Collective modes in the inner crust of a neutron star

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Activity report 2013-2014

1 Project goals

During the last years, astronomical observations have become more and more numerous and precise, and neutron stars can now be considered as laboratories for nuclear physics under extreme conditions. Among theses observations, the cooling is very relevant to constrain models of the neutron-star crust, imposing also strong constraints on nuclear-physics models. The cooling depends to a large extent on the specific heat of the crust, which in turn depends on the low-energy collective excitations. The inner crust is expected to be a crystalline lattice of neutron-rich nuclei (clusters) immersed in a gas of unbound neutrons. The existing microscopic calculations were limited to excitations with a wavelength smaller than the size of the unit cell of the lattice.

The aim of this thesis is to extend a recently developed semi-classical method for the low-energy (long-wavelength) collective modes in the so-called "lasagna" phase to the crystalline lattice and "pasta" phases with different geometries, which requires a numerical solution of the hydrodynamic equations. The parameters of the model such as the equation of state (EoS) and the surface tension between the clusters and the gas will be obtained from microscopic calculations. Once the spectrum of excitations obtained, we will be able to compute the specific heat and the cooling of the neutron stars.

2 Description of work achieved

2.1 Quasiparticle random-phase approximation in a uniform neutron gas

The neutron star crust contains a dilute gas of unbound neutrons between the nuclear clusters, consequently we started our work by the study of it. Because of the low temperature of the crust (~100 keV) far below the superfluid critical temperature (~1 MeV), we expect the neutron gas to be superfluid. This implies the existence of a low-energy collective mode, also known as Goldstone or Bogoliubov-Anderson mode. While the neutron quasiparticles are gapped and their contribution to the specific heat is therefore exponentially small, the collective mode gives an important contribution to the specific heat.

We started our work taking into account only the particle-particle (pairing) interaction, which is responsible for the superfluid properties. In order to simplify the analytical and numerical computations, we used a separable potential fitted to a low-momentum nucleon-nucleon interaction. The time-dependent Hartree-Fock–Bogoliubov (TDHFB) theory treats the dynamics of physical quantities describing a paired quantum system, such as our cold and paired neutron gas. This formalism treats also non-linear effects, but we restricted ourselves to small-amplitude oscillations around the ground state. This assumption allowed us to expand the TDHFB equations to the first order and recover the quasiparticle random-phase approximation (QRPA).

As mentioned before, there must be a Goldstone boson because the pairing field breaks the U(1) symmetry. In the case of a superfluid, the Goldstone mode is a phase oscillation of the superfluid order parameter, corresponding to a density wave (sound wave). We have seen that this Goldstone mode is found within the QRPA and that its dispersion relation at long wavelength is to a good approximation linear in the wave vector, the slope corresponding to the speed of sound of the non-interacting Fermi gas.

To improve this calculation, we included also the interaction in the particle-hole channel. We derived our particle-hole interaction from the Skyrme energy-density functional (EDF). We introduced a new formalism to describe the complete set of response functions describing the neutron gas. Actually, the full Skyrme QRPA calculation had not been performed before, and existing calculations were limited to the so-called Landau approximation. The Landau approximation assumes that the excitations occur near the Fermi level, which is only true when the momentum transfer is much smaller than the Fermi momentum.

We have confirmed that with the inclusion of the particle-hole interaction, the slope of the dispersion relation of the Goldstone mode coincides with the hydrodynamic speed of sound as calculated from the full Skyrme EoS. In this way we have shown that (at least at zero temperature) the microscopic QRPA can be approximated by much simpler hydrodynamic equations as long as one is interested in excitations with long wavelengths and low energies. This finding will be the basis for our study of collective modes in the inhomogeneous case (crystalline and pasta phases) within the hydrodynamic approach.

An interesting aspect of our treatment within the QRPA formalism is that we can also calculate the deviations of the dispersion relation of the collective mode from the hydrodynamic (linear) one. The deviation is a consequence of the coupling of the collective mode to two-quasi-particle states. When approaching the pair-breaking energy, the dispersion relation of the collective mode starts to bend, approaching slowly the pair-breaking threshold. At even higher momenta, it enters the two-quasi-particle continuum where the collective mode becomes damped.

We also compared our full QRPA response functions with those obtained within the Landau approximation, as it was used in the existing literature. Note that, in the presence of an effective mass, it is necessary to go beyond the zeroth-order Landau approximation in order to respect Galilean invariance. Not surprisingly, the response functions obtained within the full QRPA calculation and those obtained within the Landau approximation are very different except in a small range of small momentum transfers. However, the dispersion relations of the collective mode obtained from both computations are very close, i.e., even if the response function is incomplete, because of some simplifications, the pole of the response function is not strongly modified.

We mentioned that at higher momenta the dispersion relation of the collective mode is quite different between hydrodynamics and QRPA. However, in the relevant temperature range the results concerning the specific heat are very similar, because mainly low momenta and energies contribute to the specific heat. Therefore, the hydrodynamical approach stays quite efficient to describe the heat capacity of our medium.

2.2 Clusters and gas as phase coexistence

After this study of the uniform neutron gas, we turned to the description of the neutron-star crust containing neutron gas, clusters, and electrons. Because we want to use the hydrodynamic approach, the ground state should correspond to a hydrostatic equilibrium (i.e., pressure and chemical potentials in the gas and in the clusters must be equal). As interaction, we use again different parametrization of the Skyrme EDF. Imposing charge neutrality, beta equilibrium and phase coexistence, we obtain very reasonable results for the EoS of the crust, the proton fraction, and the volume fraction of the clusters.

However, these conditions are not sufficient to determine the structure of the crust. The size of the clusters and their geometry have to be determined from a minimization of Coulomb and surface energy. We checked that the Wigner-Seitz approximation to the Coulomb energy is very accurate in all cases of interest. The surface energy depends mainly on the surface tension. In the hydrodynamic approach, the surface tension is a microscopic input parameter, like the microscopic EoS. Unlike the surface tension of ordinary nuclei, the surface tension of a cluster immersed in a neutron gas is not empirically known.

In order to determine the surface tension, we follow a method established in the literature, namely to parametrize the density profile across the surface and determine its parameters (diffuseness of the surface and thickness of the so-called “neutron skin”) by minimizing the surface energy obtained with the Skyrme EDF within the extended Thomas-Fermi (ETF) approximation. What makes the calculation quite involved is that in the case of a curved surface, the surface tension leads also to a small difference between the pressure inside the cluster and the pressure of the surrounding gas (Young-Laplace formula).

This study is still work in progress.

2.3 Other activities

In addition to my research activities, I was also involved in a public outreach activity initiated by T. Foglizzo (IRFU/SAp). From January to February 2014, I made several presentations at the stand “Un chercheur, une manip: La fontaine aux supernovae” in the museum “Palais de la Découverte”, where I presented a small experiment (invented by T. Foglizzo) showing in flowing water an instability which is analogous to an instability that might appear during supernova explosions.

3 Publications

3.1 Preprint

- N. Martin and M. Urban, "Collective Modes in a Superfluid Neutron Gas within the Quasiparticle Random-Phase Approximation", eprint arXiv:1406.0335 (submitted to Phys. Rev. C).

3.2 Conference contributions

- N. Martin, "Collective modes in superfluid neutron matter", talk given at the SN2NS meeting, Paris, February 3-5, 2014.
- N. Martin, "Collective modes in superfluid neutron matter", talk given at the MODE workshop "Pulsars and their environment", IPN Lyon, May 19-21, 2014.
- M. Urban, N. Martin, and M. Oertel, "Collective modes in the inner crust of neutron stars", talk given at FUSTIPEN workshop "Structure of the neutron star crust: experimental and observational signatures", GANIL, Caen, May 26-27, 2014.

4 Relevance of the project within P2IO

This project shows very nicely the connection of the "physics of the two infinities", namely of very small (nuclear physics) and very large (astrophysics) scales. Naturally, the IPN theory group is mainly interested in the microphysics aspects, but already a few years ago a couple of members of the IPN theory group started to work on astrophysical questions. Since then, a regular and active exchange between this part of the IPN theory group and the group of T. Foglizzo at IRFU/SAp as well as the ROC (relativity and compact objects) group at LUTH (Observatoire de Paris, Meudon) has developed. For instance, regularly common journal clubs and workshops are organized. The IRFU/SAp group is, however, mainly interested in supernovae and not in neutron stars. This is why the IPN is the only P2IO laboratory which is directly involved in the project. Nevertheless the goals of our project fit perfectly the goals of P2IO as our results for the neutron-star cooling will hopefully allow us to better constrain nuclear physics models with the help of astrophysical observations, or, vice versa, to improve our understanding of astrophysical observations with the help of microscopic models.