P2IO report

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Project goals

The work described here is associated with the project QEAGE (Quantum Effects in Analogue Gravity Experiments).

Analogue gravity allows the use of experimentally accessible systems – whose dispersive and/or dissipative behaviour is well understood – to mimic wave propagation in curved spacetime. When quantum fields are introduced into such a system, certain types of inhomogeneities in the spacetime can lead to the spontaneous creation of entangled (quasi-)particles, by analogy with the following effects known from the combination of General Relativity and Quantum Field Theory:

- cosmological pair creation, an instance of the *Dynamical Casimir Effect*, which is due to variations of the metric in time; and
- black hole radiation, or the *Hawking Effect*, which is due to variations of the metric in space.

In the gravitational context, these effects are somewhat mysterious in origin, for in the standard theory the observed (quasi-)particles originate in field modes of such short wavelengths that we cannot justify the application of standard theory; this is the *trans-Planckian problem* [1]. Analogue systems, through dispersion and/or dissipation, are not subject to this issue, and thus provide a means of testing the true role of the high-frequency regime in the two effects given above.

Our project is to investigate the Dynamical Casimir Effect and Hawking Effect in $1+1$-dimensional analogue systems, with particular emphasis on the deviations from standard theory due to dissipative effects. (There is already a substantial literature examining the Hawking Effect for a dispersive field, see e.g. [2] for a review; it has been found to be quite robust.)

Work achieved

Dynamical Casimir Effect

In cosmological pair creation and its analogues, the system parameters vary monotonically in time in imitation of the expansion of the universe. However, another method to enduce pair creation (which can be more efficient) is to consider a sinusoidal oscillation of the system parameters, described by its own frequency ω_p. The system then behaves like a parametric amplifier: the field degrees of freedom whose natural frequency is near $\omega_p/2$ are in resonance with the modulation, and their particle content increases exponentially with time. Meanwhile, those field degrees of freedom that are out of resonance oscillate in time, with a frequency and amplitude that depend on how far they are detuned from the resonant frequency $\omega_p/2$.

Parametric amplification is a classical effect, and any initial content of the resonant field modes will themselves be amplified in a classical manner. This, however, is in addition to amplification of vacuum fluctuations, a purely quantum effect that spontaneously generates particles out of an initial vacuum state. Distinguishing stimulated and spontaneous creation is a difficult but necessary step if we are to be sure that the observed effects are quantum in origin. One way of doing this is to consider a measure of the separability of the final state. A separable state can be described by a classical ensemble; a non-separable state, however, contains pairs of particles so strongly correlated that it cannot be described in such a way. From the number n of created particle pairs and the strength of the correlations c between the members of each pair – both of which are standard observables – it is possible to construct a quantity, $\Delta = n - |c|$, which is positive for separable states and negative for non-separable states [3].

We considered the modulated Dynamical Casimir Effect from the point of view of this separability parameter [4]. We assumed an initial thermal state fully described by its temperature. No matter how high this initial temperature, we found that, near the resonant frequency $\omega_p/2$, the state eventually becomes non-separable if the modulation lasts long enough. The behaviour for off-resonant frequencies is a little more complicated, with the separability parameter undergoing an oscillatory behaviour much as the number of particles does. If the initial temperature is too large, the state of these off-resonant modes always remains separable; whereas, for temperatures below some critical value, the state alternates between separable and non-separable. See the left panel of Figure 1 for an illustration of this behaviour.

It is to be expected that the exponential growth at resonance will be limited in practice, particularly by dissipation – the irreversible loss of energy to an environment. Based on a previously formulated model of dissipation in which the system as a whole (the field of interest plus the environment) evolves unitarily [3], we included some weak dissipation (of rate Γ) to see how, at resonance, the final number of particles and degree of non-separability are affected. To each of these quantities is associated a critical Γ which delineates different types of behaviour. On the one hand, the number of created particles increases exponentially if the dissipation rate is below some critical value Γ_n, and approaches a finite limit for $\Gamma > \Gamma_n$; this Γ_n is found to be independent of the initial temperature. On the other hand, there is another critical dissipation rate, Γ_Δ, such that when $\Gamma < \Gamma_\Delta$ the state eventually becomes non-separable, and when $\Gamma > \Gamma_\Delta$ the state always remains separable; this Γ_Δ does depend on the initial temperature. Furthermore, it can be very small, so that even weak dissipation can be enough to destroy the non-separability of the final state.

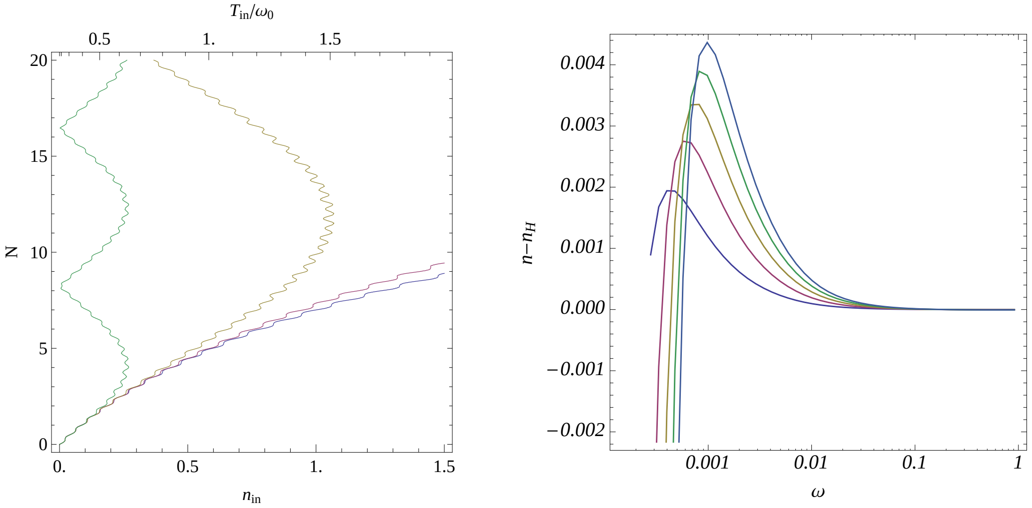

Figure 1: LEFT: For the modulated DCE, in the plane of temperature (or equivalently, initial occupation number) v. the number of oscillations (that is, the duration of the modulation) are shown the loci separating separable and non-separable states. The different curves representing different amounts of detuning from the resonance at $\omega_p/2$, with the blue curve being at resonance and the green being far from resonance. RIGHT: Regarding the Hawking Effect with constant rate of dissipation, for various values of $\Gamma = 1 - 5 \times 10^{-6}$ (in units of κ, the gradient at the horizon) is shown the difference between the calculated spectrum and the thermal Hawking spectrum. The left panel appears in Ref. [4]; the right panel is as yet unpublished.

Hawking Effect

Black hole (Hawking) radiation arises due to the anomalous scattering properties of event horizons, where the velocity of one of the waves vanishes. In the standard theoretical treatment (which is also applicable to dispersive waves), it is derived from elements of the S matrix that describes the evolution of ingoing modes into outgoing modes. In the presence of dissipation, however, these are no longer well-defined, and a new, more general computation technique must be adopted. As mentioned above in the context of the Dynamical Casimir Effect, we begin by considering a closed system of field plus environment which evolves unitarily [3]. On tracing out the environment degrees of freedom, we are left with a dissipative field sourced by a stochastic noise kernel dependent on the state (usually taken to be thermal) of the environment. The response of the field is described by a Green's function, and its state is fully described by its anticommutator, which is given by an integral over the Green's function and the noise kernel. Since the latter calculation must almost always be done numerically (at least in part), it is convenient to work with models which yield simple Green's functions that can be found easily. We have and are considering several such models, each with its own advantages and disadvantages, with computational ease and physical relevance being the two main (and often diametrically opposed) criteria. A secondary aim of our work is to compare the results of the various models, distinguishing model-dependent effects from those due generically to the presence of dissipation.

The simplest Green's function occurs for a non-dispersive field, which for any given frequency ω has only two components: a right-propagating part (denoted by u) and a left-propagating part (denoted by v). The mathematical properties of such a Green's function have been studied in some detail. It is particularly simple when Γ is constant, for then the u- and v-modes are completely decoupled, the Green's function splitting into a sum of two independent components, only one of which is sensitive to the presence of any given horizon (the u-modes, in our case) and which is therefore relevant to the black hole radiation. So long as $\omega \gg \Gamma$, we find that this model reproduces the standard Hawking spectrum very well. The deviation of the spectrum from the thermal Hawking spectrum is shown in the right panel of Figure 1. (Note that the low frequencies with $\omega \lesssim \Gamma$ are not considered physically relevant, since it is not clear how to apply a particle interpretation to a field which is so heavily damped.)

A model which is believed to be quite relevant to the physical universe is one in which Γ is proportional to k^2, where k is the wave vector. This reproduces an essentially non-dissipative model in the long-wavelength regime where field theory works well, and allows strong dissipation in the unknown short-wavelength regime. Only recently have we seen how to put this model into a workably simple form. We have, however, attempted to mimic it by allowing an x-dependent dissipation rate $\Gamma(x)$, which is small far from the horizon and large near the horizon. At first, we did not attempt to ensure the decoupling of u- and v-modes, and we found that, while at high frequencies these effectively decouple, at low frequencies (though still at $\omega \gg \Gamma_{\mathrm{as}}$, where by Γ_{as} we mean the asymptotic value of $\Gamma(x)$) an incident v-mode stimulates the emission of u-modes, one on either side of the horizon. The state of the u-modes was thus found to correspond closely to the thermal Hawking prediction at high frequencies, and to be in the same thermal state as the incident v-modes at low frequencies. We have recently decided to alter this model such that Γ remains a function of x, but such that u- and v-modes do not mix.

As well as setting up the u-v decoupled model with x-dependent Γ and the simplified model with $\Gamma \propto k^2$, our next task is to consider the correlation term between u-modes emitted on either side of the horizon, and thus to find the separability parameter Δ described above (in the context of the Dynamical Casimir Effect) and its dependence on Γ.

Publications

This work has so far yielded the following publication in relation to the modulated Dynamical Casimir Effect [4]:

- Quantum entanglement due to modulated Dynamical Casimir Effect, *Phys. Rev. A* **89**, 063606 (2014)

Relevance within P2IO

The field of analogue gravity, and in particular its quantum effects, is an attempt to probe and understand the more elusive features resulting from the fusion of General

Relativity and Quantum Field Theory, which well describe, respectively, the large-scale and the small-scale structure of the universe. While it cannot provide us with a theory of Quantum Gravity, it forces us to resolve the compatibility issues between GR and QFT that arise within specific analogue settings, and thus yields examples of how this resolution can be achieved.

Analogue black hole radiation is, of course, closely connected with the Hawking radiation from astrophysical black holes, where historically it was first discovered. The Dynamical Casimir Effect is essentially the same effect as cosmological pair creation due to the rapid expansion of the universe during inflation: the amplification of vacuum fluctuations is believed to account for the matter content of the universe and the inhomogeneities in the Cosmic Microwave Background.

Interaction with other laboratories

There has as yet been little interaction with other laboratories, though there are plans for such interactions in the near future. An experimental team at the Institut d'Optique led by Chris Westbrook work with atomic Bose-Einstein condensates, and have already performed an experiment on the modulated Dynamical Casimir Effect in which the final state was found to be separable [5]; using our theoretical work, we hope to be able to advise them on an experiment to yield a non-separable state. We have had and shall continue to have discussions with a team at the Laboratoire de Photonique et de Nanostructures led by Alberto Amo, who work with polariton fluids in optical cavities, in the hope of creating an analogue black hole in such a system; this possibility was explored theoretically in Ref. [6].

References

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