P2IO research activity report

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PhD: Compactifications of String Theory and the observable world

Project goals

Consistent superstring theories live in 10 space-time dimensions. In order to make contact with our four-dimensional world, we assume that the additional dimensions form a compact space of a size inaccesible in a direct way by current experiments, but whose geometry has important effects on the four-dimensional physics. The way of dealing with these extra dimensions is called compactification.

In a first step, we look for compactifications that preserve some amount of supersymmetry. The latter is broken later by other effects at a lower energy scale. The geometry of the six-dimensional internal manifold depends on the amount of supersymmetry preserved and on the vacuum expectation values (VEVs) of the ten-dimensional fluxes. The case of study is the one where eight real supercharges are preserved and the non-zero fluxes are all the possible ones allowed by the four-dimensional symmetry (Poincaré).

The compactification procedure is not necessarily restricted to the case where we have four large (external) dimensions. Compactifications to five dimensions are crucial in the study of the AdS/CFT correspondence. We also study this case where the supersymmetry is again eight real supercharges, all the fluxes are turned on and the external space is AdS.

The appropriate mathematical framework for describing both cases is *exceptional generalized geometry*. In this framework, all the ten-dimensional fields are given a geometric description via algebraic structures on a larger space. The group of dualities (transformations between the metric and the gauge fields) acts on this space. In the cases we study, the duality groups are the exceptional groups (and thus the name exceptional generalized geometry) $E_{7(7)}$ for the four-dimensional case and $E_{6(6)}$ for the five-dimensional case.

The moduli (massless scalar particles) of the effective theories organize themselves in objects that transform under certain representations of the exceptional groups. It is conjectured that these objects satisfy some integrability conditions. The goal of our project is to prove these conditions using the equations that come from the supersymmetry requirement and to examine whether it is possible to find a unifying language to treat both cases.

Description of work achieved

We start with the $E_{6(6)}$ case performing our calculations in type IIB string theory. According to the compactification scheme the ten-dimensional space-time “splits” in the (warped) product of a five-dimensional Anti de Sitter (AdS) space and a five-dimensional internal compact manifold. Taking into account the symmetries of the AdS space, we find the components of the ten-dimensional fields that can acquire VEVs.

The supersymmetry variations of the fermions should vanish on the supersymmetric ground state of the theory. Requiring this, we obtain a set of 3 equations (2 algebraic, 1 differential) of the schematic form

$$\underbrace{\nabla\chi}_{\substack{\text{present only}\\\text{in 1 equation}}} \quad + \quad (\text{fluxes})\chi = 0 \tag{1}$$

where χ is some internal spinor. Two of these equations come from the supersymmetry variation of the gravitino (internal and external components) and one from the dilatino variation.

The next step is to construct the appropriate structures that describe the geometry as well as the gauge fields, and which should be "closed" in some generalized sense. In theories with eight supercharges, we expect to have two algebraic structures, one for the vector multiplets and one (or rather a triplet, satisfying the $SU(2)$ N=2 algebra) for the hypermultiplets. The former structure (K) is in the vector representation (27-dimensional) of $E_{6(6)}$ and the others ($J_a, \quad a = 1,2,3$) form an $SU(2)$ triplet in the adjoint representation (78-dimensional) of $E_{6(6)}$. K and J_a can be constructed as bilinears of the internal spinors, schematically as follows

$$K = \epsilon_{IJ}\ \chi^I \otimes \chi^J \tag{2}$$

$$J_a = (\sigma_a)_I{}^J\ \chi^I \otimes \bar{\chi}_J \tag{3}$$

where $I, J = 1,2$ are fundamental $SU(2)_R$ indices and σ_a are the Pauli matrices.

The equations that express the integrability of the exceptional structures are conjectured to be

$$\mathcal{D}J_a + \epsilon_{abc}Tr(J_b\mathcal{D}J_c) \stackrel{?}{=} \lambda_a c(K) \tag{4}$$

$$(K \cdot \mathcal{D})J_a + (D \times K)J_a \stackrel{?}{=} \epsilon_{abc}\lambda_b J_c \tag{5}$$

In these equations D is the covariant derivative, which is an object in the dual vector representation of $E_{6(6)}$, $c(K)$ is some invariant of the group and the parameters λ_a should be related to the AdS radius. The first goal of my thesis is to verify (or disprove) the question mark above the equality symbol, and to find the relation between λ_a and the AdS radius.

In order to prove equations 4 and 5, we are using the supersymmetry condition 1. We have seen schematically that the equations are satisfied, but we still need to complete the proof. In this way, we will have achieved the *covariantization* of the supersymmetry condition in $E_{6(6)}$ language. After having completely verified the conjectured equations, we want to study the deforormations of the structure J_a and K such that the equations are still satisfied. These deformations amount to massless scalars in the five dimensional theory, or in other words marginal deformations of the gauge theory.

In the case where there are six internal dimensions, the relevant group is $E_{7(7)}$. The fundamental representation of $E_{7(7)}$ is 56-dimensional while the adjoint representation is 133-dimensional (dimension of the group). The supersymmetry condition has the same form as in 1 and the exceptional structures are constructed as in 2 and 3 in terms of the new internal spinors. Again, the goal in this case is to prove equations similar to 4 and 5. The general procedure which we follow in this case is the same as for $E_{6(6)}$, however the details are different ($E_{n(n)}$ exceptional grous are very different for different n, unlike the ordinary $GL(n)$ or $SO(n)$ groups where subgrous, algebra, etc can be worked out for a generic n). Most of components have been verified already.

In both cases, proving equations 4 and 5 is a non-trivial issue. In order to perform explicit calculations, one has to "split" them in components and verify each of them separately. This "splitting" is done by identifying the appropriate subgroup of $E_{6(6)}$ and $E_{7(7)}$ ($USp(8)$ and $SL(8,\mathbb{R})$ respectively) which we use in order to assign indices to the various objects in the equations.

Moreover, since internal spinors transform under different subgroups than the other geometric quantities, we have to find the appropriate transformations between the two different subgroups. These details have been worked out in our project.

In order to describe better the above procedure, let us consider Maxwell's equations in their relativistic form and let us suppose that we want to prove them using their non-relativistic expression. The procedure we would follow then is divided in two steps. In the first one, we would find which components of the electromagnetic field-strength tensor should be identified with the electric and magnetic field. In the second step, we would split the relativistic equations in space- and time-parts and we would try to verify them separately. The original equation is $SO(1,3)$-covariant, while the (manifest) symmetry in the end is just $SO(3)$.

Of course, in our project this procedure becomes much more complicated because of the complicated structure of the exceptional groups. However, the spirit is the same. Since $E_{6(6)}$ or $E_{7(7)}$ are the U-duality groups (the groups that include all the "stringy" symmetries), we attempt to write the equations of (compactified) string theory in a way covariant under these groups. For the case of the supersymmetry condition, these equations should take the form 4 and 5. Then, the above two-step procedure is followed. The first step (embedding of the non-covariant objects in the covariant ones) has been done, while a lot of progress has also been done in the second step (proving the equations).

Publications

M.Grana and P. Ntokos, "Generalized geometric vacua with eight supercharges", to be submitted.

Relevance of the project within P2IO

String theory is considered to be the best candidate for a unified theory of all phenomena in nature, from the infinitely small (particle physics) to the infinitely large (cosmology), a goal which is incorporated already in the name of P2IO. To make direct contact with experiments we must, however, know the precise way in which the Standard Model and Einstein's gravity are embedded as low-energy limits in string theory. In a broad sense, this is the research topic of the present thesis.

In the case of five-dimensional compactifications, studying the geometry of the internal manifold would give important information (through the AdS/CFT duality) for the gauge theory which lives on the boundary. In our project, we are looking for a general and elegant formalism which could describe cases where the symmetries of the internal manifold can be broken. This could lead to breaking conformal invariance on the other side of the duality and therefore to more interesting gauge theories from a phenomenological point of view.

Both of the aforementioned directions of this thesis include ideas like supersymmetry and extra dimensions of space-time, which are very popular in models of physics beyond the Standard Model. Moreover, the formalism that we use in our project is based on certain symmetries that can survive in the low-energy limit of string theory. Therefore, the project makes direct contact with the symmetries of the subatomic world, which is included in the thematics of P2IO.