LHC Physics

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Outline

Lecture I:

- Some motivation.
- Calculating LHC cross sections (Xsection).
- Parton distribution functions, parton luminosities.

Lecture II:

- Example, top-pair Xsection calculation.
- Kinematics & jets.

Lecture I:

Some motivation (SM problems, naturalness);

How to calculate Xsections @ the LHC;

Parton distribution functions (PDFs) parton luminosities.

Why the LHC? What are the problems of the Standard Model* (SM), before the LHC started?

WW/unitarity, masses	fine tuning, naturalness	neutrino masses	flavor puzzle
		dark matter	(strong CP)
		baryogenesis	unification, charge quantisation

* Let's set quantum gravity aside for simplicity …

Why the LHC? What are the problems of the Standard Model* (SM), before the LHC started?

data driven, clear scale	conceptual vague scale	data driven, no clear reachable scale	conceptual
WW/unitarity, masses	fine tuning, naturalness	neutrino masses	flavor puzzle
		dark matter	(strong CP)
		baryogenesis	unification, charge quantisation

* Let's set quantum gravity aside for simplicity …

Why the LHC? (2 subjective reasons)

- Higgs & unitarity, suggests physics < TeV.
- Given the Higgs, the fine tuning problem requires new physics at a scale, generically, within the reach of the LHC.

The SM Higgsless Unitarity Problem

$$\mathcal{L}_{mass} = M_W^2\, W_\mu^+ W^{-\,\mu} + \frac{1}{2} M_Z^2\, Z^\mu Z_\mu - \sum_{i,j} \left\{ \bar{u}_L^{(i)} M_{ij}^u u_R^{(j)} + \bar{d}_L^{(i)} M_{ij}^d d_R^{(j)} + \bar{e}_L^{(i)} M_{ij}^e e_R^{(j)} + \bar{\nu}_L^{(i)} M_{ij}^\nu \nu_R^{(j)} + h.c. \right\}$$

Mass terms are not invariant under the local $SU(2)_L \times U(1)_Y$ symmetry

The optical theorem

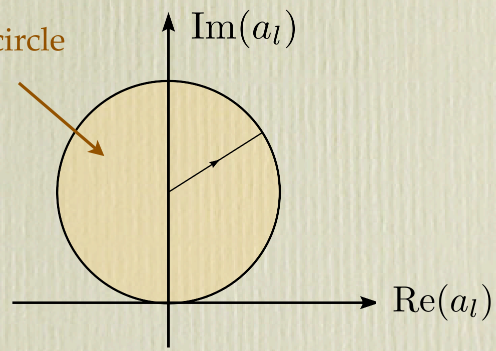
$$\frac{E}{p}\frac{1}{s}\,\mathrm{Im}\,(A(\theta=0)) = \sigma_{tot}(WW \to \text{anything})$$

requires for each partial wave:

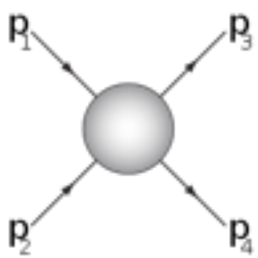
$$\mathrm{Im}(a_l(s)) = |a_l(s)|^2 + |a_l^{in}(s)|^2$$

$$\boxed{\mathrm{Re}(a_l(s)) \leq \frac{1}{2}}$$

physical amplitudes bound within the unitarity circle



The SM Higgsless Unitarity Problem



$$s = (p_1 + p_2)^2 = (p_3 + p_4)^2$$
$$t = (p_1 - p_3)^2 = (p_2 - p_4)^2$$
$$u = (p_1 - p_4)^2 = (p_2 - p_3)^2$$

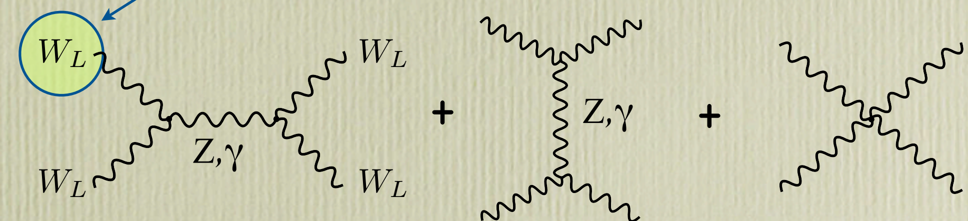
Mandelstam variables

The amplitude for scattering of longitudinal W's and Z's grows with the energy and eventually violates the unitarity bound:

Ex: $$A(W_L^+ W_L^- \to W_L^+ W_L^-) = \frac{g_2^2}{4M_W^2}(s+t)$$

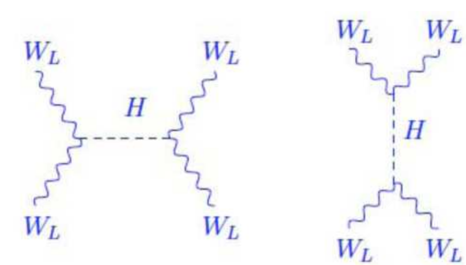
each longitudinal polarization gives a factor E

$$\epsilon_L^\mu = \frac{p^\mu}{M_W} + O\left(\frac{E}{M_W}\right)$$



Unitarity is violated at $\sqrt{s} \simeq \Lambda = 1.2\,\mathrm{TeV}$

Unitarity is restored by adding diagrams with intermediate Higgs in them as long as m_h <. 800 GeV .



The Higgs & the fine tuning/naturalness problem

't Hooft definition of technical naturalness:
a parameter is natural if when it's set to 0 there's an enhanced symmetry.

Additive renormaliztion (unnatural parameters): $d\lambda/dln\mu \propto \lambda g(\mu) + f(\mu)$
Multiplicative renormalization (natural parameters): $d\lambda/dln\mu \propto \lambda g(\mu)$

The Higgs mass parameter is subject to additive renormalisation.
Thus, it is sensitive to microscopic new physics dynamics.

Naturalness might give a hint: Higgs mass is additive, sensitive to microscopic scales. Within the SM it translates to UV sensitivity: $\frac{d\,m_H^2}{d\,\ln\mu} = \frac{3m_H^2}{8\pi^2}\left(2\lambda + y_t^2 - \frac{3g_2^2}{4} - \frac{3g_1^2}{20}\right).$

See: Giudice (13)

Beyond the SM: any scale that couples to the Higgs (or even to tops, gauge ...) will induce a large shift to the Higgs mass, $\delta m_H^2 \approx \frac{\alpha}{4\pi} M^2.$ Farina, Pappadopulo & Strumia (13)

Tunning vs. fine tuning/naturalness problem

Flavor puzzle: the parameters' are small and hierarchical.

Is the flavor sector fine tuned? $m_u/m_t \sim 10^{-5}$.

Massless fermions theory: $\mathcal{L}_{\text{fermions}} \in \bar{\psi}_L \partial_\mu \gamma_\mu \psi_L + \bar{\psi}_R \partial_\mu \gamma_\mu \psi_R$

Two separate U(1)'s: $\psi_{L,R} \to e^{\theta_{L,R}} \psi_{L,R}$

Sym' is indeed enhanced when the mass vanishes. (modulo anomalies)

Mass term breaks it to a single U(1): $\bar{\psi}_L m \psi_R$

Only invariant under transformation with $\theta_L = \theta_R = \theta$

Flavor (including neutrinos) parameters are natural

Flavor parameters are natural, subject to tuning & then radiatively stable, no UV sensitivity.

Within the SM the only exception is the Higgs mass. (& the QCD angle & the cosmological constant)

(A simple way to understand this is to realise that a massless fermion requires 2 degrees of freedom (dof) while a massive 4.
A massless vector boson requires 2 and a massive 3.
Thus, there is discontinuity in the massless to massive limit.
This does not happen for a massive scalar.)

LHC physics

Why LHC?

Need more E!

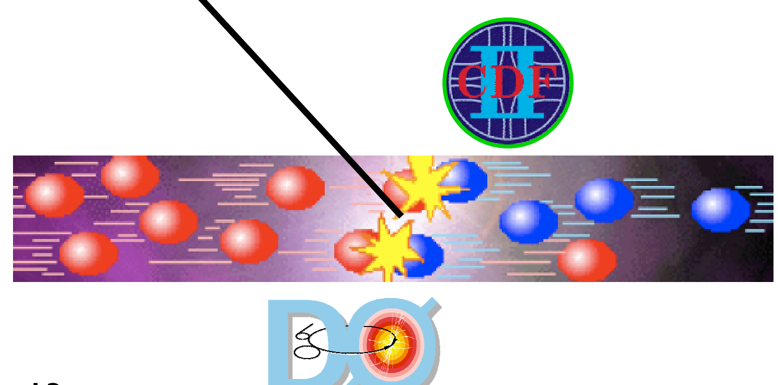
Sync' radiation,
problem for circular e-collider:

$$\frac{dW}{dt}\Big|_e \approx \left(\frac{e}{r}\right)^2 \left(\frac{E}{m_e}\right)^4 \sim 10^4\,\mathrm{GeV\,s^{-1}} \;\Rightarrow\; \times 10^{12} e \sim \textbf{MWs radiation!}$$

10^{13} improvement when e <=> proton

E~2TeV (2000GeV)

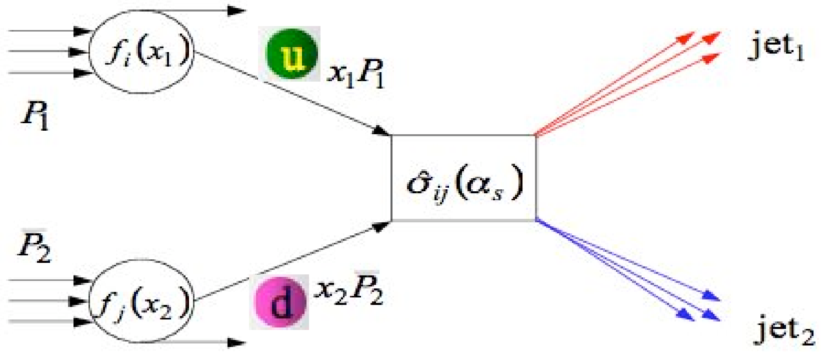
$\bar{p}$

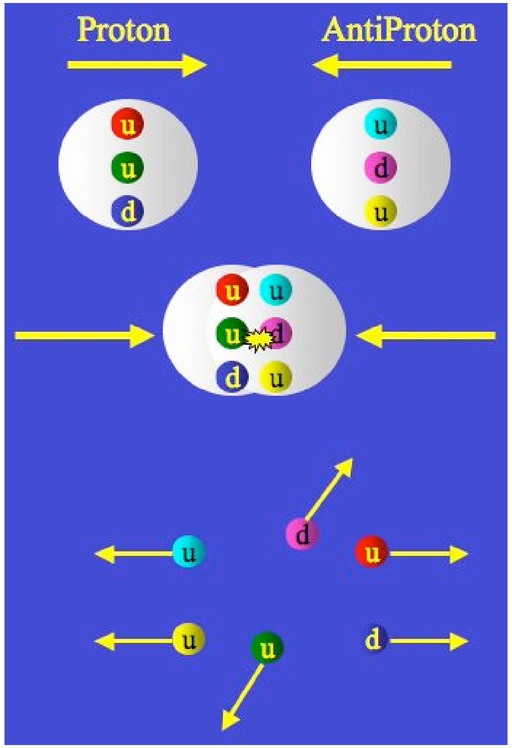


p

Nothing's free - QCD dust

- Expect $m_t = 130\text{-}200\,\mathrm{GeV}$, who needs 2TeV?
- Proton anti-proton are composite:
- Typical E's much smaller: $E^2_{\mathrm{event}} = x_1 x_2 E^2_{p\bar{p}}$





- We don't know what is E_{CM}.
- We don't know which particles interacted.
- And ...

Calculating Xsections at the LHC: Parton Distribution Functions (PDFs)

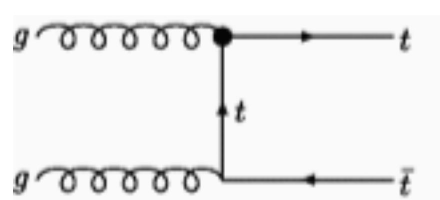
(assuming no p-rapidity or pt cuts)

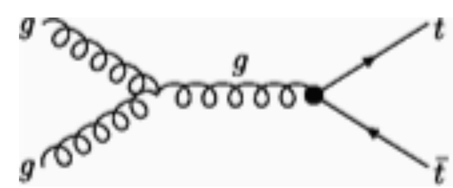
$$\frac{d\sigma(pp \to f)}{d\hat{s}} = \sum_{ij} \hat{\sigma}_{ij}(\hat{s}) \int_0^1 \int_0^1 dx_i dx_j f_i(x_i) f_j(x_j) \delta(\hat{s} - x_i x_j s)$$

$\hat{\sigma}(\hat{s})$ Corresponds to the Born/hard/local/short distance Xsection that we would like to calculate/measure.

For instance $gg \to t\bar{t}$

$\hat{s} = (p_t + p_{\bar{t}})^2 = (p_g + p_{g'})^2$

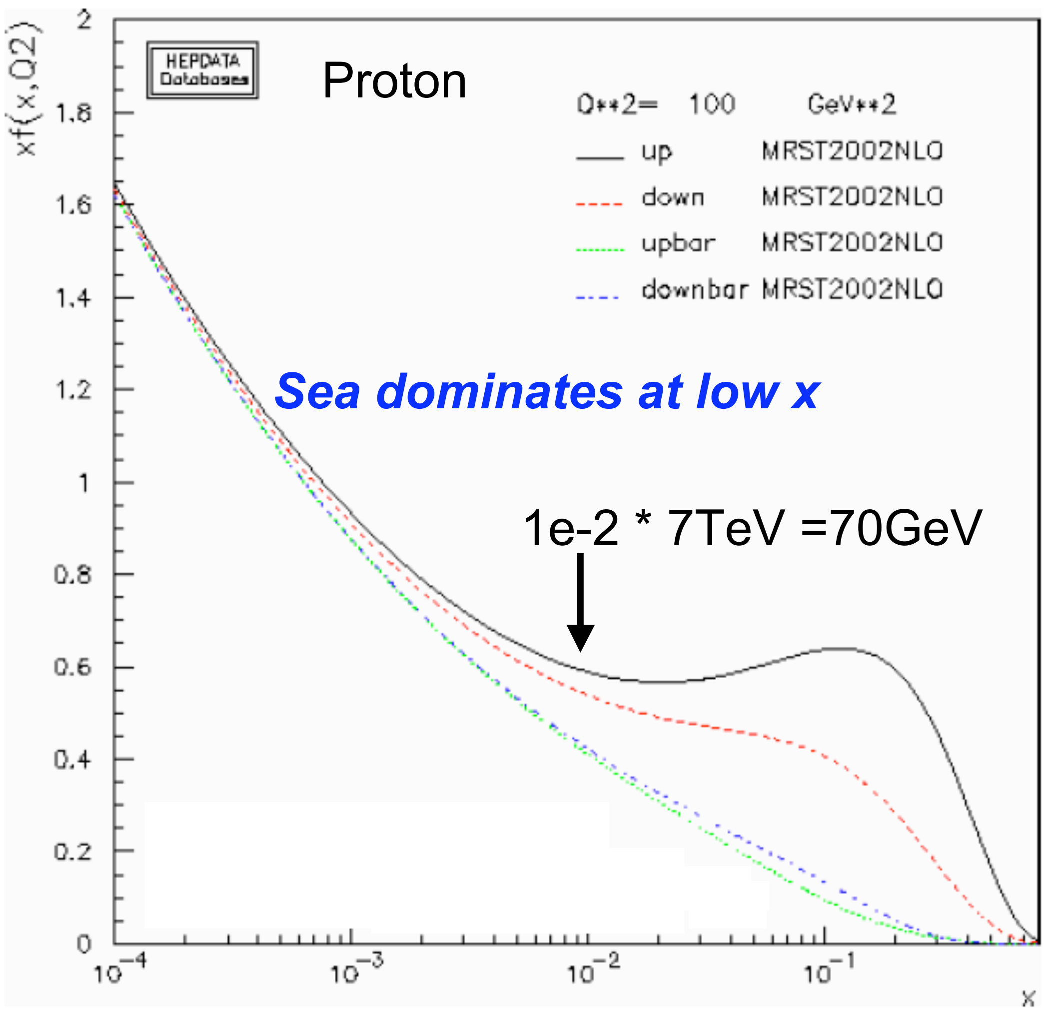




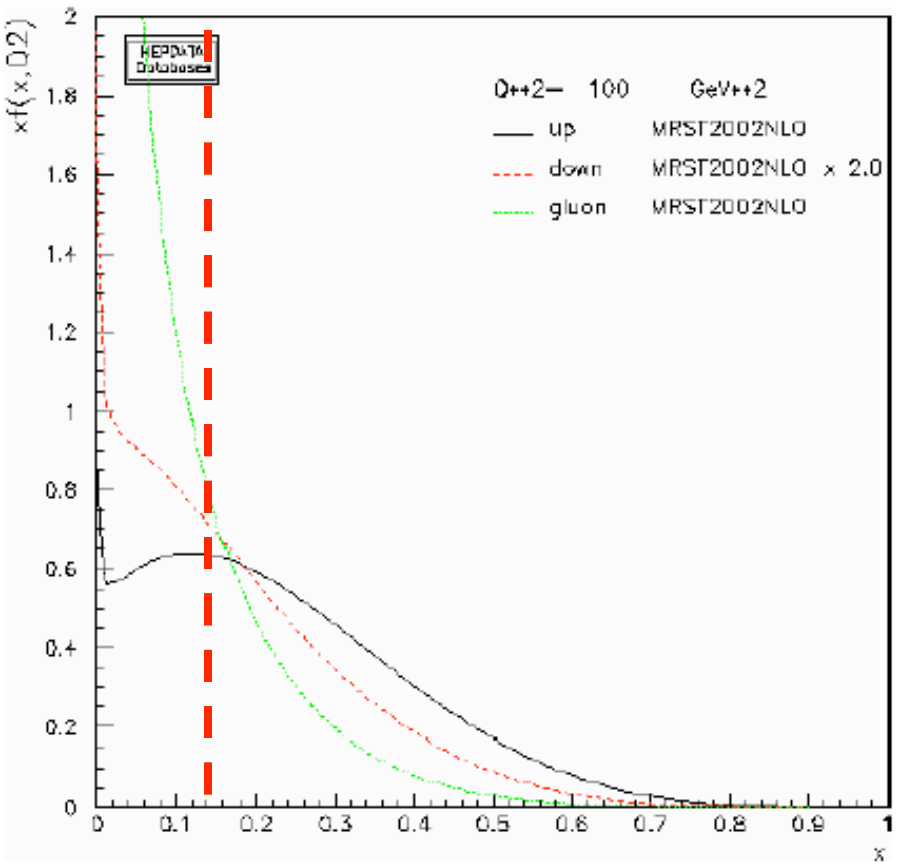
PDFs (What are they?)

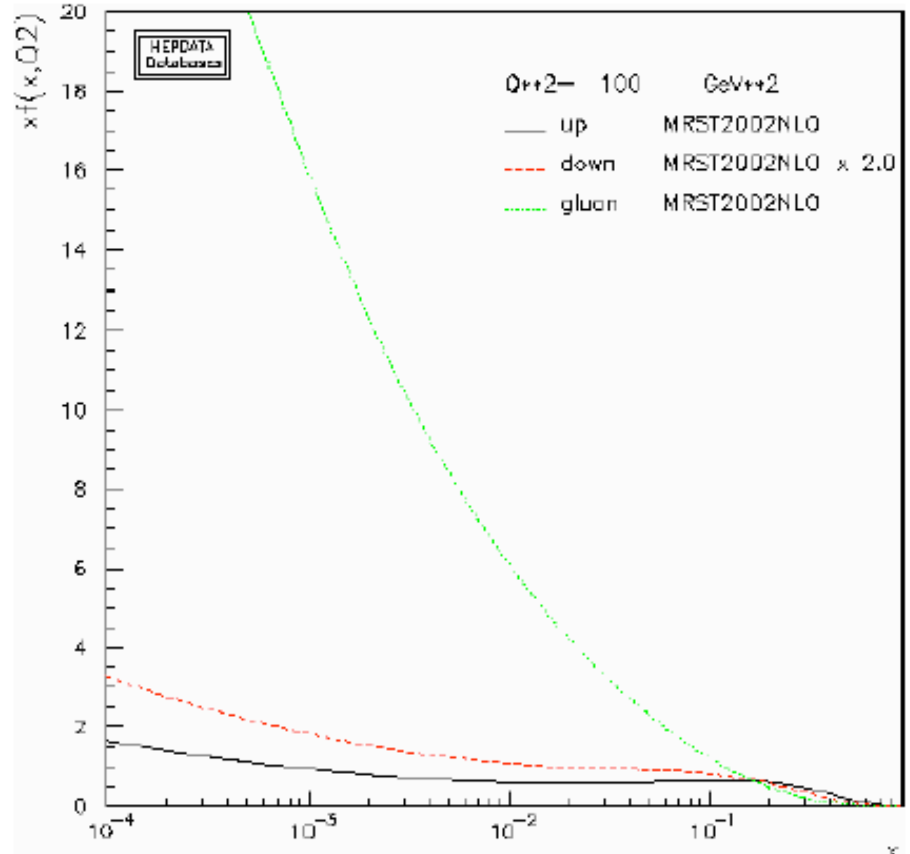
PDFs are non-perturbative objects.

Probability of finding a constituent f with a longitudinal momentum fraction of $x \Rightarrow f_f(x)dx$



PDFs at the LHC





Gluons dominate at low x .

To set the scale, x = 0.14 at LHC is 0.14 * 7TeV = 1TeV

=> The LHC is a gluon collider !!!

Lecture I:

Some motivation (SM problems, naturalness);

How to calculate Xsections @ the LHC;

Parton distribution functions (PDFs) parton luminosities.

Beginning of 2nd Lecture

- Parton Luminosities (cont').
- Example, top-pair Xsection calculation.
- Kinematics & jets.

Calculating Xsections at the LHC: Parton Distribution Functions (PDFs)

(assuming no p-rapidity or pt cuts)

$$\frac{d\sigma(pp \to f)}{d\hat{s}} = \sum_{ij} \hat{\sigma}_{ij}(\hat{s}) \int_0^1 \int_0^1 dx_i dx_j f_i(x_i) f_j(x_j) \delta(\hat{s} - x_i x_j s)$$

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For instance $gg \to t\bar{t}$

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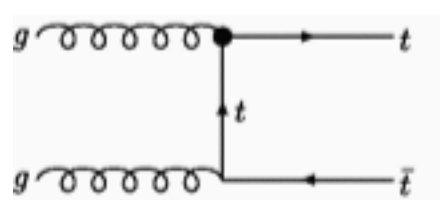
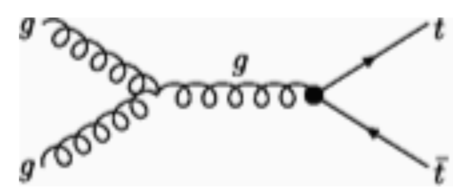




Physically only pairs of PDF are important

(assuming no p-rapidity or pt cuts)

$$\frac{d\sigma(pp \to f)}{d\hat{s}} = \sum_{ij} \hat{\sigma}_{ij}(\hat{s}) \int_0^1 \int_0^1 dx_i dx_j f_i(x_i) f_j(x_j) \delta(\hat{s} - x_i x_j s)$$

$$= \sum_{ij} \frac{\hat{\sigma}_{ij}(\hat{s})}{\hat{s}} \int_0^1 \int_0^1 dx_i dx_j f_i(x_i) f_j(x_j) \delta\left(1 - x_i x_j \frac{s}{\hat{s}}\right)$$

$$\tau = \frac{\hat{s}}{s}$$

$$\frac{d\sigma(pp \to f)}{d\tau} = \sum_{ij} \frac{\hat{\sigma}_{ij}(\hat{s})}{\tau} \int_0^1 \int_0^1 dx_i dx_j f_i(x_i) f_j(x_j) \delta\left(1 - \frac{x_i x_j}{\tau}\right)$$

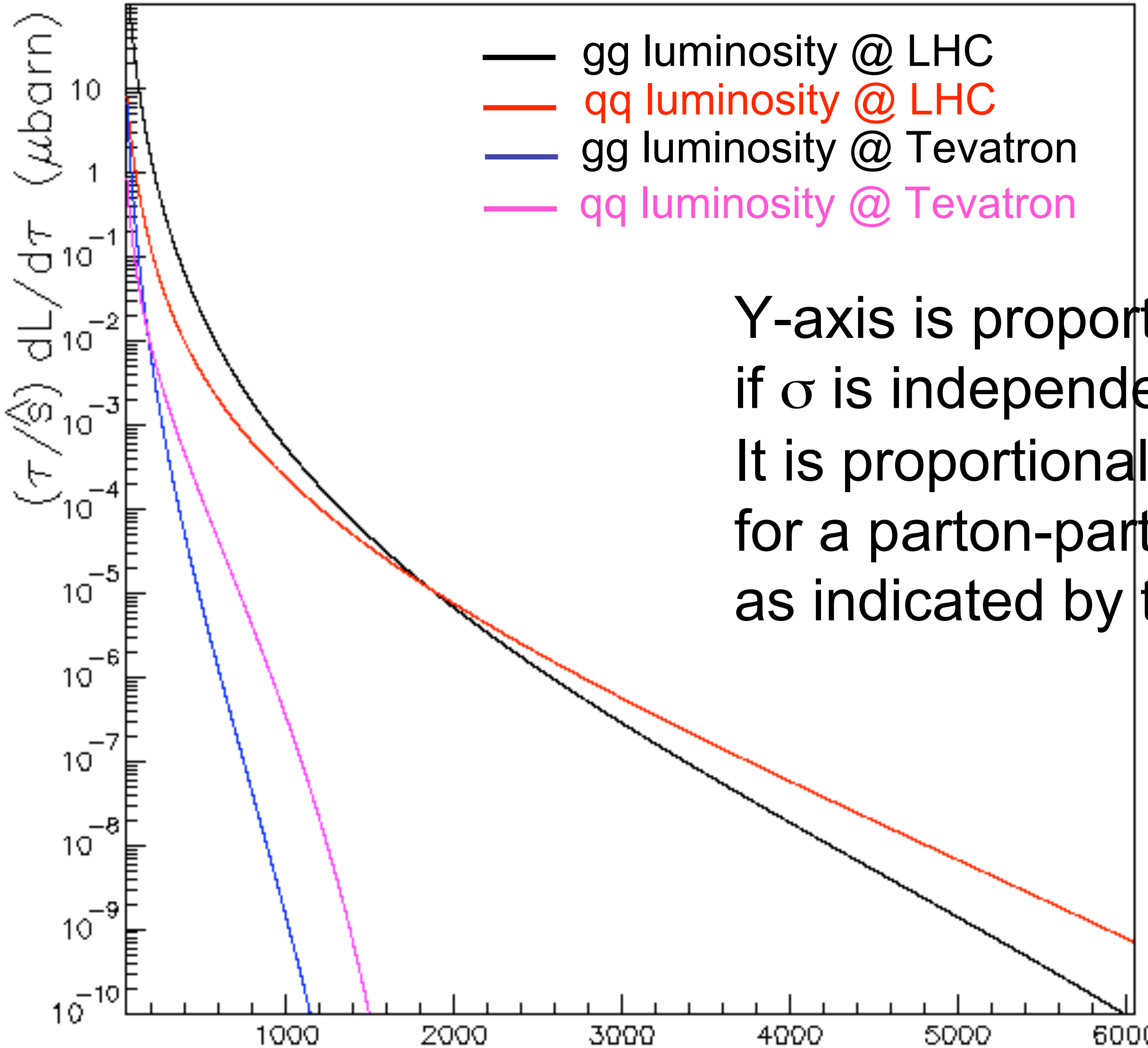
$$\frac{d\sigma(pp \to f)}{d\tau} = \sum_{ij} \frac{\hat{\sigma}_{ij}(\hat{s})}{\tau} \boxed{\int_\tau^1 dx_i \frac{\tau}{x_i} f_i(x_i) f_j\left(\frac{\tau}{x_i}\right)}$$

Parton-parton luminosities

$$\frac{dL_{ij}}{d\tau} = \frac{1}{1+\delta_{ij}} \int_{\tau}^{1} \frac{dx}{x} \left[f_i(x) f_j\left(\frac{\tau}{x}\right) + f_i\left(\frac{\tau}{x}\right) f_j(x) \right]$$

- Function of dimensionless quantity:
  - Scaling => independent of CM energy of proton proton collisions.
- However, $\hat{\sigma}_{ij}\left(\hat{s}\right) \equiv \hat{\sigma}_{ij}\left(E^2\right)$ depends on E. The collider characteristics only help us understand the energy scale E^2 accessible given an S for proton-proton collisions.

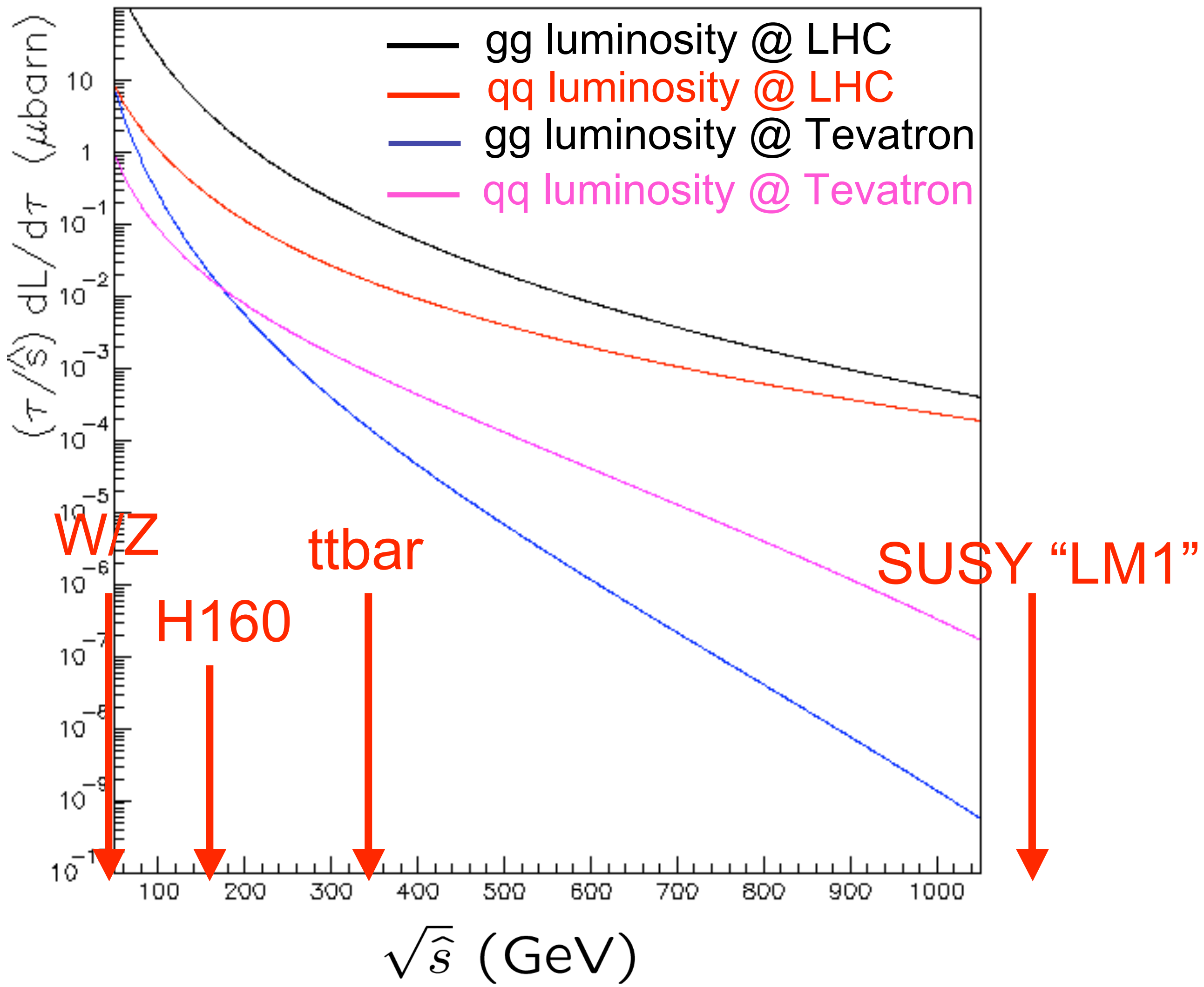
Luminosity functions, adding Xsection scale



Y-axis is proportional to σ if σ is independent of $\hat{s}$

It is proportional to probability for a parton-parton collision with $\hat{s}$ as indicated by the x-axis.

Zooming-in on the < 1 TeV region



Cross sections at 1.96TeV versus 14TeV Tevatron vs LHC

	Cross section		Ratio
$Z \rightarrow \mu\mu$	260pb	1750pb	6.7
WW	10pb	100pb	10
H_{160GeV}	0.2pb	25pb	125
$mSugra_{LM1}$	0.0006pb	50pb	80,000

At $10^{32}cm^{-2}s^{-1}$ LHC might accumulate $10pb^{-1}$ in one day!

Consider for example LHC top pair production

$$\sigma^{p(g)p(g)\to t\bar{t}} = \int_{\tau_{\min}}^{1} d\tau \, \frac{\hat{\sigma}^{t\bar{t}}(\hat{s} = \tau s)}{\tau} \, \frac{d\mathcal{L}_{gg}}{d\tau} \qquad \tau_{\min} = (2m_t/14\,\mathrm{TeV})^2$$

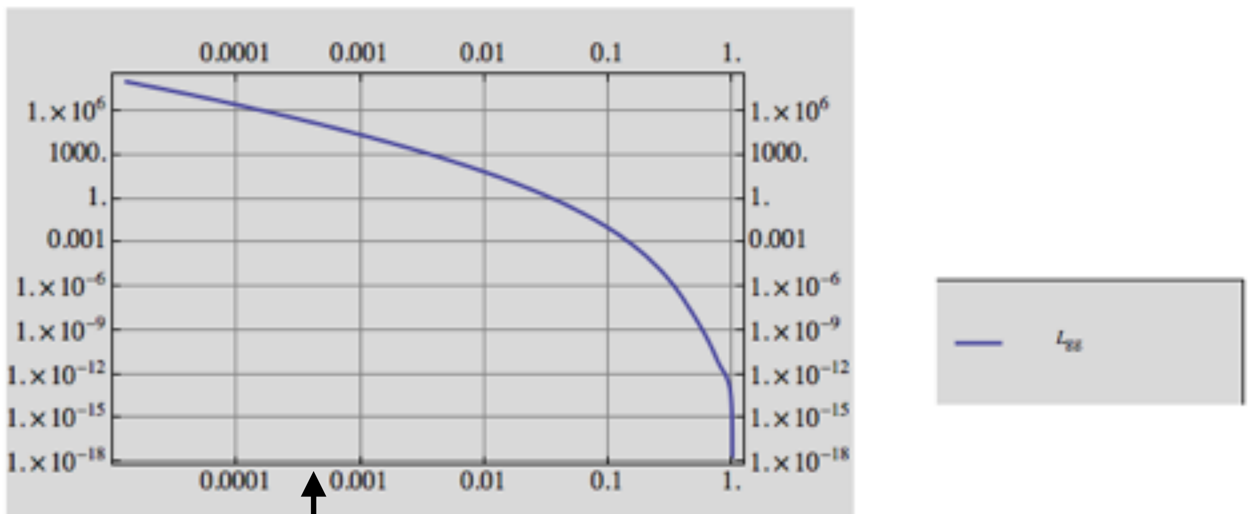
$$\frac{d\mathcal{L}_{gg}}{d\tau} = \int_{\tau}^{1} \frac{dx}{x} \, f_g(x) f_g(\tau/x)$$

$$\beta = \sqrt{1 - 4m_t^2/\hat{s}}$$
$$\hat{\sigma}_{gg\to t\bar{t}} = \frac{\pi\alpha_s^2\beta}{48\hat{s}} \left(31\beta + \left(\frac{33}{\beta} - 18\beta + \beta^3\right) \ln\left[\frac{1+\beta}{1-\beta}\right] - 59\right)$$

The gluon luminosity function at LHC14

MSTW-PDF running factorisation scale as $Q^2 = \hat{s} = \tau s = \tau \times 14^2\ \mathrm{TeV}^2$

$$\frac{1}{\tau}\frac{d\mathcal{L}_{gg}}{d\tau}$$

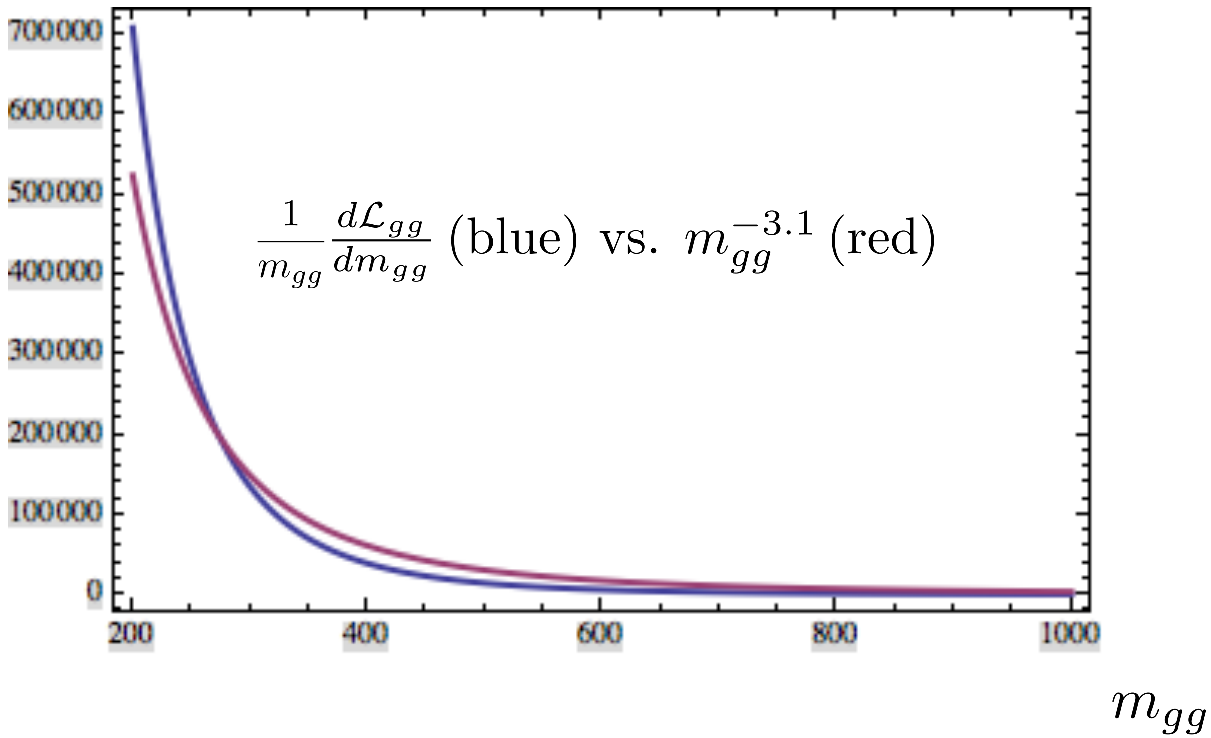


τ

Typical τ for $t\bar{t}$ proudction at LHC14: $(2m_t/14\,\mathrm{TeV})^2 \sim 6 \times 10^{-4}$.

The luminosity functions are rapidly falling

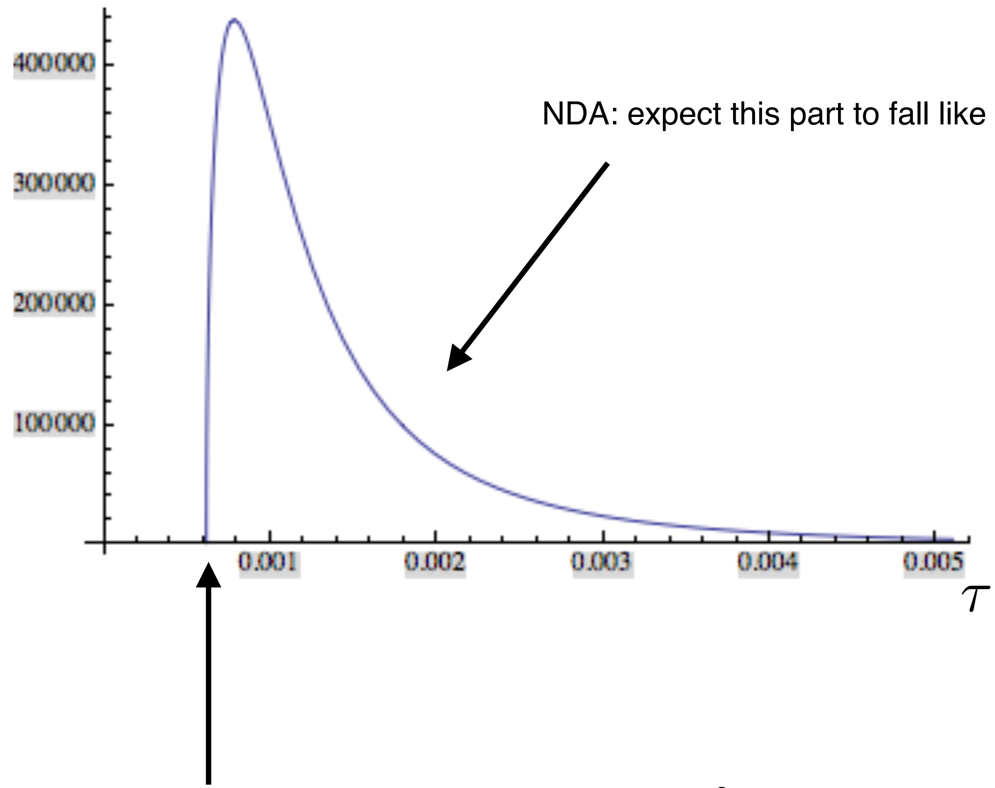
MSTW-PDF running factorisation scale as $Q^2 = \hat{s} = \tau s = \tau \times 14^2\ \text{TeV}^2$



Generically, cross section falls even faster!

MSTW-PDF running factorisation scale as $Q^2 = \hat{s} = \tau s = \tau \times 14^2\,\mathrm{TeV}^2$

$$\frac{\hat{\sigma}^{t\bar{t}}(\tau)}{\tau}\;\frac{d\mathcal{L}_{gg}}{d\tau}$$



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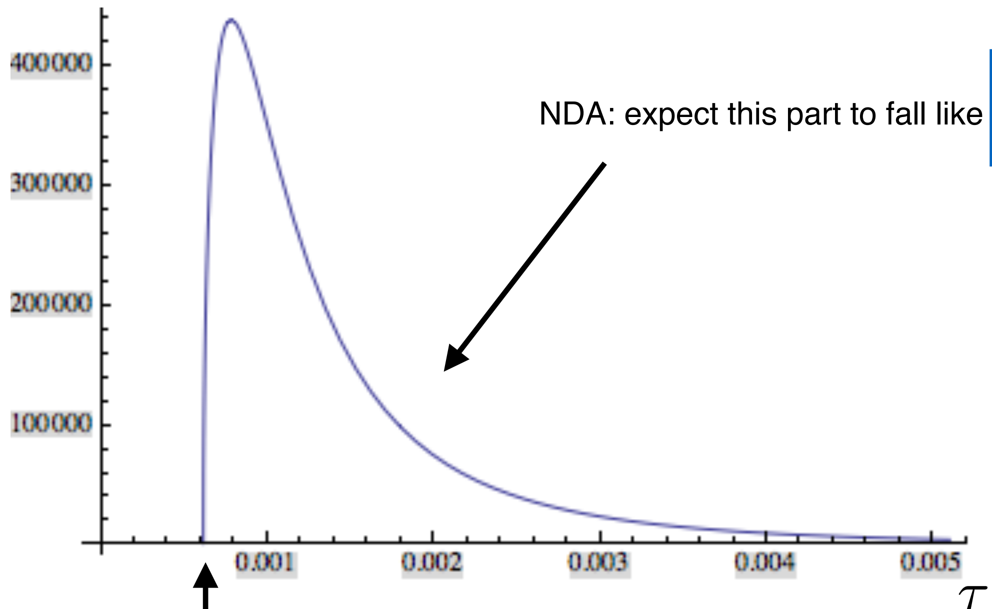
$$\frac{\hat{\sigma}^{t\bar{t}}(\tau)}{\tau}\,\frac{d\mathcal{L}_{gg}}{d\tau}$$

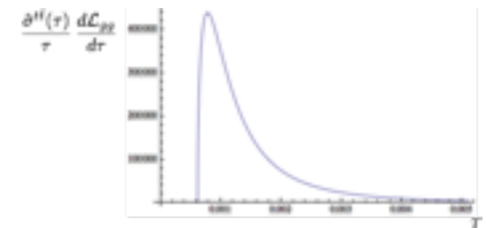


NDA: expect this part to fall like $\boxed{m_{gg}^{-5} \propto \tau^{-5/2}}$

Typical τ for $t\bar{t}$ proudction at LHC14: $(2m_t/14\,\mathrm{TeV})^2 \sim 6 \times 10^{-4}$.

Back to estimating LHC cross section

What are the implications for this rapid fall?

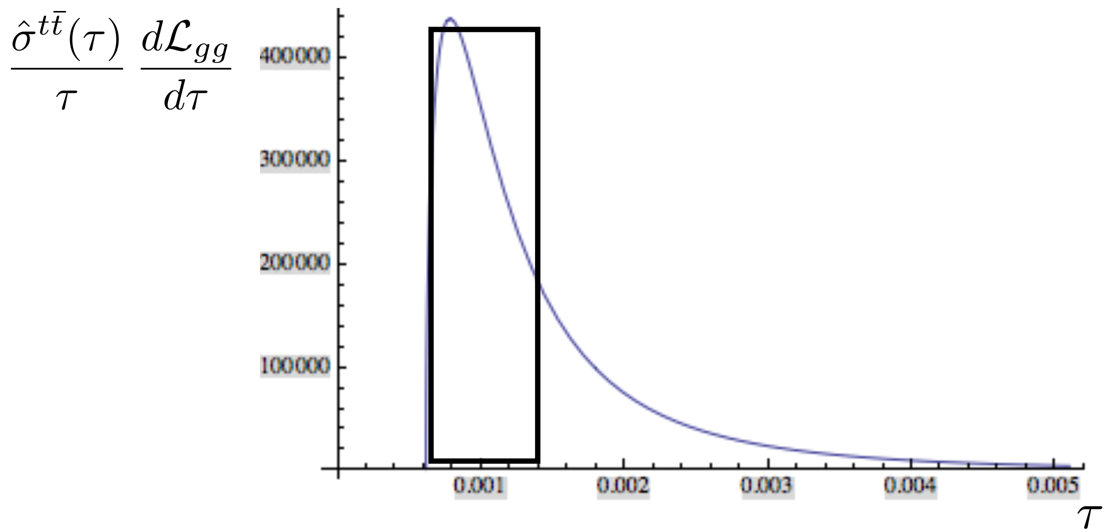


Massive particles (h,W,Z,t, *squarks*, *KK gluon* …) are produced near threshold.

Any dimensional cut (in the transverse direction), m_{xx}, p_T, *missing* E_T, H_T , implies that the signal and background distributions would peak right where the cut is located.

Maybe we can use this fact for a quick & rough estimation of the top pair Xsection?

Rough estimation for the LHC cross section step I: Replacing the integral with differential



Let's replace the integral with differential:

$$\sigma^{p(g)p(g)\to t\bar t} = \int_{\tau_{\min}}^{1} d\tau\ \frac{\hat\sigma^{t\bar t}(\hat s{=}\tau s)}{\tau}\ \frac{d\mathcal{L}_{gg}}{d\tau} \sim \Delta\tau\ \frac{\hat\sigma^{t\bar t}(\tau s)}{\tau}\ \frac{d\mathcal{L}_{gg}}{d\tau}\ \Big|_{\tau\to\frac{4}{3}\tau_{\min}}$$

$$\Delta\tau \sim \tfrac{4}{3}\tau_{\min}$$

Rough NDA estimation for the cross section step 1.1: Replacing the Born Xsection with its NDA value

NDA for 2->2 Xsection (far from threshold): $\hat{\sigma}(\hat{s}) \to \frac{1}{\hat{s}}$

$$\sigma^{p(g)p(g)\to t\bar{t}} = \int_{\tau_{\min}}^{1} d\tau \, \frac{\hat{\sigma}^{t\bar{t}}(\hat{s}{=}\tau s)}{\tau} \, \frac{d\mathcal{L}_{gg}}{d\tau}$$

$$\sim \Delta\tau \, \frac{\hat{\sigma}^{t\bar{t}}(\tau s)}{\tau} \, \frac{d\mathcal{L}_{gg}}{d\tau} \, \Big|_{\tau\to\frac{4}{3}\tau_{\min}}$$

$$\sim \Delta\tau \, \frac{\frac{\alpha_s^2}{\tau s}}{\tau} \, \frac{d\mathcal{L}_{gg}}{d\tau} \, \Big|_{\tau\to\frac{4}{3}\tau_{\min}}$$

And the results are:

$$\text{Precise}^{\text{LO}}:\ \sigma^{p(g)p(g)\to t\bar{t}} = \int_{\tau_{\min}}^{1} d\tau\, \frac{\hat{\sigma}^{t\bar{t}}(\hat{s}=\tau s)}{\tau}\, \frac{d\mathcal{L}_{gg}}{d\tau} = 398.687\ \text{pb}$$

$$\text{Approx' luminosities:}\ \Delta\tau\, \frac{\hat{\sigma}^{t\bar{t}}(\tau s)}{\tau}\, \frac{d\mathcal{L}_{gg}}{d\tau}\, \Big|_{\tau\to\frac{4}{3}\tau_{\min}} = 354.212\ \text{pb}$$

$$\text{"NDA":}\ \Delta\tau\, \frac{\frac{\alpha_s^2}{\tau s}}{\tau}\, \frac{d\mathcal{L}_{gg}}{d\tau}\, \Big|_{\tau\to\frac{4}{3}\tau_{\min}} = 940.538\ \text{pb}$$

my mathematica:

```
In[186]:= GeV2pb = 0.389 10^9 pb;
mt = 173.1;
βt[shat_] := Sqrt[1 - 4 mt^2/shat]
αs = 0.11;
σggtt[τ_] := (π αs^2 βt[τ s14])/(
  48 τ s14) (31 βt[τ s14]^2 + (33/βt[τ s14] - 18 βt[τ s14] + βt[τ s14]^3) Log[(1 + βt[τ s14])/(1 - βt[τ s14])] - 59)
In[191]:= NIntegrate[dLdtaugg14Num[τp] σggtt[τp], {τp, (2 mt)^2/s14, 1}] GeV2pb
Out[191]= 398.687 pb
In[232]:= dLdtaugg14Num[4/3 (2 mt)^2/s14] σggtt[4/3 (2 mt)^2/s14] 4/3 (2 mt)^2/
  s14 GeV2pb
Out[232]= 354.212 pb
In[233]:= dLdtaugg14Num[4/3 (2 mt)^2/s14] ( αs^2/(4/3 (2 mt)^2)) 4/3 (2 mt)^2/s14 GeV2pb
Out[233]= 940.538 pb
```

$t\bar{t}$ Xsection @ LHC14, compare with state of the art:

$$\text{Precise}^{\text{LO}}: \ \sigma^{p(g)p(g)\to t\bar{t}} = \int_{\tau_{\min}}^{1} d\tau \, \frac{\hat{\sigma}^{t\bar{t}}(\hat{s}=\tau s)}{\tau} \frac{d\mathcal{L}_{gg}}{d\tau} = 398.687 \text{ pb}$$

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Theory: Xsection (Tevatron, LHC) now known to NNLO (+NNLL resum')

Collider	σ_{tot} [pb]	scales [pb]	pdf [pb]
Tevatron	7.164	$+0.110(1.5\%)$ $-0.200(2.8\%)$	$+0.169(2.4\%)$ $-0.122(1.7\%)$
LHC 7 TeV	172.0	$+4.4(2.6\%)$ $-5.8(3.4\%)$	$+4.7(2.7\%)$ $-4.8(2.8\%)$
LHC 8 TeV	245.8	$+6.2(2.5\%)$ $-8.4(3.4\%)$	$+6.2(2.5\%)$ $-6.4(2.6\%)$
LHC 14 TeV	$\boxed{953.6}$	$+22.7(2.4\%)$ $-33.9(3.6\%)$	$+16.2(1.7\%)$ $-17.8(1.9\%)$

Mitov, CERN, 4/13

Bärnreuther, Czakon & Mitov; Czakon & Mitov x2 (12); Czakon, Fiedler & Mitov (13).

Some kinematics

LHC, longitudinal vs. transverse

Relativistic invariant phase-space element:

$d\tau = d^3p/E = dp_x dp_y dp_z/E$

Define pp collision axis along *z-axis*:

From $p^\mu = (E, p_x, p_y, p_z)$ – which are invariant under boosts along z?

the two longitudinal components: E and p_z are NOT invariant the two transverse components: p_x and p_y (and dp_x, dp_y) ARE invariant

Need all variables invariant for boost along z-axis:

For convenience, define $\mathbf{p}^\mu$ with only 1 component not Lorentz invariant Choose p_T, m, ϕ as the "transverse" (invariant) coordinates

where $p_T \equiv p\sin(\theta)$ and ϕ is the azimuthal angle

As 4th coordinate define "rapidity": $y = 1/2 \ln [(E+p_Z)/(E-p_Z)]$

Rapidity

- Form a boost of velocity β along z axis
  - $p_z \Rightarrow \gamma(p_z + \beta E)$
  - $E \Rightarrow \gamma(E + \beta p_z)$
  - Transform rapidity $\Rightarrow$

$$y = \frac{1}{2}\ln\frac{E+p_z}{E-p_z} \Rightarrow \frac{1}{2}\ln\frac{\gamma\left(E+\beta p_z\right)+\gamma\left(p_z+\beta E\right)}{\gamma\left(E+\beta p_z\right)-\gamma\left(p_z+\beta E\right)}$$

$$= \frac{1}{2}\ln\frac{\left(E+p_z\right)\left(1+\beta\right)}{\left(E-p_z\right)\left(1-\beta\right)} = y + \ln\gamma\left(1+\beta\right)$$

$$y \Rightarrow y + y_b$$

- Boosts along the beam axis change y by a constant, y_b :
  - $(p_T, y, \phi, m) \Rightarrow (p_T, y+y_b, \phi, m)$ with $y \Rightarrow y + y_b$, $y_b \equiv \ln\gamma(1+\beta)$
    rapidity is simply additive

Measure

Boosts along the beam axis change y by a constant, y_b :

y -> $y+y_b$ => rapidity is simply additive.

Can change coordinate from:
$dx_1 dx_2$ to $dy d\tau$, with identity Jacobian.

LHC: $q_1 = \frac{1}{2}\sqrt{s}\,(x_1,0,0,x_1)$ $q_2 = \frac{1}{2}\sqrt{s}\,(x_2,0,0,-x_2)$

Rapidity of system q_1+q_2 is: $y = \frac{1}{2}\ln[(E+p_z)/(E-p_z)] = \frac{1}{2}\ln(x_1/x_2)$

"Pseudo" and "Real" rapidity

The relation between y, β and θ can be seen using $p_Z = p\cos\theta$ and $p = \beta E$:

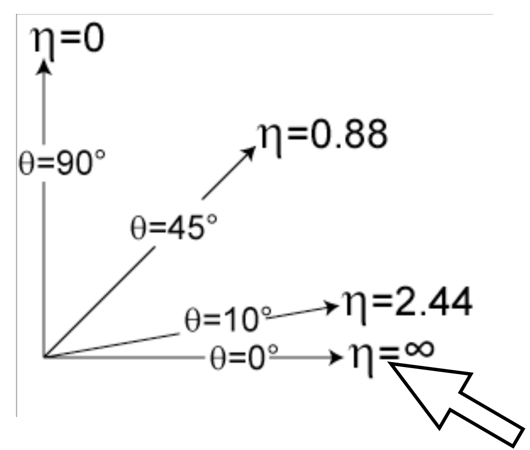
$$y = \frac{1}{2} \cdot \ln \frac{(E+p_Z)}{(E-p_Z)} = \frac{1}{2} \cdot \ln \frac{(1+\beta\cos\theta)}{(1-\beta\cos\theta)}$$

This expression can almost associate the position in the detector (θ) with the rapidity y, apart from the β terms.
However, at the LHC (and Tevatron, HERA), $\gtrsim$90% of the particles in the detector are pions with $\beta \approx 1$. Therefore we can introduce the "pseudorapidity" defined as $\eta = y(\theta)$ for $\beta=1$:

$$\eta = \frac{1}{2} \cdot \ln \frac{(1+\cos\theta)}{(1-\cos\theta)} = \ln \frac{\cos(\theta/2)}{\sin(\theta/2)} = -\ln\left(\tan \frac{\theta}{2}\right)$$

$\cos^2\theta/2 = ½ \cdot (1+\cos\theta)$
$\sin^2\theta/2 = ½ \cdot (1-\cos\theta)$

The pseudorapidity η is a good approximation of the true relativistic rapidity y when a particle is "relativistic".
It is a handy variable to approximate the rapidity y if the mass and the momentum of a particle are not known.

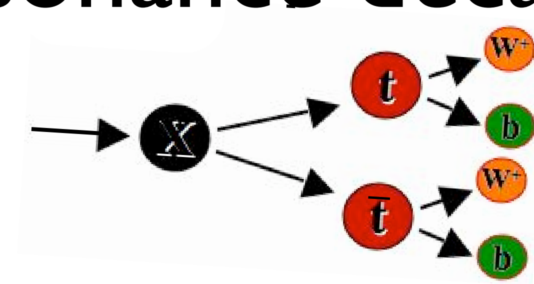


14

Few words about jets

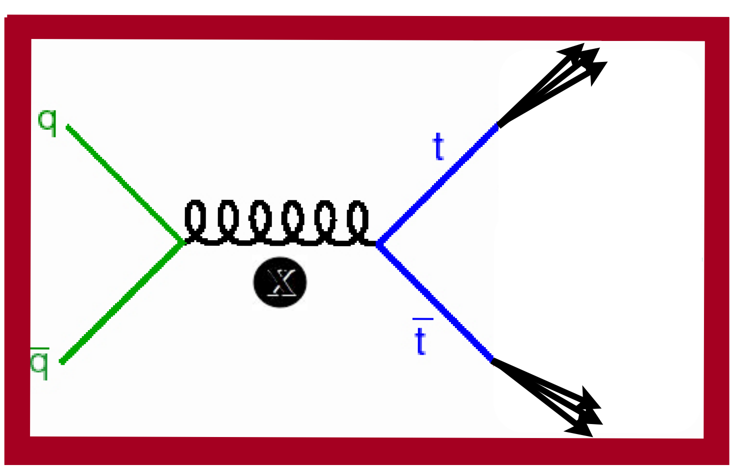
Connection to this school's theme

What if we have a heavy resonance decaying dominantly to tops $H/W/Z$?

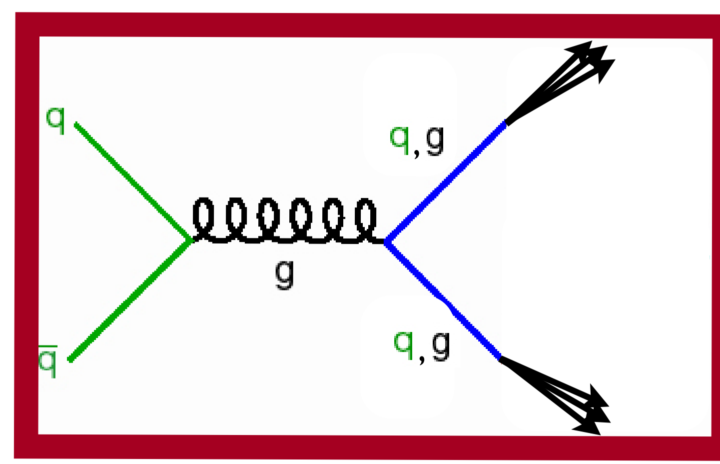


$$\Delta\theta_{ij} \sim m_J/E_J$$

Boosted tops appears as 2 jets, top jets.



Apart from mass, similar to ordinary 2-jet QCD process.



But what are jets??

Intuitive definition: spray of particles moving in the same direction.

More precise: Objects that describe differential energy flow that are sensitive to microscopic (perturbative) dynamics & insensitive to long distance (non-perturbative) physics.

Let us see an example.

Intro': $e^+e^- \rightarrow quarks$

$$R = \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}$$

Far below the Z pole: $R = \frac{\sum_q \sigma(e^+e^- \rightarrow q\bar{q})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} = 3\sum_q Q_q^2 \,.$

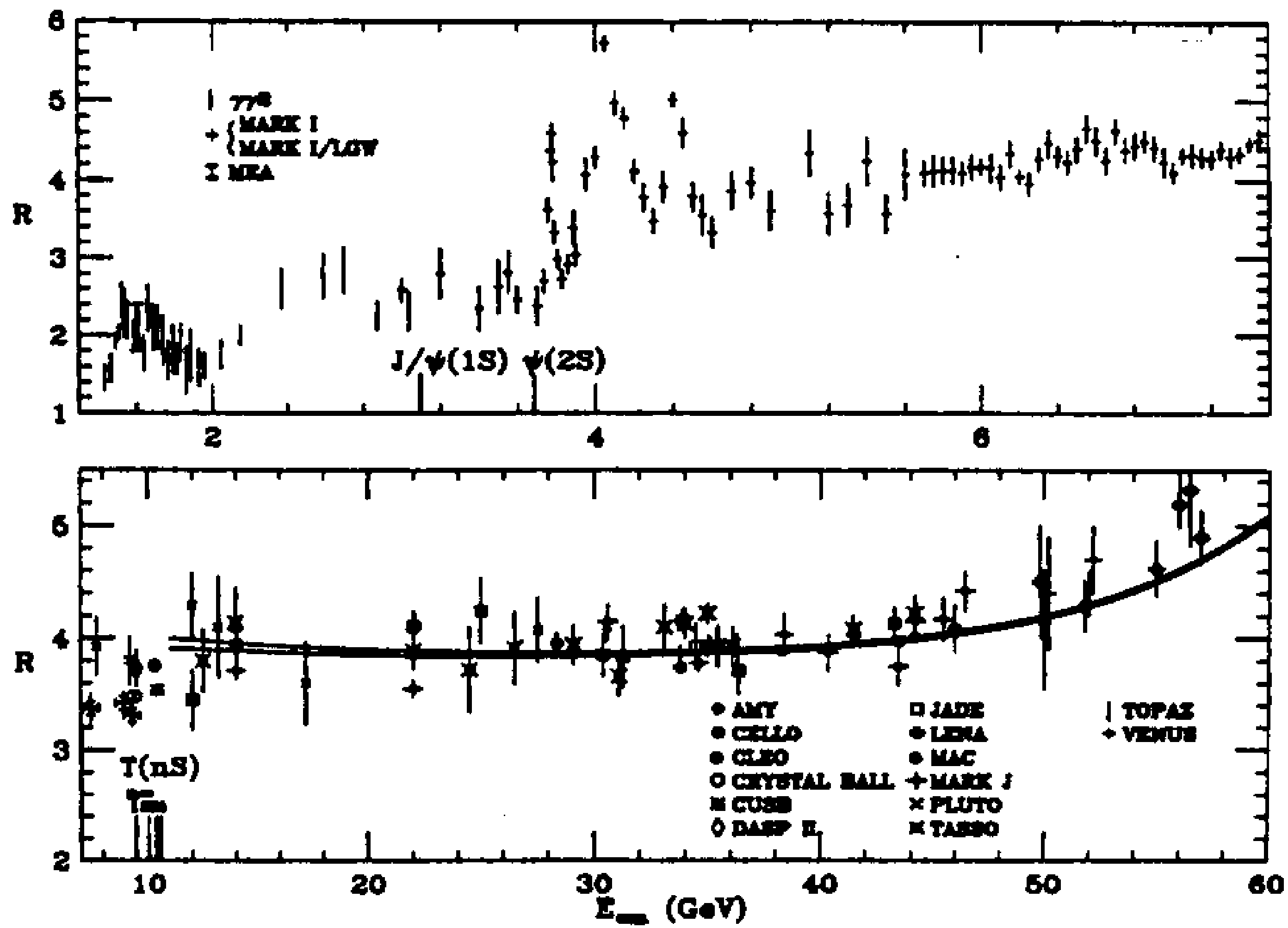
On the Z pole, the corresponding quantity is the ratio of the partial decay widths of the Z to hadrons and to muon pairs:

$$R_Z = \frac{\Gamma(Z \rightarrow \text{hadrons})}{\Gamma(Z \rightarrow \mu^+\mu^-)} = \frac{\sum_q \Gamma(Z \rightarrow q\bar{q})}{\Gamma(Z \rightarrow \mu^+\mu^-)} = \frac{3\sum_q(a_q^2 + v_q^2)}{a_\mu^2 + v_\mu^2} \,.$$

Intro’: $e^+e^- \rightarrow quarks$

For the 3 light quarks: $R = 3\left[\left(\frac{2}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 + \left(-\frac{1}{3}\right)^2\right] = 2$

Adding $c, c+b$ yield $R = 10/3, 11/3$

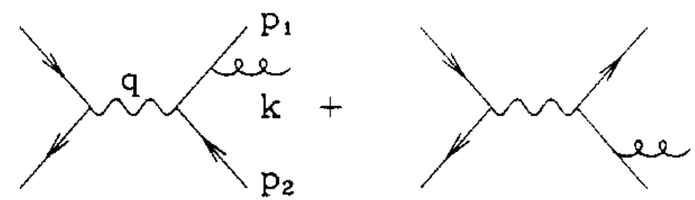


Results seem always higher??

Intro': $e^+e^- \to quarks$ @ NLO

Contribution from higher orders ...

$$e^+(q_1) + e^-(q_2) \to q(p_1) + \bar{q}(p_2) + g(k)$$



it is convenient to write the three-body phase space integration as

$$d\Phi_3 = \frac{s}{2^{10}\pi^5} d\alpha\, d\cos\beta\, d\gamma\, dx_1\, dx_2$$

where α, β, γ are Euler angles, and $x_1 = 2E_q/\sqrt{s}$ and $x_2 = 2E_{\bar{q}}/\sqrt{s}$ are the energy fractions of the final state quark and antiquark. The matrix element is obtained using the Feynman rules.

$$\sigma^{q\bar{q}g} = \sigma_0\, 3 \sum_q Q_q^2 \int dx_1 dx_2 \frac{C_F \alpha_S}{2\pi} \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)} \qquad \sigma_0 = \frac{4\pi\alpha^2}{3s} Q_f^2 \,.$$

$C_F = 4/3$

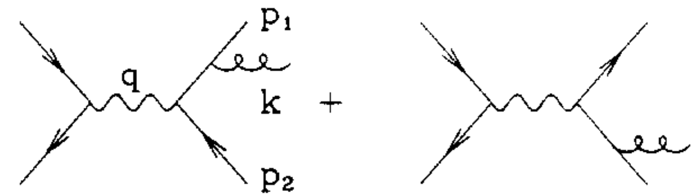
where the integration region is: $0 \le x_1, x_2 \le 1$, $x_1 + x_2 \ge 1$.

the integrals are divergent at $x_i = 1$.

Intro': $e^+e^- \to quarks$ @ NLO

Contribution from higher orders ...

$$e^+(q_1) + e^-(q_2) \to q(p_1) + \bar{q}(p_2) + g(k)$$



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$C_F = 4/3$

Question: are the x's Lorentz invariant?

Show that $s_{12} \equiv m_{12}^2 = (p_1 + p_2)^2 = s(1 - x_3)$

$e^+e^- \rightarrow quarks$: Soft & collinear singularities of QCD

Since $1 - x_1 = x_2 E_g(1 - \cos\theta_{2g})/\sqrt{s}$

and $1 - x_2 = x_1 E_g(1 - \cos\theta_{1g})/\sqrt{s}$, where E_g is the gluon energy

and θ_{ig} the angles between the gluon and the quarks,

$$\Downarrow$$

singularities come from regions

of phase space where the gluon is *collinear* with the quark or antiquark, $\theta_{ig} \rightarrow 0$,

or where the gluon is *soft*, $E_g \rightarrow 0$.

These singularities are not physical due to the IR hadronic scale of QCD. However, the corresponding IR dynamics cannot be described in perturbation theory.

$e^+e^- \rightarrow quarks$: regularization of the total Xsection

The above singularities actually don't really affect the total Xsec' if it's appropriately regularized (various ways). We use Dim' Reg', it affects both phase space & Dirac matrix trace factors.

$$\sigma^{q\bar{q}g}(\epsilon) \;=\; \sigma_0 \; 3 \sum_q Q_q^2 \, H(\epsilon) \int dx_1 dx_2 \, \frac{2\alpha_S}{3\pi} \, \frac{x_1^2 + x_2^2 - \epsilon(2 - x_1 - x_2)}{(1-x_1)^{1+\epsilon}(1-x_2)^{1+\epsilon}}$$

with $\epsilon = \frac{1}{2}(4-d)$, and $H(\epsilon) = \dfrac{3(1-\epsilon)^2}{(3-2\epsilon)\Gamma(2-2\epsilon)} = 1 + O(\epsilon)$.

$$\sigma^{q\bar{q}g}(\epsilon) \;=\; \sigma_0 \; 3 \sum_q Q_q^2 \, \frac{C_F \alpha_S}{2\pi} \, H(\epsilon) \left[\frac{2}{\epsilon^2} + \frac{3}{\epsilon} + \frac{19}{2} + O(\epsilon) \right] .$$

$e^+e^- \rightarrow quarks$: regularization of the total Xsection

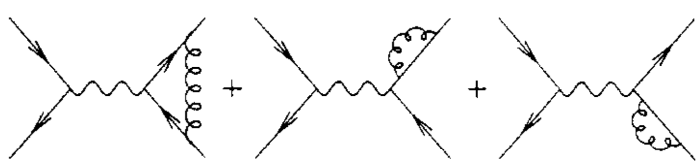
The virtual gluon contribution can be calculated in a similar fashion, with dimensional regularization again used to control the infra-red divergences in the loops. The result is

$$\sigma^{q\bar{q}(g)}(\epsilon) \;=\; \sigma_0\, 3\sum_q Q_q^2\, \frac{C_F\alpha_S}{2\pi}\, H(\epsilon)\, \left[-\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 8 + O(\epsilon)\right].$$

When the two contributions are added together, the poles exactly cancel and the result is *finite* in the limit $\epsilon \rightarrow 0$:

$$R \;=\; 3\sum_q Q_q^2\, \left\{1 + \frac{\alpha_S}{\pi} + O(\alpha_S^2)\right\}.$$

Note that the next-to-leading order correction is positive, and with a value for α_S of about 0.15, can accommodate the experimental measurement at $\sqrt{s} = 34$ GeV. In contrast, the corresponding correction is negative for a scalar gluon.



Jets

The previous success, regarding the total rate, didn't tell us anything about the distribution of energy flow / hadrons in the final state & how to linked it with the partonic Xsec':

LO - $\frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2 Q_f^2}{2s}(1+\cos^2\theta)$?? NLO - $\frac{1}{\sigma}\frac{d^2\sigma}{dx_1 dx_2} = C_F \frac{\alpha_S}{2\pi}\frac{x_1^2+x_2^2}{(1-x_1)(1-x_2)}$??

We expect the fragmented hadrons to roughly follow the parton direction, as seen in data from the 50s in cosmic ray & then latter on consistently in many exp'.

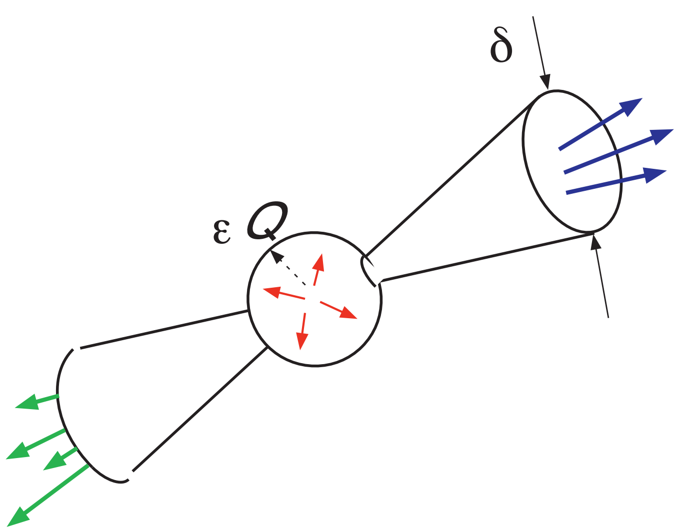
Then the soft/collinear gluons events would still have energy flow of 2 outgoing partons - "2 jets" topology.

On the other hand a well separated Xtra gluon emission is suppressed & look like an Xtra energy flow source - "3 jets"

Cone Jets, IRC safety (Sterman-Weinberg, 77)

Need to find a definition of these object, calculable in perturbation theory & yield finite rates (IRC save).

Sterman Weinberg: a final state is classified as two-jet-like if all but a fraction ϵ of the total available energy is contained in a pair of cones of half-angle δ.



Cone jets for e^+e^- annihilation.

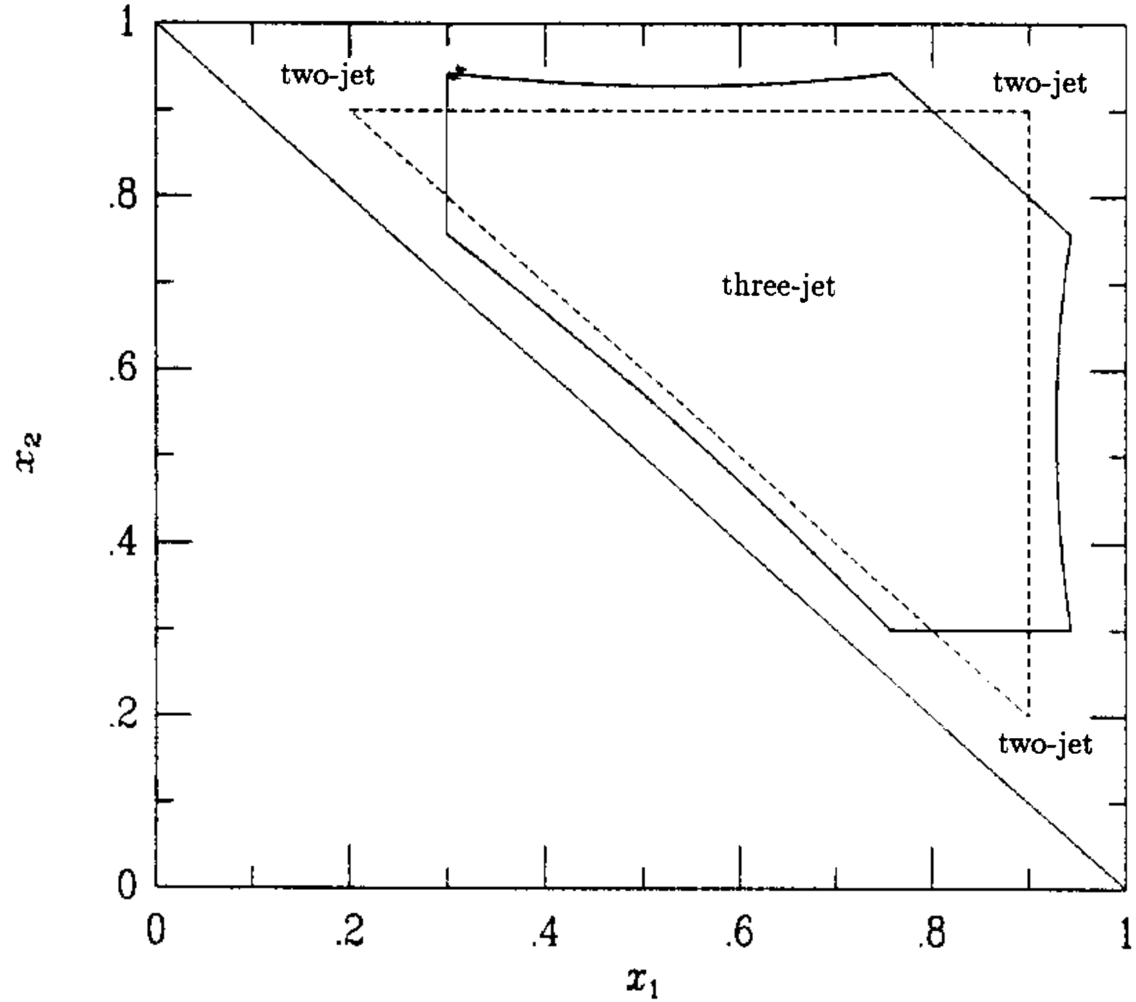
Cone Jets, IRC safety (Sterman-Weinberg, 77)

two-jet cross section is then obtained by integrating the matrix elements over the appropriate region of phase space determined by ϵ and δ.

At lowest order, the two-jet and total cross sections obviously coincide, for any values of the parameters.

At $O(\alpha_S)$, the two-jet cross section is obtained by integrating over the appropriate range of x_1 and x_2.

Cone Jets, IRC safety (Sterman-Weinberg, 77)



Boundaries between the two- and three-jet regions in the (x_1, x_2) plane for (a) Sterman-Weinberg jets with $(\epsilon, \delta) = (0.3, 30°)$ (solid lines), and (b) JADE algorithm jets with $y = 0.1$ (dashed lines).

Cone Jets, IRC safety (Sterman-Weinberg, 77)

at this order $\sigma = \sigma_2 + \sigma_3$.

σ_3 can be performed in 4 dimensions, since the matrix

element singularities are outside the three-jet region at this order.

Defining the two and three-jet fractions by $f_i = \sigma_i/\sigma$ $(i = 2, 3)$

$$
\begin{aligned}
f_2 &= 1 - 8C_F \frac{\alpha_S}{2\pi} \left\{ \log\frac{1}{\delta} \left[\log\left(\frac{1}{2\epsilon} - 1\right) - \frac{3}{4} + 3\epsilon \right] \right. \\
&\qquad \left. + \frac{\pi^2}{12} - \frac{7}{16} - \epsilon + \frac{3}{2}\epsilon^2 + O(\delta^2 \log\epsilon) \right\} , \\
f_3 &= 1 - f_2 .
\end{aligned}
$$

This is IRC safe, observables as well as derivatives, such as angular dist' etc ...

Cone Jets, IRC safety

This is IRC save, observables as well as derivatives, such as angular dist' etc ...

Notice that when the parameters ϵ and δ are small, the $O(\alpha_S)$ correction becomes logarithmically large. This is simply the vestige of the soft and collinear singularities. There are techniques for resumming terms involving $\alpha_S \log \delta$ to all orders in perturbation theory; when δ is small this should improve on the first order result.

It implies that the number of jets is not a physical parameter!
The intuitive connection between partons & jets holds only at LO.

At higher orders in perturbation theory, we can have events with more than three jets.

For example, the $O(\alpha_S^2)$ $q\bar{q}q\bar{q}$ and $q\bar{q}gg$ production processes can give rise to four jet events.

Cones in hadron colliders

Sterman-Weinberg cones give inefficient ‘tiling’ of the phase-space 4pi solid angle.

Similarly for hadronic machine one needs to use different E threshold and not COM.

And, also non trivial to implement in practice, “where to place the cone?” And, “how to deal with overlaps?”. Thus, alternatives were constructed.

One needs to find way to cluster partons (energy) in an IR safe manner.

Summary

LHC opens a new era: colliders energy > electroweak (EW) scale.

Probing the mechanism of EW symmetry breaking.

New phenomena is kinematically allowed a shot of looking at new physics related to naturalness.

Calculation at the LHC are challenging due to nature of incoming composite particles.

Yet simple concepts as parton luminosities & understanding kinematics & jets allow for semi-quantitative control.