

Basics of Neutrino Physics

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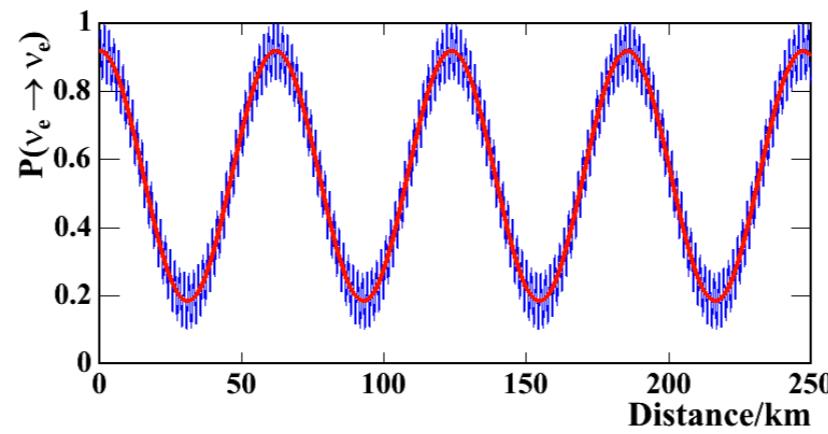
Lecture 2

Neutrino Oscillations

"In as far as the neutrino masses are negligible compared to the charged lepton masses, the observable effects of leptonic mixing angles are limited to fairly exotic effects such as neutrino oscillations."

Froggatt and Nielsen, 1978

1. Neutrino Oscillations in Vacuum



First Ideas

1957 - B. Pontecorvo suggested $\nu \rightarrow \bar{\nu}$

oscillations in analogy to $K^0 \rightarrow \bar{K}^0$ ones

1962 - Flavor transitions $\nu_e \rightarrow \nu_\mu$

considered by Maki, Sakata and Nakagawa



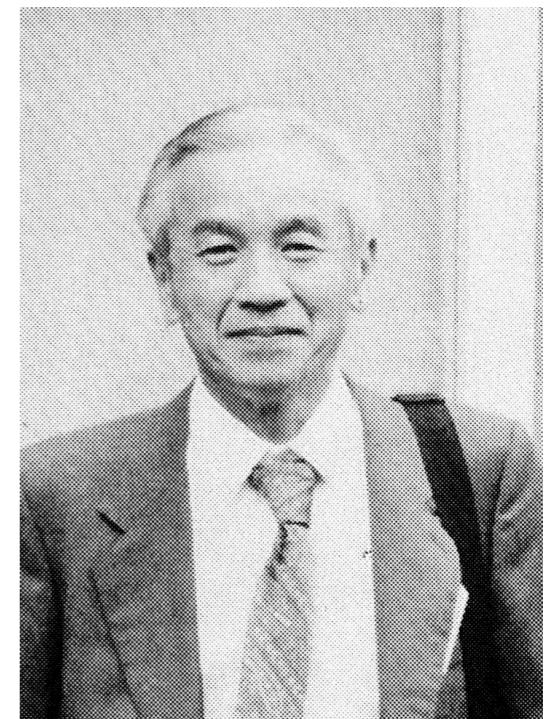
Бруно Понтецорво



B. Pontecorvo



Z. Maki



M. Nakagawa

Neutrino Masses & Mixings

$$\nu_e, \nu_\mu, \nu_\tau \neq \nu_1, \nu_2, \nu_3$$

(flavor or weak eigenstates)

(mass eigenstates)

Note: if $|\bar{\nu}\rangle : U^* \rightarrow U$

superposition of n light mass eigenstates

$$|\nu_\alpha(x)\rangle = \sum_{i=1}^n U_{\alpha i}^* \int \frac{d^3 p}{(2\pi)^3} f_j(\vec{p}) e^{-iE_i(t-t_0)} e^{i\vec{p}(\vec{x}-\vec{x}_0)} |\nu_i\rangle$$

obs: indices L omitted !

neutrinos are produced by CC weak interactions as wave packets localized around a source position $x_0 = (t_0, \vec{x}_0)$

See E. Akhmedov and A. Smirnov, arXiv :0905.1903

We will derive the oscillation probability using plane waves - conceptually wrong but gives the right result much quicker

Neutrino Masses & Mixings

$$\nu_e, \nu_\mu, \nu_\tau \neq \nu_1, \nu_2, \nu_3$$

(flavor or weak eigenstates)

(mass eigenstates)

$$|\nu_\alpha\rangle = \sum_{i=1}^n U_{\alpha i}^* |\nu_i\rangle \quad \alpha = e, \mu, \tau$$

mixing matrix

state produced by CC interaction
superposition of n light mass
eigenstates

$$|\nu_\alpha(t)\rangle = \sum_{i=1}^n U_{\alpha i}^* |\nu_i(t)\rangle$$

after propagating for a time t
after traveling a distance $L \approx ct$

$$P_{\alpha\beta}(t) = |A_{\alpha\beta}(t)|^2 = |\langle\nu_\beta|\nu_\alpha(t)\rangle|^2 = \left| \sum_{i=1}^n \sum_{j=1}^n U_{\alpha i}^* U_{\beta j} \langle\nu_j(0)|\nu_i(t)\rangle \right|^2$$

probability of detecting it as ν_β

Neutrino Oscillations in Vacuum

$$P_{\alpha\beta}(t) = |\mathcal{A}_{\alpha\beta}(t)|^2 = |\langle \nu_\beta | \nu_\alpha(t) \rangle|^2 = \left| \sum_{i=1}^n \sum_{j=1}^n U_{\alpha i}^* U_{\beta j} \langle \nu_j(0) | \nu_i(t) \rangle \right|^2$$

$$|\nu_i(t)\rangle = e^{-i\frac{E_i}{\hbar}t} |\nu_i(0)\rangle \quad \text{all with the same momentum}$$

$$P_{\alpha\beta}(t) = \left| \sum_{i=1}^n U_{\alpha i}^* U_{\beta i} e^{-i\frac{E_i}{\hbar}t} \right|^2$$

[put back c and \hbar for now]

very relativistic neutrinos

$$E_i = \sqrt{p^2c^2 + m_i^2c^4} \approx pc + \frac{m_i^2c^3}{2p} = E + \frac{m_i^2c^4}{2E}$$

$$P_{\alpha\beta}(L) = \left| \sum_i^n U_{\alpha i}^* U_{\beta i} e^{-i\frac{\Delta m_{i1}^2 c^3}{2E\hbar} L} \right|^2$$

$$\Delta m_{ij}^2 = m_i^2 - m_j^2$$

to very good approximation

The Mixing Matrix U

$(n \times n)$ unitary matrix $\Leftrightarrow n^2$ real parameters

$n(n-1)/2$ mixing angles

$n(n+1)/2$ phases

Dirac ν : $n + (n-1) = 2n-1$ phases can be absorbed

in redefinitions of lepton fields

$n(n+1)/2 - (2n-1) = (n-1)(n-2)/2$ Dirac physical phases

Majorana ν : only n phases can be absorbed in

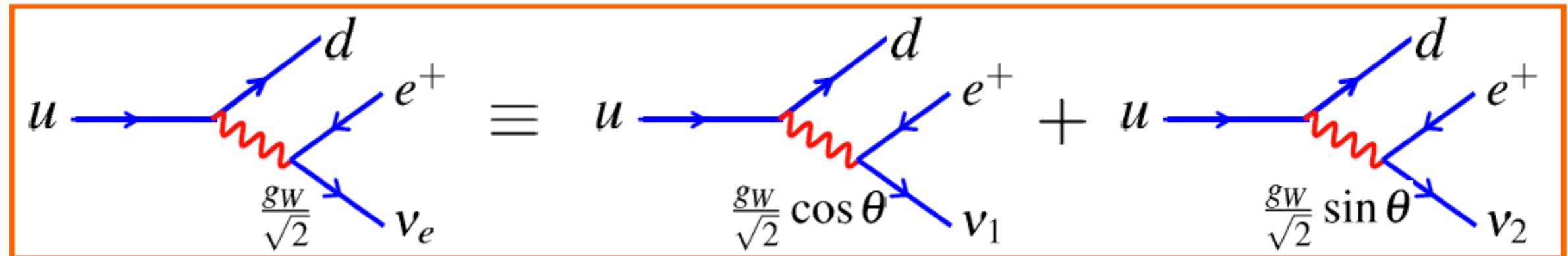
redefinitions of charged lepton fields

Dirac physical phases + $(n-1)$ Majorana physical phases

do not enter
oscillations

E7.

For Two Flavors



$$U = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}$$

1 angle + 0 Dirac phases

oscillation probability

$$P_{e\mu}(L) = \sin^2 2\theta \sin^2 \left(\frac{\Delta m_{21}^2 L}{4E} \right)$$

natural units ($c = \hbar = 1$)

survival probability

$$P_{ee} = 1 - P_{e\mu}$$

phase difference

equal masses \rightarrow no oscillation

$$= \sin^2 2\theta \sin^2 \left(1.27 \frac{\Delta m_{21}^2}{eV^2} \frac{L}{m} \frac{MeV}{E} \right)$$

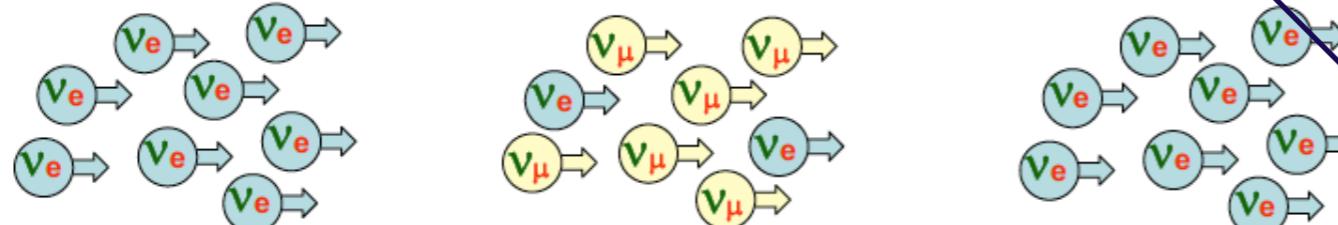
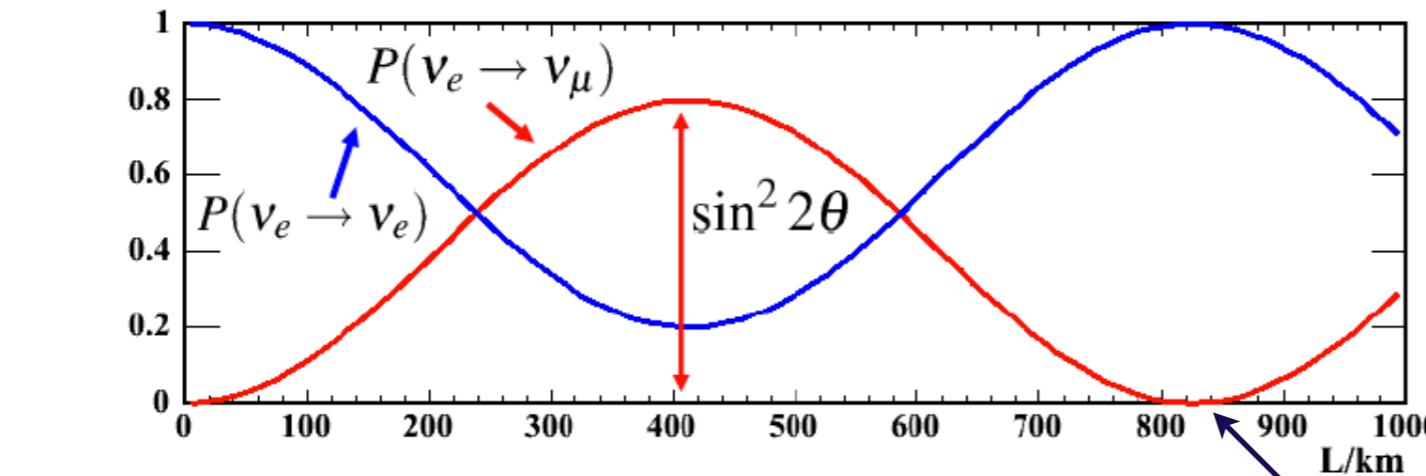
L = baseline

For Two Flavors

e.g. $\Delta m^2 = 0.003 \text{ eV}^2$, $\sin^2 2\theta = 0.8$, $E_\nu = 1 \text{ GeV}$

oscillation length

$$L_{\text{osc}} = \frac{4\pi E}{\Delta m_{21}^2} \sim 2.5 \text{ km} \frac{E (\text{GeV})}{\Delta m_{21}^2 (\text{eV}^2)}$$



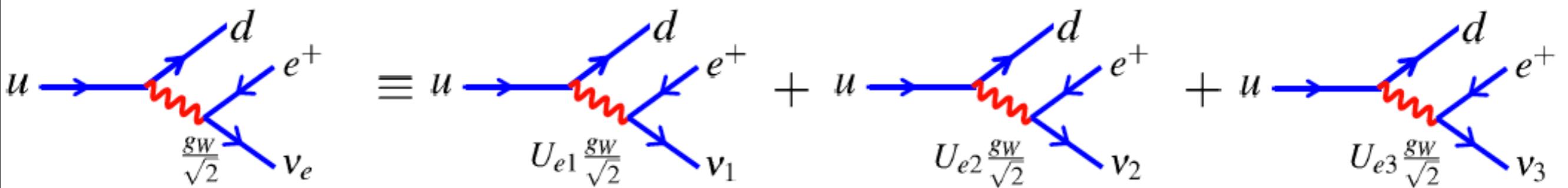
$$L_{\text{osc}} \sim 830 \text{ km}$$

$$P_{e\mu}(L) = \sin^2 2\theta \sin^2 \left(\frac{\pi L}{L_{\text{osc}}} \right) \rightarrow \frac{1}{2} \sin^2 2\theta$$

average regime

when can one observe oscillations?

For Three Flavors



$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

standard parametrization - PMNS matrix

3 angles + 1 phase

$$c_{ij} \equiv \cos \theta_{ij} \quad s_{ij} \equiv \sin \theta_{ij} \quad \theta_{ij} \in [0, \pi/2] \quad \delta \in [0, 2\pi]$$

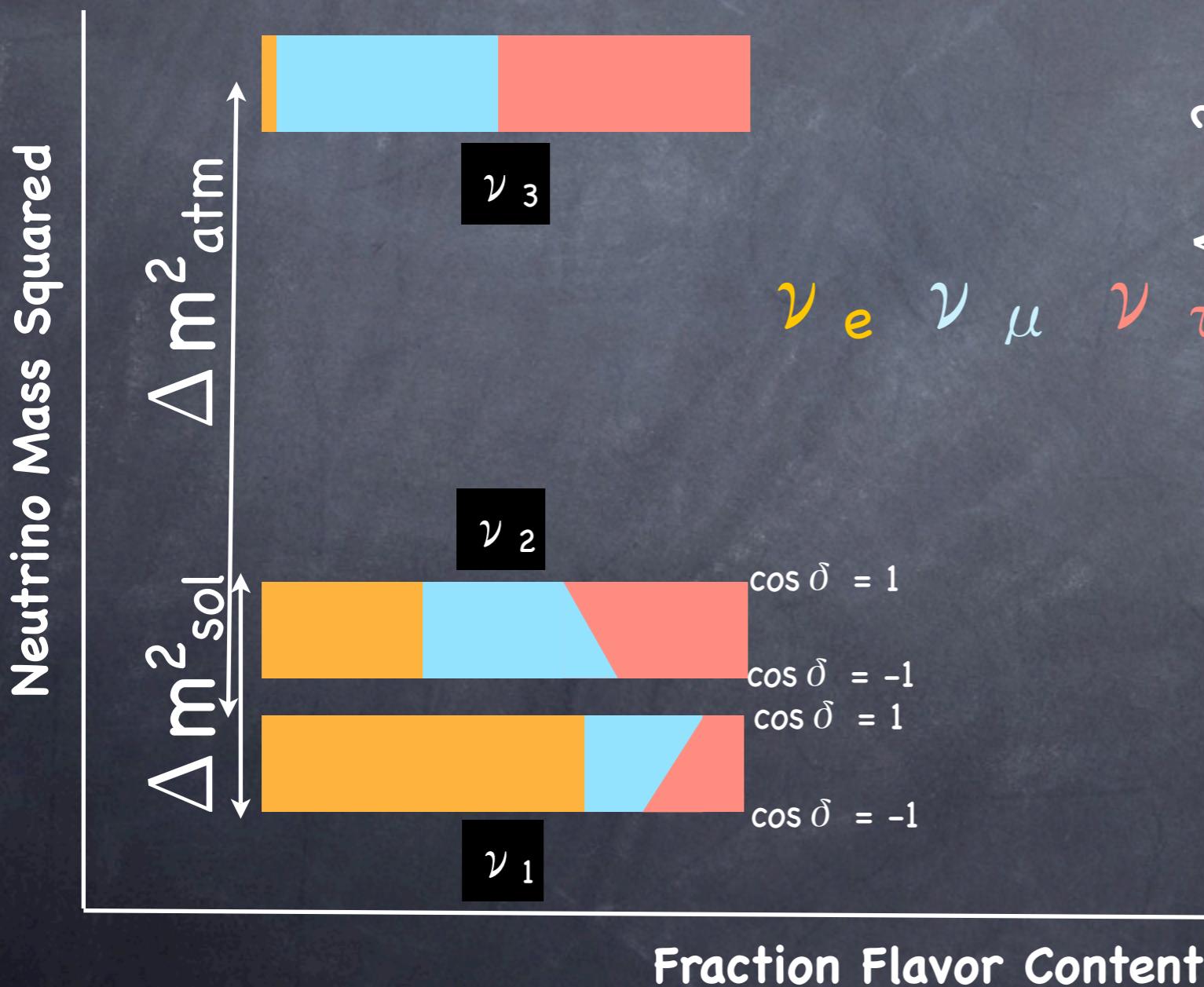
$$\Delta m_{31}^2 = \Delta m_{32}^2 + \Delta m_{21}^2$$

if $\Delta m_{21}^2 \ll |\Delta m_{31}^2|$ and $s_{13} \rightarrow 0$ 12 and 23 sub-systems decouple

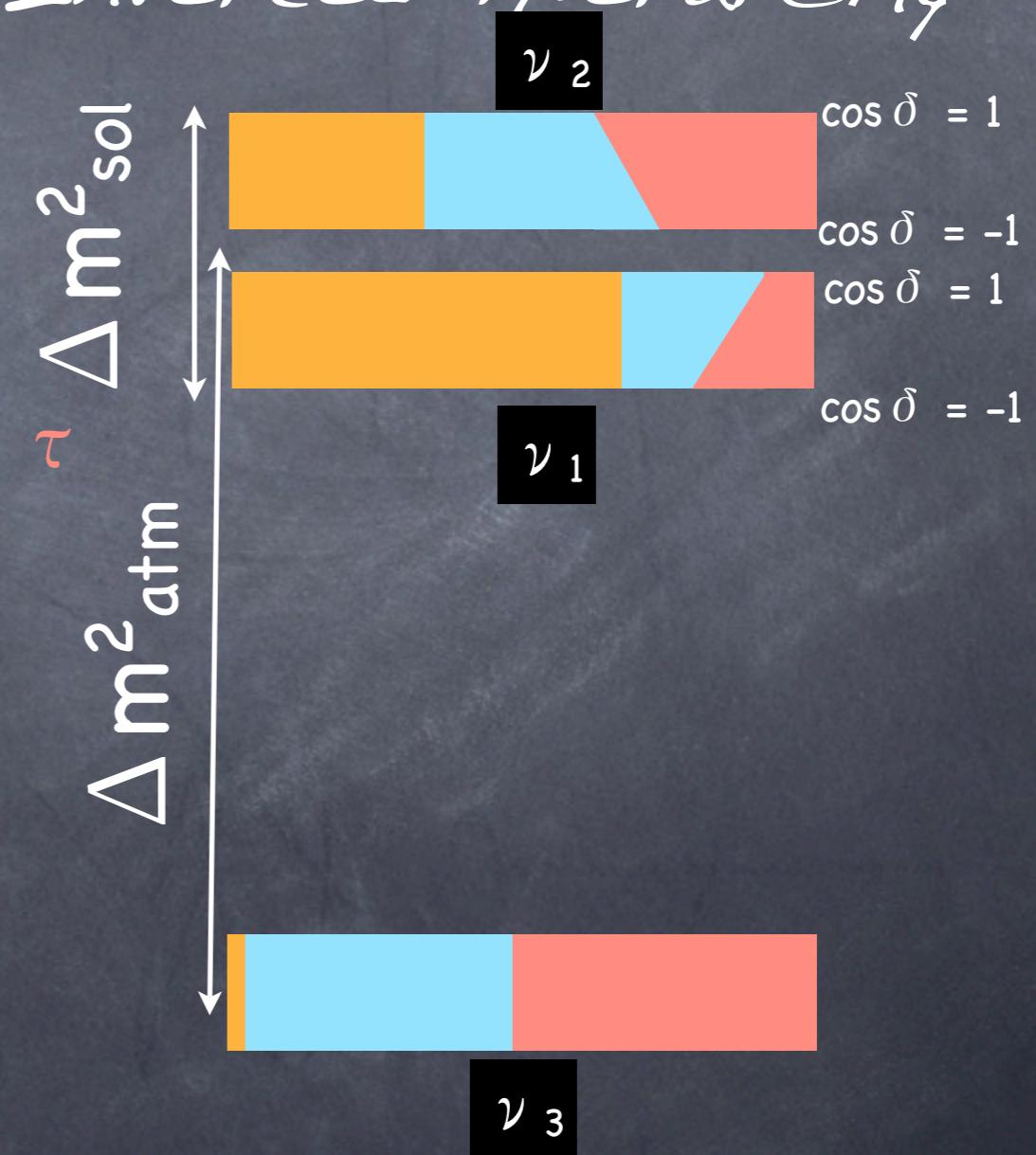
The Standard Framework

$$\Delta m_{21}^2 = \Delta m^2_{sol} > 0 \quad \Delta m_{21}^2 \ll |\Delta m^2_{32}| \approx |\Delta m^2_{31}|$$

Normal Hierarchy



Inverted Hierarchy



we will see why soon

CPT in Neutrino Oscillations

CP symmetry

LH Particles \Leftrightarrow RH Antiparticles

RH Particles \Leftrightarrow LH Antiparticles

All Lorentz invariant, local Quantum Field Theories can be shown to be invariant under CPT (charge conjugation + parity + time reversal)

G. Lüders (1954), W. Pauli (1955), J.S.Bell (1954)

No reason to think CPT is not conserved ...

So if CP is conserved (violated) T is conserved (violated)

$$CP : \nu_{\alpha L} \Leftrightarrow \bar{\nu}_{\alpha R} \Rightarrow U^*_{\alpha i} \Leftrightarrow U_{\alpha i}; \quad \delta \Leftrightarrow -\delta$$

$$T : t \Leftrightarrow t_0 \Rightarrow \nu_{\alpha L} \Leftrightarrow \nu_{\beta L}$$

CP and T absent for 2 flavors. Its is a ≥ 3 flavor effect !

CPT in Neutrino Oscillations

Measures CP:

$$\Delta P_{\alpha\beta}^{\text{CP}} \equiv P(\nu_\alpha \rightarrow \nu_\beta) - P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta)$$

Measures T:

$$\Delta P_{\alpha\beta}^T \equiv P(\nu_\alpha \rightarrow \nu_\beta) - P(\nu_\beta \rightarrow \nu_\alpha)$$

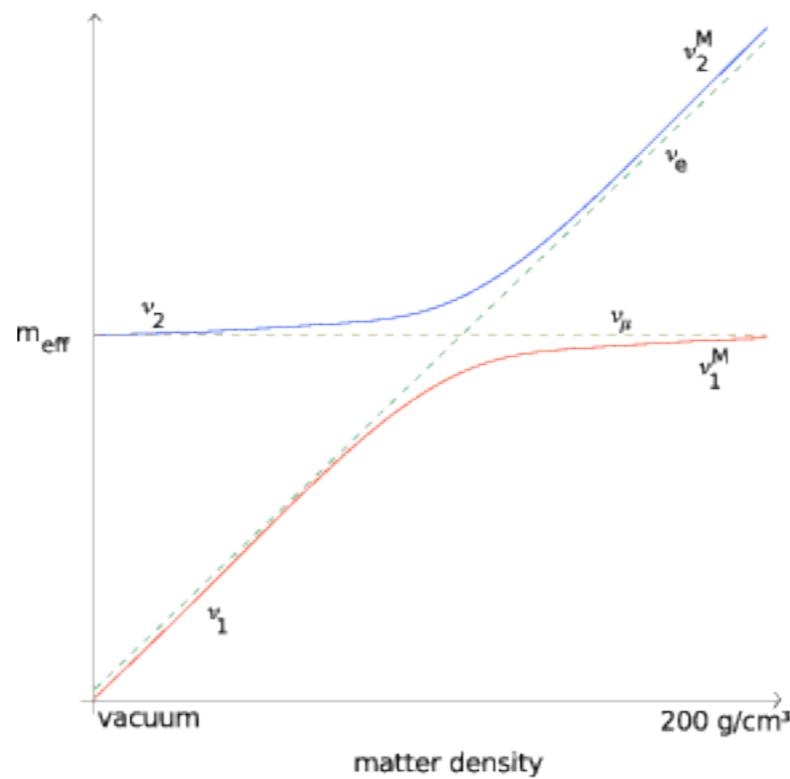
Measures CPT:

$$\Delta P_{\alpha\beta}^{\text{CPT}} \equiv P(\nu_\alpha \rightarrow \nu_\beta) - P(\bar{\nu}_\beta \rightarrow \bar{\nu}_\alpha)$$

For 3 flavors $\Delta P_{e\mu}^{\text{CP}} = \Delta P_{\mu\tau}^{\text{CP}} = \Delta P_{\tau e}^{\text{CP}} \equiv \Delta P$

$$\begin{aligned} \Delta P = & -4 s_{12} c_{12} s_{13} c_{13}^2 s_{23} c_{23} \sin \delta & \text{Max. for } \delta = \pi/2 \text{ or } 3\pi/2 \\ & \times \left[\sin \left(\frac{\Delta m_{12}^2}{2E} L \right) + \sin \left(\frac{\Delta m_{23}^2}{2E} L \right) + \sin \left(\frac{\Delta m_{31}^2}{2E} L \right) \right] \end{aligned}$$

Very Difficult to measure ...



2. Neutrino Oscillations in Matter

The MSW Effect

1978 - Wolfenstein suggests matter can drastically impact neutrino oscillations

1985 - Mikheyev & Smirnov observe the possibility of resonance in flavor conversion



L. Wolfenstein



S. Mikheyev

A. Yu Smirnov

How can matter affect
neutrinos ?

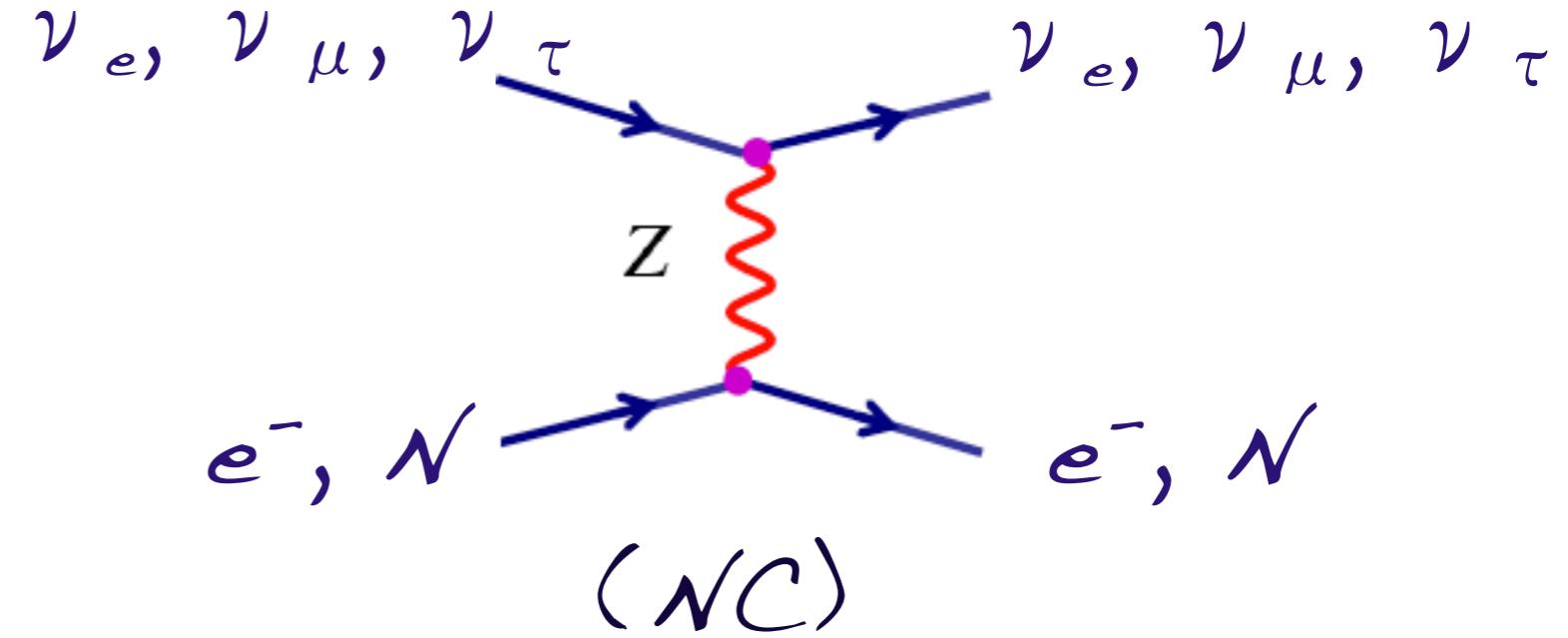
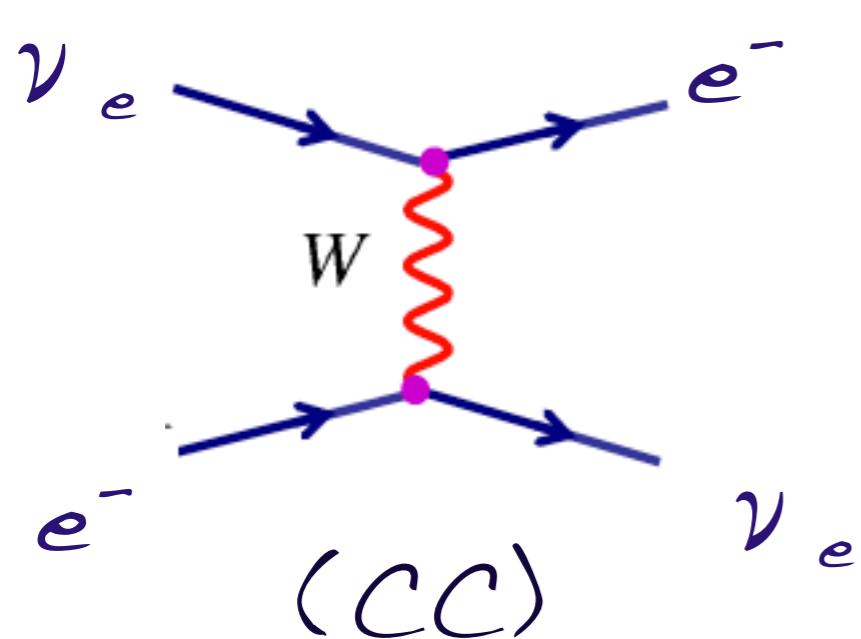
incoherent process (capture, finite angle
scattering) $\sigma \propto G_F^2$

coherent forward scattering

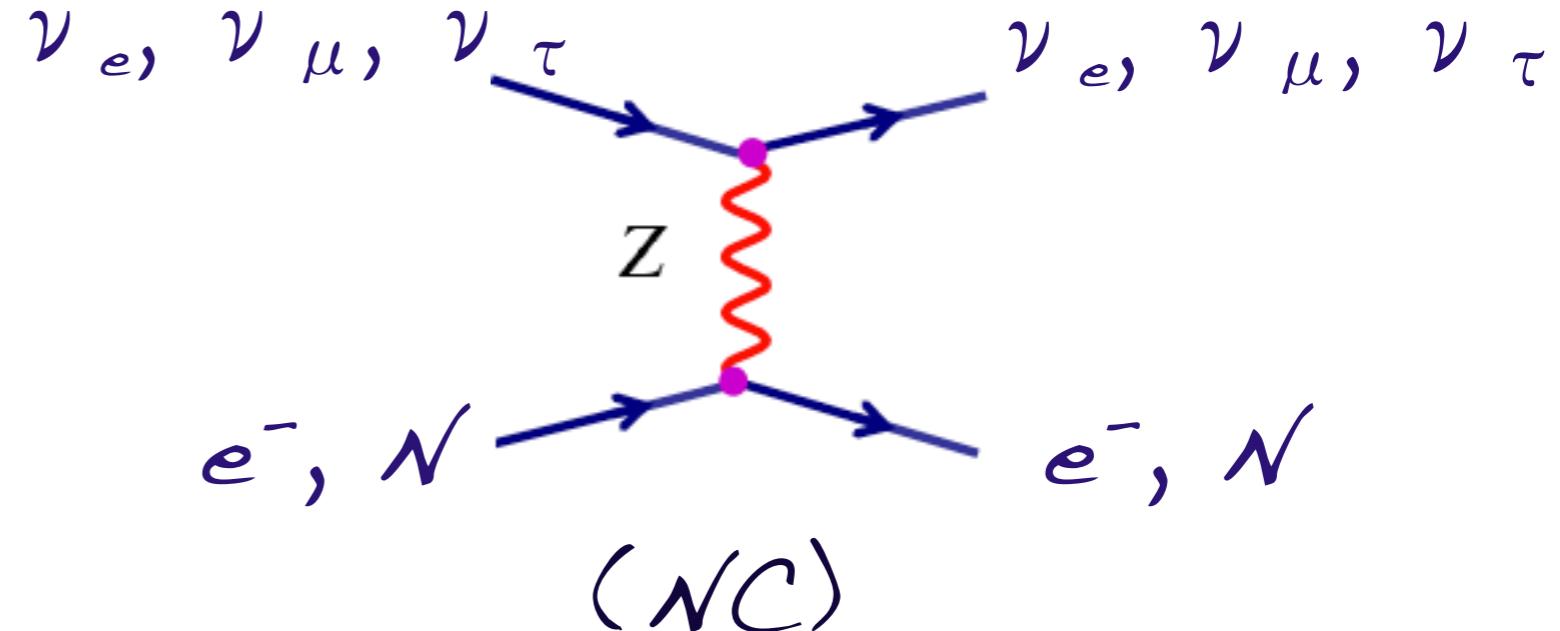
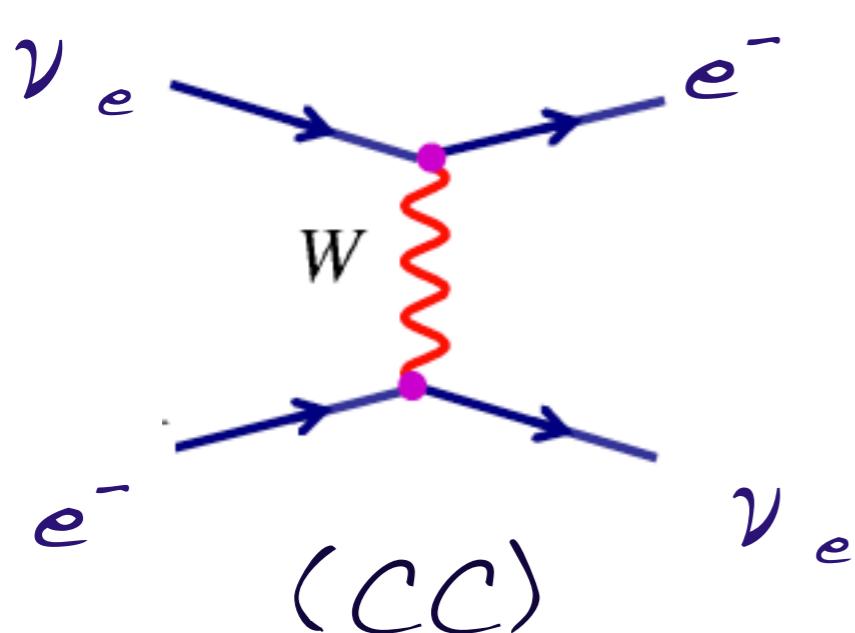
effects $\propto G_F$ a lot larger!

lead to effective potentials for ν 's in
matter $\propto G_F n$

Coherent Forward Scattering



Coherent Forward Scattering



averaged over e^- spin and summed over all e^- in the medium

$$H_{CC}^{(e)} = \sqrt{2} G_F \int d^3 p_e f(E_e, T) \times \langle \langle \bar{e}(s, p_e) | \bar{e}(x) \gamma^\mu P_L \nu_e(x) \bar{\nu}_e(x) \gamma_\mu P_L e(x) | e(s, p_e) \rangle \rangle$$

$$= \sqrt{2} G_F \bar{\nu}_e(x) \gamma_\mu P_L \nu_e(x) \int d^3 p_e f(E_e, T) \langle \langle e(s, p_e) | \bar{e}(x) \gamma^\mu P_L e(x) | e(s, p_e) \rangle \rangle$$

$f(E_e, T)$ homogeneous, isotropic and normalized
 coherence \rightarrow same $e(s, p_e)$ @ start and end

Coherent Forward Scattering

$$\langle e(s, p_e) | \bar{e}(x) \gamma^\mu P_L e(x) | e(s, p_e) \rangle =$$

$$\frac{1}{V} \langle e(s, p_e) | \bar{u}_s(p_e) a_s^\dagger(p_e) \gamma^\mu P_L a_s(p_e) u_s(p_e) | e(s, p_e) \rangle$$

$$\langle \dots \rangle = n_e(p_e) \frac{1}{2} \sum_s = n_e(p_e) \frac{p_e^\mu}{E_e}$$

*n_e = electron
density*

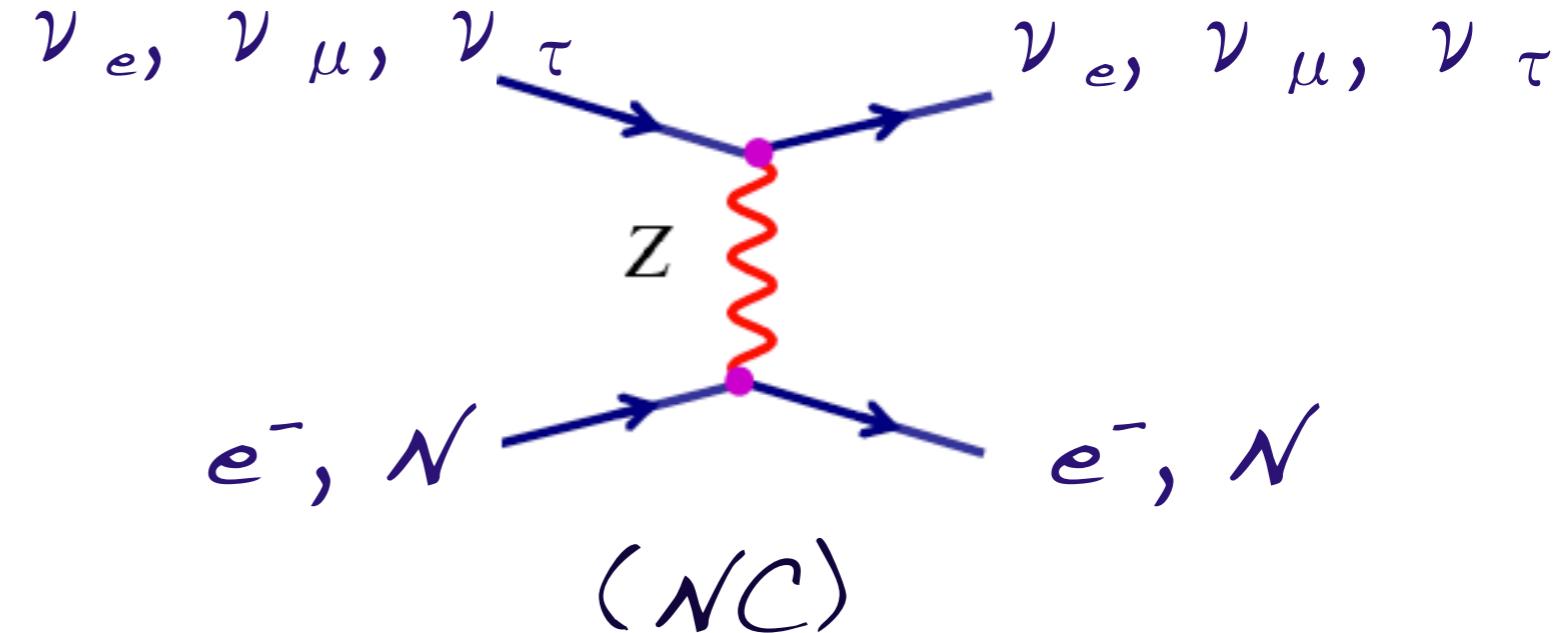
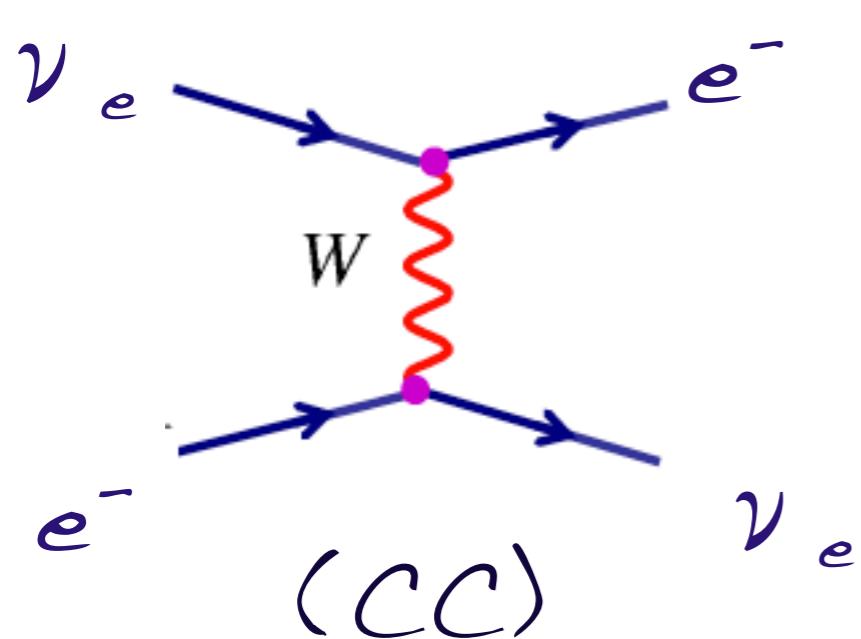
isotropic medium $\int d^3 p_e \vec{p}_e f(E_e, T) = 0$

$$H_c^{(e)} = \sqrt{2} G_F n_e \bar{\nu}_e(x) \gamma_0 P_L \nu_e(x)$$

$$V_c = \langle \nu_e | \int d^3 x H_c^{(e)} | \nu_e \rangle = \sqrt{2} G_F n_e$$

*effective
potential*

Coherent Forward Scattering



$$V^{(e)}_{NC} = - V^{(p)}_{NC}$$

$$V_{NC} = V^{(n)}_{NC}$$

$$V_e = V_c + V_{NC}$$

common phase

$$\text{antineutrinos : } V \rightarrow -V$$

ν Oscillations in Matter

$$i \frac{d}{dt} |\nu_\alpha(\mathbf{p}, t)\rangle = \mathbf{H} |\nu_\alpha(\mathbf{p}, t)\rangle \quad |\nu_\alpha(\mathbf{p}, 0)\rangle \equiv |\nu_\alpha(\mathbf{p})\rangle$$

Schrödinger picture

$$\mathcal{A}_{\alpha\beta}(\mathbf{p}, t) = \langle \nu_\beta(\mathbf{p}) | \nu_\alpha(\mathbf{p}, t) \rangle \quad \mathcal{A}_{\alpha\beta}(\mathbf{p}, 0) = \delta_{\alpha\beta}$$

flavor transition amplitude

$$i \frac{d}{dt} \mathcal{A}_{\alpha\beta}(\mathbf{p}, t) = \langle \nu_\beta(\mathbf{p}) | \mathbf{H} | \nu_\alpha(\mathbf{p}, t) \rangle = \langle \nu_\beta(\mathbf{p}) | \mathbf{H}_0 | \nu_\alpha(\mathbf{p}, t) \rangle + \langle \nu_\beta(\mathbf{p}) | \mathbf{H}_I | \nu_\alpha(\mathbf{p}, t) \rangle$$

$$\langle \nu_\beta(\mathbf{p}) | \mathbf{H}_0 | \nu_\alpha(\mathbf{p}, t) \rangle = \sum_{\rho} \langle \nu_\beta(\mathbf{p}) | \mathbf{H}_0 | \nu_\rho(\mathbf{p}) \rangle \langle \nu_\rho(\mathbf{p}) | \nu_\alpha(\mathbf{p}, t) \rangle$$

$$= \sum_{\rho} \sum_{\mathbf{j}} \mathbf{U}_{\rho\mathbf{j}}^* \mathbf{U}_{\beta\mathbf{j}} \mathbf{E}_{\mathbf{j}} \mathcal{A}_{\alpha\rho}(\mathbf{p}, t)$$

$$\langle \nu_\beta(\mathbf{p}) | \mathbf{H}_I | \nu_\alpha(\mathbf{p}, t) \rangle = \sum_{\rho} \underbrace{\langle \nu_\beta(\mathbf{p}) | \mathbf{H}_I | \nu_\rho(\mathbf{p}) \rangle}_{\delta_{\beta\rho} \mathbf{V}_\beta} \mathcal{A}_{\alpha\rho}(\mathbf{p}, t)$$

ν Oscillations in Matter

$$i \frac{d}{dt} A_{\alpha\beta} = \sum_{\rho} \left(\sum_j U_{\rho j}^* U_{\beta j} E_j + \delta_{\beta\rho} V_{\beta} \right) A_{\alpha\rho}(p, t)$$

ultra-relativistic ν : $E_j = E + m_j^2/(2E)$ $t \approx r$

$$V_e = V_c + V_{NC}$$

$$V_{\mu} = V_{\tau} = V_{NC}$$

$$i \frac{d}{dr} A_{\alpha\beta} = (E + V_{NC}) A_{\alpha\beta}(p, r) + \sum_{\rho} \left(\sum_j U_{\beta j} \frac{m_j^2}{2E} U_{\rho j}^* + \delta_{\rho e} \delta_{\beta e} V_c \right) A_{\alpha\rho}(p, r)$$

$$A'_{\alpha\beta}(p, r) = A_{\alpha\beta}(p, r) e^{iE r + i \int_0^r V_{NC}(x') dx'} \quad \text{global phase}$$

ν Oscillations in Matter

$$i \frac{d}{dr} \mathcal{A}_{\alpha\beta} = (E + V_{NC}) \mathcal{A}_{\alpha\beta}(p, r) + \sum_{\rho} \left(\sum_j U_{\beta j} \frac{m_j^2}{2E} U_{\rho j}^* + \delta_{\rho e} \delta_{\beta e} V_c \right) \mathcal{A}_{\alpha\rho}(p, r)$$

define

$$\mathcal{A}'_{\alpha\beta}(p, r) = \mathcal{A}_{\alpha\beta}(p, r) e^{iE r + i \int_0^r V_{NC}(x') dx'} \quad \text{global phase}$$

$$i \frac{d}{dr} \mathcal{A}'_{\alpha\beta}(p, r) = e^{iE r + i \int_0^r V_{NC}(x') dx'} \left(-E - V_{NC} + i \frac{d}{dr} \right) \mathcal{A}_{\alpha\beta}$$

$$i \frac{d}{dr} \mathcal{A}'_{\alpha\beta} = \sum_{\rho} \left(\sum_j U_{\beta j} \frac{m_j^2}{2E} U_{\rho j}^* + \delta_{\rho e} \delta_{\beta e} V_c \right) \mathcal{A}'_{\alpha\rho}(p, r)$$

so $P_{\alpha\beta} = |\mathcal{A}_{\alpha\beta}|^2 = |\mathcal{A}'_{\alpha\beta}|^2$

The Standard Framework

$$i \frac{d}{dr} \begin{pmatrix} A_{\alpha e} \\ A_{\alpha \mu} \\ A_{\alpha \tau} \end{pmatrix} = \left(\frac{1}{2E} \mathbf{U} \mathbf{M}^2 \mathbf{U}^\dagger + \mathbf{A} \right) \begin{pmatrix} A_{\alpha e} \\ A_{\alpha \mu} \\ A_{\alpha \tau} \end{pmatrix}$$

evolution of neutrino amplitudes

$$\mathbf{M} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \Delta m_{21}^2 & 0 \\ 0 & 0 & \Delta m_{31}^2 \end{pmatrix}$$

$$\mathbf{A} = \begin{pmatrix} V_e & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$V_e = \sqrt{2} G_F n_e \sim 7.6 Y_e \frac{\rho}{10^{14} \text{g/cm}^3} \text{eV}$$

$$\begin{array}{ll} \rho \sim 10 \text{g/cm}^3 V_e \sim 10^{-13} \text{eV} & \text{Earth's core} \\ \rho \sim 100 \text{g/cm}^3 V_e \sim 10^{-12} \text{eV} & \text{Sun's core} \end{array}$$

Two Flavors in Matter

$$i \frac{d}{dr} \begin{pmatrix} A_{\alpha e} \\ A_{\alpha \mu} \end{pmatrix} = \begin{pmatrix} -\frac{\Delta m_{21}^2}{4E} \cos 2\theta + V_e & \frac{\Delta m_{21}^2}{4E} \sin 2\theta \\ \frac{\Delta m_{21}^2}{4E} \sin 2\theta & \frac{\Delta m_{21}^2}{4E} \cos 2\theta \end{pmatrix} \begin{pmatrix} A_{\alpha e} \\ A_{\alpha \mu} \end{pmatrix}$$

constant matter density

$$\sin^2 \theta_m = \frac{1}{2} [1 + \frac{(A - \Delta m_{21}^2 \cos 2\theta)}{\Delta m_m^2}]$$

$$\Delta m_m^2 = \sqrt{(\Delta m_{21}^2 \cos 2\theta - A)^2 + (\Delta m_{21}^2 \sin 2\theta)^2}$$

will only happen for neutrinos if
 $\Delta m_{21}^2 > 0$

$$A \equiv 2\sqrt{2} E G_F n_e = 2 E V_e$$

$$\sqrt{2} G_F n_e = \frac{\Delta m_{21}^2}{2E} \cos 2\theta$$

*MSW resonance
conditions*

Two Flavors in Matter

$$\sin^2 \theta_m = \frac{1}{2} [1 + \frac{(A - \Delta m_{21}^2 \cos 2\theta)}{\Delta m_m^2}]$$

MSW resonance conditions

$$\theta_m = 45^\circ \quad \text{maximal mixing}$$

$$\sqrt{2} G_F n e = \frac{\Delta m_{21}^2}{2E} \cos 2\theta$$

$$n_e = n_e^{\text{res}}$$

variable matter density

$$|\nu_e\rangle = \cos\theta_m |\nu_{1m}\rangle + \sin\theta_m |\nu_{2m}\rangle$$

↓ interaction ↓ instantaneous ↓ eigenstates in matter
 $|\nu_\mu\rangle = -\sin\theta_m |\nu_{1m}\rangle + \cos\theta_m |\nu_{2m}\rangle$

$$n_e \gg n_e^{\text{res}} \quad \theta_m = 90^\circ$$

$$\nu_3 \rightarrow \nu_e$$

$$n_e \ll n_e^{\text{res}} \quad \theta_m \approx 0^\circ$$

Two Flavors in Matter

variable matter density

instantaneous eigenstates in matter

$$i \frac{d}{dr} \begin{pmatrix} \nu_{1m} \\ \nu_{2m} \end{pmatrix} = \frac{1}{4E} \begin{pmatrix} -\Delta m_m^2 & -4iE d\theta_m(r)/dr \\ 4iE d\theta_m(r)/dr & \Delta m_m^2 \end{pmatrix} \begin{pmatrix} \nu_{1m} \\ \nu_{2m} \end{pmatrix}$$

adiabatic transitions:

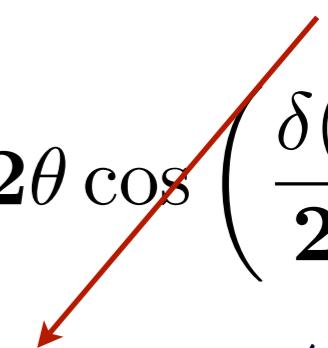
if $\Delta m_m^2 \gg 4E d\theta_m(r)/dr$

instantaneous mass eigenstates behave like energy eigenstates \rightarrow they do not mix on evolution

$$P(\nu_e \rightarrow \nu_e) = \cos^2 \theta_m \cos^2 \theta + \sin^2 \theta_m \sin^2 \theta + \frac{1}{2} \sin 2\theta_m \sin 2\theta \cos \left(\frac{\delta(r)}{2E} \right)$$

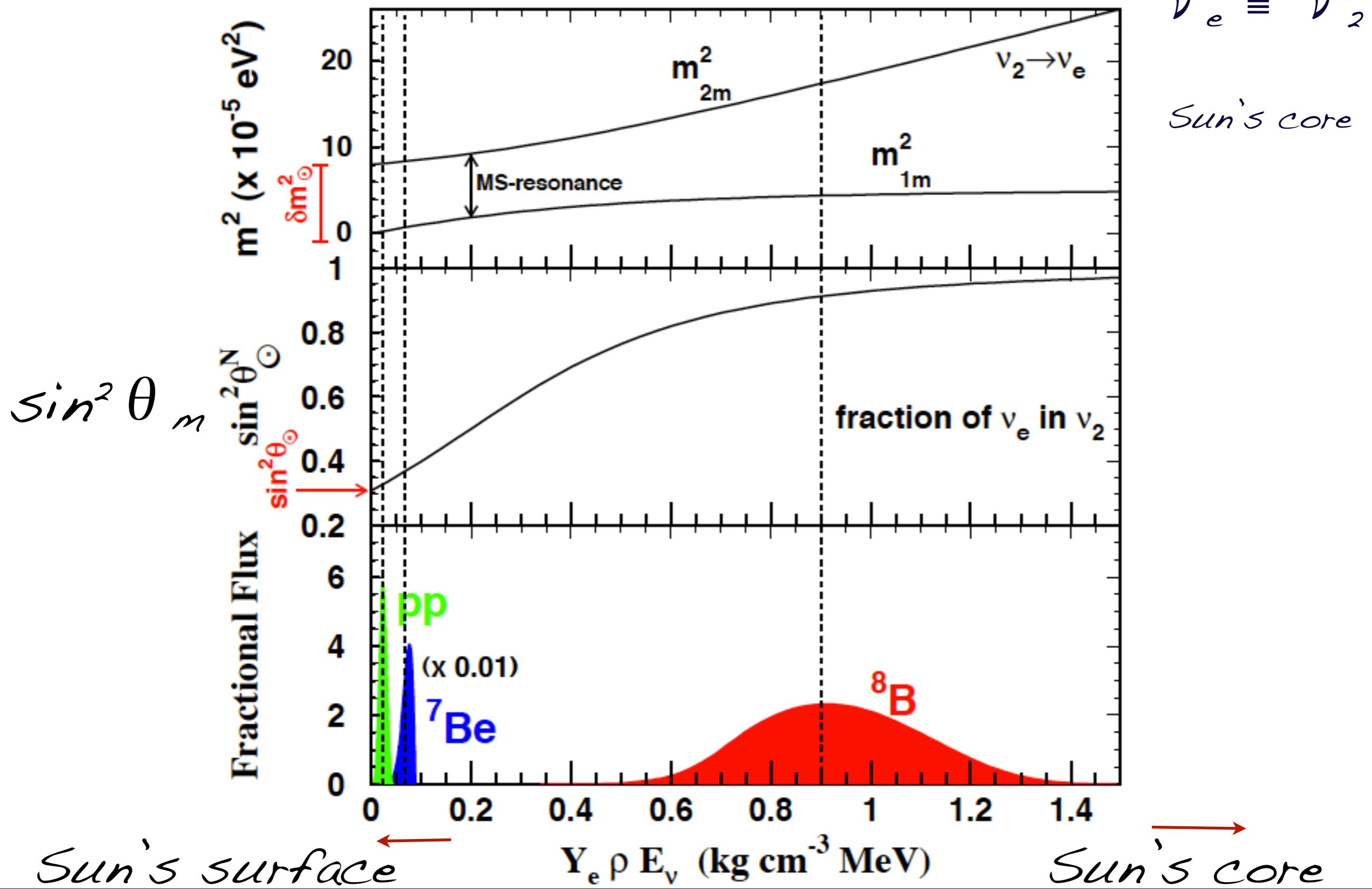
$\delta(r) = \int_{r_0}^r \Delta m_m^2(r') dr'$

Sun : $\delta(r) \gg E$

averaged 

Two Flavors in the Sun

$$P(\nu_e \rightarrow \nu_e) = \cos^2 \theta_m \cos^2 \theta + \sin^2 \theta_m \sin^2 \theta$$



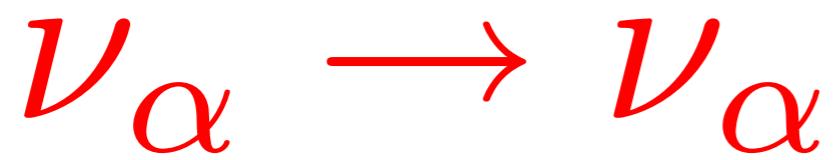
3. Linking with Experiments

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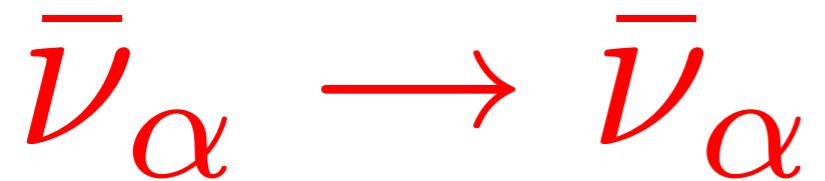
+ D. Harris
Lectures

+ T. Schwetz
Lectures

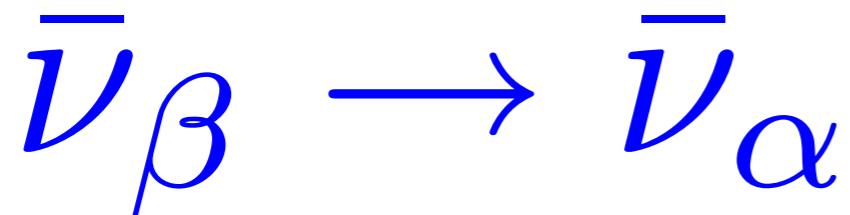
Two Types of Oscillation Experiments



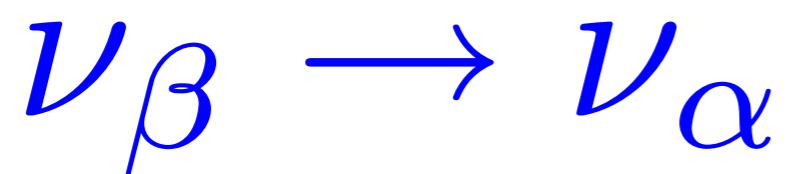
disappearance



experiments



appearance



experiments

$$\beta \neq \alpha$$

From Theory to Experiment

[Experiment]

[Theory]

of neutrinos

ν Flux

survival probability

$$N_\alpha(L) = A \int \Phi(E) \sigma(E) P(\nu_\alpha \rightarrow \nu_\alpha; E, L) \epsilon(E)$$

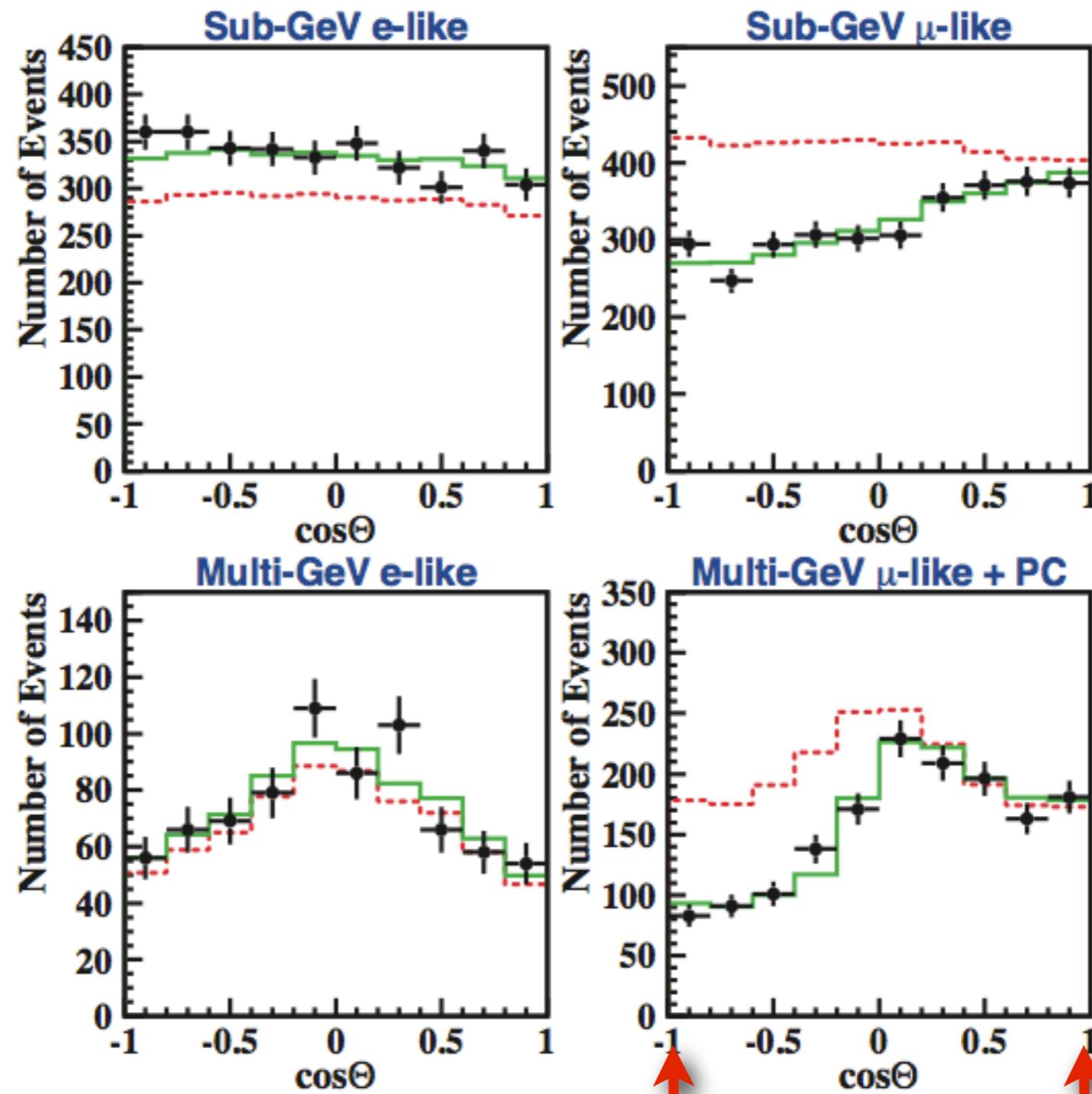
number of
targets x time

[Experiment]

x-section
[Theory]

detector
efficiency
[Experiment]

Atmospheric Neutrinos

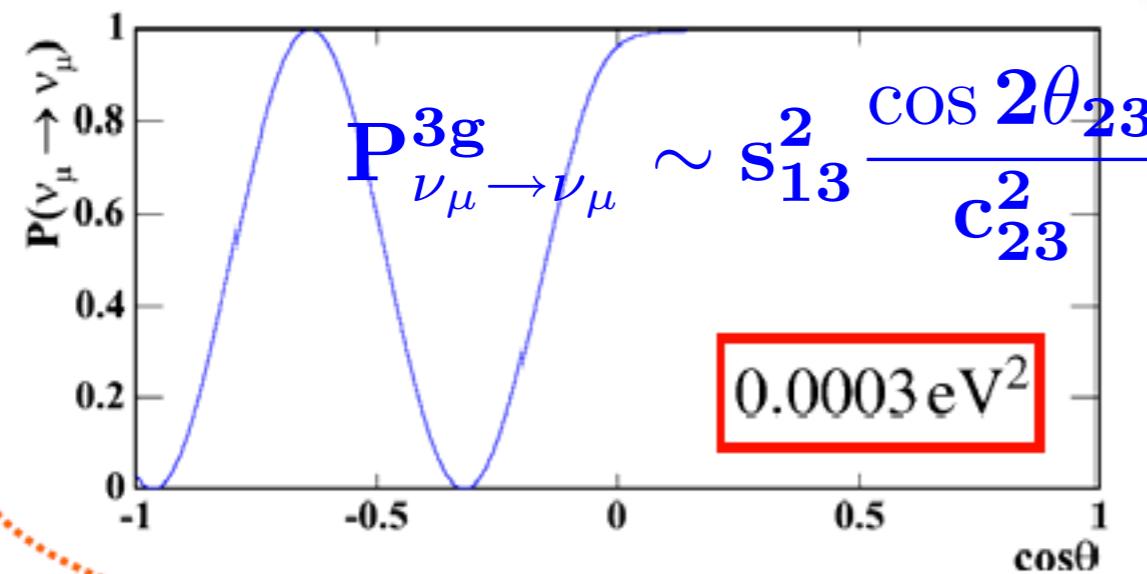


up going

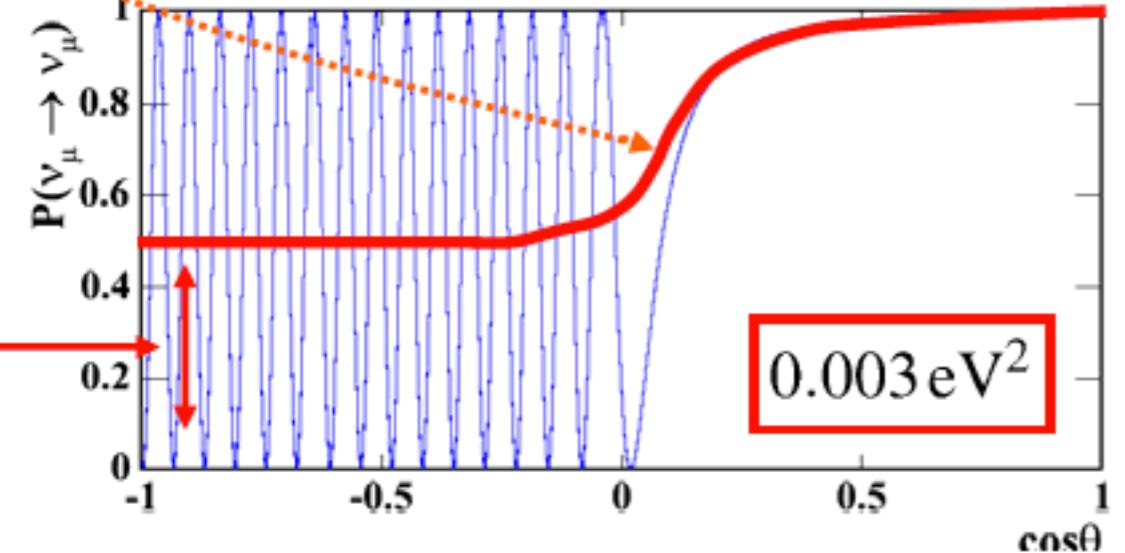
down going

Super-Kamiokande

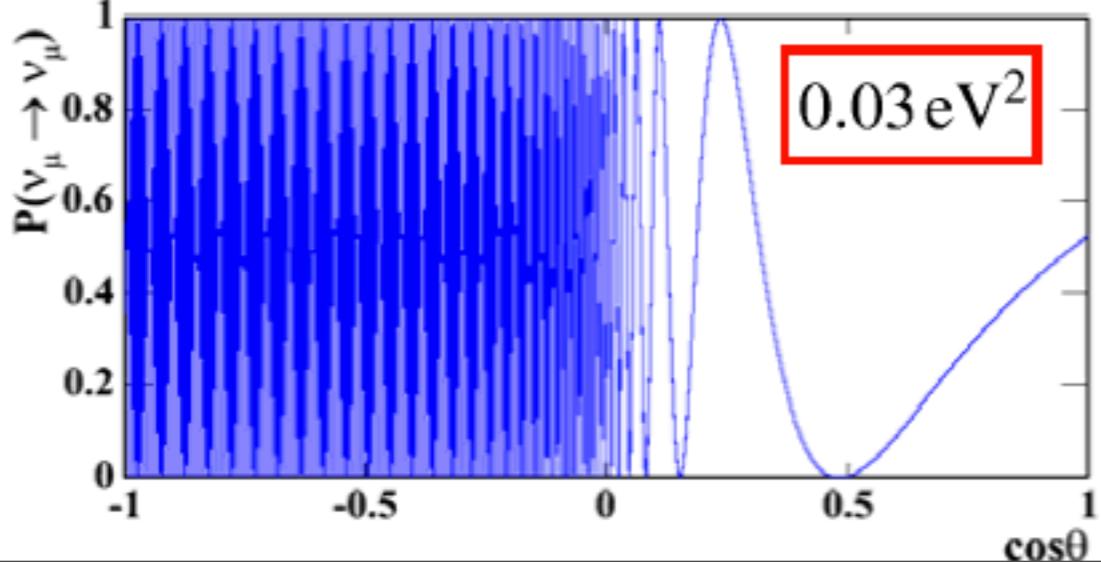
Atmospheric Neutrinos



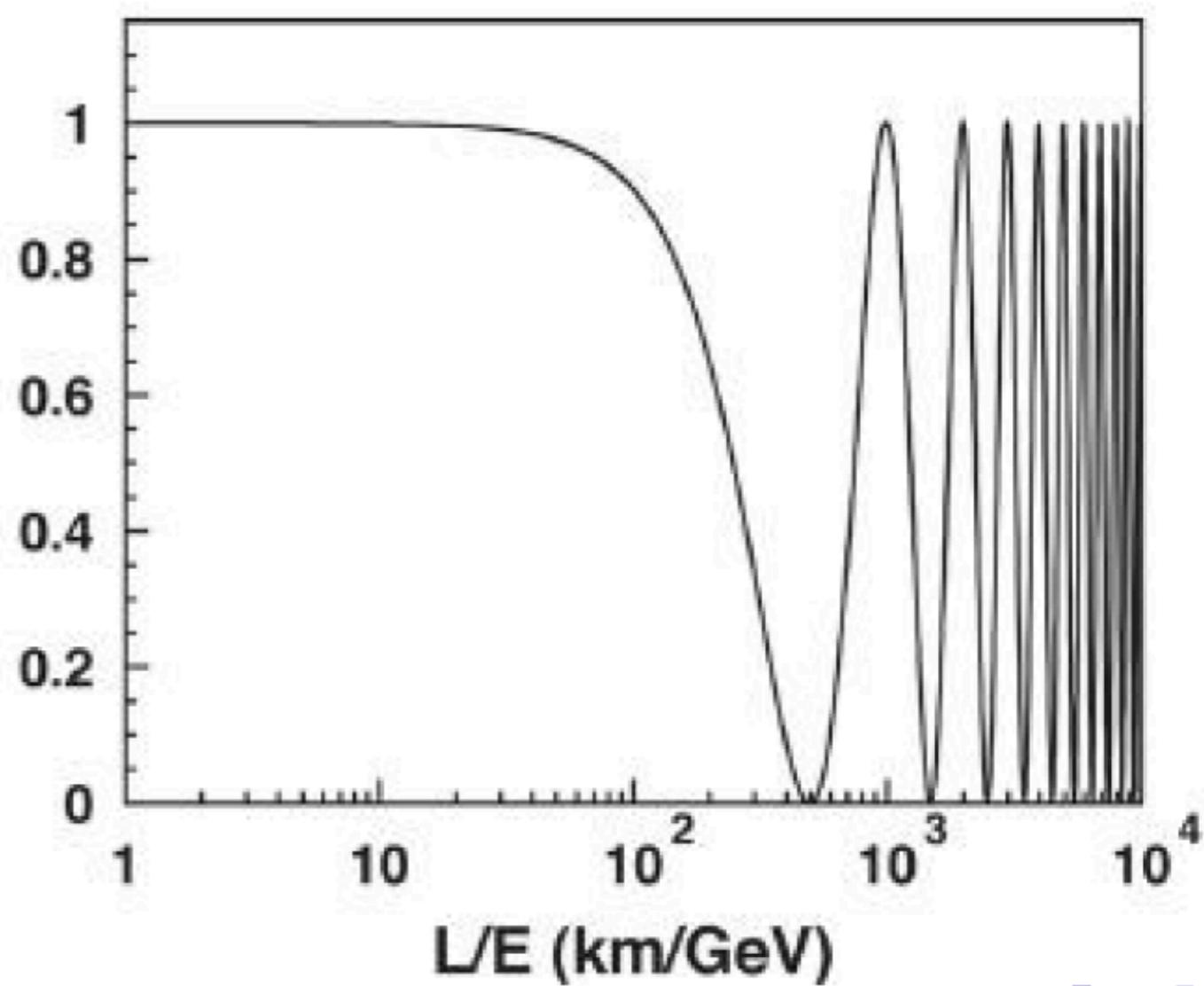
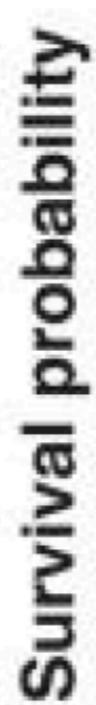
$$+ \left(1 - s_{13}^2 \frac{\cos 2\theta_{23}}{c_{23}^2}\right) P_{\nu_\mu \rightarrow \nu_\mu}^{2g}(\Delta m_{31}^2, \theta_{23})$$



$$0.003 \text{ eV}^2$$



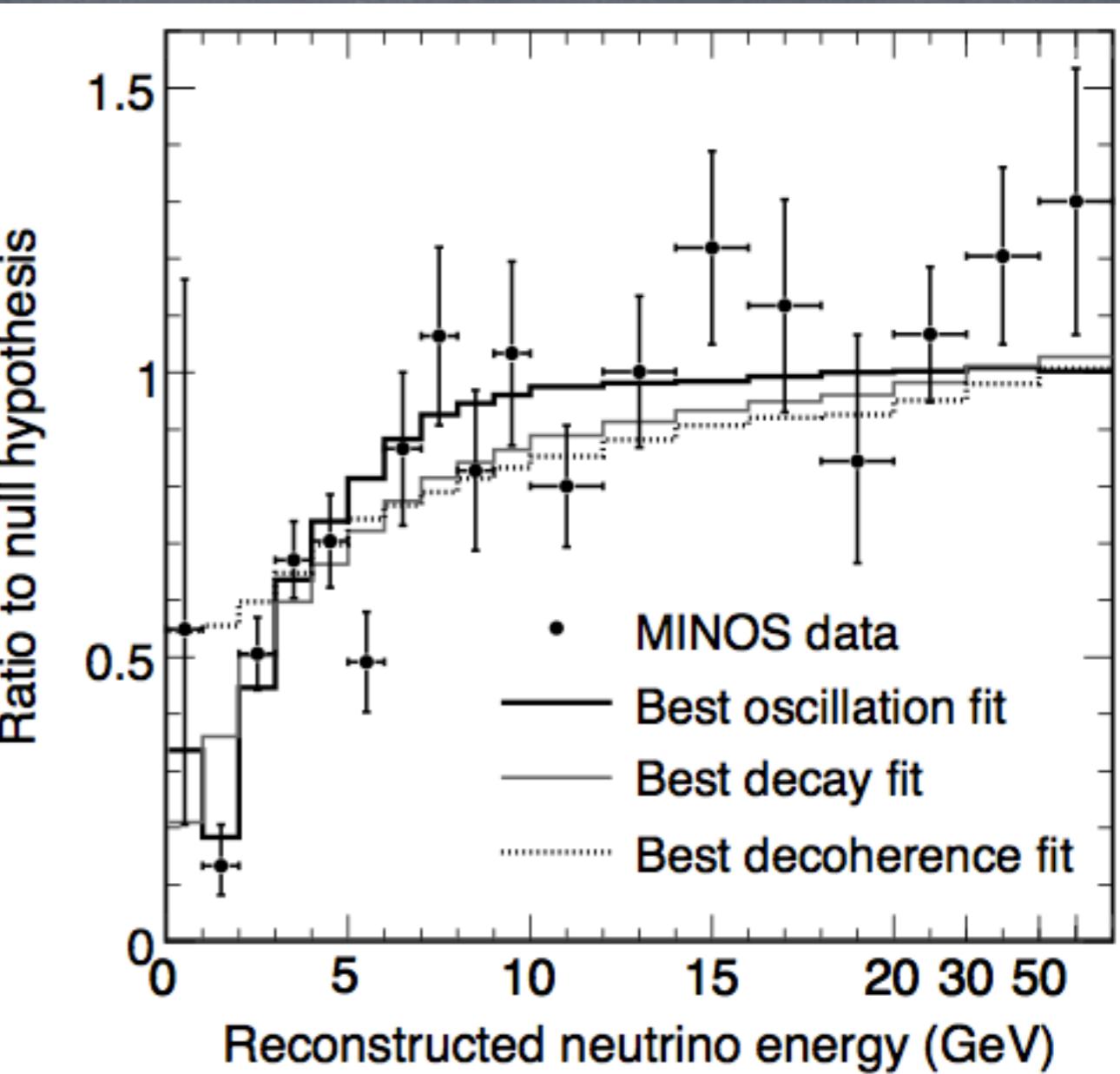
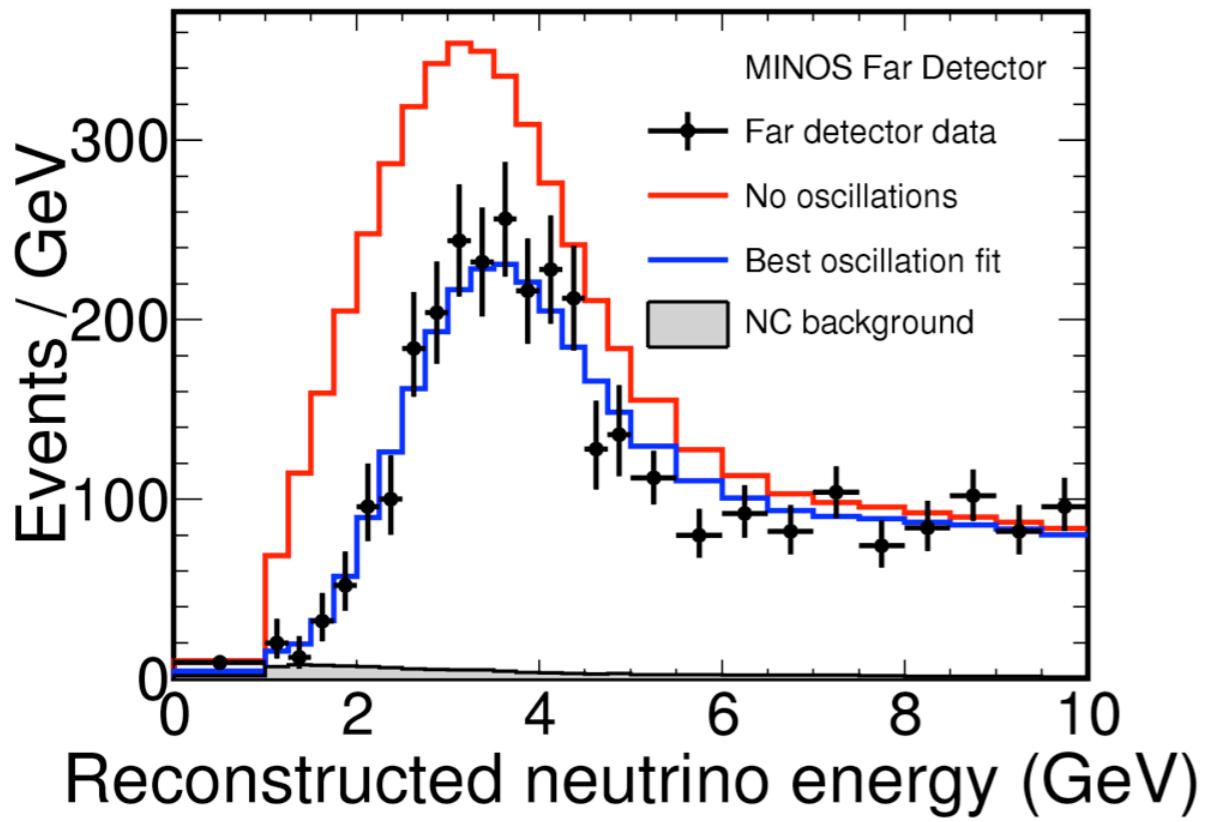
| 0.03 eV²



Accelerator Neutrinos

$$\pi^+ \rightarrow \mu^+ + \nu_\mu$$

MINOS :
 $\nu_\mu \rightarrow \nu_\mu$



$$P_{\nu_\mu \rightarrow \nu_\mu}^{3g} \sim s_{13}^2 \frac{\cos 2\theta_{23}}{c_{23}^2} + (1 - s_{13}^2 \frac{\cos 2\theta_{23}}{c_{23}^2}) P_{\nu_\mu \rightarrow \nu_\mu}^{2g}(\Delta m_{31}^2, \theta_{23})$$

$$\bar{\nu}_e \rightarrow \bar{\nu}_e$$

Reactor Experiments

$$P_{\bar{\nu}_e \rightarrow \bar{\nu}_e}^{3g} = 1 - c_{13}^4 \sin^2 2\theta_{12} \sin^2 \Delta_{21} - s_{12}^2 \sin^2 2\theta_{13} \sin^2 \Delta_{32} \\ - c_{12}^2 \sin^2 2\theta_{13} \sin^2 \Delta_{32}$$

$$\Delta_{ij}=\frac{\Delta m_{ij}^2 L}{4E}$$

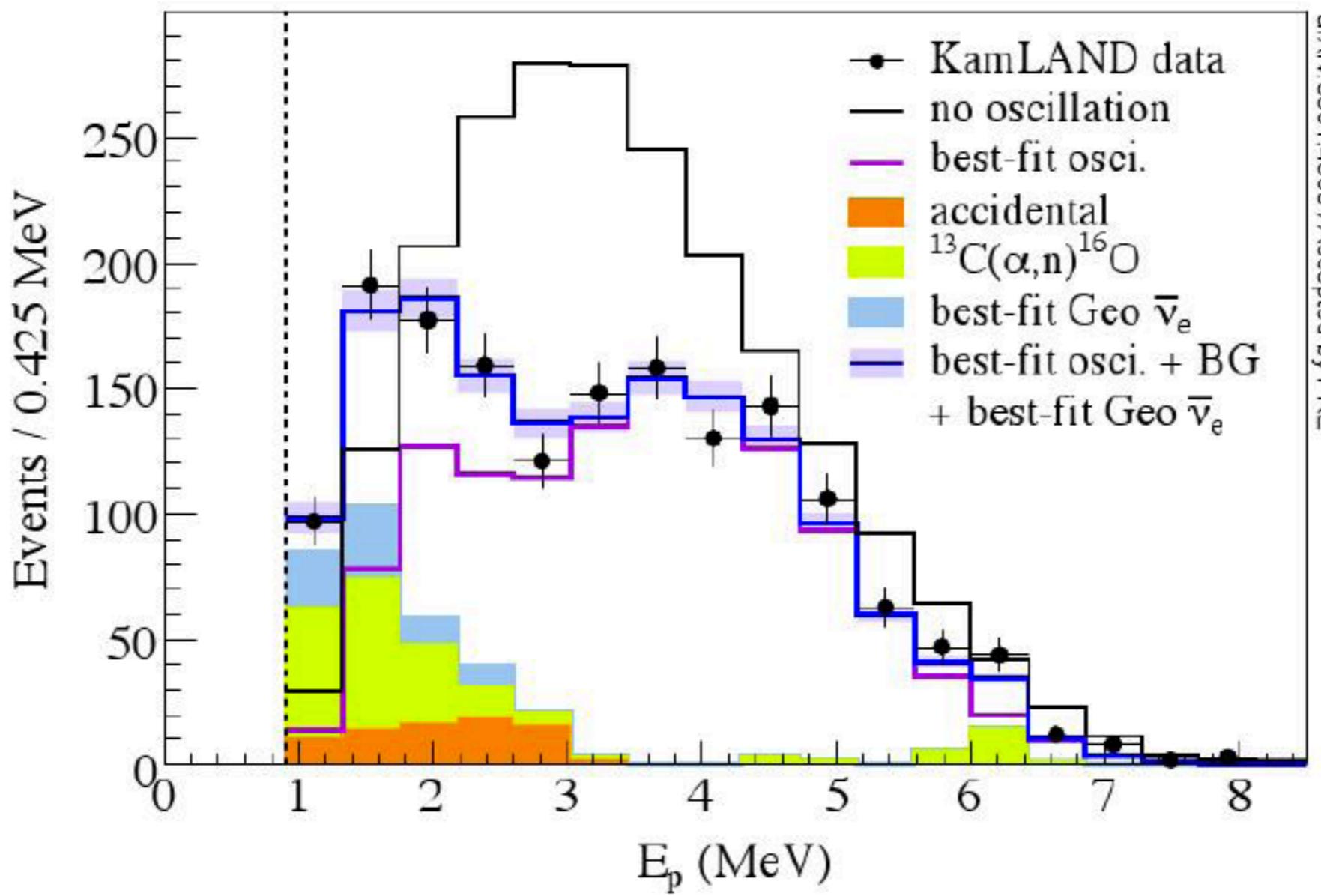
$$P_{\bar{\nu}_e \rightarrow \bar{\nu}_e}^{3g} \sim 1 - \sin^2 2\theta_{13} \sin^2 \left(\frac{\Delta m_{32}^2 L}{2E} \right)$$

$\bar{\nu}_e \rightarrow \bar{\nu}_e$

KamLAND

$$P_{\nu_e \rightarrow \nu_e}^{3g} = \sin^4 \theta_{13} + \cos^4 \theta_{13} P_{\nu_e \rightarrow \nu_e}^{2g}(\Delta m_{12}^2, \theta_{12})$$

From Mar 9, 2002 to May 12, 2007
1491 live days, 2881 ton-year exposure (3.8x KL2004)



baseline

180 km

$E \sim 3$ MeV

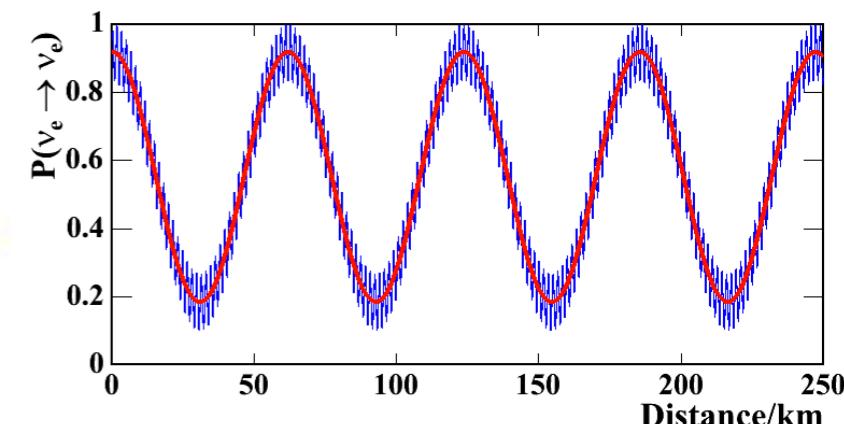
$|\Delta m_{31}^2| \sim 3 \times 10^{-3} \text{ eV}^2$

$L_{\text{osc}}^{31} \sim 2.5 \text{ km}$

averaged !

$\Delta m_{21}^2 \sim 8 \times 10^{-5} \text{ eV}^2$

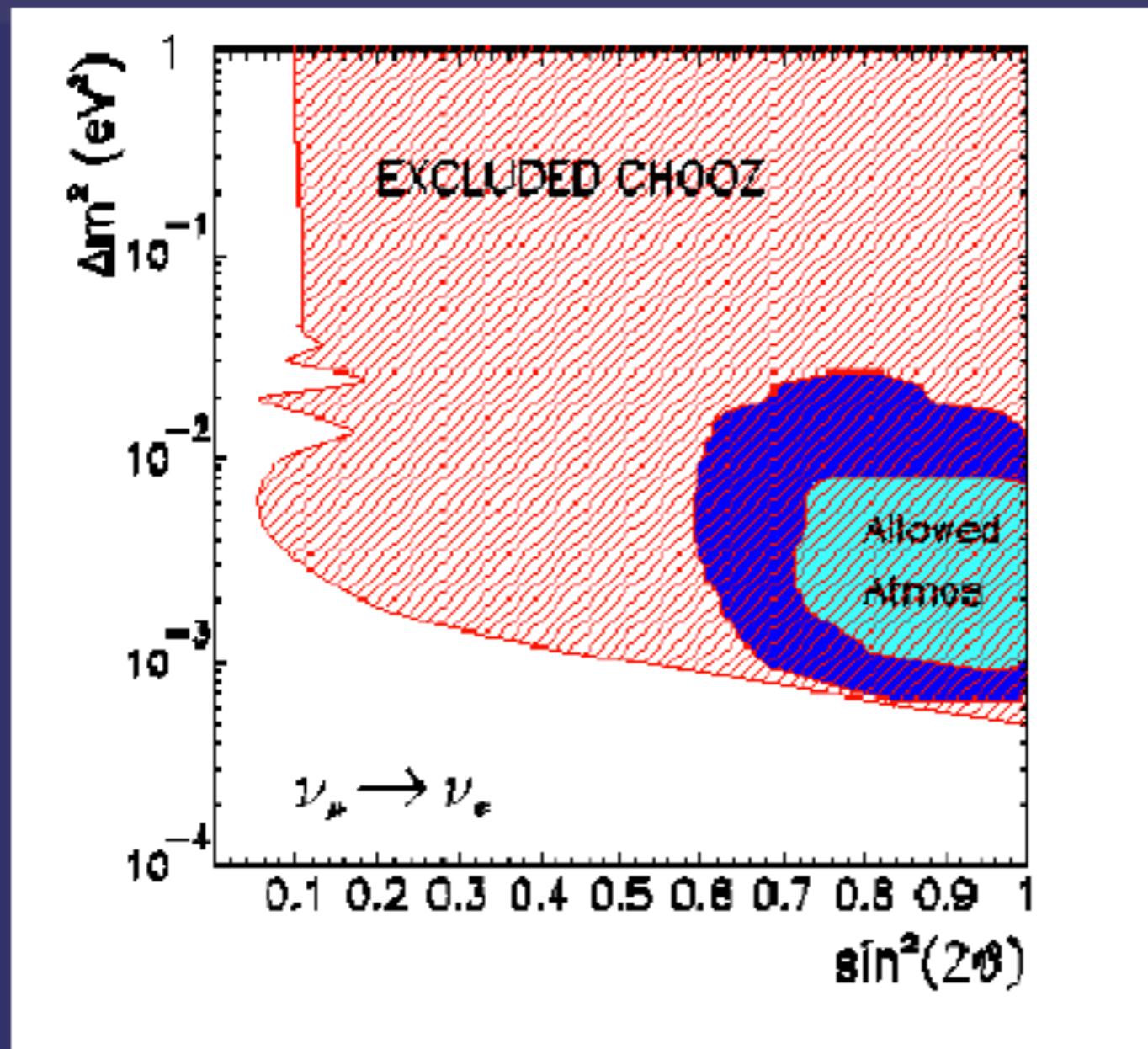
$L_{\text{osc}}^{21} \sim 100 \text{ km}$



Reactors

$\bar{\nu}_e \rightarrow \bar{\nu}_e$

CHOOZ (1999)



$$P^{\text{osc}} < 0,05$$

$$\begin{aligned} \sin^2 2\theta_{13} &< 0,15 \\ \sin^2 \theta_{13} &< 0,04 \end{aligned}$$

$$L_{ij}^{\text{osc}} = \frac{4\pi E}{\Delta m_{ij}^2}$$

$$P_{ee}^{\text{CHOOZ}} \approx 1 - \sin^2 2\theta_{13} \sin^2 \left(\frac{\pi L}{L_{32}^{\text{osc}}} \right)$$

baseline

1 km

$E \sim 3 \text{ MeV}$

$|\Delta m_{31}^2| \sim 3 \times 10^{-3} \text{ eV}^2$

$L_{\text{osc}}^{\text{31}} \sim 2.5 \text{ km}$

$|\Delta m_{21}^2| \sim 8 \times 10^{-5} \text{ eV}^2$

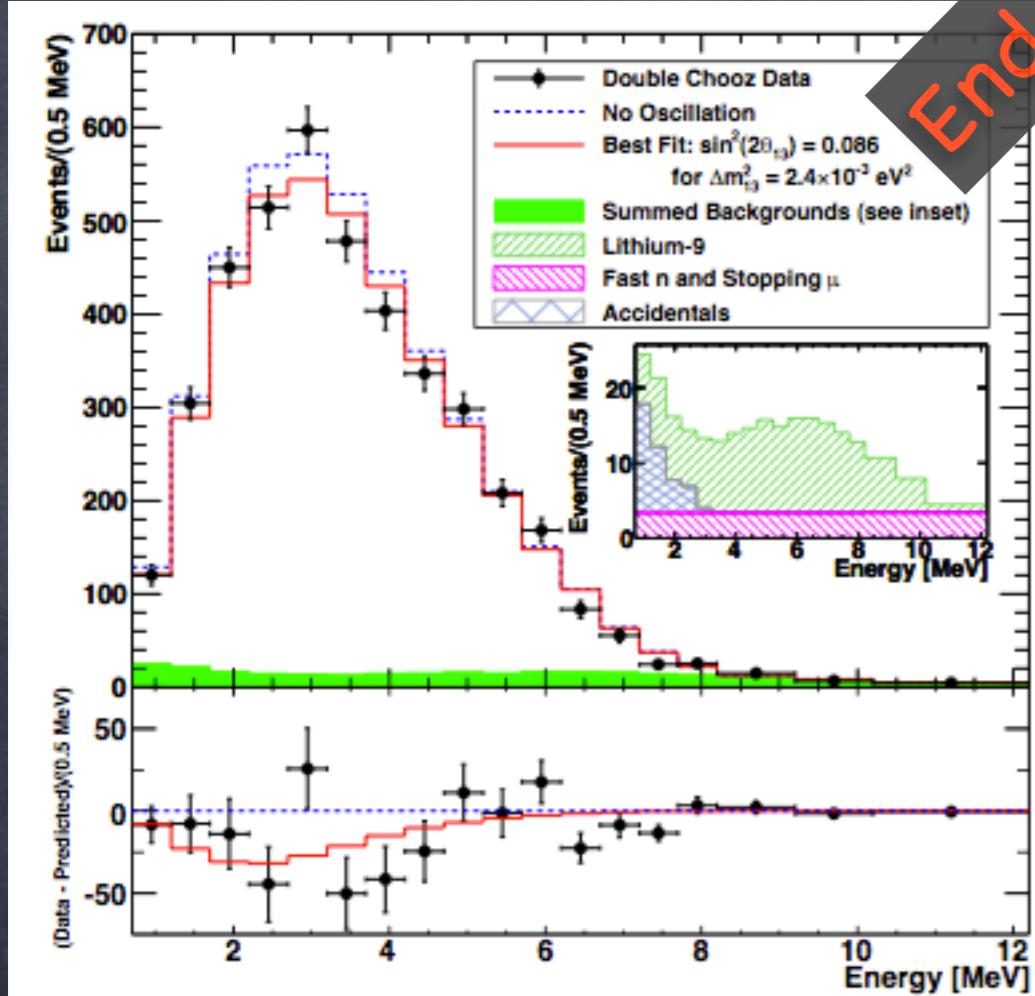
$L_{\text{osc}}^{\text{21}} \sim 100 \text{ km}$

do not
contribute at all

$\bar{\nu}_e$

Double Chooz

End of 2011



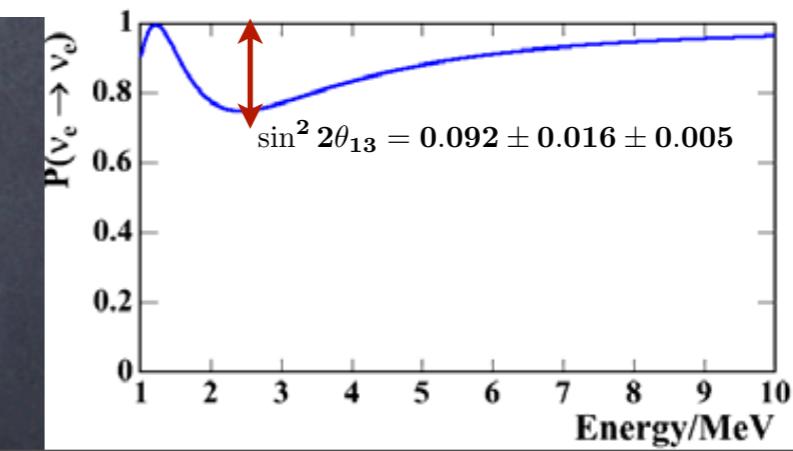
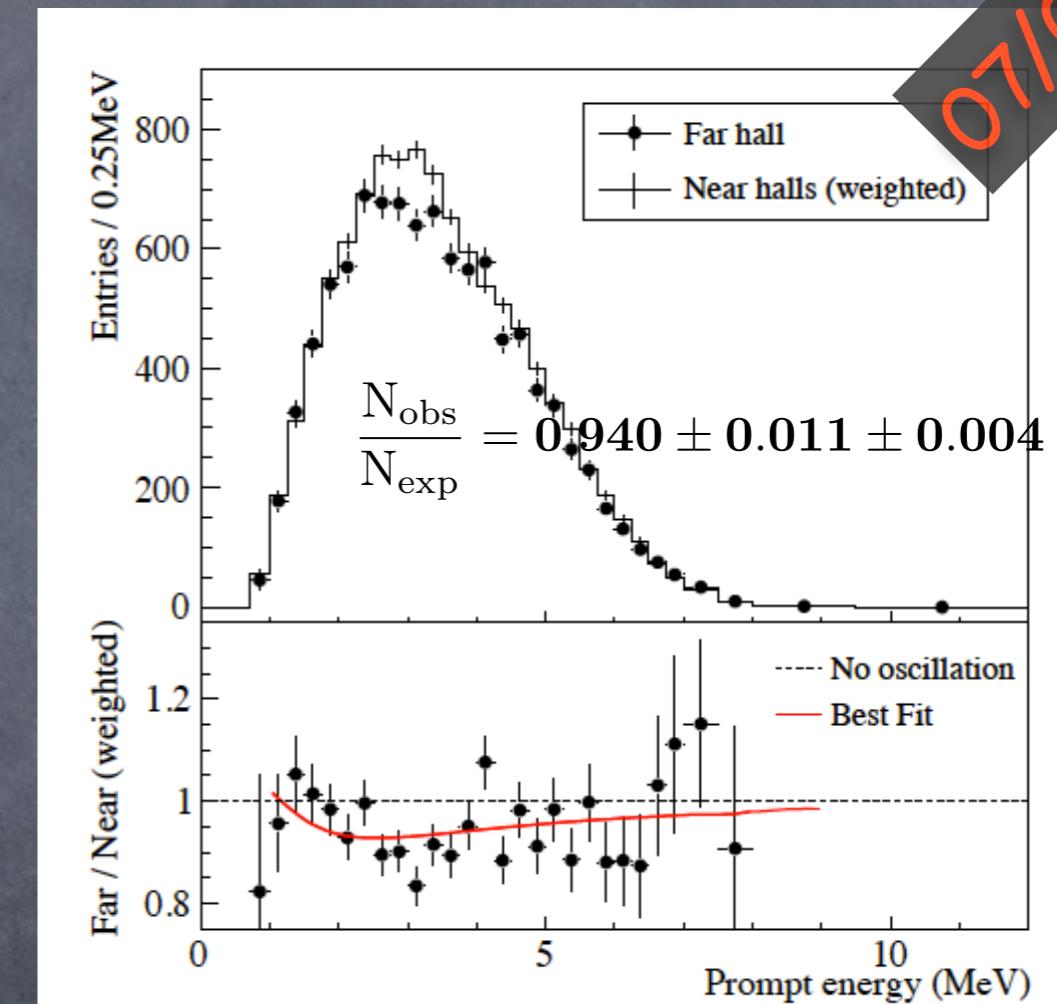
$$\frac{N_{\text{obs}}}{N_{\text{exp}}} = 0.944 \pm 0.016 \pm 0.040$$

$$\sin^2 2\theta_{13} = 0.086 \pm 0.041 \pm 0.030$$

$\bar{\nu}_e$

RENO, Daya Bay

01/03/2012



$$\nu_\mu \rightarrow \nu_e$$

Accelerator Experiments

$$P_{\nu_\mu \rightarrow \nu_e}^{3g} = |2U_{\mu 3}^* U_{e3} \sin \Delta_{31} e^{-i\Delta_{32}} + 2U_{\mu 2}^* U_{e2} \sin \Delta_{21}|^2$$

Sensitivity to δ

$$P_{\nu_\mu \rightarrow \nu_e}^{3g} \sim P_{\text{atm}} + 2\sqrt{P_{\text{atm}}} \sqrt{P_{\text{sol}}} \cos(\Delta_{32} + \delta) + P_{\text{sol}}$$

$$\sqrt{P_{\text{sol}}} \equiv c_{23} c_{13} \sin 2\theta_{12} \sin \Delta_{21}$$

$$\sqrt{P_{\text{atm}}} \equiv s_{23} \sin 2\theta_{13} \sin \Delta_{31}$$

P($\nu_\mu \rightarrow \nu_e$) @ T2K

P($\nu_\mu \rightarrow \nu_e$) @ T2K

atmospheric term

$$P(\nu_\mu \rightarrow \nu_e) \approx \sin^2 2\theta_{13} \sin^2 \theta_{23} \frac{\sin^2 \Delta(1-a)}{(1-a)^2}$$

$$\Delta \equiv \frac{\Delta m_{31}^2 L}{4 E}$$

$$a \equiv \frac{2\sqrt{2} G_F N_e E}{\Delta m_{31}^2} \quad |a| \approx 0.06$$

$$\epsilon \equiv \frac{\Delta m_{21}^2}{\Delta m_{31}^2} \approx 0.03$$

P($\nu_\mu \rightarrow \nu_e$) @ T2K

atmospheric term

$$P(\nu_\mu \rightarrow \nu_e) \approx \sin^2 2\theta_{13} \sin^2 \theta_{23} \frac{\sin^2 \Delta(1-a)}{(1-a)^2}$$

solar term

$$+ \epsilon^2 \sin^2 2\theta_{12} \cos^2 \theta_{23} \frac{\sin^2 \Delta a}{a^2}$$

$$\Delta \equiv \frac{\Delta m_{31}^2 L}{4 E}$$

$$a \equiv \frac{2\sqrt{2} G_F N_e E}{\Delta m_{31}^2} \quad |a| \approx 0.06$$

$$\epsilon \equiv \frac{\Delta m_{21}^2}{\Delta m_{31}^2} \approx 0.03$$

P($\nu_\mu \rightarrow \nu_e$) @ T2K

atmospheric term

$$P(\nu_\mu \rightarrow \nu_e) \approx \sin^2 2\theta_{13} \sin^2 \theta_{23} \frac{\sin^2 \Delta(1-a)}{(1-a)^2}$$

interference term

$$+ \epsilon \sin 2\theta_{13} \sin 2\theta_{12} \sin 2\theta_{23} \cos(\Delta + \delta) \frac{\sin \Delta a}{a} \frac{\sin \Delta(1-a)}{1-a}$$

solar term

$$+ \epsilon^2 \sin^2 2\theta_{12} \cos^2 \theta_{23} \frac{\sin^2 \Delta a}{a^2}$$

$$\Delta \equiv \frac{\Delta m_{31}^2 L}{4 E} \quad a \equiv \frac{2\sqrt{2} G_F N_e E}{\Delta m_{31}^2} \quad |a| \approx 0.06$$

$$\epsilon \equiv \frac{\Delta m_{21}^2}{\Delta m_{31}^2} \approx 0.03$$

P($\nu_\mu \rightarrow \nu_e$) @ T2K

atmospheric term

$$P(\nu_\mu \rightarrow \nu_e) \approx \sin^2 2\theta_{13} \sin^2 \theta_{23} \frac{\sin^2 \Delta(1-a)}{(1-a)^2}$$

interference term

$$+ \epsilon \sin 2\theta_{13} \sin 2\theta_{12} \sin 2\theta_{23} \cos(\Delta + \delta) \frac{\sin \Delta a}{a} \frac{\sin \Delta(1-a)}{1-a}$$

solar term

$$+ \epsilon^2 \sin^2 2\theta_{12} \cos^2 \theta_{23} \frac{\sin^2 \Delta a}{a^2}$$

$$\Delta \equiv \frac{\Delta m_{31}^2 L}{4 E}$$

$$\epsilon \equiv \frac{\Delta m_{21}^2}{\Delta m_{31}^2} \approx 0.03$$

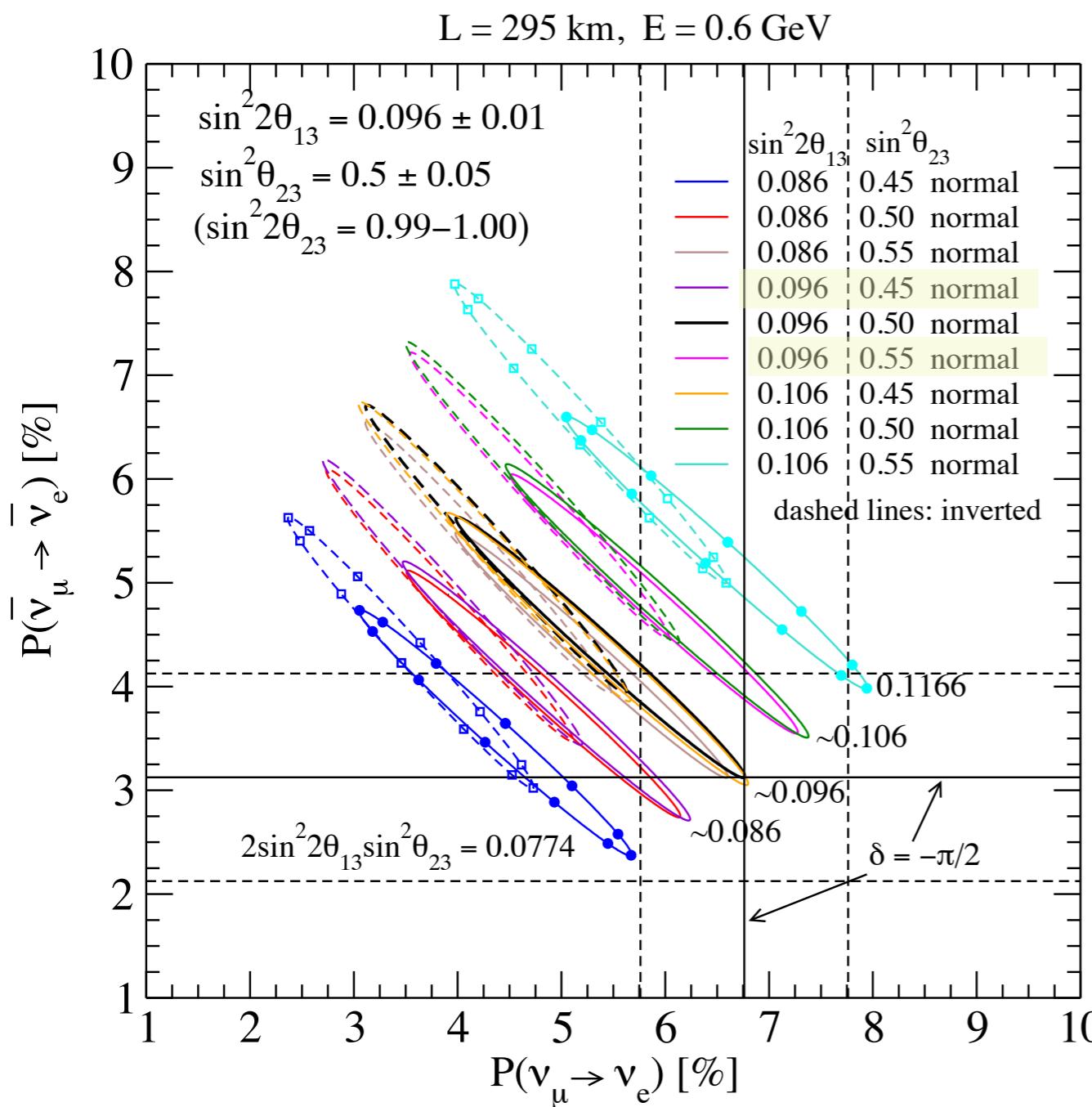
$$a \equiv \frac{2\sqrt{2} G_F N_e E}{\Delta m_{31}^2} \quad |a| \approx 0.06$$

$$\nu \rightarrow \bar{\nu}$$

$$a \rightarrow -a \quad \delta \rightarrow -\delta$$

$$P(\nu_\mu \rightarrow \nu_e) \text{ & } P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$$

Plagued with degeneracies



CP degeneracy (δ , θ_{13})

solved by reactor experiments

$$\text{sgn } \Delta m^2_{31}$$

$$\sin \delta > 0 \quad \text{NH}$$

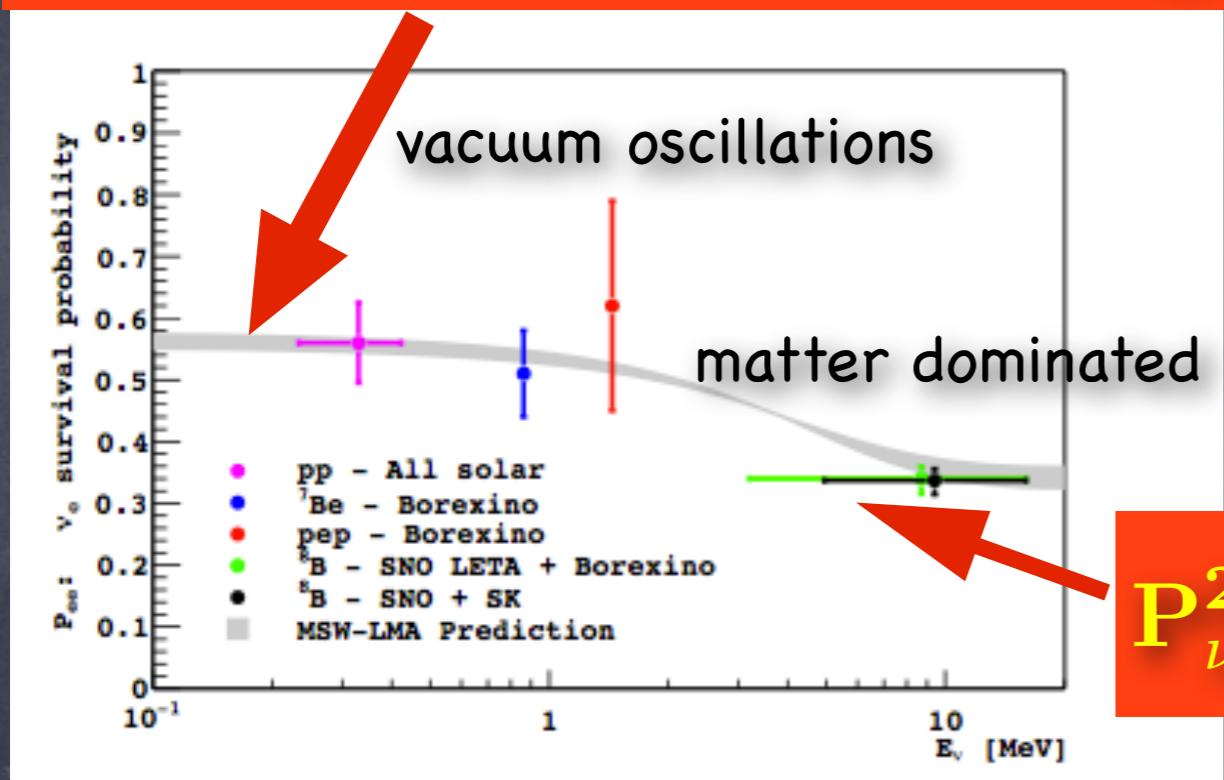
$\sin \delta < 0$ IH

$$\clubsuit (\theta_{23}, \pi/2 - \theta_{23})$$

octant degeneracy

Solar Neutrinos

$$P_{\nu_e \rightarrow \nu_e}^{2g}(\Delta m_{12}^2, \theta_{12}) \sim 1 - \frac{1}{2} \sin^2 2\theta_{12}$$



MSW Effect

$$P_{\nu_e \rightarrow \nu_e}^{2g-mat}(\Delta m_{12}^2, \theta_{12}) \sim \sin^2 \theta_{12}$$

$$P_{\nu_e \rightarrow \nu_e}^{3g} = \sin^4 \theta_{13} + \cos^4 \theta_{13} P_{\nu_e \rightarrow \nu_e}^{2g-mat}(\Delta m_{12}^2, \theta_{12})$$

$$V_e = \sqrt{2} G_F n_e \quad n_e \rightarrow n_e \cos^2 \theta_{13}$$

$$L_{31,32}^{\text{osc}} = \frac{4\pi E}{|\Delta m_{31,32}^2|} \ll L_{\text{Sun-Earth}} \quad \text{those are averaged}$$

Proposed Problems:

E7.

Show explicitly that the Majorana phases do not enter in the neutrino oscillation probabilities

E8.

Show how we get the 1.27 factor in the probability

$$P_{e\mu}(L) = \sin^2 2\theta \sin^2 \left(1.27 \frac{\Delta m_{21}^2}{\text{eV}^2} \frac{L}{m} \frac{\text{MeV}}{E} \right)$$

E9.

Why neutrino oscillation might exhibit CP violation in matter even though they might conserve CP in vacuum?

Proposed Problems:

E10.

Neutrinos and anti-neutrinos of all flavors are produced inside a Supernova. Discuss the level crossing diagrams for neutrinos and anti-neutrinos for the normal and inverse mass hierarchy. Show there is a low and a high resonance and estimate the matter density at the these two resonance points.

A landscape painting featuring warm autumnal tones of orange, yellow, and red. A path or road leads from the bottom left towards a dense, dark forest on a hillside. The sky is filled with soft, hazy clouds.

Thank You!