Testing quantum mechanics with cosmology

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Quantum cosmology

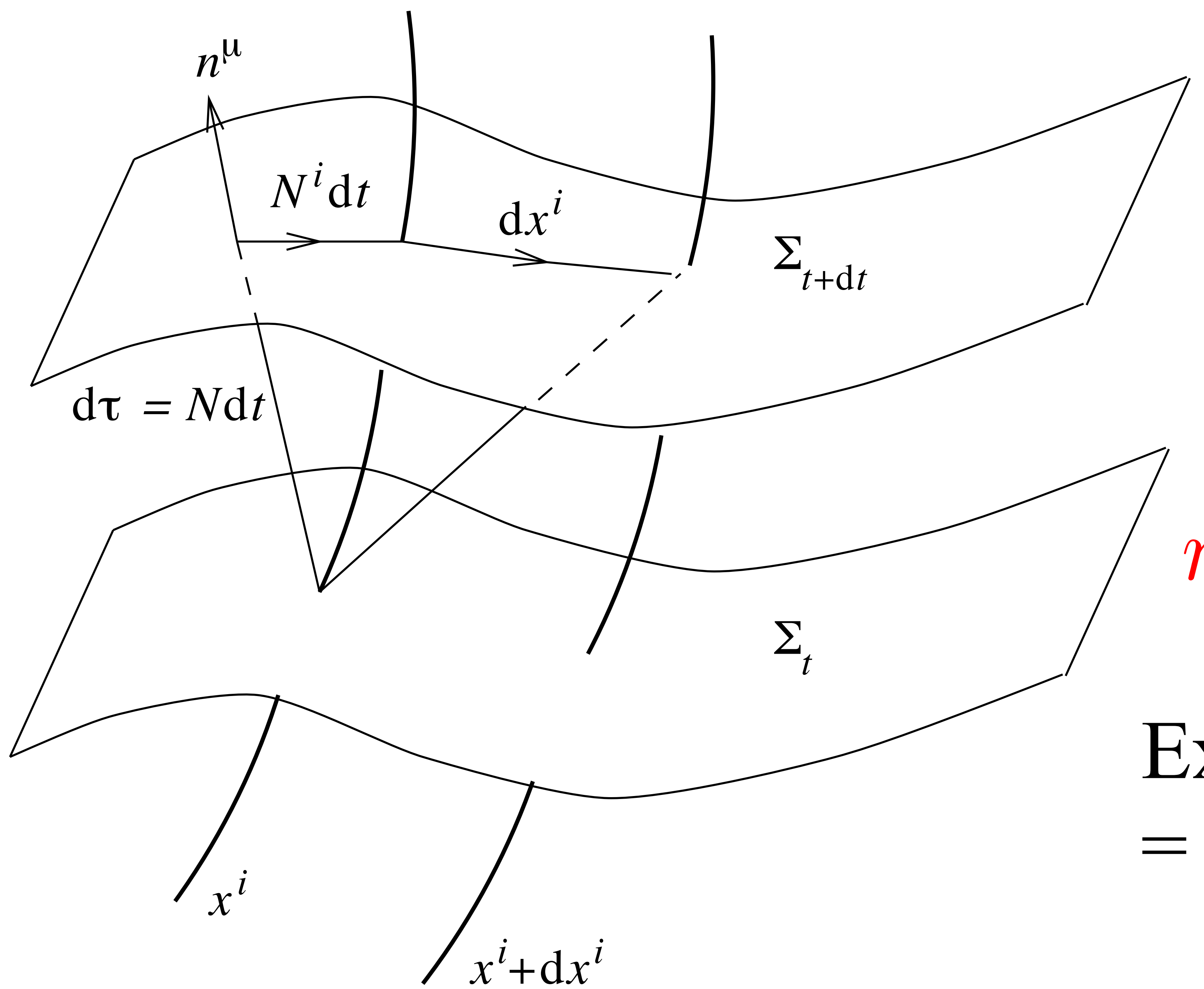
- Hamiltonian GR

$$\mathrm{d}s^2 = g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = -\mathcal{N}^2\mathrm{d}t^2 + h_{ij}\left(\mathrm{d}x^i + \mathcal{N}^i\mathrm{d}t\right)\left(\mathrm{d}x^j + \mathcal{N}^j\mathrm{d}t\right)$$

Lapse function

Intrinsic metric
= first fundamental form

Shift vector



n^μ Normal to Σ_t

Intrinsic curvature tensor ${}^3R^i{}_{jkl}(h)$

Extrinsic curvature
= second fundamental form

$$\begin{aligned} K_{ij} &\equiv -\nabla_j n_i = -\Gamma^0{}_{ij} n_0 \\ &= \frac{1}{2\mathcal{N}}\left(\nabla_j \mathcal{N}_i + \nabla_i \mathcal{N}_j - \frac{\partial h_{ij}}{\partial t}\right) \end{aligned}$$

Action: $$\mathcal{S} = \frac{1}{16\pi G_\mathrm{N}}\left[\int_\mathcal{M} \mathrm{d}^4x\sqrt{-g}\left({}^4R - 2\Lambda\right) + 2\int_{\partial\mathcal{M}} \mathrm{d}^3x\sqrt{h}K^i{}_i\right] + \mathcal{S}_\mathrm{matter}$$

In 3+1 expansion: $\mathcal{S} \equiv \int \mathrm{d}t L = \frac{1}{16\pi G_{\mathrm{N}}} \int \mathrm{d}t \mathrm{d}^3x \, \mathcal{N}\sqrt{h} \left(K_{ij}K^{ij} - K^2 + {}^3R - 2\Lambda\right) + \mathcal{S}_{\mathrm{matter}}$

Canonical momenta

$$\pi^{ij} \equiv \frac{\delta L}{\delta \dot{h}_{ij}} = -\frac{\sqrt{h}}{16\pi G_{\mathrm{N}}} \left(K^{ij} - h^{ij}K\right)$$

$$\pi_\Phi \equiv \frac{\delta L}{\delta \dot{\Phi}} = \frac{\sqrt{h}}{\mathcal{N}} \left(\dot{\Phi} - \mathcal{N}^i \frac{\partial \Phi}{\partial x^i}\right)$$

$$\left.\begin{aligned} \pi^0 &\equiv \frac{\delta L}{\delta \dot{\mathcal{N}}} = 0 \\ \pi^i &\equiv \frac{\delta L}{\delta \dot{\mathcal{N}}_i} = 0 \end{aligned}\right\} \text{Primary constraints}$$

Hamiltonian $H \equiv \int \mathrm{d}^3x \left(\pi^0\dot{\mathcal{N}} + \pi^i\dot{\mathcal{N}}_i + \pi^{ij}\dot{h}_{ij} + \pi_\Phi\dot{\Phi}\right) - L = \int \mathrm{d}^3x \left(\pi^0\dot{\mathcal{N}} + \pi^i\dot{\mathcal{N}}_i + \mathcal{N}\mathcal{H} + \mathcal{N}_i\mathcal{H}^i\right)$

$$\left.\begin{aligned} &\text{Variation wrt lapse } \mathcal{H} = 0 \text{ Hamiltonian constraint} \\ &\text{Variation wrt shift } \mathcal{H}^i = 0 \text{ momentum constraint} \end{aligned}\right\} \text{Secondary constraints}$$

$\Longrightarrow$ Classical description

- Superspace & canonical quantisation

Relevant configuration space?

matter fields

$$\mathrm{Riem}(\Sigma) \equiv \left\{ h_{ij}\,(x^\mu), \Phi\,(x^\mu) \;\mid x \in \Sigma \right\}$$

parameters

GR $\Longrightarrow$ invariance / diffeomorphisms $\Longrightarrow \mathrm{Conf} = \dfrac{\mathrm{Riem}(\Sigma)}{\mathrm{Diff}_0(\Sigma)}$ superspace

Wave functional $\Psi\left[h_{ij}(x), \Phi(x)\right]$

Dirac canonical quantisation

$$\pi^{ij} \to -i\frac{\delta}{\delta h_{ij}} \qquad \pi_\Phi \to -i\frac{\delta}{\delta \Phi} \qquad \pi^0 \to -i\frac{\delta}{\delta \mathcal{N}} \qquad \pi^i \to -i\frac{\delta}{\delta \mathcal{N}_i}$$

Primary constraints

$$\hat{\pi}\Psi = -i\frac{\delta\Psi}{\delta\mathcal{N}} = 0$$

$$\hat{\pi}^i\Psi = -i\frac{\delta\Psi}{\delta\mathcal{N}_i} = 0$$

Momentum constraint $\hat{\mathcal{N}}^i\Psi = 0 \quad \Longrightarrow \quad i\nabla_j^{(h)}\left(\frac{\delta\Psi}{\delta h_{ij}}\right) = 8\pi G_{\mathrm{N}}\hat{T}^{0i}\Psi$

$\Longrightarrow$ Ψ is the same for configurations $\{h_{ij}(x), \Phi(x)\}$ related by a coordinate transformation

Hamiltonian constraint

$$\boxed{\hat{\mathcal{H}}\Psi = \left[-16\pi G_{\mathrm{N}}\mathcal{G}_{ijkl}\frac{\delta^2}{\delta h_{ij}\delta h_{kl}} + \frac{\sqrt{h}}{16\pi G_{\mathrm{N}}}\left(-{}^3R + 2\Lambda + 16\pi G_{\mathrm{N}}\hat{T}^{00}\right)\right]\Psi = 0}$$

Wheeler - De Witt equation

$$\mathcal{G}_{ijkl} = \frac{1}{2}h^{-1/2}\left(h_{ik}h_{jl} + h_{il}h_{jk} - h_{ij}h_{kl}\right)$$

DeWitt metric...

- Minisuperspace

Restrict attention from an infinite dimensional configuration space to 2 dimensional space = mini - superspace

$$h_{ij}\mathrm{d}x^i\mathrm{d}x^j = a^2(t)\left[\frac{\mathrm{d}r^2}{1-kr^2} + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\varphi^2\right)\right] \qquad \Phi(x) = \phi(t)$$

WDW equation becomes Schrödinger-like for $\Psi\left[a(t), \phi(t)\right]$

Conceptual and technical problems:

Infinite number of dof $\longrightarrow$ a few: mathematical consistency?
Freeze momenta? Heisenberg uncertainties?
QM = minisuperspace of QFT

- Minisuperspace

Restrict attention from an infinite dimensional configuration space to 2 dimensional space = mini - superspace

$$h_{ij}\mathrm{d}x^i\mathrm{d}x^j = a^2(t)\left[\frac{\mathrm{d}r^2}{1-kr^2} + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\varphi^2\right)\right] \qquad \Phi(x) = \phi(t)$$

WDW equation becomes Schrödinger-like for $\Psi\left[a(t),\phi(t)\right]$

However, one can actually make calculations!

Exemple : Quantum cosmology of a perfect fluid

$$\mathrm{d}s^2 = N^2(\tau)\mathrm{d}\tau - a^2(\tau)\gamma_{ij}\mathrm{d}x^i\mathrm{d}x^j$$

Perfect fluid: Schutz formalism ('70)

$$p = p_0\left[\frac{\dot\varphi + \theta\dot s}{N(1+\omega)}\right]^{\frac{1+\omega}{\omega}}$$

$(\varphi, \theta, s) =$ Velocity potentials

canonical transformation: $T = -p_s \mathrm{e}^{-s/s_0} p_\varphi^{-(1+\omega)} s_0 \rho_0^{-\omega}$ ...

+ rescaling (volume…) + units… : simple Hamiltonian:

$$H = \left(-\frac{p_a^2}{4a} - \mathcal{K}a + \frac{p_T}{a^{3\omega}}\right)N$$

$a^{3\omega}$

Wheeler-De Witt $H\Psi = 0$

$$\mathcal{K} = 0 \Longrightarrow \chi \equiv \frac{2a^{3(1-\omega)/2}}{3(1-\omega)} \Longrightarrow \boxed{i\frac{\partial\Psi}{\partial T} = \frac{1}{4}\frac{\partial^2\Psi}{\partial\chi^2}}$$

space defined by $\chi > 0$ $\longrightarrow$ constraint $\bar{\Psi}\dfrac{\partial\Psi}{\partial\chi} = \Psi\dfrac{\partial\bar{\Psi}}{\partial\chi}$

Gaussian wave packet

$$\Rightarrow \quad \Psi = \left[\frac{8T_0}{\pi\left(T_0^2+T^2\right)^2}\right]^{\frac{1}{4}} \exp\left(-\frac{T_0\chi^2}{T_0^2+T^2}\right) \mathrm{e}^{-iS(\chi,T)}$$

phase $S = \dfrac{T\chi^2}{T_0^2+T^2} + \dfrac{1}{2}\arctan\dfrac{T_0}{T} - \dfrac{\pi}{4}$

What do we do with the wave function of the Universe???

Quantum mechanics of closed systems

Physical system = Hilbert space of configurations
State vectors
Observables = self-adjoint operators
Measurement = eigenvalue $A|a_n\rangle = a_n|a_n\rangle$

Evolution = Schrödinger equation (time translation invariance) $i\hbar\frac{\mathrm{d}}{\mathrm{d}t}|\psi(t)\rangle = \hat{H}|\psi(t)\rangle$

Hamiltonian

Born rule $\mathrm{Prob}[a_n;t] = |\langle a_n|\psi(t)\rangle|^2$

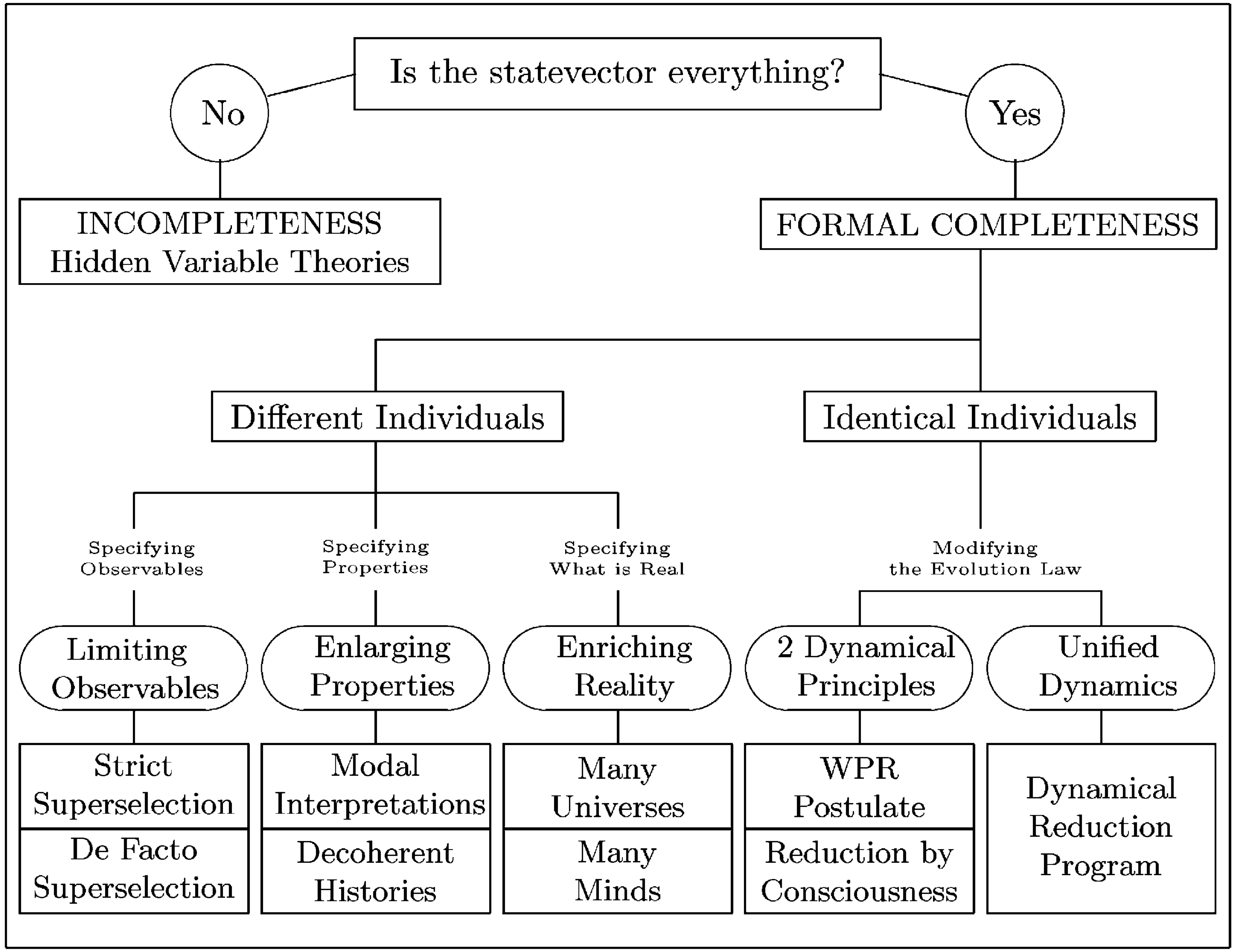
Collapse of the wavefunction: $|\psi(t)\rangle$ before measurement, $|a_n\rangle$ after

Schrödinger equation = linear (superposition principle) / unitary evolution
Wavepacket reduction = non linear / stochastic

Mutually incompatible

+ External observer

- Possible solutions and a criterion: the Born rule



A. Bassi and G.C. Ghirardi, *Phys. Rep.* **379**, 257 (2003)

▲ ***Superselection rules***

▲ ***Modal interpretation***

▲ ***Consistent histories***

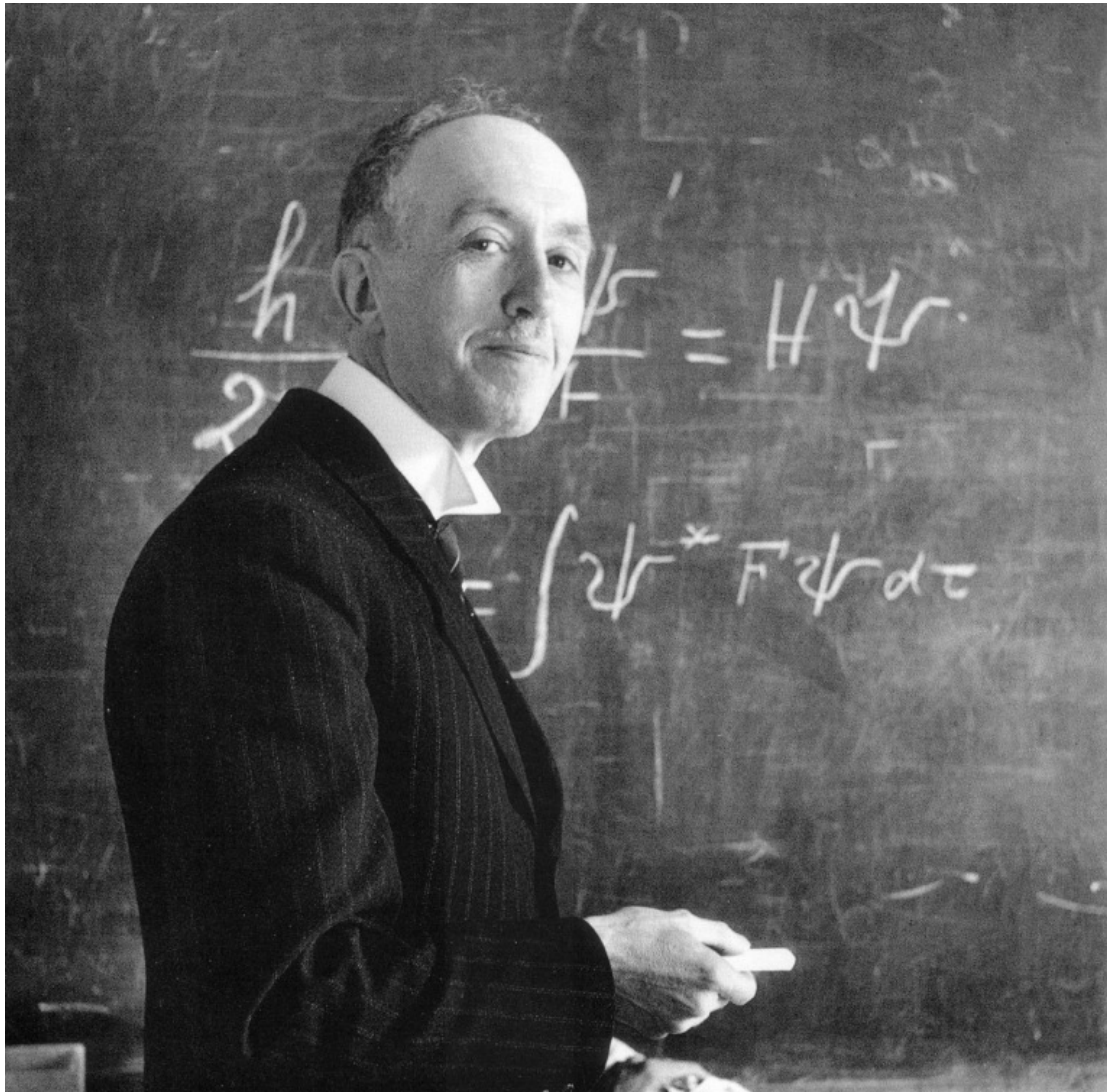
▲ ***Many worlds / many minds***

▲ ***Hidden variables***

▲ ***Modified Schrödinger dynamics***

} Born rule not put by hand!

Ontological *formulation* (dBB)



Louis de Broglie (Prince, duke ...)

David Bohm (Communist)

1927 Solvay meeting and von Neuman mistake ... ‘*In 1952, I saw the impossible done*’ (J. Bell)

Hidden Variable Theories

Schrödinger $$i\frac{\partial\Psi}{\partial t} = \left[-\frac{\nabla^2}{2m} + V\left(\boldsymbol{r}\right)\right]\Psi$$

Polar form of the wave function $$\Psi = A\left(\boldsymbol{r}, t\right)\mathrm{e}^{iS\left(\boldsymbol{r}, t\right)}$$

Hamilton-Jacobi $$\frac{\partial S}{\partial t} + \frac{\left(\nabla S\right)^2}{2m} + V\left(\boldsymbol{r}\right) + Q\left(\boldsymbol{r}, t\right) = 0$$

quantum potential

$$\equiv -\frac{1}{2m}\frac{\nabla^2 A}{A}$$

$$\Psi = A(\boldsymbol{r},t)\,\mathrm{e}^{iS(\boldsymbol{r},t)}$$

Ontological *formulation* (BdB) $\exists\,\boldsymbol{x}(t)$

Trajectories satisfy (Bohm)

$$m\frac{\mathrm{d}^2\boldsymbol{x}}{\mathrm{d}t^2} = -\boldsymbol{\nabla}(V+Q) \qquad Q \equiv -\frac{1}{2m}\frac{\boldsymbol{\nabla}^2|\Psi|}{|\Psi|}$$

$$\Psi = A(\boldsymbol{r},t)\,\mathrm{e}^{iS(\boldsymbol{r},t)}$$

Ontological *formulation* (dBB) $\exists\, \boldsymbol{x}(t)$

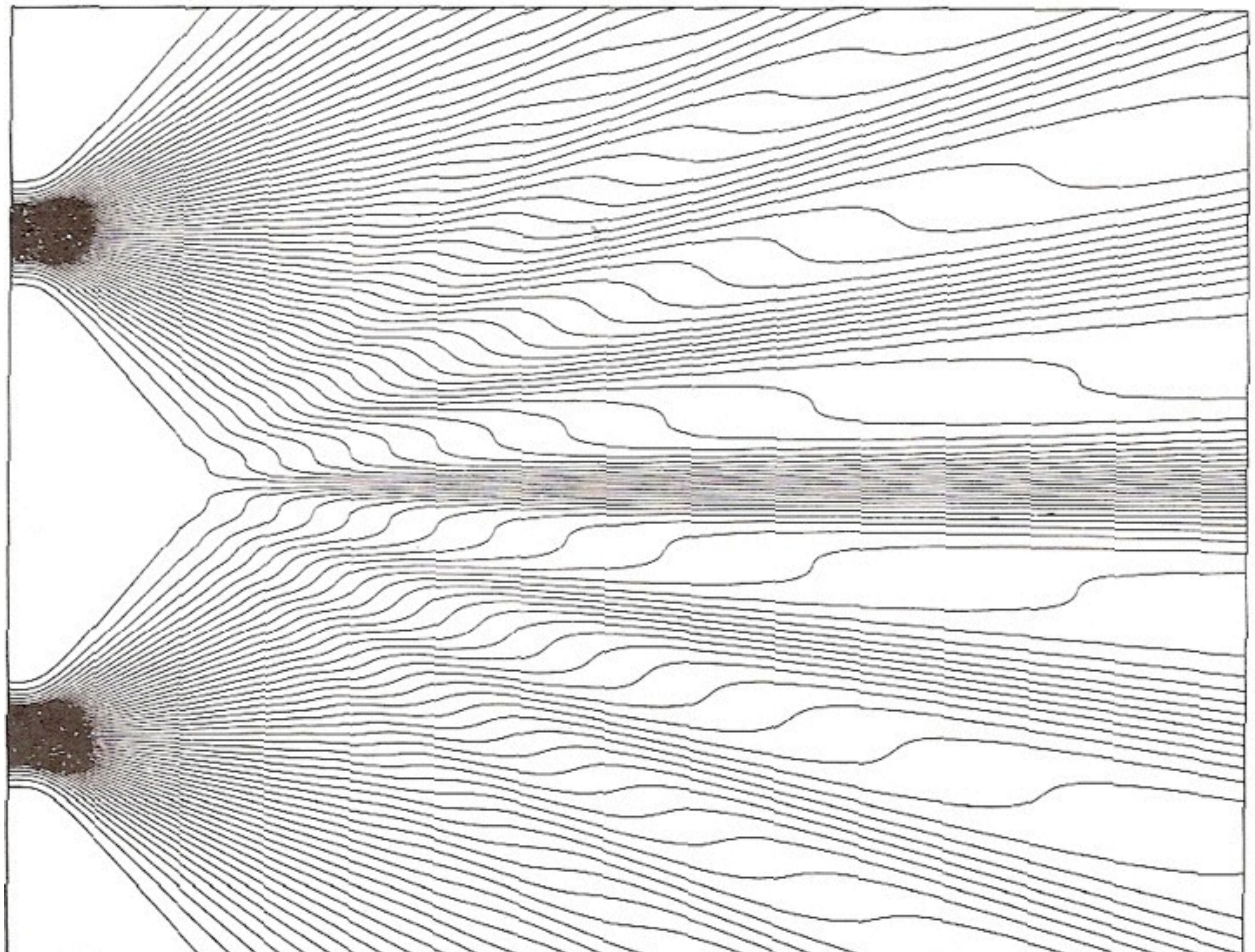
Trajectories satisfy (de Broglie) $$m\frac{\mathrm{d}\boldsymbol{x}}{\mathrm{d}t} = \Im\mathrm{m}\,\frac{\Psi^* \boldsymbol{\nabla}\Psi}{|\Psi(\boldsymbol{x},t)|^2} = -\boldsymbol{\nabla} S$$

Properties:

- ☺ strictly equivalent to Copenhagen QM
  - probability distribution (attractor)

    $$\exists t_0;\, \rho(\boldsymbol{x},t_0) = |\Psi(\boldsymbol{x},t_0)|^2$$
- ☺ classical limit well defined $Q \longrightarrow 0$
- ☺ state dependent
- ☺ $\exists$ intrinsic reality
  - non local …
- ☺ no need for external classical domain/observer!

The two-slit experiment:



Surrealistic trajectories?

Non straight in vacuum...

$$m\frac{\mathrm{d}^2x(t)}{\mathrm{d}t^2} = -\nabla\left(\cancel{V} + Q\right)$$

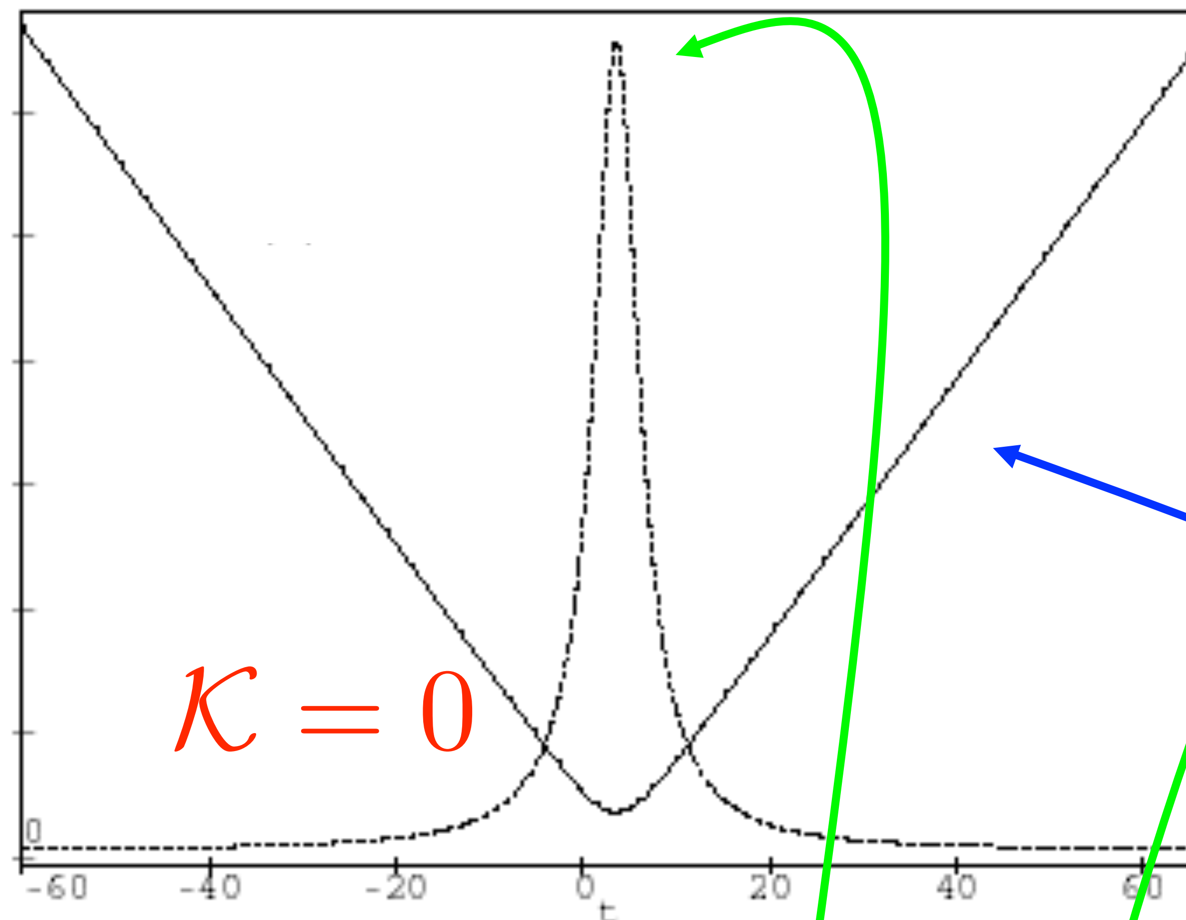
Back to the QC wave function

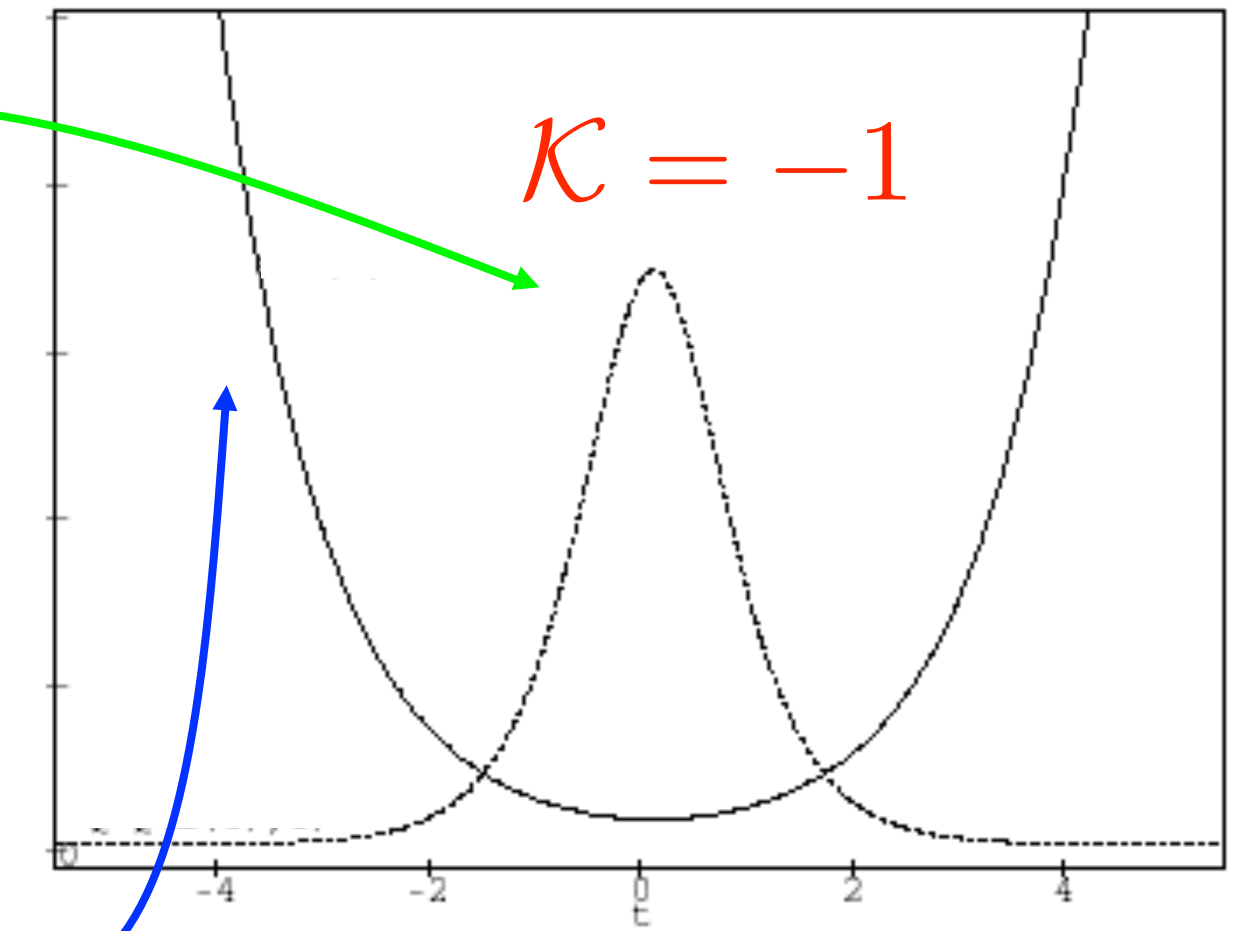
Gaussian wave packet

$$\Psi = \left[\frac{8T_0}{\pi\left(T_0^2+T^2\right)^2}\right]^{\frac{1}{4}} \exp\left(-\frac{T_0\chi^2}{T_0^2+T^2}\right) \mathrm{e}^{-iS(\chi,T)}$$

phase $$S = \frac{T\chi^2}{T_0^2+T^2} + \frac{1}{2}\arctan\frac{T_0}{T} - \frac{\pi}{4}$$

Hidden trajectory $$a = a_0\left[1+\left(\frac{T}{T_0}\right)^2\right]^{\frac{1}{3(1-\omega)}}$$

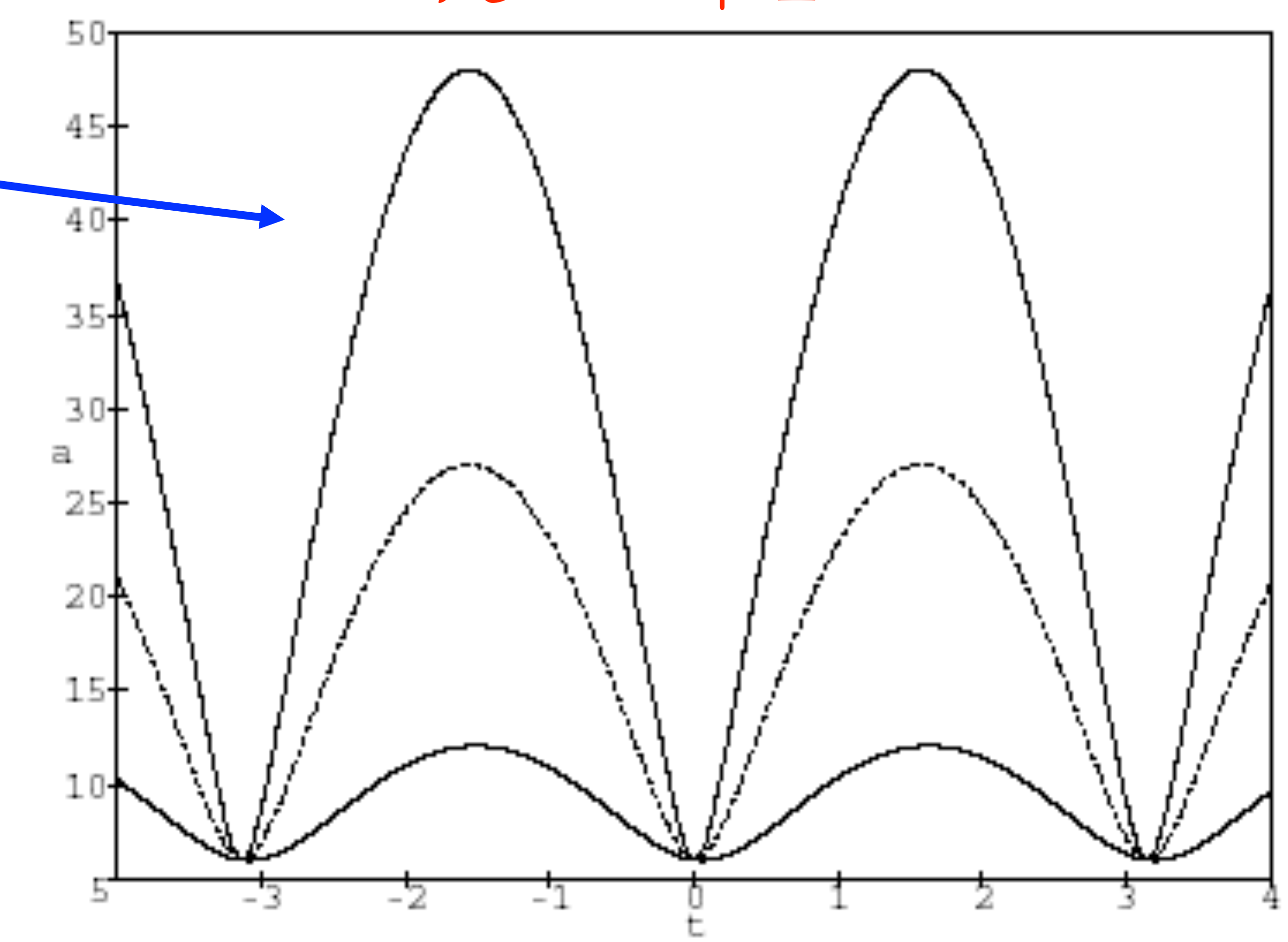




$a(T)$

Trajectory

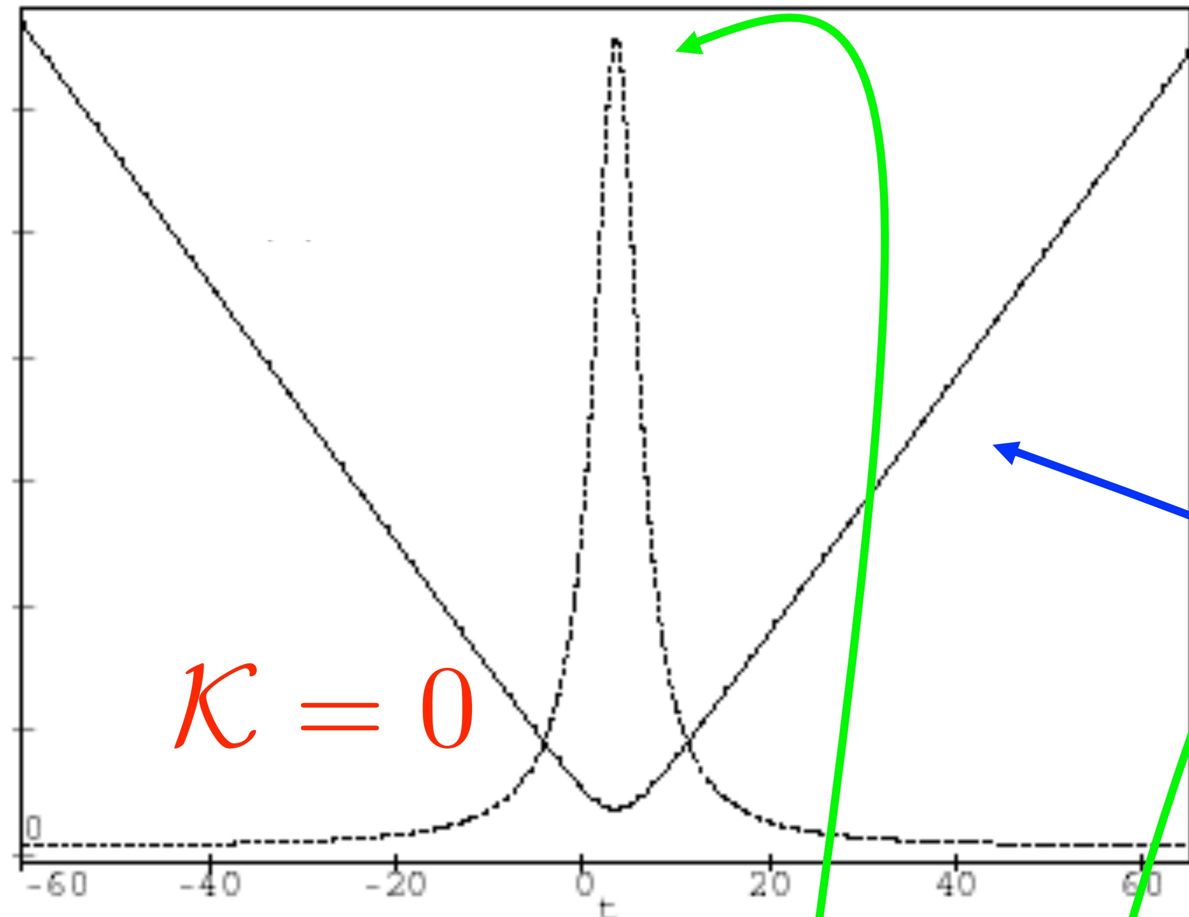
$\mathcal{K} = +1$

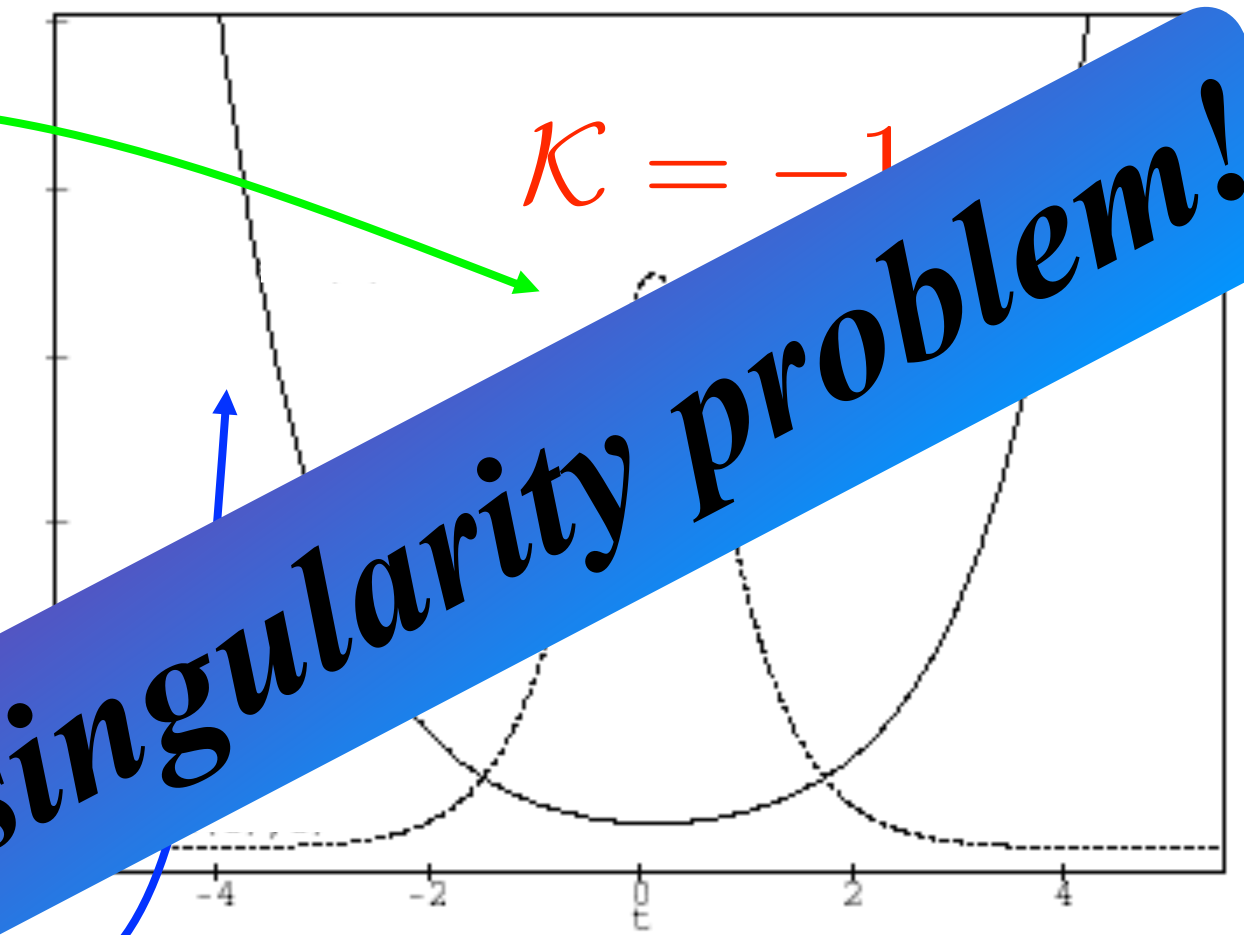


$Q(T)$

quantum potential

J. Acacio de Barros, N. Pinto-Neto & M. A. Sagorio-Leal,, *Phys. Lett.* **A241**, 229 (1998)



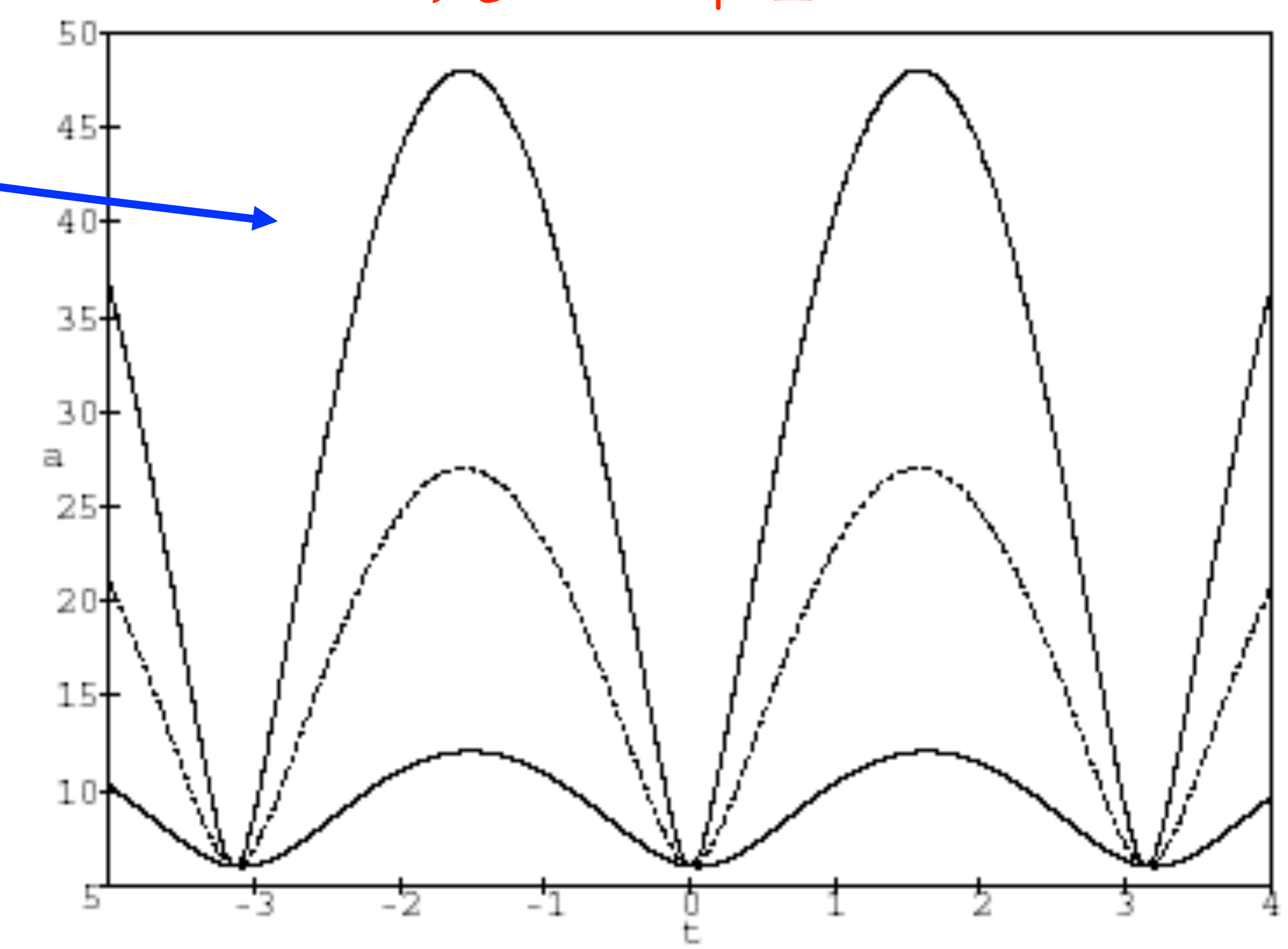


Natural quantum solution to the singularity problem!

(T)

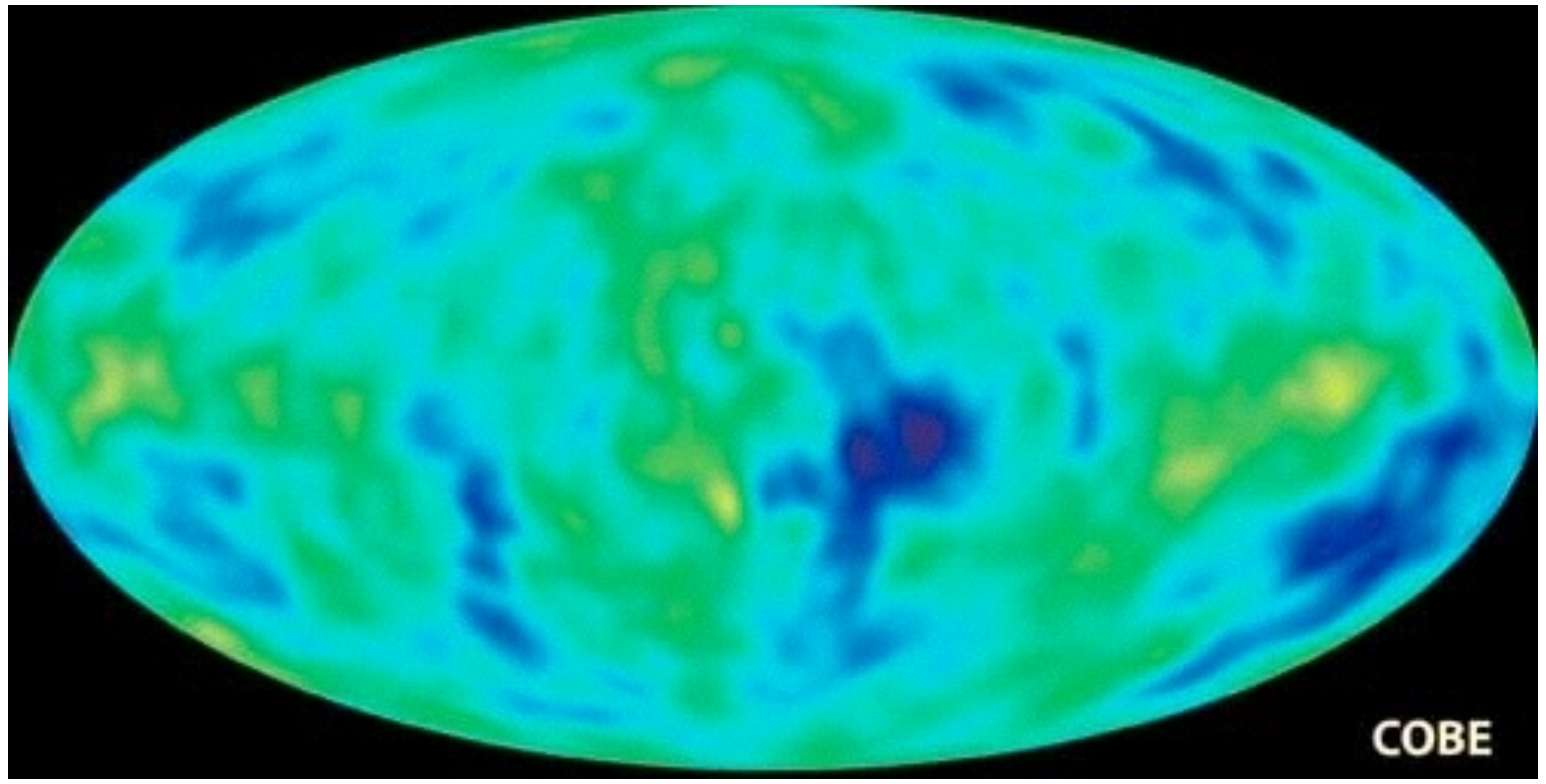
tum potential

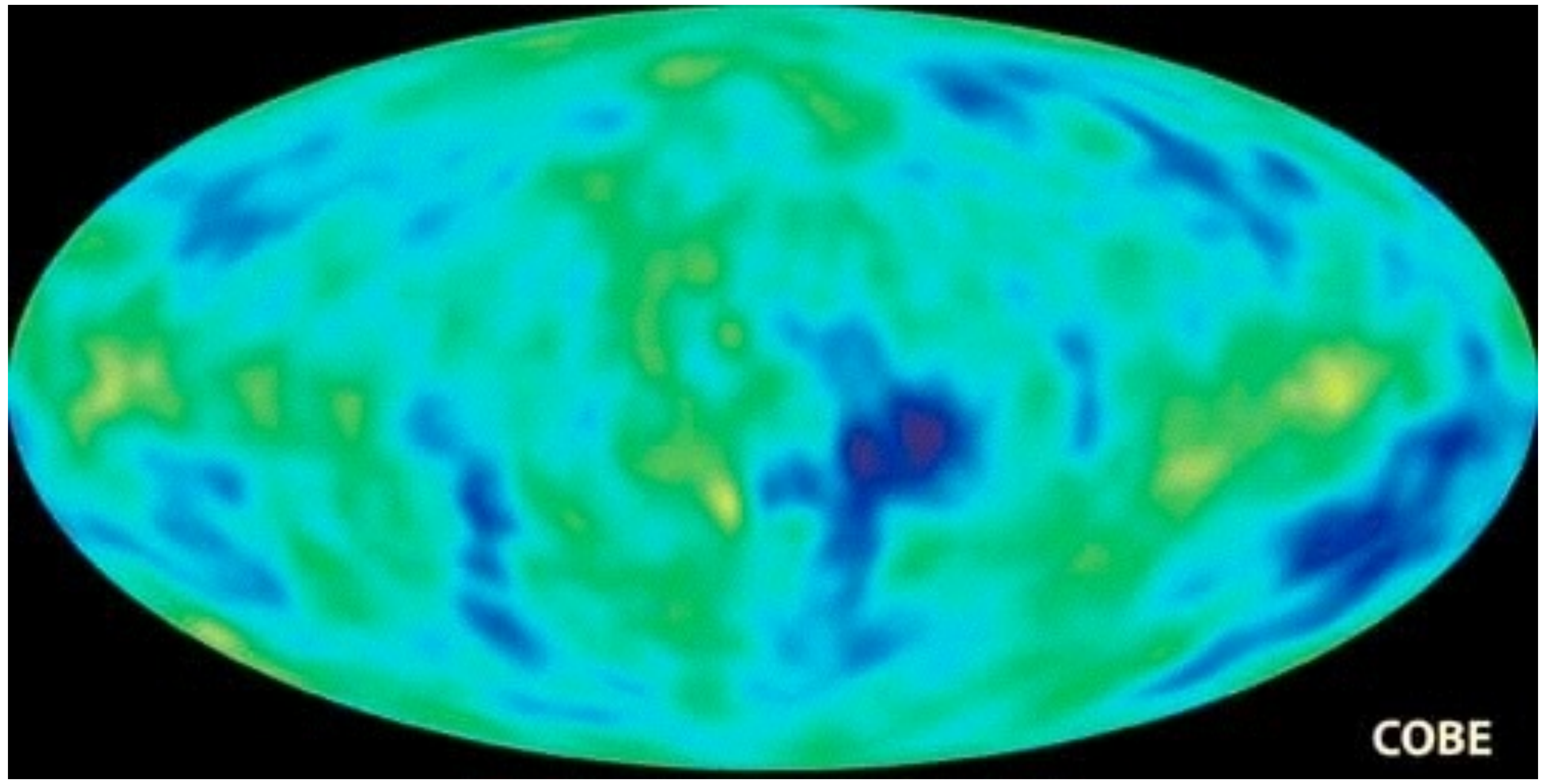
$\mathcal{K} = +1$



J. Acacio de Barros, N. Pinto-Neto & M. A. Sagorio-Leal,, *Phys*. *Lett*. **A241**, 229 (1998)

What about perturbations?


COBE



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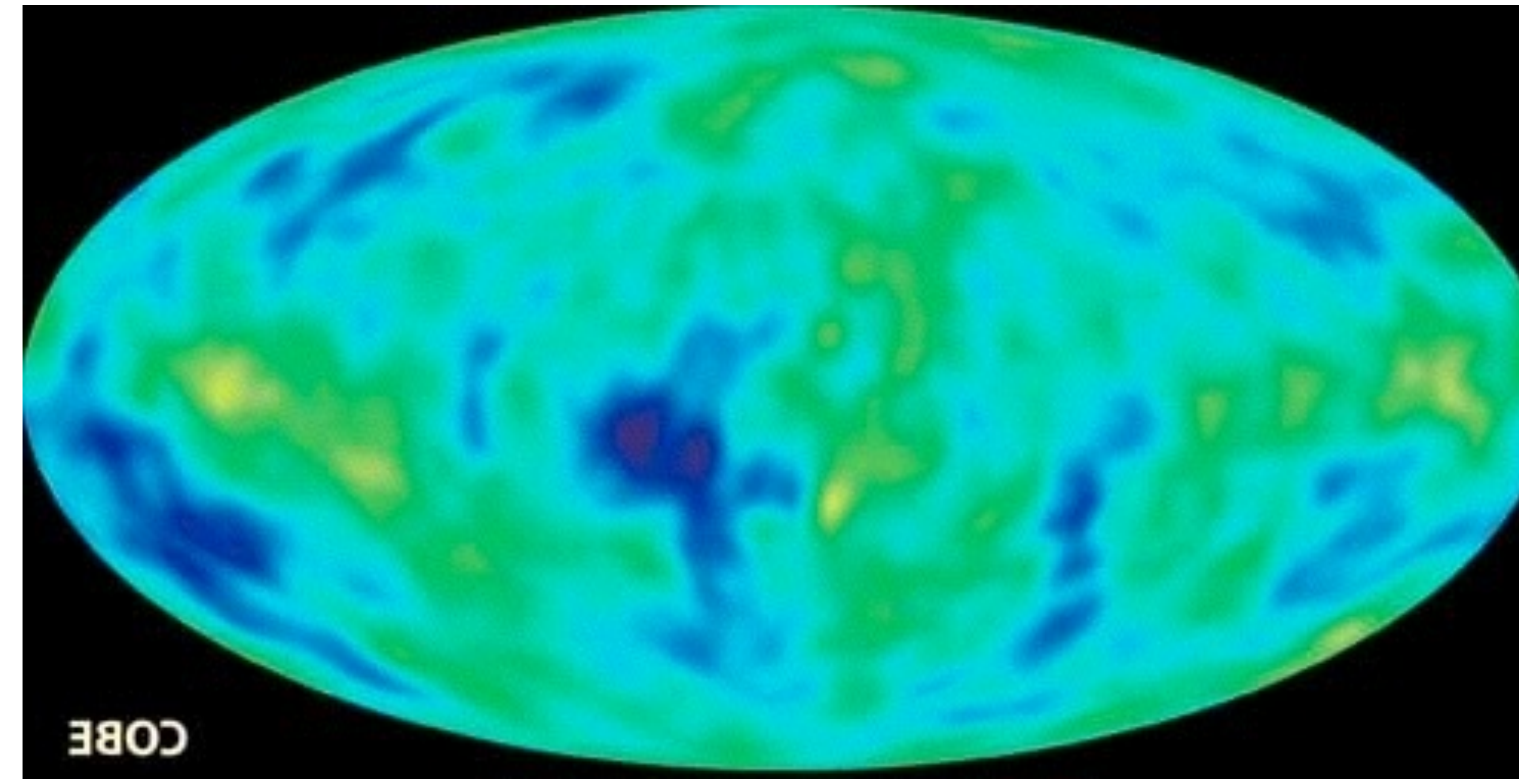

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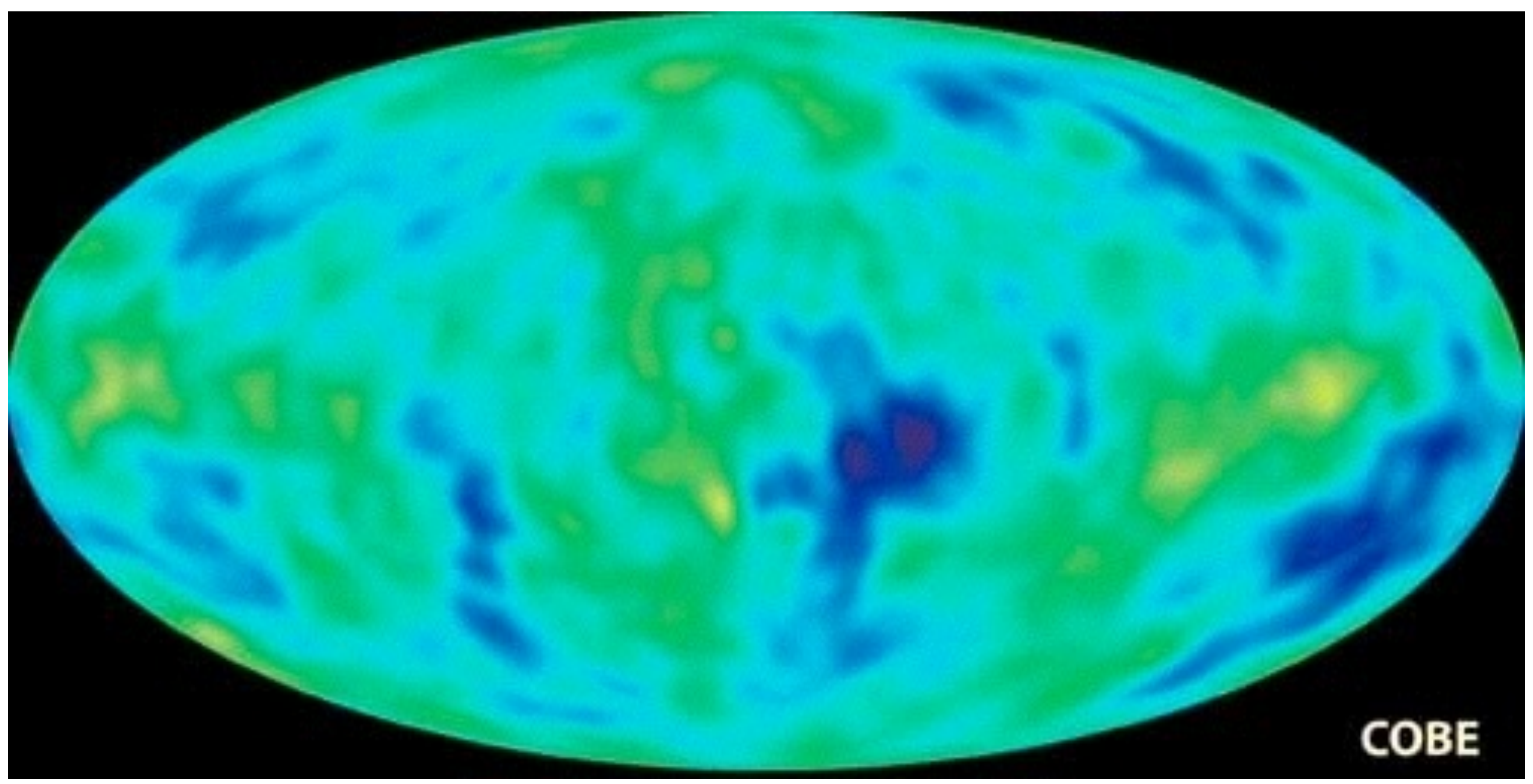

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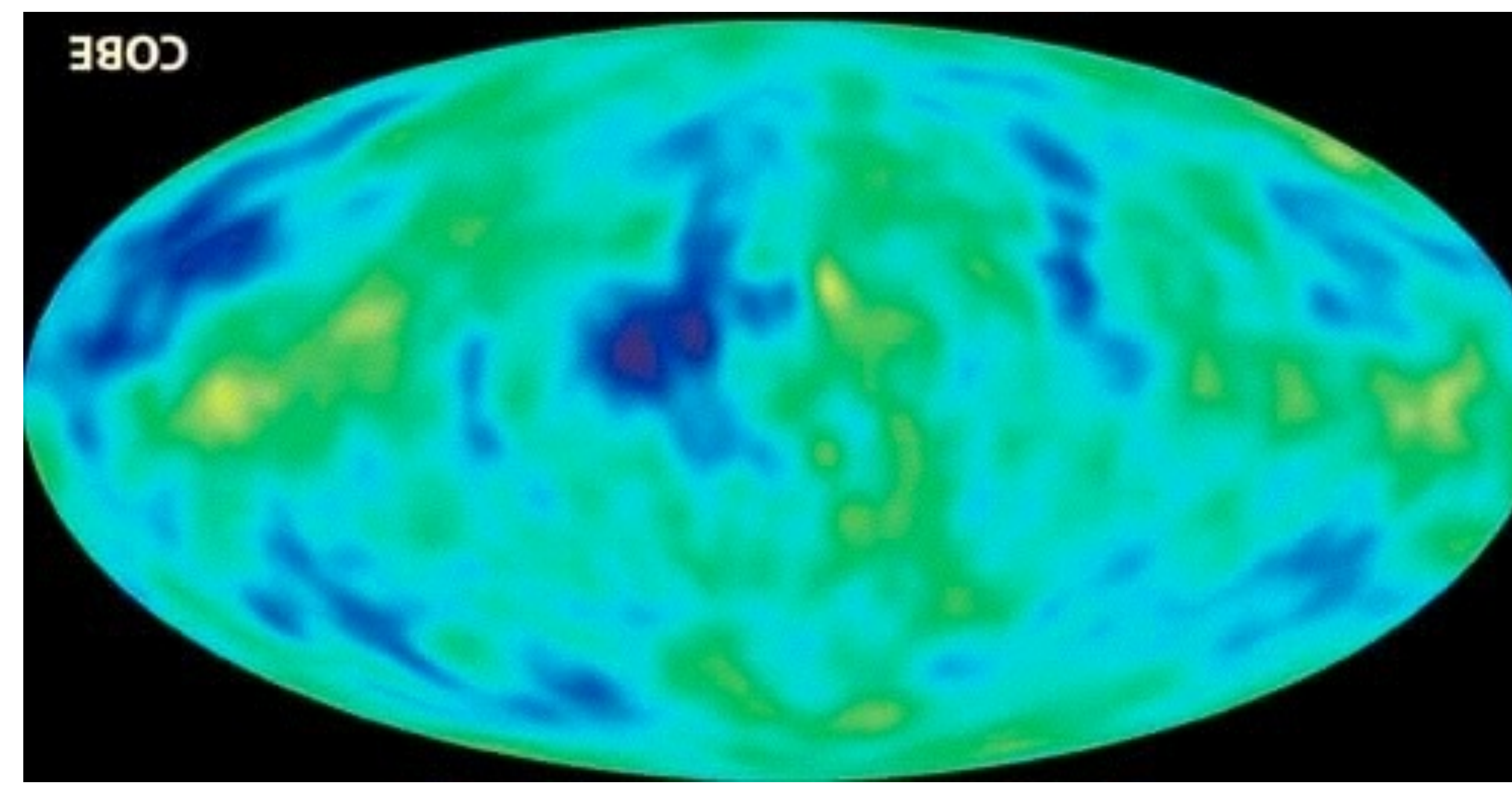
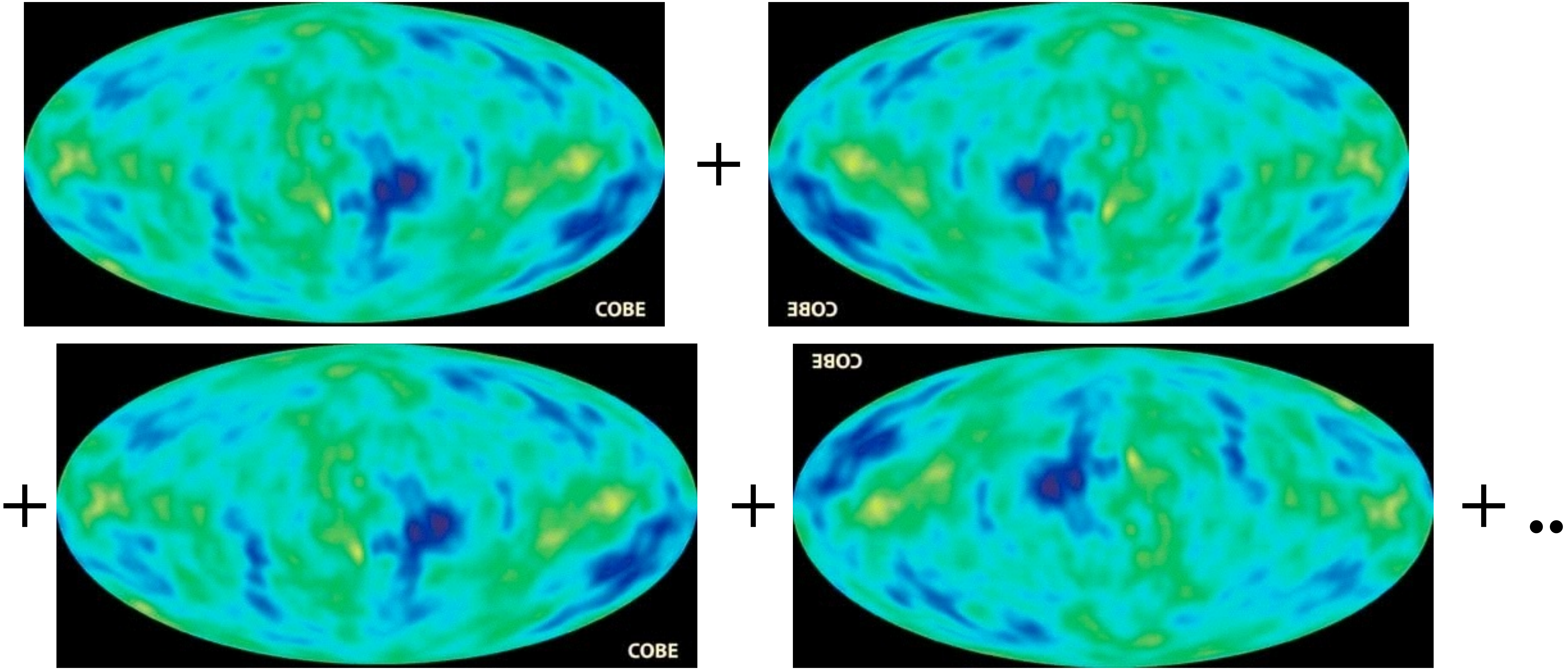


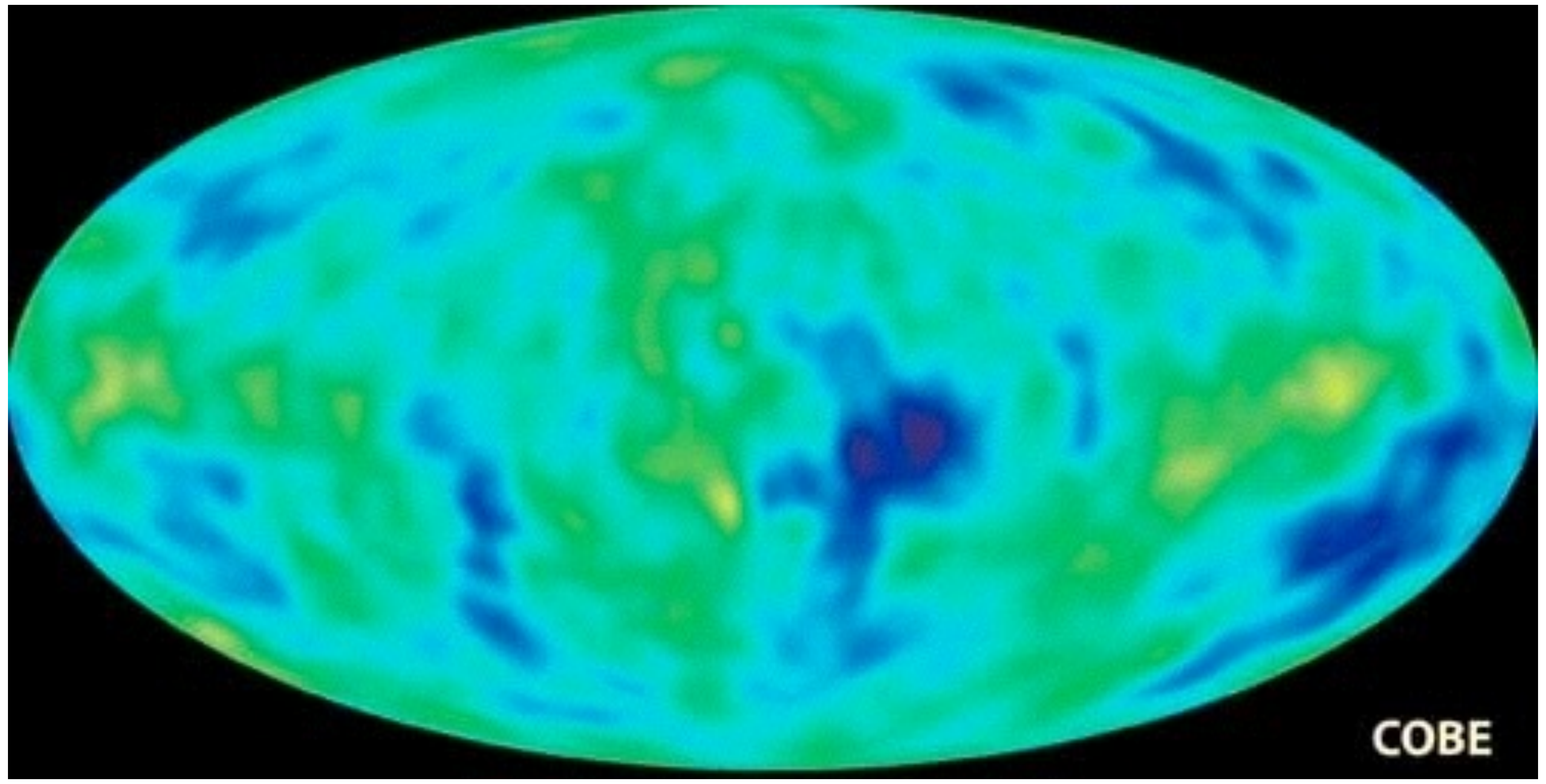
+ ...

Superposition



Superposition

Collapse in 1992 ???



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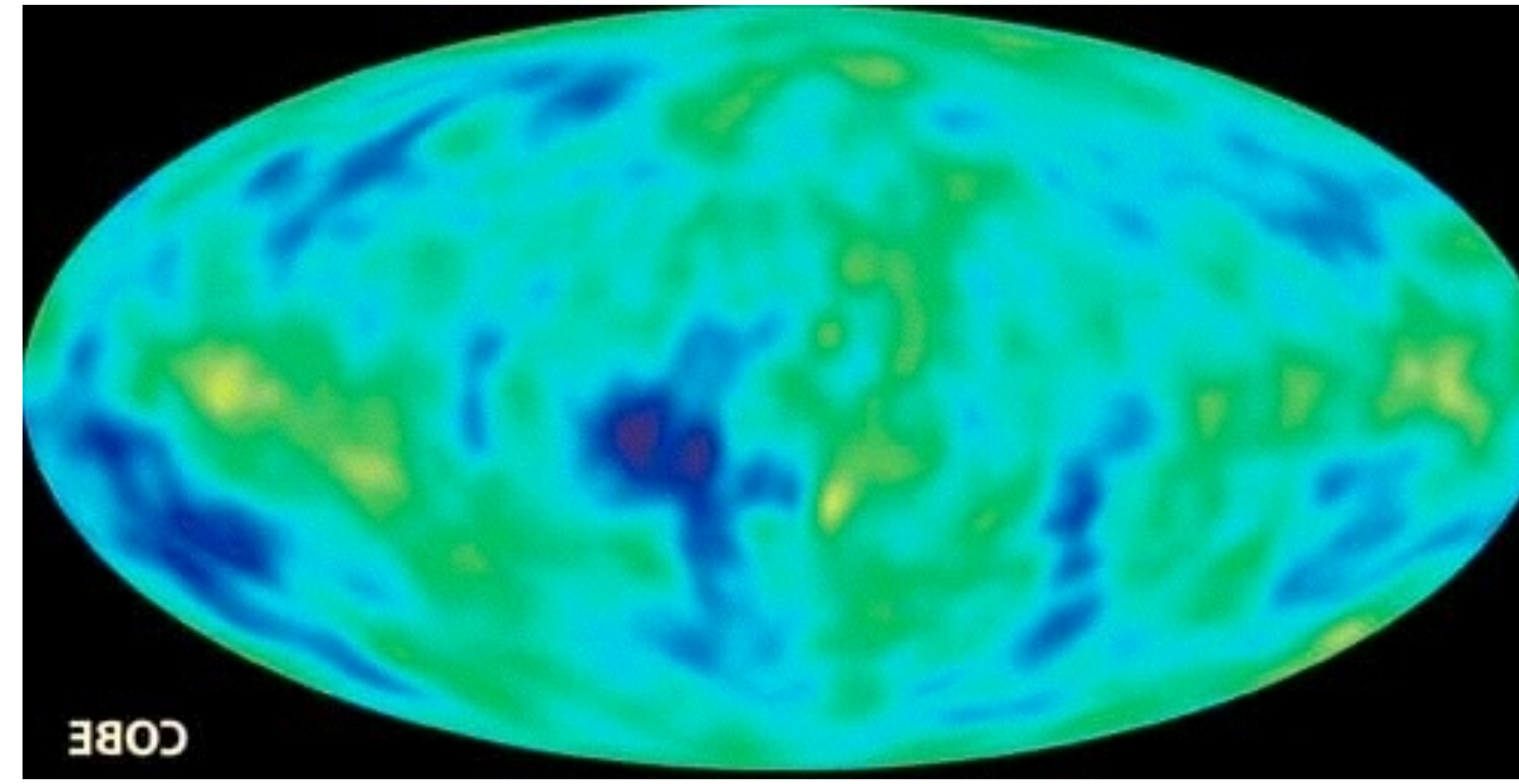

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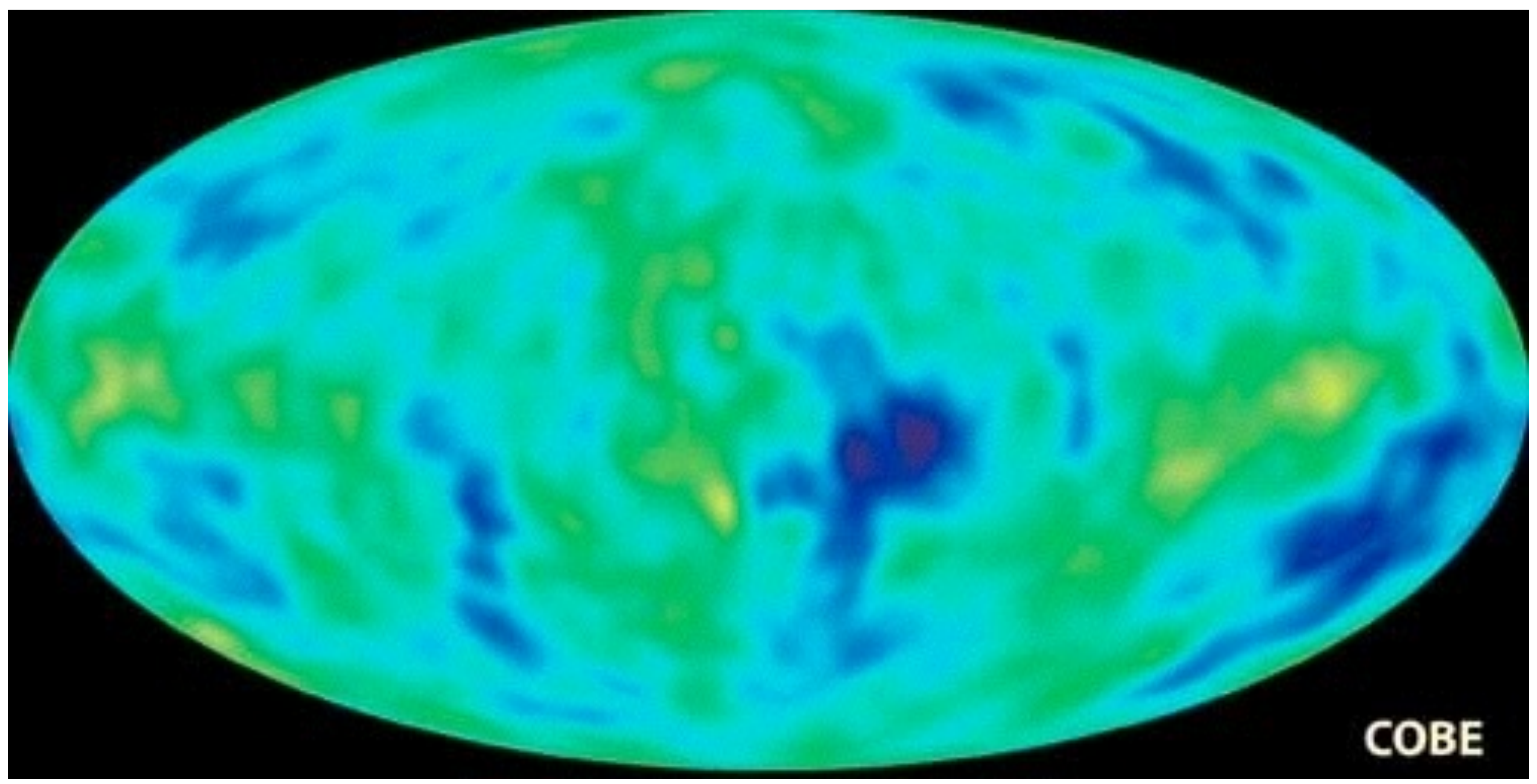

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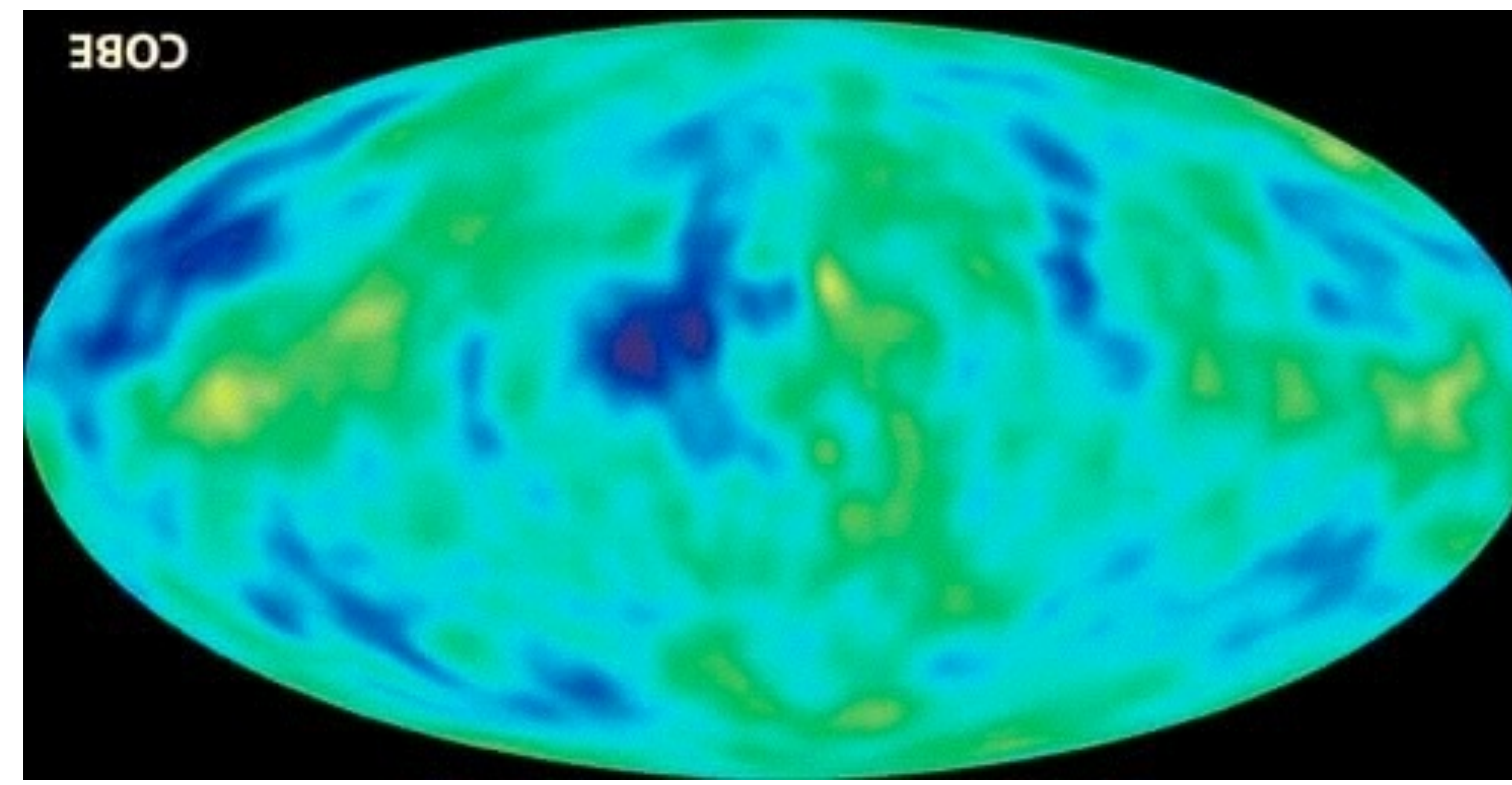
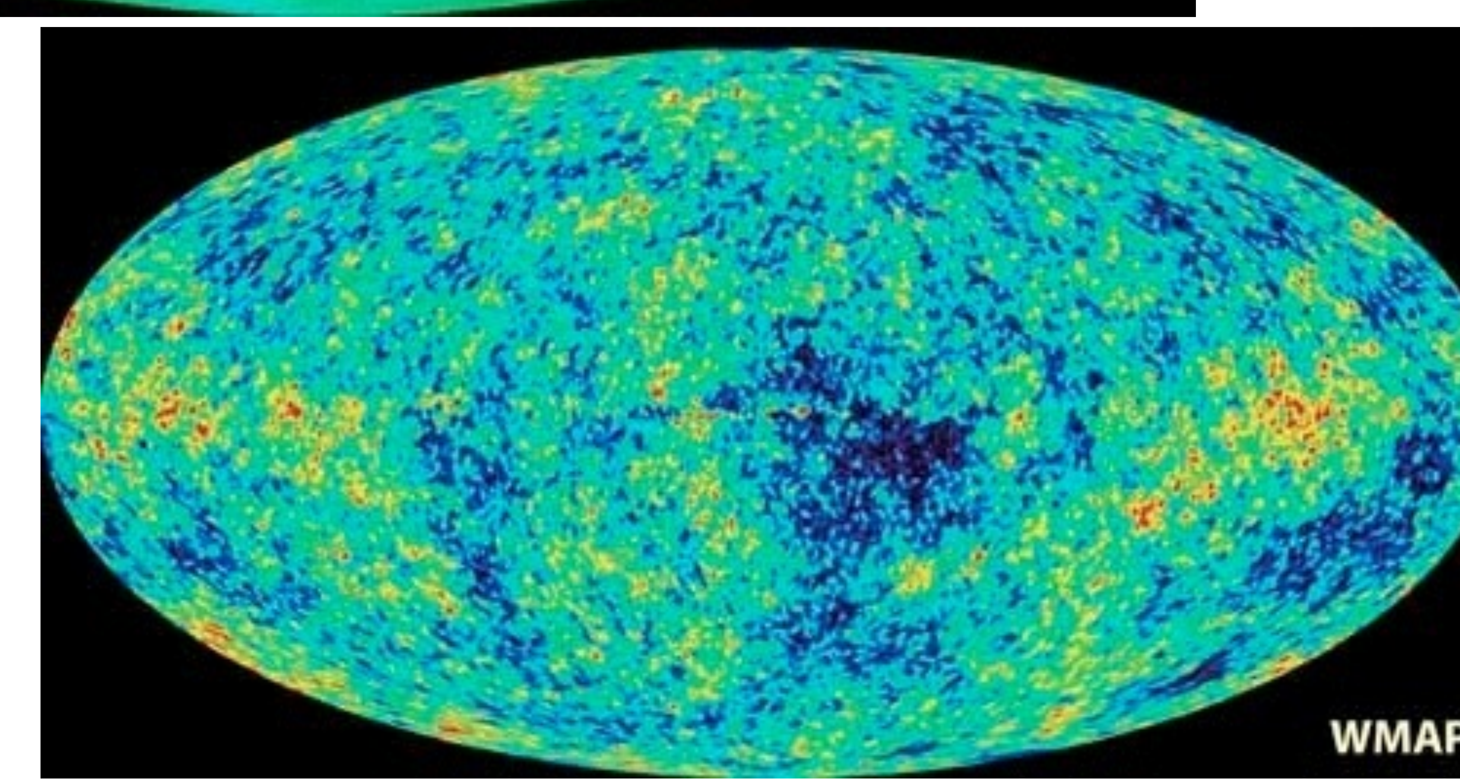


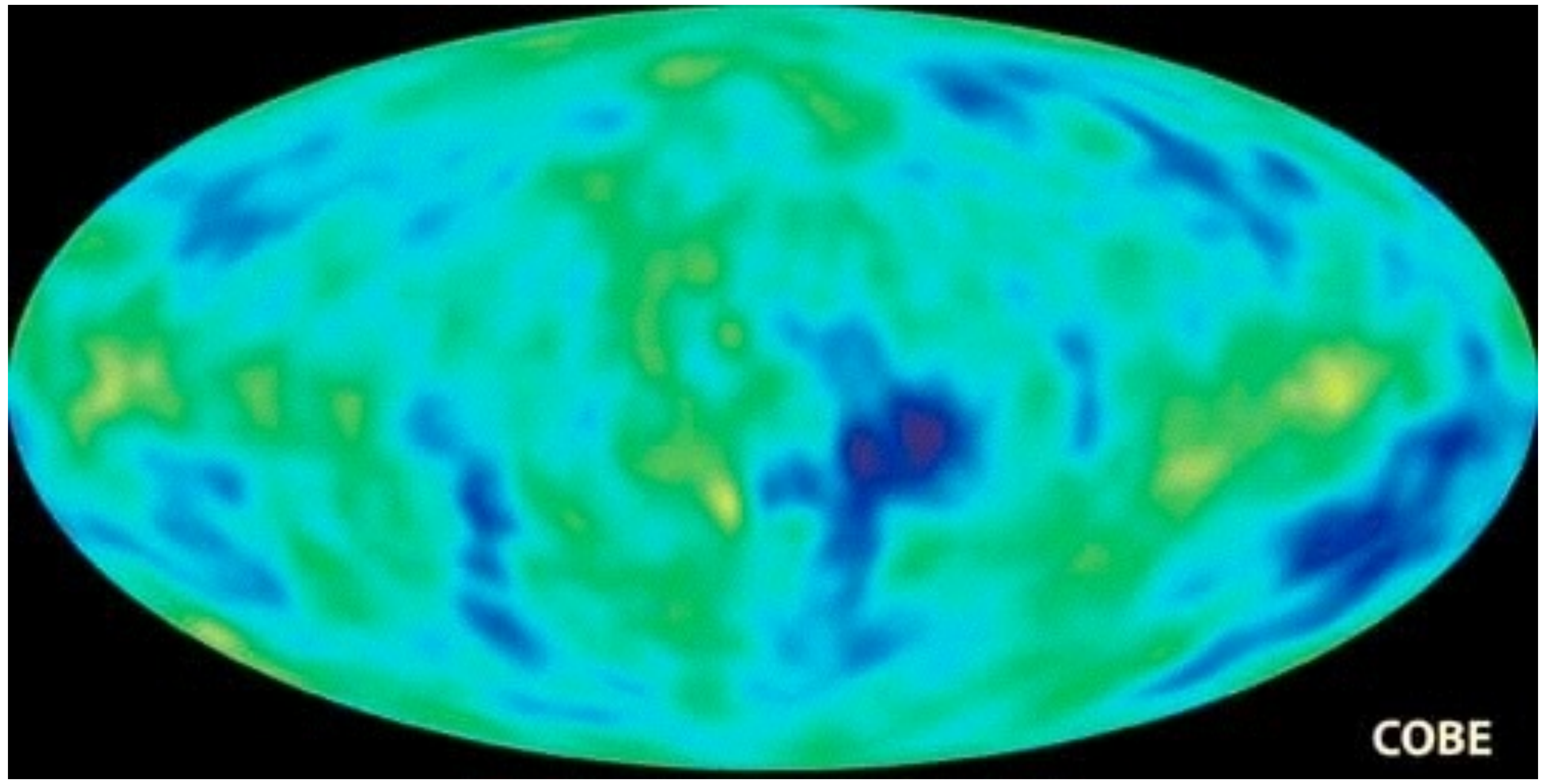
+ ...

Superposition



Collapse in 1992 ???

Further collapse in 2003
on smaller scales???



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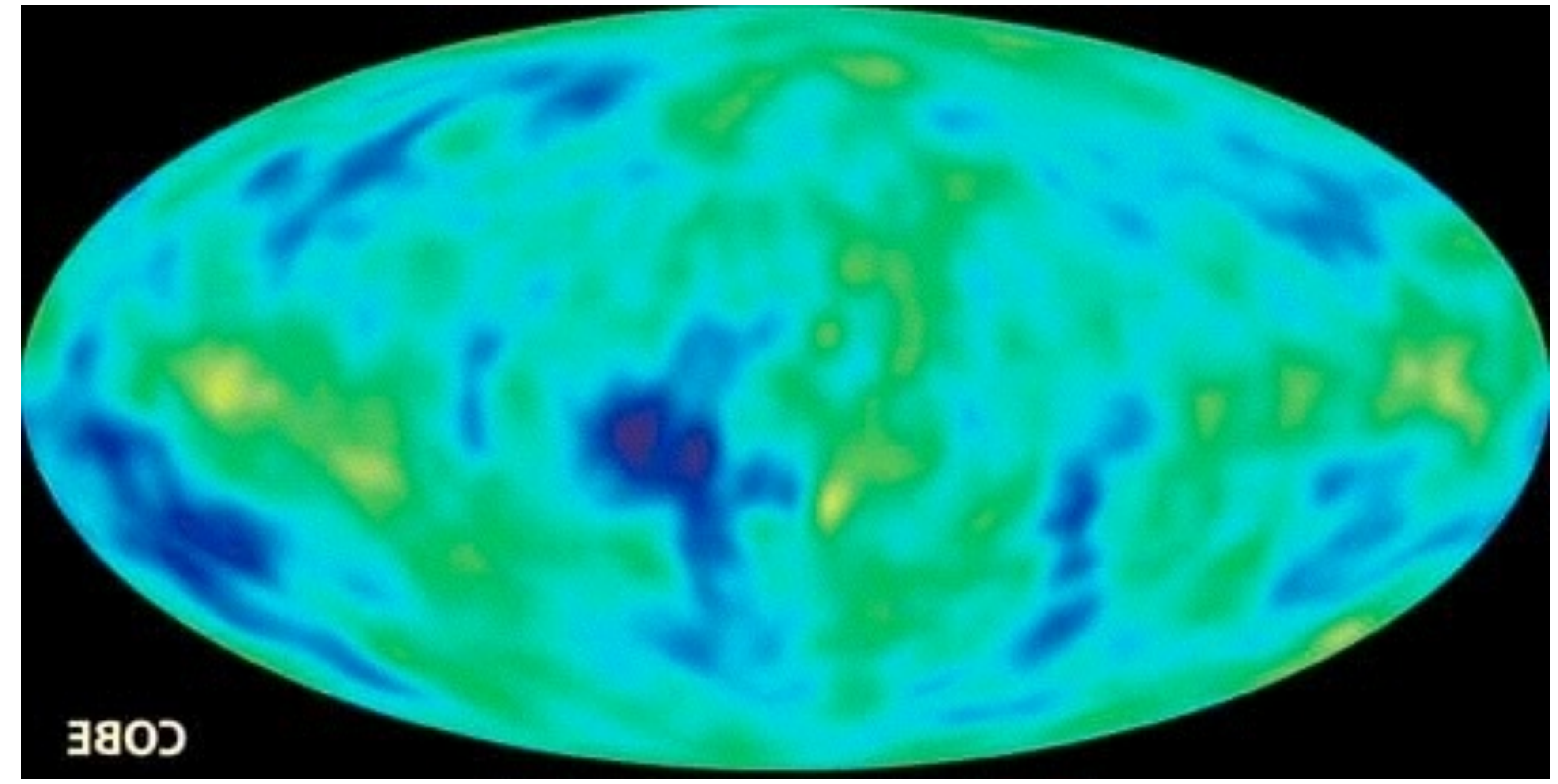

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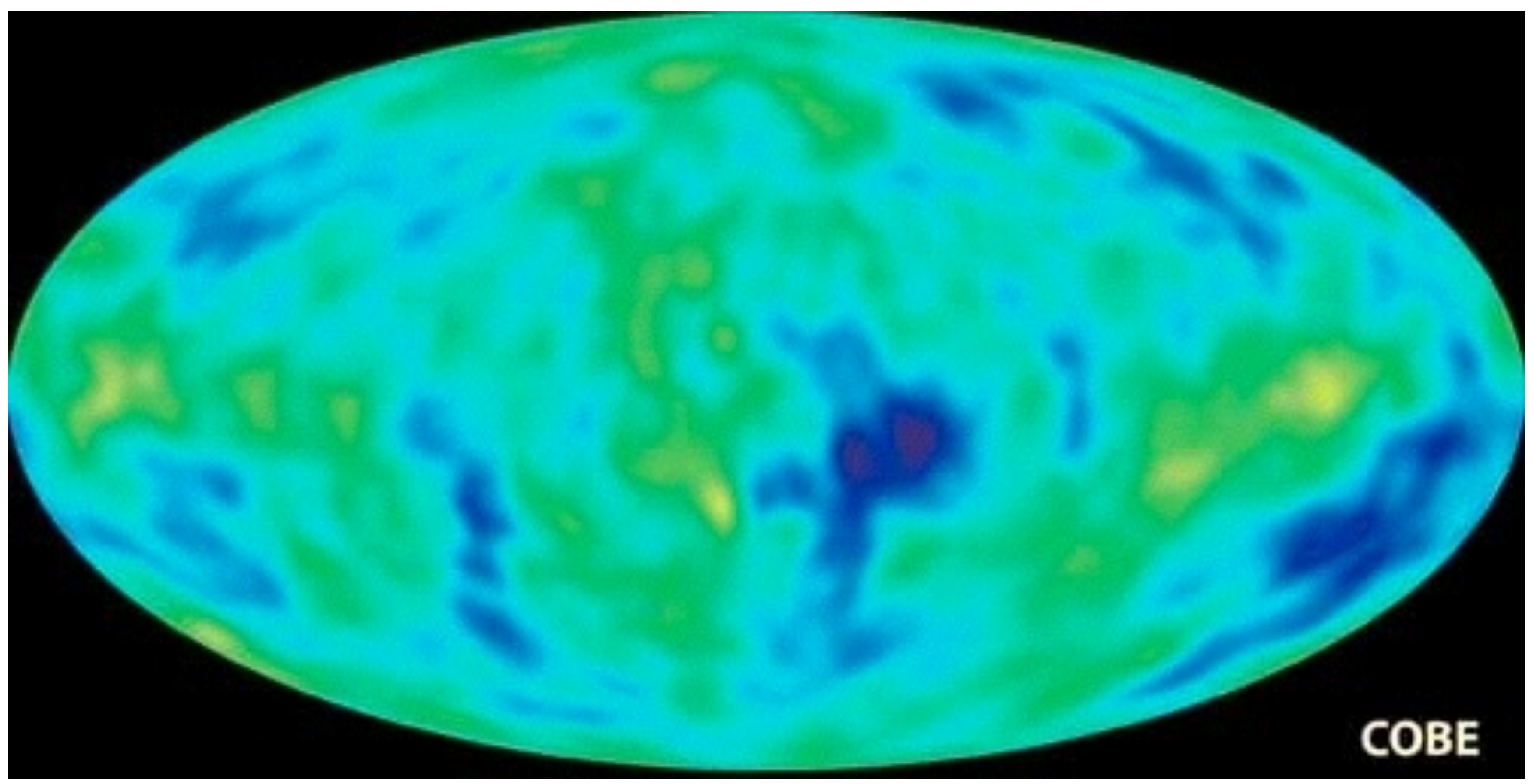

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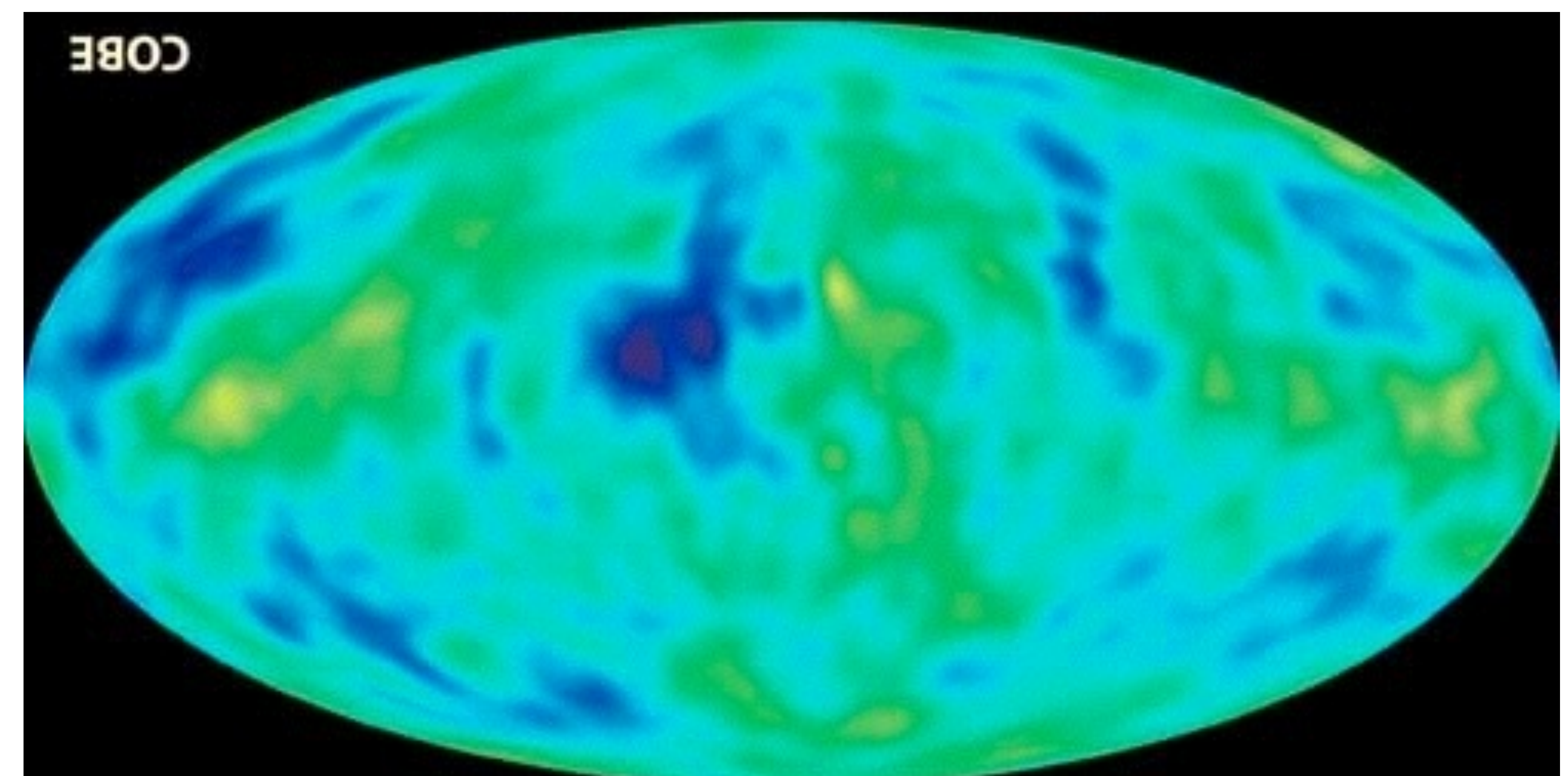
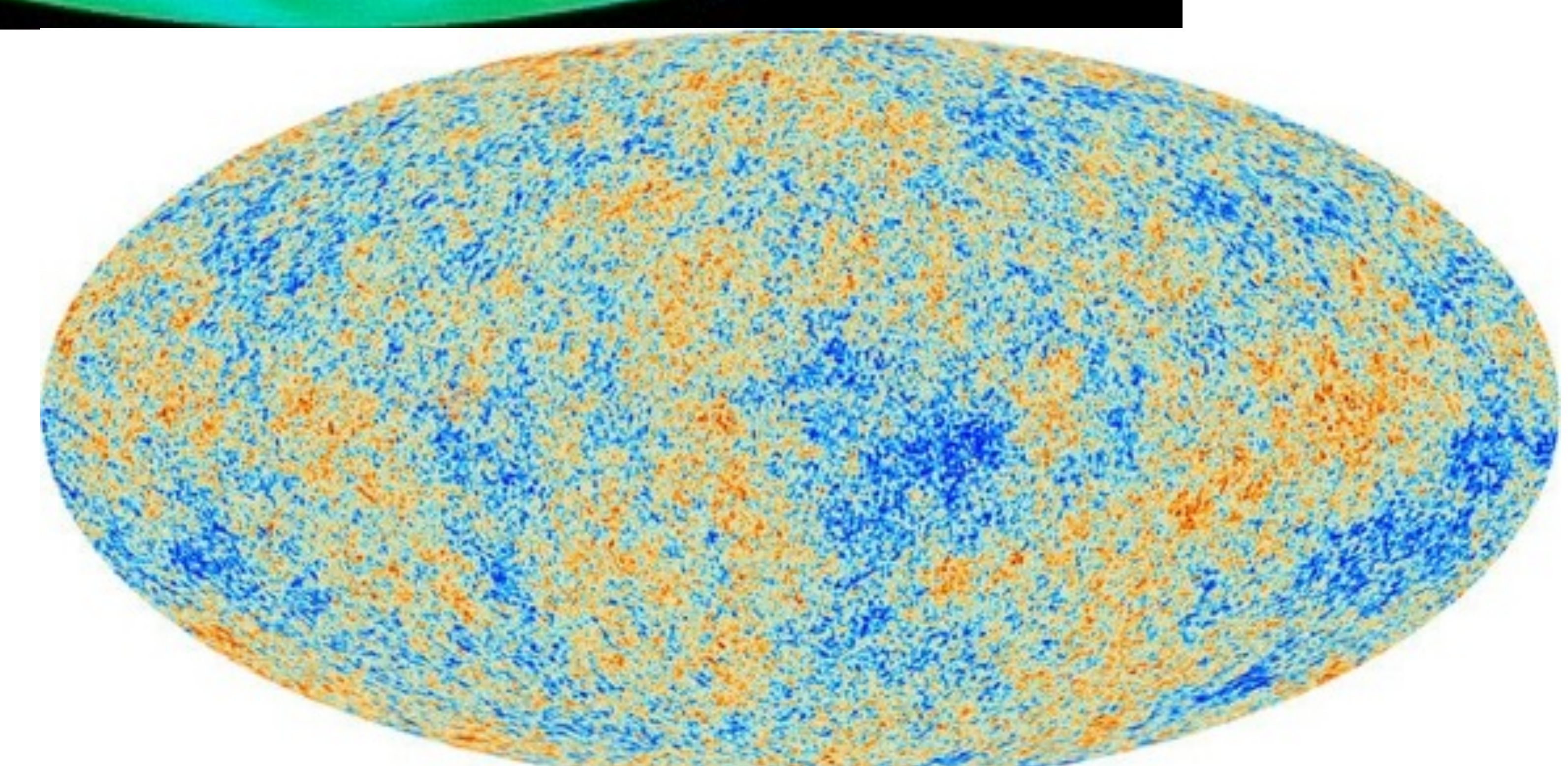


+ ...

Superposition

Final (ultimate!) collapse in 2012?

Collapse in 1992 ???



- Both background and perturbations are quantum

Usual treatment of the perturbations?

Einstein-Hilbert action up to 2nd order $\mathcal{S}_{\mathrm{E-H}} = \int \mathrm{d}^4x \left[R^{(0)} + \delta^{(2)} R\right]$

Classical Quantum

Bardeen (Newton) gravitational potential

$$\mathrm{d}s^2 = a^2(\eta)\left\{(1+2\Phi)\,\mathrm{d}\eta^2 - \left[(1-2\Phi)\,\gamma_{ij} + h_{ij}\right]\mathrm{d}x^i\mathrm{d}x^j\right\}$$

conformal time $\mathrm{d}\eta = a(t)^{-1}\mathrm{d}t$

$$\Delta\Phi = -\frac{3\ell_{\mathrm{Pl}}^2}{2}\sqrt{\frac{\rho+p}{\omega}}\,a\frac{\mathrm{d}}{\mathrm{d}\eta}\left(\frac{v}{a}\right)$$

Mukhanov-Sasaki variable

$$\int \mathrm{d}^4x\,\delta^{(2)}\mathcal{L} = \frac{1}{2}\int\sqrt{\gamma}\mathrm{d}^3\boldsymbol{x}\,\mathrm{d}\eta\left[(\partial_\eta v)^2 - \gamma^{ij}\partial_i v\partial_j v + \frac{z''}{z}v^2\right]$$

V. F. Mukhanov, H. A. Feldman & R. H. Brandenberger, *Phys. Rep.* **215**, 203 (1992)

Simple scalar field with varying mass in Minkowski space!!! $z = z[a(\eta)]$

Self-consistent treatment of the perturbations?

Hamiltonian up to 2nd order $H = H_{(0)} + H_{(2)} + \cdots$

$$\Delta\Phi = -\frac{3\ell_{\mathrm{Pl}}^2}{2}\sqrt{\frac{\rho+p}{\omega}}\,a\frac{\mathrm{d}}{\mathrm{d}\eta}\left(\frac{v}{a}\right)$$

factorization of the wave function

$$\Psi = \Psi_{(0)}\left(a,T\right)\Psi_{(2)}\left[v,T;a\left(T\right)\right]$$

comes from 0th order

Use dBB or...

The GRW dynamical collapse model

Ghirardi - Rimini - Weber

Modified Schrödinger equation with collapse towards $\hat{C}$ eigenstates

$$\mathrm{d}|\Psi\rangle = -i\hat{H}|\Psi\rangle\mathrm{d}t + \sqrt{\gamma}\left(\hat{C} - \langle\hat{C}\rangle\right)\mathrm{d}W_t|\Psi\rangle \underbrace{- \frac{\gamma}{2}\left(\hat{C} - \langle\hat{C}\rangle\right)^2 \mathrm{d}t|\Psi\rangle}_{\text{normalization}}$$

Hamiltonian

non linear stochastic

$$\langle\hat{C}\rangle \equiv \langle\Psi|\hat{C}|\Psi\rangle$$

random outcomes

Born rule

$$\mathbb{E}\left(\mathrm{d}W_t\right) = 0$$

$$\mathbb{E}\left(\mathrm{d}W_t \mathrm{d}W_{t'}\right) = \mathrm{d}t\mathrm{d}t'\delta(t - t')$$

break superposition principle

Wiener process

BONUS: Amplification mechanism

Grown perturbations

Big objects are classical
small objects are quantum!

Primordial perturbations

Year	first author [ref.]	interfering object	m/m_p	τ	d	in GRW $\lambda <$	in GRW $\lambda/\sigma^2 <$	in CSL $\lambda <$	in CSL $\lambda/\sigma^2 <$
1927	Davisson [13]	electron	5×10^{-4}	N/A	2×10^{-10}m	10^{14} s^{-1}	3×10^{33} m^{-2}s^{-1}	10^{17} s^{-1}	5×10^{36} m^{-2}s^{-1}
1930	Estermann [15]	He	4	N/A	4×10^{-10}m	10^{11} s^{-1}	6×10^{29} m^{-2}s^{-1}	3×10^{10} s^{-1}	10^{29} m^{-2}s^{-1}
1959	Möllenstedt [28]	electron	5×10^{-4}	3×10^{-9} s	2×10^{-6} m	7×10^{11} s^{-1}	10^{23} m^{-2}s^{-1}	10^{15} s^{-1}	3×10^{26} m^{-2}s^{-1}
1987	Tonomura [37]	electron	5×10^{-4}	10^{-8} s	10^{-4} m	2×10^{11} s^{-1}	2×10^{19} m^{-2}s^{-1}	4×10^{14} s^{-1}	4×10^{22} m^{-2}s^{-1}
1988	Zeilinger [40]	neutron	1	10^{-2} s	10^{-4} m	2×10^{2} s^{-1}	2×10^{10} m^{-2}s^{-1}	2×10^{2} s^{-1}	2×10^{10} m^{-2}s^{-1}
1991	Carnal [9]	He	4	6×10^{-4} s	10^{-5} m	4×10^{2} s^{-1}	4×10^{12} m^{-2}s^{-1}	10^{2} s^{-1}	10^{12} m^{-2}s^{-1}
1999	Arndt [4]	C_{60}	720	6×10^{-3} s	10^{-7} m	2×10^{-1} s^{-1}	2×10^{13} m^{-2}s^{-1}	3×10^{-4} s^{-1}	3×10^{10} m^{-2}s^{-1}
2001	Nairz [29]	C_{70}	840	10^{-2} s	3×10^{-7} m	10^{-1} s^{-1}	10^{12} m^{-2}s^{-1}	10^{-4} s^{-1}	10^{9} m^{-2}s^{-1}
2004	Hackermüller [24]	C_{70}	840	2×10^{-3} s	10^{-6} m	10^{0} s^{-1}	10^{12} m^{-2}s^{-1}	10^{-3} s^{-1}	10^{9} m^{-2}s^{-1}
2007	Gerlich [17]	$C_{30}H_{12}F_{30}N_2O_4$	10^3	10^{-3} s	3×10^{-7} m	10^{0} s^{-1}	10^{13} m^{-2}s^{-1}	10^{-3} s^{-1}	10^{10} m^{-2}s^{-1}
2011	Gerlich [18]	$C_{60}[C_{12}F_{25}]_{10}$	7×10^3	10^{-3} s	3×10^{-7} m	10^{-1} s^{-1}	10^{12} m^{-2}s^{-1}	10^{-5} s^{-1}	10^{8} m^{-2}s^{-1}
	Proposed future experiments								
	Romero-Isart [35]	$[SiO_2]_{150,000}$	10^7	10^{-1} s	4×10^{-7} m	10^{-6} s^{-1}	6×10^{6} m^{-2}s^{-1}	10^{-13} s^{-1}	6×10^{-1} m^{-2}s^{-1}
	Nimmrichter [30]	$Au_{500,000}$	10^8	6×10^{0} s	10^{-7} m	2×10^{-9} s^{-1}	2×10^{5} m^{-2}s^{-1}	2×10^{-17} s^{-1}	2×10^{-3} m^{-2}s^{-1}

Table 1: Bounds on σ, λ obtained from different diffraction experiments. For each experiment, m = mass of the interfering object, m_p = proton mass, τ = time of flight between grating and image plane, d = period of grating (or transverse coherence length in [37]), N/A = not applicable. For each theory (GRW or CSL), two bounds are obtained. This table is the basis for Fig. 3.

Feldmann & Tumulka (2011)

γ constrained...

Spontaneous collapse amplification mechanism

N identical particles

collapse operator: $\hat{C} = \sum_{i=1}^{N} \hat{x}_i$ acting on $|\Psi(\{x_i\})\rangle = |\Psi_{\text{CM}}(R)\rangle \otimes |\Psi_{\text{rel}}(\{r_i\})\rangle$

$$\mathrm{d}|\Psi_{\text{rel}}(\{r_i\})\rangle = \left\{ \left[-i\hat{H}_{\text{rel}} - \frac{\gamma}{2} \sum_{i=1}^{N-1} (\hat{r}_i - \langle \hat{r}_i \rangle)^2 \right] \mathrm{d}t + \sqrt{\gamma} \sum_{i=1}^{N-1} (\hat{r}_i - \langle \hat{r}_i \rangle) \, \mathrm{d}W_t^{(i)} \right\} |\Psi_{\text{rel}}(\{r_i\})\rangle$$

usual quantum behavior

$$\mathrm{d}|\Psi_{\text{CM}}(R)\rangle = \left\{ \left[-i\hat{H}_{\text{CM}} - \frac{N\gamma}{2} \left(\hat{R} - \langle \hat{R} \rangle \right)^2 \right] \mathrm{d}t + \sqrt{N\gamma} \left(\hat{R} - \langle \hat{R} \rangle \right) \mathrm{d}W_t \right\} |\Psi_{\text{CM}}(R)\rangle$$

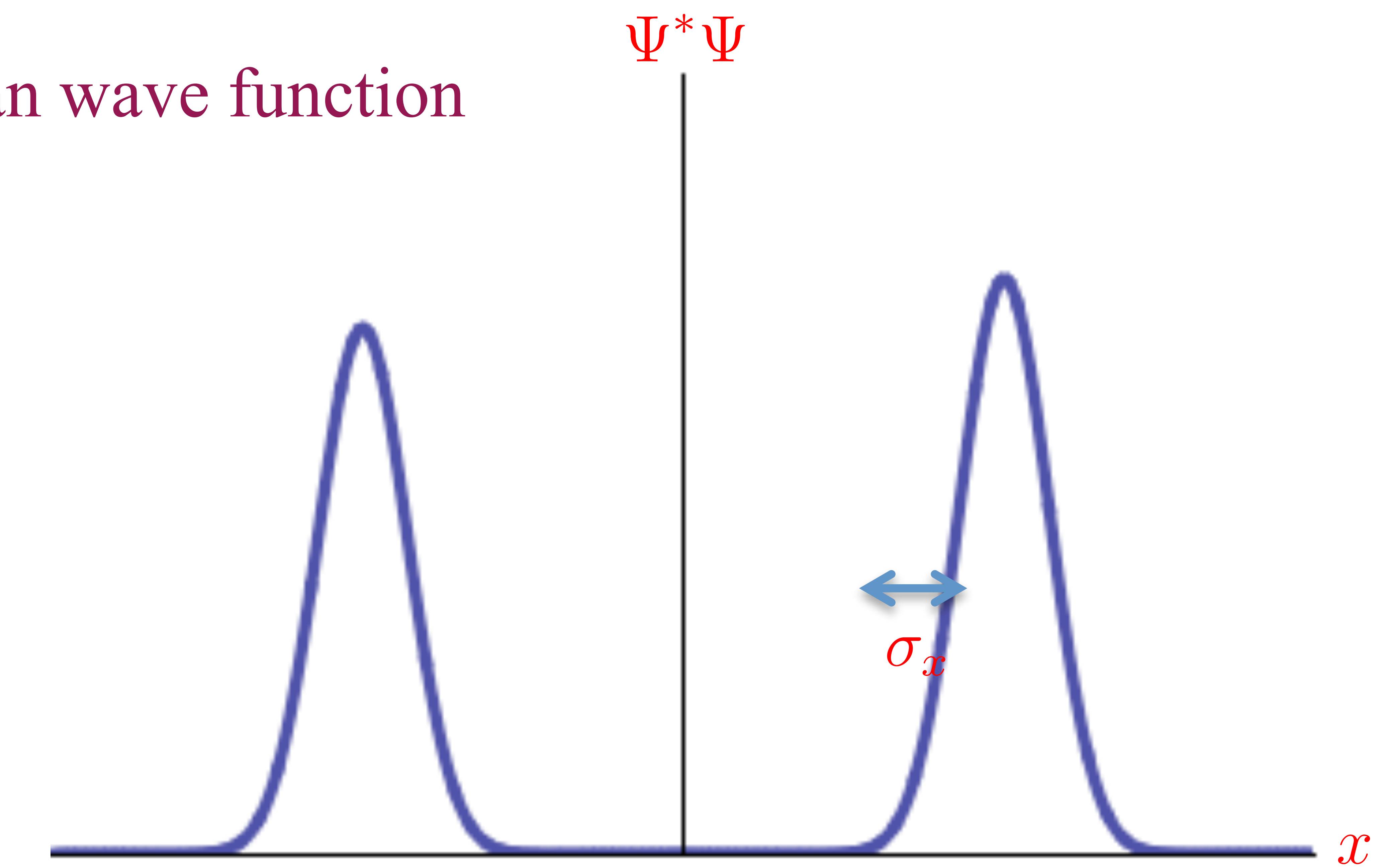
macro objectification

Example: free particle evolution $\hat{H} = \frac{\hat{p}^2}{2m}$

Animations provided by V. Vennin... thx!

and projection on position operator $\hat{C} = \hat{x}$

initial double gaussian wave function



Amplification mechanism $\Longrightarrow$ $\gamma \propto N$ (number of particles)

$$\sigma_x(\infty) = \left(\frac{\hbar}{4m\gamma}\right)^{\frac{1}{4}}$$

- 4.7 cm for a proton
- 4.6×10^{-14} m for 1g object
- 5.9×10^{-28} m for the Earth

Constraints:

(falsifiable theory!)

- Atomic energy levels
- Nuclear energy levels
- Diffraction Experiments
- Proton Decay
- Spontaneous Xray emission
- Spontaneous IGM warming
- Dissociation of cosmic H
- Decay of supercurrents
- Latent image formation
- Thermalized spectral distorsions
- Neutrino and kaon oscillations

Constraints:

(falsifiable theory!)

Cosmological perturbations: different test by orders of magnitude!

Classicalization of Cosmological Perturbations

Predictions of the theory:

Calculated by quantum average $\langle \Psi | \hat{O} | \Psi \rangle$

Usually in a lab:
repeat the experiment

Ensemble average over experiments $\Longleftrightarrow$ Quantum average

Here one has a single experiment (a single universe)

$\hookrightarrow$ Ergodicity

Spatial average over directions in the sky $\Longleftrightarrow$ Quantum average

Inflationary perturbations: quantum fluctuations / expanding background

Classical temperature fluctuations promoted to quantum operators

$$\frac{\widehat{\Delta T}}{T} \propto \hat{v}$$

variable-mass scalar fields in Minkowski spacetime

second order perturbed Einstein action $\quad {}^{(2)}\delta S = \frac{1}{2}\int \mathrm{d}^4 x \left[\left(v'\right)^2 - \delta^{ij}\partial_i v \partial_j v + \frac{\left(a\sqrt{\epsilon_1}\right)''}{a\sqrt{\epsilon_1}} v^2 \right]$

$\epsilon_1 = 1 - \mathcal{H}'/\mathcal{H}^2$

slow-roll parameter

+ Fourier transform $\quad v\left(\eta, \boldsymbol{x}\right) = \frac{1}{\left(2\pi\right)^{3/2}} \int_{\mathbb{R}^3} \mathrm{d}^3 \boldsymbol{k}\, v_{\boldsymbol{k}}\left(\eta\right) \mathrm{e}^{i\boldsymbol{k}\cdot\boldsymbol{x}}$

$$\Longrightarrow \quad {}^{(2)}\delta S = \int \mathrm{d}\eta \int \mathrm{d}^3 \boldsymbol{k} \left\{ v_{\boldsymbol{k}}' v_{\boldsymbol{k}}^{*\prime} + v_{\boldsymbol{k}} v_{\boldsymbol{k}}^{*} \left[\frac{\left(a\sqrt{\epsilon_1}\right)''}{a\sqrt{\epsilon_1}} - k^2 \right] \right\}$$

Lagrangian formulation...

Hamiltonian

$$H = \int \mathrm{d}^3\boldsymbol{k} \left\{ p_{\boldsymbol{k}} p^*_{\boldsymbol{k}} + v_{\boldsymbol{k}} v^*_{\boldsymbol{k}} \left[\overbrace{k^2 - \frac{\left(a\sqrt{\epsilon_1}\right)''}{a\sqrt{\epsilon_1}}}^{\omega^2(\eta,\boldsymbol{k})} \right] \right\}$$

collection of parametric oscillators with time dependent frequency

factorization of the full wave function

real and imaginary parts

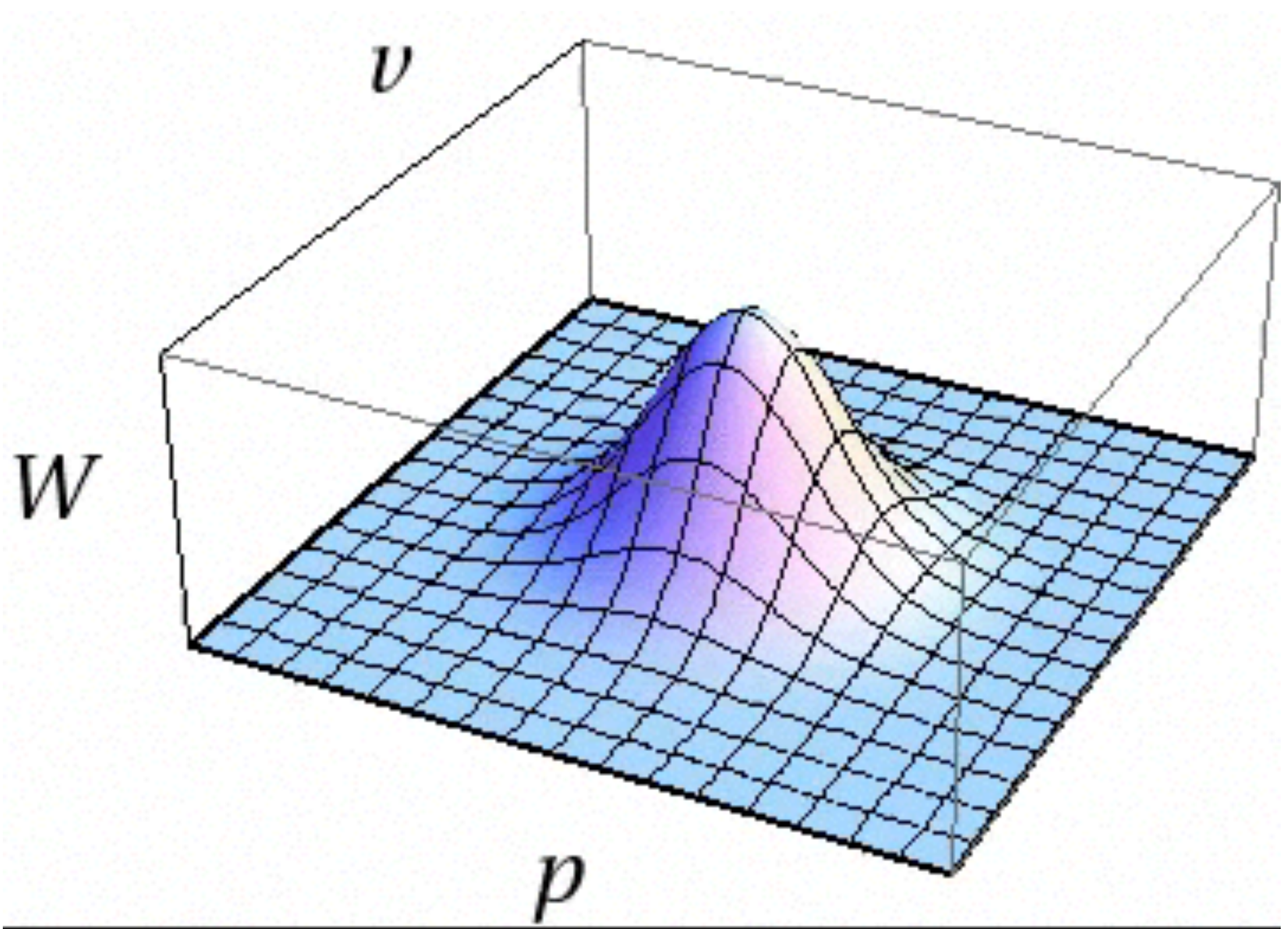
$$\Psi\left[v(\eta,\boldsymbol{x})\right] = \prod_{\boldsymbol{k}} \Psi_{\boldsymbol{k}}\left(v^{\mathrm{R}}_{\boldsymbol{k}}, v^{\mathrm{I}}_{\boldsymbol{k}}\right) = \prod_{\boldsymbol{k}} \Psi^{\mathrm{R}}_{\boldsymbol{k}}\left(v^{\mathrm{R}}_{\boldsymbol{k}}\right) \Psi^{\mathrm{I}}_{\boldsymbol{k}}\left(v^{\mathrm{I}}_{\boldsymbol{k}}\right)$$

$$\left.\begin{aligned} i\frac{\Psi^{\mathrm{R,I}}_{\boldsymbol{k}}}{\partial\eta} &= \hat{\mathcal{H}}^{\mathrm{R,I}}_{\boldsymbol{k}} \Psi^{\mathrm{R,I}}_{\boldsymbol{k}} \\ \hat{\mathcal{H}}^{\mathrm{R,I}}_{\boldsymbol{k}} &= -\frac{1}{2}\frac{\partial^2}{\partial\left(v^{\mathrm{R,I}}_{\boldsymbol{k}}\right)^2} + \frac{1}{2}\omega^2(\eta,\boldsymbol{k})\left(\hat{v}^{\mathrm{R,I}}_{\boldsymbol{k}}\right)^2 \end{aligned}\right\}$$

Gaussian state solution $\Psi\left(\eta, v_{\boldsymbol{k}}\right) = \left[\frac{2\Re\mathrm{e}\,\Omega_{\boldsymbol{k}}\left(\eta\right)}{\pi}\right]^{1/4} \mathrm{e}^{-\Omega_{\boldsymbol{k}}(\eta) v_{\boldsymbol{k}}^2}$

Wigner function $W\left(v_{\boldsymbol{k}}, p_{\boldsymbol{k}}\right) = \int \frac{\mathrm{d}x}{2\pi^2} \Psi^*\left(v_{\boldsymbol{k}} - \frac{x}{2}\right) \mathrm{e}^{-ip_{\boldsymbol{k}}x} \Psi\left(v_{\boldsymbol{k}} + \frac{x}{2}\right)$

large squeezing limit $\Rightarrow$ $W \propto \delta\left(p_{\boldsymbol{k}} + k \tan\phi_{\boldsymbol{k}} v_{\boldsymbol{k}}\right)$



Stochastic distribution of classical processes

Ergodicity

realization

spatial direction

$$\left\langle \frac{\Delta T(\xi, \boldsymbol{e})}{T} \right\rangle_{\xi} \simeq \left\langle \frac{\Delta T(\xi, \boldsymbol{e})}{T} \right\rangle_{\boldsymbol{e}}$$

Animations provided by V. Vennin... thx!

Primordial Power Spectrum

Standard case

Quantization in the Schrödinger picture
(functional representation)

$$i\frac{\mathrm{d}|\Psi_{\boldsymbol{k}}\rangle}{\mathrm{d}\eta} = \hat{\mathcal{H}}_{\boldsymbol{k}}\,|\Psi_{\boldsymbol{k}}\rangle$$

with $$\hat{\mathcal{H}}_{\boldsymbol{k}} = \frac{\hat{p}_{\boldsymbol{k}}^2}{2} + \omega^2(\boldsymbol{k},\eta)\hat{v}_{\boldsymbol{k}}^2$$

and $$\omega^2(\boldsymbol{k},\eta) = k^2 - \frac{(a\sqrt{\epsilon_1})''}{a\sqrt{\epsilon_1}}$$

$$= k^2 - \frac{\beta(\beta+1)}{\eta^2}$$

$$\hat{v}_{\boldsymbol{k}} = v_{\boldsymbol{k}}$$

$$\hat{p}_{\boldsymbol{k}} = i\frac{\partial}{\partial v_{\boldsymbol{k}}}$$

Power-law inflation example

$$a(\eta) = \ell_0(-\eta)^{1+\beta}$$

$$\beta \lesssim -2$$

(de Sitter: $\beta = -2$)

Parametric Oscillator System

Primordial Power Spectrum

Standard case

Quantization in the Schrödinger picture (functional representation)

$$\Psi_{\boldsymbol{k}}(\eta, v_{\boldsymbol{k}}) = \left[\frac{2\,\Re\mathrm{e}\Omega_{\boldsymbol{k}}(\eta)}{\pi}\right]^{1/4} \mathrm{e}^{-\Omega_{\boldsymbol{k}}(\eta)\, v_{\boldsymbol{k}}^2}$$

$$i\frac{\mathrm{d}|\Psi_{\boldsymbol{k}}\rangle}{\mathrm{d}\eta} = \hat{\mathcal{H}}_{\boldsymbol{k}}\,|\Psi_{\boldsymbol{k}}\rangle$$

with

$$\hat{\mathcal{H}}_{\boldsymbol{k}} = \frac{\hat{p}_{\boldsymbol{k}}^2}{2} + \omega^2(\boldsymbol{k}, \eta)\hat{v}_{\boldsymbol{k}}^2$$

$$\Omega_{\boldsymbol{k}}' = -2i\Omega_{\boldsymbol{k}}^2 + \frac{i}{2}\omega^2(\eta, \boldsymbol{k})$$

$$\Omega_{\boldsymbol{k}} = -\frac{i}{2}\frac{f_{\boldsymbol{k}}'}{f_{\boldsymbol{k}}}$$

$$\boxed{f_{\boldsymbol{k}}'' + \omega^2(\boldsymbol{k}, \eta)\, f_{\boldsymbol{k}} = 0}$$

Primordial Power Spectrum

Standard case

Two physical scales

Hubble radius $H^{-1} = \dfrac{a^2}{a'} \underset{\beta\sim-2}{\simeq} \ell_0$

wavelength $\lambda = \dfrac{a}{k} \underset{\beta\sim-2}{\simeq} \dfrac{\ell_0}{-k\eta}$

Sub-Hubble regime

$\lambda \ll H^{-1}$

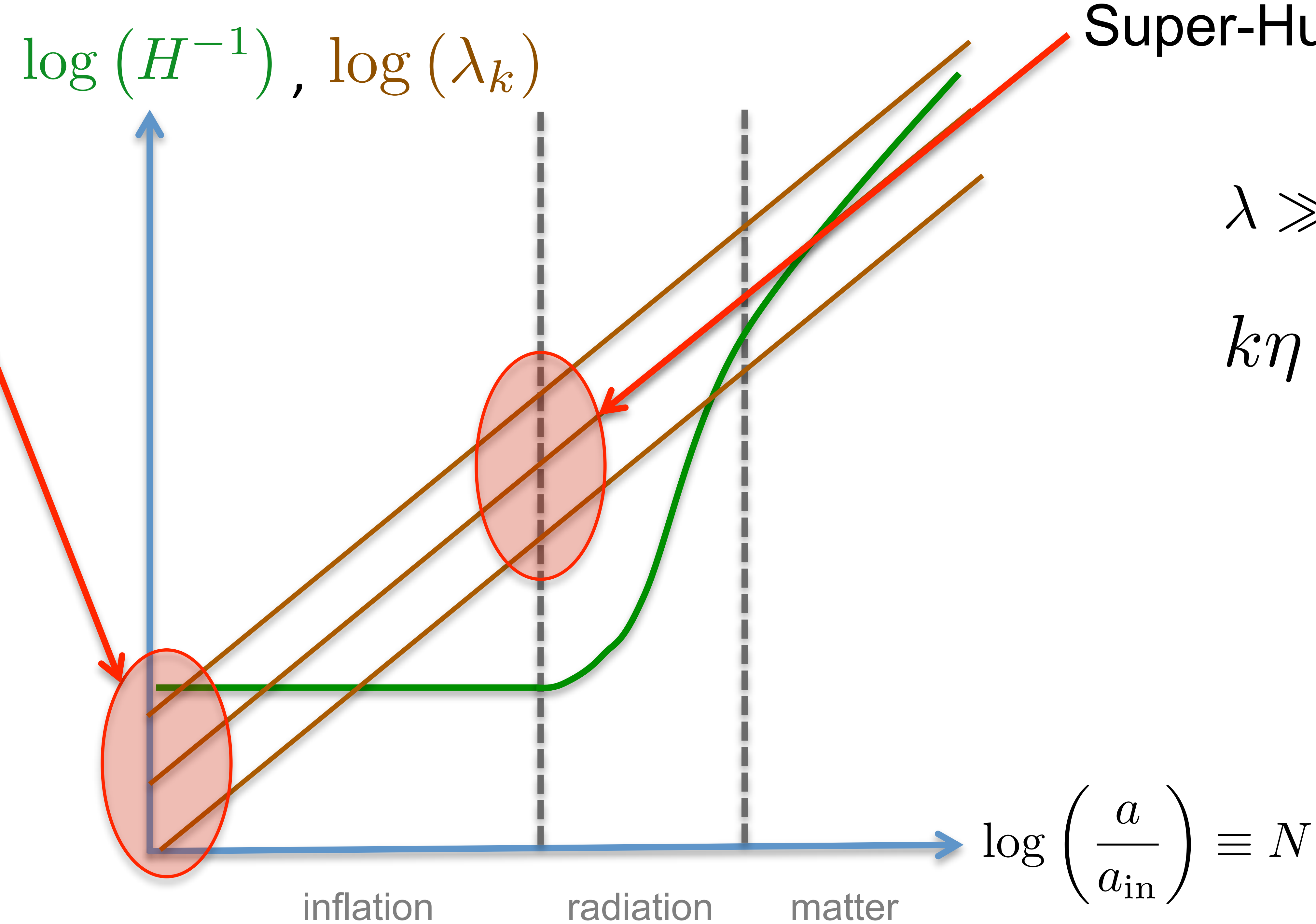
$k\eta \to -\infty$

$\omega \simeq k$

harmonic oscillator

$$\Psi_{\boldsymbol{k}} = \left(\frac{k}{\pi}\right)^{\frac{1}{4}} \mathrm{e}^{-\frac{k}{2}v_{\boldsymbol{k}}^2}$$

Bunch Davis vacuum



Super-Hubble regime

$\lambda \gg H^{-1}$

$k\eta \to 0^-$

sets initial conditions $f_{\boldsymbol{k}}(k\eta \to -\infty) = \mathrm{e}^{ik\eta}/\sqrt{2k}$

Primordial Power Spectrum

Standard case

$\boxed{f''_{\boldsymbol{k}} + \omega^2(\boldsymbol{k},\eta) f_{\boldsymbol{k}} = 0}$ with $\omega^2(\boldsymbol{k},\eta) = k^2 - \frac{\beta(\beta+1)}{\eta^2}$ and $f_{\boldsymbol{k}}(k\eta \to -\infty) = \mathrm{e}^{ik\eta}/\sqrt{2k}$

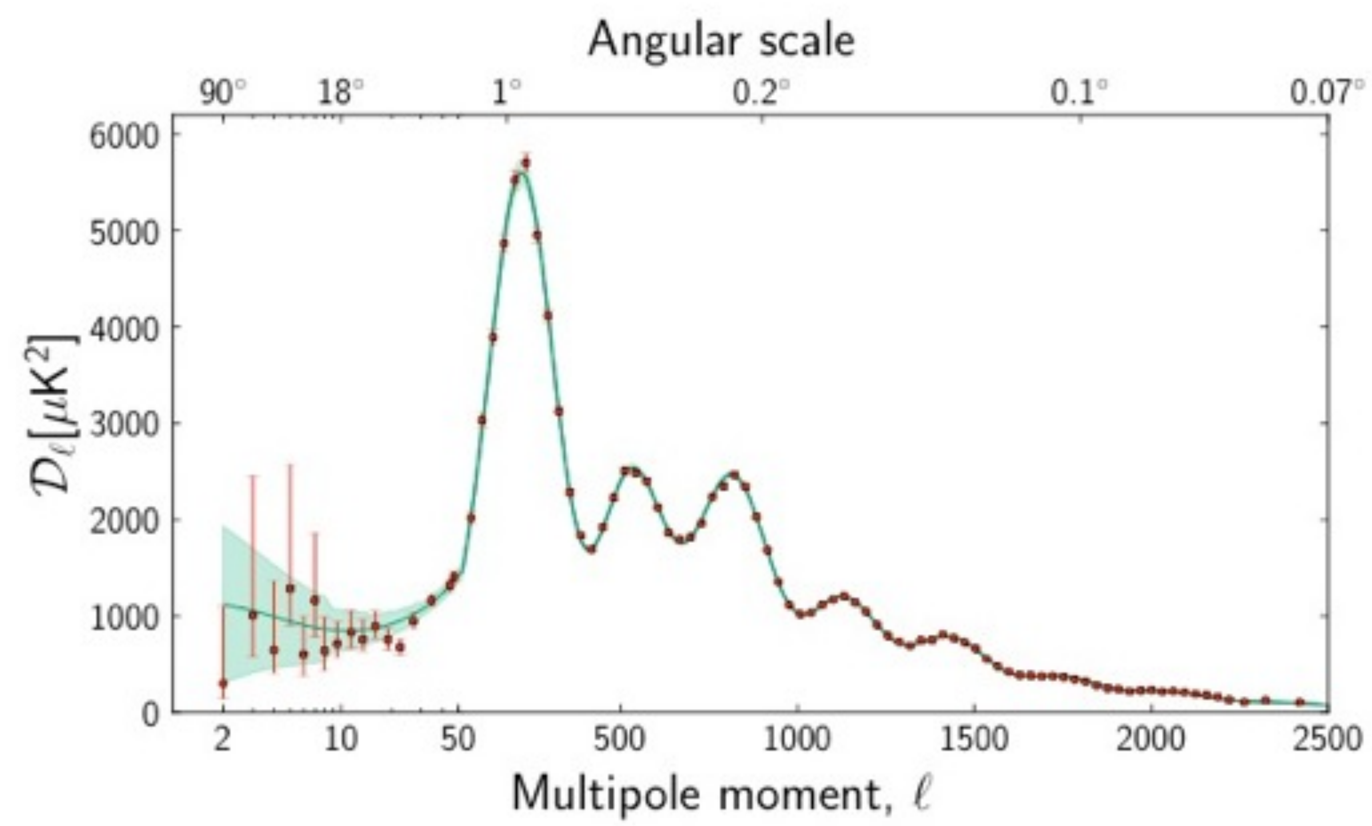
Uniquely determines $f_{\boldsymbol{k}}$ $\xrightarrow{\Omega_{\boldsymbol{k}} = -\frac{i}{2}\frac{f'_{\boldsymbol{k}}}{f_{\boldsymbol{k}}}}$ $\Re\mathrm{e}\,\Omega_{\boldsymbol{k}} = \langle \hat{v}_{\boldsymbol{k}}^2 \rangle - \langle \hat{v}_{\boldsymbol{k}} \rangle^2$

Evaluated at the end of inflation($k\eta \to 0^-$), this gives $P_v(k) = \frac{k^3}{2\pi^3}\left(\langle \hat{v}_{\boldsymbol{k}}^2 \rangle - \langle \hat{v}_{\boldsymbol{k}} \rangle^2\right)$

and eventually $P_\zeta(k) = \frac{1}{2a^2 M_{\mathrm{Pl}}^2 \epsilon_1} P_v(k) = A_S k^{n_\mathrm{S}-1}$

with $n_\mathrm{S} = 2\beta + 5 \underset{\beta\sim -2}{\simeq} 1$

Planck: $1 - n_\mathrm{s} = 0.0389 \pm 0.0054$



Primordial Power Spectrum

Modified Theory

Modified Schrödinger equation

$$\mathrm{d}|\Psi_{\boldsymbol{k}}\rangle = -i\hat{\mathcal{H}}_{\boldsymbol{k}}\,|\Psi\rangle\mathrm{d}\eta + \sqrt{\gamma}\left(\hat{v}_{\boldsymbol{k}} - \langle\hat{v}_{\boldsymbol{k}}\rangle\right)\mathrm{d}W_{\eta}\,|\Psi_{\boldsymbol{k}}\rangle - \frac{\gamma}{2}\left(\hat{v}_{\boldsymbol{k}} - \langle\hat{v}_{\boldsymbol{k}}\rangle\right)^2\mathrm{d}\eta\,|\Psi_{\boldsymbol{k}}\rangle$$

Extended Gaussian wave function

$$\Psi_{\boldsymbol{k}}\left(\eta, v_{\boldsymbol{k}}\right) = \left[\frac{2\,\Re\mathrm{e}\Omega_{\boldsymbol{k}}(\eta)}{\pi}\right]^{1/4}\exp\left\{-\Re\mathrm{e}\Omega_{\boldsymbol{k}}\left(\eta\right)\left[v_{\boldsymbol{k}} - \bar{v}_{\boldsymbol{k}}\left(\eta\right)\right]^2 + i\sigma_{\boldsymbol{k}}(\eta) + i\chi_{\boldsymbol{k}}(\eta)v_{\boldsymbol{k}} - i\Im\mathrm{m}\Omega_{\boldsymbol{k}}(\eta)\left(v_{\boldsymbol{k}}\right)^2\right\}$$

Modified equation of motion

$$\Omega_{\boldsymbol{k}}' = -2i\Omega_{\boldsymbol{k}}^2 + \frac{i}{2}\omega^2(\eta,\boldsymbol{k}) + \gamma \quad \xrightarrow{\Omega_{\boldsymbol{k}} = -\frac{i}{2}\frac{f_{\boldsymbol{k}}'}{f_{\boldsymbol{k}}}} \quad f_{\boldsymbol{k}}'' + \left[\omega^2(\eta,k) - 2i\gamma\right]f_{\boldsymbol{k}} = 0$$

Primordial Power Spectrum

Modified Theory

$$f_{\boldsymbol{k}}'' + \left[k^2 - \frac{\beta(\beta+1)}{\eta^2} - 2i\gamma\right] f_{\boldsymbol{k}} = 0$$

Ability to fix Bunch Davis vacuum as an initial condition ?

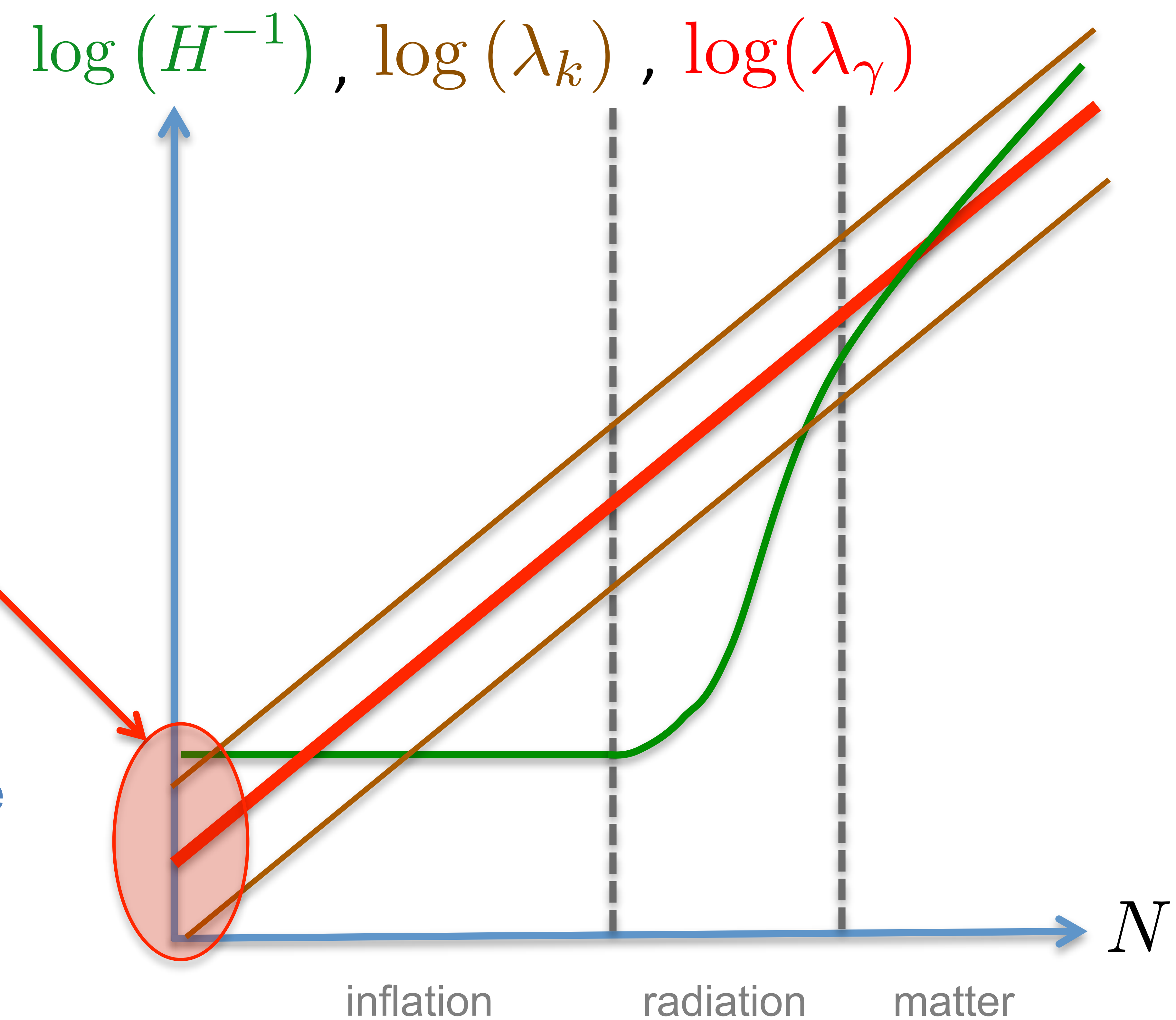
$$\omega^2(\eta, k) \underset{k\eta \to -\infty}{\longrightarrow} k^2 - 2i\gamma$$

$$f_{\boldsymbol{k}} \underset{k\eta \to -\infty}{=} A_{\boldsymbol{k}} \mathrm{e}^{i\omega\eta} + \cancel{B_{\boldsymbol{k}} \mathrm{e}^{-i\omega\eta}}$$

Bunch Davis vacuum

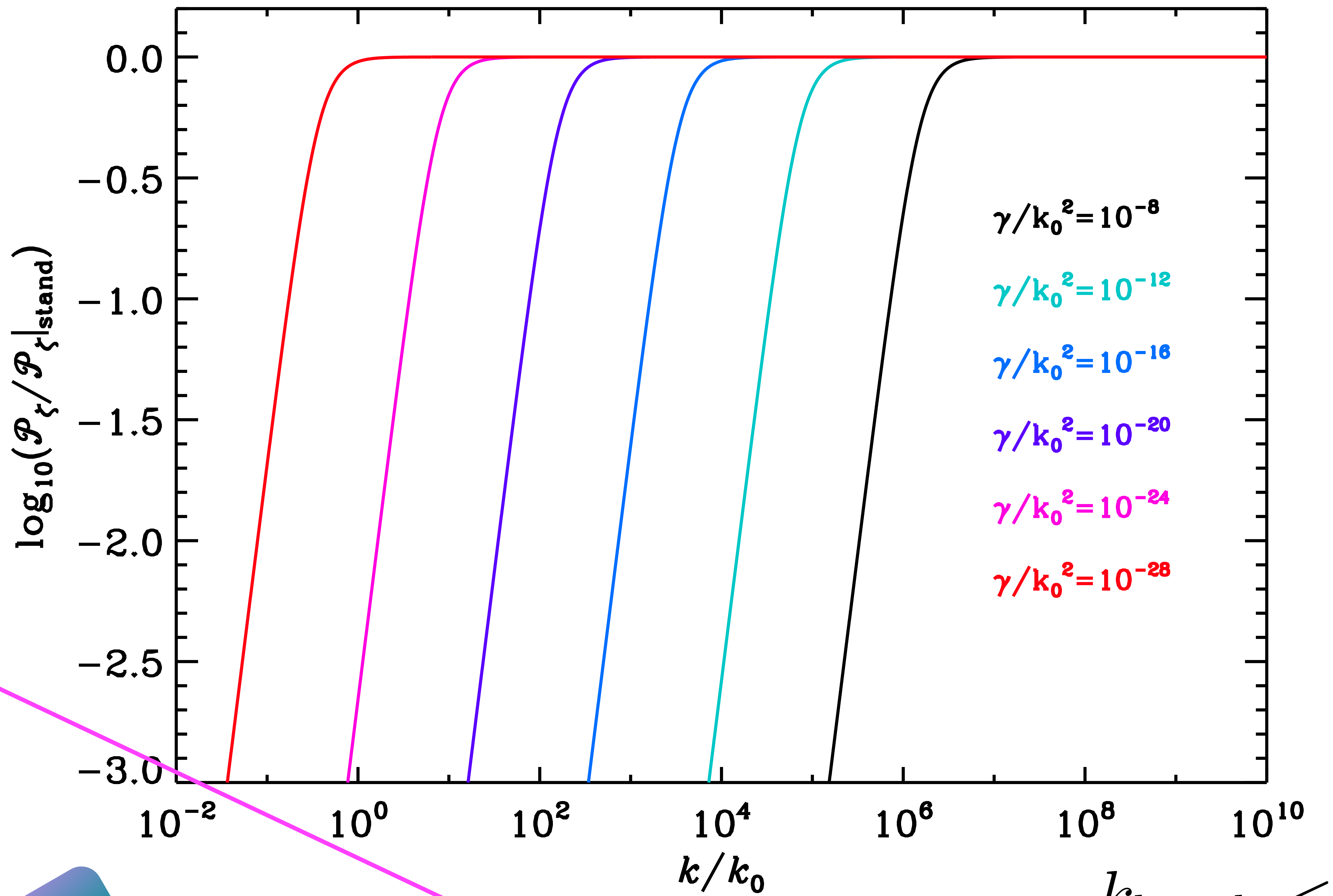
leads to non normalizable wavefunction

$(\Re\mathrm{e}\, \Omega_{\boldsymbol{k}} < 0)$



Primordial Power Spectrum

Modified Theory



comoving Hubble wavenumber now

$\ell_\gamma \gg 10^{13}\ell_{\rm H}$

$k_{\rm break} < k_0$

$$\boxed{\frac{\gamma}{k_0^2} \ll {\rm e}^{-\Delta N_*} \simeq 10^{-28}}$$

$k < k_{\rm br}\ldots = 4$

$k > k_{\rm break} : n_{\rm S} = 2\beta + 5 \simeq 1$

Conclusions (1)

Quantum measurement problem very severe in cosmology

Test?
(non equilibrium...)

Two possible extensions of QM can be used
(Born rule not set by hand)

- dBB ontology
- Spontaneous collapse

Constraint on γ

- collapse time ✓
- final spread

Plenty of new effects awaiting to be discovered/understood...