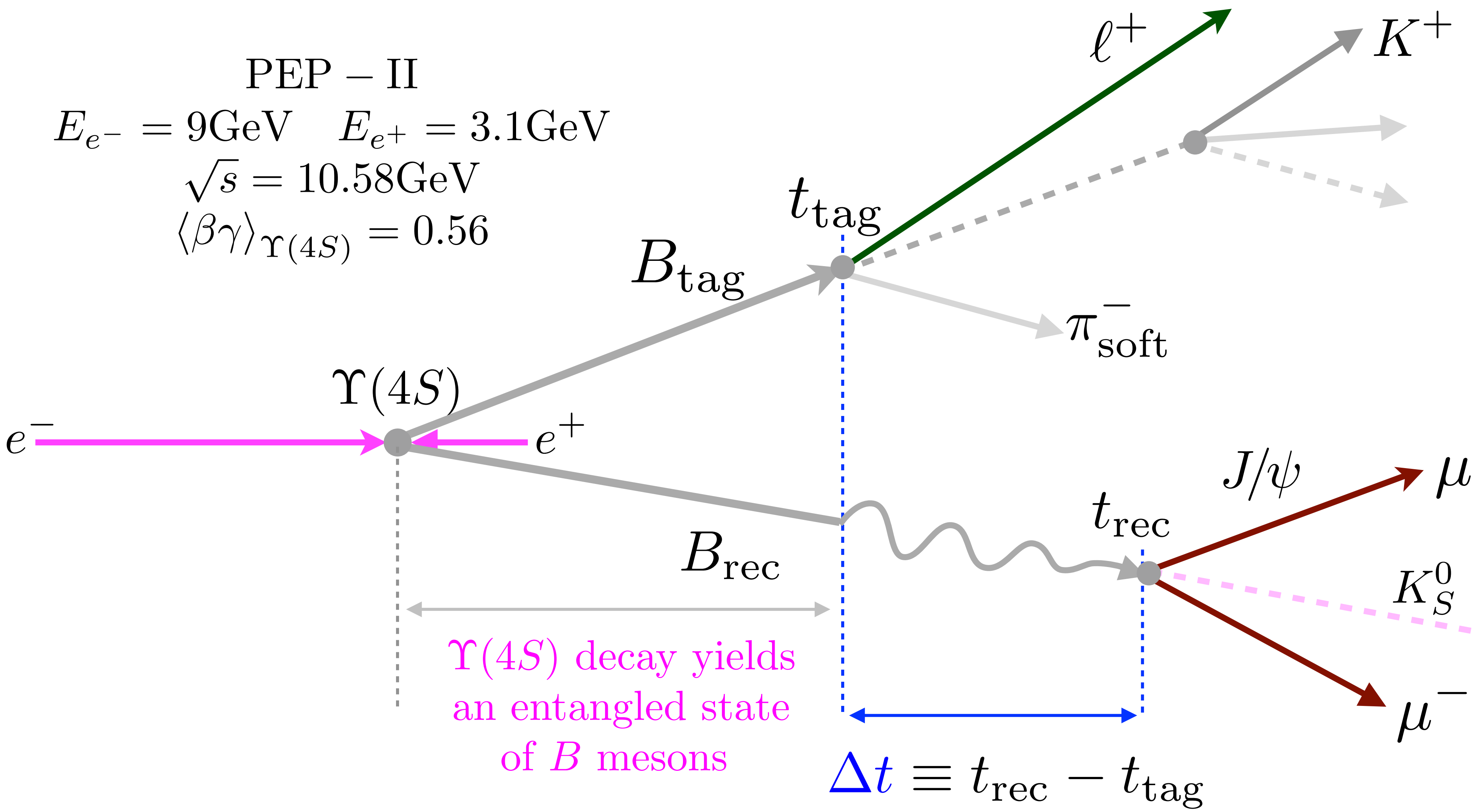


Time-Dependent CP Asymmetry
Ingredients of the Measurement

- **Flavor tagging and time measurement:**
  the "golden channel" $B^0 \to J/\psi\, K^0_S$ as an example



- **Flavor tagging and time measurement:**
  the "golden channel" $B^0 \to J/\psi\, K^0_S$ as an example



Inclusive B-meson flavor identification

$B_{\text{tag}}(t_{\text{tag}}) = B^0$

$B_{\text{rec}}(t_{\text{tag}}) = \overline{B}^0$

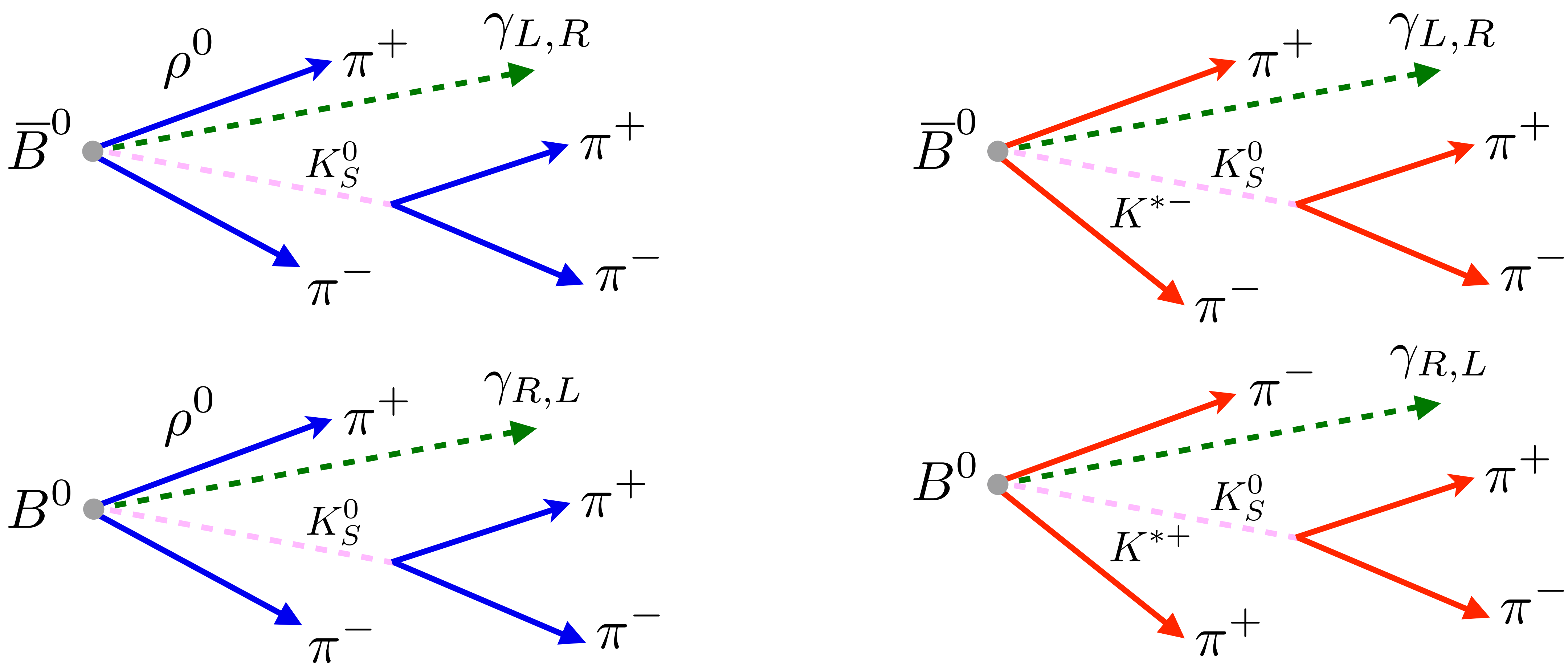
Exclusive B-meson reconstruction

$$\Delta t \approx \frac{\Delta z}{\beta\gamma c} \qquad \langle \Delta z \rangle_{B\overline{B}} = 257\mu\text{m}$$

Good vertexing required for time difference determination

Analysis Goal
Measure S in $B^0 \to K_S \rho^0 \gamma$ decays

- Want to extract the CP violation parameters from a time-dependent analysis of $B^0 \to K_S\, \rho^0(\to \pi^-\pi^+)\, \gamma$ decays
  $\Rightarrow$ $K_S\, \rho^0$ a CP eigenstate $\to$ $K_S\, \rho^0\, \gamma$ can contribute to S in presence of NP
- Irreductible contribution of $B^0 \to K^{*\pm}(\to K_S \pi^\pm)\pi^\mp \gamma$ events
  $\Rightarrow$ $K^{*\pm}\pi^\mp$ not a CP eigenstate $\to$ $K^{*\pm}\pi^\mp\gamma$ dilutes S in all scenarios



Analysis Goal
Measure S in $B^0 \rightarrow K_S \rho^0 \gamma$ decays

- Want to extract the CP violation parameters from a time-dependent analysis of $B^0 \rightarrow K_S \rho^0 (\rightarrow \pi^- \pi^+) \gamma$ decays
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- Irreductible contribution of $B^0 \rightarrow K^{*\pm} (\rightarrow K_S \pi^\pm) \pi^\mp \gamma$ events
  $\Rightarrow$ $K^{*\pm} \pi^\mp$ not a CP eigenstate $\rightarrow$ $K^{*\pm} \pi^\mp \gamma$ dilutes S in all scenarios

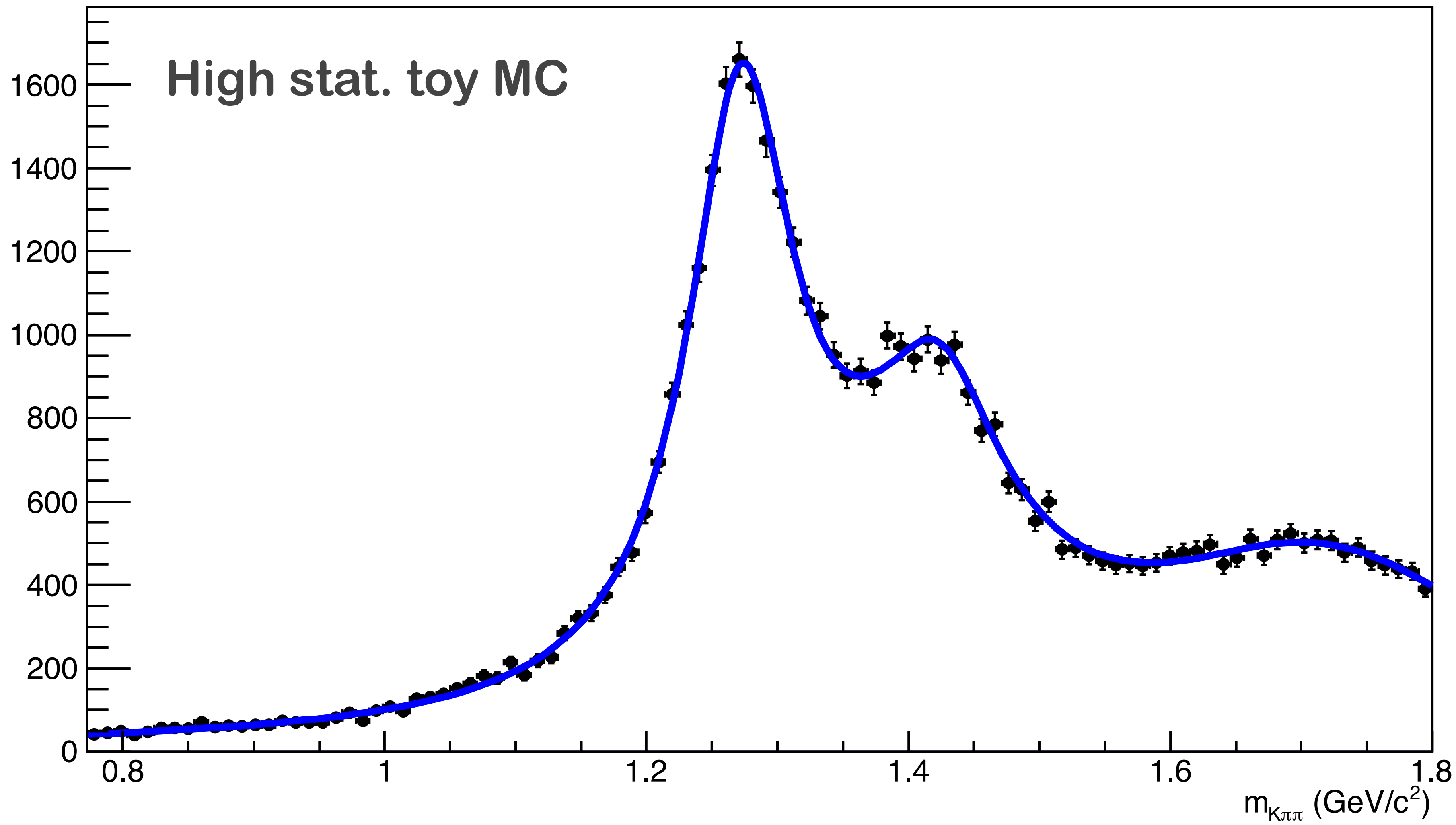
- A time-dependent measurement of $B^0 \rightarrow K_S \pi^- \pi^+ \gamma$ decays yields an effective value of S: S_{eff}
- The true value of S (S_ρ) is diluted by the factor $\mathcal{D}_\rho$ such as:

$$S_{eff} = S_\rho \times \mathcal{D}_\rho$$

$\Rightarrow$ At first order, the dilution effect is function of the amplitudes of $K^*(892)\,\pi$ and $K\,\rho^0(770)$ decay modes

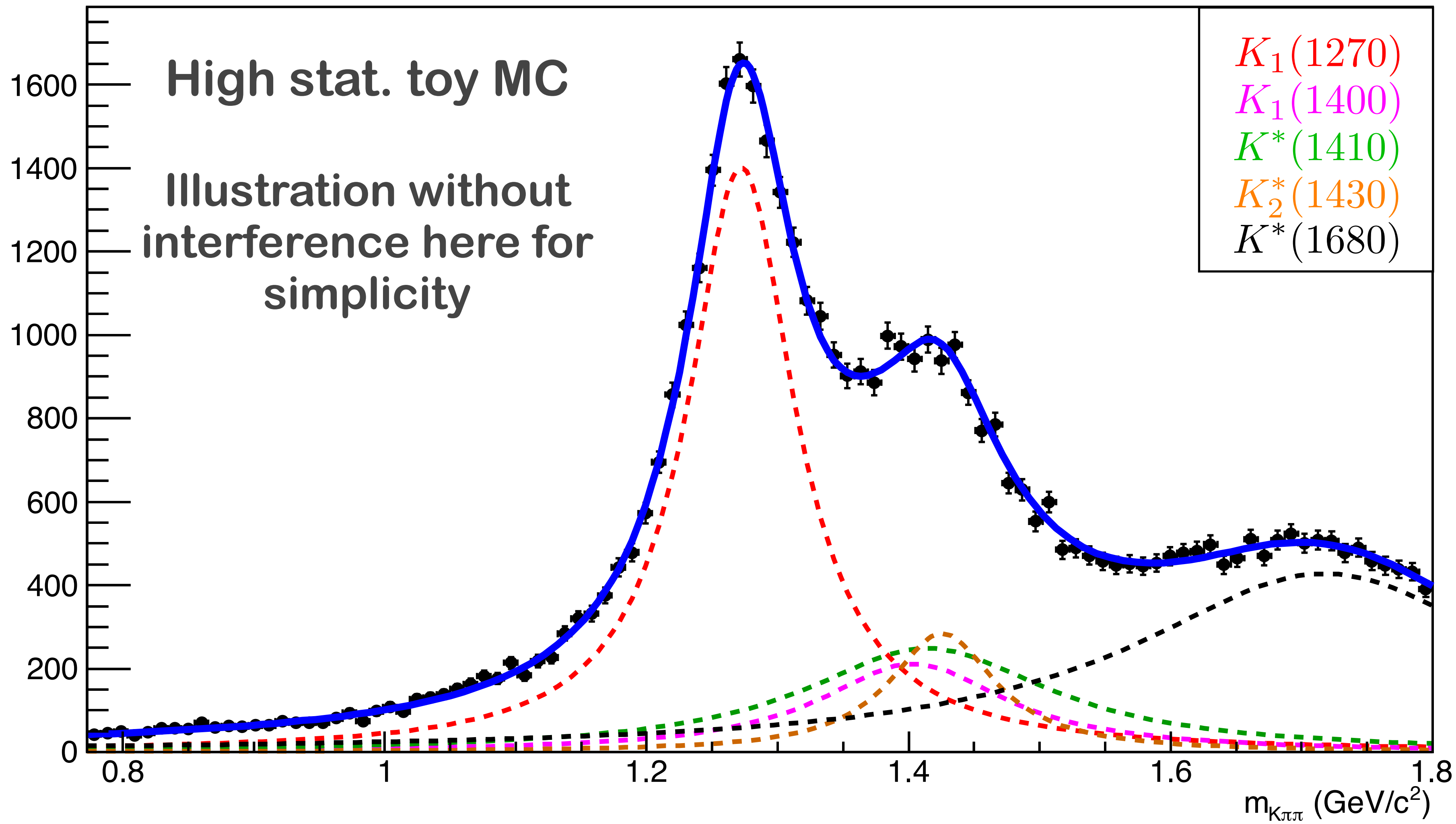
In an Ideal World…

- Perform a time-dependent amplitude analysis of $B^0 \rightarrow K_S \pi^- \pi^+ \gamma$ decays to directly extract the CP violation parameters



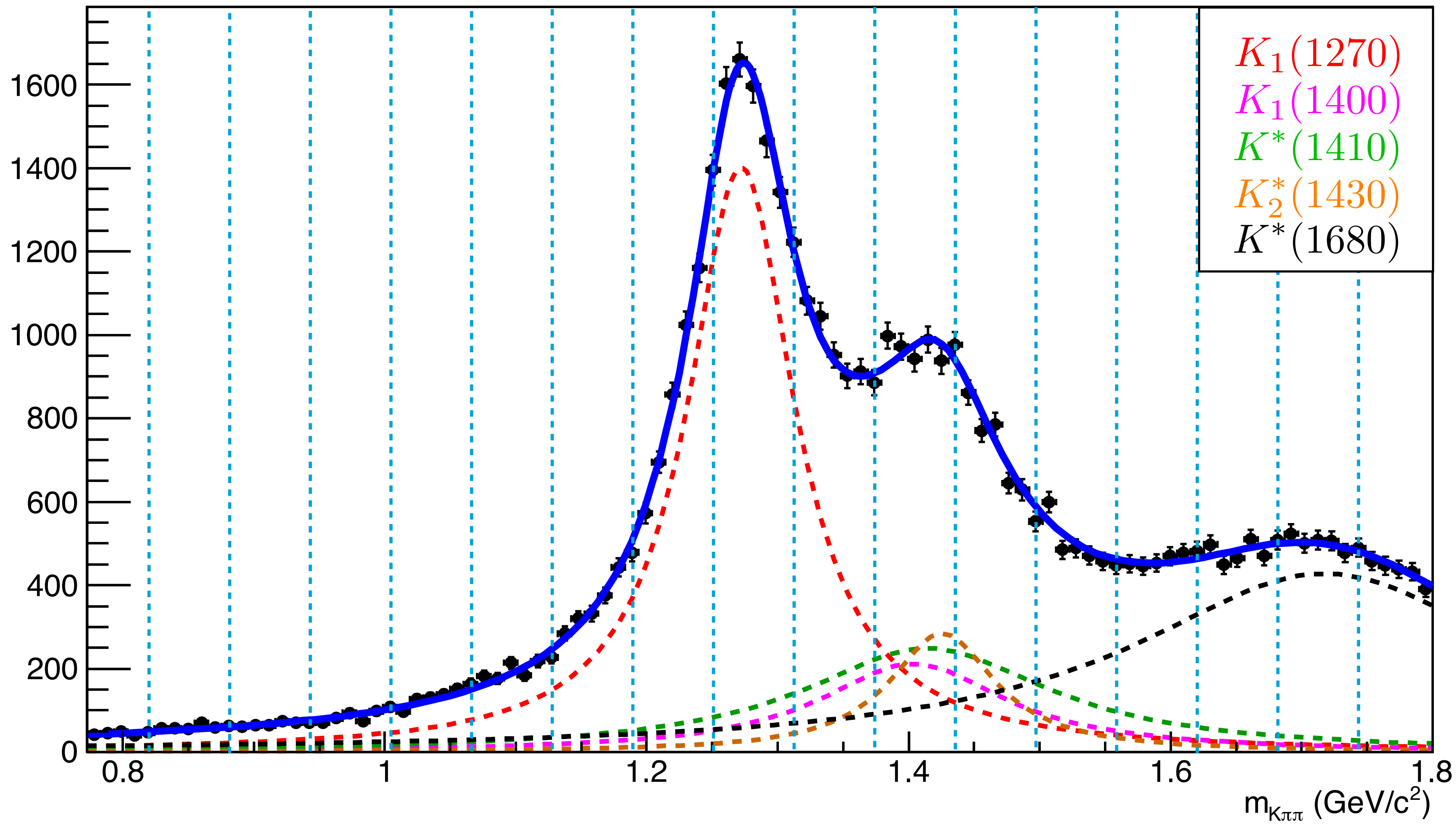
In an Ideal World...

- Perform a time-dependent amplitude analysis of $B^0 \to K_S \pi^- \pi^+ \gamma$ decays to directly extract the CP violation parameters
  - Five Kaonic resonances contribute to the signal via intermediate state resonances



In an Ideal World…

- Perform a time-dependent amplitude analysis of $B^0 \to K_S \pi^- \pi^+ \gamma$ decays to directly extract the CP violation parameters
  - Five Kaonic resonances contribute to the signal via intermediate state resonances
  - Look at the invariant-mass plane ("Dalitz-plot") separately in fine bins of $m_{K\pi\pi}$



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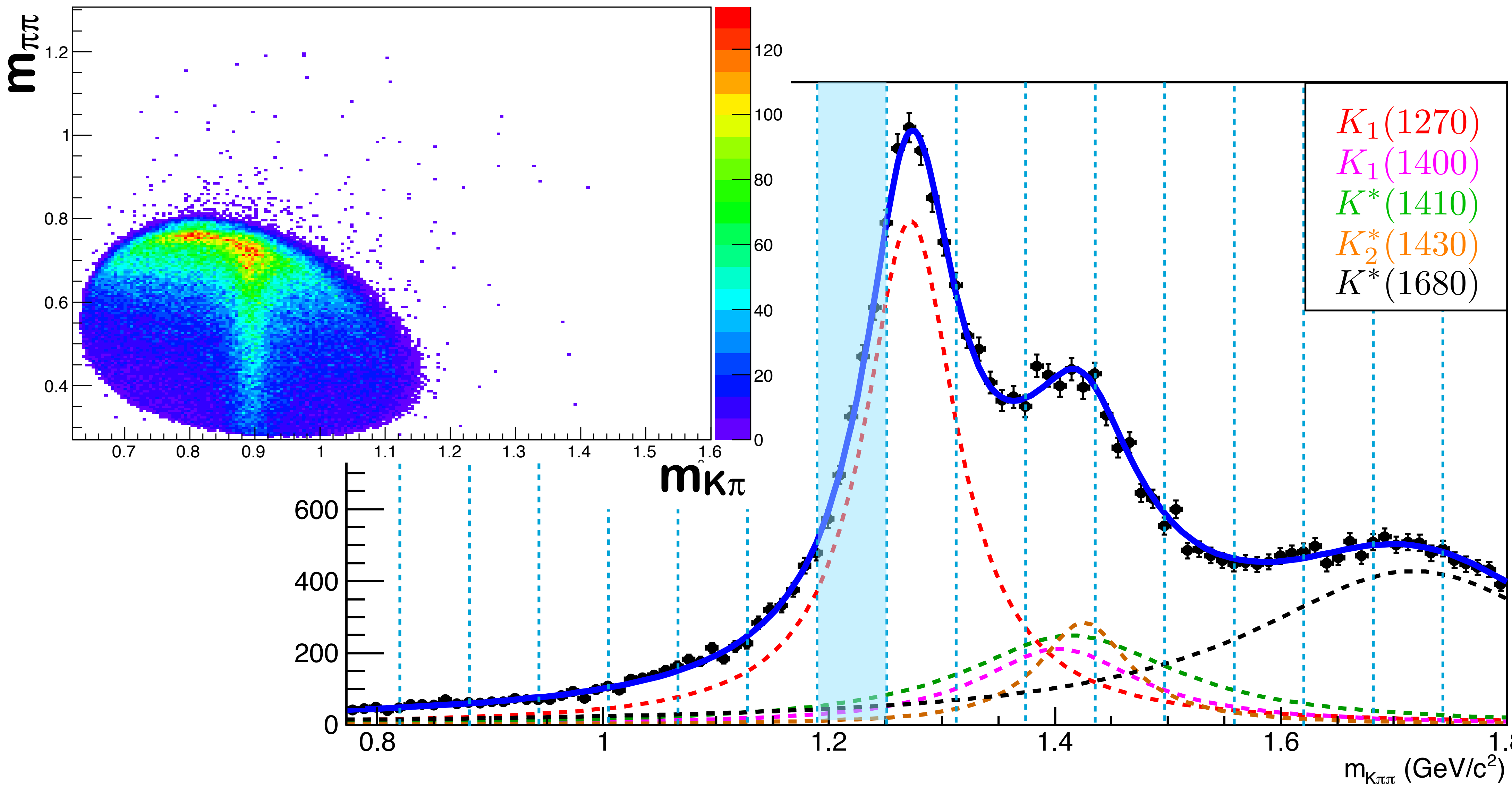
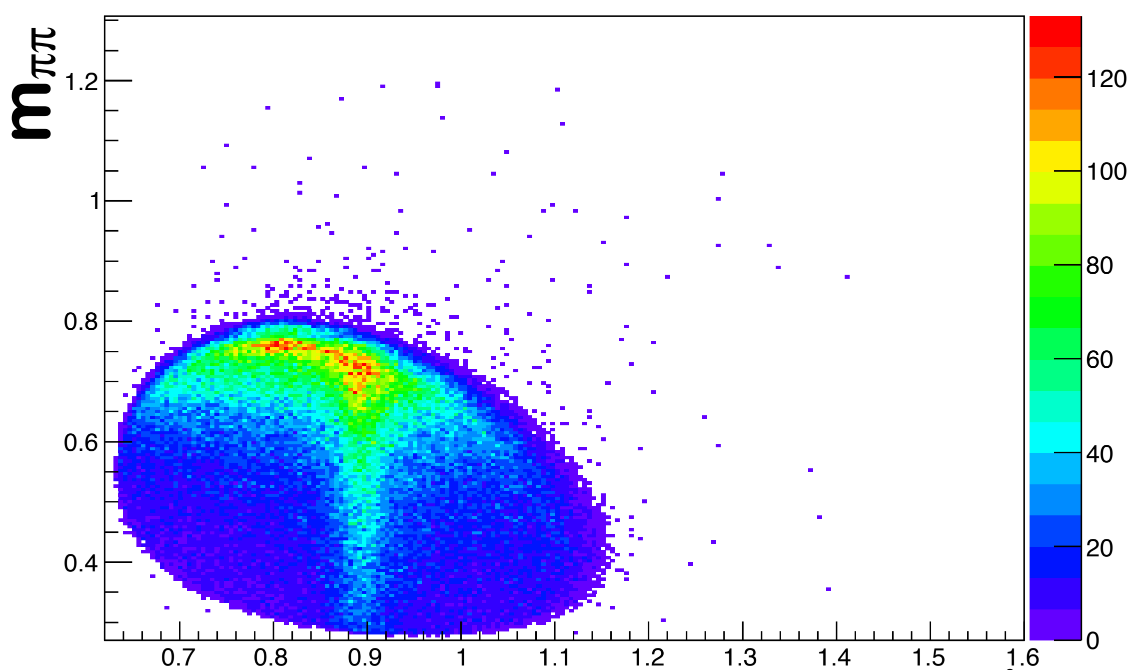
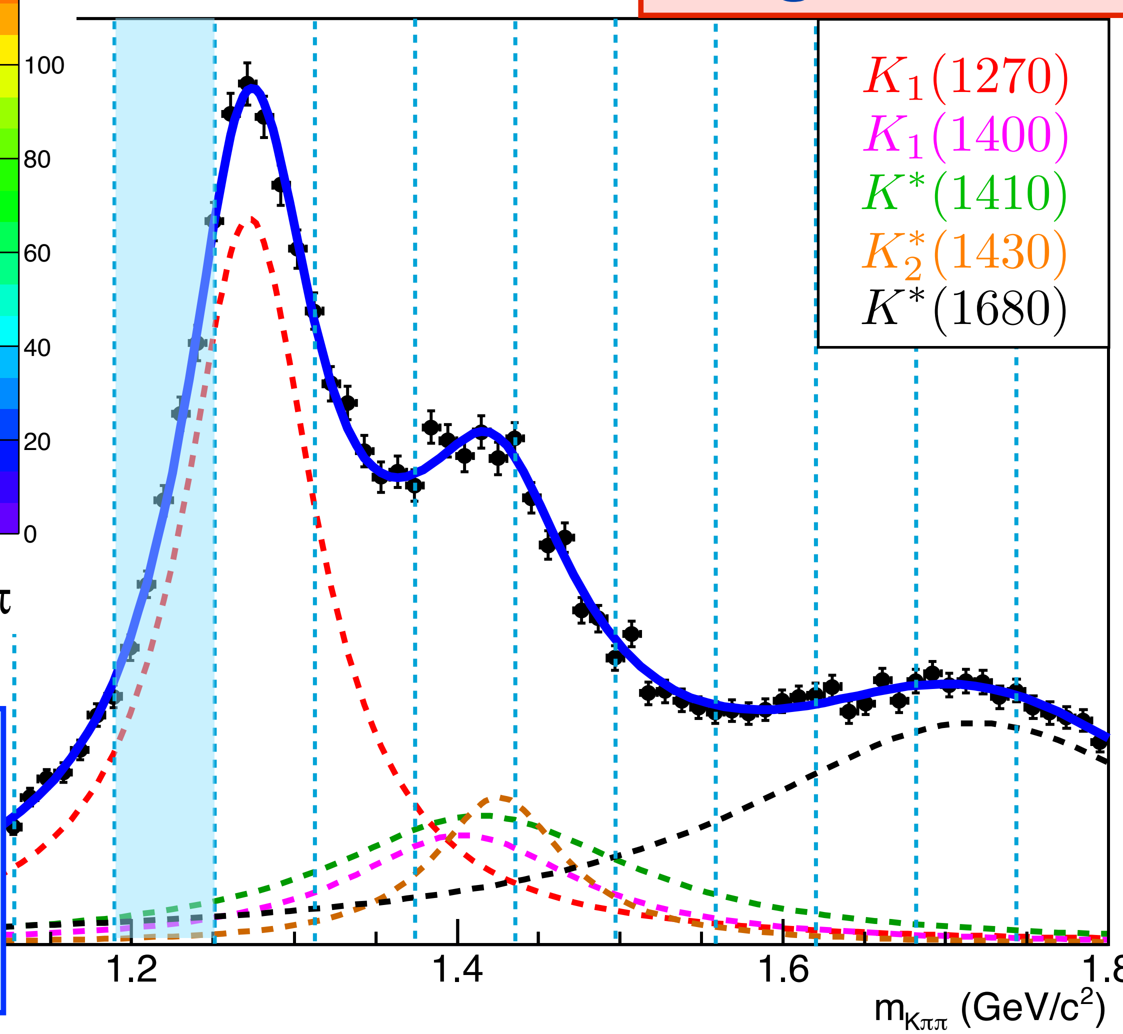


In an Ideal World…

- Perform a time-dependent amplitude analysis of $B^0 \to K_S \pi^- \pi^+ \gamma$ decays to directly extract the CP violation parameters
  - Five Kaonic resonances contribute to the signal via intermediate state resonances
  - Look at the invariant-mass plane ("Dalitz-plot") separately in fine bins of $m_{K\pi\pi}$
    → the amplitudes of $K^*(892)\,\pi$ and $K\,\rho^0(770)$ decay modes vary in each bin



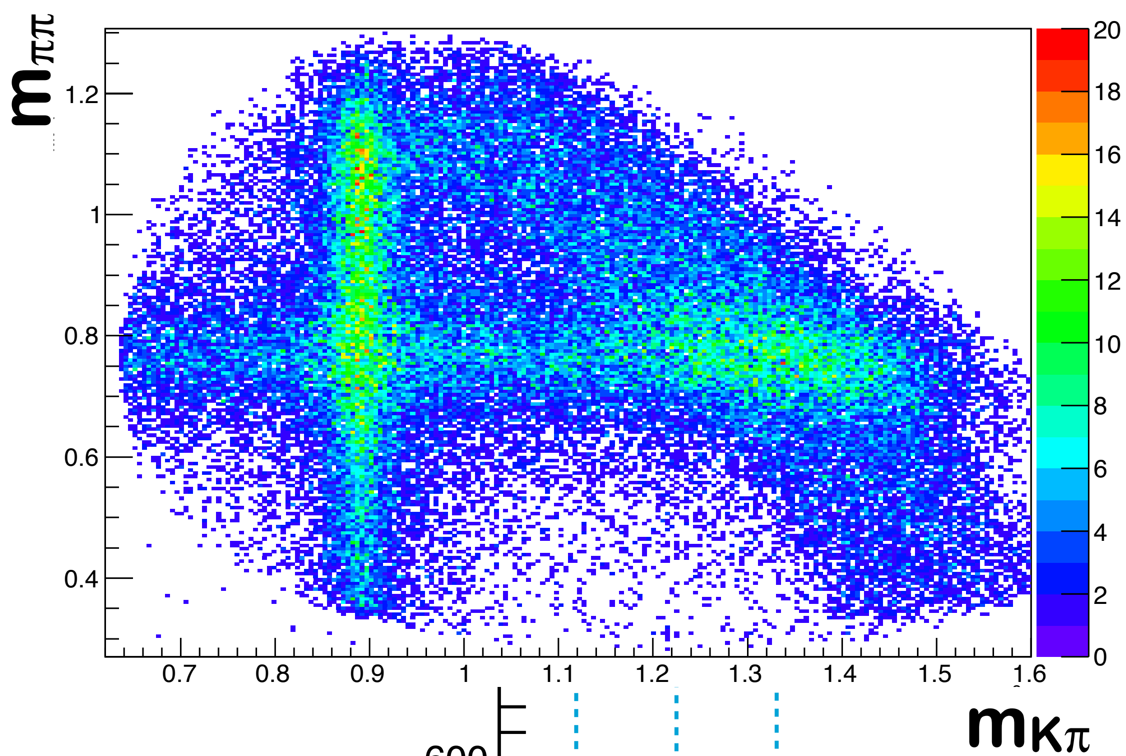
A given dilution of S

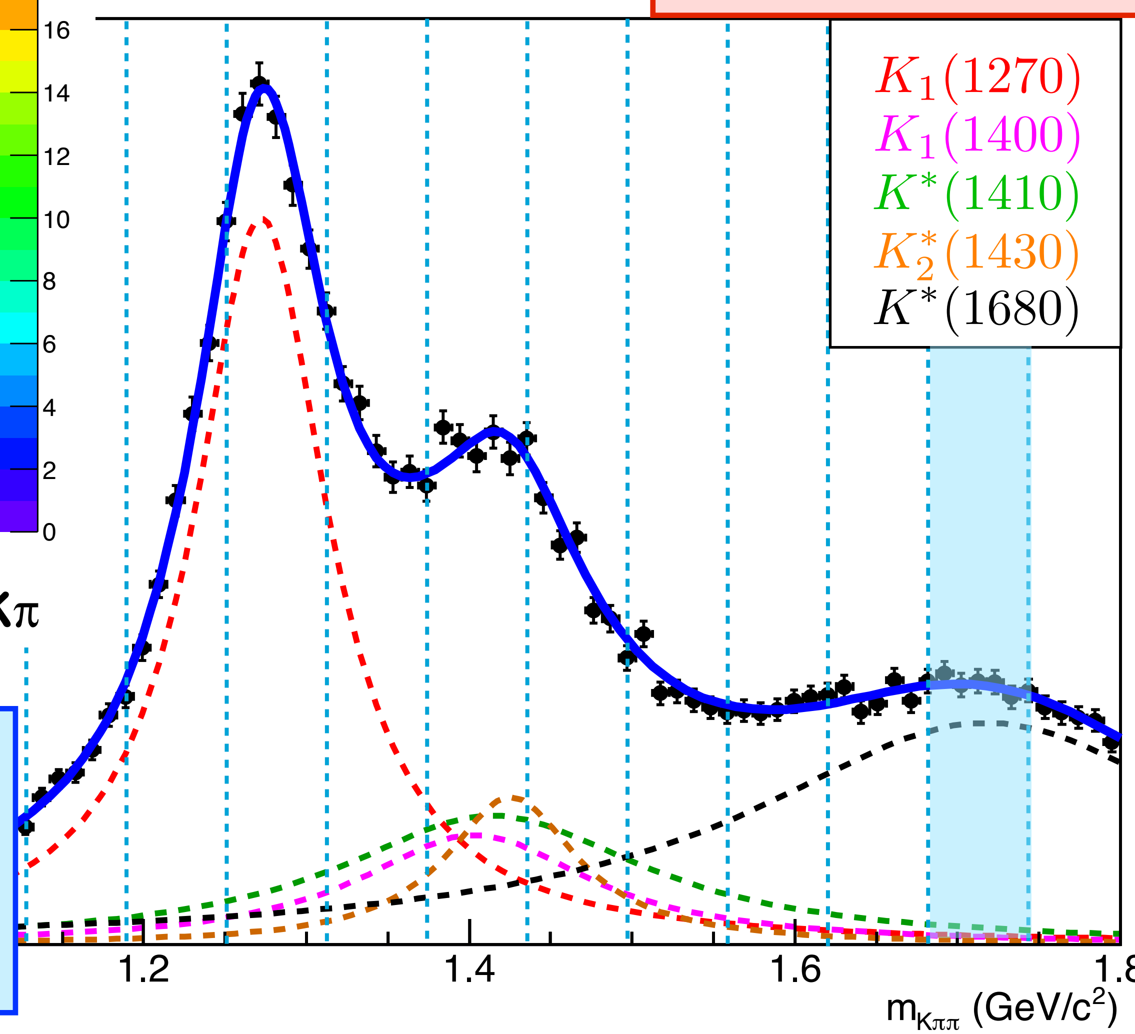


$K_1(1270)$ dominates:

$$\frac{\Gamma(K\,\rho^0(770))}{\Gamma(K\,\rho^0(770) + K^*(892)\,\pi)} \sim 0.7$$

- Perform a time-dependent amplitude analysis of $B^0 \to K_S \pi^- \pi^+ \gamma$ decays to directly extract the CP violation parameters
  - Five Kaonic resonances contribute to the signal via intermediate state resonances
  - Look at the invariant-mass plane ("Dalitz-plot") separately in fine bins of $m_{K\pi\pi}$
    - → the amplitudes of $K^*(892)\,\pi$ and $K\,\rho^0(770)$ decay modes vary in each bin





Another dilution of S

$K^*(1680)$ dominates:

$$\frac{\Gamma(K\,\rho^0(770))}{\Gamma(K\,\rho^0(770) + K^*(892)\,\pi)} \sim 0.6$$

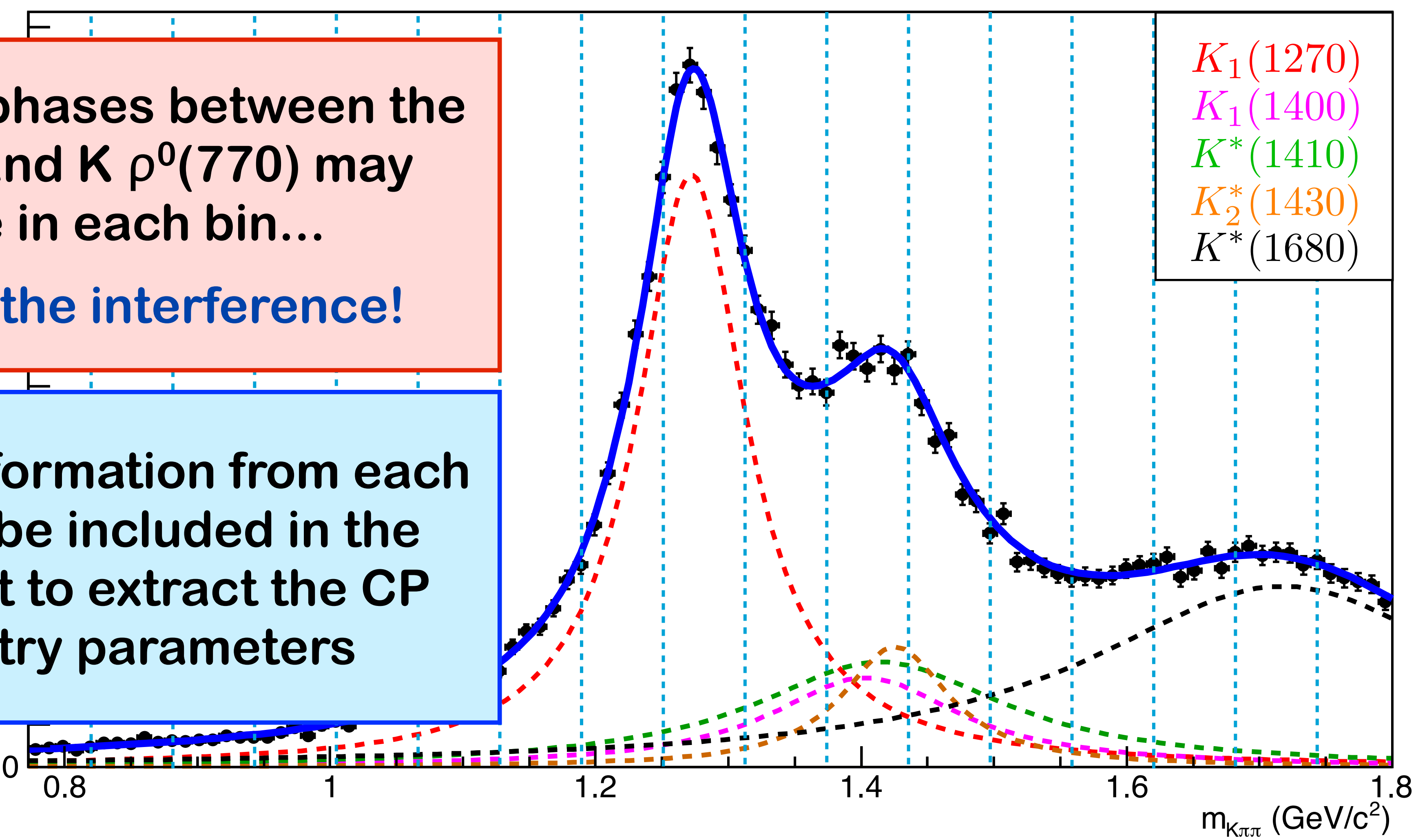
In an Ideal World...

- Perform a time-dependent amplitude analysis of $B^0 \to K_S \pi^- \pi^+ \gamma$ decays to directly extract the CP violation parameters
  - Five Kaonic resonances contribute to the signal via intermediate state resonances
  - Look at the invariant-mass plane ("Dalitz-plot") separately in fine bins of $m_{K\pi\pi}$
    → the amplitudes of $K^*(892)\,\pi$ and $K\,\rho^0(770)$ decay modes vary in each bin

The relative phases between the $K^*(892)\,\pi$ and $K\,\rho^0(770)$ may change in each bin...

Impact on the interference!

Finally the information from each bin should be included in the likelihood fit to extract the CP asymmetry parameters



In the Real World Now...

- Now what the $m_{K\pi\pi}$ distribution would look like with our number of expected events in $B^0 \rightarrow K_S \pi^- \pi^+ \gamma$ decays

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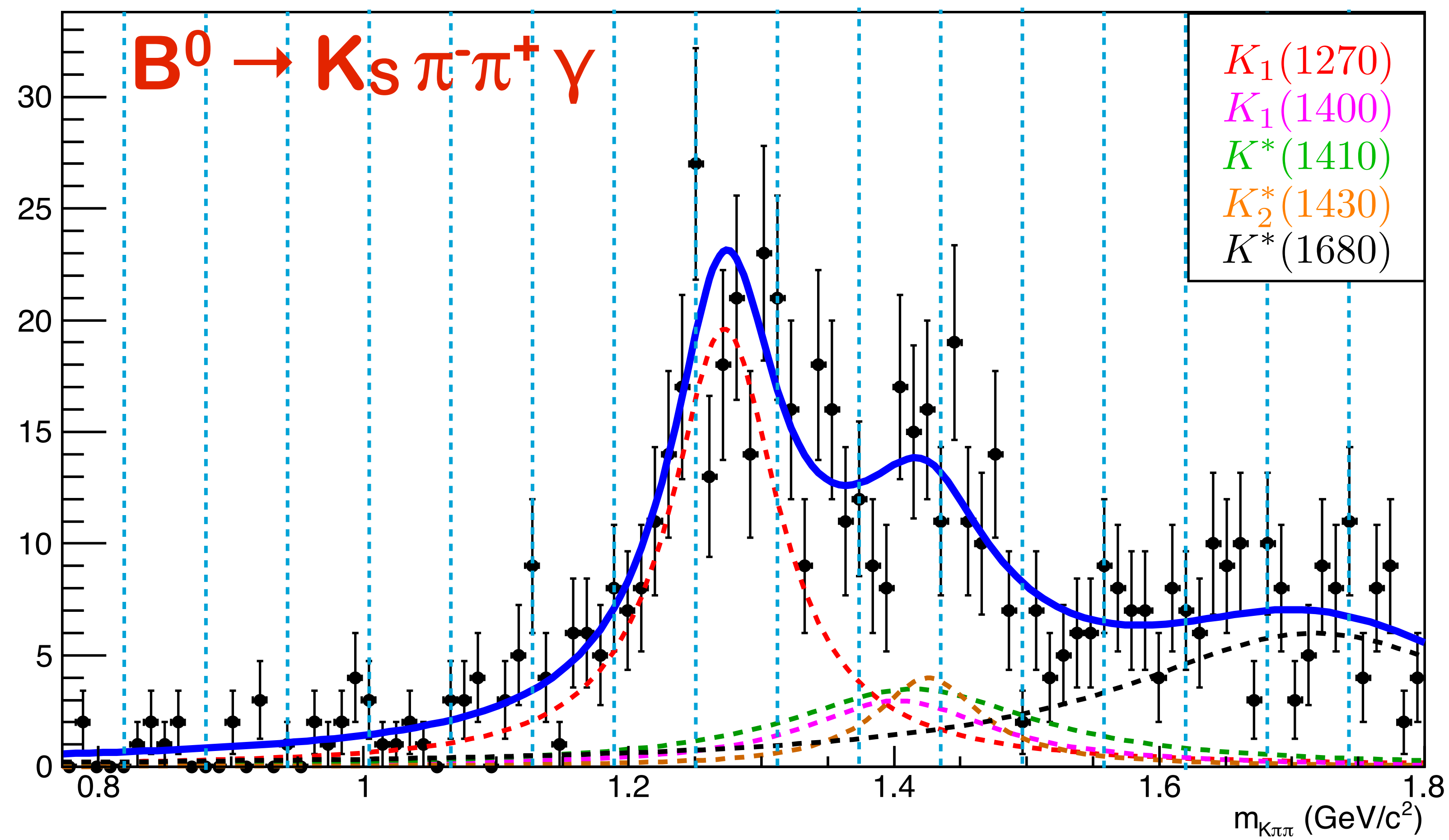

In the Real World Now…

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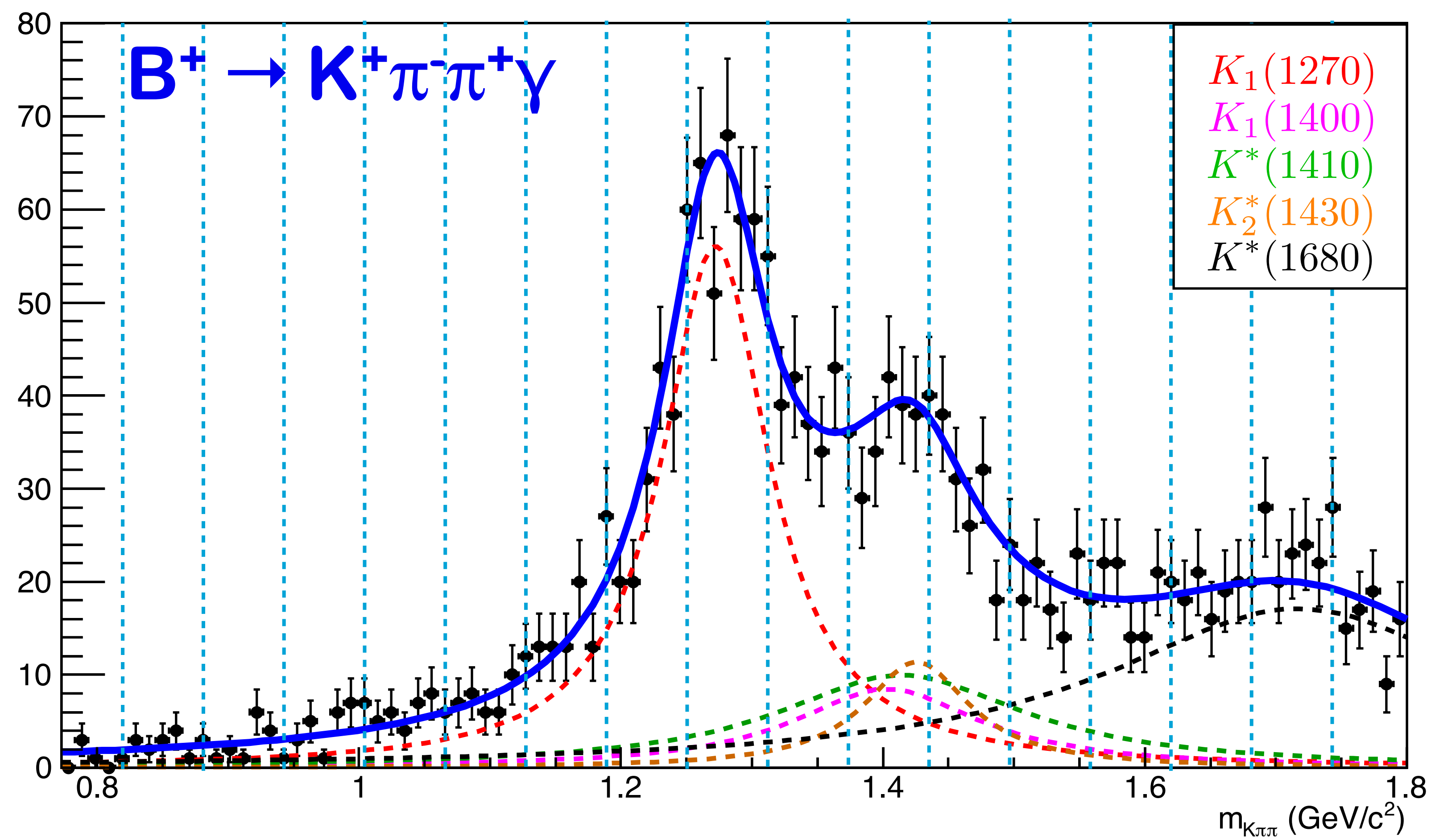
- Instead, assuming Isospin symmetry:
  - Extract the amplitudes from the charged $B^+ \rightarrow K^+ \pi^- \pi^+ \gamma$ decays (more statistics)



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- Instead, assuming Isospin symmetry:
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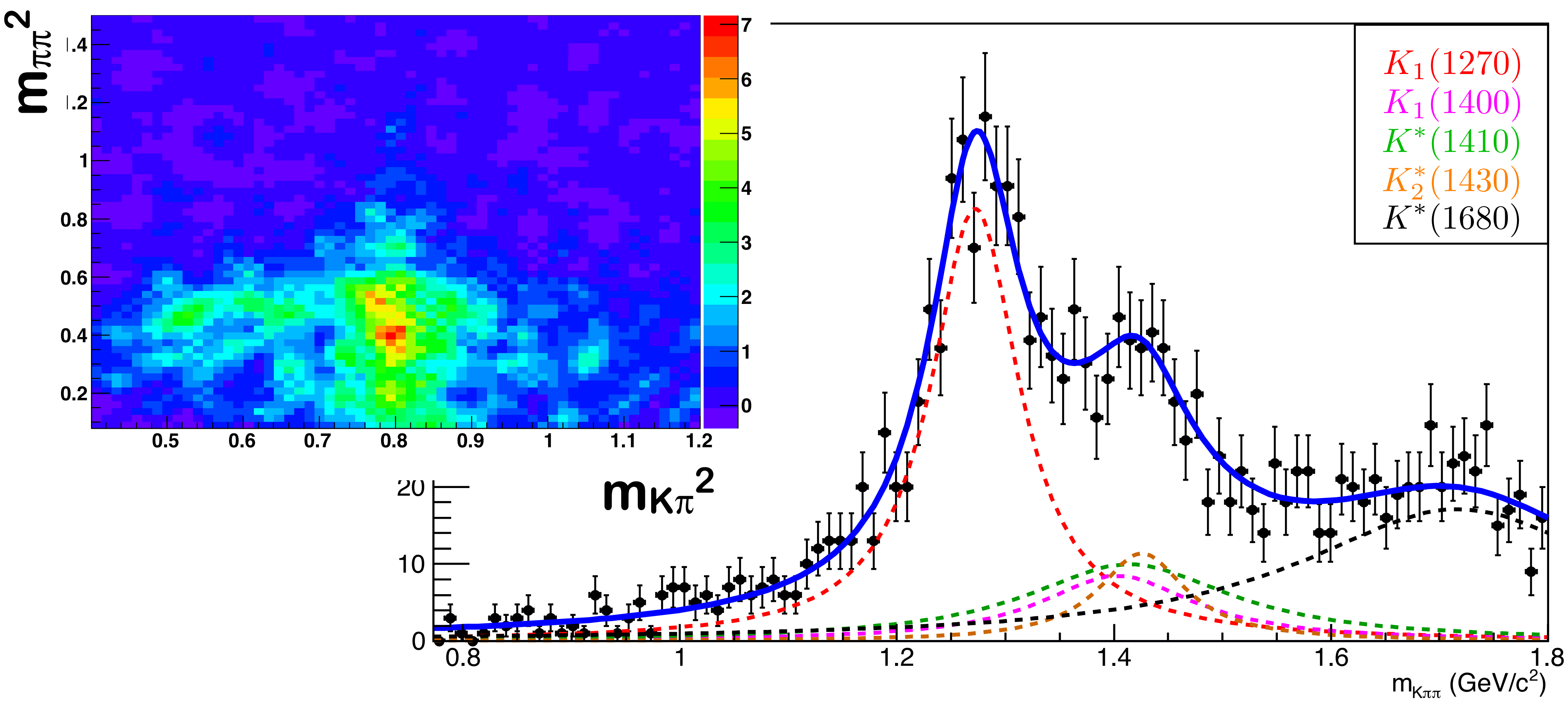
  - Even in the **whole $m_{K\pi\pi}$ range** difficult to perform an analysis in Pseudo-Dalitz plot



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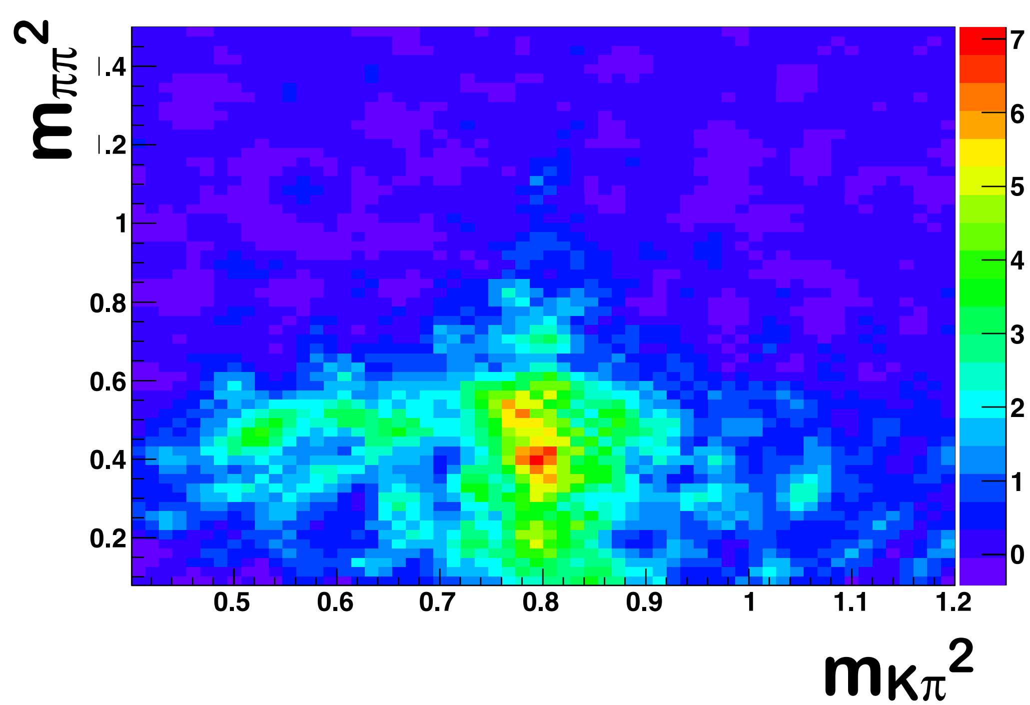
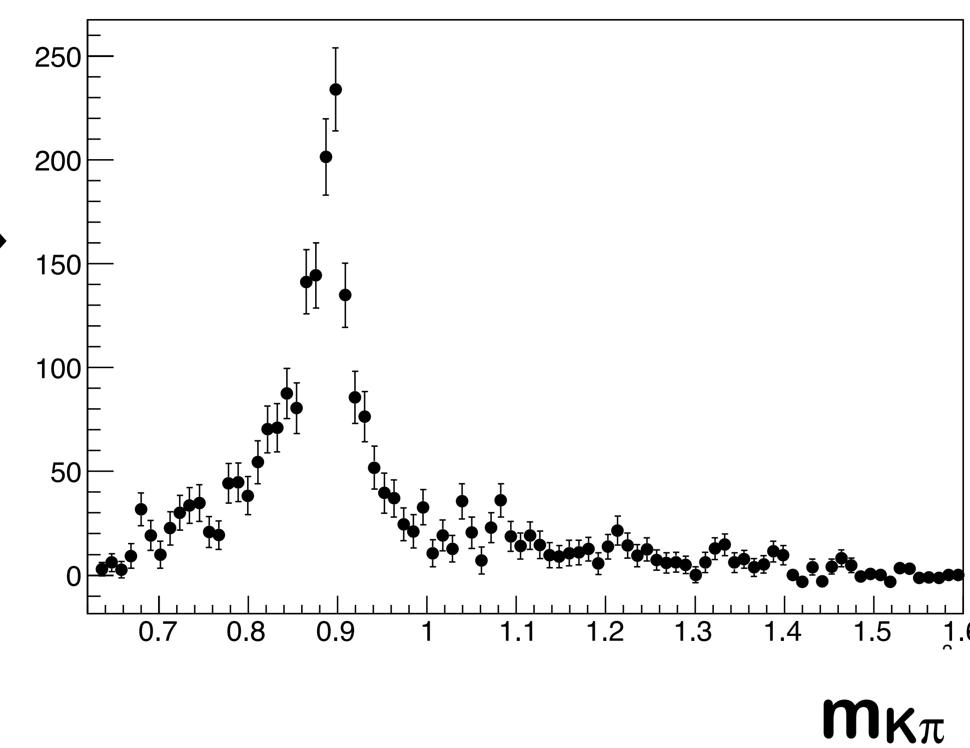
  - Extract the amplitudes in the projection over one dimension ($m_{K\pi}$ for instance)



projection



In the Real World Now…

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  - Even in the whole $m_{K\pi\pi}$ range difficult to perform an analysis in Pseudo-Dalitz plot
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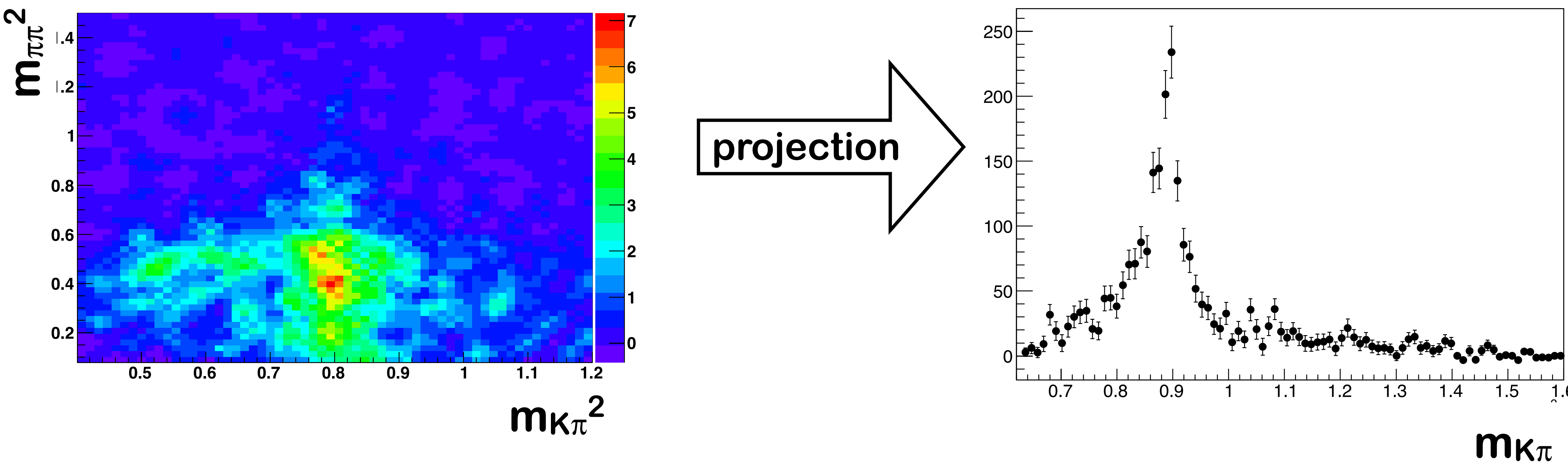


  - First perform a fit to the $m_{K\pi\pi}$ spectrum to extract the K_{res} amplitudes and their relative weights

- **Defined as the ratio:**

$$\mathcal{D}_{K^0_S\rho\gamma} \equiv \frac{\mathcal{S}_{K^0_S\pi^+\pi^-\gamma}}{\mathcal{S}_{K^0_S\rho\gamma}}$$

- **CP asymmetry when considering all the resonances** ρ^0, $K^{*\pm}$ or $(K\pi)^{\pm}$ S-wave **in the total amplitude:**

$$\mathcal{A}_{CP}^{K^0_S\pi^+\pi^-\gamma}(t) = \mathcal{C}_{K^0_S\pi^+\pi^-\gamma}\cos(\Delta\mathrm{M}t) + \mathcal{S}_{K^0_S\pi^+\pi^-\gamma}\sin(\Delta\mathrm{M}t)$$

- **CP asymmetry when considering only the ρ⁰ resonance in the total amplitude:**

$$\mathcal{A}_{CP}^{K^0_S\rho\gamma}(t) = \mathcal{C}_{K^0_S\rho\gamma}\cos(\Delta\mathrm{M}t) + \mathcal{S}_{K^0_S\rho\gamma}\sin(\Delta\mathrm{M}t)$$

The Dilution Factor
Analytical Expression

- **In terms of amplitudes, the dilution factor can be expressed as:**

$$\mathcal{D}_{K^0_S\rho\gamma} = \frac{\mathcal{S}_{K^0_S\pi^+\pi^-\gamma}}{\mathcal{S}_{K^0_S\rho\gamma}}$$

$$= \frac{\int \left[|A_\rho|^2 + \Re(A^*_\rho A_{K^{*+}}) + \Re(A^*_\rho A_{K^{*-}}) + \Re(A^*_{K^{*+}} A_{K^{*-}}) + \Re(A^*_{(K\pi)^+} A_{(K\pi)^-})\right]}{\int \left[|A_\rho|^2 + \Re(A^*_\rho A_{K^{*+}}) + \Re(A^*_\rho A_{K^{*-}}) + \frac{|A_{K^{*+}}|^2 + |A_{K^{*-}}|^2}{2} + \frac{|A_{(K\pi)^+}|^2 + |A_{(K\pi)^-}|^2}{2}\right]}$$

Integration performed over phase-space region

The amplitudes entering in the dilution factor expression are extracted from a fit to the $m_{K\pi}$ spectrum

I II III **IV** V VI

Study of $B \to K\pi\pi\gamma$ decays with the BaBar detector:

$B^+ \to K^+\pi^-\pi^+\gamma$ Analysis

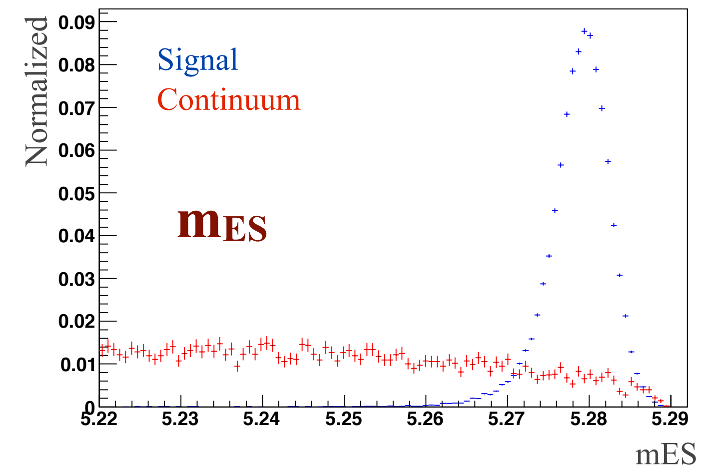
(1) Fit to m_{ES}, ΔE and Fisher: extraction of invariant mass spectra
(2) Fit to the $m_{K\pi\pi}$ spectrum: extraction of K_{res} amplitudes
(3) Fit to the $m_{K\pi}$ spectrum: extraction of $K^*(892)$, $\rho^0(770)$ and $(K\pi)$ S-wave amplitudes
(4) The dilution factor: computation

Fit to m_{ES}, ΔE and Fisher
Discriminating Variables

Using three discriminating variables to separate signal from backgrounds:

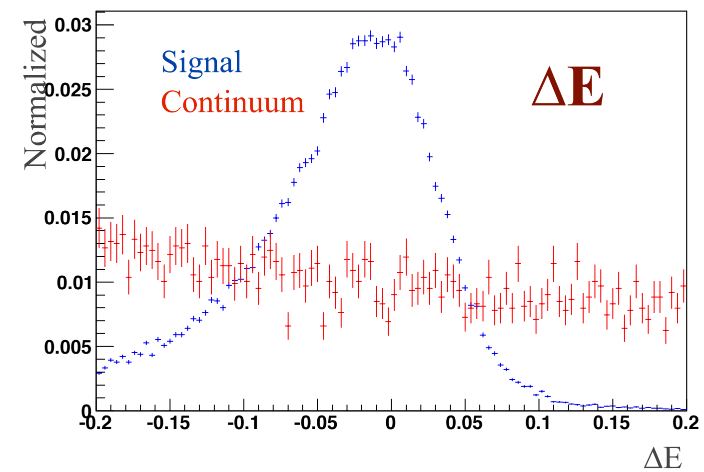
- **Energy-substituted mass** (m_{ES}):

$$m_{ES} \overset{\text{LAB}}{=} \sqrt{\left(s/2 + \mathbf{p}_B \cdot \mathbf{p}_0\right)^2 / E_0^2 - p_B^2} \overset{\text{CM}}{=} \sqrt{E^{*2}_{\text{beam}} - p^{*2}_B}$$

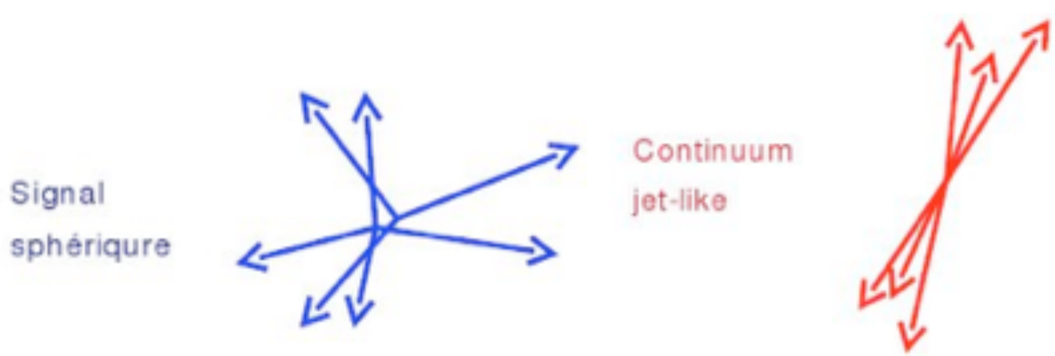
→ 

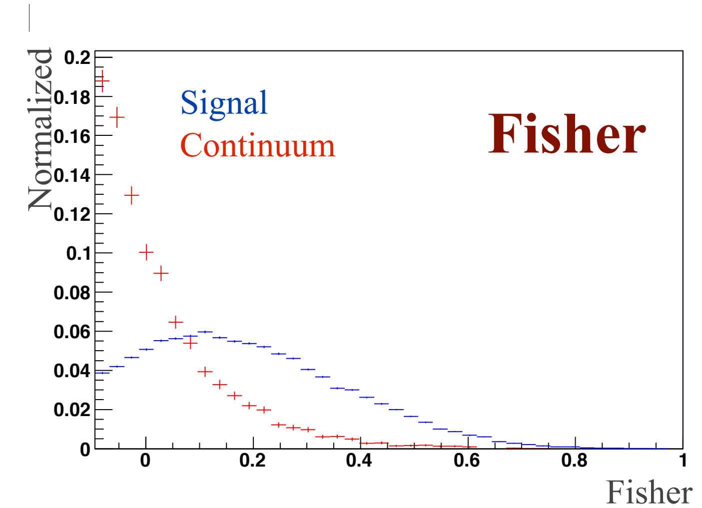
- **Energy difference** (ΔE):

$$\Delta E \overset{\text{LAB}}{=} (2q_B q_0 - s)/2\sqrt{s} \overset{\text{CM}}{=} E^*_B - E^*_{\text{beam}}$$

→ 

- **Fisher discriminant**:
  - ↳ Linear combination of 6 event-shape variables

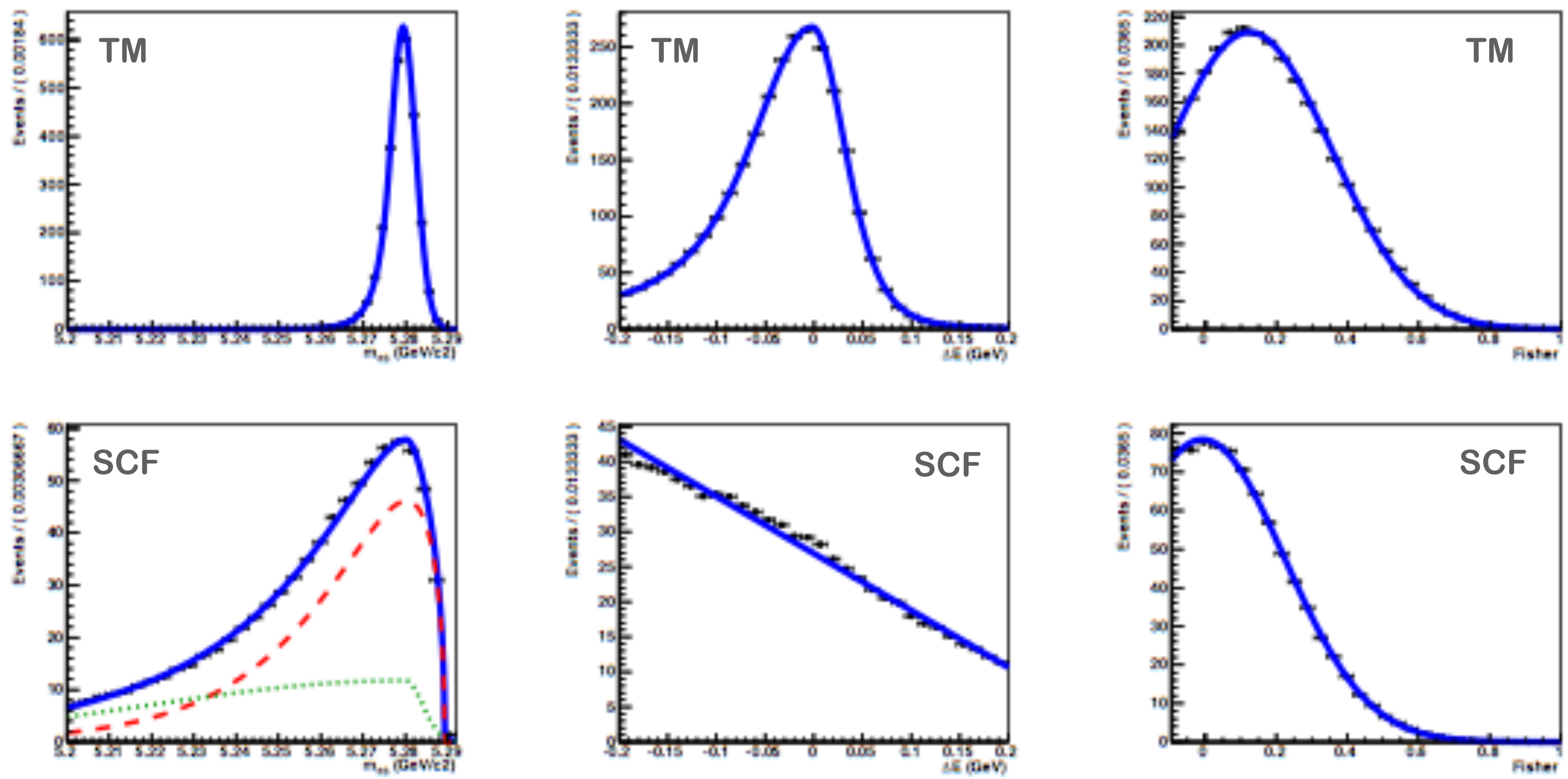


→ 

- **From $B^+ \rightarrow K^+ \pi^+ \pi^- \gamma$ <u>MC samples</u> separate signal events in two categories:**
  - ↳ **Truth Matched (TM):** Correctly reconstructed signal candidates (using MC truth information)
  - ↳ **Self Cross Feed (SCF):** Mis-reconstructed signal candidates
  - ↳ **We use a MC cocktail of different $B \rightarrow K_{res}(\rightarrow K\pi\pi)\,\gamma$**

Category	Expected Yield
Signal TM	**2295**
Signal SCF	686
Total Signal	2981

mES **ΔE** **Fisher discriminant**

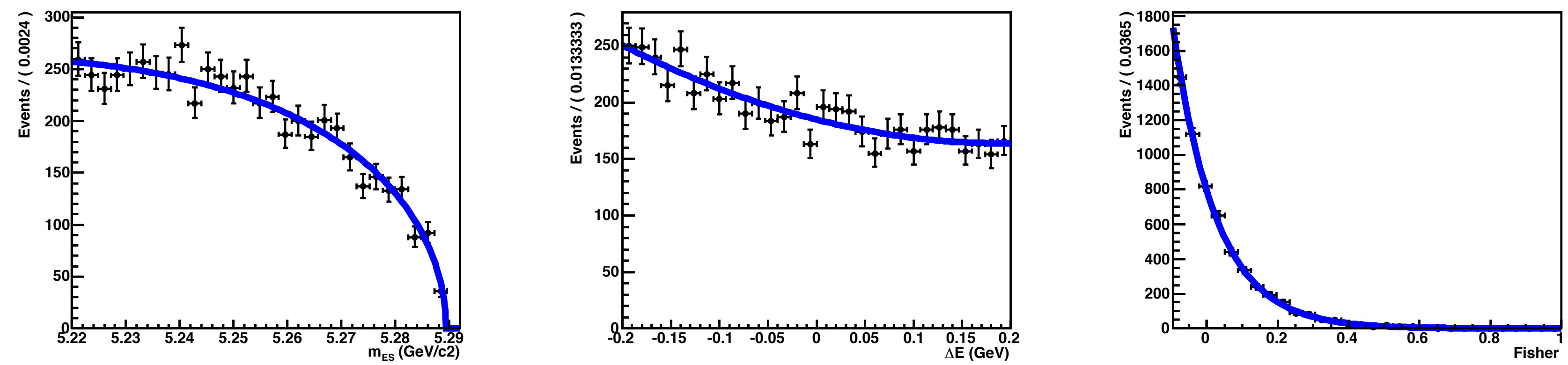


Fit to m_{ES}, ΔE and Fisher
Backgrounds (2)

Distribution of discriminating variables for a few backgrounds

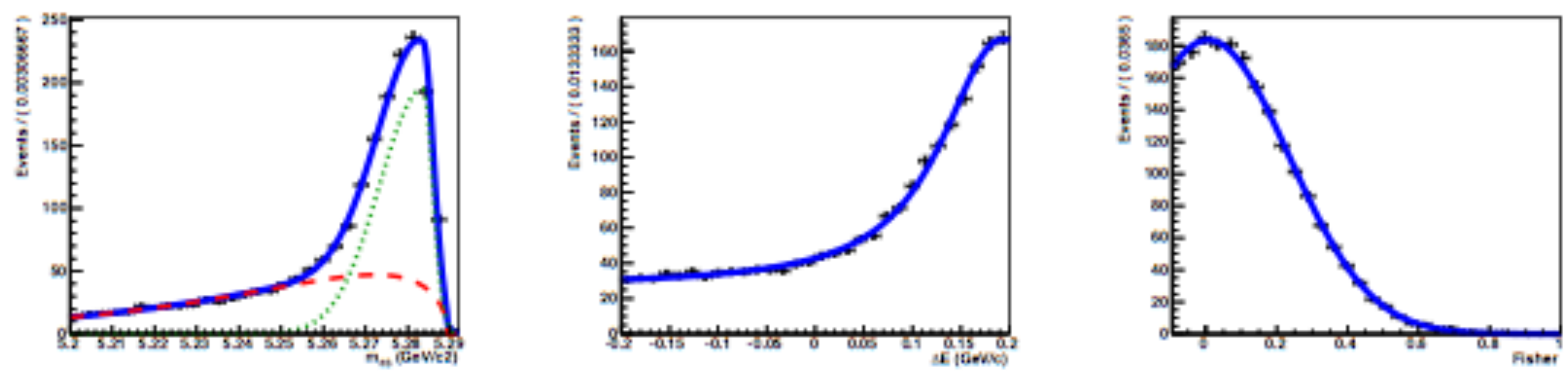
mES | ΔE | Fisher discriminant

Continuum
($e^+e^- \rightarrow q\bar{q}$; q = u,d,s,c) →

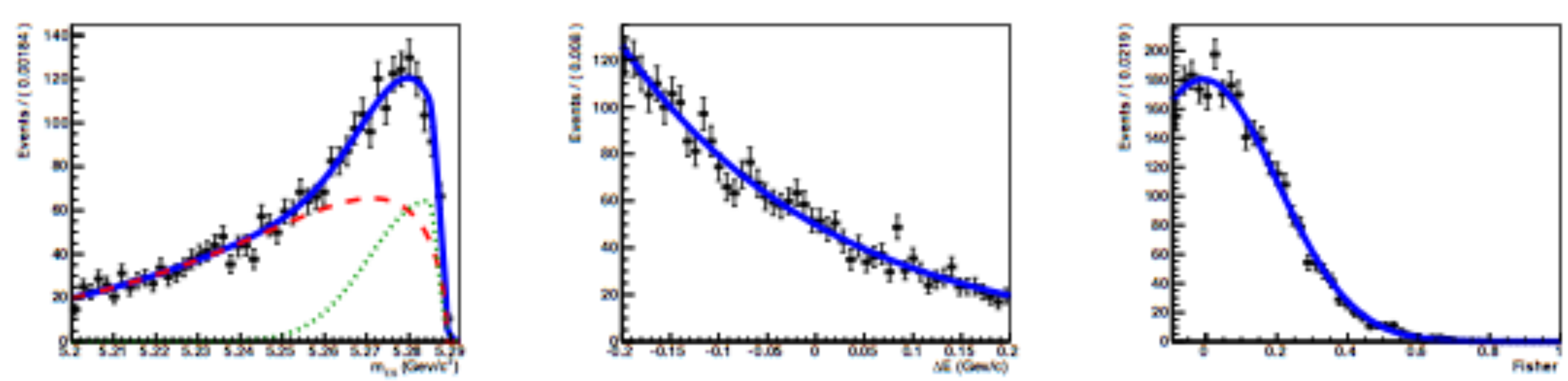


From B decays:

$B^0 \rightarrow K^{*0} (\rightarrow K\pi) \gamma$
one π originates from the other B →



$B^0 \rightarrow X_s (\nrightarrow K\pi) \gamma$
high multiplicity final state: one particle (or more) are missed →



Expected yields of all the backgrounds in the fit model

Continuum $(udsc)$	70983
$B^0 \to X_{sd}(\nrightarrow K\pi)\gamma$ $B^+ \to X_{su}(\nrightarrow K\pi)\gamma$	2872
$B^0 \to K^{*0}(\to K\pi)\gamma$ $B^0 \to X_{sd}(\to K\pi)\gamma$	1930
Generic B-background	1065
$B^+ \to K^{*+}(\to K\pi)\gamma$ $B^+ \to X_{su}(\to K\pi)\gamma$	442
$B^0 \to K^{*0}\eta$	56
$B^\pm \to a_1^\pm(\to \rho^0\pi^\pm)\pi^0\gamma$ $B^\pm \to K^{*0}(\to K\pi)\pi^\pm\pi^0\gamma$	17
Total Bkg	77365

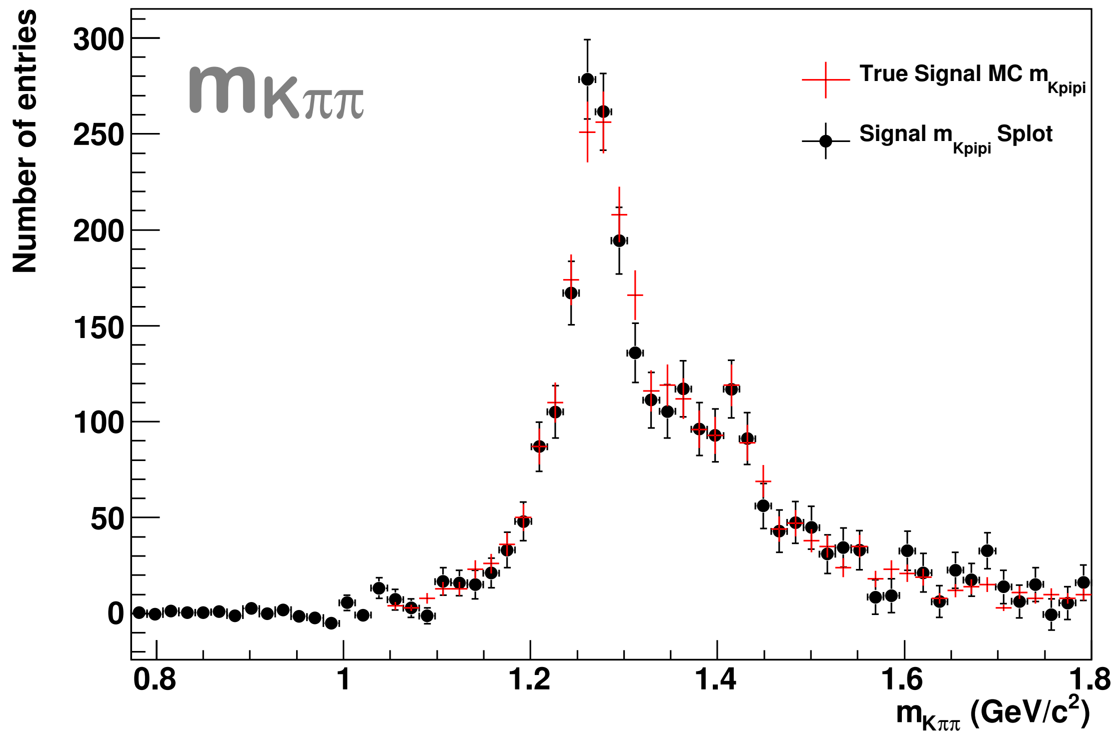
Category	Expected Yield
Signal	2295
SCF	686
Total Signal	2981

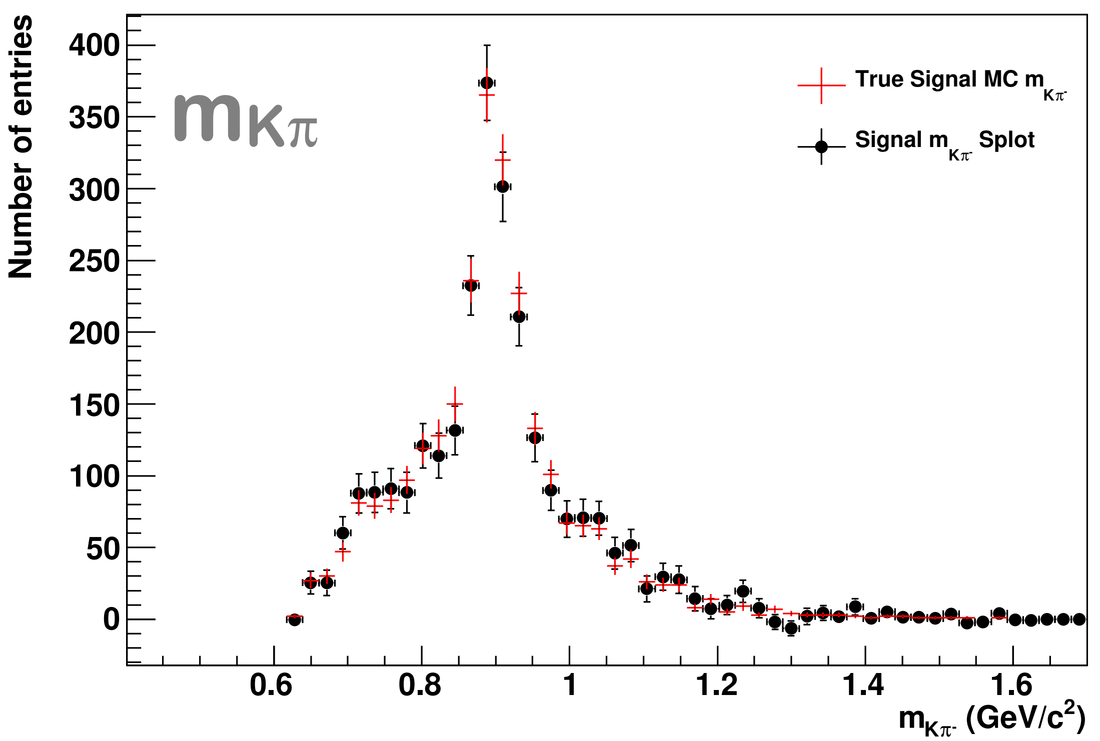
$B^+ \to K^+\pi^-\pi^+\gamma$:

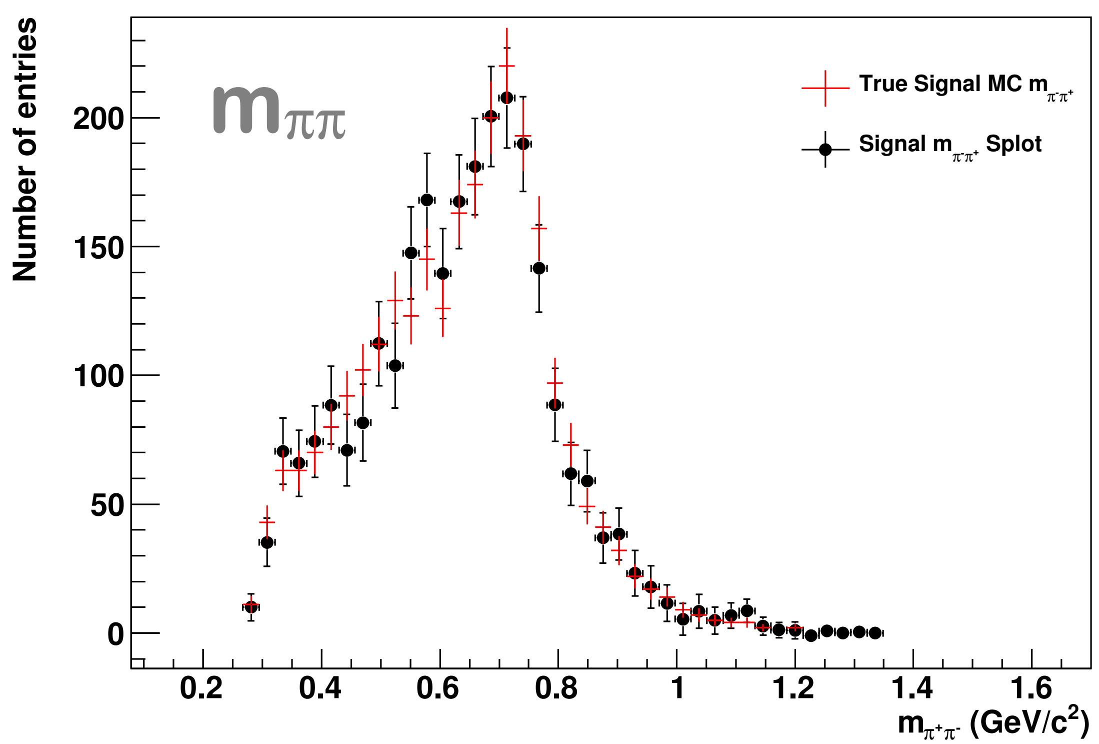
S / B ~ 1/34

Fit to M_{ES}, ΔE and Fisher
Invariant Mass Spectra Extraction (1)

- **sPlot** technique allows to **reconstruct a variable distribution without a priori knowledge** on this variable
- Use in the context of a maximum **Likelihood** method making use of the discriminating variables
- Apply event-by-event weights (**sWeights**) based on the likelihood function to extract the distributions for signal events

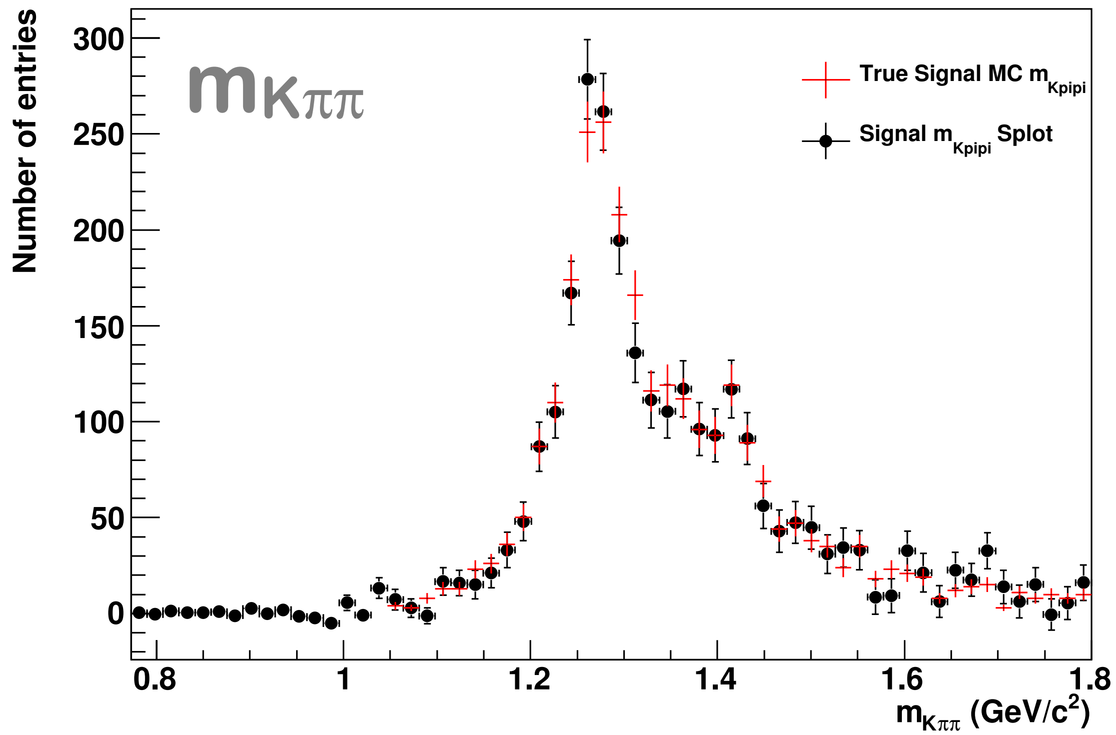


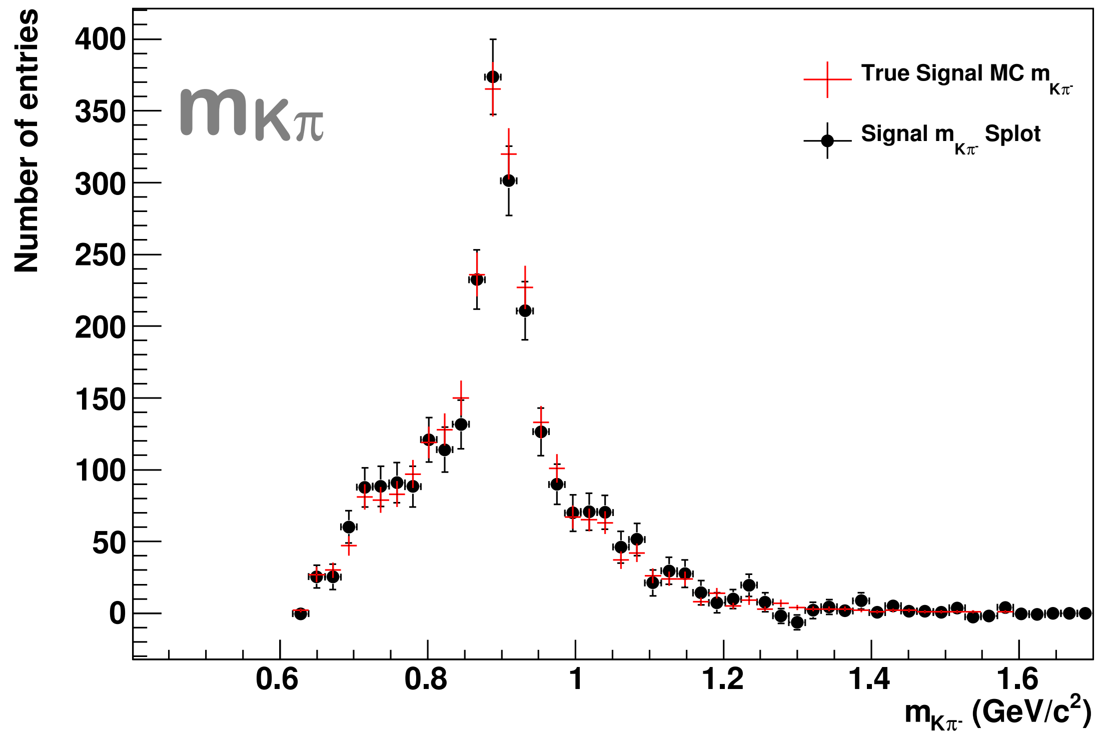


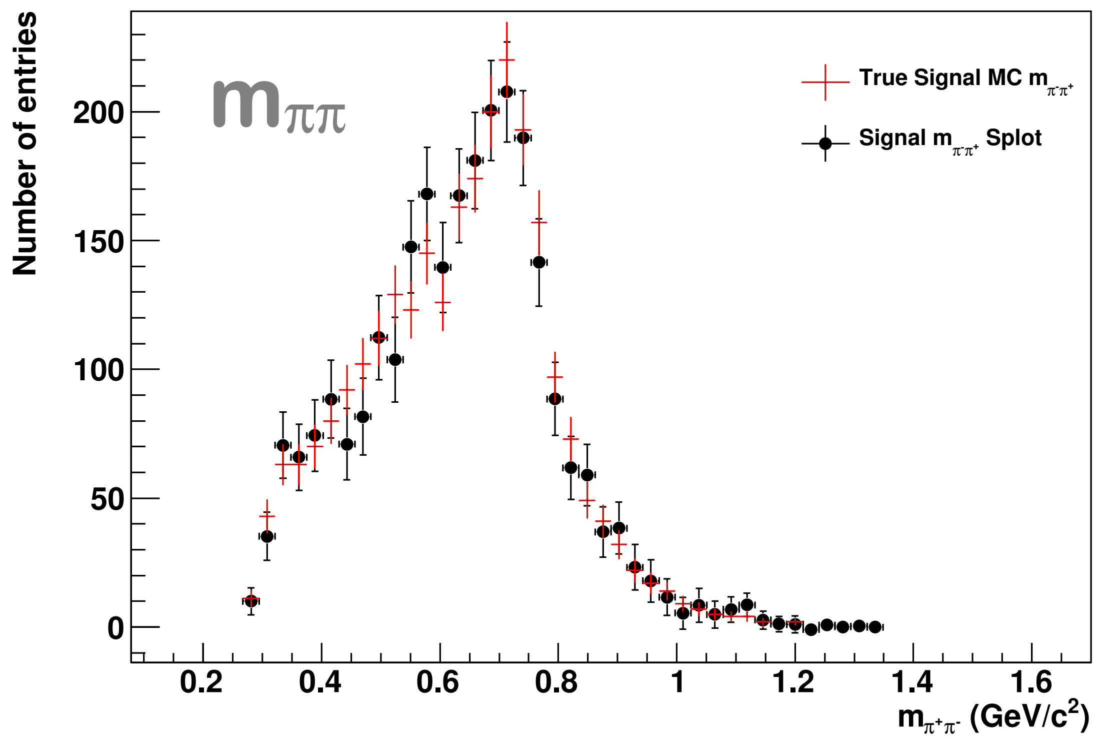


FIT TO M_{ES}, ΔE AND FISHER
INVARIANT MASS SPECTRA EXTRACTION (2)

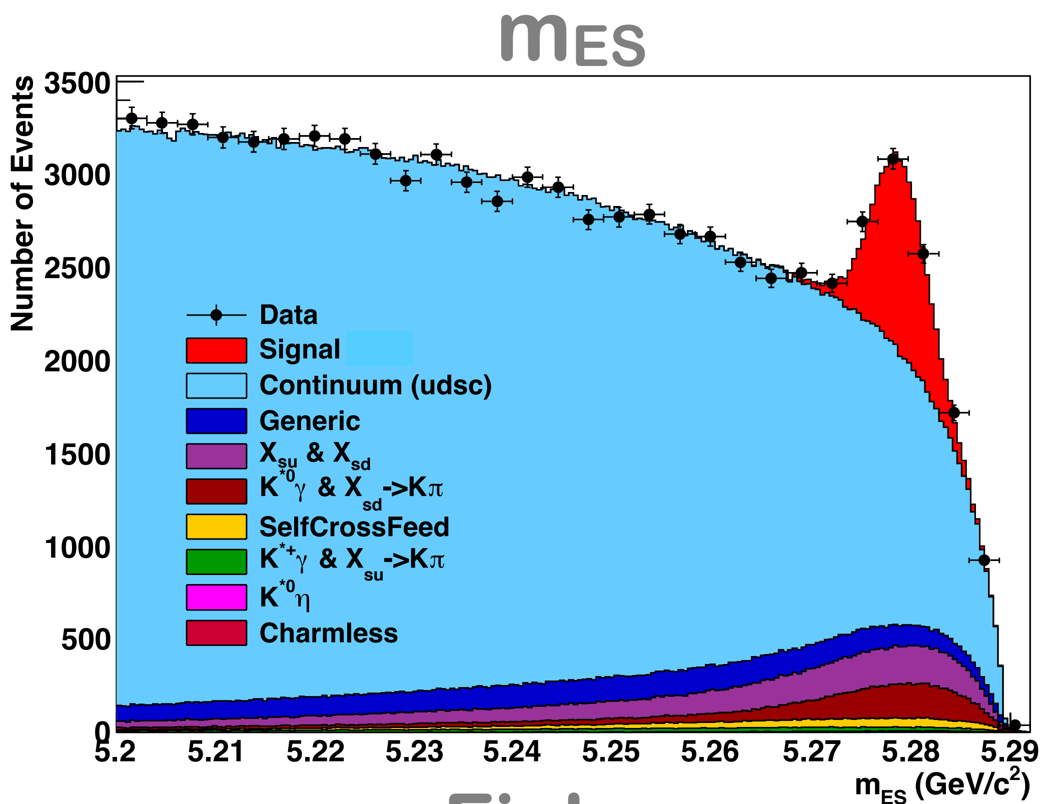
- To **validate the method**, we extract the invariant mass spectra **sPlot** from a fit to MC samples and compare to the **true MC distributions**

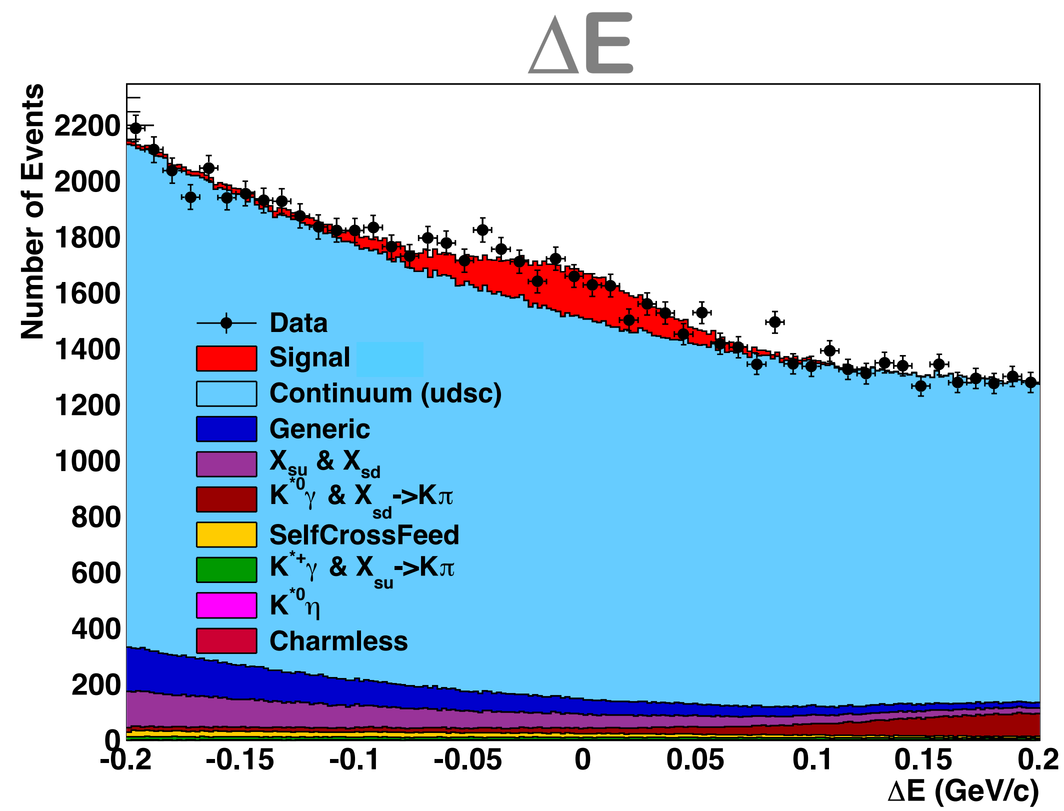


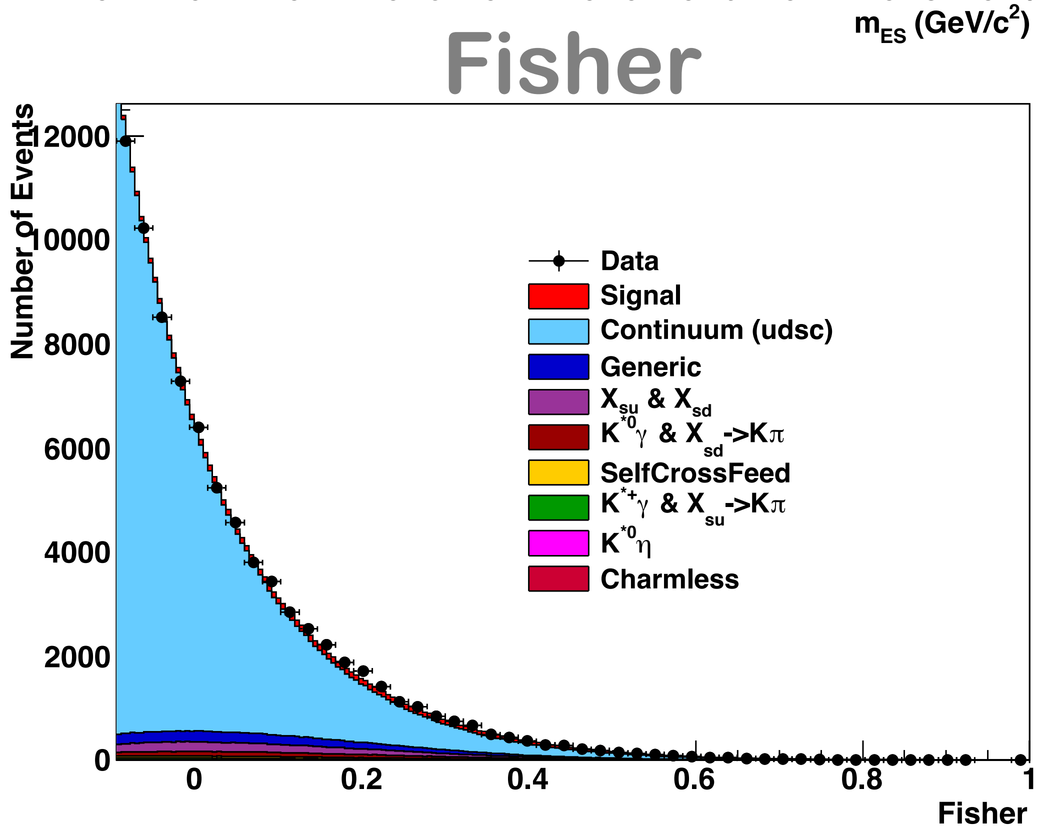




Fit to m_{ES}, ΔE and Fisher
Fit Projections & Yields







Yields from fit to data

	Value	Error (stat.)
Signal	2441.5	90.8
qqbar	70036.9	446.1
Generic	3266.6	385.0
K	1463.2	115.3

$M_{K\pi\pi}$ Spectrum Fit
Model

- **Model:**
  - **Five resonances modeled by BW** (mean and width fixed to PDG values):

J^P	$K_{\rm res}$	Mass m_j^0 (MeV/c^2)	Width Γ_j^0 (MeV/c^2)
1^+	$K_1(1270)$	1272 ± 7	90 ± 20
	$K_1(1400)$	1403 ± 7	174 ± 13
1^-	$K^*(1410)$	1414 ± 15	232 ± 21
	$K^*(1680)$	1717 ± 27	322 ± 110
2^+	$K_2^*(1430)$	1425.6 ± 1.5	98.5 ± 2.7

$$\mathrm{BW}_j^J(m) = \left.\frac{1}{(m_j^0)^2 - m^2 - i m_j^0 \Gamma_j^0}\right|_{m=m_{K\pi\pi}}$$

$$|A(m;c_j)|^2 = \sum_J \left|\sum_j c_j\, \mathrm{BW}_j^J(m)\right|^2 \Bigg|_{m=m_{K\pi\pi}}$$

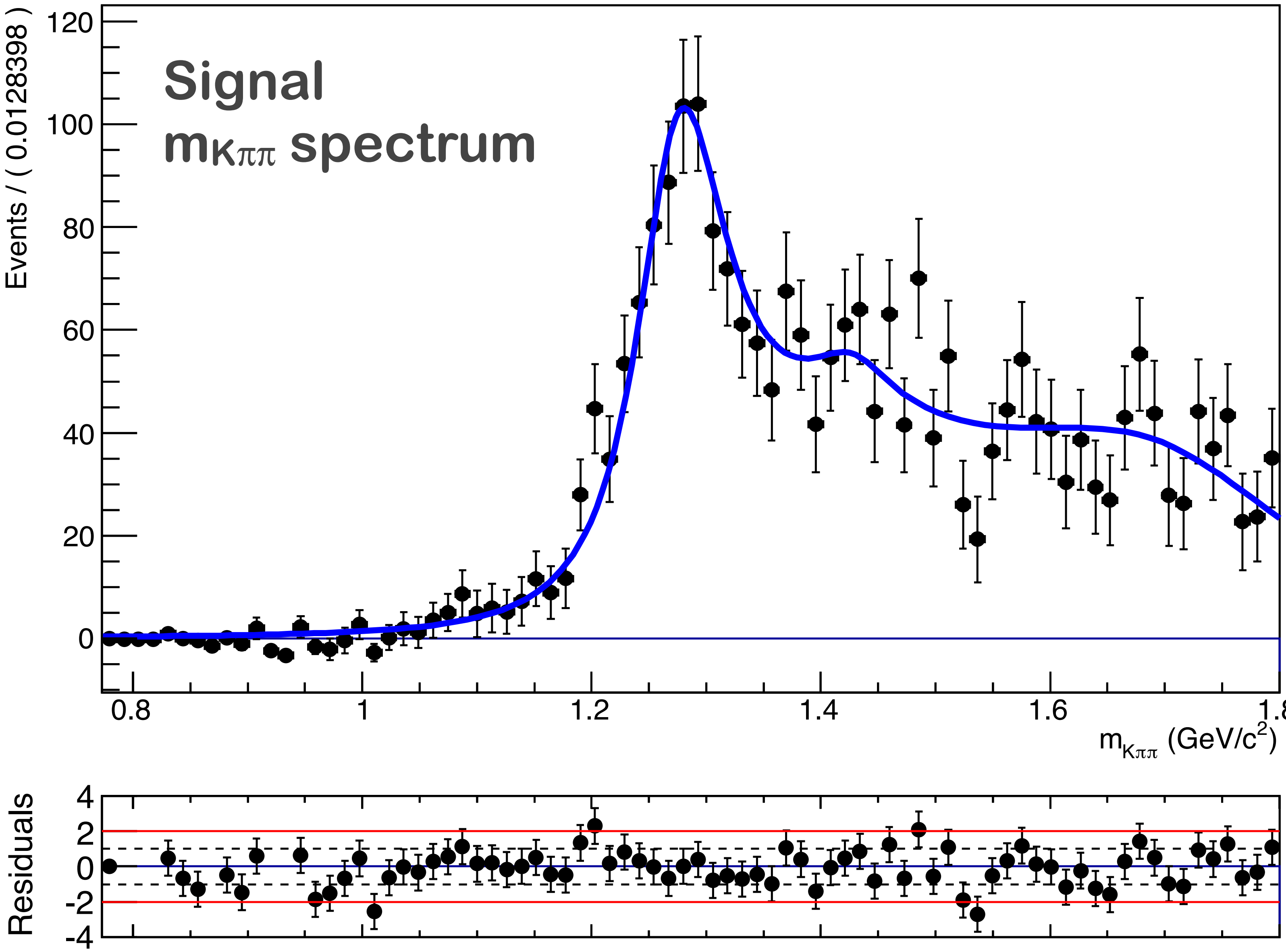
$$c_j = \alpha_j\, e^{i\phi_j}$$

- **Fit to Kππ invariant mass sPlot (binned) distribution**
  - **8 fitted parameters:**
    - **➜ 4 magnitudes, 2 relative phases**
    - **➜ 2 widths (K_1(1270) and K*(1680))**
  - **Due to the integration over the angular variables, only resonances with same J^P interfere**
  - **Randomized initial parameter values**
- **Fit fractions computed from magnitudes and phases**

$M_{K\pi\pi}$ Spectrum Fit
Results (1)

- **Errors on fit fractions :**
  - **From the fit results of the magnitudes and phases:** generated 10^5 random sets of magnitudes and phases (from nominal fit result) and computed corresponding fit fractions:

    ➜ value at ± 34.1% of distribution integral



$M_{K\pi\pi}$ Spectrum Fit
Results (2)

J^P	K_{res}	Magnitude α	Phase ϕ (rad.)	Fit fraction
1^+	$K_1(1270)$	1.0 (fixed)	0.0 (fixed)	$0.61^{+0.08}_{-0.05}$(stat.)$^{+0.05}_{-0.05}$(syst.)
	$K_1(1400)$	0.71 ± 0.10(stat.)$^{+0.12}_{-0.08}$(syst.)	2.97 ± 0.17(stat.)$^{+0.11}_{-0.12}$(syst.)	$0.17^{+0.08}_{-0.05}$(stat.)$^{+0.05}_{-0.03}$(syst.)
1^-	$K^*(1410)$	1.25 ± 0.16(stat.)$^{+0.18}_{-0.13}$(syst.)	3.15 ± 0.12(stat.)$^{+0.03}_{-0.02}$(syst.)	$0.37^{+0.08}_{-0.07}$(stat.)$^{+0.06}_{-0.02}$(syst.)
	$K^*(1680)$	2.02 ± 0.28(stat.)$^{+0.32}_{-0.21}$(syst.)	0.0 (fixed)	$0.43^{+0.05}_{-0.04}$(stat.)$^{+0.09}_{-0.06}$(syst.)
2^+	$K_2^*(1430)$	0.33 ± 0.09(stat.)$^{+0.07}_{-0.14}$(syst.)	0.0 (fixed)	$0.06^{+0.04}_{-0.03}$(stat.)$^{+0.04}_{-0.05}$(syst.)
Sum of fit fractions				$1.64^{+0.18}_{-0.14}$(stat.)$^{+0.14}_{-0.07}$(syst.)
Interference		$J^P = 1^+ : \{K_1(1270) - K_1(1400)\}$		$-0.35^{+0.10}_{-0.16}$(stat.)$^{+0.05}_{-0.06}$(syst.
		$J^P = 1^- : \{K^*(1410) - K^*(1680)\}$		$-0.29^{+0.08}_{-0.11}$(stat.)$^{+0.06}_{-0.12}$(syst.

From the fit fractions:
we calculate the BF, used in the fit to the $m_{K\pi}$ spectrum and in the $B^0 \rightarrow K_S\pi^-\pi^+\gamma$ analysis

$M_{K\pi\pi}$ Spectrum Fit
Results (3)

BABAR **not yet approved**

Mode	$\mathcal{B}(B^+ \to \text{Mode})\times \mathcal{B}(K_{res} \to K^+\pi^+\pi^-) \times 10^{-6}$	$\mathcal{B}(B^+ \to \text{Mode}) \times 10^{-6}$	PDG values ($\times 10^{-6}$)
Inclusive $B^+ \to K^+\pi^+\pi^-\gamma$	$\cdots$	$27.21 \pm 1.01^{+1.14}_{-1.25}$	27.6 ± 2.2
$K_1(1270)^+\gamma$	$14.47^{+1.97+1.14}_{-1.30-1.23}$	$44.04^{+6.00+3.48}_{-3.97-3.73} \pm 4.58$	43 ± 13
$K_1(1400)^+\gamma$	$4.07^{+1.92+1.29}_{-1.21-0.76}$	$9.65^{+4.55+3.05}_{-2.86-1.80} \pm 0.61$	< 15 CL$= 90\%$
$K^*(1410)^+\gamma$	$9.71^{+2.13+2.42}_{-1.87-0.68}$	$23.83^{+5.23+5.94}_{-4.59-1.43} \pm 2.38$	$\emptyset$
$K_2^*(1430)^+\gamma$	$1.45^{+1.21+0.87}_{-0.97-1.38}$	$10.41^{+8.68+6.34}_{-6.95-9.88} \pm 0.54$	14 ± 4
$K^*(1680)^+\gamma$	$17.03^{+1.71+3.49}_{-1.35-2.99}$	$71.67^{+7.18+14.70}_{-5.67-12.58} \pm 5.83$	< 1900 CL$= 90\%$

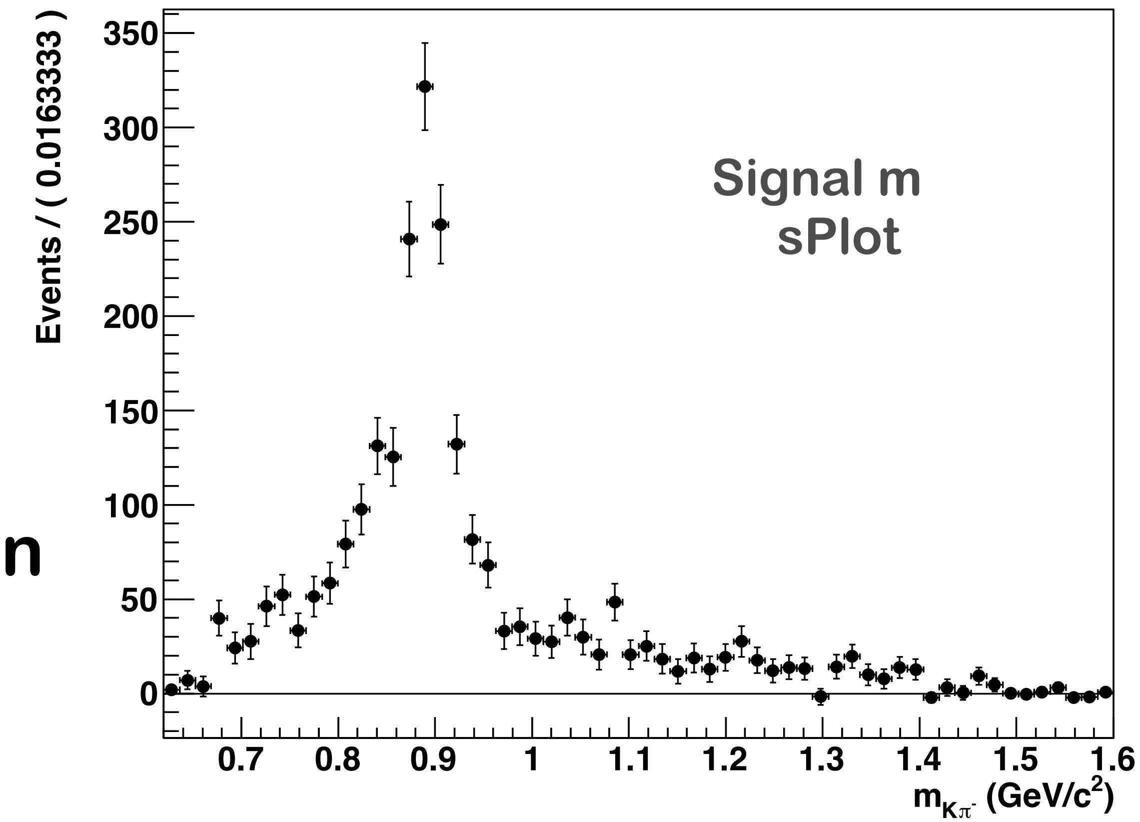
Good agreement with the existing measurements

$M_{K\pi}$ Spectrum Fit
Strategy

- Extraction of the dilution factor with a full amplitude analysis in $m_{K\pi}$-$m_{\pi\pi}$ plane is difficult (small sample)

- Instead: perform a one-dimensional fit to signal $m_{K\pi}$ sPlot corrected for efficiency:

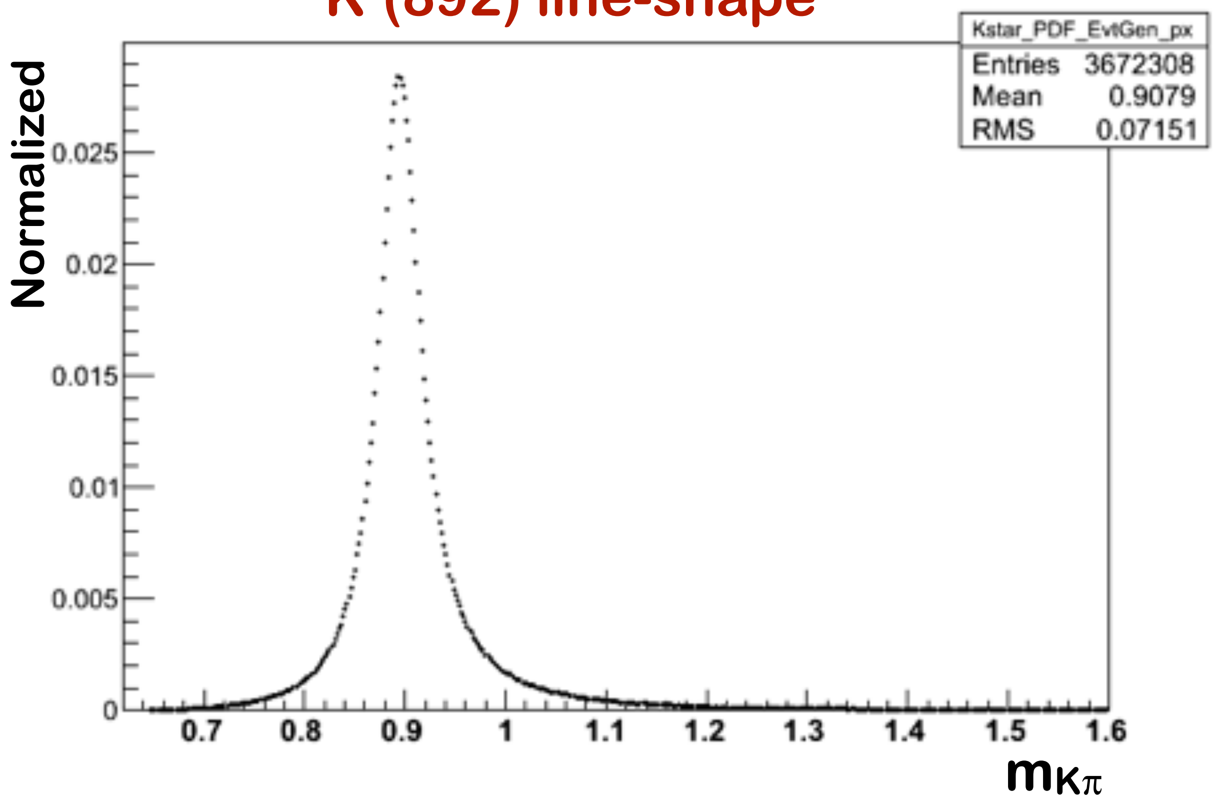
  - Built efficiency maps in $m_{K\pi}$ - $m_{\pi\pi}$ plane
  - A unique PDF: coherent sum of $K^*(892)$, $\rho^0(770)$ and $K\pi$ S-wave.
    ↳ All projected on the $m_{K\pi}$ dimension



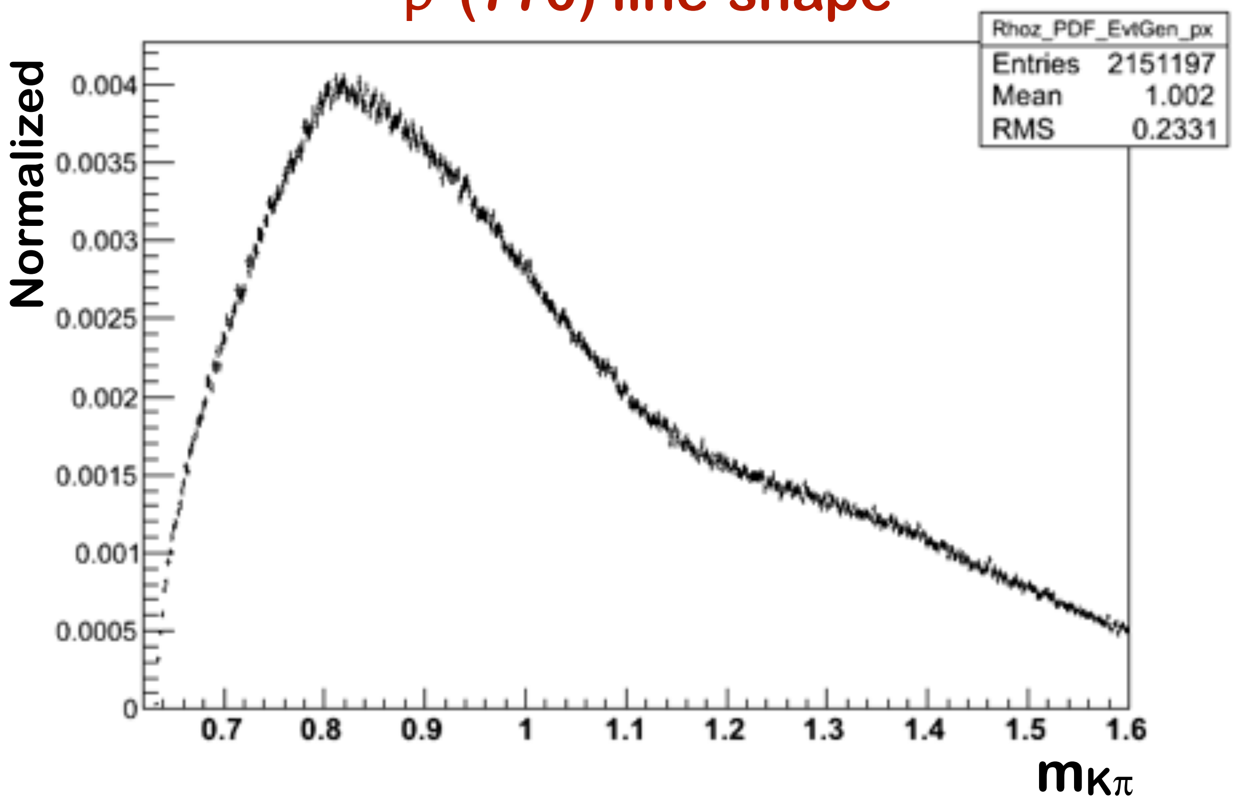
Line-shapes:

- Line-shapes significantly distorted due to phase-space effects
- Extracted from MC distributions at generator level using EvtGen:
  - Take phase-space corrections into account
  - To be used to fit efficiency-corrected TM signal sPlot
- Used fit based BR of the different $B \to K_{res}\,\gamma$

$K^{*}(892)$ line-shape



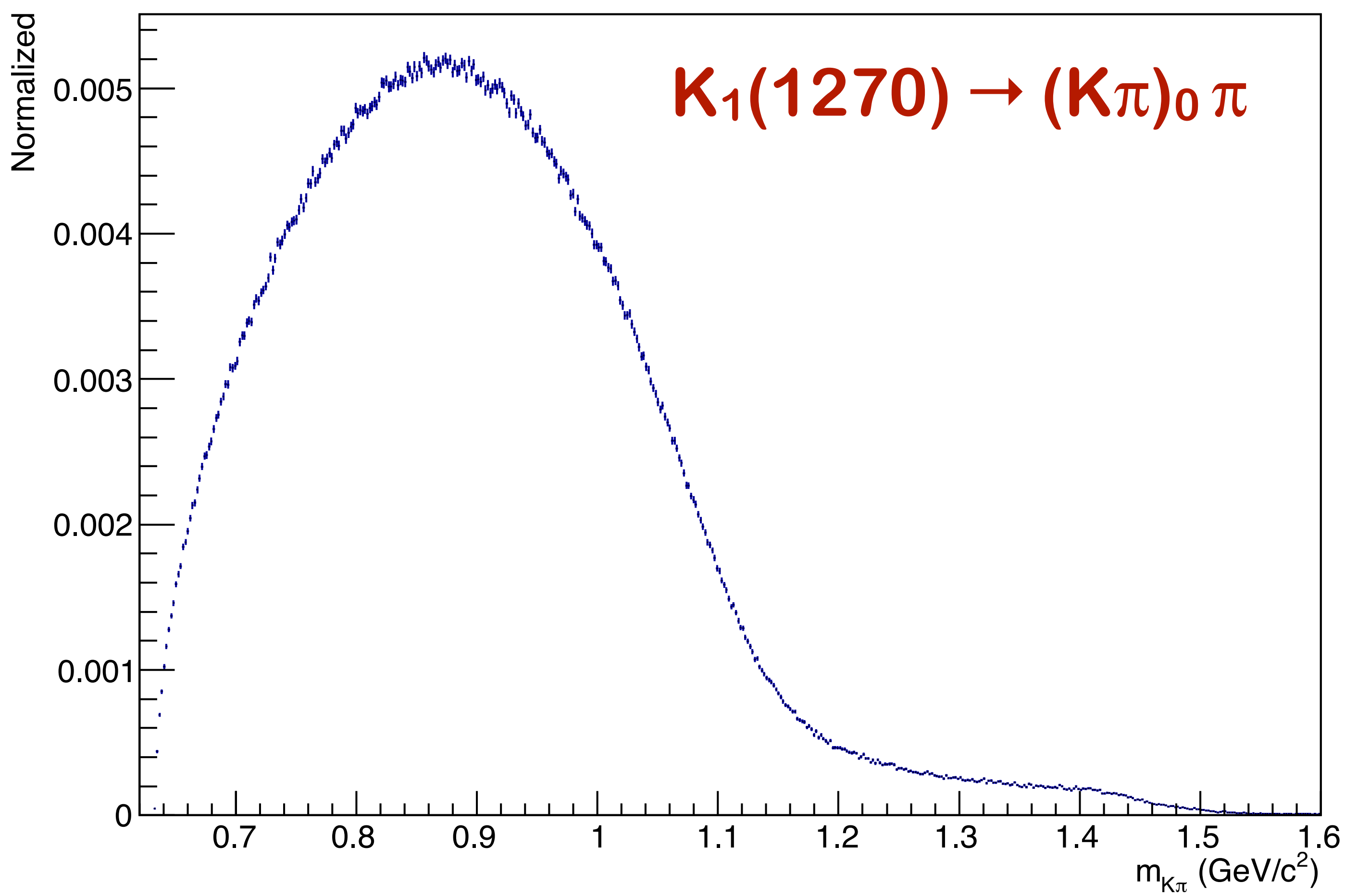
$\rho^{0}(770)$ line-shape



Line-shapes:

- Line-shapes significantly distorted due to phase-space effects
- Extracted from MC distributions at generator level using EvtGen:
  - Take phase-space corrections into account
  - To be used to fit efficiency-corrected TM signal sPlot

S-wave line-shape (using LASS)



Total PDF:

- **Coherent sum of K*(892), ρ⁰(770) and Kπ S-wave component:**

$$|A(m_{K\pi};c_j)|^2 = \left| \int_{m_{\pi\pi}^{min}}^{m_{\pi\pi}^{max}} \left(\sum_j c_j \sqrt{\mathrm{H}_{\mathrm{R}_j}(m_{K\pi}, m_{\pi\pi})}\, e^{i\Phi_{\mathrm{R}_j}(m)} \right) dm_{\pi\pi} \right|^2 , \quad c_j = \alpha_j\, e^{i\phi_j}$$

$$= \left|c_{K^*}\right|^2 \mathcal{H}_{K^*} + \left|c_{\rho^0}\right|^2 \mathcal{H}_{\rho^0} + \left|c_{(K\pi)_0}\right|^2 \mathcal{H}_{(K\pi)_0} + I$$

- **Invariant-mass-dependent magnitude defined as the projection of two-dimensional histograms:**

$$\mathcal{H}_{R_j}(m_{K\pi}) = \int_{m_{\pi\pi}^{min}}^{m_{\pi\pi}^{max}} H_{R_j}(m_{K\pi}, m_{\pi\pi})\, dm_{\pi\pi}.$$

- **The invariant-mass-dependent phase is taken from the analytical expression of the corresponding line shape:**

$$\Phi_{\mathrm{R}_j}(m) = \arccos\left(\frac{\Re[\mathrm{R}_j(m)]}{|\mathrm{R}_j(m)|} \right) \Leftrightarrow \begin{cases} m = m_{K\pi} \ \Rightarrow & \mathrm{R}_j(m_{K\pi}) \text{ is taken as RBW for } K^{*0}(892) \text{ and as LASS for S-wave ,} \\ m = m_{\pi\pi} \ \Rightarrow & \mathrm{R}_j(m_{\pi\pi}) \text{ is taken as a GS line shape for } \rho^0(770)\, , \end{cases}$$

Total PDF:

- **Coherent sum of K*(892), ρ⁰(770) and Kπ S-wave component:**

$$|A(m_{K\pi};c_j)|^2 = \left|\int_{m_{\pi\pi}^{min}}^{m_{\pi\pi}^{max}} \left(\sum_j c_j \sqrt{\mathrm{H}_{\mathrm{R}_j}(m_{K\pi},m_{\pi\pi})}\, e^{i\Phi_{\mathrm{R}_j}(m)}\right) dm_{\pi\pi}\right|^2 , \quad c_j = \alpha_j\, e^{i\phi_j}$$

$$= |c_{K^*}|^2\,\mathcal{H}_{K^*} + |c_{\rho^0}|^2\,\mathcal{H}_{\rho^0} + |c_{(K\pi)_0}|^2\,\mathcal{H}_{(K\pi)_0} + I$$

Parameters in the fit:
2 fixed as reference - 4 free

- **Invariant-mass-dependent magnitude defined as the projection of two-dimensional histograms:**

$$\mathcal{H}_{R_j}(m_{K\pi}) = \int_{m_{\pi\pi}^{min}}^{m_{\pi\pi}^{max}} H_{R_j}(m_{K\pi},m_{\pi\pi})\, dm_{\pi\pi}.$$

- **The invariant-mass-dependent phase is taken from the analytical expression of the corresponding line shape:**

$$\Phi_{\mathrm{R}_j}(m) = \arccos\left(\frac{\Re[\mathrm{R}_j(m)]}{|\mathrm{R}_j(m)|}\right) \Leftrightarrow \begin{cases} m = m_{K\pi} & \Rightarrow \begin{array}{l}\mathrm{R}_j(m_{K\pi}) \text{ is taken as} \\ \text{RBW for } K^{*0}(892) \text{ and} \\ \text{as LASS for S-wave ,}\end{array} \\ m = m_{\pi\pi} & \Rightarrow \begin{array}{l}\mathrm{R}_j(m_{\pi\pi}) \text{ is taken as a GS} \\ \text{line shape for } \rho^0(770)\,,\end{array}\end{cases}$$

Total PDF:

- **Coherent sum of K*(892), ρ⁰(770) and Kπ S-wave component:**

$$|A(m_{K\pi};c_j)|^2 = \left| \int_{m_{\pi\pi}^{min}}^{m_{\pi\pi}^{max}} \left(\sum_j c_j \sqrt{\mathrm{H}_{\mathrm{R}_j}(m_{K\pi}, m_{\pi\pi})}\, e^{i\Phi_{\mathrm{R}_j}(m)} \right) dm_{\pi\pi} \right|^2 , \quad c_j = \alpha_j\, e^{i\phi_j}$$

$$= \left|c_{K^*}\right|^2 \mathcal{H}_{K^*} + \left|c_{\rho^0}\right|^2 \mathcal{H}_{\rho^0} + \left|c_{(K\pi)_0}\right|^2 \mathcal{H}_{(K\pi)_0} + I$$

Interference term described in next slide

- **Invariant-mass-dependent magnitude defined as the projection of two-dimensional histograms:**

$$\mathcal{H}_{R_j}(m_{K\pi}) = \int_{m_{\pi\pi}^{min}}^{m_{\pi\pi}^{max}} H_{R_j}(m_{K\pi}, m_{\pi\pi})\, dm_{\pi\pi}.$$

- **The invariant-mass-dependent phase is taken from the analytical expression of the corresponding line shape:**

$$\Phi_{\mathrm{R}_j}(m) = \arccos\left(\frac{\Re[\mathrm{R}_j(m)]}{|\mathrm{R}_j(m)|}\right) \Leftrightarrow \begin{cases} m = m_{K\pi} \Rightarrow & \mathrm{R}_j(m_{K\pi}) \text{ is taken as RBW for } K^{*0}(892) \text{ and as LASS for S-wave ,} \\ m = m_{\pi\pi} \Rightarrow & \mathrm{R}_j(m_{\pi\pi}) \text{ is taken as a GS line shape for } \rho^0(770) \text{ ,} \end{cases}$$

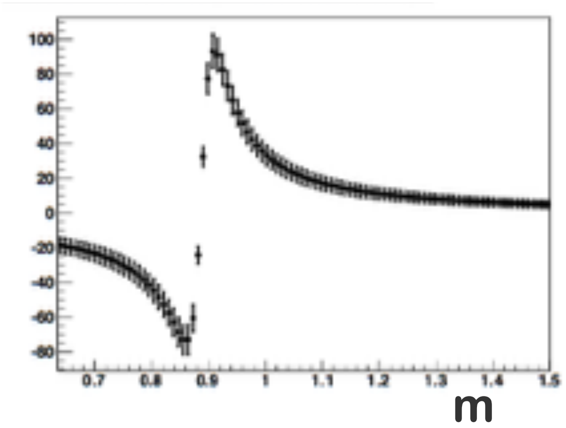
$M_{K\pi}$ Spectrum Fit
Model (4)

Interference:

- **Interference terms:**

Illustration:

RBW+GS interf. (ϕ_{ρ^0}= π/2)



$$\begin{aligned} I(m_{K\pi}; c_{\rho^0}, c_{(K\pi)_0}) \quad = \quad & 2\alpha_{\rho^0}\Bigg[\cos(\phi_{\rho^0} - \Phi_{\mathrm{RBW}})\int_{m_{\pi\pi}^{min}}^{m_{\pi\pi}^{max}} \sqrt{\mathrm{H}_{\rho^0}\mathrm{H}_{K^*}}\cos(\Phi_{\mathrm{GS}})\,dm_{\pi\pi} \\ & - \sin(\phi_{\rho^0} - \Phi_{\mathrm{RBW}})\int_{m_{\pi\pi}^{min}}^{m_{\pi\pi}^{max}} \sqrt{\mathrm{H}_{\rho^0}\mathrm{H}_{K^*}}\sin(\Phi_{\mathrm{GS}})\,dm_{\pi\pi}\Bigg] \\ & + 2\alpha_{\rho^0}\alpha_{(K\pi)_0}\Bigg[\cos(\phi_{\rho^0} - \phi_{(K\pi)_0} - \Phi_{\mathrm{LASS}})\int_{m_{\pi\pi}^{min}}^{m_{\pi\pi}^{max}} \sqrt{\mathrm{H}_{\rho^0}\mathrm{H}_{(K\pi)_0}}\cos(\Phi_{\mathrm{GS}})\,dm_{\pi\pi} \\ & - \sin(\phi_{\rho^0} - \phi_{(K\pi)_0} - \Phi_{\mathrm{LASS}})\int_{m_{\pi\pi}^{min}}^{m_{\pi\pi}^{max}} \sqrt{\mathrm{H}_{\rho^0}\mathrm{H}_{(K\pi)_0}}\sin(\Phi_{\mathrm{GS}})\,dm_{\pi\pi}\Bigg] \; . \end{aligned}$$

Term describing interference between the K*(892) and ρ^0(770) amplitudes

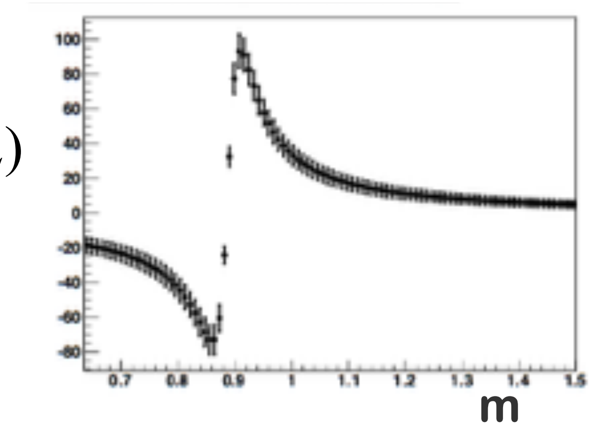
Term describing interference between the ρ^0(770) and (Kπ) S-wave amplitudes

Interference:

- **Interference terms:**

Illustration:

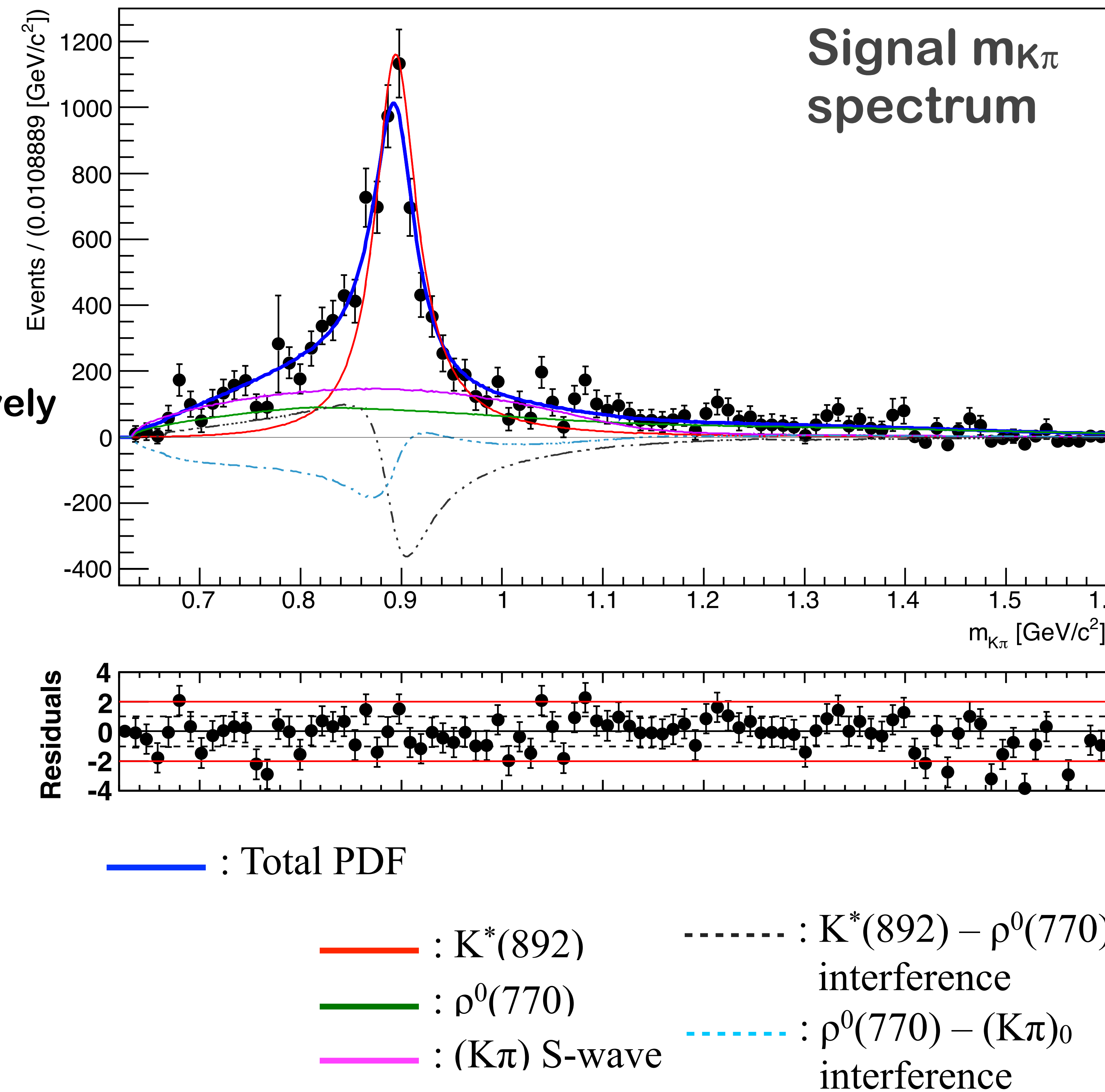
RBW+GS interf. ($\phi_{\rho^0}=\pi/2$)



$$
\begin{aligned}
I(m_{K\pi}; c_{\rho^0}, c_{(K\pi)_0}) \; = \;& 2\alpha_{\rho^0}\Bigg[\cos(\phi_{\rho^0}-\Phi_{\mathrm{RBW}})\int_{m_{\pi\pi}^{min}}^{m_{\pi\pi}^{max}} \sqrt{\mathrm{H}_{\rho^0}\mathrm{H}_{K^*}}\cos(\Phi_{\mathrm{GS}})\,dm_{\pi\pi} \\
& - \sin(\phi_{\rho^0}-\Phi_{\mathrm{RBW}})\int_{m_{\pi\pi}^{min}}^{m_{\pi\pi}^{max}} \sqrt{\mathrm{H}_{\rho^0}\mathrm{H}_{K^*}}\sin(\Phi_{\mathrm{GS}})\,dm_{\pi\pi}\Bigg] \\
& + 2\alpha_{\rho^0}\alpha_{(K\pi)_0}\Bigg[\cos(\phi_{\rho^0}-\phi_{(K\pi)_0}-\Phi_{\mathrm{LASS}})\int_{m_{\pi\pi}^{min}}^{m_{\pi\pi}^{max}} \sqrt{\mathrm{H}_{\rho^0}\mathrm{H}_{(K\pi)_0}}\cos(\Phi_{\mathrm{GS}})\,dm_{\pi\pi} \\
& - \sin(\phi_{\rho^0}-\phi_{(K\pi)_0}-\Phi_{\mathrm{LASS}})\int_{m_{\pi\pi}^{min}}^{m_{\pi\pi}^{max}} \sqrt{\mathrm{H}_{\rho^0}\mathrm{H}_{(K\pi)_0}}\sin(\Phi_{\mathrm{GS}})\,dm_{\pi\pi}\Bigg] \; .
\end{aligned}
$$

The interference between the K*(892) and (Kπ) S-wave amplitudes vanishes due to the integration over the $m_{\pi\pi}$ dimension

- **Free parameters:**
  - modules and phases of ρ^0 and S-wave
- **Fixed parameters:**
  - module and phase of $K^*(892)$ fixed as reference to 1 and 0 respectively
- **Check for multiple solutions:**
  - Performed ~2x10^4 fits with random initial parameter values
    ↳ All converged to the same solution
- **Errors of decay fractions:**
  - Stat: 10^4 random generation from nominal fit result
    ↳ value at ± 34.1% of distribution integral



$M_{K\pi}$ Spectrum Fit
Results (2)

	Module α	Phase ϕ (rad.)	Fit Fraction
$K^{*0}(892)$	1.0 (fixed)	0.0 (fixed)	$0.636^{+0.011}_{-0.009}(\text{stat.})^{+0.017}_{-0.012}(\text{syst.})$
$\rho^0(770)$	$0.725^{+0.015}_{-0.015}(\text{stat.})^{+0.013}_{-0.022}(\text{syst.})$	$3.110^{+0.036}_{-0.035}(\text{stat.})^{+0.060}_{-0.048}(\text{syst.})$	$0.335^{+0.015}_{-0.013}(\text{stat.})^{+0.033}_{-0.032}(\text{syst.})$
$(K\pi)$ S-wave	$0.808^{+0.044}_{-0.050}(\text{stat.})^{+0.044}_{-0.058}(\text{syst.})$	$3.197^{+0.132}_{-0.125}(\text{stat.})^{+0.126}_{-0.101}(\text{syst.})$	$0.416^{+0.039}_{-0.041}(\text{stat.})^{+0.056}_{-0.072}(\text{syst.})$
Sum of all fit fractions			$1.387^{+0.048}_{-0.042}(\text{stat.})^{+0.106}_{-0.088}(\text{syst.})$
Interferences			
$K^{*0}(892)$—$\rho^0(770)$			$-0.178^{+0.004}_{-0.006}(\text{stat.})^{+0.008}_{-0.010}(\text{syst.})$
$(K\pi)$ S-wave—$\rho^0(770)$			$-0.208^{+0.029}_{-0.044}(\text{stat.})^{+0.032}_{-0.049}(\text{syst.})$

Fit fractions to be used in the computation of the dilution factor value.

Take the opportunity to calculate the corresponding branching fractions

$M_{K\pi}$ Spectrum Fit
Results (3)

BABAR **not yet approved**

Mode	$\mathcal{B}(B^+ \to \text{Mode})\times$ $\mathcal{B}(R \to hh) \times 10^{-6}$	$\mathcal{B}(B^+ \to \text{Mode}) \times 10^{-6}$	PDG values ($\times 10^{-6}$)
Inclusive $B^+ \to K^+\pi^+\pi^-\gamma$	$\cdots$	$27.21 \pm 1.01^{+1.14}_{-1.25}$	27.6 ± 2.2
$K^{*0}(892)\pi^+\gamma$	$17.31^{+0.94+1.19}_{-0.89-1.12}$	$25.96^{+1.42+1.79}_{-1.34-1.68}$	20^{+7}_{-6}
$K^+\rho(770)^0\gamma$	$9.12^{+0.75+1.30}_{-0.69-1.31}$	$9.21^{+0.76+1.31}_{-0.70-1.32} \pm 0.02$	< 20 CL$=90\%$
$(K\pi)^{*0}_0\pi^+\gamma$	$11.32^{+1.48+2.00}_{-1.54-2.60}$	$\cdots$	$\emptyset$
$(K\pi)^0_0\pi^+\gamma$ (NR)	$\cdots$	$10.81^{+1.42+1.91}_{-1.47-2.48}$	< 9.2 CL$=90\%$
$K^*_0(1430)^0\pi^+\gamma$	$0.51 \pm 0.07^{+0.09}_{-0.12}$	$0.82 \pm 0.11^{+0.15}_{-0.19} \pm 0.08$	$\emptyset$

Good agreement with the existing measurements

The Dilution Factor
Computation (1)

- **Relations between fit fractions (FF) and the terms appearing in the dilution factor expression:**

"overlap" between $(K\pi)^+$ and $(K\pi)^-$ $\propto \mathrm{FF}_{(K\pi)_0}$

"overlap" between K^{*+} and K^{*-} $\propto \mathrm{FF}_{K^*}$

$$\mathcal{D}_{K_S^0\rho\gamma} = \frac{\int \left[|A_\rho|^2 + \Re(A_\rho^* A_{K^{*+}}) + \Re(A_\rho^* A_{K^{*-}}) + \Re(A_{K^{*+}}^* A_{K^{*-}}) + \Re(A_{(K\pi)^+}^* A_{(K\pi)^-}) \right]}{\int \left[|A_\rho|^2 + \Re(A_\rho^* A_{K^{*+}}) + \Re(A_\rho^* A_{K^{*-}}) + \frac{|A_{K^{*+}}|^2 + |A_{K^{*-}}|^2}{2} + \frac{|A_{(K\pi)^+}|^2 + |A_{(K\pi)^-}|^2}{2} \right]}$$

$\propto \mathrm{FF}_\rho$ $\quad \propto \mathrm{FF}^{\mathrm{interf.}}_{K^*-\rho}$ $\quad \propto \mathrm{FF}_{K^*}$ $\quad \propto \mathrm{FF}_{(K\pi)_0}$

- **Optimize the final error on S:**

from the time-dependent fit

$$\sigma_{\mathcal{S}_{K^0_S\rho\gamma}} \propto \frac{\sigma_{\mathcal{S}_{K^0_S\pi^+\pi^-\gamma}}}{\mathcal{D}_{K^0_S\rho\gamma}} \Rightarrow \mathcal{D}_{K^0_S\rho\gamma} \searrow \Rightarrow \sigma_{\mathcal{S}_{K^0_S\rho\gamma}} \nearrow$$

Need to have D as large as possible

⇨ **Apply a posteriori cuts on $m_{\pi\pi}$ and $m_{K\pi}$ to enhance the proportion of ρ**

- $m_{\pi\pi} \in [0.600, 0.900]$ (GeV/c^2) ρ selection
- $m_{K\pi} \in [m_{K\pi}^{\mathrm{min}}, 0.845] \cup [0.945, m_{K\pi}^{\mathrm{max}}]$ (GeV/c^2) K* veto

The Dilution Factor
Computation (3)

- **Value of the dilution factor:**

  - $|A_\rho|^2 = 0.270^{+0.030}_{-0.028}$;
  - $\frac{|A_{K*+}|^2+|A_{K*-}|^2}{2} = 0.078^{+0.002}_{-0.001}$;
  - $\frac{|A_{(K\pi)+}|^2+|A_{(K\pi)-}|^2}{2} = 0.139^{+0.023}_{-0.027}$;
  - $\Re(A^*_\rho A_{K*+}) + \Re(A^*_\rho A_{K*-}) = -0.092^{+0.005}_{-0.006}$;
  - $\Re(A^*_{K*+} A_{K*-}) = 0.002^{+0.001}_{-0.000}$;
  - $\Re\left(A^*_{(K\pi)+} A_{(K\pi)-}\right) = 0.037^{+0.008}_{-0.010}$.

- Calculated from the fit fractions extracted from a fit performed to the $m_{K\pi}$ spectrum
- Take into account the cuts on $m_{\pi\pi}$ and $m_{K\pi}$ while integrating over the phase space regions

Total uncertainties (stat. + syst.)

$$\mathcal{D}_{K^0_S\rho\gamma} = 0.549^{+0.096}_{-0.094}$$

Dominated by systematic uncertainties

Study of $B \rightarrow K \pi \pi \gamma$ decays with the BaBar detector:

$B^0 \rightarrow K_S \pi^- \pi^+ \gamma$ Time-Dependent Analysis

(1) Extraction of effective CP violation parameters

(2) Extraction of CP violation parameters for $B^0 \rightarrow K_S \rho^0 \gamma$

$B^0 \to K_S \pi^- \pi^+ \gamma$ TDCP Analysis

- **Blind analysis**
- **Standard strategy:**
  - Optimized the cuts to maximize the sensitivity to the CP asymmetry parameters
  - Use weights extracted from the $m_{K\pi\pi}$ fit in $B^+ \to K^+\pi^-\pi^+\gamma$ for signal cocktail and yield estimation
  - Build the PDFs for signal and each background categories
  - Δt PDF parameters:
    - Dilution, delta dilution and asymmetry factors extracted from MC
    - Signal resolution function parameters taken from charmonium $\sin(2\beta)$ analyses

      Phys. Rev. Lett. 99, 171803 (2007)

- **Many technical aspects identical to the $B^+ \to K^+\pi^-\pi^+\gamma$ analysis**

$B^0 \to K_S \pi^- \pi^+ \gamma$ TDCP Analysis
Yields - PDFs

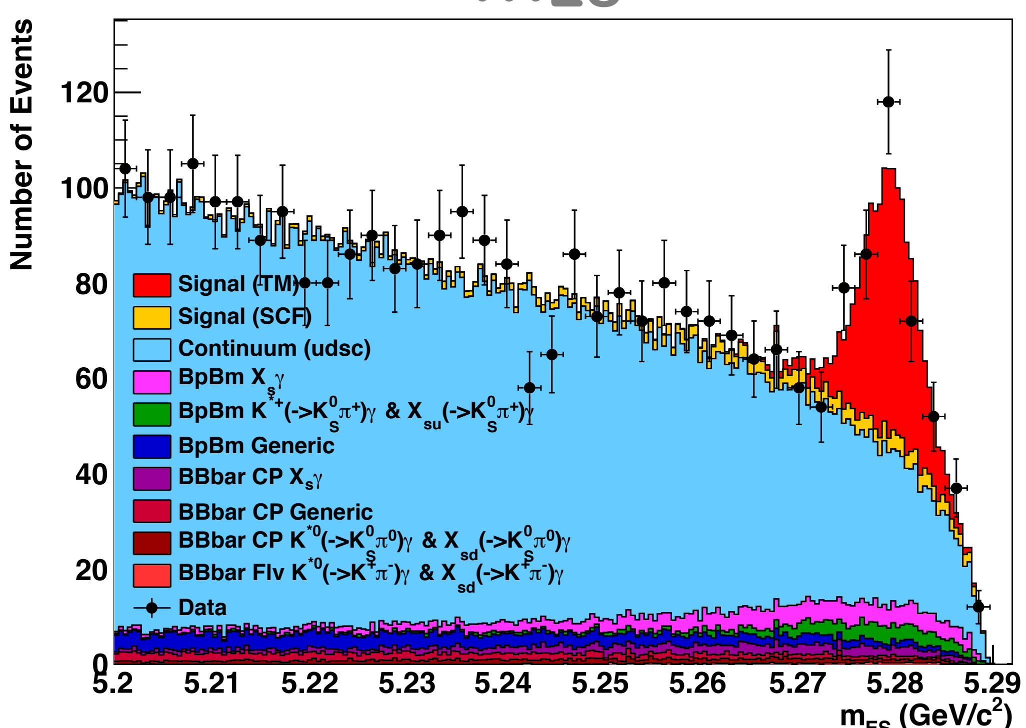
Category	Time-dependent model	Estimated yield
Signal TM	Signal	210
Signal SCF		$f_{SCF} = 0.272$
Continuum (*udsc*)	Continuum	2236
$B^+ \to X_{su}(\nrightarrow K\pi)\gamma$	Charged	94
$B^+ \to K^{*+}(\to K_S^0\pi^+)\gamma$ $B^+ \to X_{su}(\to K_S^0\pi^+)\gamma$	Charged	54
$B^0 \to X_{sd}(\nrightarrow K\pi)\gamma$	Neutral *CP*	51
$B^0\overline{B}^0$ generic B background	Neutral *CP*	35
B^+B^- generic B background	Charged	34
$B^0 \to K^{*0}(\to K_S^0\pi^0)\gamma$ $B^0 \to X_{sd}(\to K_S^0\pi^0)\gamma$	Neutral *CP*	30
$B^0 \to K^{*0}(\to K^\pm\pi^\mp)\gamma$ $B^0 \to X_{sd}(\to K^\pm\pi^\mp)\gamma$	Neutral flavor	4
Total Bkg		2538

- **Use the cuts on $m_{\pi\pi}$ and $m_{K\pi}$:**
  - $m_{\pi\pi} \in [0.600, 0.900]$ (GeV/c^2)
  - $m_{K\pi} \in [m_{K\pi}^{\min}, 0.845] \cup [0.945, m_{K\pi}^{\max}]$ (GeV/c^2)
- **Added selection criteria on the K_S:**
  - $|m_{\pi^+\pi^-} - m_{K_S^0}| < 11$ MeVc^2
  - $\cos\theta_{\text{flight}} > 0.995$
  - $d_{K_S^0} > 5\sigma(d_{K_S^0})$
- **Identified 7 B-background categories:**
  - 3 B^+B^-
  - 3 $B^0\overline{B}^0$ (CP)
  - 1 $B^0\overline{B}^0$ (Flavor)
- **Taken m_{ES}-ΔE correlations into account for Signal TM and $B^+ \to K_S\pi^+\gamma$**
- **S/B ~ 1/12** (~1/34 in $B^+ \to K^+\pi^-\pi^+\gamma$)

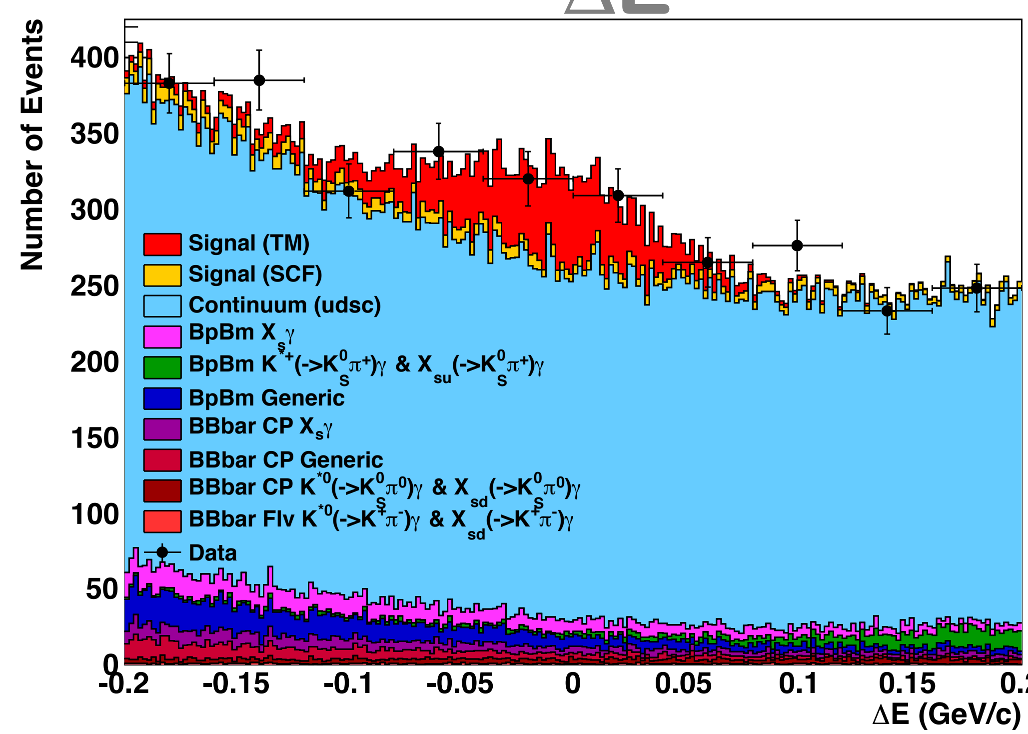
$B^0 \to K_S \pi^- \pi^+ \gamma$ TDCP Analysis

Fit Projections & Yields

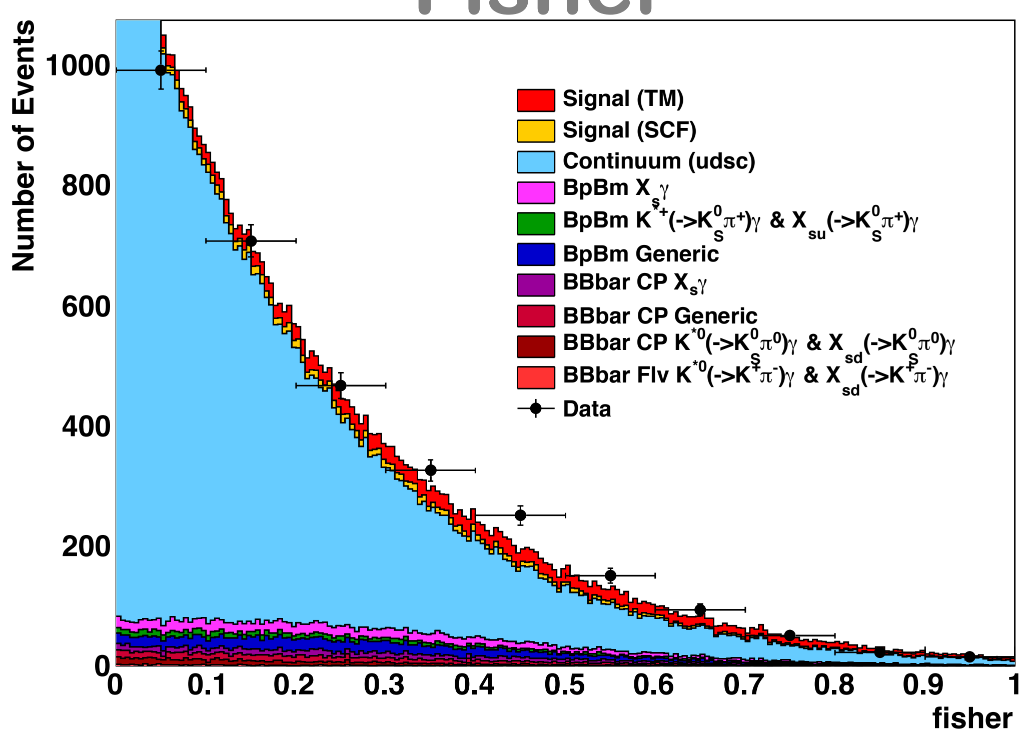
m_{ES}



ΔE



Fisher



Category	Fitted yield	Fit error (stat.)
Signal	245.0	24.3
Continuum *udsc*	2446.4	56.8
$B^+ \to K^{*+}(\to K^0_S\pi^+)\gamma$ $B^+ \to X_{su}(\to K^0_S\pi^+)\gamma$	41.7	21.8

$B^0 \to K_S \pi^- \pi^+ \gamma$ TDCP Analysis
CP Asymmetry Parameters

- **CP asymmetry parameters from the fit to data ($B^0 \to K_S \pi^- \pi^+ \gamma$):**

$$\mathcal{S}_{K_S^0 \pi^+ \pi^- \gamma} = 0.137 \pm 0.249(\text{stat.})^{+0.042}_{-0.033}(\text{syst.})$$
$$\mathcal{C}_{K_S^0 \pi^+ \pi^- \gamma} = -0.390 \pm 0.204(\text{stat.})^{+0.045}_{-0.050}(\text{syst.})$$

- **CP asymmetry parameters for $B^0 \to K_S \rho^0 \gamma$:**

$$\mathcal{S}_{K_S^0 \rho \gamma} = \frac{\mathcal{S}_{K_S^0 \pi^+ \pi^- \gamma}}{\mathcal{D}_{K_S^0 \rho \gamma}} = 0.249 \pm 0.455^{+0.076}_{-0.060}$$

$$\mathcal{D}_{K_S^0 \rho \gamma} = 0.549^{+0.096}_{-0.094}$$

Summary and Conclusion

Summary and Conclusion

- **$B^+ \to K^+\pi^-\pi^+\gamma$ analysis:**
  - Measured the overall branching fraction:

BABAR not yet approved

$$\mathcal{B}(B^+ \to K^+\pi^+\pi^-\gamma) = (27.21 \pm 1.01^{+1.14}_{-1.25}) \times 10^{-6}$$

  - The kaonic resonances branching fractions:

BABAR not yet approved

$$\begin{aligned}
\mathcal{B}(B^+ \to K_1(1270)^+\gamma) &= (44.04^{+6.00+3.80}_{-3.97-3.98} \pm 4.58) \times 10^{-6} \\
\mathcal{B}(B^+ \to K_1(1400)^+\gamma) &= (9.65^{+4.55+2.85}_{-2.86-1.74} \pm 0.61) \times 10^{-6} \\
\mathcal{B}(B^+ \to K^*(1410)^+\gamma) &= (23.83^{+5.23+3.92}_{-4.59-1.58} \pm 2.38) \times 10^{-6} \\
\mathcal{B}(B^+ \to K^*(1680)^+\gamma) &= (71.67^{+7.18+7.03}_{-5.67-8.83} \pm 5.83) \times 10^{-6}
\end{aligned}$$

First measurements of $K^*(1410)$ and $K^*(1680)$ BF in $B^+ \to K_{res}\gamma$

  - The $K^*(892)$, $\rho^0(770)$ and $K\pi$ S-wave branching fractions:

BABAR not yet approved

$$\begin{aligned}
\mathcal{B}(B^+ \to K^{*0}\pi^+\gamma) &= (25.96^{+1.42+1.79}_{-1.34-1.68}) \times 10^{-6} \\
\mathcal{B}(B^+ \to K^+\rho^0\gamma) &= (9.21^{+0.76+1.31}_{-0.70-1.32} \pm 0.02) \times 10^{-6} \\
\mathcal{B}(B^+ \to (K\pi)^0_0\pi^+\gamma) &= (10.81^{+1.42+1.91}_{-1.47-2.48}) \times 10^{-6} \\
\mathcal{B}(B^+ \to K_0^*(1430)^0\pi^+\gamma) &= (0.82 \pm 0.11^{+0.15}_{-0.19} \pm 0.08) \times 10^{-6}
\end{aligned}$$

- Used an original approach given the small available statistics
- Used LASS parametrization to describe the $(K\pi)$ S-wave in $K_1(1270)$ decays

Adding experimental information about resonances in this mode. Might help theoreticians improving the predictions for the extraction of the photon polarization in angular $B \to K\pi\pi\gamma$ analysis

Summary and Conclusion

- **$B^0 \to K_S\pi^-\pi^+\gamma$ TDCP analysis:**
  - **The mixing induced CP violation parameter for $B^0 \to K_S\rho^0\gamma$ decays:**

$$\mathcal{S}_{K^0_S\rho\gamma} = \frac{\mathcal{S}_{K^0_S\pi^+\pi^-\gamma}}{\mathcal{D}_{K^0_S\rho\gamma}} = 0.249 \pm 0.455^{+0.076}_{-0.060}$$

Paper in prep.

  - **Compared with other CPV measurements in radiative decays:**

$$\mathcal{S}^{\text{Belle}}_{K^0_S\rho\gamma} = 0.11 \pm 0.33^{+0.05}_{-0.09}$$

PhysRevLett.101.251601

$$\mathcal{S}^{\text{BABAR}}_{K^0_S\pi^0\gamma} = -0.78 \pm 0.59 \pm 0.09$$

PhysRevD.78.071102

$$\mathcal{S}^{\text{Belle}}_{K^0_S\pi^0\gamma} = -0.10 \pm 0.31 \pm 0.07$$

PhysRevD.74.111104

Summary and Conclusion

- $B^0 \to K_S\pi^-\pi^+\gamma$ **TDCP analysis:**
  - The mixing induced CP violation parameter for $B^0 \to K_S\rho^0\gamma$ decays:

$$\mathcal{S}_{K_S^0\rho\gamma} = \frac{\mathcal{S}_{K_S^0\pi^+\pi^-\gamma}}{\mathcal{D}_{K_S^0\rho\gamma}} = 0.249 \pm 0.455^{+0.076}_{-0.060}$$

Paper in prep.

- In agreement with SM predictions
- Compatible with the results published by Belle
- With the current statistics: does not allow constraining NP models

- Would benefit from better knowledge on the various K_{res}
- Such measurement difficult in LHCb...but other methods to probe the photon polarization are being exploited

With ~50000 expected events in $B^0 \to K_S\pi^-\pi^+\gamma$ mode such analysis in Belle-II's world provides a promising way to search for NP!

Merci de Votre Attention

The Dilution Factor
Analytical Expression

- **Defined as the ratio:**

$$\mathcal{D}_{K^0_S\rho\gamma} \equiv \frac{\mathcal{S}_{K^0_S\pi^+\pi^-\gamma}}{\mathcal{S}_{K^0_S\rho\gamma}}$$

- **CP asymmetry when considering all the resonances** ρ^0, $K^{*\pm}$ or $(K\pi)^{\pm}$ S-wave **in the total amplitude:**

$$\mathcal{A}_{CP}^{K^0_S\pi^+\pi^-\gamma}(t) = \mathcal{C}_{K^0_S\pi^+\pi^-\gamma}\cos(\Delta \mathrm{M}t) + \mathcal{S}_{K^0_S\pi^+\pi^-\gamma}\sin(\Delta \mathrm{M}t)$$

- **CP asymmetry when considering only the** ρ^0 **resonance in the total amplitude:**

$$\mathcal{A}_{CP}^{K^0_S\rho\gamma}(t) = \mathcal{C}_{K^0_S\rho\gamma}\cos(\Delta \mathrm{M}t) + \mathcal{S}_{K^0_S\rho\gamma}\sin(\Delta \mathrm{M}t)$$

- **In terms of amplitudes:**

$$B^0(t)\to H_{\rm res}P_{\rm scal}\gamma \qquad H_{\rm res}=\rho^0,\ K^{*\pm}\ \text{or}\ (K\pi)^\pm\ \text{S-wave}\ ; \qquad P_{\rm scal}=K^0_S\ \text{or}\ \pi^\pm$$

$$\begin{aligned}
A_R^{H_{\rm res}}(B^0\to H_{\rm res}P_{\rm scal}\gamma_L) &= \xi_1 A_{H_{\rm res}}\sin\psi e^{-i\phi_R^{H_{\rm res}}}e^{i\delta^{H_{\rm res}}}\\
A_L^{H_{\rm res}}(B^0\to H_{\rm res}P_{\rm scal}\gamma_R) &= \xi_2 A_{H_{\rm res}}\cos\psi e^{-i\phi_L^{H_{\rm res}}}e^{i\delta^{H_{\rm res}}}\\
\overline{A}_L^{H_{\rm res}}(\overline{B}^0\to H_{\rm res}P_{\rm scal}\gamma_L) &= \xi_3 A_{H_{\rm res}}\cos\psi e^{i\phi_L^{H_{\rm res}}}e^{i\delta^{H_{\rm res}}}\\
\overline{A}_R^{H_{\rm res}}(\overline{B}^0\to H_{\rm res}P_{\rm scal}\gamma_R) &= \xi_4 A_{H_{\rm res}}\sin\psi e^{i\phi_R^{H_{\rm res}}}e^{i\delta^{H_{\rm res}}}
\end{aligned}$$

$$\begin{aligned}
\tan\psi &= C'_{7_\gamma}/C_{7_\gamma}\\
\phi_{L/R}^{H_{\rm res}} &\Rightarrow CP\text{-odd weak phases}\\
\delta^{H_{\rm res}} &\Rightarrow CP\text{-even strong phases}\\
\xi_i &\equiv CP(H_{\rm res}P_{\rm scal})=\pm1\\
(\xi_1,\xi_2,\xi_3,\xi_4) &= (+,-,+,-)\ \text{for}\ \rho\ \text{and}\ K^{*\pm}\\
(\xi_1,\xi_2,\xi_3,\xi_4) &= (+,+,+,+)\ \text{for}\ (K\pi)^\pm\ \text{S-wave}
\end{aligned}$$

$$\mathcal{A}_{CP}(t)=\frac{\Gamma_{\overline{B}^0}(t)-\Gamma_{B^0}(t)}{\Gamma_{\overline{B}^0}(t)+\Gamma_{B^0}(t)}\equiv\mathcal{C}\cos(\Delta{\rm M}t)+\mathcal{S}\sin(\Delta{\rm M}t)$$

$$\begin{aligned}
\Gamma_{B^0}(t) &= |\mathcal{M}_L(t)|^2+|\mathcal{M}_R(t)|^2\\
\Gamma_{\overline{B}^0}(t) &= |\overline{\mathcal{M}}_L(t)|^2+|\overline{\mathcal{M}}_R(t)|^2
\end{aligned}$$

$$\begin{aligned}
\mathcal{M}_L(t) &= \sum_{H_{\rm res}}\left(A_L^{H_{\rm res}}{\rm f}_+(t)+\overline{A}_L^{H_{\rm res}}\frac{q}{p}{\rm f}_-(t)\right)\ ; \quad \overline{\mathcal{M}}_L(t)=\sum_{H_{\rm res}}\left(\overline{A}_L^{H_{\rm res}}{\rm f}_+(t)+A_L^{H_{\rm res}}\frac{q}{p}{\rm f}_-(t)\right)\\
\mathcal{M}_R(t) &= \sum_{H_{\rm res}}\left(A_R^{H_{\rm res}}{\rm f}_+(t)+\overline{A}_R^{H_{\rm res}}\frac{q}{p}{\rm f}_-(t)\right)\ ; \quad \overline{\mathcal{M}}_R(t)=\sum_{H_{\rm res}}\left(\overline{A}_R^{H_{\rm res}}{\rm f}_+(t)+A_R^{H_{\rm res}}\frac{q}{p}{\rm f}_-(t)\right)
\end{aligned}$$

$${\rm f}_\pm(t)\equiv\frac{1}{2}\left(e^{-i{\rm M}_Lt}e^{-\frac{1}{2}\Gamma_Lt}\pm e^{-i{\rm M}_Ht}e^{-\frac{1}{2}\Gamma_Ht}\right) \qquad \frac{q}{p}=e^{-i2\beta}$$

- **In terms of amplitudes:**

$$B^0(t) \to H_{\rm res}P_{\rm scal}\gamma \qquad H_{\rm res} = \rho^0,\ K^{*\pm} \text{ or } (K\pi)^\pm \text{ S-wave} ; \qquad P_{\rm scal} = K_S^0 \text{ or } \pi^\pm$$

$$\begin{aligned}
A_R^{H_{\rm res}}(B^0 \to H_{\rm res}P_{\rm scal}\gamma_L) &= \xi_1 A_{H_{\rm res}} \sin\psi e^{-i\phi_R^{H_{\rm res}}} e^{i\delta^{H_{\rm res}}} \\
A_L^{H_{\rm res}}(B^0 \to H_{\rm res}P_{\rm scal}\gamma_R) &= \xi_2 A_{H_{\rm res}} \cos\psi e^{-i\phi_L^{H_{\rm res}}} e^{i\delta^{H_{\rm res}}} \\
\overline{A}_L^{H_{\rm res}}(\overline{B}^0 \to H_{\rm res}P_{\rm scal}\gamma_L) &= \xi_3 A_{H_{\rm res}} \cos\psi e^{i\phi_L^{H_{\rm res}}} e^{i\delta^{H_{\rm res}}} \\
\overline{A}_R^{H_{\rm res}}(\overline{B}^0 \to H_{\rm res}P_{\rm scal}\gamma_R) &= \xi_4 A_{H_{\rm res}} \sin\psi e^{i\phi_R^{H_{\rm res}}} e^{i\delta^{H_{\rm res}}}
\end{aligned}$$

$$\begin{aligned}
\tan\psi &= C'_{7_\gamma}/C_{7_\gamma} \\
\phi_{L/R}^{H_{\rm res}} &\Rightarrow \mathit{CP}\text{-odd weak phases} \\
\delta^{H_{\rm res}} &\Rightarrow \mathit{CP}\text{-even strong phases} \\
\xi_i &\equiv \mathit{CP}(H_{\rm res}P_{\rm scal}) = \pm 1 \\
(\xi_1,\xi_2,\xi_3,\xi_4) &= (+,-,+,-) \text{ for } \rho \text{ and } K^{*\pm} \\
(\xi_1,\xi_2,\xi_3,\xi_4) &= (+,+,+,+) \text{ for } (K\pi)^\pm \text{ S-wave}
\end{aligned}$$

$$\mathcal{A}_{CP}(t) = \frac{\Gamma_{\overline{B}^0}(t) - \Gamma_{B^0}(t)}{\Gamma_{\overline{B}^0}(t) + \Gamma_{B^0}(t)} \equiv \mathcal{C}\cos(\Delta M t) + \mathcal{S}\sin(\Delta M t)$$

$$\begin{aligned}
\Gamma_{B^0}(t) &= |\mathcal{M}_L(t)|^2 + |\mathcal{M}_R(t)|^2 \\
\Gamma_{\overline{B}^0}(t) &= |\overline{\mathcal{M}}_L(t)|^2 + |\overline{\mathcal{M}}_R(t)|^2
\end{aligned}$$

$$\begin{aligned}
\mathcal{M}_L(t) &= \sum_{H_{\rm res}}\left(A_L^{H_{\rm res}} f_+(t) + \overline{A}_L^{H_{\rm res}}\frac{q}{p} f_-(t)\right) ; \quad \overline{\mathcal{M}}_L(t) = \sum_{H_{\rm res}}\left(\overline{A}_L^{H_{\rm res}} f_+(t) + A_L^{H_{\rm res}}\frac{q}{p} f_-(t)\right) \\
\mathcal{M}_R(t) &= \sum_{H_{\rm res}}\left(A_R^{H_{\rm res}} f_+(t) + \overline{A}_R^{H_{\rm res}}\frac{q}{p} f_-(t)\right) ; \quad \overline{\mathcal{M}}_R(t) = \sum_{H_{\rm res}}\left(\overline{A}_R^{H_{\rm res}} f_+(t) + A_R^{H_{\rm res}}\frac{q}{p} f_-(t)\right)
\end{aligned}$$

$$f_\pm(t) \equiv \frac{1}{2}\left(e^{-iM_L t}e^{-\frac{1}{2}\Gamma_L t} \pm e^{-iM_H t}e^{-\frac{1}{2}\Gamma_H t}\right) \qquad \frac{q}{p} = e^{-i2\beta}$$

- **In terms of amplitudes:**

$$B^0(t) \to H_{\rm res} P_{\rm scal}\gamma \qquad H_{\rm res} = \rho^0,\ K^{*\pm} \text{ or } (K\pi)^\pm \text{ S-wave}\ ; \qquad P_{\rm scal} = K^0_S \text{ or } \pi^\pm$$

$$\begin{aligned}
A_R^{H_{\rm res}}(B^0 \to H_{\rm res} P_{\rm scal}\gamma_L) &= \xi_1 A_{H_{\rm res}} \sin\psi e^{-i\phi_R^{H_{\rm res}}} e^{i\delta^{H_{\rm res}}} \\
A_L^{H_{\rm res}}(B^0 \to H_{\rm res} P_{\rm scal}\gamma_R) &= \xi_2 A_{H_{\rm res}} \cos\psi e^{-i\phi_L^{H_{\rm res}}} e^{i\delta^{H_{\rm res}}} \\
\overline{A}_L^{H_{\rm res}}(\overline{B}^0 \to H_{\rm res} P_{\rm scal}\gamma_L) &= \xi_3 A_{H_{\rm res}} \cos\psi e^{i\phi_L^{H_{\rm res}}} e^{i\delta^{H_{\rm res}}} \\
\overline{A}_R^{H_{\rm res}}(\overline{B}^0 \to H_{\rm res} P_{\rm scal}\gamma_R) &= \xi_4 A_{H_{\rm res}} \sin\psi e^{i\phi_R^{H_{\rm res}}} e^{i\delta^{H_{\rm res}}}
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(\xi_1,\xi_2,\xi_3,\xi_4) &= (+,+,+,+) \text{ for } (K\pi)^\pm \text{ S-wave}
\end{aligned}$$

$$\mathcal{A}_{CP}(t) = \frac{\Gamma_{\overline{B}^0}(t) - \Gamma_{B^0}(t)}{\Gamma_{\overline{B}^0}(t) + \Gamma_{B^0}(t)} \equiv \mathcal{C}\cos(\Delta M t) + \mathcal{S}\sin(\Delta M t)$$

$$\begin{aligned}
\Gamma_{B^0}(t) &= |\mathcal{M}_L(t)|^2 + |\mathcal{M}_R(t)|^2 \\
\Gamma_{\overline{B}^0}(t) &= |\overline{\mathcal{M}}_L(t)|^2 + |\overline{\mathcal{M}}_R(t)|^2
\end{aligned}$$

$$\begin{aligned}
\mathcal{M}_L(t) &= \sum_{H_{\rm res}} \left(A_L^{H_{\rm res}} f_+(t) + \overline{A}_L^{H_{\rm res}} \frac{q}{p} f_-(t) \right) ; \quad \overline{\mathcal{M}}_L(t) = \sum_{H_{\rm res}} \left(\overline{A}_L^{H_{\rm res}} f_+(t) + A_L^{H_{\rm res}} \frac{q}{p} f_-(t) \right) \\
\mathcal{M}_R(t) &= \sum_{H_{\rm res}} \left(A_R^{H_{\rm res}} f_+(t) + \overline{A}_R^{H_{\rm res}} \frac{q}{p} f_-(t) \right) ; \quad \overline{\mathcal{M}}_R(t) = \sum_{H_{\rm res}} \left(\overline{A}_R^{H_{\rm res}} f_+(t) + A_R^{H_{\rm res}} \frac{q}{p} f_-(t) \right)
\end{aligned}$$

$$f_\pm(t) \equiv \frac{1}{2}\left(e^{-iM_L t} e^{-\frac{1}{2}\Gamma_L t} \pm e^{-iM_H t} e^{-\frac{1}{2}\Gamma_H t} \right) \qquad \frac{q}{p} = e^{-i2\beta}$$

- **In terms of amplitudes:**

$$B^0(t) \to H_{\rm res} P_{\rm scal}\gamma \qquad H_{\rm res} = \rho^0,\ K^{*\pm} \text{ or } (K\pi)^\pm \text{ S-wave}\ ; \qquad P_{\rm scal} = K^0_S \text{ or } \pi^\pm$$

$$\begin{aligned}
A_R^{H_{\rm res}}(B^0 \to H_{\rm res}P_{\rm scal}\gamma_L) &= \xi_1 A_{H_{\rm res}} \sin\psi e^{-i\phi_R^{H_{\rm res}}} e^{i\delta^{H_{\rm res}}}\\
A_L^{H_{\rm res}}(B^0 \to H_{\rm res}P_{\rm scal}\gamma_R) &= \xi_2 A_{H_{\rm res}} \cos\psi e^{-i\phi_L^{H_{\rm res}}} e^{i\delta^{H_{\rm res}}}\\
\overline{A}_L^{H_{\rm res}}(\overline{B}^0 \to H_{\rm res}P_{\rm scal}\gamma_L) &= \xi_3 A_{H_{\rm res}} \cos\psi e^{i\phi_L^{H_{\rm res}}} e^{i\delta^{H_{\rm res}}}\\
\overline{A}_R^{H_{\rm res}}(\overline{B}^0 \to H_{\rm res}P_{\rm scal}\gamma_R) &= \xi_4 A_{H_{\rm res}} \sin\psi e^{i\phi_R^{H_{\rm res}}} e^{i\delta^{H_{\rm res}}}
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\Gamma_{\overline{B}^0}(t) &= |\overline{\mathcal{M}}_L(t)|^2 + |\overline{\mathcal{M}}_R(t)|^2
\end{aligned}$$

$$\begin{aligned}
\mathcal{M}_L(t) &= \sum_{H_{\rm res}}\left(A_L^{H_{\rm res}} f_+(t) + \overline{A}_L^{H_{\rm res}}\frac{q}{p} f_-(t)\right)\ ; \quad \overline{\mathcal{M}}_L(t) = \sum_{H_{\rm res}}\left(\overline{A}_L^{H_{\rm res}} f_+(t) + A_L^{H_{\rm res}}\frac{q}{p} f_-(t)\right)\\
\mathcal{M}_R(t) &= \sum_{H_{\rm res}}\left(A_R^{H_{\rm res}} f_+(t) + \overline{A}_R^{H_{\rm res}}\frac{q}{p} f_-(t)\right)\ ; \quad \overline{\mathcal{M}}_R(t) = \sum_{H_{\rm res}}\left(\overline{A}_R^{H_{\rm res}} f_+(t) + A_R^{H_{\rm res}}\frac{q}{p} f_-(t)\right)
\end{aligned}$$

$$f_\pm(t) \equiv \frac{1}{2}\left(e^{-iM_L t}e^{-\frac{1}{2}\Gamma_L t} \pm e^{-iM_H t}e^{-\frac{1}{2}\Gamma_H t}\right) \qquad \frac{q}{p} = e^{-i2\beta}$$

- **In terms of amplitudes:**

$$B^0(t) \to H_{\text{res}} P_{\text{scal}} \gamma \qquad H_{\text{res}} = \rho^0,\ K^{*\pm} \text{ or } (K\pi)^\pm \text{ S-wave} ; \qquad P_{\text{scal}} = K^0_S \text{ or } \pi^\pm$$

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A_L^{H_{\text{res}}}(B^0 \to H_{\text{res}} P_{\text{scal}} \gamma_R) &= \xi_2 A_{H_{\text{res}}} \cos\psi e^{-i\phi_L^{H_{\text{res}}}} e^{i\delta^{H_{\text{res}}}} \\
\overline{A}_L^{H_{\text{res}}}(\overline{B}^0 \to H_{\text{res}} P_{\text{scal}} \gamma_L) &= \xi_3 A_{H_{\text{res}}} \cos\psi e^{i\phi_L^{H_{\text{res}}}} e^{i\delta^{H_{\text{res}}}} \\
\overline{A}_R^{H_{\text{res}}}(\overline{B}^0 \to H_{\text{res}} P_{\text{scal}} \gamma_R) &= \xi_4 A_{H_{\text{res}}} \sin\psi e^{i\phi_R^{H_{\text{res}}}} e^{i\delta^{H_{\text{res}}}}
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\delta^{H_{\text{res}}} &\Rightarrow CP\text{-even strong phases} \\
\xi_i &\equiv CP(H_{\text{res}} P_{\text{scal}}) = \pm 1 \\
(\xi_1, \xi_2, \xi_3, \xi_4) &= (+,-,+,-) \text{ for } \rho \text{ and } K^{*\pm} \\
(\xi_1, \xi_2, \xi_3, \xi_4) &= (+,+,+,+) \text{ for } (K\pi)^\pm \text{ S-wave}
\end{aligned}$$

$$\mathcal{A}_{CP}(t) = \frac{\Gamma_{\overline{B}^0}(t) - \Gamma_{B^0}(t)}{\Gamma_{\overline{B}^0}(t) + \Gamma_{B^0}(t)} \equiv \mathcal{C}\cos(\Delta M t) + \mathcal{S}\sin(\Delta M t)$$

$$\begin{aligned}
\Gamma_{B^0}(t) &= |\mathcal{M}_L(t)|^2 + |\mathcal{M}_R(t)|^2 \\
\Gamma_{\overline{B}^0}(t) &= |\overline{\mathcal{M}}_L(t)|^2 + |\overline{\mathcal{M}}_R(t)|^2
\end{aligned}$$

$$\begin{aligned}
\mathcal{M}_L(t) &= \sum_{H_{\text{res}}} \left(A_L^{H_{\text{res}}} f_+(t) + \overline{A}_L^{H_{\text{res}}} \frac{q}{p} f_-(t) \right) ; \quad \overline{\mathcal{M}}_L(t) = \sum_{H_{\text{res}}} \left(\overline{A}_L^{H_{\text{res}}} f_+(t) + A_L^{H_{\text{res}}} \frac{q}{p} f_-(t) \right) \\
\mathcal{M}_R(t) &= \sum_{H_{\text{res}}} \left(A_R^{H_{\text{res}}} f_+(t) + \overline{A}_R^{H_{\text{res}}} \frac{q}{p} f_-(t) \right) ; \quad \overline{\mathcal{M}}_R(t) = \sum_{H_{\text{res}}} \left(\overline{A}_R^{H_{\text{res}}} f_+(t) + A_R^{H_{\text{res}}} \frac{q}{p} f_-(t) \right)
\end{aligned}$$

$$f_\pm(t) \equiv \frac{1}{2}\left(e^{-iM_L t} e^{-\frac{1}{2}\Gamma_L t} \pm e^{-iM_H t} e^{-\frac{1}{2}\Gamma_H t} \right) \qquad \frac{q}{p} = e^{-i2\beta}$$

- **In terms of amplitudes:**

$B^0(t) \to H_{\rm res} P_{\rm scal} \gamma \qquad H_{\rm res} = \rho^0,\ K^{*\pm}$ or $(K\pi)^\pm$ S-wave ; $\quad P_{\rm scal} = K^0_S$ or $\pi^\pm$

$$\begin{aligned}
A_R^{H_{\rm res}}(B^0 \to H_{\rm res} P_{\rm scal} \gamma_L) &= \xi_1 A_{H_{\rm res}} \sin\psi e^{-i\phi_R^{H_{\rm res}}} e^{i\delta^{H_{\rm res}}} \\
A_L^{H_{\rm res}}(B^0 \to H_{\rm res} P_{\rm scal} \gamma_R) &= \xi_2 A_{H_{\rm res}} \cos\psi e^{-i\phi_L^{H_{\rm res}}} e^{i\delta^{H_{\rm res}}} \\
\overline{A}_L^{H_{\rm res}}(\overline{B}^0 \to H_{\rm res} P_{\rm scal} \gamma_L) &= \xi_3 A_{H_{\rm res}} \cos\psi e^{i\phi_L^{H_{\rm res}}} e^{i\delta^{H_{\rm res}}} \\
\overline{A}_R^{H_{\rm res}}(\overline{B}^0 \to H_{\rm res} P_{\rm scal} \gamma_R) &= \xi_4 A_{H_{\rm res}} \sin\psi e^{i\phi_R^{H_{\rm res}}} e^{i\delta^{H_{\rm res}}}
\end{aligned}$$

$$\begin{aligned}
\tan\psi &= C'_{7_\gamma}/C_{7_\gamma} \\
\phi_{L/R}^{H_{\rm res}} &\Rightarrow CP\text{-odd weak phases} \\
\delta^{H_{\rm res}} &\Rightarrow CP\text{-even strong phases} \\
\xi_i &\equiv CP(H_{\rm res} P_{\rm scal}) = \pm 1 \\
(\xi_1,\xi_2,\xi_3,\xi_4) &= (+,-,+,-) \text{ for } \rho \text{ and } K^{*\pm} \\
(\xi_1,\xi_2,\xi_3,\xi_4) &= (+,+,+,+) \text{ for } (K\pi)^\pm \text{ S-wave}
\end{aligned}$$

$$\mathcal{A}_{CP}(t) = \frac{\Gamma_{\overline{B}^0}(t) - \Gamma_{B^0}(t)}{\Gamma_{\overline{B}^0}(t) + \Gamma_{B^0}(t)} \equiv \mathcal{C}\cos(\Delta M t) + \mathcal{S}\sin(\Delta M t)$$

$$\begin{aligned}
\Gamma_{B^0}(t) &= |\mathcal{M}_L(t)|^2 + |\mathcal{M}_R(t)|^2 \\
\Gamma_{\overline{B}^0}(t) &= |\overline{\mathcal{M}}_L(t)|^2 + |\overline{\mathcal{M}}_R(t)|^2
\end{aligned}$$

$$\begin{aligned}
\mathcal{M}_L(t) &= \sum_{H_{\rm res}} \left(A_L^{H_{\rm res}} f_+(t) + \overline{A}_L^{H_{\rm res}} \frac{q}{p} f_-(t) \right) ; \quad \overline{\mathcal{M}}_L(t) = \sum_{H_{\rm res}} \left(\overline{A}_L^{H_{\rm res}} f_+(t) + A_L^{H_{\rm res}} \frac{q}{p} f_-(t) \right) \\
\mathcal{M}_R(t) &= \sum_{H_{\rm res}} \left(A_R^{H_{\rm res}} f_+(t) + \overline{A}_R^{H_{\rm res}} \frac{q}{p} f_-(t) \right) ; \quad \overline{\mathcal{M}}_R(t) = \sum_{H_{\rm res}} \left(\overline{A}_R^{H_{\rm res}} f_+(t) + A_R^{H_{\rm res}} \frac{q}{p} f_-(t) \right)
\end{aligned}$$

$$f_\pm(t) \equiv \frac{1}{2}\left(e^{-iM_L t} e^{-\frac{1}{2}\Gamma_L t} \pm e^{-iM_H t} e^{-\frac{1}{2}\Gamma_H t} \right) \qquad \frac{q}{p} = e^{-i2\beta}$$

The Dilution Factor
Analytical Expression

- **In terms of amplitudes:**

$$B^0(t) \to H_{\rm res} P_{\rm scal} \gamma \qquad H_{\rm res} = \rho^0,\ K^{*\pm} \text{ or } (K\pi)^\pm \text{ S-wave}\ ; \qquad P_{\rm scal} = K^0_S \text{ or } \pi^\pm$$

$$\begin{aligned}
A_R^{H_{\rm res}}(B^0 \to H_{\rm res} P_{\rm scal} \gamma_L) &= \xi_1 A_{H_{\rm res}} \sin\psi e^{-i\phi_R^{H_{\rm res}}} e^{i\delta^{H_{\rm res}}} \\
A_L^{H_{\rm res}}(B^0 \to H_{\rm res} P_{\rm scal} \gamma_R) &= \xi_2 A_{H_{\rm res}} \cos\psi e^{-i\phi_L^{H_{\rm res}}} e^{i\delta^{H_{\rm res}}} \\
\overline{A}_L^{H_{\rm res}}(\overline{B}^0 \to H_{\rm res} P_{\rm scal} \gamma_L) &= \xi_3 A_{H_{\rm res}} \cos\psi e^{i\phi_L^{H_{\rm res}}} e^{i\delta^{H_{\rm res}}} \\
\overline{A}_R^{H_{\rm res}}(\overline{B}^0 \to H_{\rm res} P_{\rm scal} \gamma_R) &= \xi_4 A_{H_{\rm res}} \sin\psi e^{i\phi_R^{H_{\rm res}}} e^{i\delta^{H_{\rm res}}}
\end{aligned}$$

$$\begin{aligned}
\tan\psi &= C'_{7_\gamma}/C_{7_\gamma} \\
\phi_{L/R}^{H_{\rm res}} &\Rightarrow CP\text{-odd weak phases} \\
\delta^{H_{\rm res}} &\Rightarrow CP\text{-even strong phases} \\
\xi_i &\equiv CP(H_{\rm res} P_{\rm scal}) = \pm 1 \\
(\xi_1, \xi_2, \xi_3, \xi_4) &= (+,-,+,-) \text{ for } \rho \text{ and } K^{*\pm} \\
(\xi_1, \xi_2, \xi_3, \xi_4) &= (+,+,+,+) \text{ for } (K\pi)^\pm \text{ S-wave}
\end{aligned}$$

$$\mathcal{A}_{CP}(t) = \frac{\Gamma_{\overline{B}^0}(t) - \Gamma_{B^0}(t)}{\Gamma_{\overline{B}^0}(t) + \Gamma_{B^0}(t)} \equiv \mathcal{C}\cos(\Delta M t) + \mathcal{S}\sin(\Delta M t)$$

$$\begin{aligned}
\Gamma_{B^0}(t) &= |\mathcal{M}_L(t)|^2 + |\mathcal{M}_R(t)|^2 \\
\Gamma_{\overline{B}^0}(t) &= |\overline{\mathcal{M}}_L(t)|^2 + |\overline{\mathcal{M}}_R(t)|^2
\end{aligned}$$

$$\begin{aligned}
\mathcal{M}_L(t) &= \sum_{H_{\rm res}} \left(A_L^{H_{\rm res}} f_+(t) + \overline{A}_L^{H_{\rm res}} \frac{q}{p} f_-(t) \right) ; \quad \overline{\mathcal{M}}_L(t) = \sum_{H_{\rm res}} \left(\overline{A}_L^{H_{\rm res}} f_+(t) + A_L^{H_{\rm res}} \frac{q}{p} f_-(t) \right) \\
\mathcal{M}_R(t) &= \sum_{H_{\rm res}} \left(A_R^{H_{\rm res}} f_+(t) + \overline{A}_R^{H_{\rm res}} \frac{q}{p} f_-(t) \right) ; \quad \overline{\mathcal{M}}_R(t) = \sum_{H_{\rm res}} \left(\overline{A}_R^{H_{\rm res}} f_+(t) + A_R^{H_{\rm res}} \frac{q}{p} f_-(t) \right)
\end{aligned}$$

$$f_\pm(t) \equiv \frac{1}{2}\left(e^{-iM_L t} e^{-\frac{1}{2}\Gamma_L t} \pm e^{-iM_H t} e^{-\frac{1}{2}\Gamma_H t} \right) \qquad \frac{q}{p} = e^{-i2\beta}$$

The Dilution Factor
Analytical Expression

- **In terms of amplitudes, the dilution factor can be expressed as:**

$$\mathcal{D}_{K^0_S\rho\gamma} = \frac{\mathcal{S}_{K^0_S\pi^+\pi^-\gamma}}{\mathcal{S}_{K^0_S\rho\gamma}}$$

$$= \frac{\int \left[|A_\rho|^2 + \Re\left(A_\rho^* A_{K^{*+}}\right) + \Re\left(A_\rho^* A_{K^{*-}}\right) + \Re\left(A_{K^{*+}}^* A_{K^{*-}}\right) + \Re\left(A_{(K\pi)^+}^* A_{(K\pi)^-}\right) \right]}{\int \left[|A_\rho|^2 + \Re\left(A_\rho^* A_{K^{*+}}\right) + \Re\left(A_\rho^* A_{K^{*-}}\right) + \frac{|A_{K^{*+}}|^2 + |A_{K^{*-}}|^2}{2} + \frac{|A_{(K\pi)^+}|^2 + |A_{(K\pi)^-}|^2}{2} \right]}$$

Integration performed over phase-space region

The amplitudes entering in the dilution factor expression are extracted from a fit to the $m_{K\pi}$ spectrum

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Total PDF:

- Coherent sum of K*(892), ρ⁰(770) and Kπ S-wave component:

$$|A(m_{K\pi};c_j)|^2 = \left|\int_{m_{\pi\pi}^{min}}^{m_{\pi\pi}^{max}} \left(\sum_j c_j \sqrt{\mathrm{H}_{\mathrm{R}_j}(m_{K\pi},m_{\pi\pi})}\, e^{i\Phi_{\mathrm{R}_j}(m)}\right) dm_{\pi\pi}\right|^2 , \quad c_j = \alpha_j\, e^{i\phi_j}$$

$$= |c_{K^*}|^2\, \mathcal{H}_{K^*} + |c_{\rho^0}|^2\, \mathcal{H}_{\rho^0} + |c_{(K\pi)_0}|^2\, \mathcal{H}_{(K\pi)_0} + I$$

Interference term described in next slide

- Invariant-mass-dependent magnitude defined as the projection of two-dimensional histograms:

$$\mathcal{H}_{R_j}(m_{K\pi}) = \int_{m_{\pi\pi}^{min}}^{m_{\pi\pi}^{max}} H_{R_j}(m_{K\pi},m_{\pi\pi})\, dm_{\pi\pi}.$$

- The invariant-mass-dependent phase is taken from the analytical expression of the corresponding line shape:

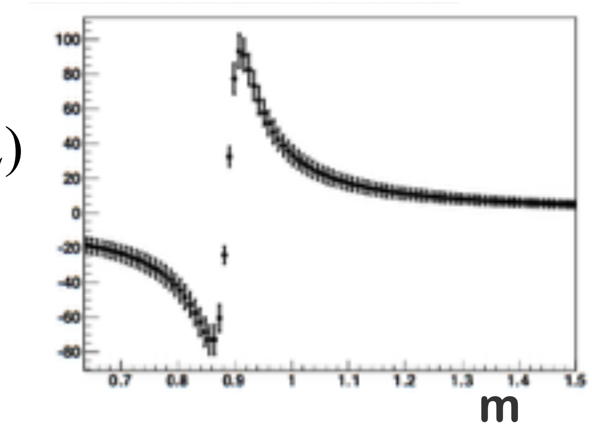
$$\Phi_{\mathrm{R}_j}(m) = \arccos\left(\frac{\Re[\mathrm{R}_j(m)]}{|\mathrm{R}_j(m)|}\right) \Leftrightarrow \begin{cases} m = m_{K\pi} \Rightarrow & \mathrm{R}_j(m_{K\pi}) \text{ is taken as RBW for } K^{*0}(892) \text{ and as LASS for S-wave ,} \\ m = m_{\pi\pi} \Rightarrow & \mathrm{R}_j(m_{\pi\pi}) \text{ is taken as a GS line shape for } \rho^0(770)\text{ ,} \end{cases}$$

Interference:

- **Interference terms:**

Illustration:

RBW+GS interf. (ϕ_{ρ^0}= π/2)



$$
\begin{aligned}
I(m_{K\pi}; c_{\rho^0}, c_{(K\pi)_0}) \;=\; & 2\alpha_{\rho^0}\Bigg[\cos(\phi_{\rho^0} - \Phi_{\rm RBW}) \int_{m_{\pi\pi}^{min}}^{m_{\pi\pi}^{max}} \sqrt{{\rm H}_{\rho^0}{\rm H}_{K^*}}\cos(\Phi_{\rm GS})\, dm_{\pi\pi} \\
& - \sin(\phi_{\rho^0} - \Phi_{\rm RBW}) \int_{m_{\pi\pi}^{min}}^{m_{\pi\pi}^{max}} \sqrt{{\rm H}_{\rho^0}{\rm H}_{K^*}}\sin(\Phi_{\rm GS})\, dm_{\pi\pi}\Bigg] \\
& + 2\alpha_{\rho^0}\alpha_{(K\pi)_0}\Bigg[\cos(\phi_{\rho^0} - \phi_{(K\pi)_0} - \Phi_{\rm LASS}) \int_{m_{\pi\pi}^{min}}^{m_{\pi\pi}^{max}} \sqrt{{\rm H}_{\rho^0}{\rm H}_{(K\pi)_0}}\cos(\Phi_{\rm GS})\, dm_{\pi\pi} \\
& - \sin(\phi_{\rho^0} - \phi_{(K\pi)_0} - \Phi_{\rm LASS}) \int_{m_{\pi\pi}^{min}}^{m_{\pi\pi}^{max}} \sqrt{{\rm H}_{\rho^0}{\rm H}_{(K\pi)_0}}\sin(\Phi_{\rm GS})\, dm_{\pi\pi}\Bigg] .
\end{aligned}
$$

Term describing interference between the K*(892) and ρ^0(770) amplitudes

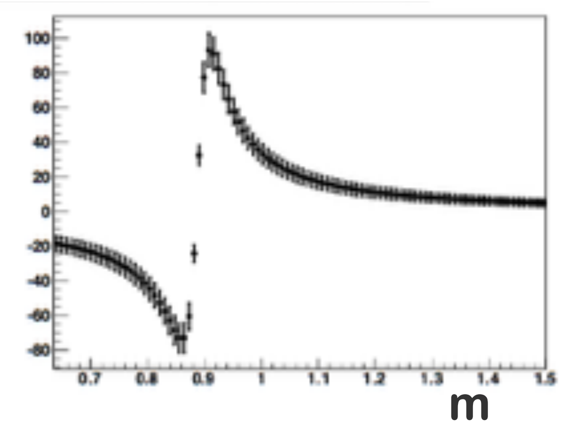
Term describing interference between the ρ^0(770) and (Kπ) S-wave amplitudes

Interference:

- **Interference terms:**

Illustration:

RBW+GS interf. (ϕ_{ρ^0}= π/2)

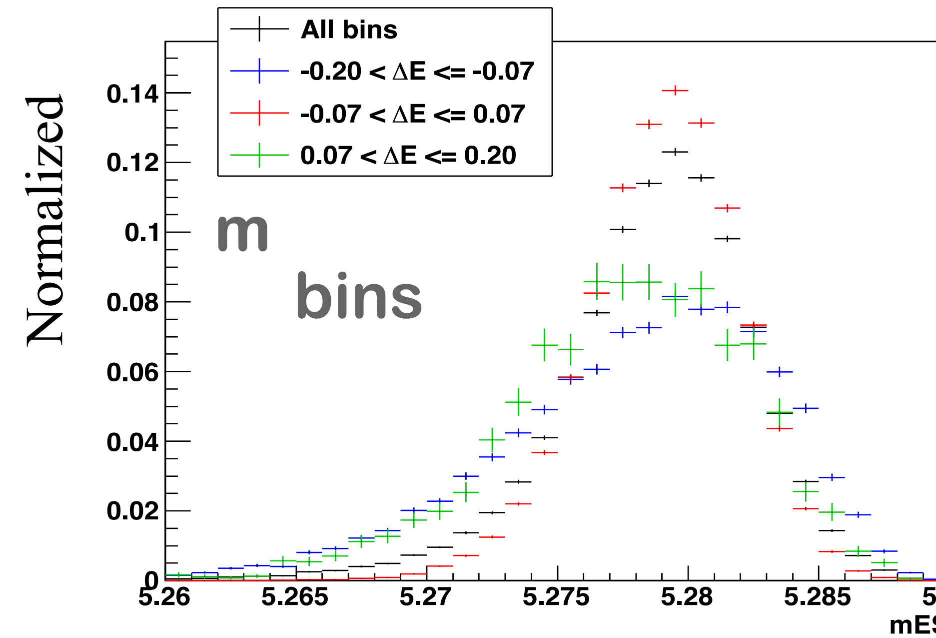


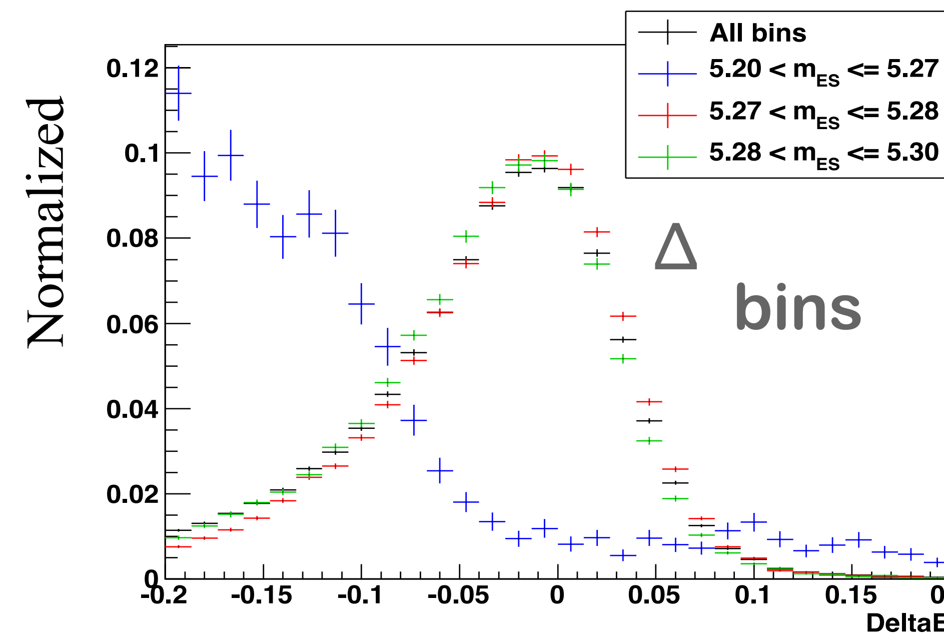
$$I(m_{K\pi}; c_{\rho^0}, c_{(K\pi)_0}) = 2\alpha_{\rho^0}\left[\cos(\phi_{\rho^0} - \Phi_{\rm RBW})\int_{m_{\pi\pi}^{min}}^{m_{\pi\pi}^{max}} \sqrt{{\rm H}_{\rho^0}{\rm H}_{K^*}}\cos(\Phi_{\rm GS})\, dm_{\pi\pi} - \sin(\phi_{\rho^0} - \Phi_{\rm RBW})\int_{m_{\pi\pi}^{min}}^{m_{\pi\pi}^{max}} \sqrt{{\rm H}_{\rho^0}{\rm H}_{K^*}}\sin(\Phi_{\rm GS})\, dm_{\pi\pi}\right]$$
$$+2\alpha_{\rho^0}\alpha_{(K\pi)_0}\left[\cos(\phi_{\rho^0} - \phi_{(K\pi)_0} - \Phi_{\rm LASS})\int_{m_{\pi\pi}^{min}}^{m_{\pi\pi}^{max}} \sqrt{{\rm H}_{\rho^0}{\rm H}_{(K\pi)_0}}\cos(\Phi_{\rm GS})\, dm_{\pi\pi} - \sin(\phi_{\rho^0} - \phi_{(K\pi)_0} - \Phi_{\rm LASS})\int_{m_{\pi\pi}^{min}}^{m_{\pi\pi}^{max}} \sqrt{{\rm H}_{\rho^0}{\rm H}_{(K\pi)_0}}\sin(\Phi_{\rm GS})\, dm_{\pi\pi}\right].$$

The interference between the K*(892) and (Kπ) S-wave amplitudes vanishes due to the integration over the $m_{\pi\pi}$ dimension

$B^+ \to K^+\pi^-\pi^+\gamma$ Analysis
m_{ES}-ΔE correlations

- We use a **2D conditional m_{ES}-ΔE PDF** for signal TM:
  - Parametrized signal TM m_{ES}-ΔE dependence from fits to several sub-samples of MC events in various sized bins of m_{ES} and ΔE
  - Used the dependence: $m_{ES}(\Delta E)$
  - (m_{ES} Crystal Ball — ΔE Cruijff)
  - Crystal Ball mean and sigma:
    - 2nd order polynomial function
  - Crystal Ball alpha and order:
    - 1st order polynomial function





Time-Dependent CP Asymmetry
Ingredients of the Measurement

- **Time-dependent CP asymmetry including experimental effects :**

$$\mathcal{A}_{CP}(\Delta t) = \frac{\Gamma\left(B_{\mathrm{tag}=B^0}(\Delta t) \to f_{CP}\right) - \Gamma\left(B_{\mathrm{tag}=\overline{B}^0}(\Delta t) \to f_{CP}\right)}{\Gamma\left(B_{\mathrm{tag}=B^0}(\Delta t) \to f_{CP}\right) + \Gamma\left(B_{\mathrm{tag}=\overline{B}^0}(\Delta t) \to f_{CP}\right)}$$

$$= \langle \mathcal{D} \rangle \left(\mathcal{S}\sin(\Delta m_d \Delta t) - \mathcal{C}\cos(\Delta m_d \Delta t)\right)$$

Dilution related to the probability to mistag the B meson

- **Time-dependent CP asymmetry including experimental effects :**

$$\mathcal{A}_{CP}(\Delta t) = \frac{\Gamma\left(B_{\text{tag}=B^0}(\Delta t) \to f_{CP}\right) - \Gamma\left(B_{\text{tag}=\overline{B}^0}(\Delta t) \to f_{CP}\right)}{\Gamma\left(B_{\text{tag}=B^0}(\Delta t) \to f_{CP}\right) + \Gamma\left(B_{\text{tag}=\overline{B}^0}(\Delta t) \to f_{CP}\right)}$$

$$= \langle \mathcal{D} \rangle \left(\mathcal{S} \sin(\Delta m_d \Delta t) - \mathcal{C} \cos(\Delta m_d \Delta t)\right)$$

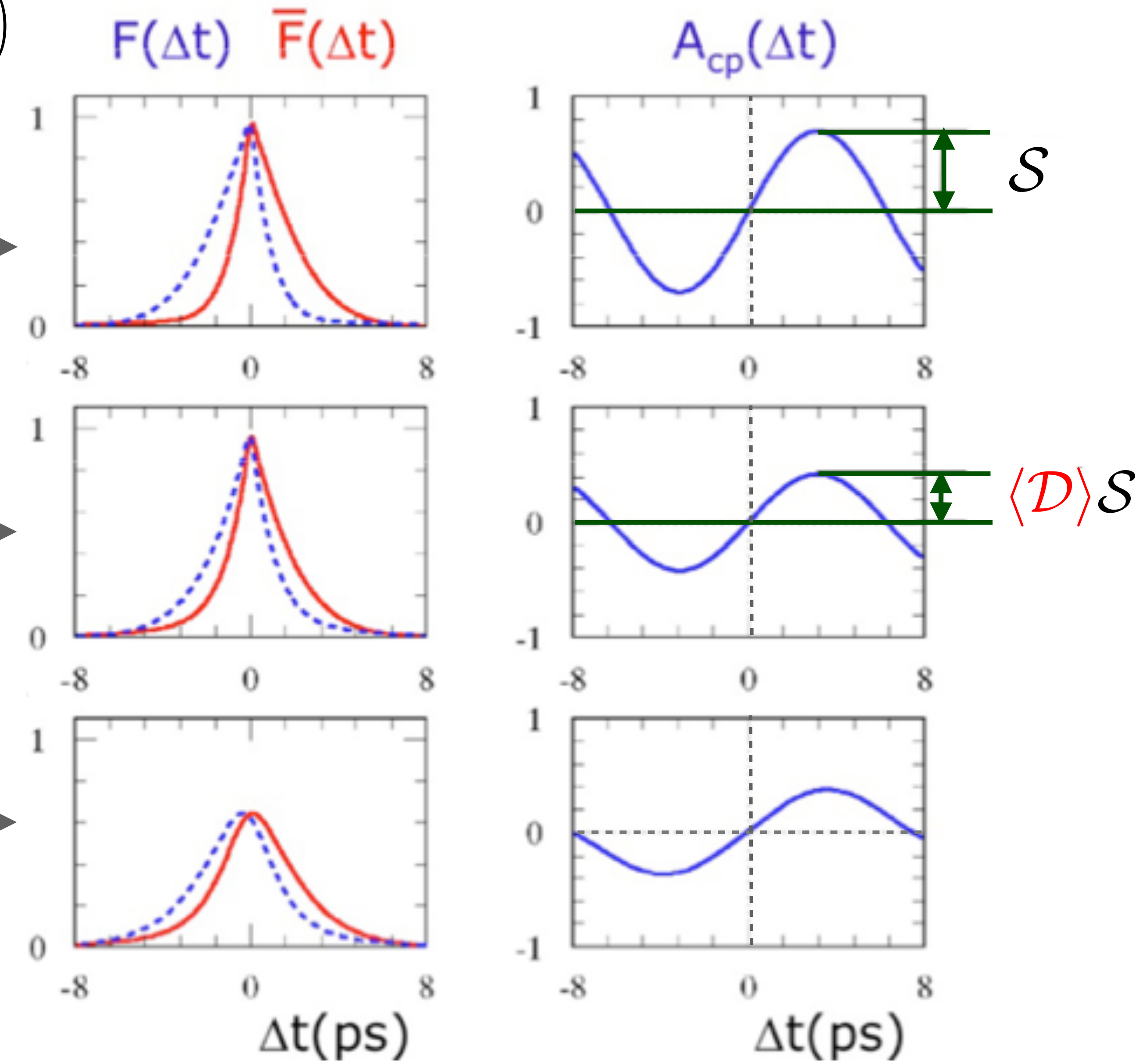
Everything perfect →

Adding mis-tag effects:

$$\langle \mathcal{D} \rangle = 1 - 2\omega$$

ω: mistag probability →

Adding imperfect Δt resolution effects →



Validation tests (1)
pseudo-experiments

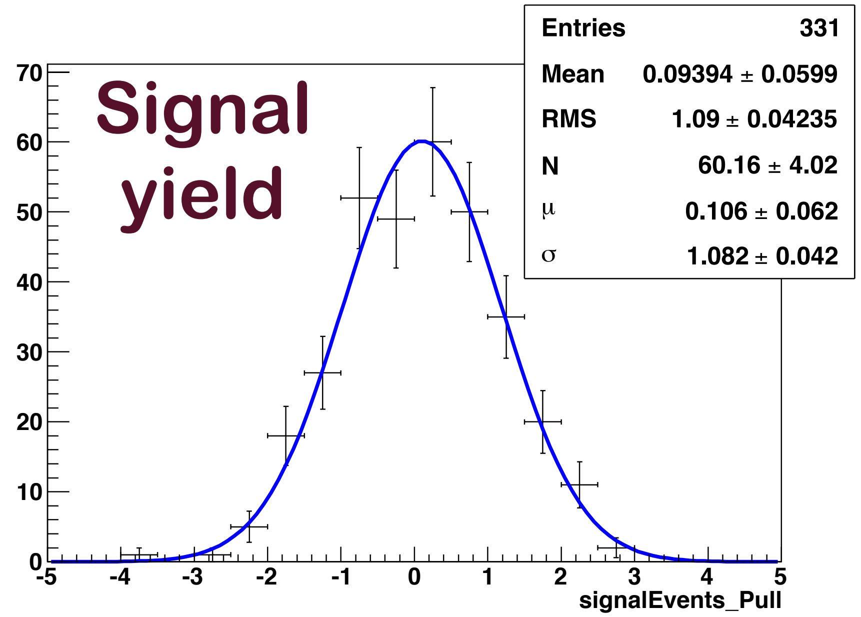
A tool for validation tests:

- From this model, we perform consistency tests:
  - ↳ Create simulated pseudo-experiments based on our model and test the "**pulls**"
- For unbiased fit parameters:
  - ↳ Pull of θ_i is a **gaussian** with:
    - **mean = 0**
    - **width = 1**

$$pull_i = \frac{\theta_i^{true} - \theta_i^{fit}}{\sigma_i^{fit}}$$

★ **Pure toys:**

- **100% convergence**

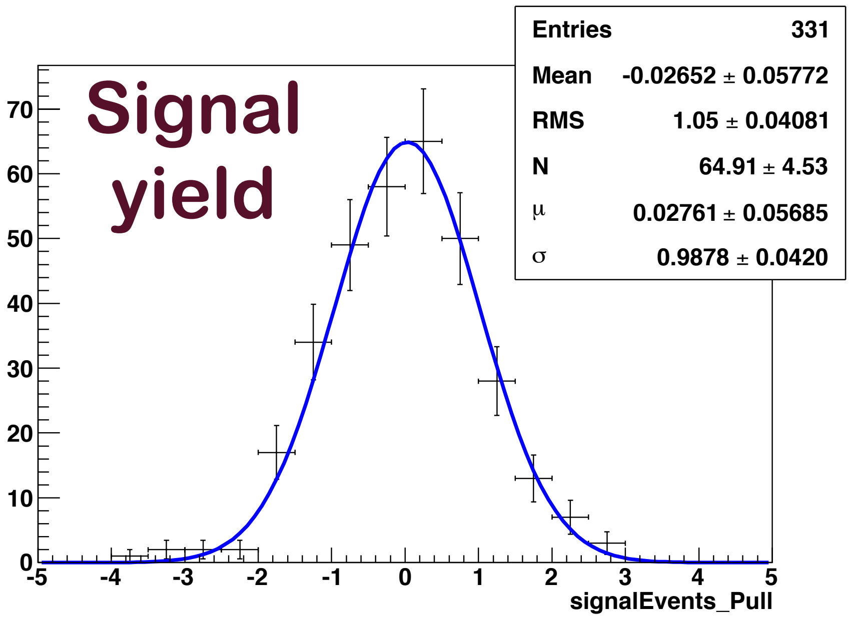


No bias on the signal yield

	Fit variable	Fit Parameter	Pull Mean	Pull Width
Signal TM	m_{ES}	CB_μ(Coeff0)	0.079 ± 0.052	0.946 ± 0.037
		CB_μ(Coeff1)	0.035 ± 0.052	0.950 ± 0.037
		CB_μ(Coeff2)	0.009 ± 0.056	1.001 ± 0.039
		CB_σ(Coeff0)	-0.060 ± 0.047	1.085 ± 0.042
		CB_σ(Coeff1)	-0.098 ± 0.054	0.980 ± 0.038
		CB_σ(Coeff2)	-0.116 ± 0.058	1.053 ± 0.041
	ΔE	Cr_μ	0.004 ± 0.048	0.870 ± 0.034
		Cr_{σ_L}	0.067 ± 0.053	0.958 ± 0.037
	Fisher	G_μ	-0.037 ± 0.054	0.991 ± 0.039
		G_σ	-0.012 ± 0.052	0.943 ± 0.037
udsc	m_{ES}	Arg_ξ	-0.031 ± 0.054	0.989 ± 0.038
	ΔE	Chebychev(Coeff0)	-0.037 ± 0.056	1.017 ± 0.040
		Chebychev(Coeff1)	-0.090 ± 0.055	1.000 ± 0.039
Yields		Signal	0.094 ± 0.059	1.090 ± 0.042
		Continuum *udsc*	0.034 ± 0.057	1.032 ± 0.040
		Generic B-background	-0.063 ± 0.056	1.027 ± 0.040
		$B^0 \to K^{*0}(\to K\pi)\gamma$ $B^0 \to X_{sd}(\to K\pi)\gamma$	0.006 ± 0.054	0.980 ± 0.038

$B^+ \to K^+\pi^-\pi^+\gamma$ Analysis
Toy studies (2)

- **Embedded toys:**
  - 100% convergence



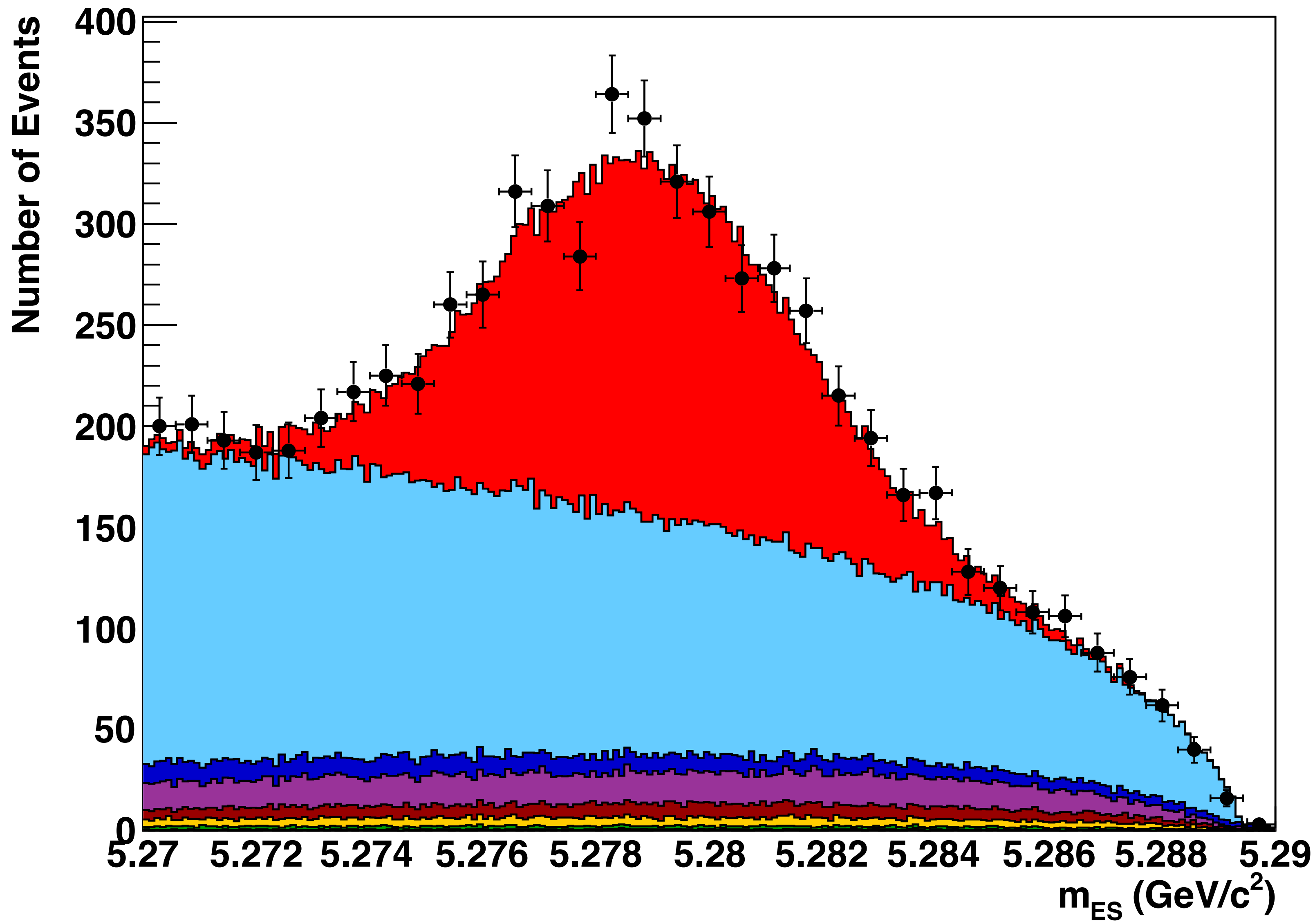
	Fit variable	Fit Parameter	Pull Mean	Pull Width
Signal TM	$m_{\rm ES}$	$\rm CB_\mu$(Coeff0)	0.706 ± 0.057	1.042 ± 0.040
		$\rm CB_\mu$(Coeff1)	-0.338 ± 0.058	1.048 ± 0.040
		$\rm CB_\mu$(Coeff2)	-0.328 ± 0.064	1.180 ± 0.046
		$\rm CB_\sigma$(Coeff0)	1.033 ± 0.054	0.983 ± 0.038
		$\rm CB_\sigma$(Coeff1)	-0.901 ± 0.053	0.961 ± 0.037
		$\rm CB_\sigma$(Coeff2)	-0.681 ± 0.065	1.174 ± 0.046
	ΔE	$\rm Cr_\mu$	-0.061 ± 0.053	0.971 ± 0.038
		$\rm Cr_{\sigma_L}$	-0.457 ± 0.063	1.154 ± 0.045
	Fisher	$\rm G_\mu$	0.117 ± 0.057	1.030 ± 0.040
		$\rm G_\sigma$	0.116 ± 0.055	0.994 ± 0.039
udsc	$m_{\rm ES}$	$\rm Arg_\xi$	0.125 ± 0.052	0.948 ± 0.037
	ΔE	Chebychev(Coeff0)	-0.642 ± 0.058	1.056 ± 0.041
		Chebychev(Coeff1)	0.052 ± 0.053	0.957 ± 0.037
Yields		Signal	-0.027 ± 0.058	1.050 ± 0.041
		Continuum *udsc*	0.229 ± 0.058	1.054 ± 0.041
		Generic B background	-0.354 ± 0.057	1.035 ± 0.040
		$B^0 \to K^{*0}(\to K\pi)\gamma$ $B^0 \to X_{sd}(\to K\pi)\gamma$	-0.026 ± 0.055	1.002 ± 0.039

No bias on the signal yield

Signal region:

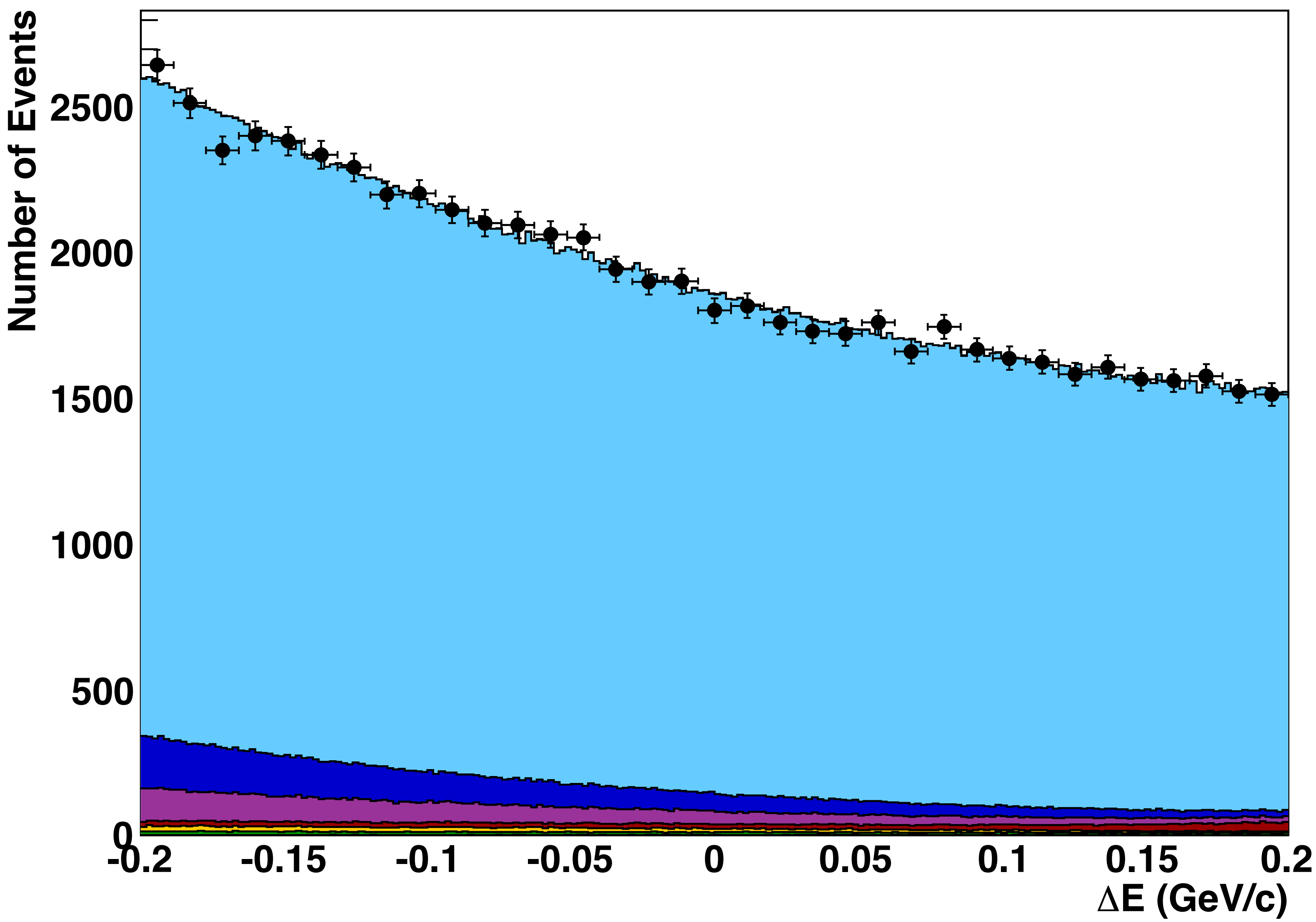
$-0.15 \leq \Delta E \leq 0.10$

$5.27 < M_{ES} \leq 5.292$



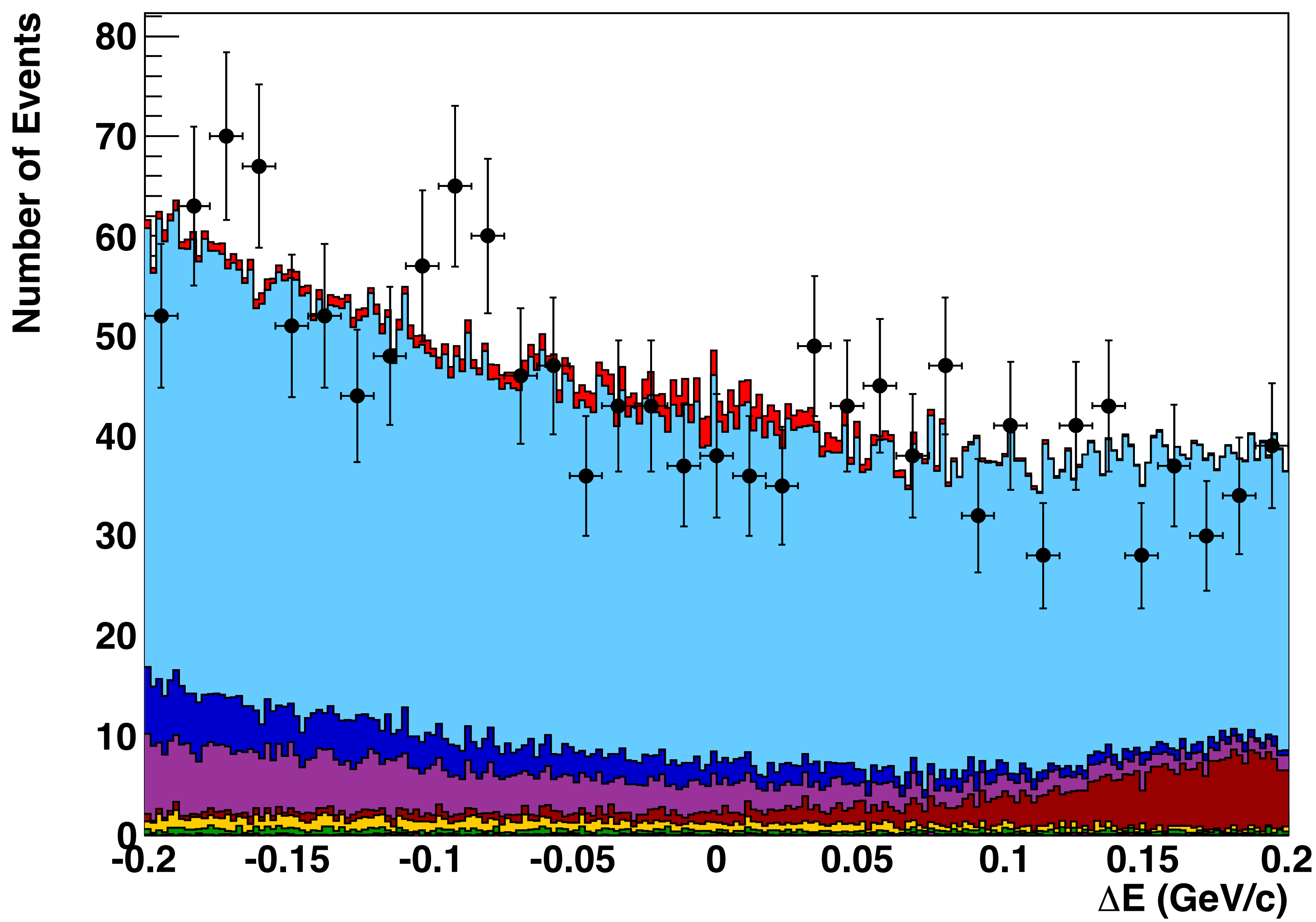
$B^+ \to K^+\pi^-\pi^+\gamma$ Analysis
ΔE Projection (1)

5.200 < mES ≤ 5.270

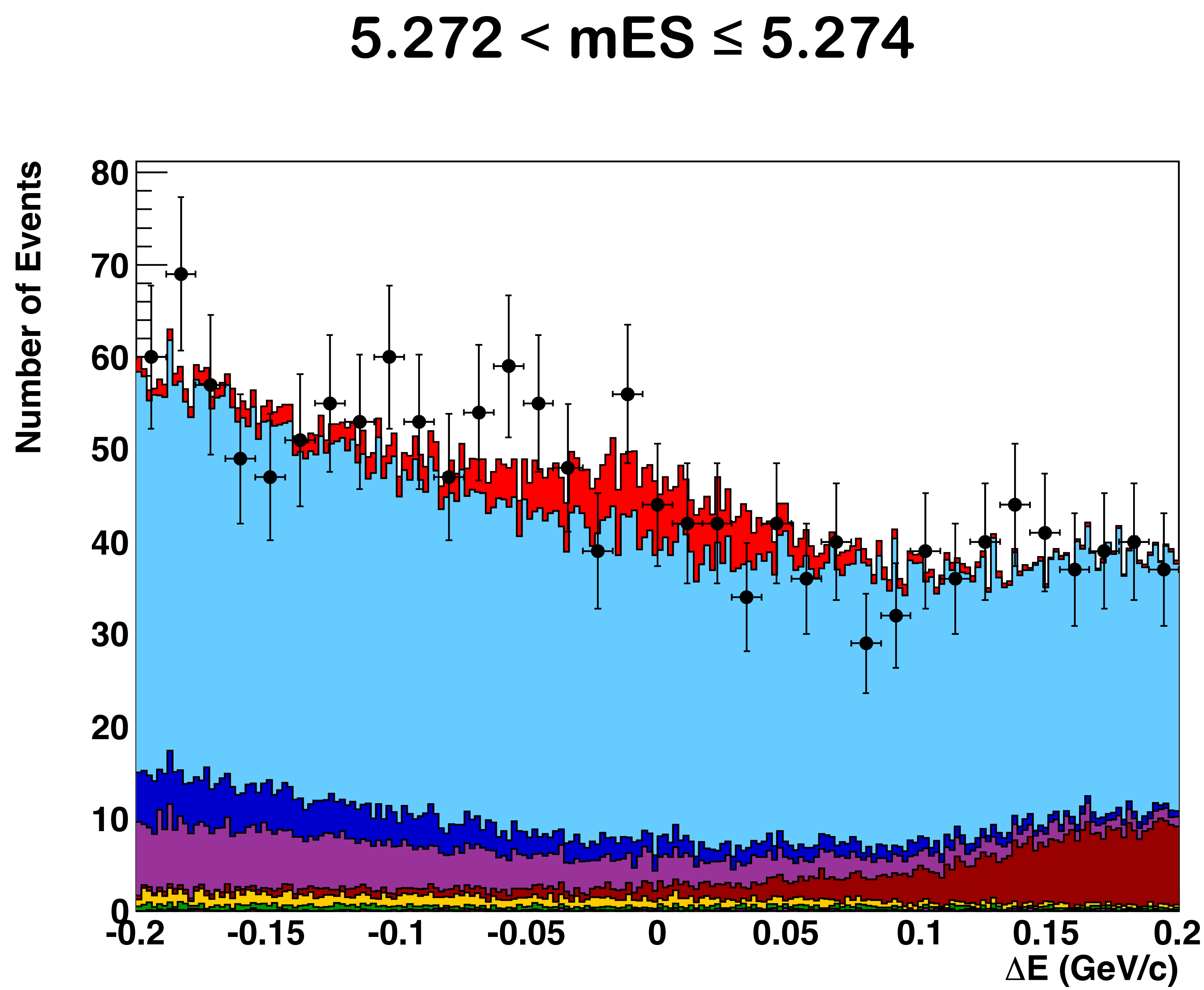


$B^+ \to K^+\pi^-\pi^+\gamma$ Analysis
ΔE Projection (2)

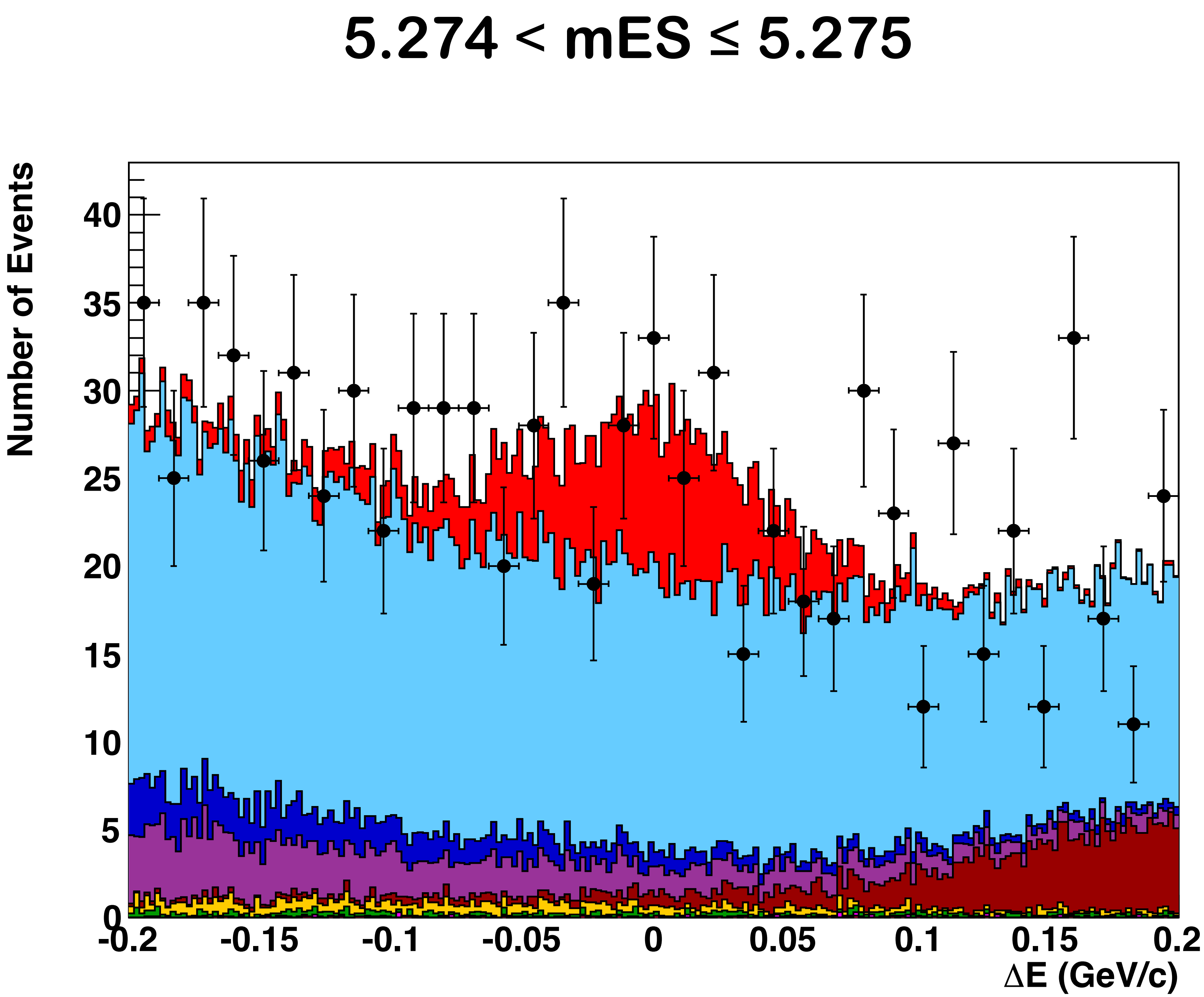
5.270 < mES ≤ 5.272



$B^+ \to K^+\pi^-\pi^+\gamma$ Analysis
ΔE Projection (3)

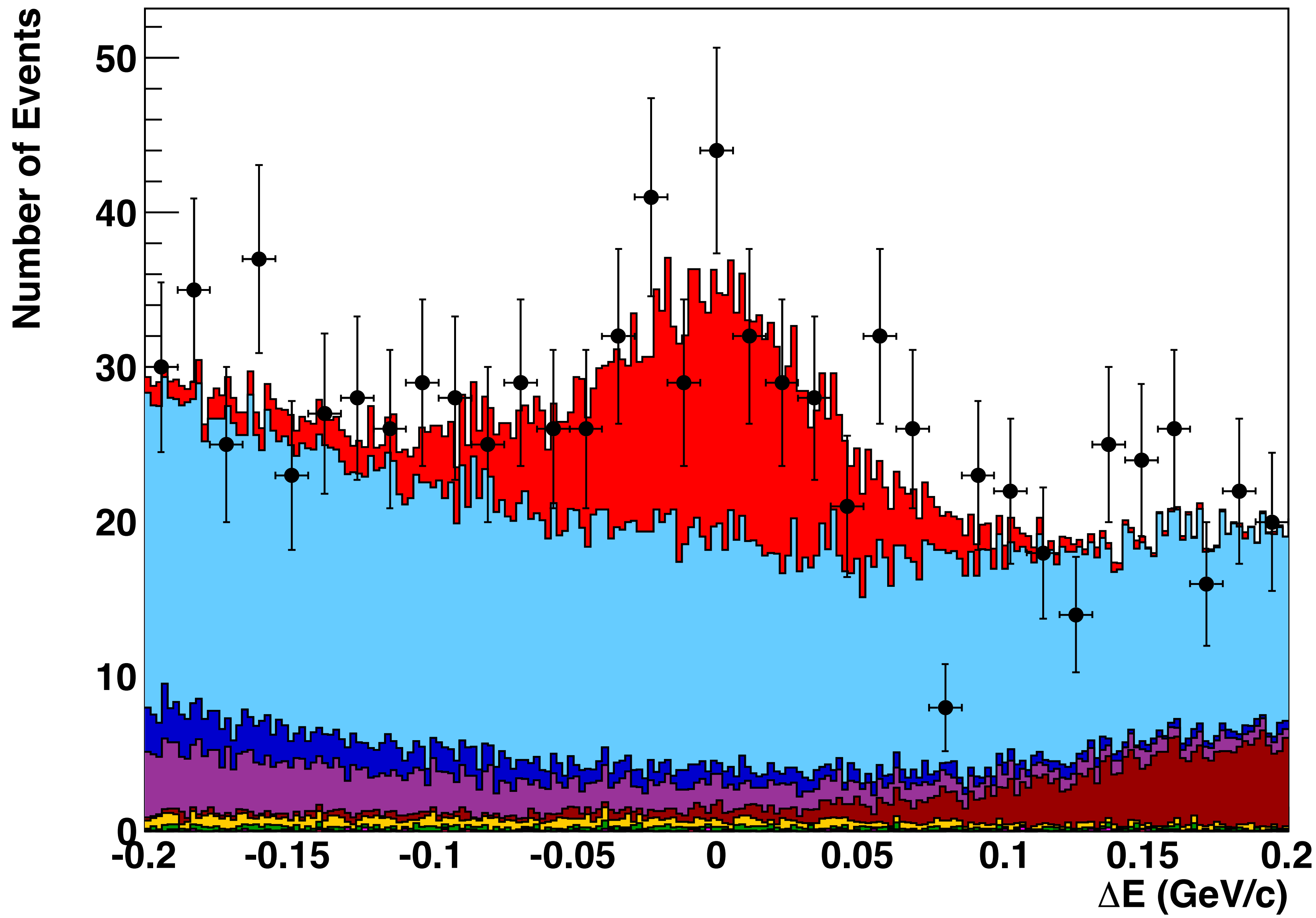


$B^+ \to K^+\pi^-\pi^+\gamma$ Analysis
ΔE Projection (4)



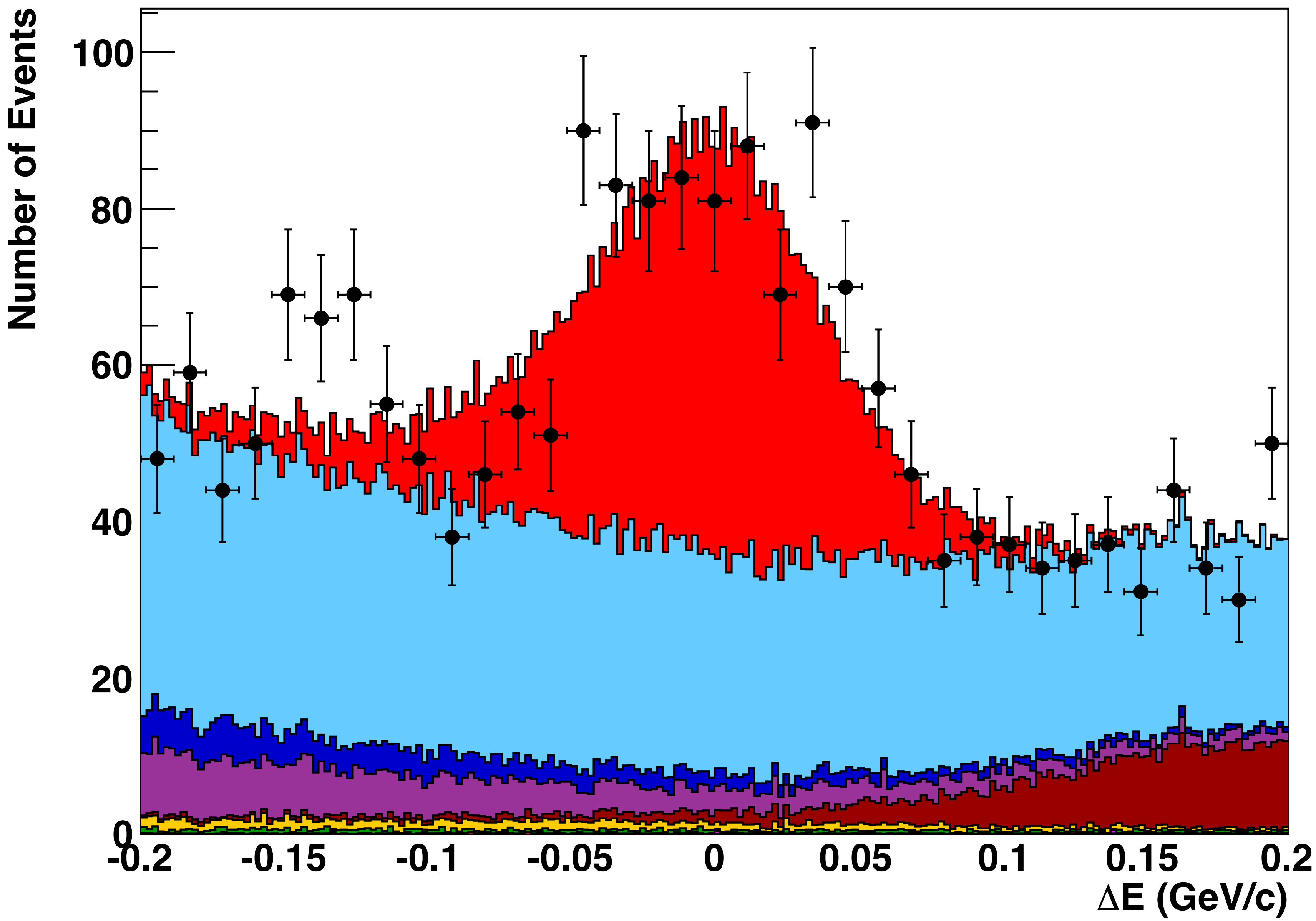
$B^+ \to K^+\pi^-\pi^+\gamma$ Analysis
ΔE Projection (5)

5.275 < mES ≤ 5.276

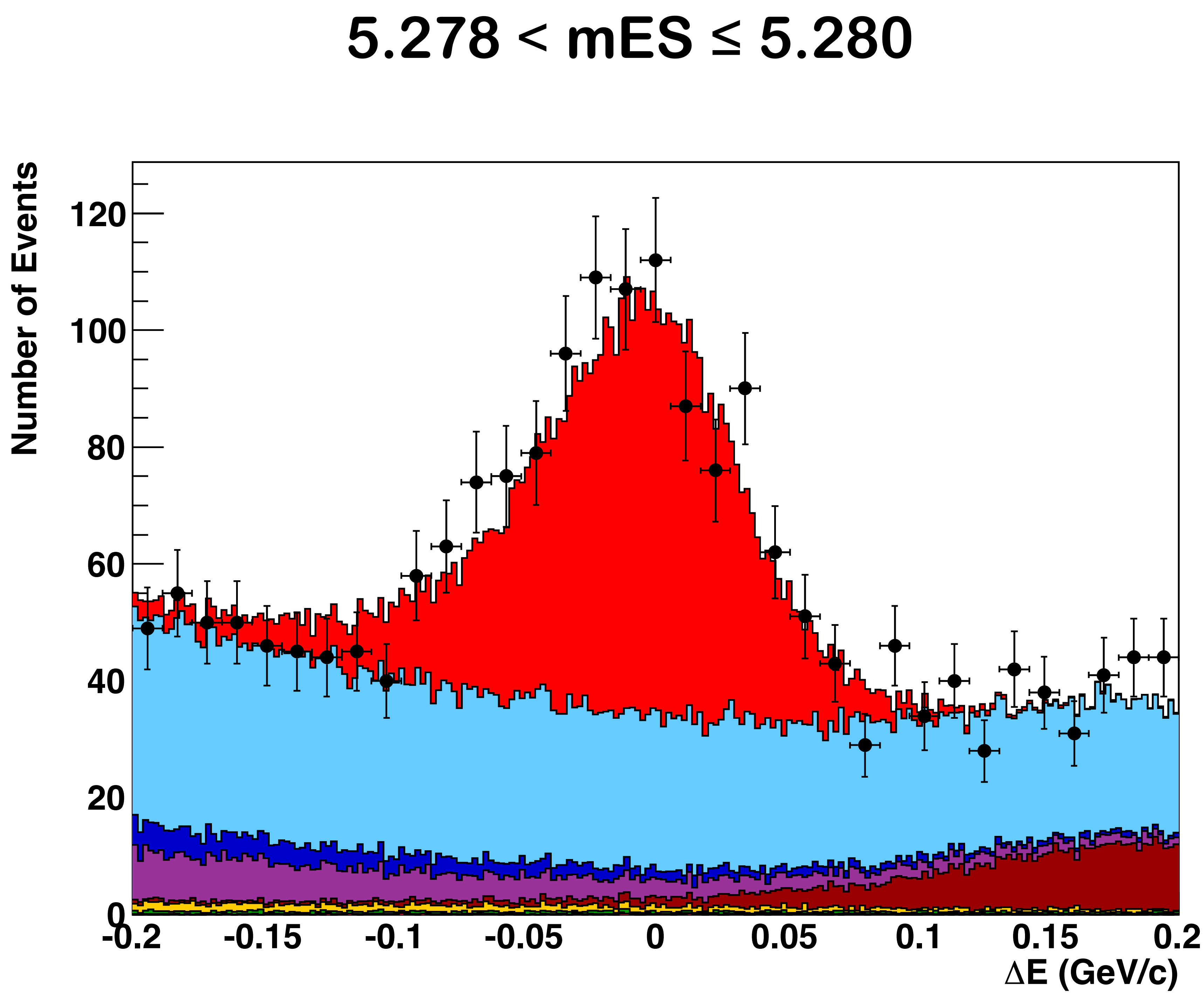


$B^+ \to K^+\pi^-\pi^+\gamma$ Analysis
ΔE Projection (6)

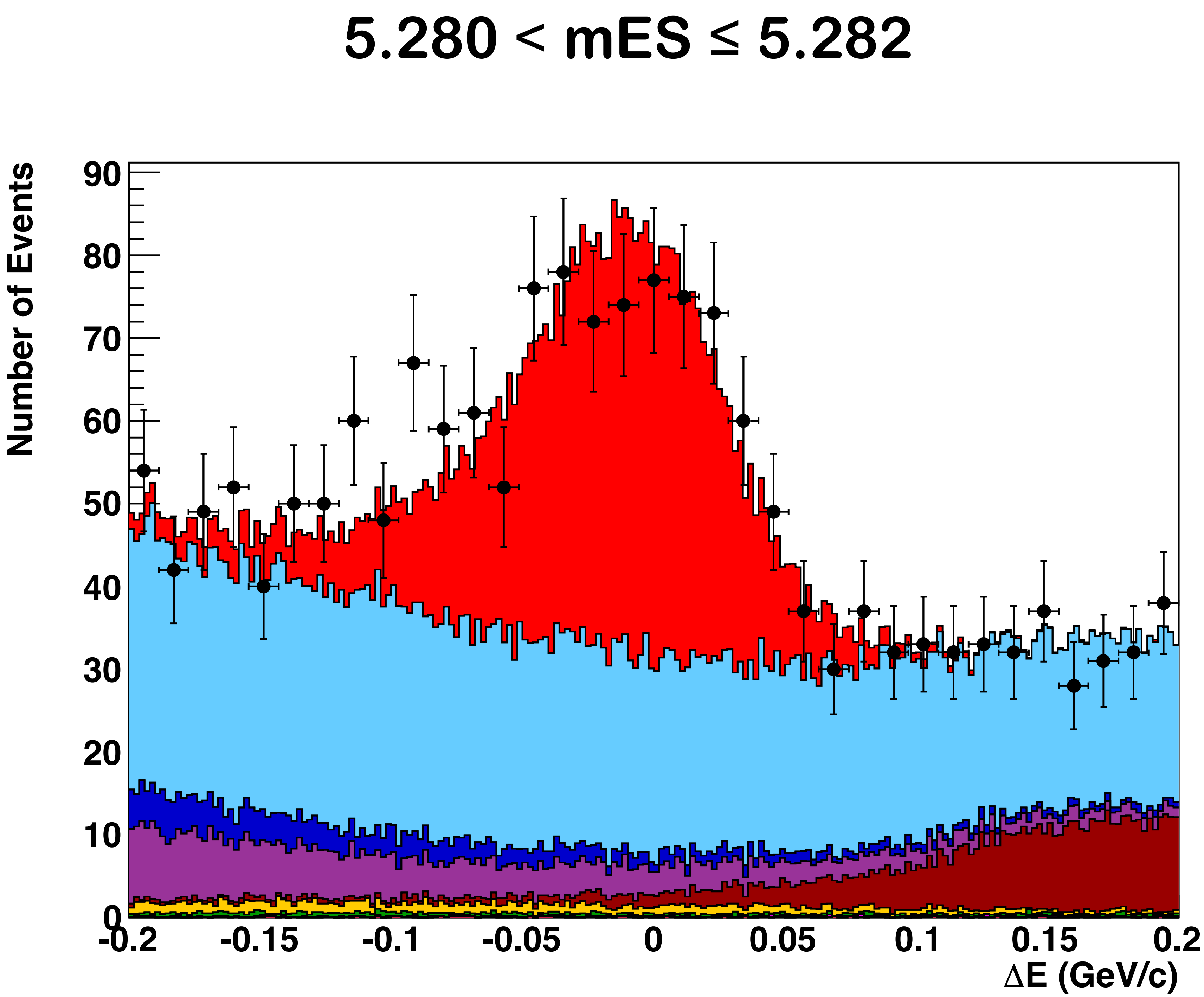
5.276 < mES ≤ 5.278



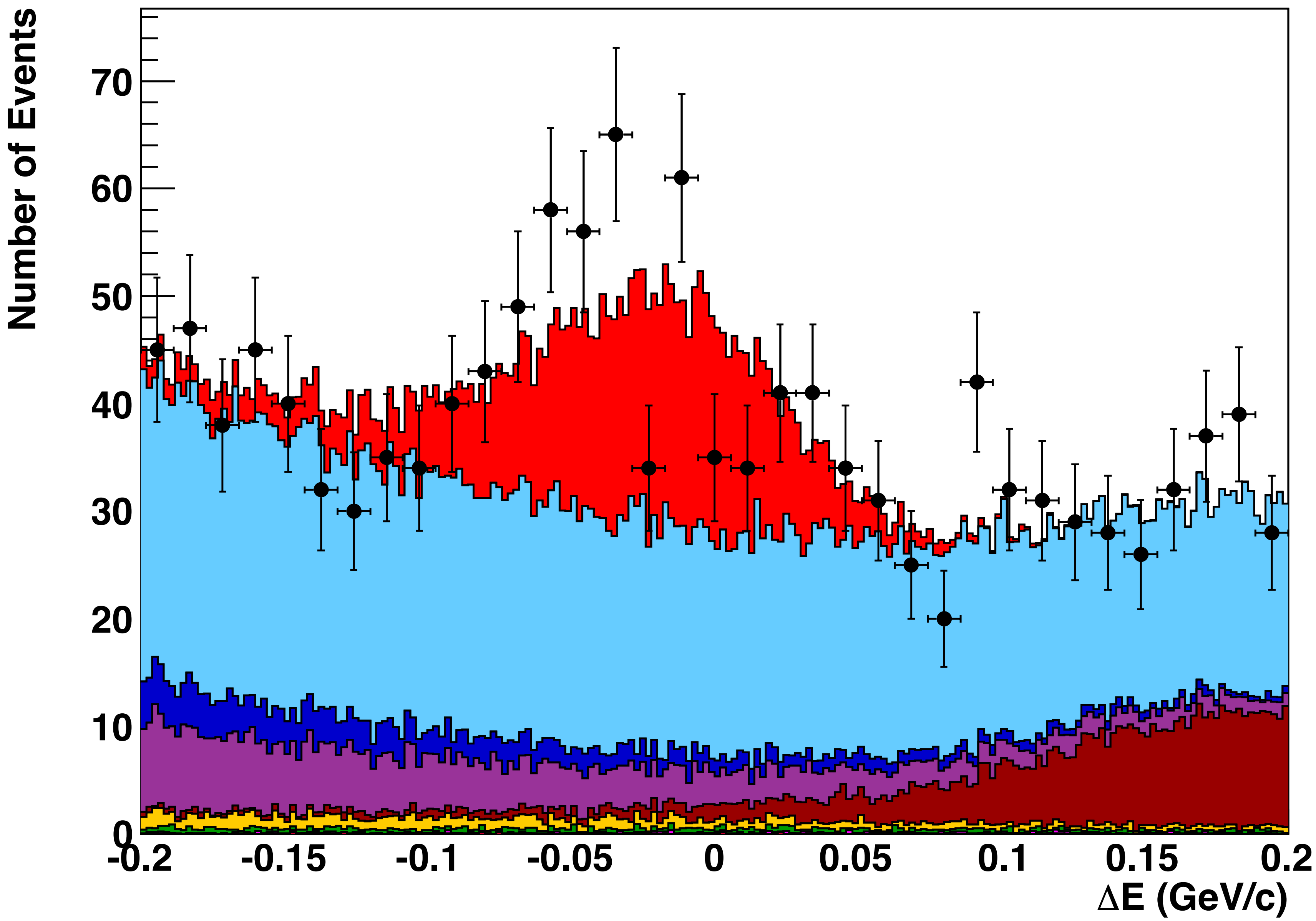
$B^+ \to K^+\pi^-\pi^+\gamma$ Analysis
ΔE Projection (7)



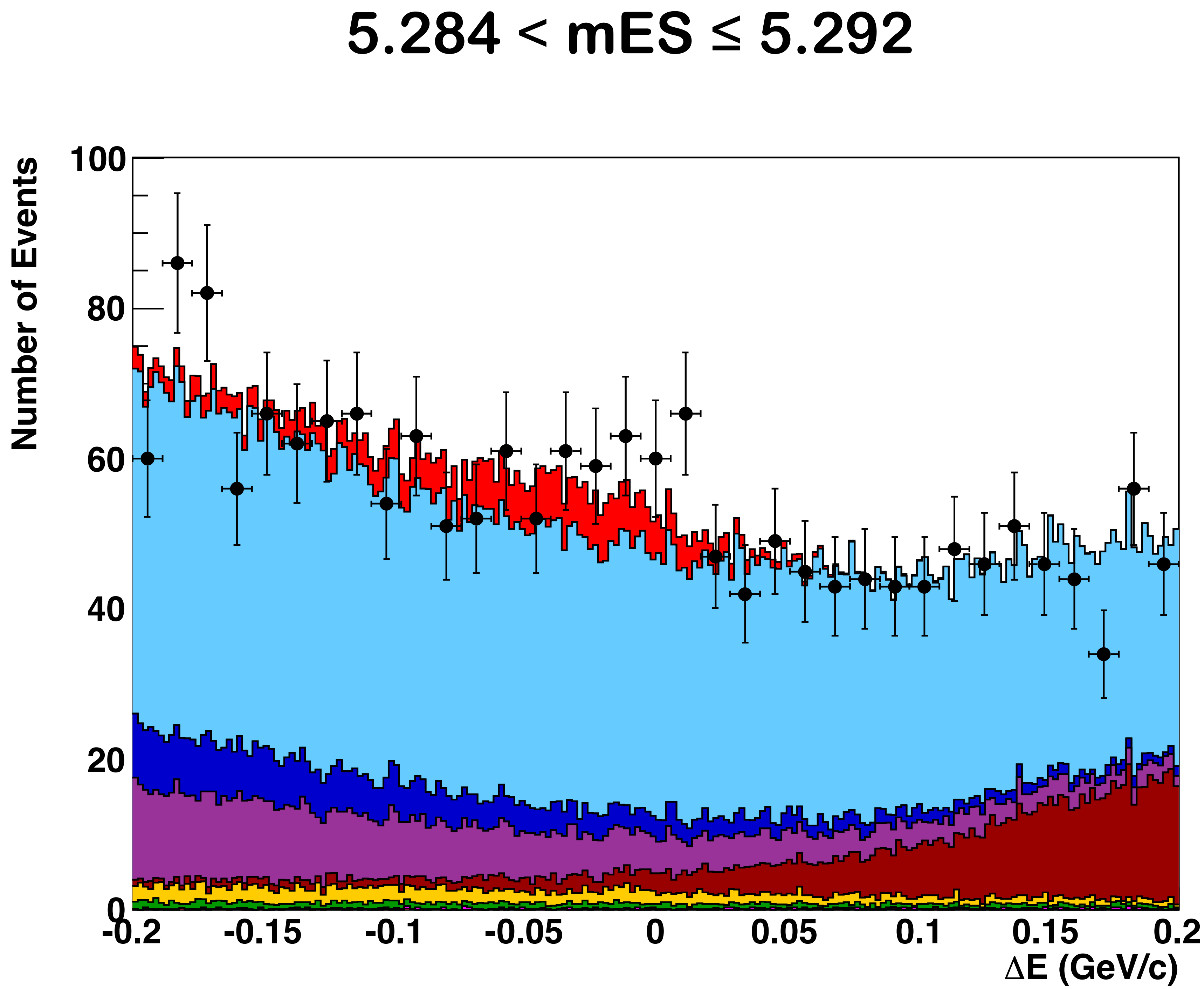
$B^+ \to K^+\pi^-\pi^+\gamma$ Analysis
ΔE Projection (8)



5.282 < mES ≤ 5.284



$B^+ \to K^+\pi^-\pi^+\gamma$ Analysis
ΔE Projection (10)

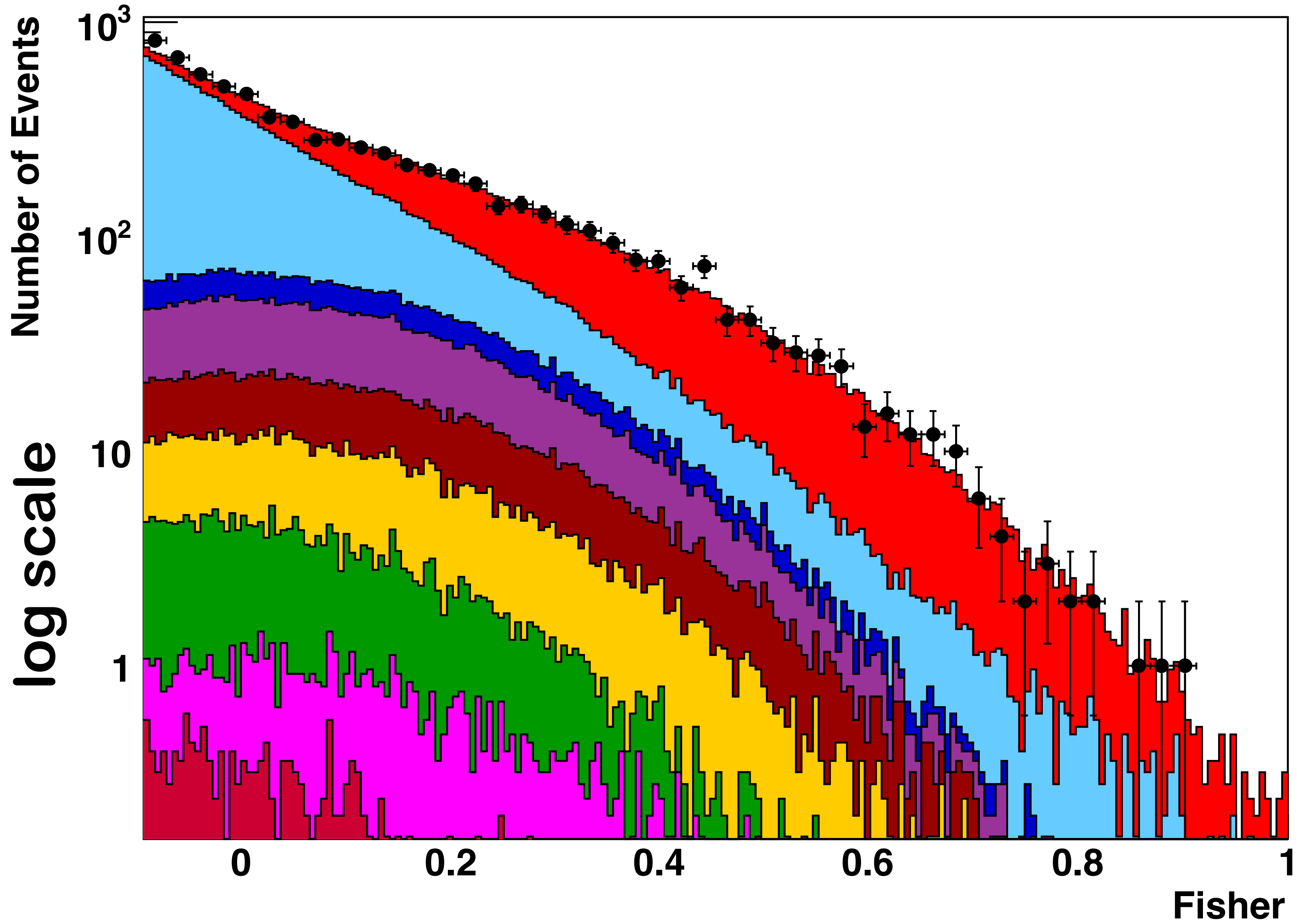


$B^+ \to K^+\pi^-\pi^+\gamma$ Analysis

Fisher Projection

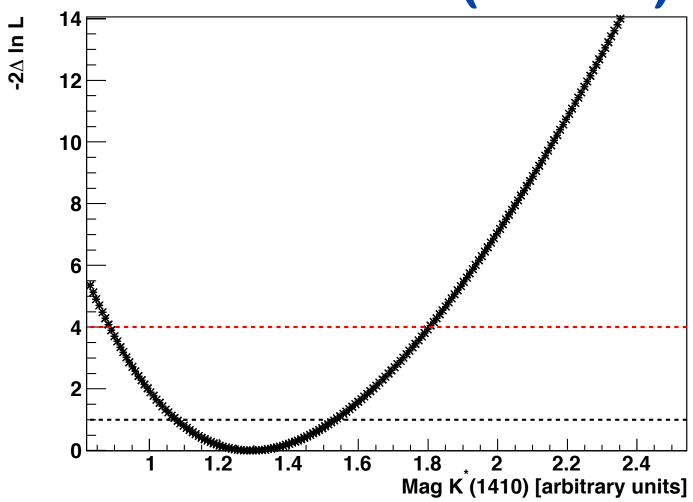
Signal region:

$$-0.15 \leq \Delta E \leq 0.10$$
$$5.27 < M_{ES} \leq 5.292$$

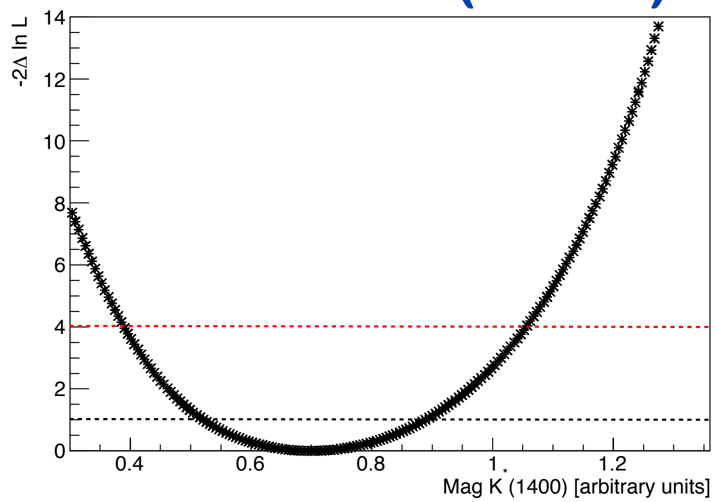


$B^+ \to K^+\pi^-\pi^+\gamma$ Analysis
$M_{K\pi\pi}$ Fit: Likelihood Scans

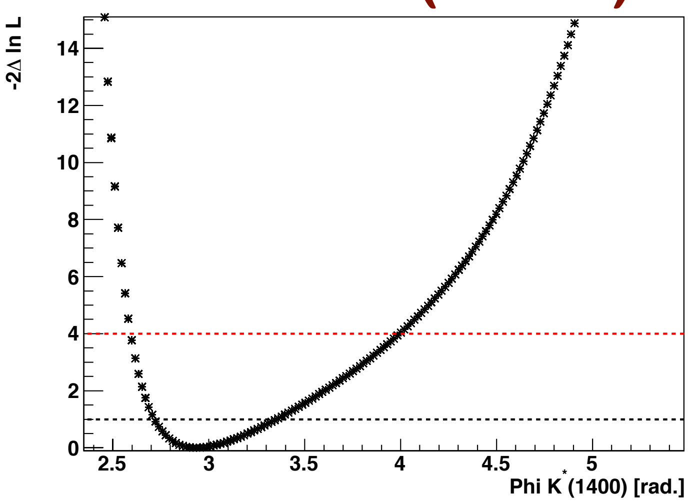
Module $K^*(1410)$



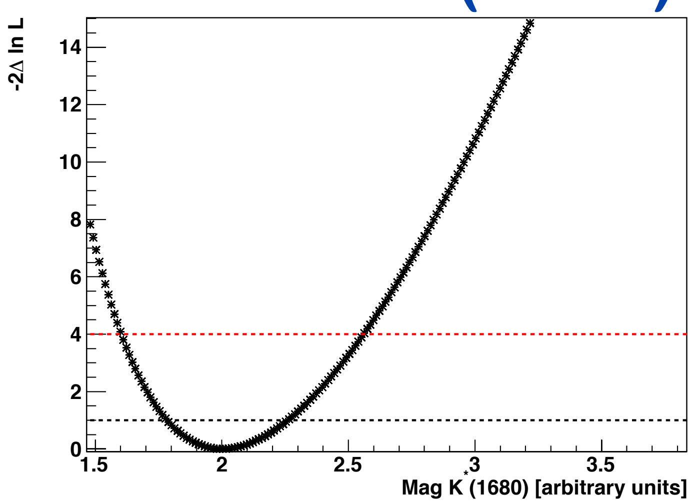
Module $K^*(1400)$



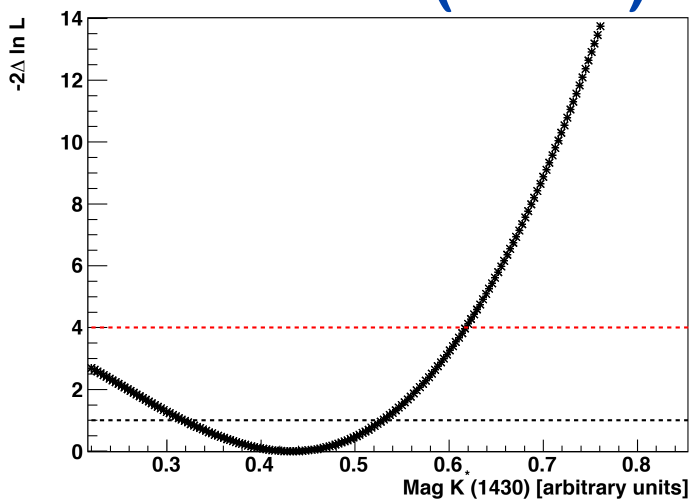
Phase $K^*(1400)$



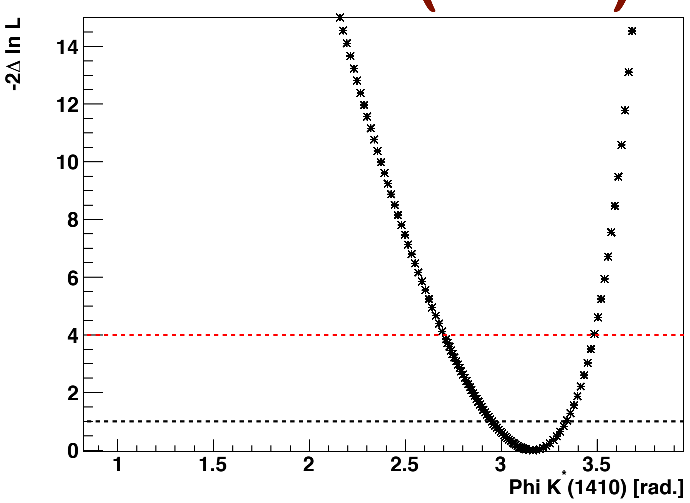
Module $K^*(1680)$



Module $K^*(1430)$



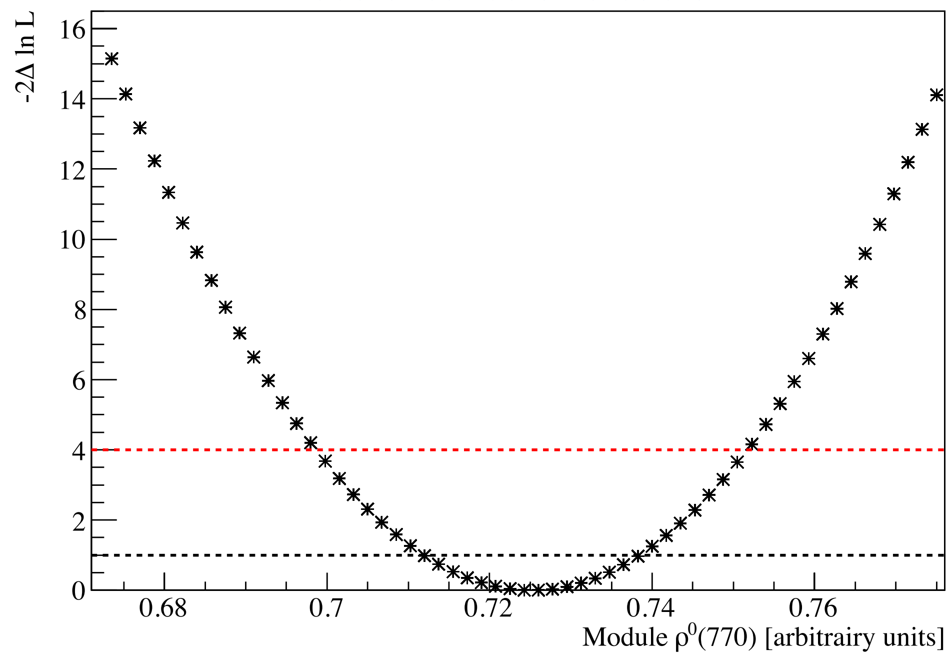
Phase $K^*(1410)$



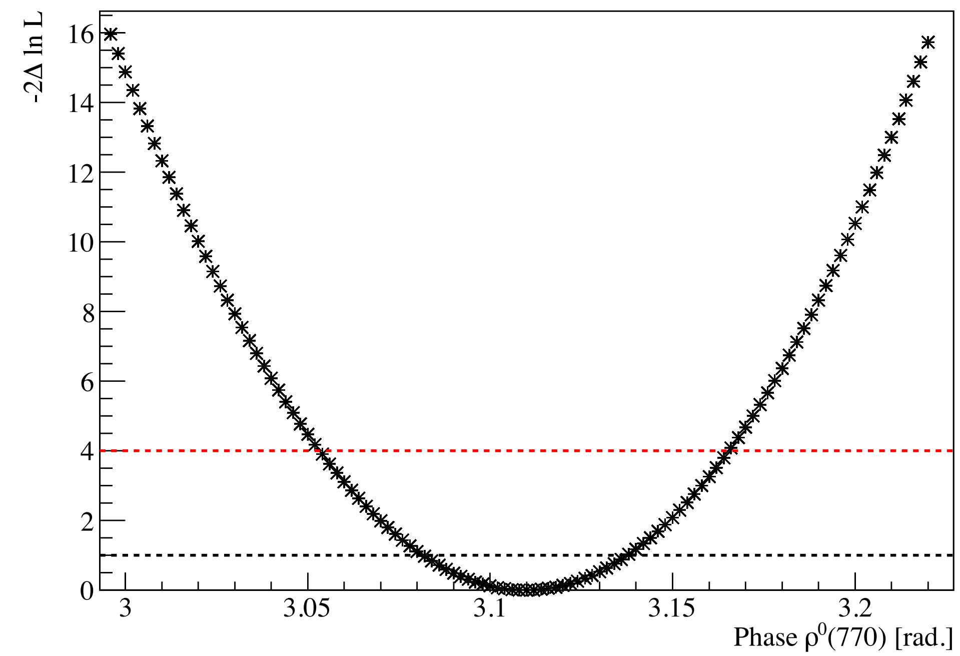
$B^+ \to K^+\pi^-\pi^+\gamma$ Analysis

$M_{K\pi}$ Fit: Likelihood Scans

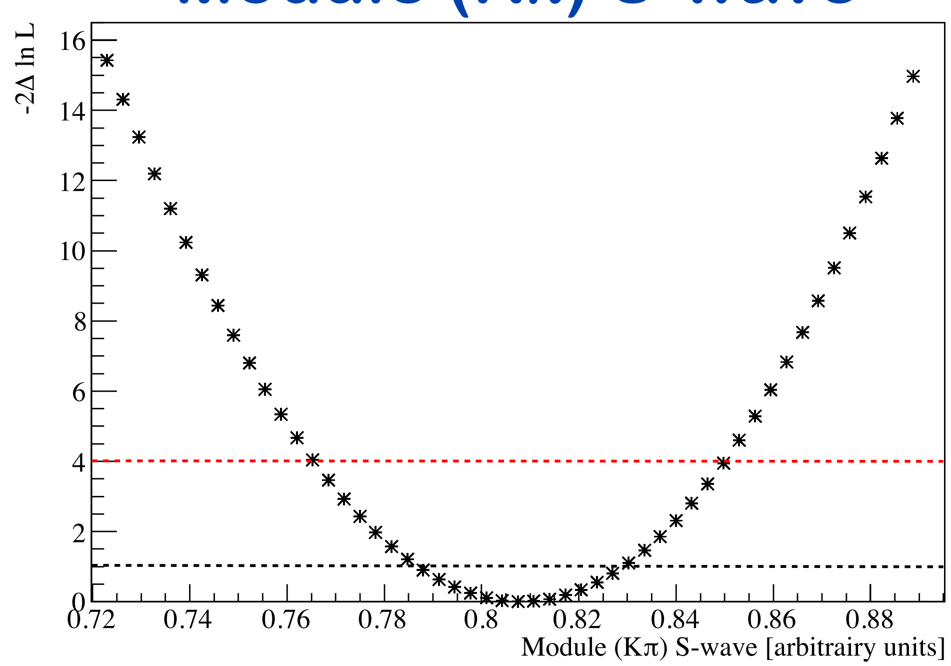
Module Rho



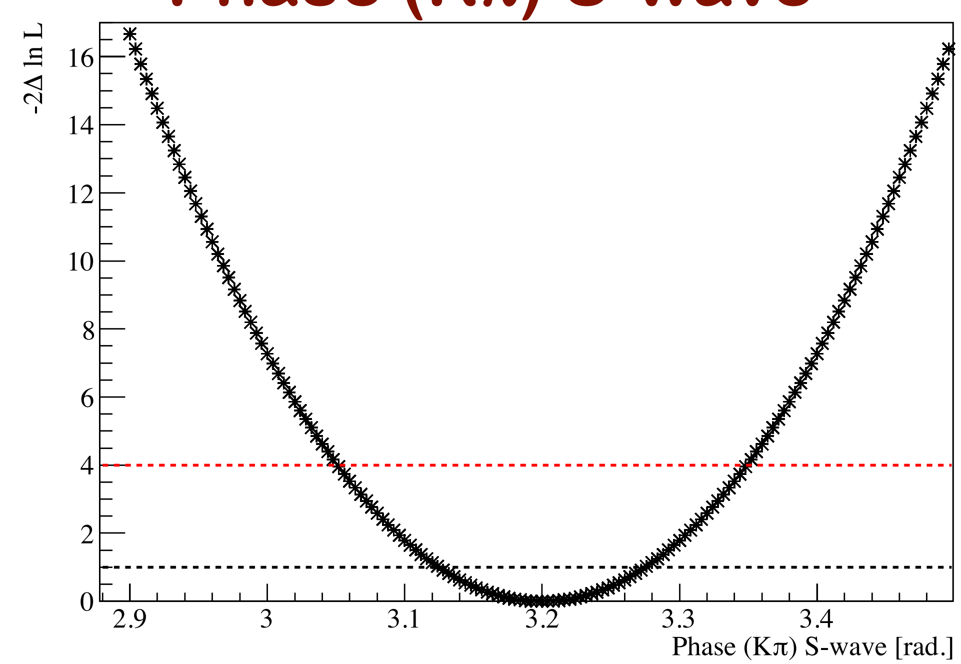
Phase Rho



Module (Kπ) S-wave



Phase (Kπ) S-wave



CENTRE DE PHYSIQUE DES PARTICULES DE MARSEILLE
CPPM

1000 pseudo-experiments with nominal fit model

Fit fraction	Pull Mean	Pull Width
$FF_{K^{*0}(892)}$	-0.027 ± 0.034	1.046 ± 0.026
$FF_{\rho^0(770)}$	0.080 ± 0.035	1.077 ± 0.027
$FF_{(K\pi)_{\mathrm{S-wave}}}$	0.109 ± 0.032	0.987 ± 0.024
FF_{Sum}	0.055 ± 0.034	1.050 ± 0.026
$FF^{\mathrm{Interf.}}_{K^{*0}(892)-\rho^0(770)}$	0.018 ± 0.033	1.034 ± 0.025
$FF^{\mathrm{Interf.}}_{(K\pi)_{\mathrm{S-wave}}-\rho^0(770)}$	-0.076 ± 0.036	1.095 ± 0.027

1000 pseudo-experiments with different values of the phase of the rho

$\phi_{\rho^0(770)}$ generated value	Pull Mean	Pull Width
$0 \times (\pi/5)$	-0.042 ± 0.032	1.046 ± 0.024
$1 \times (\pi/5)$	-0.072 ± 0.034	0.938 ± 0.026
$2 \times (\pi/5)$	0.295 ± 0.033	1.471 ± 0.025
$3 \times (\pi/5)$	0.122 ± 0.035	1.138 ± 0.025
$4 \times (\pi/5)$	-0.032 ± 0.035	0.974 ± 0.027
$5 \times (\pi/5)$	0.012 ± 0.033	1.052 ± 0.023
$6 \times (\pi/5)$	-0.027 ± 0.032	1.031 ± 0.025
$7 \times (\pi/5)$	-0.006 ± 0.033	0.890 ± 0.026
$8 \times (\pi/5)$	0.057 ± 0.034	0.999 ± 0.025
$9 \times (\pi/5)$	0.039 ± 0.032	1.052 ± 0.027
$10 \times (\pi/5)$	-0.021 ± 0.036	1.019 ± 0.025

Bad pulls: (rows $2 \times (\pi/5)$ and $3 \times (\pi/5)$)

Interference term cancels K* contribution...

Likelihood sometimes negative in K* region!

Nominal value: 3.110 (between $4 \times (\pi/5)$ and $5 \times (\pi/5)$)

Model appears to be robust in almost the entire parameter space

$B^+ \to K^+\pi^-\pi^+\gamma$ Analysis
$M_{K\pi}$ Fit: Pure Toys (3)

1000 pseudo-experiments with different values of the phase of the (Kπ) S-wave

$\phi_{(K\pi)\,\mathrm{S-wave}}$ generated value	Pull Mean	Pull Width
$0 \times (\pi/5)$	0.013 ± 0.034	1.027 ± 0.023
$1 \times (\pi/5)$	-0.045 ± 0.035	1.031 ± 0.025
$2 \times (\pi/5)$	-0.031 ± 0.033	1.054 ± 0.024
$3 \times (\pi/5)$	0.008 ± 0.036	1.023 ± 0.026
$4 \times (\pi/5)$	0.042 ± 0.034	1.021 ± 0.025
$5 \times (\pi/5)$	-0.039 ± 0.034	0.982 ± 0.024
$6 \times (\pi/5)$	-0.061 ± 0.032	1.041 ± 0.023
$7 \times (\pi/5)$	0.057 ± 0.036	1.091 ± 0.027
$8 \times (\pi/5)$	-0.048 ± 0.035	1.055 ± 0.026
$9 \times (\pi/5)$	-0.080 ± 0.036	0.912 ± 0.024
$10 \times (\pi/5)$	0.071 ± 0.033	1.019 ± 0.025

No bad pull behavior observed:

Interference term smoother than the one for K*-Rho

Nominal value: 3.197

Model appears to be robust in almost the entire parameter space

Fixed line-shape parameters of the kaonic resonances

Parameter	+ signed deviation	− signed deviation
Magnitude $K_1(1400)$	0.117	0.075
Magnitude $K^*(1410)$	0.174	0.127
Magnitude $K_2^*(1430)$	0.073	0.135
Magnitude $K^*(1680)$	0.303	0.202
Phase $K_1(1400)$	0.104	0.118
Phase $K^*(1410)$	0.024	0.022
Width $K_1(1270)$	0.004	0.003
Width $K^*(1680)$	0.043	0.025
FF $K_1(1270)$	0.050	0.050
FF $K_1(1400)$	0.052	0.030
FF $K^*(1410)$	0.053	0.023
FF $K_2^*(1430)$	0.035	0.049
FF $K^*(1680)$	0.089	0.056
FF $\{K_1(1270) - K_1(1400)\}$	0.050	0.060
FF $\{K^*(1410) - K^*(1680)\}$	0.057	0.115
FF Sum	0.140	0.072

CENTRE DE PHYSIQUE DES PARTICULES DE MARSEILLE
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Fixed parameter in the M_{ES}, ΔE and Fisher fit

Parameter	+ signed deviation	− signed deviation
Magnitude $K_1(1400)$	0.027	0.014
Magnitude $K^*(1410)$	0.060	0.025
Magnitude $K_2^*(1430)$	0.015	0.026
Magnitude $K^*(1680)$	0.108	0.047
Phase $K_1(1400)$	0.018	0.022
Phase $K^*(1410)$	0.013	0.005
Width $K_1(1270)$	0.002	0.004
Width $K^*(1680)$	0.012	0.004
FF $K_1(1270)$	0.010	0.009
FF $K_1(1400)$	0.009	0.003
FF $K^*(1410)$	0.019	0.005
FF $K_2^*(1430)$	0.008	0.012
FF $K^*(1680)$	0.010	0.005
FF $\{K_1(1270)-K_1(1400)\}$	0.011	0.013
FF $\{K^*(1410)-K^*(1680)\}$	0.004	0.020
FF Sum	0.023	0.001

sPlot extraction procedure

Parameter	+ signed deviation	− signed deviation
Magnitude $K_1(1400)$	0.003	$\emptyset$
Magnitude $K^*(1410)$	$\emptyset$	0.016
Magnitude $K_2^*(1430)$	$\emptyset$	0.007
Magnitude $K^*(1680)$	$\emptyset$	0.051
Phase $K_1(1400)$	0.004	$\emptyset$
Phase $K^*(1410)$	$\emptyset$	0,000
Width $K_1(1270)$	0.001	$\emptyset$
Width $K^*(1680)$	$\emptyset$	0.002
FF $K_1(1270)$	0.009	$\emptyset$
FF $K_1(1400)$	0.005	$\emptyset$
FF $K^*(1410)$	$\emptyset$	0.001
FF $K_2^*(1430)$	$\emptyset$	0.001
FF $K^*(1680)$	$\emptyset$	0.009
FF $\{K_1(1270) - K_1(1400)\}$	$\emptyset$	0.009
FF $\{K^*(1410) - K^*(1680)\}$	0.005	$\emptyset$
FF Sum	$\emptyset$	0.004

CENTRE DE PHYSIQUE DES PARTICULES DE MARSEILLE
CPPM

Number of bins in the fitted dataset

Parameter	+ signed deviation	− signed deviation
Magnitude $K_1(1400)$	0.003	0.001
Magnitude $K^*(1410)$	0.005	0.002
Magnitude $K_2^*(1430)$	0.002	0.006
Magnitude $K^*(1680)$	0.008	0.005
Phase $K_1(1400)$	0.002	0.002
Phase $K^*(1410)$	0.002	0.001
Width $K_1(1270)$	0.001	0.003
Width $K^*(1680)$	0.005	0.001
FF $K_1(1270)$	0.001	0.009
FF $K_1(1400)$	0.007	0.001
FF $K^*(1410)$	0.005	0.005
FF $K_2^*(1430)$	0.003	0.002
FF $K^*(1680)$	0.006	0.001
FF $\{K_1(1270) - K_1(1400)\}$	0.002	0.001
FF $\{K^*(1410) - K^*(1680)\}$	0.001	0.001
FF Sum	0.002	0.002

CENTRE DE PHYSIQUE DES PARTICULES DE MARSEILLE
CPPM

Weights of the kaonic resonances used to construct the total PDF

Parameter	+ signed deviation	− signed deviation
Magnitude $\rho^0(770)$	0.011	0.003
Magnitude $(K\pi)$ S-wave	0.009	0.046
Phase $\rho^0(770)$	0.030	0.030
Phase $(K\pi)$ S-wave	0.077	0.042
FF $K^{*0}(892)$	0.013	0.007
FF $\rho^0(770)$	0.027	0.006
FF $(K\pi)$ S-wave	0.012	0.065
FF $\{K^{*0}(892) - \rho^0(770)\}$	0.005	0.008
FF $\{(K\pi)$ S-wave $- \rho^0(770)\}$	0.020	0.038
FF Sum	0.087	0.030

sPlot extraction procedure

Parameter	+ signed deviation	− signed deviation
Magnitude $\rho^0(770)$	$\emptyset$	0.019
Magnitude $(K\pi)$ S-wave	0.030	$\emptyset$
Phase $\rho^0(770)$	$\emptyset$	0.014
Phase $(K\pi)$ S-wave	$\emptyset$	0.042
FF $K^{*0}(892)$	0.001	$\emptyset$
FF $\rho^0(770)$	$\emptyset$	0.027
FF $(K\pi)$ S-wave	0.043	$\emptyset$
FF $\{K^{*0}(892) - \rho^0(770)\}$	0.004	$\emptyset$
FF $\{(K\pi)$ S-wave $- \rho^0(770)\}$	0.014	$\emptyset$
FF Sum	$\emptyset$	0.048

Number of bins in the fitted dataset

Parameter	+ signed deviation	− signed deviation
Magnitude $\rho^0(770)$	0.006	0.000
Magnitude $(K\pi)$ S-wave	0.000	0.035
Phase $\rho^0(770)$	0.000	0.009
Phase $(K\pi)$ S-wave	0.000	0.016
FF $K^{*0}(892)$	0.011	0.000
FF $\rho^0(770)$	0.014	0.000
FF $(K\pi)$ S-wave	0.000	0.029
FF $\{K^{*0}(892) - \rho^0(770)\}$	0.000	0.006
FF $\{(K\pi)$ S-wave $- \rho^0(770)\}$	0.000	0.020
FF Sum	0.053	0.000

Number of bins in the PDF

Parameter	+ signed deviation	− signed deviation
Magnitude $\rho^0(770)$	0.000	0.004
Magnitude $(K\pi)$ S-wave	0.020	0.000
Phase $\rho^0(770)$	0.012	0.000
Phase $(K\pi)$ S-wave	0.013	0.000
FF $K^{*0}(892)$	0.000	0.006
FF $\rho^0(770)$	0.000	0.003
FF $(K\pi)$ S-wave	0.015	0.000
FF $\{K^{*0}(892) - \rho^0(770)\}$	0.001	0.000
FF $\{(K\pi)$ S-wave $- \rho^0(770)\}$	0.007	0.000
FF Sum	0.000	0.022

Fixed parameter in the M_{ES}, ΔE and Fisher fit

Parameter	+ signed deviation	− signed deviation
Magnitude $\rho^0(770)$	0.005	0.009
Magnitude $(K\pi)$ S-wave	0.018	0.005
Phase $\rho^0(770)$	0.045	0.003
Phase $(K\pi)$ S-wave	0.073	0.006
FF $K^{*0}(892)$	0.002	0.003
FF $\rho^0(770)$	0.011	0.009
FF $(K\pi)$ S-wave	0.020	0.013
FF $\{K^{*0}(892) - \rho^0(770)\}$	0.003	0.001
FF $\{(K\pi)$ S-wave $- \rho^0(770)\}$	0.013	0.007
FF Sum	0.024	0.045

Line-shape parameters of the resonances used in the $m_{K\pi}$ fit model

Parameter	+ signed deviation	− signed deviation
Magnitude $\rho^0(770)$	0.002	0.001
Magnitude $(K\pi)$ S-wave	0.007	0.001
Phase $\rho^0(770)$	0.012	0.017
Phase $(K\pi)$ S-wave	0.018	0.042
FF $K^{*0}(892)$	0.000	0.004
FF $\rho^0(770)$	0.003	0.011
FF $(K\pi)$ S-wave	0.021	0.001
FF $\{K^{*0}(892) - \rho^0(770)\}$	0.002	0.001
FF $\{(K\pi)$ S-wave $- \rho^0(770)\}$	0.014	0.004
FF Sum	0.007	0.039

CENTRE DE PHYSIQUE DES PARTICULES DE MARSEILLE
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Line-shape parameters of the kaonic resonances in MC

Parameter	+ signed deviation	− signed deviation
Magnitude $\rho^0(770)$	0.000	0.003
Magnitude $(K\pi)$ S-wave	0.014	0.001
Phase $\rho^0(770)$	0.021	0.030
Phase $(K\pi)$ S-wave	0.025	0.063
FF $K^{*0}(892)$	0.000	0.005
FF $\rho^0(770)$	0.000	0.005
FF $(K\pi)$ S-wave	0.006	0.001
FF $\{K^{*0}(892) - \rho^0(770)\}$	0.002	0.001
FF $\{(K\pi)$ S-wave $- \rho^0(770)\}$	0.008	0.003
FF Sum	0.011	0.021

PDFs tagCat dependence

Tagging category ID reminder:

Tag	ID
None	tagCat 0
Lepton	tagCat 63
Kaon I	tagCat 64
Kaon II	tagCat 65
Kaon-Pion	tagCat 66
Pion	tagCat 67
Other	tagCat 68

sub-tagger	NN	Discriminating variables	Training target
Electron	4:12:1	p^*, E_{90}^{W}, $\cos(\theta_{miss})$, q	Classify B^0 vs. $\bar{B}^0$
Muon	4:12:1	p^*, E_{90}^{W}, $\cos(\theta_{miss})$, q	Classify B^0 vs. $\bar{B}^0$
KinLep	3:3:1	p^*, E_{90}^{W}, $\cos(\theta_{miss})$	Recognize leptons from "direct" decays
Kaon	5:10:1	K1, K2, K3, nK_s^0, $\sum p_t^2$	Classify B^0 vs. $\bar{B}^0$
SlowPion	3:10:1	p^*, $\cos(\theta_{thr})$, $\mathcal{L}_K$	Recognize true slow pions
MaxPstar	3:6:1	p^*, $DOCA_{xy}$, $\cos(\theta)$	Recognize fast tracks
KPi	3:10:1	Kaon tag, SlowPion tag, $cos(\theta_{K,\pi})$	Recognize pairs of true kaons and slow pions
FSC	6:12:1	$\cos(\theta_{SlowFast})$, p^*_{Slow}, p^*_{Fats}, $\cos(\theta_{SlowThrust})$, $\cos(\theta_{FastThrust})$ $\mathcal{L}_{K\ Slow}$	
Lambda	6:14:1	M_Λ, χ^2, $\cos(\theta)$ flight length, p_Λ, p_p	Recognize Λ decays
Tag04	9:20:1	All of the above tags	Classify B^0 vs. $\bar{B}^0$

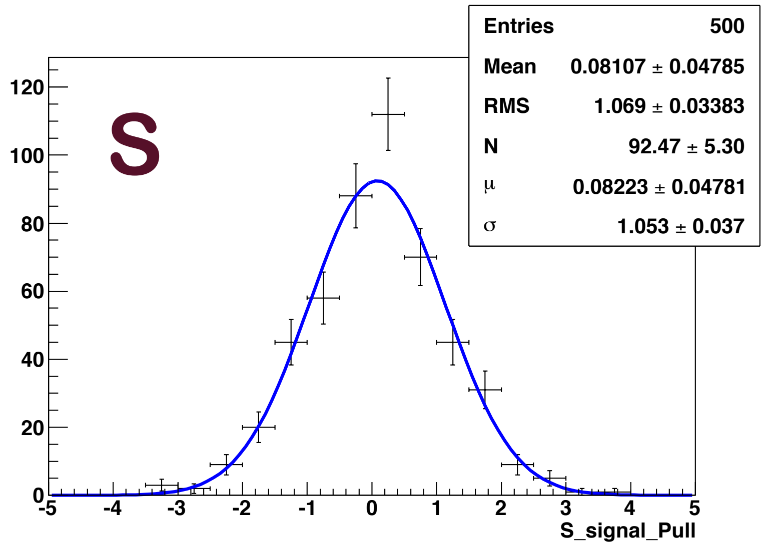
Table 7.2: Summary of the different sub-taggers. The second row indicates the number of input layers, hidden layers and output layer respectively for the NNs. The third row gives the input variables that are defined in the text, the last row details the target of the training

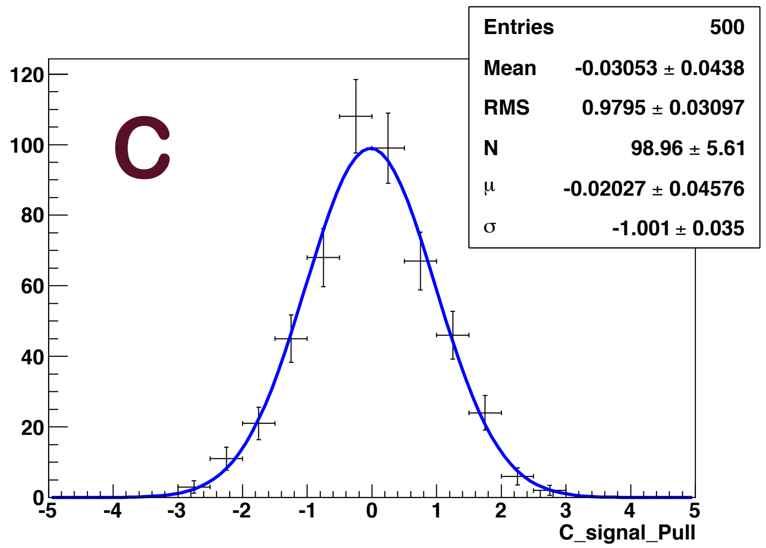
$B^0 \to K_S \pi^- \pi^+ \gamma$ TDCP Analysis

Toy studies (1)

★ **Pure toys:**

- **100% convergence**



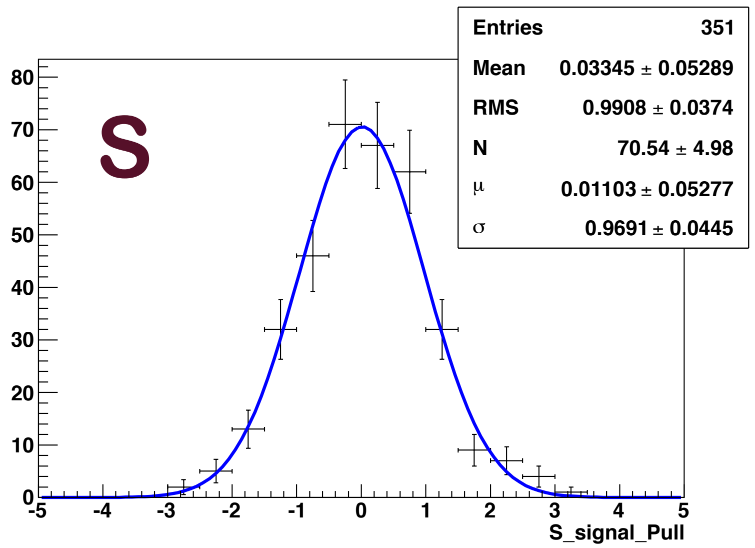


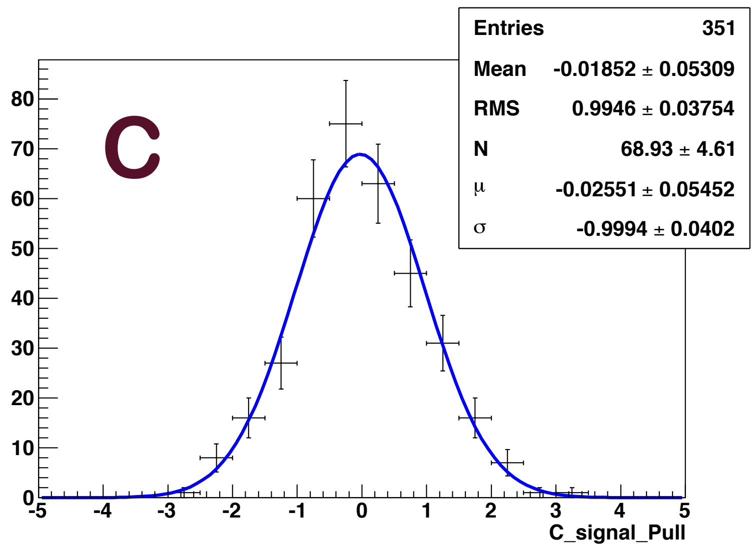
No biases on S and C

	Fit variable	Fit Parameter	Pull Mean	Pull Width
Signal TM	m_{ES}	$CB_\mu(Coeff0)$	0.071 ± 0.049	1.059 ± 0.037
		$CB_\mu(Coeff1)$	-0.078 ± 0.050	1.062 ± 0.038
		$CB_\mu(Coeff2)$	-0.187 ± 0.055	1.103 ± 0.046
		$CB_\sigma(Coeff0)$	-0.095 ± 0.054	1.053 ± 0.035
		$CB_\sigma(Coeff1)$	0.064 ± 0.044	0.898 ± 0.041
		$CB_\sigma(Coeff2)$	-0.002 ± 0.045	0.960 ± 0.038
	ΔE	Cr_{σ_R}	0.025 ± 0.044	0.941 ± 0.031
		Cr_{σ_L}	0.038 ± 0.046	0.826 ± 0.038
	Fisher	G_μ	0.027 ± 0.045	0.996 ± 0.037
udsc	*DeltaE*	Chebychev(Coeff0)	-0.003 ± 0.045	0.976 ± 0.038
		Chebychev(Coeff1)	0.001 ± 0.045	0.987 ± 0.031
	$\mathcal{R}_{bg}$	b_{core}	0.043 ± 0.044	0.964 ± 0.032
		s_{core}	-0.104 ± 0.046	0.997 ± 0.037
		$s_{outlier}$	0.166 ± 0.061	1.247 ± 0.056
		$f_{outlier}$	0.086 ± 0.045	0.961 ± 0.045
Yields		Signal	0.084 ± 0.049	0.977 ± 0.034
		Continuum udsc	0.032 ± 0.046	0.985 ± 0.036
		$B^+ \to K^{*+}(\to K^0_S \pi^+)\gamma$ $B^+ \to X_{su}(\to K^0_S \pi^+)\gamma$	-0.049 ± 0.046	1.003 ± 0.041
		$\mathcal{S}_{K^0_S \pi^+ \pi^- \gamma}$	0.082 ± 0.048	1.053 ± 0.037
		$\mathcal{C}_{K^0_S \pi^+ \pi^- \gamma}$	-0.020 ± 0.046	1.001 ± 0.035

★ **Embedded toys:**

- **100% convergence**





No biases on S and C

	Fit variable	Fit Parameter	Pull Mean	Pull Width
Signal TM	$m_{\rm ES}$	$\rm CB_\mu(Coeff0)$	0.118 ± 0.059	1.085 ± 0.046
		$\rm CB_\mu(Coeff1)$	-0.156 ± 0.064	1.155 ± 0.048
		$\rm CB_\mu(Coeff2)$	-0.188 ± 0.066	1.171 ± 0.051
		$\rm CB_\sigma(Coeff0)$	-0.153 ± 0.062	1.085 ± 0.060
		$\rm CB_\sigma(Coeff1)$	0.052 ± 0.054	0.969 ± 0.044
		$\rm CB_\sigma(Coeff2)$	-0.062 ± 0.057	0.992 ± 0.048
	ΔE	$\rm Cr_{\sigma_R}$	0.012 ± 0.053	0.955 ± 0.039
		$\rm Cr_{\sigma_L}$	0.057 ± 0.055	0.821 ± 0.040
	Fisher	$\rm G_\mu$	0.164 ± 0.054	0.991 ± 0.043
udsc	ΔE	Chebychev(Coeff0)	-0.047 ± 0.054	0.983 ± 0.035
		Chebychev(Coeff1)	0.055 ± 0.053	0.952 ± 0.040
	$\mathcal{R}_{\rm bg}$	$b_{\rm core}$	0.020 ± 0.052	0.962 ± 0.038
		$s_{\rm core}$	-0.157 ± 0.054	0.995 ± 0.039
		$s_{\rm outlier}$	0.123 ± 0.068	1.186 ± 0.053
		$f_{\rm outlier}$	0.148 ± 0.054	0.923 ± 0.043
Yields		Signal	-0.001 ± 0.054	0.967 ± 0.042
		Continuum udsc	0.050 ± 0.055	1.010 ± 0.044
		$B^+ \to K^{*+}(\to K^0_S\pi^+)\gamma$ $B^+ \to X_{su}(\to K^0_S\pi^+)\gamma$	-0.151 ± 0.054	0.990 ± 0.040
		$\mathcal{S}_{K^0_S\pi^+\pi^-\gamma}$	0.011 ± 0.053	0.969 ± 0.044
		$\mathcal{C}_{K^0_S\pi^+\pi^-\gamma}$	-0.026 ± 0.055	0.999 ± 0.040

★ **Embedded toys:**

- **No significant biases of S and C in the allowed parameter space**

Generated value in MC		Fit Parameter	Pull Mean	Pull Width
$\mathcal{S}=-0.8$	$\mathcal{C}=-0.8$	$\mathcal{S}$	$\varnothing$	$\varnothing$
		$\mathcal{C}$	$\varnothing$	$\varnothing$
	$\mathcal{C}=-0.4$	$\mathcal{S}$	0.067 ± 0.050	0.882 ± 0.035
		$\mathcal{C}$	0.030 ± 0.058	1.020 ± 0.041
	$\mathcal{C}=0.0$	$\mathcal{S}$	-0.031 ± 0.057	0.977 ± 0.040
		$\mathcal{C}$	-0.023 ± 0.062	1.070 ± 0.044
	$\mathcal{C}=+0.4$	$\mathcal{S}$	-0.007 ± 0.056	0.989 ± 0.040
		$\mathcal{C}$	0.069 ± 0.055	0.978 ± 0.039
	$\mathcal{C}=+0.8$	$\mathcal{S}$	$\varnothing$	$\varnothing$
		$\mathcal{C}$	$\varnothing$	$\varnothing$
$\mathcal{S}=-0.4$	$\mathcal{C}=-0.8$	$\mathcal{S}$	-0.032 ± 0.063	1.110 ± 0.044
		$\mathcal{C}$	-0.135 ± 0.060	1.060 ± 0.042
	$\mathcal{C}=-0.4$	$\mathcal{S}$	0.021 ± 0.059	1.040 ± 0.041
		$\mathcal{C}$	0.006 ± 0.058	1.028 ± 0.041
	$\mathcal{C}=0.0$	$\mathcal{S}$	0.011 ± 0.057	1.024 ± 0.040
		$\mathcal{C}$	0.062 ± 0.060	1.067 ± 0.042
	$\mathcal{C}=+0.4$	$\mathcal{S}$	0.061 ± 0.063	1.101 ± 0.045
		$\mathcal{C}$	0.081 ± 0.061	1.062 ± 0.043
	$\mathcal{C}=+0.8$	$\mathcal{S}$	-0.146 ± 0.060	1.047 ± 0.043
		$\mathcal{C}$	0.161 ± 0.062	1.076 ± 0.044

$B^0 \to K_S \pi^- \pi^+ \gamma$ TDCP ANALYSIS

TOY STUDIES (4)

★ **Embedded toys:**

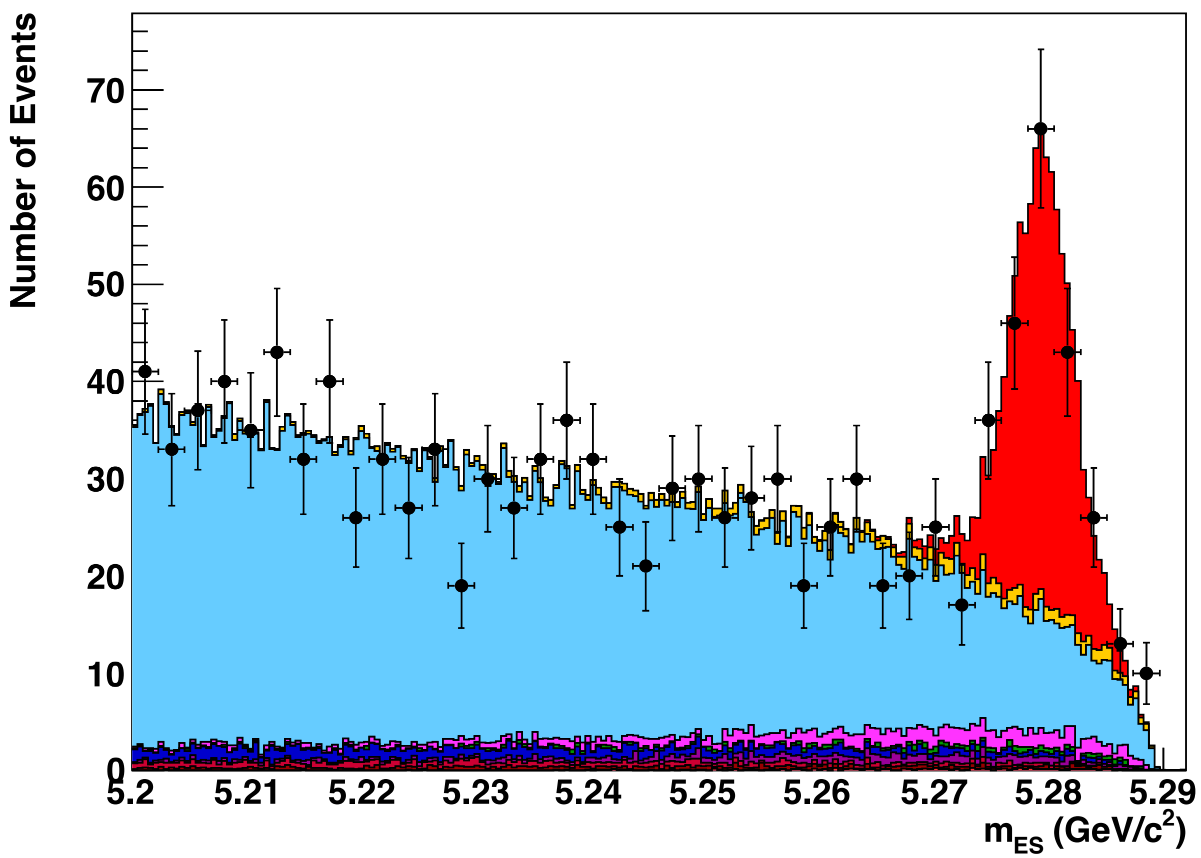
- **No significant biases of S and C in the allowed parameter space**

	Generated value in MC	Fit Parameter	Pull Mean	Pull Width
$\mathcal{S}=0.0$	$\mathcal{C}=-0.8$	$\mathcal{S}$	-0.091 ± 0.059	1.097 ± 0.046
		$\mathcal{C}$	-0.116 ± 0.057	1.041 ± 0.044
	$\mathcal{C}=-0.4$	$\mathcal{S}$	-0.058 ± 0.060	1.048 ± 0.042
		$\mathcal{C}$	-0.001 ± 0.059	1.039 ± 0.042
	$\mathcal{C}=0.0$	$\mathcal{S}$	0.011 ± 0.053	0.969 ± 0.044
		$\mathcal{C}$	-0.026 ± 0.055	0.999 ± 0.040
	$\mathcal{C}=+0.4$	$\mathcal{S}$	0.006 ± 0.058	1.02 ± 0.041
		$\mathcal{C}$	0.074 ± 0.059	1.041 ± 0.042
	$\mathcal{C}=+0.8$	$\mathcal{S}$	0.104 ± 0.058	1.025 ± 0.041
		$\mathcal{C}$	-0.060 ± 0.057	1.008 ± 0.040
$\mathcal{S}=0.4$	$\mathcal{C}=-0.8$	$\mathcal{S}$	0.077 ± 0.062	1.086 ± 0.044
		$\mathcal{C}$	-0.074 ± 0.057	1.004 ± 0.041
	$\mathcal{C}=-0.4$	$\mathcal{S}$	0.005 ± 0.061	1.058 ± 0.043
		$\mathcal{C}$	0.012 ± 0.060	1.045 ± 0.043
	$\mathcal{C}=0.0$	$\mathcal{S}$	0.066 ± 0.063	1.113 ± 0.045
		$\mathcal{C}$	0.114 ± 0.056	0.976 ± 0.039
	$\mathcal{C}=+0.4$	$\mathcal{S}$	0.066 ± 0.063	1.097 ± 0.045
		$\mathcal{C}$	0.081 ± 0.057	0.986 ± 0.040
	$\mathcal{C}=+0.8$	$\mathcal{S}$	0.052 ± 0.062	1.083 ± 0.044
		$\mathcal{C}$	0.105 ± 0.065	1.163 ± 0.046

	Generated value in MC	Fit Parameter	Pull Mean	Pull Width
$\mathcal{S}=0.8$	$\mathcal{C}=-0.8$	$\mathcal{S}$	$\varnothing$	$\varnothing$
		$\mathcal{C}$	$\varnothing$	$\varnothing$
	$\mathcal{C}=-0.4$	$\mathcal{S}$	-0.062 ± 0.058	0.992 ± 0.041
		$\mathcal{C}$	-0.102 ± 0.060	1.024 ± 0.042
	$\mathcal{C}=0.0$	$\mathcal{S}$	-0.014 ± 0.058	1.013 ± 0.041
		$\mathcal{C}$	0.028 ± 0.064	1.095 ± 0.045
	$\mathcal{C}=+0.4$	$\mathcal{S}$	-0.141 ± 0.056	0.994 ± 0.039
		$\mathcal{C}$	0.036 ± 0.058	1.039 ± 0.041
	$\mathcal{C}=+0.8$	$\mathcal{S}$	$\varnothing$	$\varnothing$
		$\mathcal{C}$	$\varnothing$	$\varnothing$

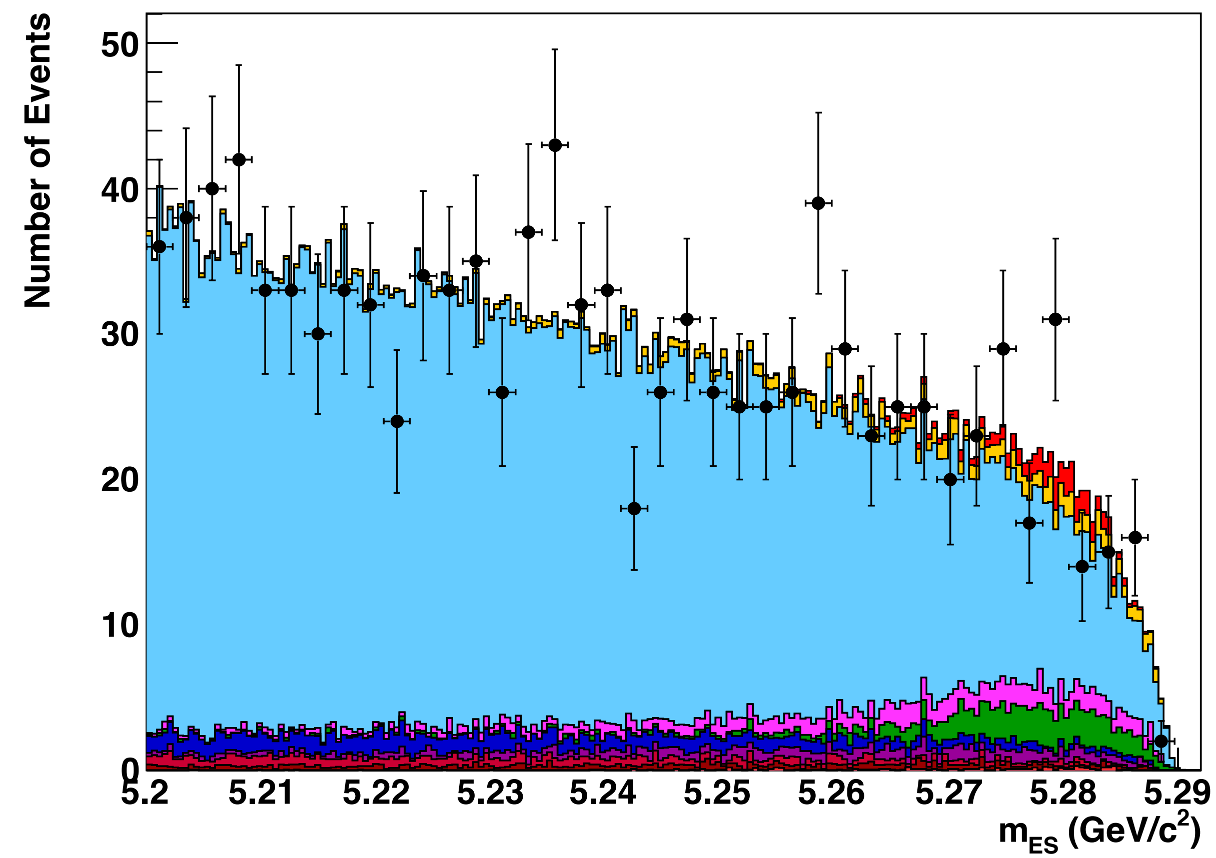
$B^0 \to K_S \pi^- \pi^+ \gamma$ TDCP Analysis

m_{ES} Projection



Signal region

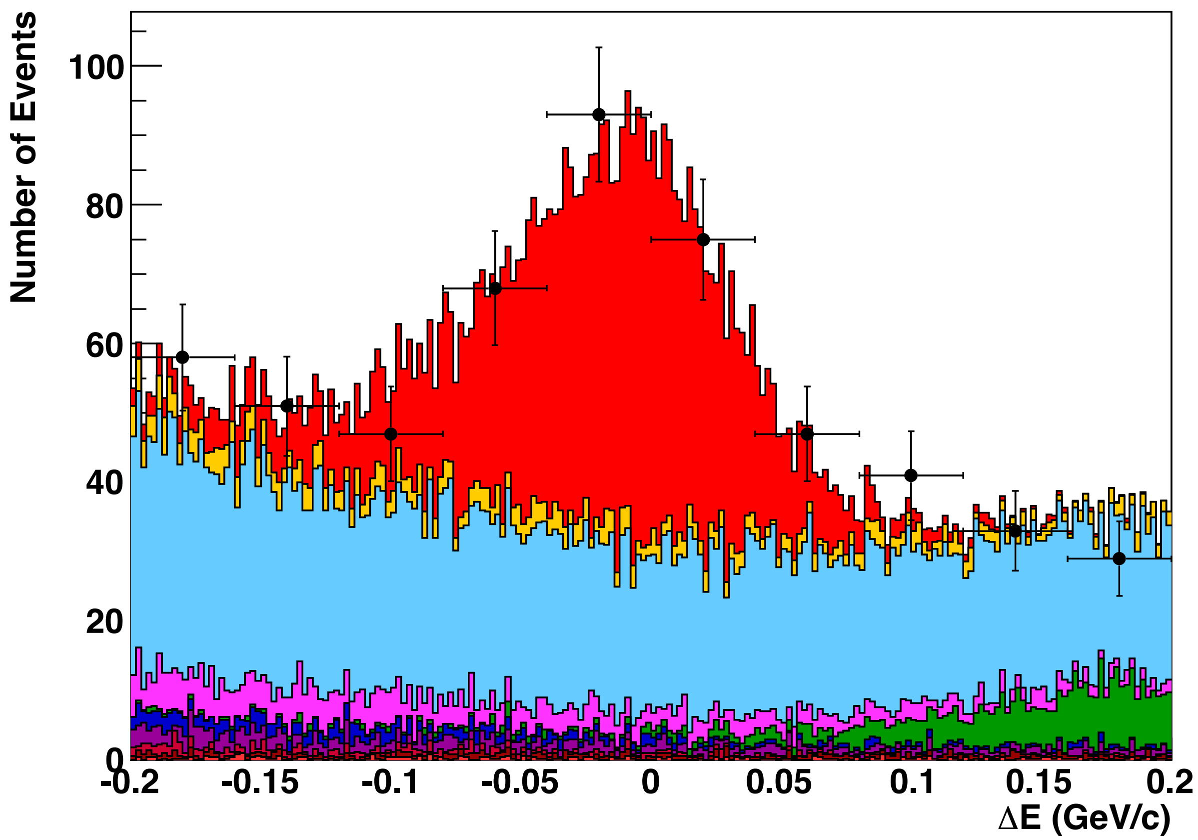
$-0.15 \leq \Delta E \leq 0.10$



Bkg region

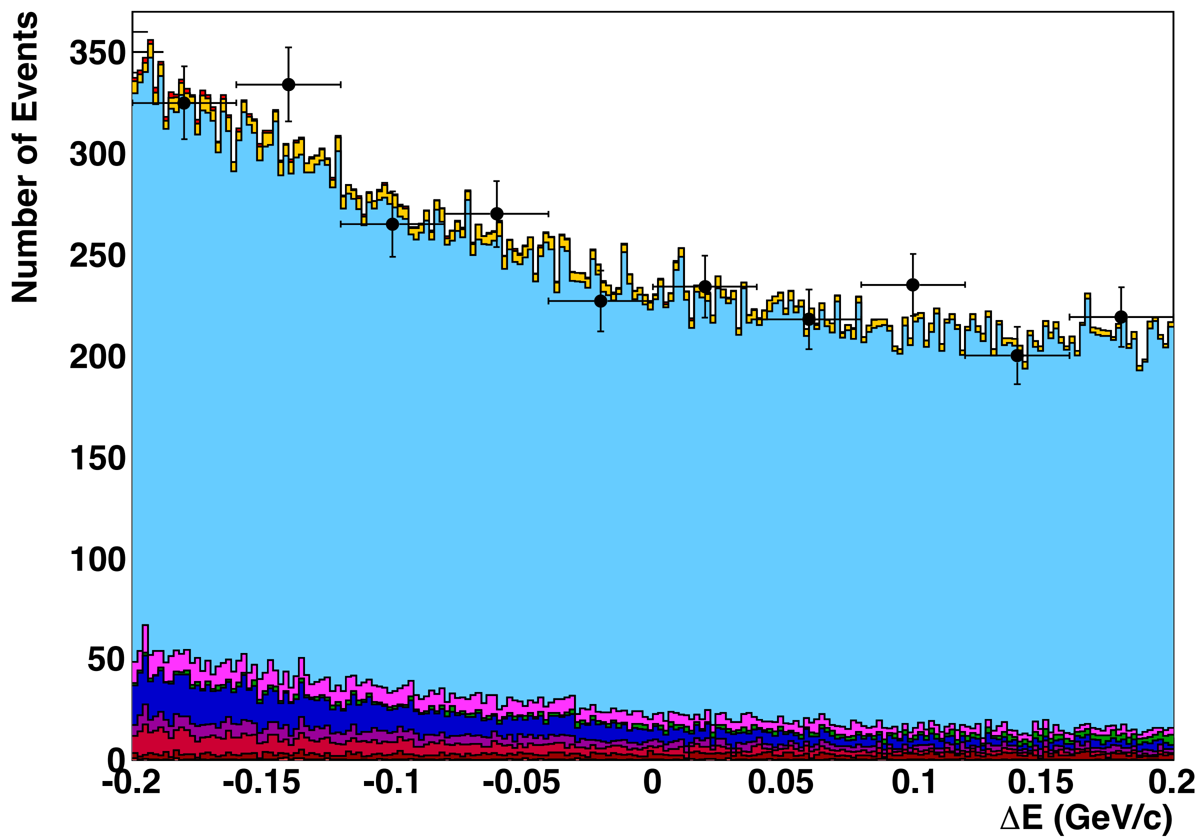
$\Delta E < -0.15$ & $0.10 < \Delta E$

$B^0 \to K_S \pi^- \pi^+ \gamma$ TDCP Analysis
ΔE Projection



Signal region

$5.27 < M_{ES} \leq 5.292$

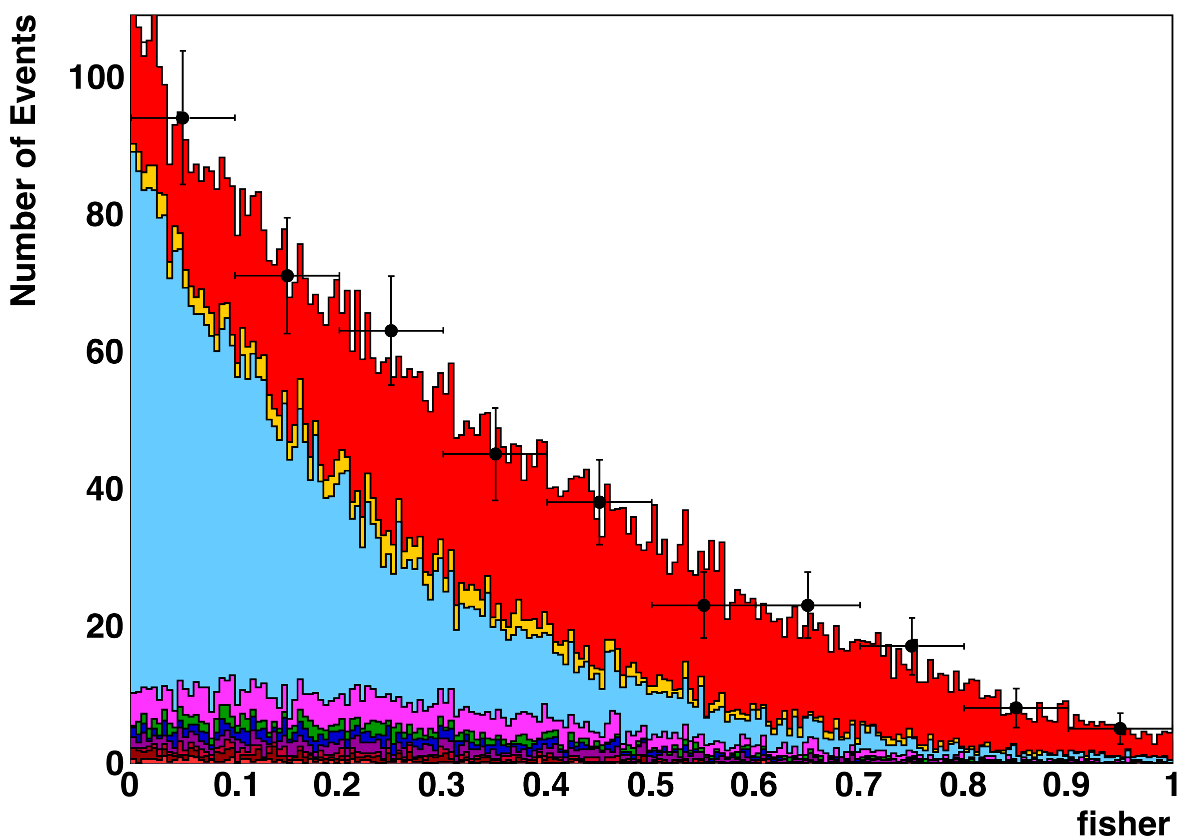


Bkg region

$5.20 < mES \leq 5.27$

$B^0 \to K_S \pi^- \pi^+ \gamma$ TDCP Analysis

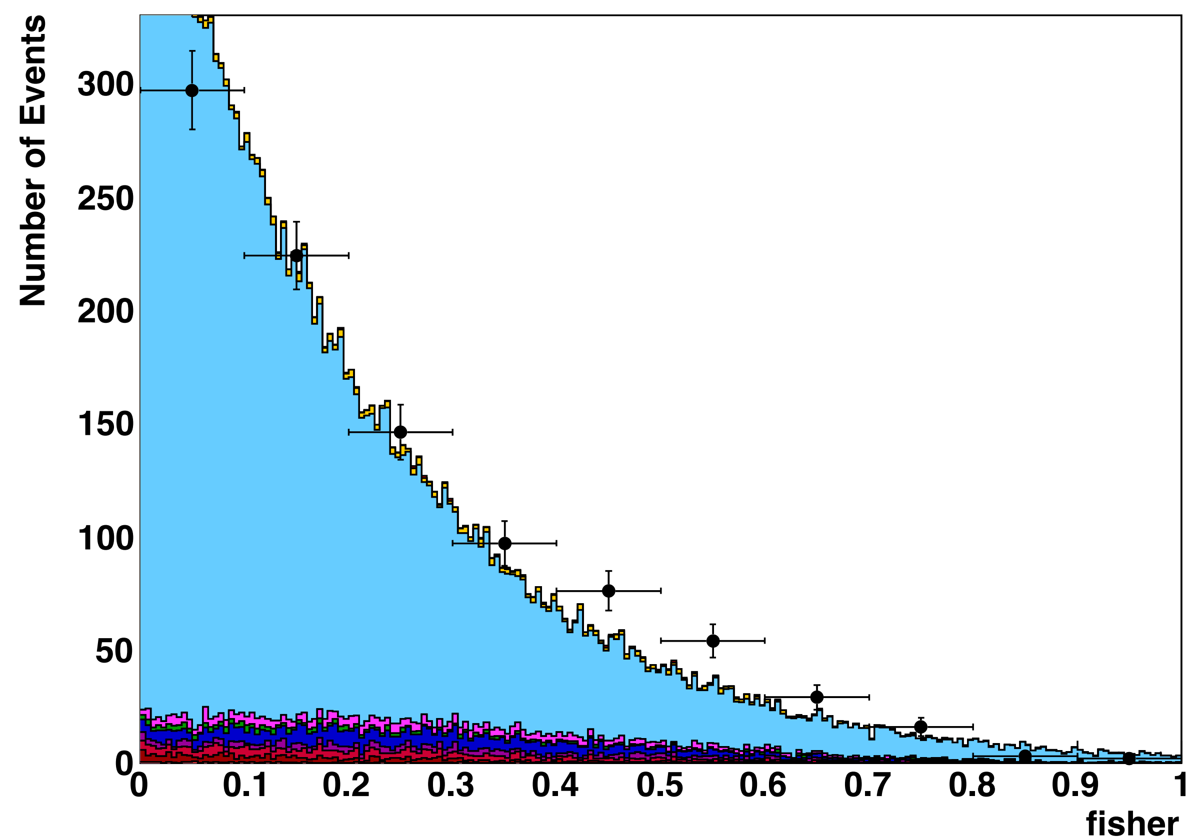
Fisher Projection



Signal region

$5.27 < M_{ES} \leq 5.292$

$-0.15 \leq \Delta E \leq 0.10$



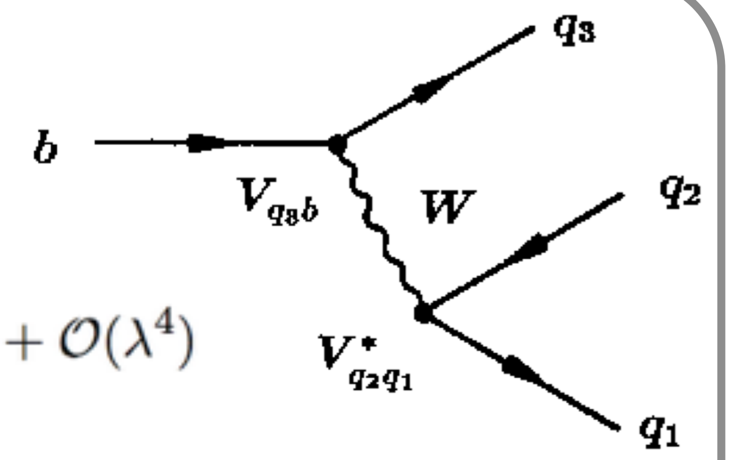
Bkg region

$5.20 < mES \leq 5.27$

$\Delta E < -0.15$ & $0.10 < \Delta E$

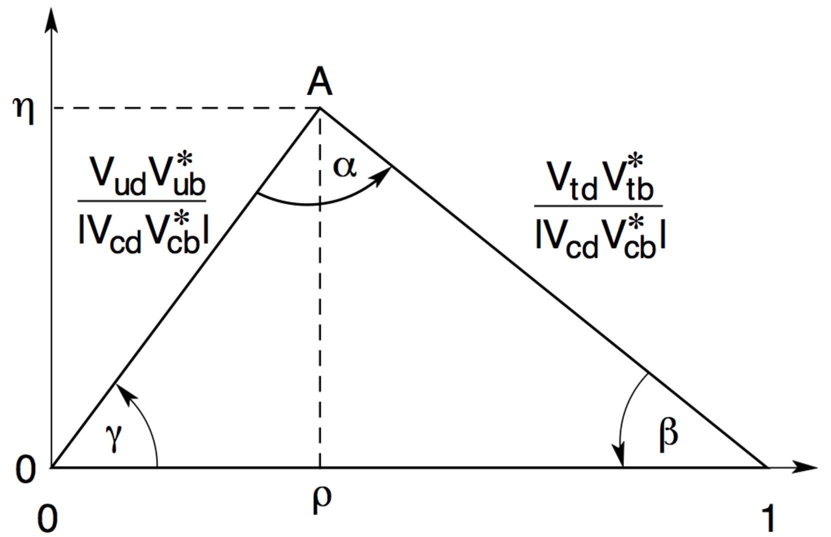
CKM Matrix and UT

- **CKM matrix** ➜ 3x3, unitary



$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

- **Unitarity Triangle (UT)** ➜ geometric representation of one CKM unitarity:



- **One of flavor physics goals:**
put constraints on UT sides and angles to test Standard Model (SM) and set constraints on New Physics (NP) models ➜