

Basics of Neutrino Physics

Renata Zukanovich Funchal
Universidade de São Paulo, Brazil

"False facts are highly injurious to
the progress of science, for they
often endure long; but false views, if
supported by some evidence, do little
harm, for every one takes a salutary
pleasure in proving their falseness."

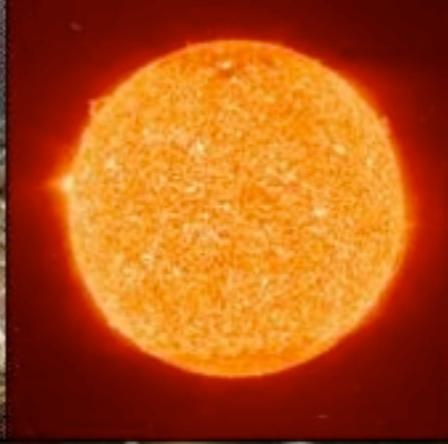
Charles Darwin

Neutrinos are everywhere ...

Nuclear Reactors



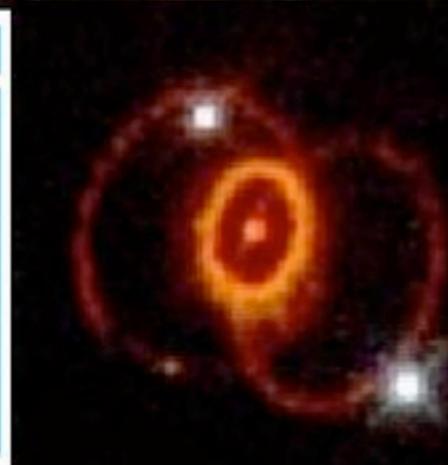
SUN



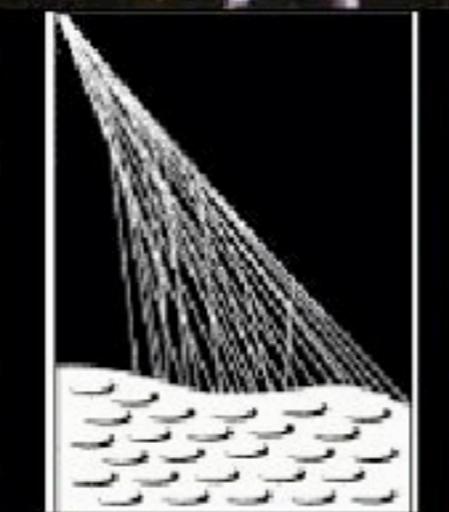
Accelerators



Supernova
(Stellar Collapse)



Atmospheric
(Cosmic Rays)

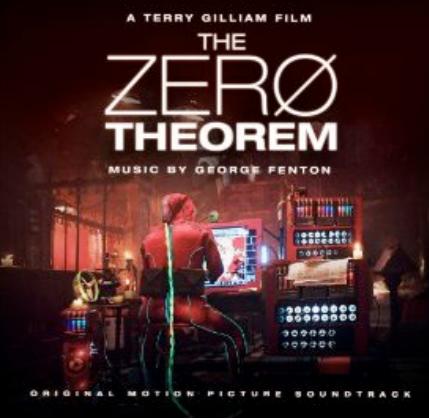


Astrophysical
Accelerators

Earth's
Crust/Mantle



Big Bang
($330 \nu /cm^3$)



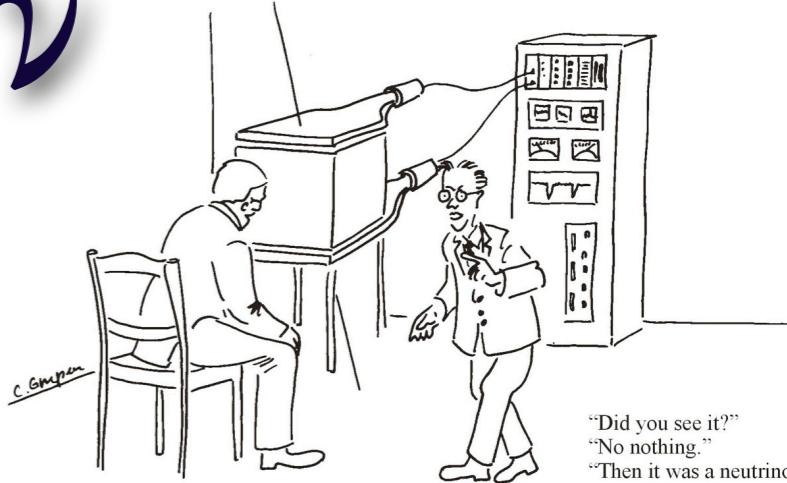
The Imponderable Lightness of ν

Cosmic Gall
by John Updike (1963)

Neutrinos, they are very small.
They have no charge and have no mass,
And do not interact at all.
The earth is just a silly ball
To them, through which they simply pass
Like dirt maids down a drafty hall,
Or photons through a sheet of glass.
They snub the most exquisite gas,
Ignore the most substantial wall,
Cold shoulder steel and sounding brass,

Insult the stallion in his stall,
And, scorning barriers of class,
Infiltrate you and me! Like tall
And painless guillotines, they fall
Down through our heads into the
grass.

At night, they enter from Nepal
And pierce the lover and his lass
From underneath the bed. You call
It wonderful; I call it crass.



Lectures :

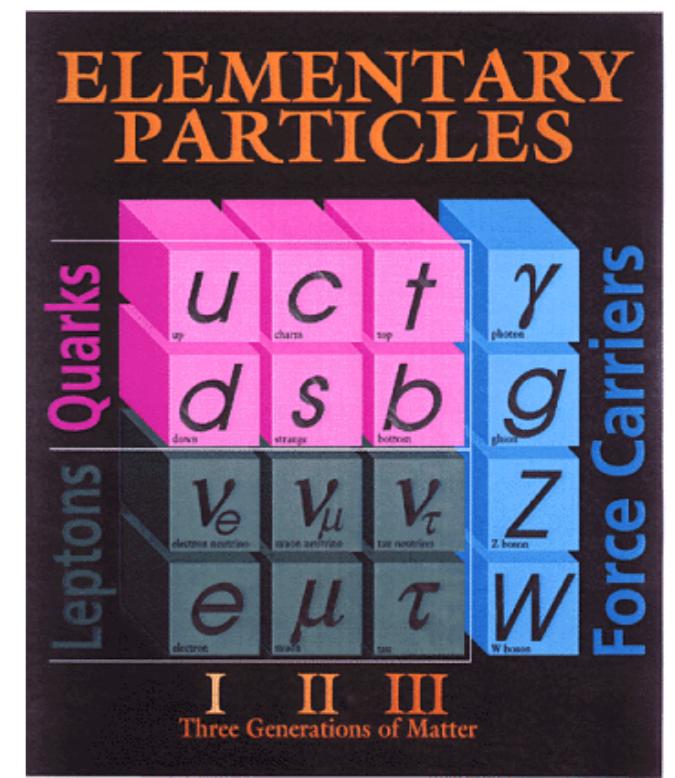
1. Basic Neutrino Properties
2. Neutrino Oscillations

Lecture 1

Basic Neutrino
Properties



1. From Discovery to the Standard Model

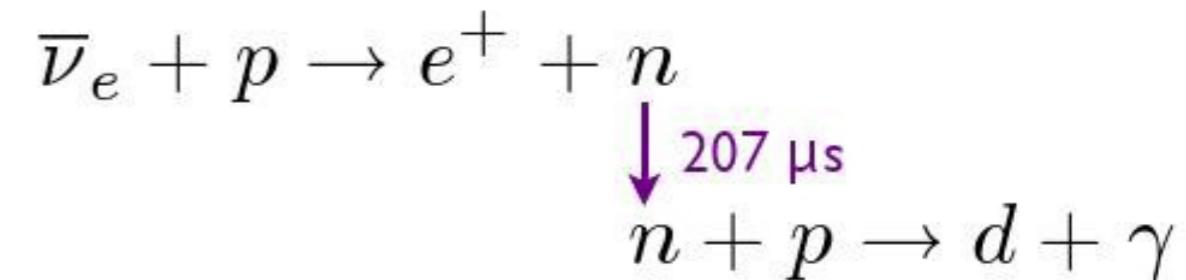
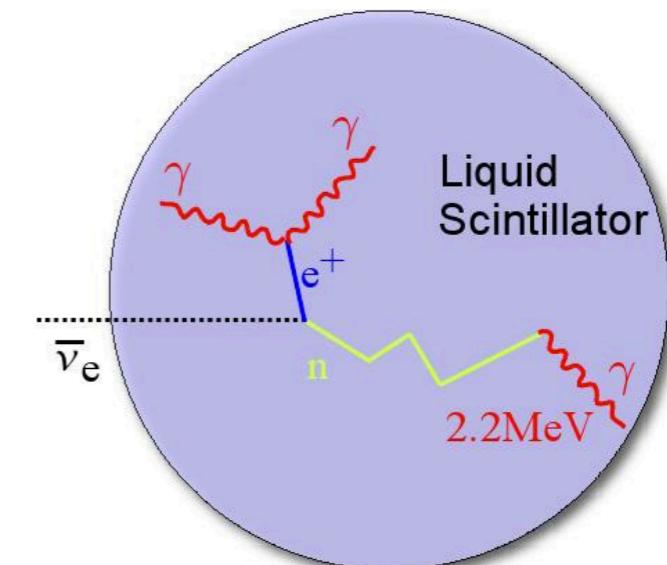


ν_e

Savannah River, South Carolina, EUA

Discovery of the First Neutrino

the electron neutrino



1956 : Fred Reines & Clyde Cowan

C.L. Cowan Jr, et al. Science 124, 103 (1956)

F. Reines and C.L. Cowan Jr, Nature 178, 446 (1956)

“We are happy to inform you that we have definitely detected neutrinos ...”

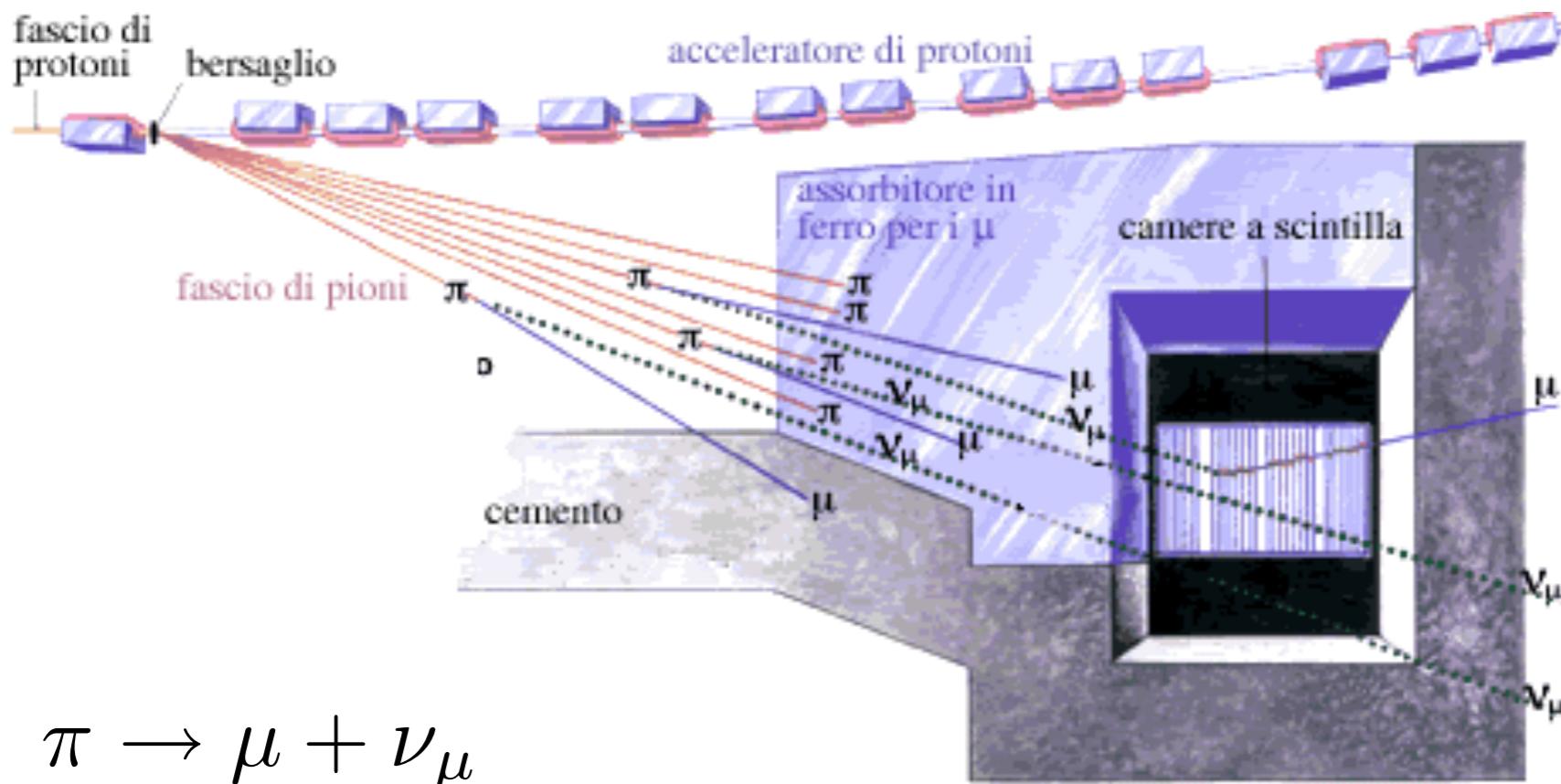
ν_μ

Discovery of the Second Neutrino

1962 : Steinberger, Lederman &
Schwartz

the muon neutrino

$$p + p \rightarrow \pi + X$$



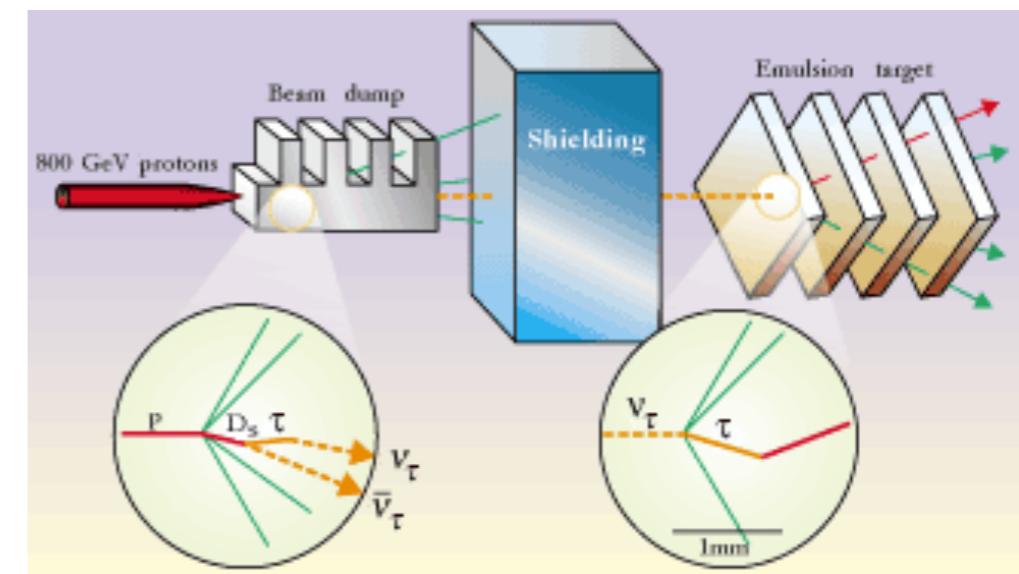
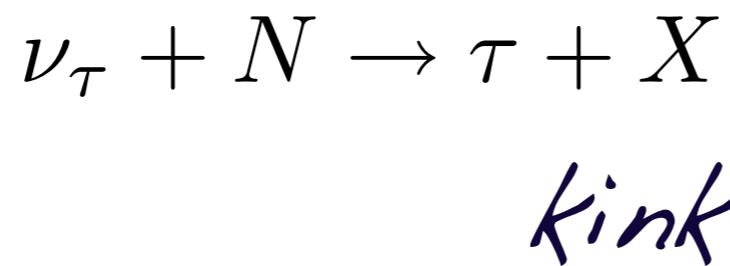
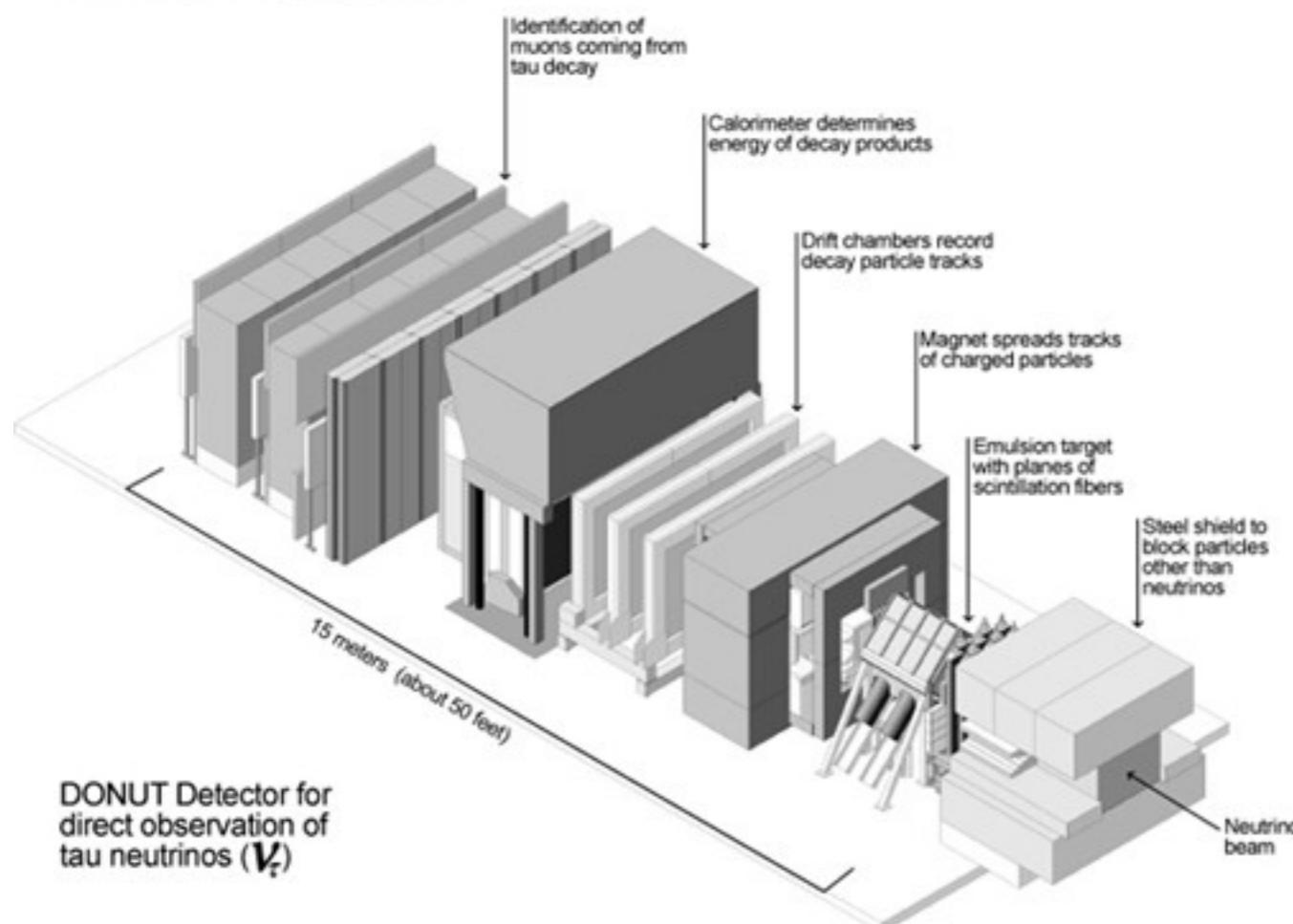
ν_τ

Discovery of the Third Neutrino

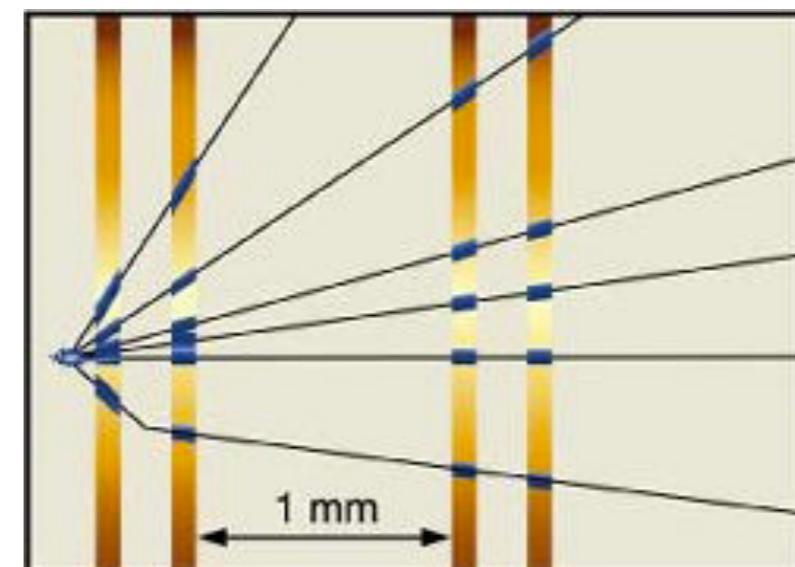
the tau neutrino

2000 : DONUT Collaboration

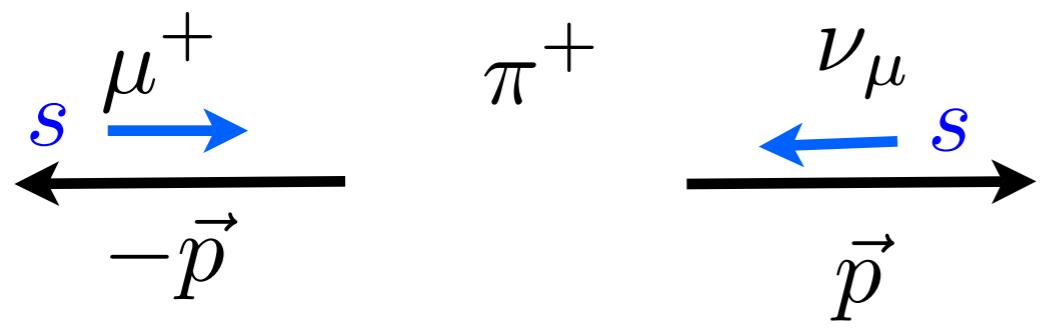
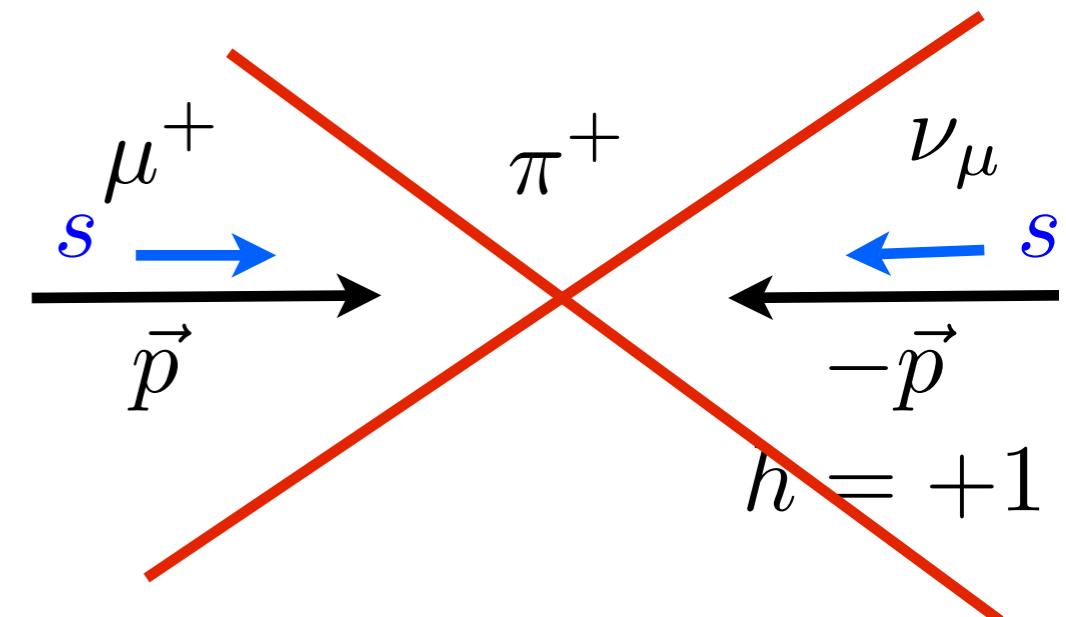
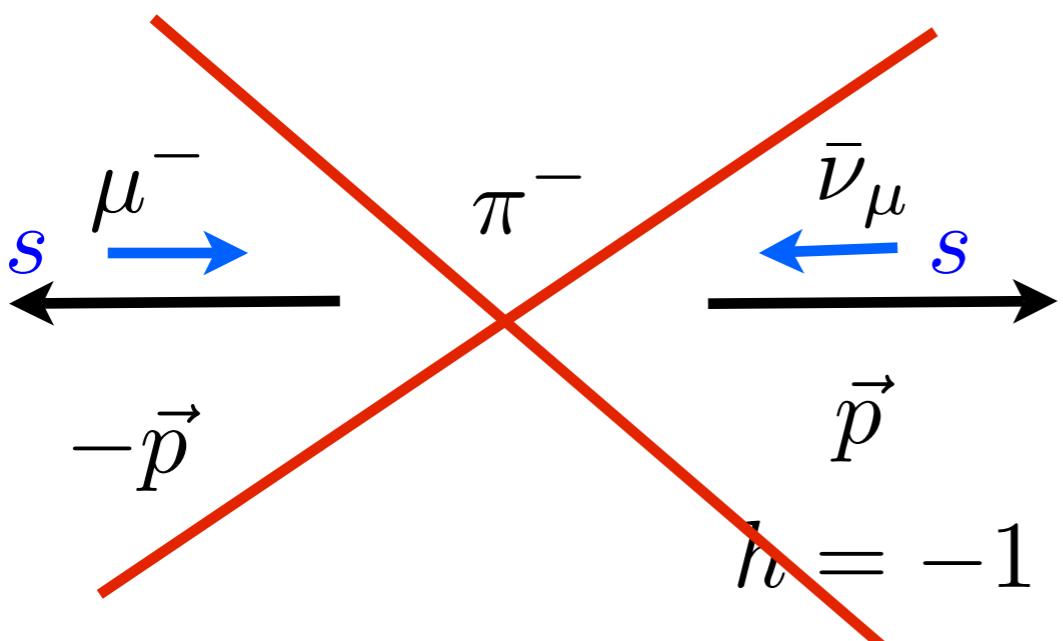
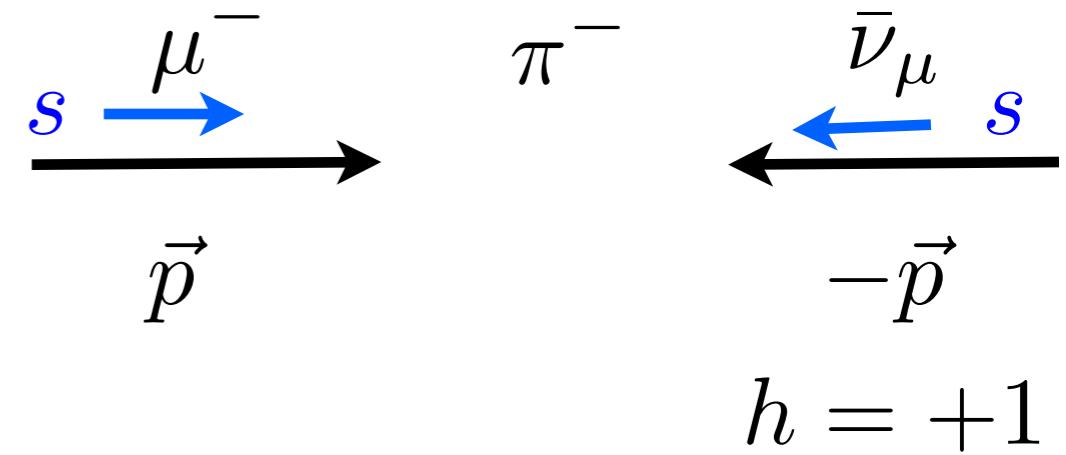
DONUT Detector



$$\tau \rightarrow \nu_\tau + \pi$$



Parity & Charge Conjugation Violation


 \hat{P}

 \hat{C}

 \hat{P}


V-A interaction

$$\bar{\Psi} \gamma^\mu \Psi \rightarrow \bar{\Psi} \gamma^\mu \frac{1}{2}(1 - \gamma^5) \Psi$$

The Standard Model

$$SU(3)_C \times SU(2)_L \times U(1)_Y$$

Electroweak Symmetry Group

- $SU(2)_L$: weak isospin Group Generators: I_a ($a = 1, 2, 3$)

with $[I_a, I_b] = i \epsilon_{abc} I_c$

e.g. in 2D representation $I_a = \tau_a / 2$

- $U(1)_Y$: hypercharge Group Generator: Y

The action of Y on fermion fields is constrained by

Gell-Mann-Nishijima Relation

$$Q = I_3 + Y$$

The Standard Model

Representations of the fermion fields (which lead to the correct phenomenology) is

left-handed (L) chiral components: weak isospin doublets

LEPTONS

$$L_e = \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}$$

$$L_\mu = \begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix}$$

$$L_\tau = \begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix}$$

$$L_\alpha \equiv (2, -1/2)$$

QUARKS

$$Q_u = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$$

$$Q_c = \begin{pmatrix} c_L \\ s_L \end{pmatrix}$$

$$Q_t = \begin{pmatrix} t_L \\ b_L \end{pmatrix}$$

$$Q_\alpha \equiv (2, 1/6)$$

The Standard Model

Representations of the fermion fields (which lead to the correct phenomenology) is

right-handed (R) chiral components: weak isospin singlets

LEPTONS

$$e_R, \mu_R, \tau_R$$

$$E_\alpha = (1, -1)$$

~~$\nu_{eR}, \nu_{\mu R}, \nu_{\tau R}$~~

not present in the SM

QUARKS

$$u_R, c_R, t_R$$

$$U_\alpha \equiv (1, 2/3)$$

$$\alpha = u, c, t$$

$$d_R, s_R, b_R$$

$$D_\alpha \equiv (1, -1/3)$$

$$\alpha = d, s, b$$

Standard Model

Quarks

<i>u</i>	<i>c</i>	<i>t</i>
up	charm	top

<i>d</i>	<i>s</i>	<i>b</i>
down	strange	bottom

Forces

<i>Z</i>	γ
Z boson	photon

<i>W</i>	<i>g</i>
W boson	gluon

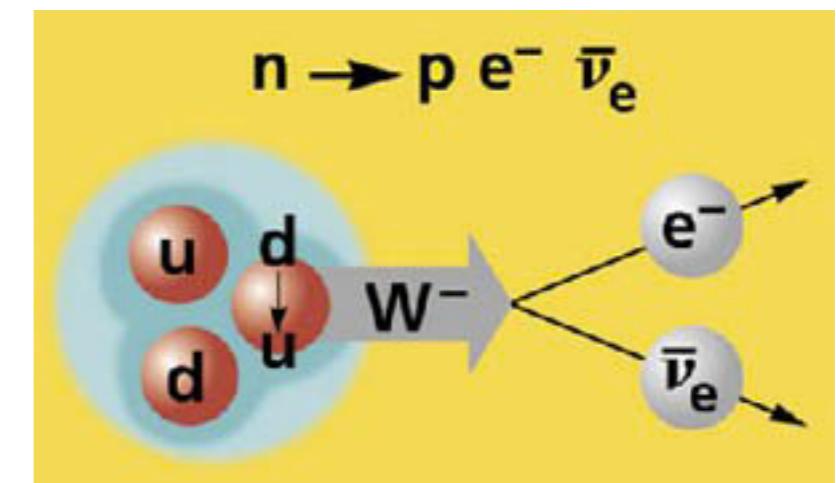
Higgs
boson

<i>e</i>	μ	τ
electron	muon	tau

ν_e	ν_μ	ν_τ
electron neutrino	muon neutrino	tau neutrino

Leptons

$$\mathcal{L}_{cc} = -\frac{g}{\sqrt{2}} j_{cc}^\mu W_\mu + \text{h.c.}$$

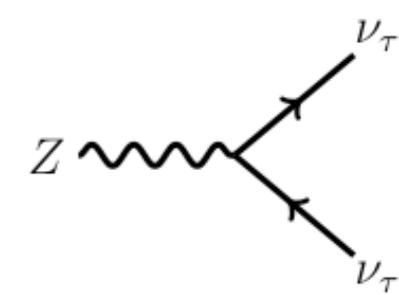
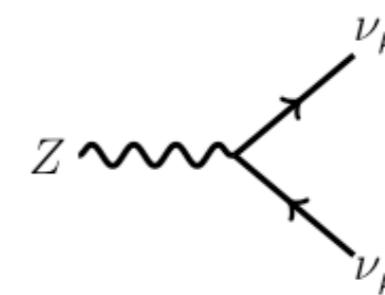
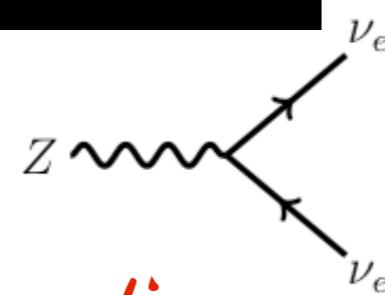


$$\mathcal{L}_{nc} = -\frac{g}{\cos \theta_W} j_{nc}^\mu Z_\mu + \text{h.c.}$$

$$j_{cc}^\mu = \bar{f}_\alpha \gamma^\mu P_L f'_\alpha$$

$$j_{nc}^\mu = \bar{f}_\alpha \gamma^\mu P_L f_\alpha$$

flavor diagonal



The Standard Model

Since L and R components of the fermion fields transform in different way, the presence of a bare mass term

$$\mathcal{L}_{\text{mass}} \propto \bar{f}f = \bar{f}_L f_R + \bar{f}_R f_L$$

in the SM Lagrangian is forbidden by $SU(2)_L \times U(1)_Y$

symmetry \Rightarrow Fermion masses generated by the
HIGGS MECHANISM

$$SU(2)_L \times U(1)_Y \Rightarrow U(1)_Q$$

after spontaneous symmetry breaking
EWSB

The Standard Model

fermion masses arise from Yukawa interactions

$$-\mathcal{L}_Y = y_{\alpha\beta}^d \bar{\mathbf{Q}}_\alpha \Phi \mathbf{D}_\beta + y_{\alpha\beta}^u \bar{\mathbf{Q}}_\alpha \tilde{\Phi} \mathbf{U}_\beta + y_{\alpha\beta}^\ell \bar{\mathbf{L}}_\alpha \Phi \mathbf{E}_\beta + \text{h.c.}$$

$$\Phi(\mathbf{x}) = \begin{pmatrix} \Phi^+(x) \\ \Phi^0(x) \end{pmatrix} \equiv (2, 1/2) \quad \tilde{\Phi}(\mathbf{x}) = i\tau_2 \Phi(\mathbf{x})^* \equiv (2, -1/2)$$

$$\Phi \rightarrow \langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

Higgs acquires a vev

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h \end{pmatrix}$$

Unitary Gauge

since we do not ν_R have no mass @tree-level

but can they acquire mass by loop corrections ?

Can we have $m_\nu \neq 0$ in the
SM ?

a loop correction could induce an effective mass
term like

$$\frac{y_{\alpha\beta}^\nu}{v} \Phi \Phi L_\alpha L_\beta$$

But the SM has an accidental global
symmetry

$$G_{\text{SM}} = U(1)_B \times U(1)_{L_e} \times U(1)_{L_\mu} \times U(1)_{L_\tau}$$

No ! Neutrinos have no mass in the SM!

Can we have $m_\nu \neq 0$ in the
SM ?

a loop correction could induce an effective mass
term like

$$\frac{y_{\alpha\beta}^\nu}{v} \Phi \Phi L_\alpha L_\beta$$

this term violates G_{SM}

But the SM has an accidental global
symmetry

$$G_{SM} = U(1)_B \times U(1)_{L_e} \times U(1)_{L_\mu} \times U(1)_{L_\tau}$$

No ! Neutrinos have no mass in the SM!

LEPTON NUMBER

$$L \equiv L_e + L_\mu + L_\tau$$

	L_e	L_μ	L_τ
e, ν_e	+1	0	0
μ, ν_μ	0	+1	0
τ, ν_τ	0	0	+1

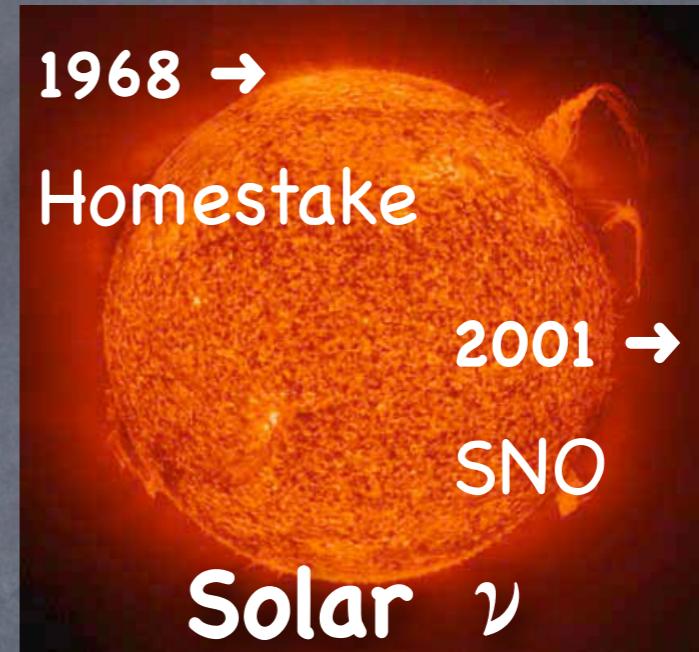
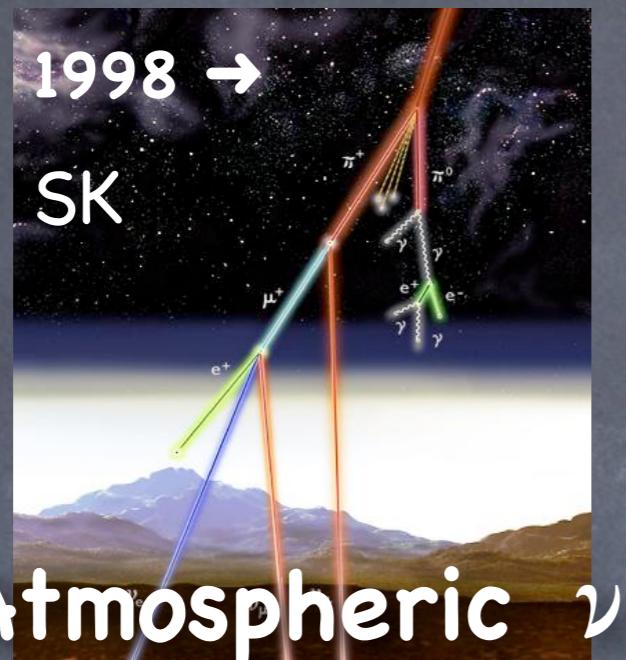
	L_e	L_μ	L_τ
$\bar{e}, \bar{\nu}_e$	-1	0	0
$\bar{\mu}, \bar{\nu}_\mu$	0	-1	0
$\bar{\tau}, \bar{\nu}_\tau$	0	0	-1

conservation of each lepton number L_α related through Noether's Theorem to the invariance of the Lagrangian under a global $U(1)$ transformation

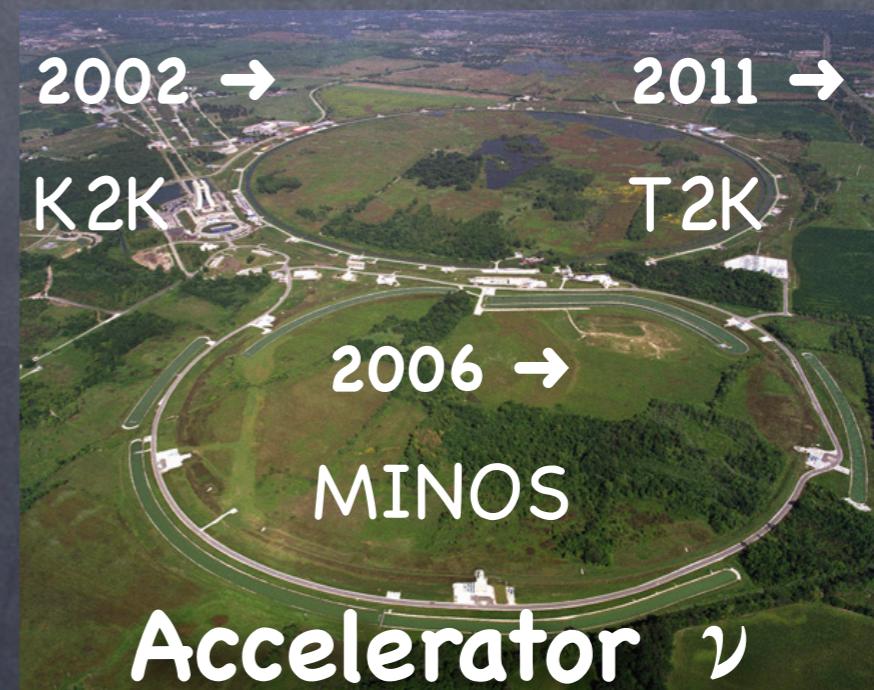
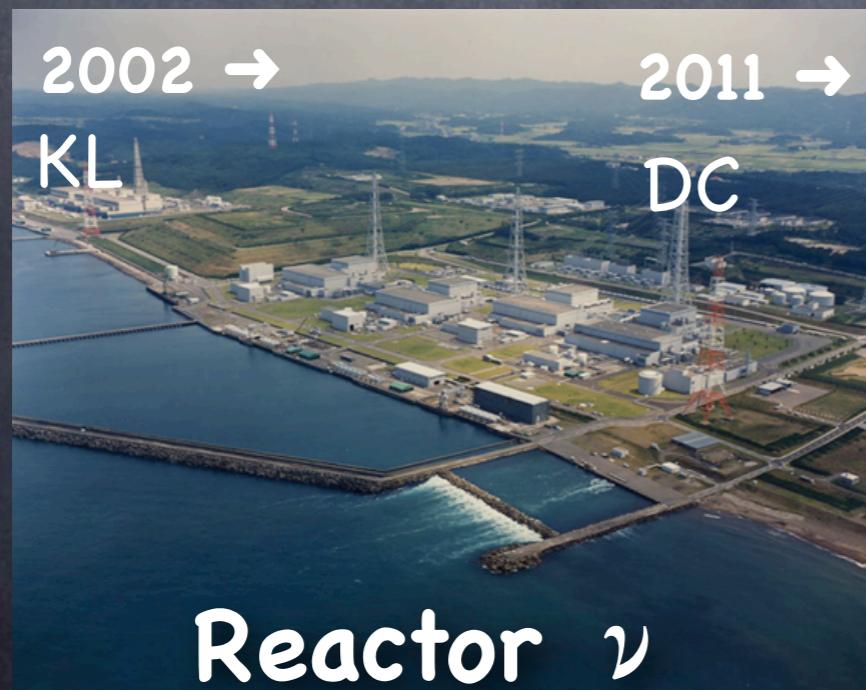
$$\ell_\alpha \rightarrow e^{i\theta_\alpha} \ell_\alpha$$

$\partial_\mu j^\mu = 0$ associated to a conserved current and the conserved charge L_α

Surprise!
However in Nature ...

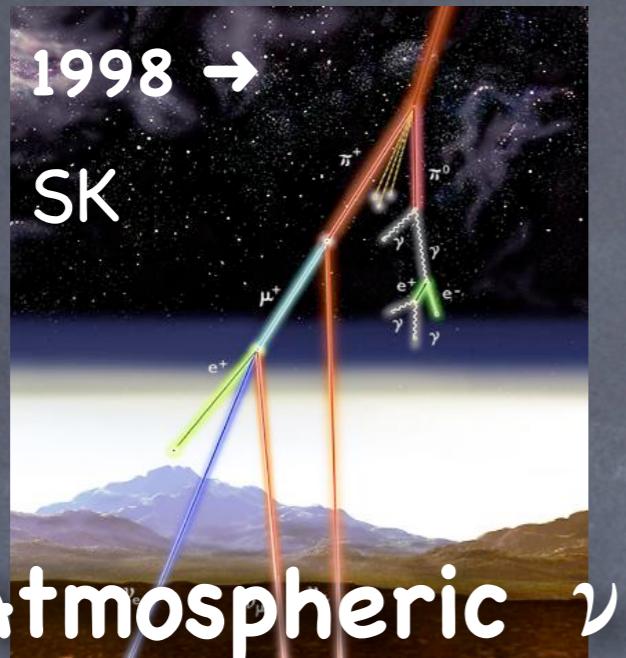


Neutrino Oscillations \rightleftarrows Need Masses & Mixings

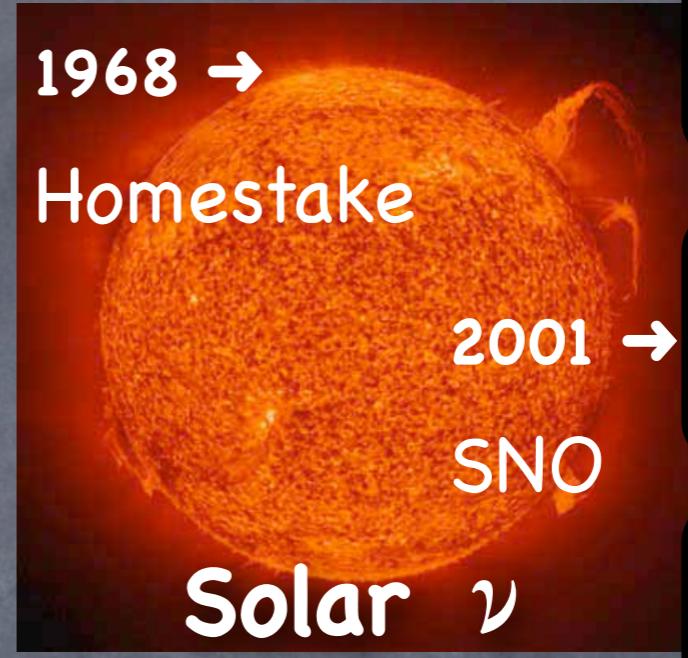


However in Nature ...

Surprise!



Atmospheric ν

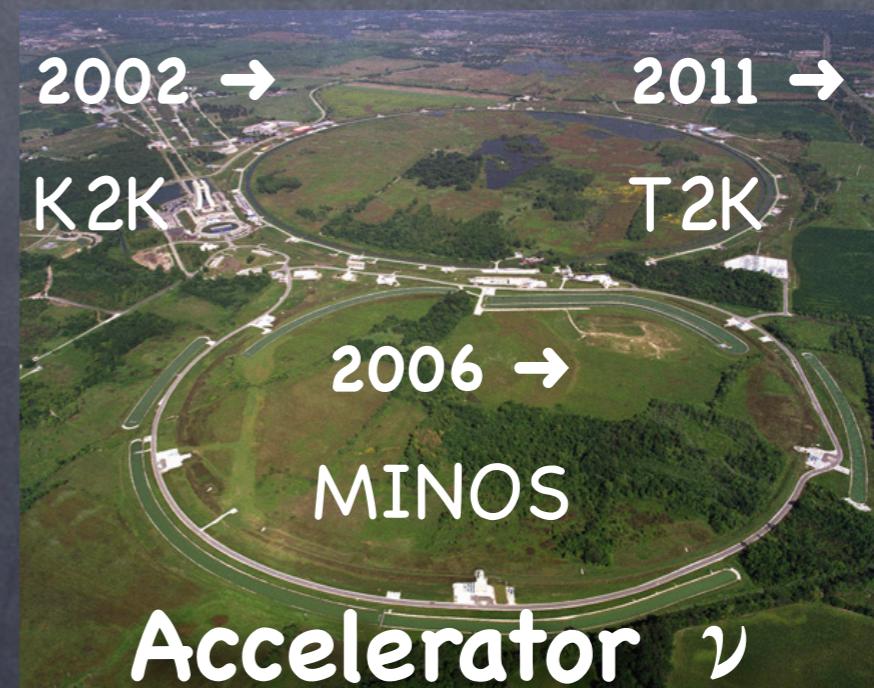
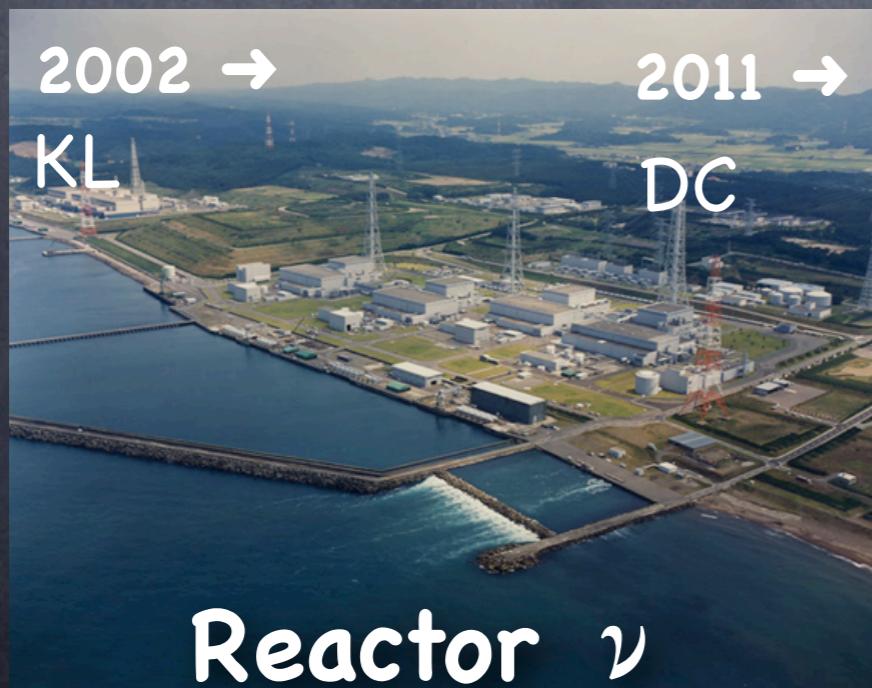


See my next
Lecture

+ D. Harris
Lectures

+ T. Schwetz
Lectures

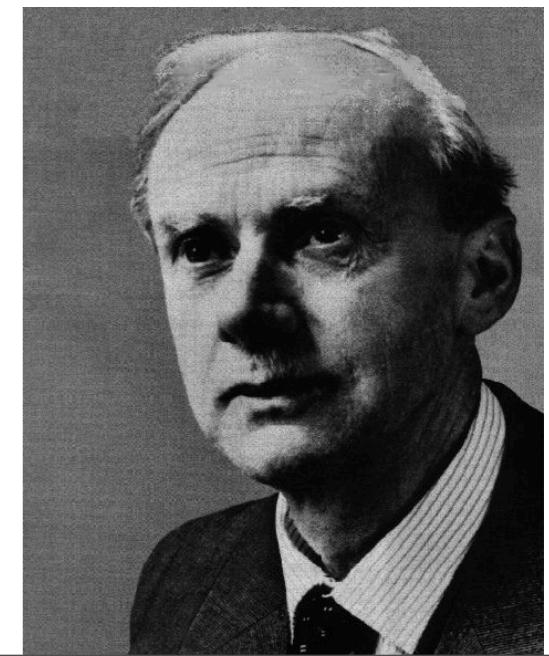
Neutrino Oscillations \rightleftarrows Need Masses & Mixings





Fotografia di Ettore Majorana tratta dalla banca universitaria
datata 3 novembre 1923.

2. Dirac versus Majorana



Majorana x Dirac ν

Dirac spinor

$$\Psi = P_L \Psi + P_R \Psi = \Psi_L + \Psi_R \quad 4 \text{ independent components}$$

Dirac equation

$$i\gamma_\mu \partial^\mu \Psi_L = m \Psi_R$$

$$i\gamma_\mu \partial^\mu \Psi_R = m \Psi_L$$

Majorana x Dirac ν

Dirac spinor

$$\Psi = P_L \Psi + P_R \Psi = \Psi_L + \Psi_R \quad 4 \text{ independent components}$$

Dirac equation

$$i\gamma_\mu \partial^\mu \Psi_L = m \Psi_R \quad \text{if } m = 0$$

$$i\gamma_\mu \partial^\mu \Psi_R = m \Psi_L$$

Weyl (1929)

2-component spinor is enough (Ψ_L or Ψ_R)

Pauli (1933) rejected this idea because leads to Parity Violation

Landau, Lee-Yang, Salam (1957) propose to describe the massless neutrino by a Weyl spinor ν_L introduced in the SM in the 60's

Majorana x Dirac ν

Can we also describe a massive fermion using a
2-component spinor?

(E. Majorana, 1937)

Majorana x Dirac ν

Can we also describe a massive fermion using a
2-component spinor?

(E. Majorana, 1937)

$$\Psi^c = C \bar{\Psi}^\tau \quad \text{charge conjugate field}$$

$$(\Psi_L)^c = (\Psi^c)_R \quad (\Psi_R)^c = (\Psi^c)_L$$

charge conjugation change chirality

$$i\gamma_\mu \partial^\mu (\Psi_L)^c = m(\Psi_R)^c \iff i\gamma_\mu \partial^\mu (\Psi_R)^c = m(\Psi_L)^c$$

$$\Psi_{L,R} \equiv \xi (\Psi_{R,L})^c = \xi C \bar{\Psi}_{R,L}^\tau$$

$$\xi \equiv e^{-i\alpha} \quad \text{phase factor}$$

Majorana x Dirac ν

Can we also describe a massive fermion using a
2-component spinor? Yes! (E. Majorana, 1937)

ξ is unphysical - can be eliminated by rephasing

Majorana Condition: $\Psi \equiv (\Psi)^C$ particle = antiparticle

Majorana Field: $\Psi = \Psi_L + \Psi_R = \Psi_L + (\Psi_L)^C$

Majorana Equation: $i\gamma_\mu \partial^\mu \Psi_L = m C \overline{\Psi_L}^T$

Majorana x Dirac ν

Can we also describe a massive fermion using a 2-component spinor? Yes!
(E. Majorana, 1937)

ξ is unphysical - can be eliminated by rephasing

Majorana Condition: $\Psi \equiv (\Psi)^c$ particle = antiparticle

Majorana Field: $\Psi = \Psi_L + \Psi_R = \Psi_L + (\Psi_L)^c$

Majorana Equation: $i\gamma_\mu \partial^\mu \Psi_L = m C \overline{\Psi_L}^T$

e.m. current vanishes $Q \equiv 0$ neutral particle

$$\overline{\Psi} \gamma^\mu \Psi = \overline{\Psi^c} \gamma^\mu \Psi^c = -\Psi^T C^\dagger \gamma^\mu C \overline{\Psi}^T = \overline{\Psi} C^T \gamma^{\mu T} C^* \Psi = -\overline{\Psi} \gamma^\mu \Psi$$

Some Properties

$$\gamma^0 \gamma^\mu \gamma^t = \gamma^\mu \gamma^0$$

$$C^\tau = C^t = C^{-1} = -C$$

$$C^{-1} \gamma^\mu = -\gamma^{\mu\tau} C^{-1}$$

$$\therefore \overline{\Psi}^c = (C \gamma^0 \Psi^*)^t \gamma^0 = \Psi^\tau \gamma^0 C^t \gamma^0 = \Psi^\tau C$$

$$C^\tau \gamma^{\mu\tau} C^* = (-C) \gamma^{\mu\tau} (-C^{-1}) = C \gamma^{\mu\tau} C^{-1}$$

$$\therefore C^\tau \gamma^{\mu\tau} C^* = -CC^{-1} \gamma^\mu = -\gamma^\mu$$

Majorana x Dirac ν

Dirac:

$$\nu(\vec{p}, h) \xrightarrow{\hat{P}} \nu(-\vec{p}, -h) \xrightarrow{\hat{C}} \bar{\nu}(-\vec{p}, -h) \xrightarrow{\hat{T}} \bar{\nu}(\vec{p}, -h)$$

LH neutrino ($h = -1$)

RH antineutrino ($h = +1$)

Majorana:

$$\nu(\vec{p}, h) \xrightarrow{\hat{P}} \nu(-\vec{p}, -h) \xrightarrow{\hat{C}} \nu(-\vec{p}, -h) \xrightarrow{\hat{T}} \nu(\vec{p}, -h)$$

LH neutrino ($h = -1$)

RH neutrino ($h = +1$)

interactions involve on LH fields

Dirac

ν_L	\swarrow destroys LH neutrino \searrow creates RH antineutrino
$\bar{\nu}_L$	\swarrow destroys RH antineutrino \searrow creates LH neutrino

Majorana x Dirac ν

Dirac:

$$\nu(\vec{p}, h) \xrightarrow{\hat{P}} \nu(-\vec{p}, -h) \xrightarrow{\hat{C}} \bar{\nu}(-\vec{p}, -h) \xrightarrow{\hat{T}} \bar{\nu}(\vec{p}, -h)$$

LH neutrino ($h = -1$)

RH antineutrino ($h = +1$)

Majorana:

$$\nu(\vec{p}, h) \xrightarrow{\hat{P}} \nu(-\vec{p}, -h) \xrightarrow{\hat{C}} \nu(-\vec{p}, -h) \xrightarrow{\hat{T}} \nu(\vec{p}, -h)$$

LH neutrino ($h = -1$)

RH neutrino ($h = +1$)

interactions involve on LH fields

$$\begin{aligned} \nu_L &\quad \text{destroys LH neutrino} \\ &\quad \text{creates RH neutrino} \\ \bar{\nu}_L &\quad \text{destroys RH neutrino} \\ &\quad \text{creates LH neutrino} \end{aligned}$$

Majorana

3. Mass Terms for Neutrinos

$$\begin{array}{c} M \\ \hline L \quad X \quad R \end{array}$$

① "Poor man's" extension of the SM

If

$$\nu \neq \nu^c = C \bar{\nu}^T$$

Dirac Particle

symmetrize the model, offers no explanation to the smallness of M_ν

$$L_\alpha \equiv (2, -1/2) \quad E_\alpha \equiv (1, -1) \quad N_\alpha \equiv (1, 0)$$

$$-\mathcal{L}_Y = y_{\alpha\beta}^d \bar{Q}_\alpha \Phi D_\beta + y_{\alpha\beta}^u \bar{Q}_\alpha \tilde{\Phi} U_\beta + y_{\alpha\beta}^\ell \bar{L}_\alpha \Phi E_\beta + \text{h.c.}$$

$$+ y_{\alpha\beta}^\nu \bar{L}_\alpha \tilde{\Phi} N_\beta + \text{h.c.}$$

EWSB

Dirac Mass Term

Higgs acquires a vev



$$- m_D \bar{\nu}_L \nu_R + \text{h.c.}$$

① "Poor man's" extension of the SM

$$-\mathcal{L}_Y = \left(\frac{v+h}{\sqrt{2}} \right) \left[\overline{\ell'_L} y^{\ell'} \ell'_R + \overline{N'_L} y^{\nu'} N'_R \right] + \text{h.c.}$$

$$\ell'_{L,R} \equiv \begin{pmatrix} e' \\ \mu' \\ \tau' \end{pmatrix}_{L,R}$$

$$\ell_{L,R} = V_{L,R}^{\ell\dagger} \ell'_{L,R}$$

real positive
numbers

$$y^\ell = V_L^{\ell\dagger} y^{\ell'} V_R^\ell \quad y_{\alpha\beta}^\ell = y_\alpha^\ell \delta_{\alpha\beta}$$

unitary matrices

$$N'_{L,R} \equiv \begin{pmatrix} \nu'_e \\ \nu'_\mu \\ \nu'_\tau \end{pmatrix}_{L,R}$$

$$N_{L,R} = V_{L,R}^{\nu\dagger} N'_{L,R}$$

real positive
numbers

$$y^\nu = V_L^{\nu\dagger} y^{\nu'} V_R^\nu \quad y_{\alpha\beta}^\nu = y_\alpha^\nu \delta_{\alpha\beta}$$

unitary matrices

① "Poor man's" extension of the SM

$$-\mathcal{L}_{\text{mass}}^D = \frac{v}{\sqrt{2}} y_\alpha^\ell \overline{e_{\alpha L}} e_{\alpha R} + \frac{v}{\sqrt{2}} y_i^\nu \overline{\nu_{i L}} \nu_{i R} + \text{h.c.}$$

charged fermions
 masses

neutrino
 masses

$$\ell_{L,R} \equiv \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}_{L,R} \equiv \begin{pmatrix} e_e \\ e_\mu \\ e_\tau \end{pmatrix}_{L,R}$$

new
 fields

$$\mathbf{N}_{L,R} \equiv \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_{L,R}$$

Yukawas have to be fine-tuned to explain smallness of neutrino masses

Ok. But what happens to the CC
and NC ?

① "Poor man's" extension of the SM
charged current for leptons

$$j_{W,L}^\mu = 2 \bar{\nu}'_\alpha \gamma^\mu P_L e'_\alpha = 2 \bar{\nu}'_{\alpha L} \gamma^\mu e'_{\alpha L} = 2 \bar{N}'_L \gamma^\mu \ell'_L$$

chiral flavor diagonal interaction

$$= 2 \bar{N}_L \boxed{V_L^{\nu^\dagger} V_L^\ell} \gamma^\mu \ell_L = 2 \bar{\nu}_{iL} \boxed{U_{\alpha i}^*} \gamma^\mu e_{\alpha L}$$

Mixing Matrix (Pontecorvo, Maki, Sakata, Nakagawa)

define LH flavor neutrinos as

$$\nu_{\alpha L} = U_{\alpha i} \nu_{iL}$$

Mixing \Rightarrow family Lepton Number (L_e, L_μ, L_τ) violated
but Total Lepton Number (L) conserved

① "Poor man's" extension of the SM
neutral current for neutrinos

$$\begin{aligned} j_{Z,\nu}^\mu &= \bar{\nu}_{\alpha L} \gamma^\mu \nu_{\alpha L} && \text{chiral flavor diagonal interaction} \\ &= \bar{\nu}_{iL} \gamma^\mu \nu_{iL} && \text{No Mixing here !} \end{aligned}$$

NC is the same (GIM Mechanism)

[S.L.Glashow, J. Iliopoulos, L. Maiani, Phys. Rev. D 2, 1285 (1970)]

ν_R is sterile !

L_α Violating Processes

Dirac mass term allows for $\cancel{L}_e, \cancel{L}_\mu, \cancel{L}_\tau$

processes such as: $\mu^\pm \rightarrow e^\pm \gamma$ or $\mu^\pm \rightarrow e^\pm e^+ e^-$

eg. $\mu^\pm \rightarrow e^\pm \gamma$

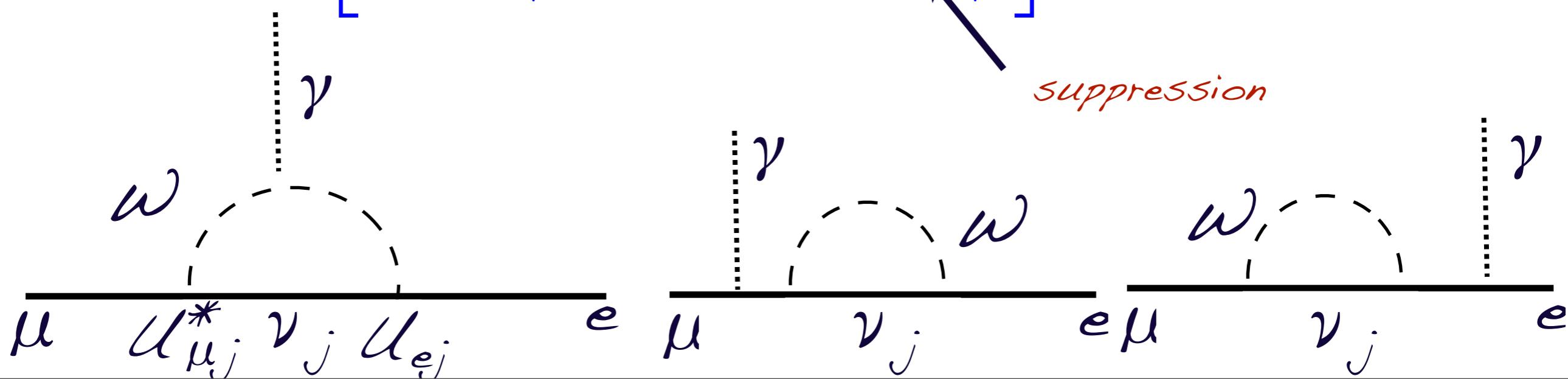
$$\sum_j U_{\mu j}^* U_{ej} = 0$$

GIM Mechanism

$$\Gamma = \frac{G_F m_\mu^5}{192\pi^3} \left[\frac{3\alpha_{em}}{32\pi} \left| \sum_j U_{\mu j}^* U_{ej} \frac{m_{\nu_i}^2}{M_W^2} \right|^2 \right]$$

SM: $BR \leq 10^{-25}$

$BR_{exp} \leq 2.4 \times 10^{-12}$



Phases of \mathcal{U}

$$j_{W,L}^\mu = 2 \overline{\nu_{iL}} U_{\alpha i}^* \gamma^\mu e_{\alpha L}$$

Can re-phase $e_{\alpha L} \rightarrow e^{i\phi_\alpha} e_{\alpha L}$ $\nu_{iL} \rightarrow e^{i\phi_i} \nu_{iL}$

$$j_{W,L}^\mu = 2 \overline{\nu_{iL}} e^{-i(\phi_1 - \phi_e)} e^{-i(\phi_i - \phi_1)} e^{i(\phi_\alpha - \phi_e)} U_{\alpha i}^* \gamma^\mu e_{\alpha L}$$

1 $N-1$ $N-1$

$1 + 2(N-1) = 2N-1$ phases can be arbitrarily chosen

$N=3 \rightarrow 5$ phases can be eliminated from \mathcal{U}
only 1 physical phase

Basic Points :

- we need to introduce singlet R neutrino fields (ν_R)
- we make use of the SM Higgs Mechanism
- $\mathcal{L}_{\text{mass}}^D = -m\bar{\nu}\nu = -m(\bar{\nu}_R \nu_L + \bar{\nu}_L \nu_R)$
- mass hierarchy problem remains $m_j^\nu = \frac{y_j^\nu v}{\sqrt{2}}$
- L_e, L_μ, L_τ are violated
- L is conserved (exact global symmetry at the classical level, just like B)
- generates a mixing matrix analogous to V_{CKM}

② + Clever extensions of the SM

If

$$\nu = \nu^c = C \bar{\nu}^T$$

Majorana Particle

if we introduce ν_R we can have

Majorana Mass Term

EI.

$$-\frac{1}{2} m_R \overline{\nu_R^c} \nu_R + h.c.$$

L is violated
by 2 units

$$P_L \nu_R^c = \nu_R^c$$

this is invariant under $SU(2)_L \times U(1)_Y$

Dirac-Majorana

$$M^{D+M} = \begin{pmatrix} 0 & (M^D)^T \\ M^D & M^R \end{pmatrix}$$

complex symmetric matrix

M_D is a $3 \times m$ complex matrix M_R is a $m \times m$ symmetric matrix

(a) mass eigenvalues of $M^R \gg v \Rightarrow$ framework of seesaw mechanism sterile neutrinos integrated out & get a low energy effective theory with 3 light active Majorana neutrinos

(b) some mass eigenvalues of $M^R \leq v \Rightarrow$ more than 3 light Majorana neutrinos

(c) $M^R = 0 \Rightarrow$ equivalent to impose L conservation, $m=3$ and we can identify the 3 sterile neutrinos c/ RH components of the LH fields (Dirac Neutrinos)

② + Clever extensions of the SM

If

$$\nu = \nu^c = C \bar{\nu}^T$$

Majorana Particle

if we don't introduce ν_R we can have

Majorana Mass Term

E1.

$$-\frac{1}{2} m_L \bar{\nu}_L^c \nu_L + h.c.$$

L is violated
by 2 units

$$P_R \nu_L^c = \nu_L^c$$

but not invariant under $SU(2)_L \times U(1)_Y$
need to extend the SM ...

Majorana Mass Term

we can write a Majorana mass term with only ν_L

(or ν_R)

$$P_R \nu_L^c = \nu_L^c$$

$$\nu^c = \nu \implies \nu = \nu_L + \nu_L^c \implies \mathcal{L}_{\text{mass}}^{\text{ML}} = -\frac{1}{2} m_L \bar{\nu}_L^c \nu_L + \text{h.c.}$$

the $1/2$ factor avoids double counting since ν_L and ν_L^c

are not independent

$$\mathcal{L}^{\text{ML}} = \frac{1}{2} [\bar{\nu}_L i \not{\partial} \nu_L + \bar{\nu}_L^c i \not{\partial} \nu_L^c - m_L (\bar{\nu}_L^c \nu_L + \bar{\nu}_L \nu_L^c)]$$

$$\mathcal{L}_{\text{mass}}^{\text{ML}} = \frac{m_L}{2} \left(\nu_L^T C^\dagger \nu_L + \nu_L^\dagger C \nu_L^* \right)$$

Basic Points:

- no need to introduce singlet R fields (ν_R)
- use $\nu_R \rightarrow \nu_L^c = C \bar{\nu}_L^T$ and $\nu = \nu^c$
- $\nu = \nu_L + \nu_R = \nu_L + C \bar{\nu}_L^T$
- $$\mathcal{L}_{\text{mass}}^{\text{ML}} = -\frac{m}{2} (\bar{\nu}_L^c \nu_L + \text{h.c.})$$
- need a Higgs triplet ($Y=1$) to form a $SU(2)_L \otimes U(1)_Y$ invariant term ($L \Delta L$)
- L_e, L_μ, L_τ are violated
- L is also violated by 2 units

The most general mass term is a
Dirac-Majorana Mass Term

E2.

$$\mathcal{L}_{\text{mass}}^{\text{D+M}} = \mathcal{L}_{\text{mass}}^{\text{D}} + \mathcal{L}_{\text{mass}}^{\text{ML}} + \mathcal{L}_{\text{mass}}^{\text{MR}}$$

$$\mathcal{L}_{\text{mass}}^{\text{D}} = -\mathbf{m}_D \bar{\nu}_R \nu_L + \text{h.c.} \quad \text{Dirac Mass Term}$$

$$\mathcal{L}_{\text{mass}}^{\text{ML}} = \frac{1}{2} \mathbf{m}_L \nu_L^T \mathbf{C}^\dagger \nu_L + \text{h.c.} \quad \text{Majorana Mass Term}$$

$$\mathcal{L}_{\text{mass}}^{\text{MR}} = \frac{1}{2} \mathbf{m}_R \nu_R^T \mathbf{C}^\dagger \nu_R + \text{h.c.} \quad \text{Majorana Mass Term}$$

Mixing in General

$$\mathbf{N}'_L \equiv \begin{pmatrix} \nu'_L \\ \nu'_R^c \end{pmatrix} \quad \nu'_L \equiv \begin{pmatrix} \nu'_{eL} \\ \nu'_{\mu L} \\ \nu'_{\tau L} \end{pmatrix} \quad \nu'_R^c \equiv \begin{pmatrix} \nu'_{1R}^c \\ \vdots \\ \nu'_{N_s R}^c \end{pmatrix}$$

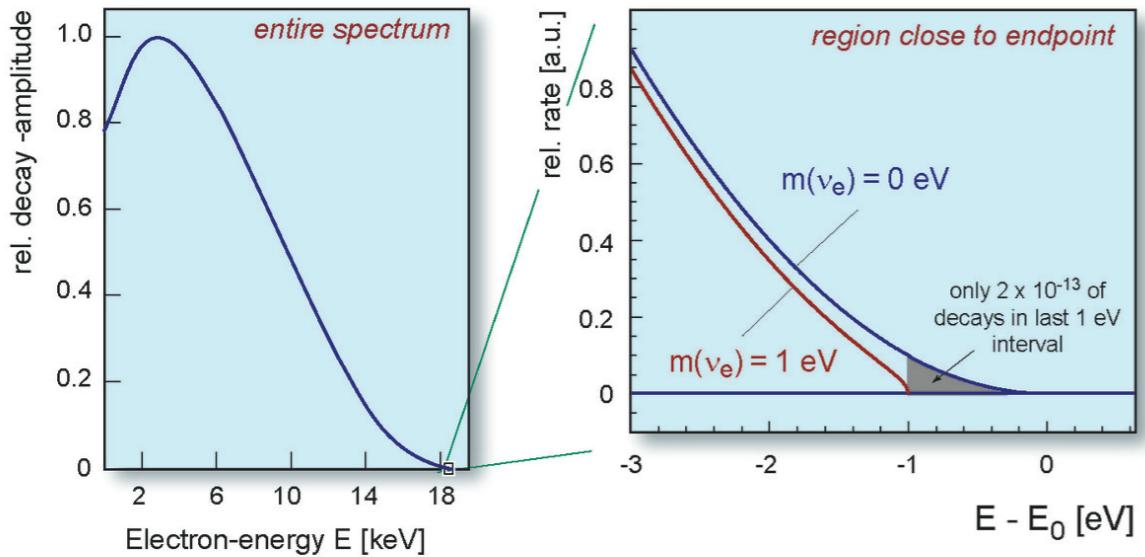
$$\mathcal{L}_{\text{mass}}^{\text{D+M}} \equiv \frac{1}{2} \mathbf{N}_L'^T \mathbf{C}^\dagger \mathbf{M}^{\text{D+M}} \mathbf{N}_L' + \text{h.c.} \quad \mathbf{M}^{\text{D+M}} = \begin{pmatrix} M^L & (M^D)^T \\ M^D & M^R \end{pmatrix}$$

- Diagonalization of the Dirac-Majorana Mass Term \Rightarrow Massive Majorana Neutrinos

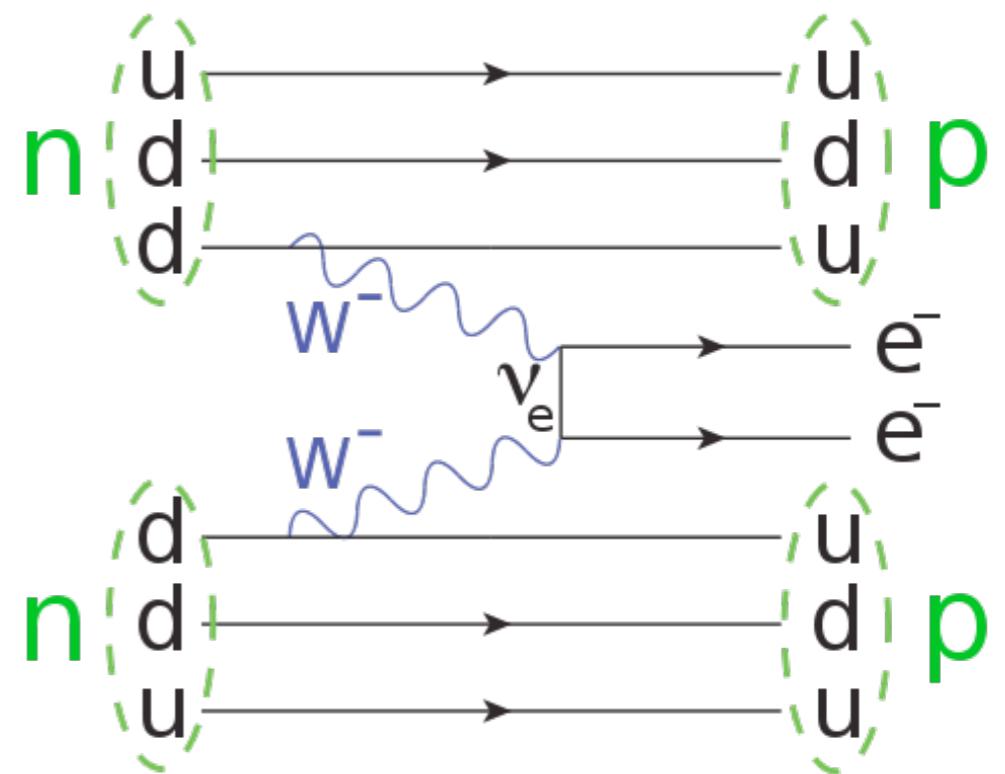
E3.

E4.

+ F. Feruglio
Lectures



4. Some Consequences of Masses and Mixings



Single β -Decay - Effective ν_e Mass



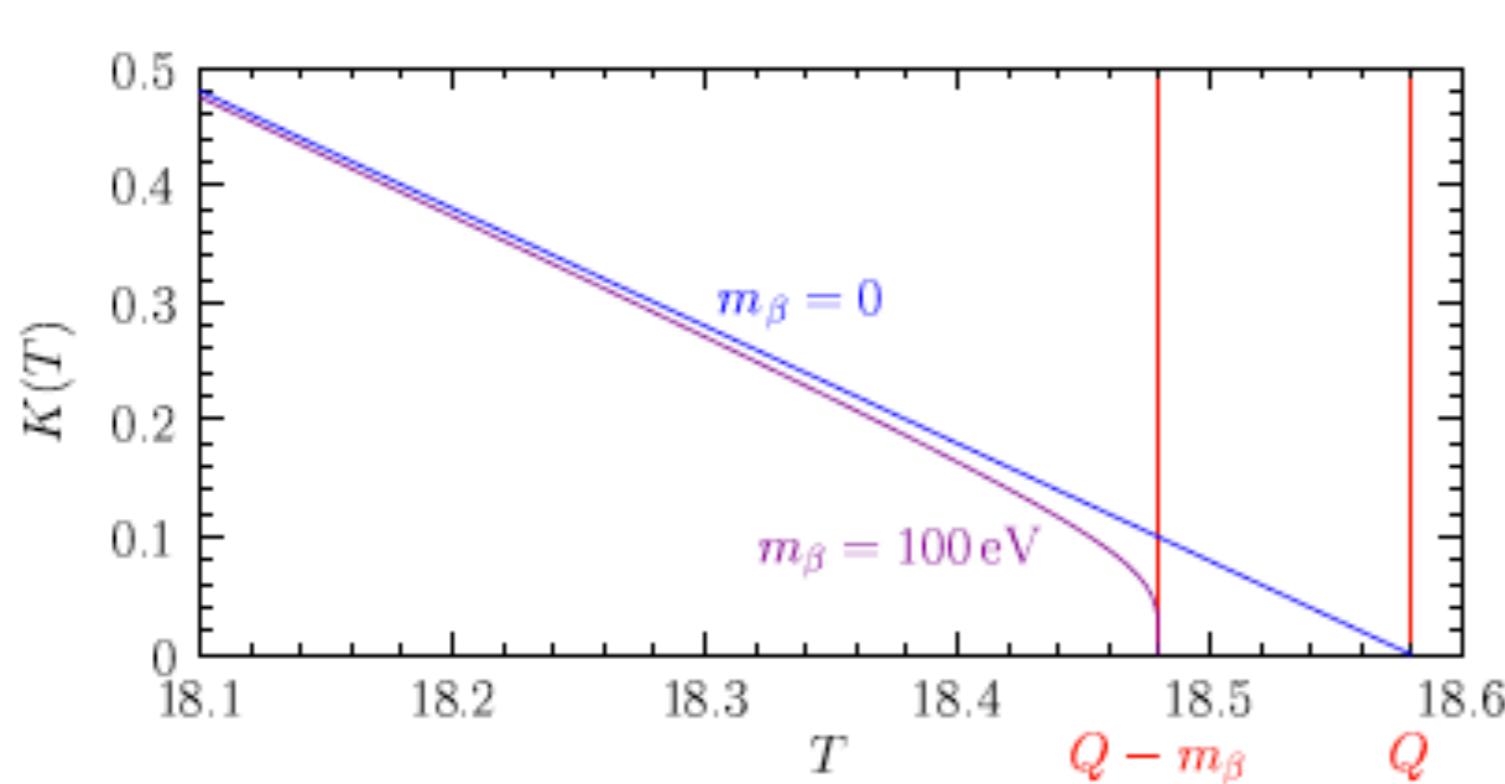
$$Q = M_{\text{H}} - M_{\text{He}} - m_e = 18.58 \text{ keV}$$

$$\frac{d\Gamma}{dT} \propto |\mathcal{M}|^2 F(E) p E(Q-T) \sqrt{(Q-T)^2 - m_{\bar{\nu}_e}^2}$$

Kurie plot:

$$K(T) = \sqrt{(Q-T) \sqrt{(Q-T)^2 - m_{\bar{\nu}_e}^2}}$$

$$\mathbf{m}_{\bar{\nu}_e} \rightarrow \mathbf{m}_\beta$$



$$m_{\nu_e} < 2.2 \text{ eV}$$

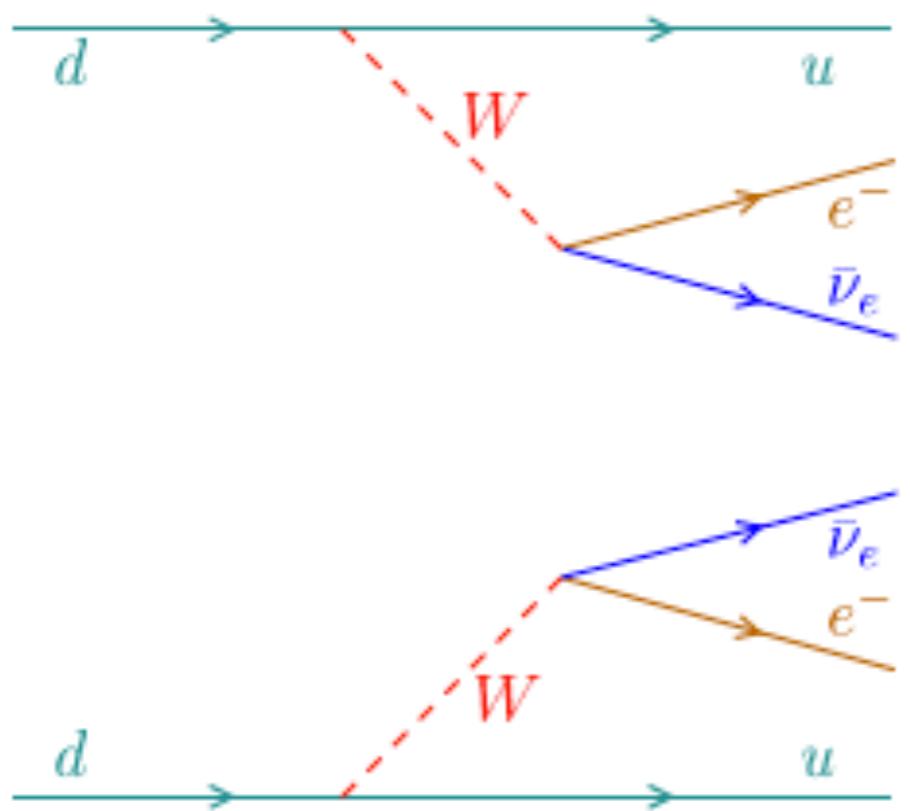
Troitsk & Mainz (2002)

$$\mathbf{m}_\beta = \sqrt{\sum_{i=1}^3 m_i^2 |U_{ei}|^2}$$

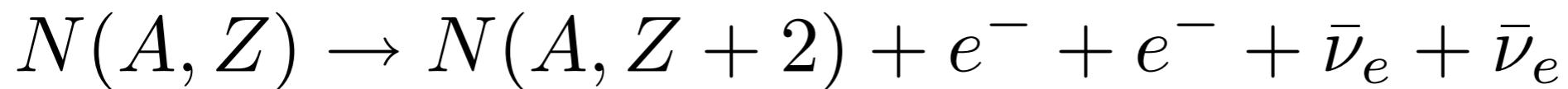
Future KATRIN sensitivity 0.2 eV

Two Neutrino Double- β Decay

$$\Delta L = 0$$

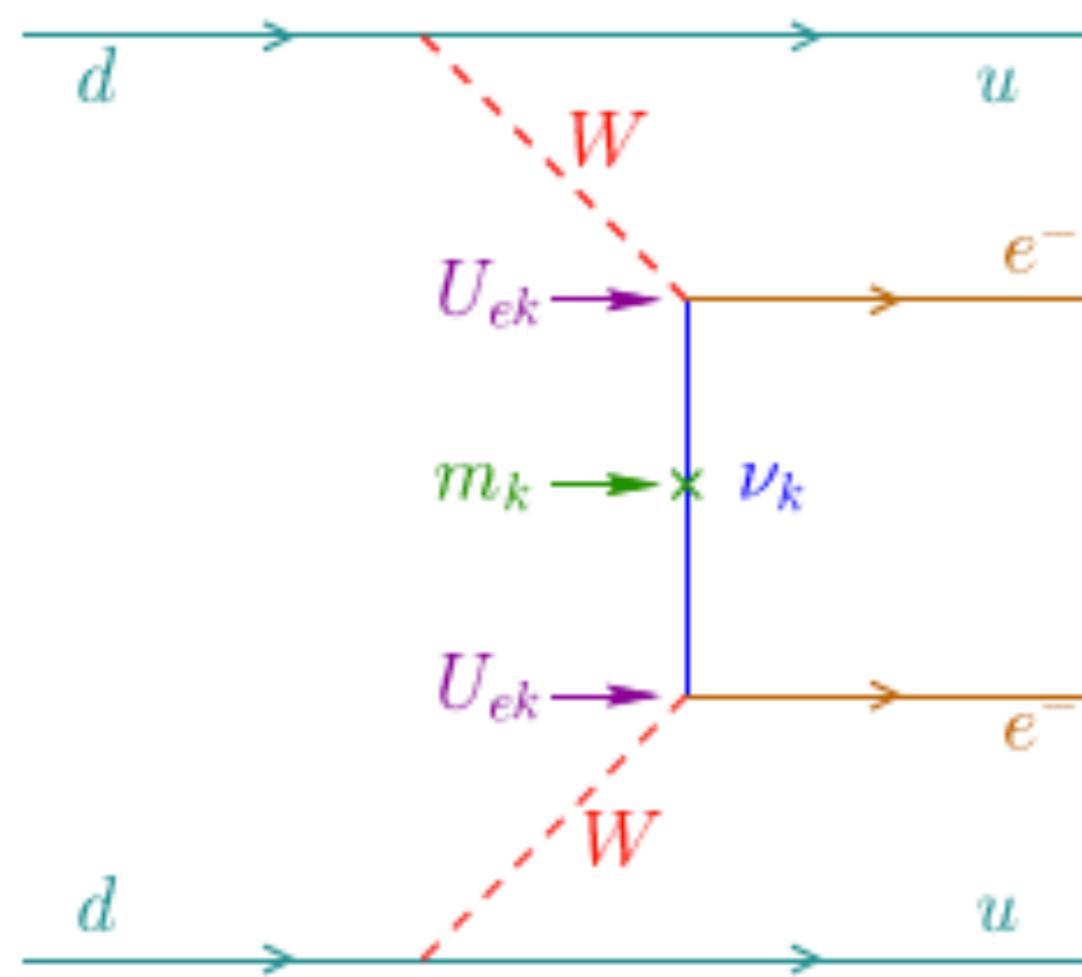


$$(T_{1/2}^{2\nu})^{-1} = G_{2\nu} |M_{2\nu}|^2$$



second order weak interaction process

Neutrinoless Double- β Decay



$$\Delta L = 2$$

if Majorana particle

$$(T_{1/2}^{0\nu})^{-1} = G_{0\nu} |M_{0\nu}|^2 |m_{\beta\beta}|^2$$

$$N(A, Z) \rightarrow N(A, Z + 2) + e^- + e^-$$

effective Majorana mass

sensitive to phases

$$m_{\beta\beta} = \sum_k U_{ek}^2 m_k$$

5. Effective Lagrangian Approach and Seesaw Mechanism

Effective Lagrangian Perspective

+ H. Georgi
Lectures

SM is an Effective Lower Energy Theory

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \delta\mathcal{L}^{d=5} + \delta\mathcal{L}^{d=6} + \dots$$

non-renormalizable higher-dimension operators invariant under $SU(2)_L \times U(1)_Y$
made of SM fields active @ low energies with coefficients (model dependent)
weighted by inverse powers of Λ (new physics scale)

$$\delta\mathcal{L}^{d=5} = \frac{g}{\Lambda} (\mathbf{L}^T \sigma_2 \Phi) \mathbf{C}^\dagger (\Phi^T \sigma_2 \mathbf{L}) + \text{h.c.}$$

[S. Weinberg, Phys. Rev. Lett. 43, 1566 (1979)]

only possible $d=5$ operator

$$\delta\mathcal{L}^{d=5} \xrightarrow{\text{EWSB}} \delta\mathcal{L}_{\text{mass}}^M = \frac{1}{2} \left(\frac{gv^2}{\Lambda} \right) \nu_L^T \mathbf{C}^\dagger \nu_L + \text{h.c.}$$

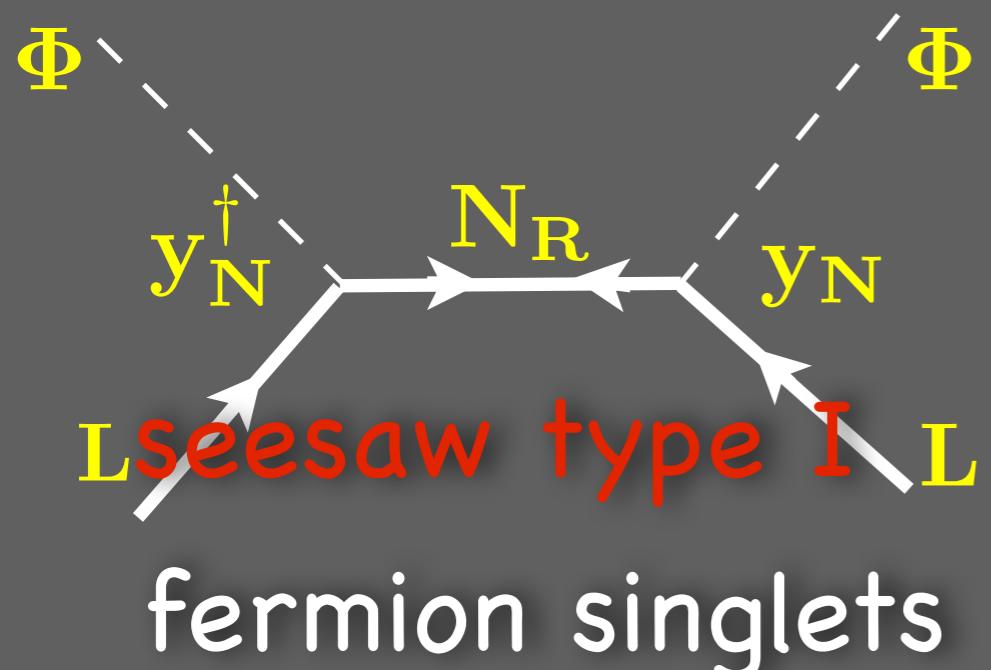
Majorana Mass

Tree-level Realizations

[E. Ma, Phys. Rev. Lett. 81, 1171 (1998)]

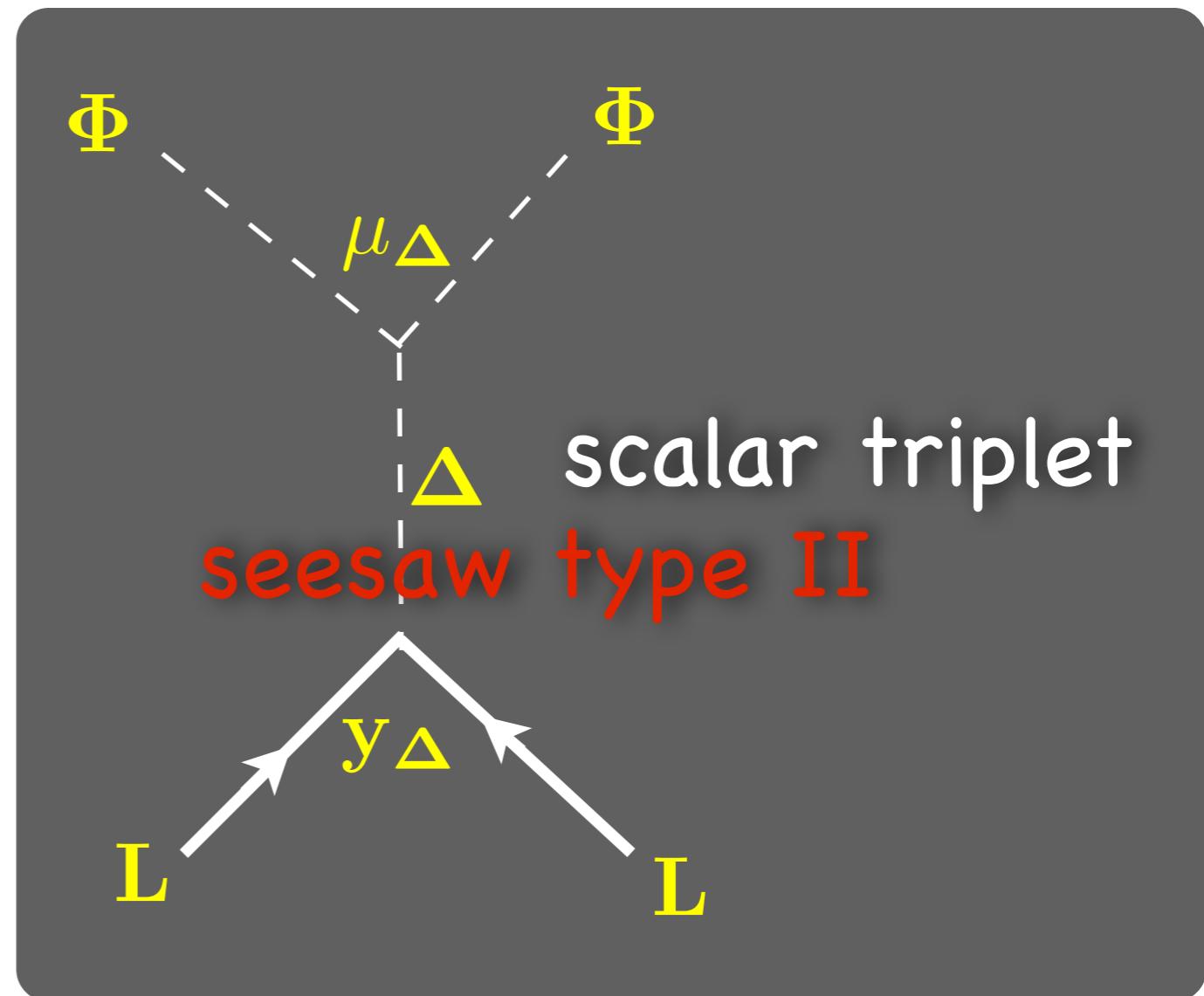
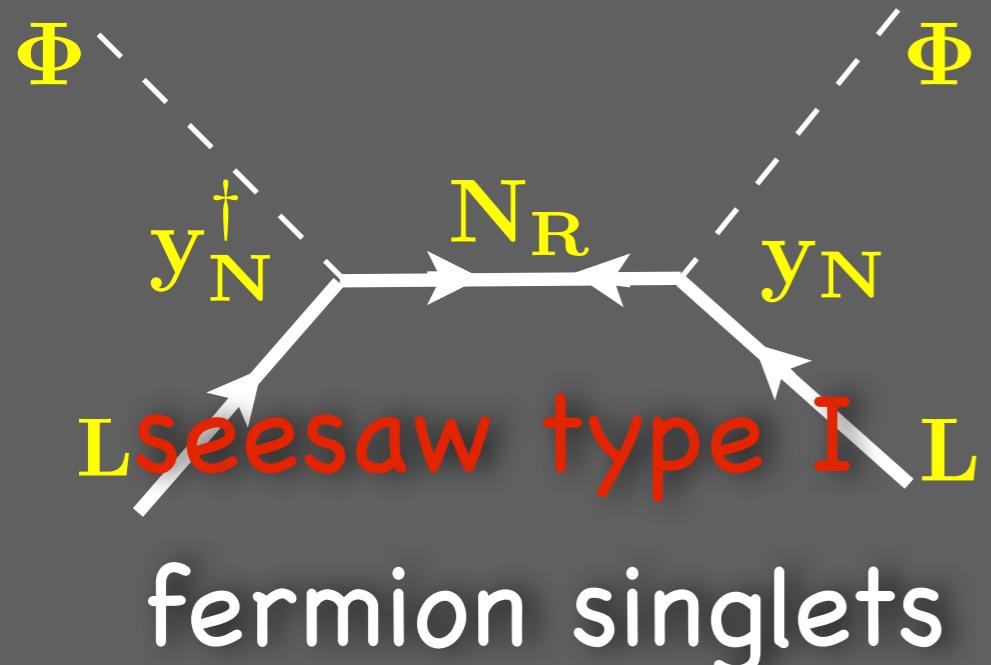
Tree-level Realizations

[E. Ma, Phys. Rev. Lett. 81, 1171 (1998)]



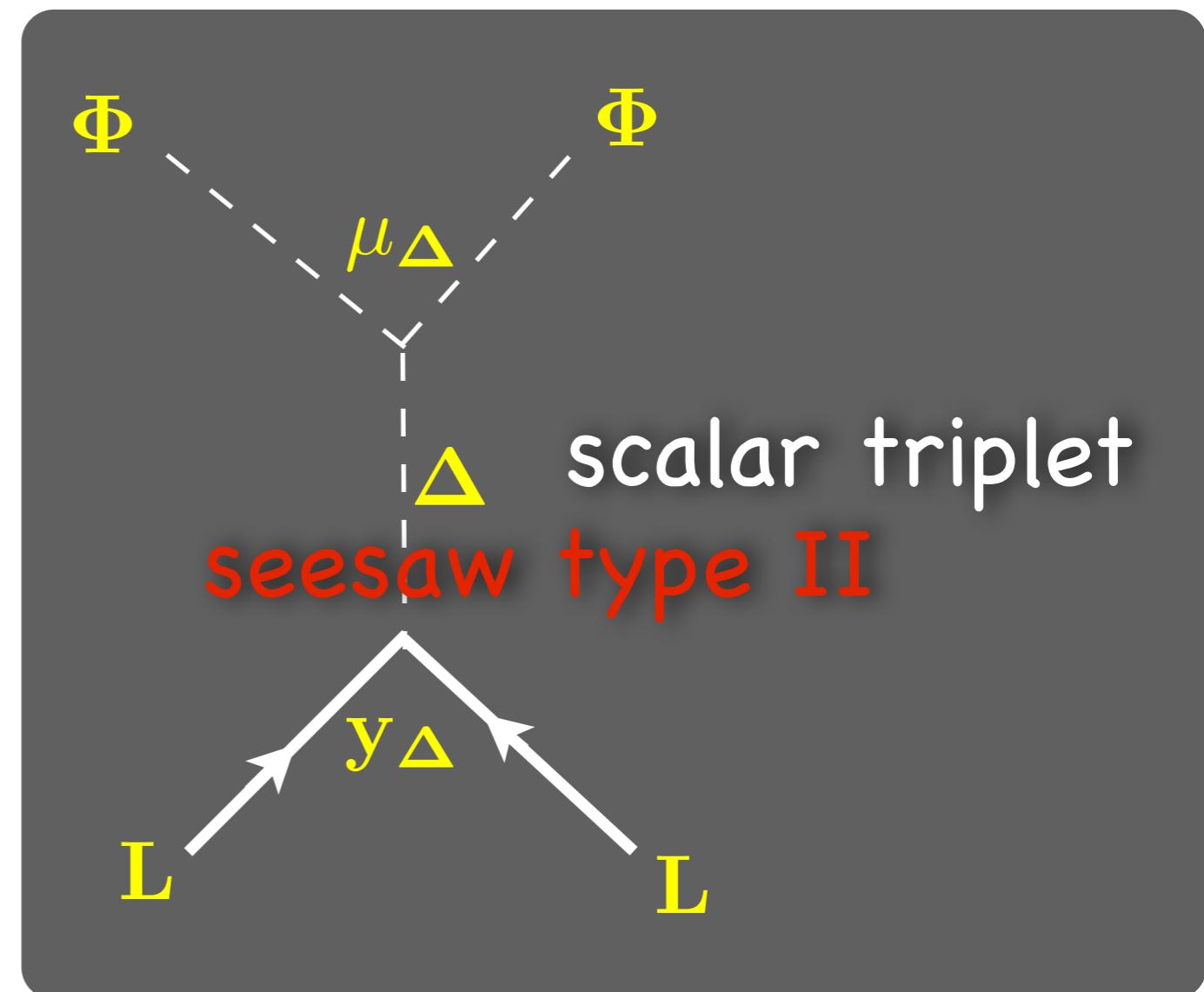
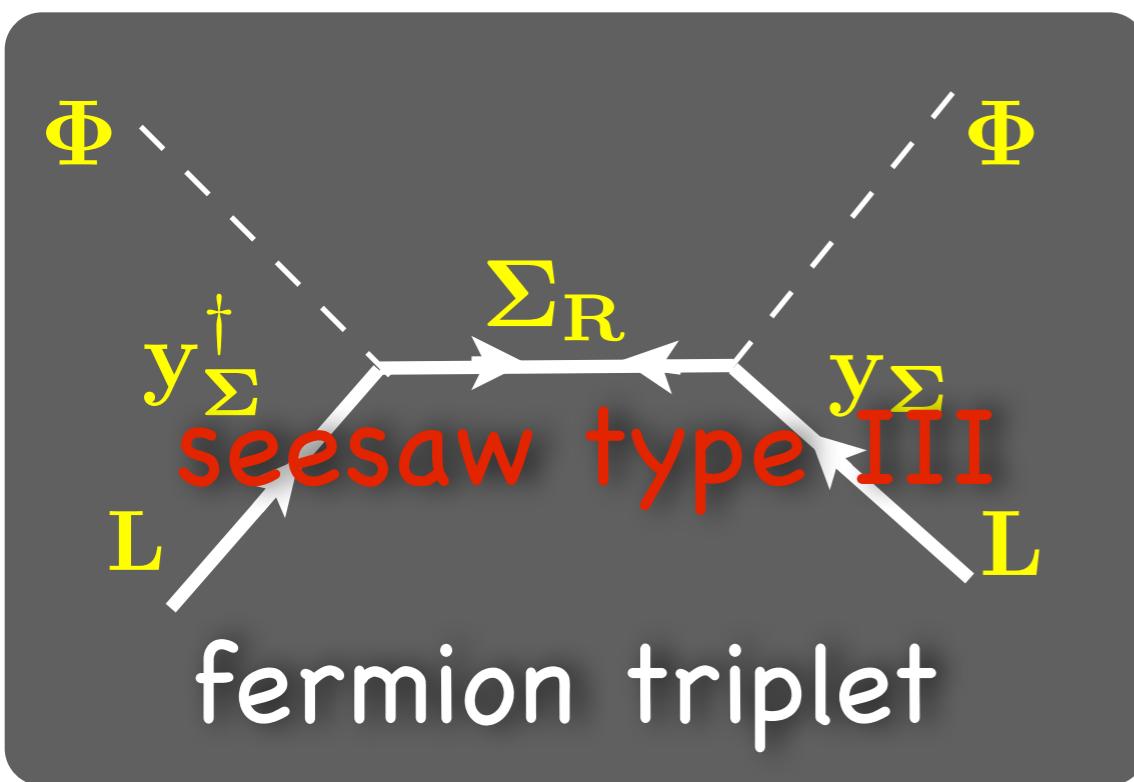
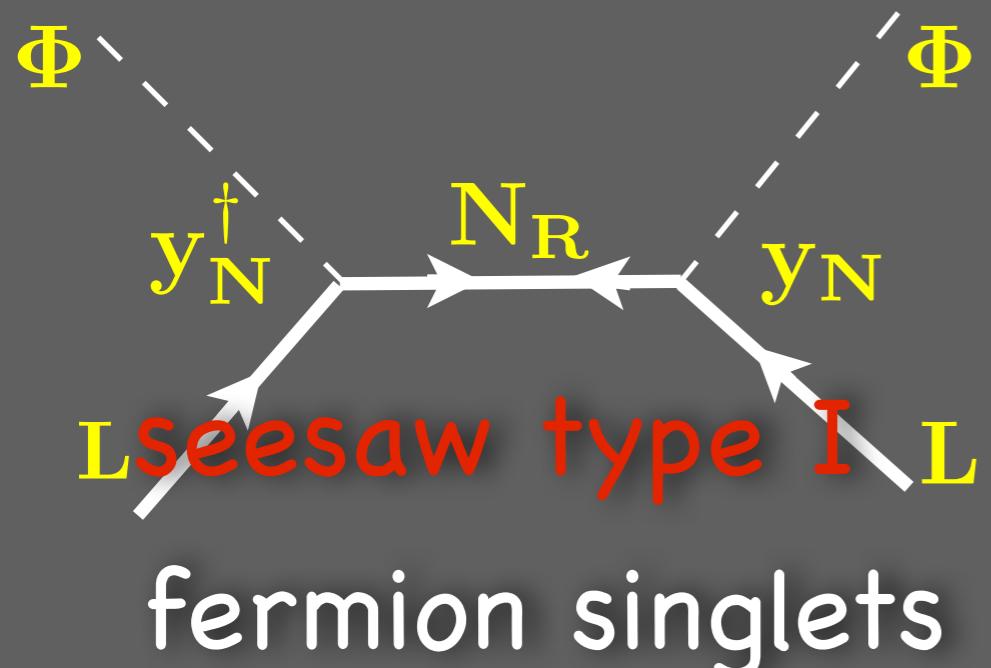
Tree-level Realizations

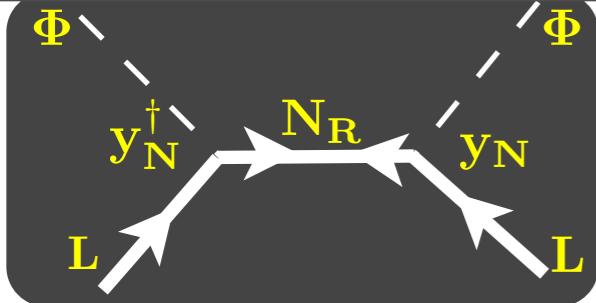
[E. Ma, Phys. Rev. Lett. 81, 1171 (1998)]



Tree-level Realizations

[E. Ma, Phys. Rev. Lett. 81, 1171 (1998)]

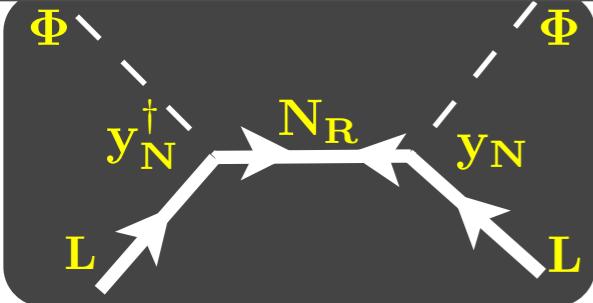




Seesaw Type I

[P. Minkowski, Phys. Lett. B 67, 421 (1977); M. Gell-Mann, P. Ramond and R. Slansky, Supergravity (1979); T. Yanagida, Proc. Workshop on the Univ. Theo. and the Baryon Numb. in the Univ. (1979); R.N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. 44, 912 (1980)]

minimal seesaw Lagrangian: only add R neutrinos to SM



Seesaw Type I

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minimal Seesaw Lagrangian: only add R neutrinos to SM

$$\mathcal{L}_{KE} = \imath \overline{L} \not{\partial} L + \imath \overline{R} \not{\partial} R + \imath \overline{N}_R \not{\partial} N_R$$

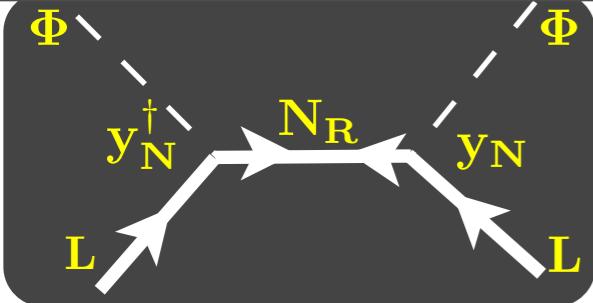
SM lepton doublets

SM lepton singlets

R neutrino singlets

New Physics Scale

$$\mathcal{L}_Y = -\overline{L} \Phi y_\ell R - \overline{L} \tilde{\Phi} y_N^\dagger N_R - \frac{1}{2} \overline{N}_R M_R N_R^c + h.c.$$



Seesaw Type I

[P. Minkowski, Phys. Lett. B 67, 421 (1977); M. Gell-Mann, P. Ramond and R. Slansky, Supergravity (1979); T. Yanagida, Proc. Workshop on the Univ. Theo. and the Baryon Numb. in the Univ. (1979); R.N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. 44, 912 (1980)]

minimal Seesaw Lagrangian: only add R neutrinos to SM

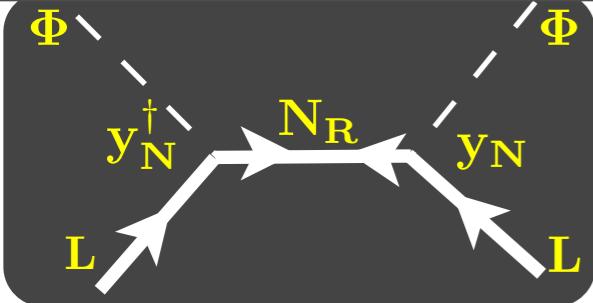
$$\mathcal{L}_{KE} = i \bar{L} \not{\partial} L + i \bar{R} \not{\partial} R + i \bar{N}_R \not{\partial} N_R$$

SM lepton doublets SM lepton singlets R neutrino singlets

$$\mathcal{L}_Y = -\bar{L} \Phi y_\ell R - \bar{L} \tilde{\Phi} y_N^\dagger N_R - \frac{1}{2} \bar{N}_R M_R N_R^c + h.c.$$

New Physics Scale

$$m_\nu \equiv \frac{g}{\Lambda} v^2 = -\frac{1}{2} \boxed{y_N^T} \frac{1}{M_R} \boxed{y_N} v^2$$



Seesaw Type I

[P. Minkowski, Phys. Lett. B 67, 421 (1977); M. Gell-Mann, P. Ramond and R. Slansky, Supergravity (1979); T. Yanagida, Proc. Workshop on the Univ. Theo. and the Baryon Numb. in the Univ. (1979); R.N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. 44, 912 (1980)]

minimal Seesaw Lagrangian: only add R neutrinos to SM

$$\mathcal{L}_{KE} = i \bar{L} \not{\partial} L + i \bar{R} \not{\partial} R + i \bar{N}_R \not{\partial} N_R$$

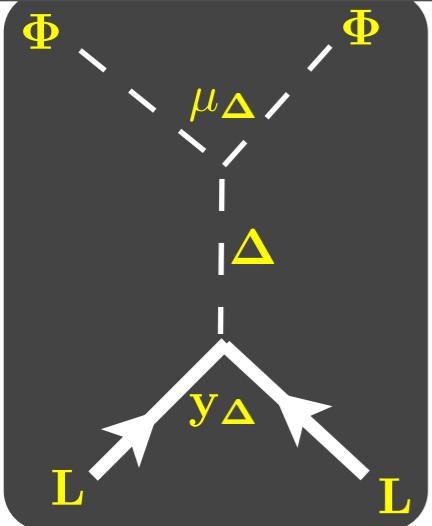
SM lepton doublets SM lepton singlets R neutrino singlets

$$\mathcal{L}_Y = -\bar{L} \Phi y_\ell R - \bar{L} \tilde{\Phi} y_N^\dagger N_R - \frac{1}{2} \bar{N}_R M_R N_R^c + h.c.$$

New Physics Scale

$$m_\nu \equiv \frac{g}{\Lambda} v^2 = -\frac{1}{2} \bar{y}_N^T \frac{1}{M_R} y_N v^2$$

M_R should be of the order 10^{11} TeV (10^5 TeV) for $y_N \approx 1$ ($y_N \approx 10^{-3}$)



Seesaw Type II

[M. Magg and C. Wetterich, Phys. Lett. B 94, 61 (1980); J. Schechter and J.W.F. Valle, Phys. Rev. D 22, 2227 (1980); R.N. Mohapatra and G. Senjanovic, Phys. Rev. D 23, 165 (1981)]

add $SU(2)_L$ triplet scalar field $\vec{\Delta}$ ($Y=1$)

minimal Lagrangian gauge invariant allows for

$$\mathcal{L}_{\Delta, (L, \Phi)} = \tilde{\bar{L}} y_{\Delta} (\vec{\sigma} \cdot \vec{\Delta}) L + \mu_{\Delta} \tilde{\Phi}^{\dagger} (\vec{\sigma} \cdot \vec{\Delta})^{\dagger} \Phi + \text{h.c.}$$

coupling to SM lepton doublets coupling to SM Higgs doublet

$$\vec{\Delta} = (\Delta_1, \Delta_2, \Delta_3)$$

where

$$\tilde{\bar{L}} = i \sigma_2 (L)^c$$

physical fields

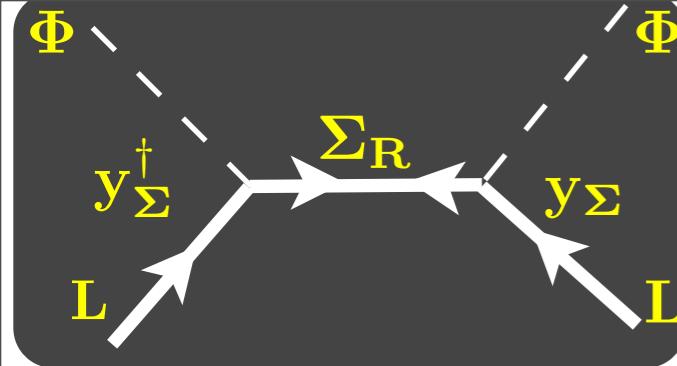
$$\langle \Delta^0 \rangle = \mathbf{u}/\sqrt{2} = \mu_{\Delta} \mathbf{v}^2 / (\sqrt{2} M_{\Delta}^2)$$

$$\Delta^{++} \equiv \frac{1}{\sqrt{2}}(\Delta_1 - i\Delta_2) \quad \Delta^+ \equiv \Delta_3 \quad \Delta^0 \equiv \frac{1}{\sqrt{2}}(\Delta_1 + i\Delta_2)$$

$$m_{\nu} \equiv \frac{g}{\Lambda} v^2 = -2 y_{\Delta} u = -2 y_{\Delta} \frac{\mu_{\Delta}}{M_{\Delta}^2} v^2$$

New Physics Scale

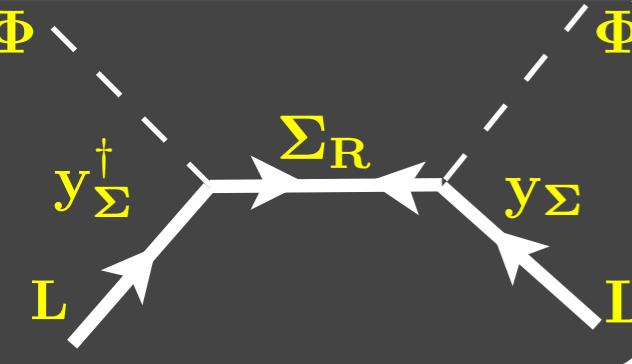
Majorana Mass
Matrix for light
neutrinos



Seesaw Type III

[R. Foot, H. Lew, X.G. He and G.C. Joshi, Z. Phys. C 44, 441 (1989); E. Ma, Phys. Rev. Lett. 81, 1171 (1998)]

add $SU(2)_L$ fermion triplet $\vec{\Sigma}$ ($Y=0$)



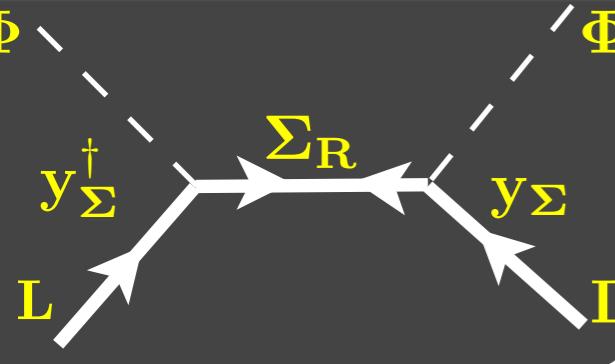
Seesaw Type III

[R. Foot, H. Lew, X.G. He and G.C. Joshi, Z. Phys. C 44, 441 (1989); E. Ma, Phys. Rev. Lett. 81, 1171 (1998)]

add $SU(2)_L$ fermion triplet $\vec{\Sigma}$ ($Y=0$)

$$\vec{\Sigma} = (\Sigma_1, \Sigma_2, \Sigma_3)$$

$$\Sigma^\pm \equiv \frac{1}{\sqrt{2}}(\Sigma_1 \mp i \Sigma_2) \quad \Sigma^0 \equiv \Sigma_3$$



Seesaw Type III

[R. Foot, H. Lew, X.G. He and G.C. Joshi, Z. Phys. C 44, 441 (1989); E. Ma, Phys. Rev. Lett. 81, 1171 (1998)]

add $SU(2)_L$ fermion triplet $\vec{\Sigma}$ ($Y=0$)

$$\mathcal{L}_\Sigma = \imath \overline{\vec{\Sigma}_R} \not{\partial} \vec{\Sigma}_R - \left[\frac{1}{2} \overline{\vec{\Sigma}_R} M_\Sigma \vec{\Sigma}_R^c + \overline{\vec{\Sigma}_R} y_\Sigma (\tilde{\Phi}^\dagger \vec{\sigma} L) + \text{h.c.} \right]$$

Majorana Mass Term coupling with L and Φ

$$\vec{\Sigma} = (\Sigma_1, \Sigma_2, \Sigma_3)$$

$$\Sigma^\pm \equiv \frac{1}{\sqrt{2}}(\Sigma_1 \mp i \Sigma_2) \quad \Sigma^0 \equiv \Sigma_3$$

if $M_\Sigma \gg v$
EWSB


$$\mathbf{m}_\nu \equiv \frac{\mathbf{g}}{\Lambda} \mathbf{v}^2 = - \mathbf{y}_\Sigma^\text{T} \frac{1}{2 M_\Sigma} \mathbf{y}_\Sigma \mathbf{v}^2$$

New Physics Scale

Majorana Mass Matrix for light neutrinos

Proposed Problems:

E1.

Show explicitly why we need $1/2$ in the Majorana mass term

E2.

Show that the most general mass term Lagrangian for a 4-component field

$$\mathcal{L}_{D+M} = -m_D \overline{\Psi_L} \Psi_R - a \overline{\Psi_L^c} \Psi_L - b \overline{\Psi_R^c} \Psi_R + h.c.$$

describes two Majorana particles with different masses.
Discuss in what limit the 4-component Dirac formalism can be recovered.

E3.

How many physical phases there are in the mixing matrix if neutrinos are Majorana particles?

Proposed Problems:

- E4. Show that a Majorana mass matrix is, in general, a complex symmetric matrix that can be diagonalized by an orthogonal matrix.
- E5. Show that neutrinoless double beta decay implies in Majorana neutrinos. Can we distinguish Majorana from Dirac using neutrino interactions?
- E6. Consider the two body decay $\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$ at rest. Calculate the momentum p_i and energy E_i of mass m_i as a function of the masses of the pion, the muon and m_i . Estimate, to first non-zero order in m_i , the difference between E_i and p_i . Can we assume the neutrinos produced in this decay have the same energy or momentum?