$$\hat{\hat{\boldsymbol{\nu}}}(\boldsymbol{\theta}) = \underset{\boldsymbol{\nu}}{\operatorname{argmax}} L(\boldsymbol{\theta}, \boldsymbol{\nu})$$

$$\lambda(\boldsymbol{\theta}) = \frac{L(\boldsymbol{\theta}, \hat{\hat{\boldsymbol{\nu}}}(\boldsymbol{\theta}))}{L(\hat{\boldsymbol{\theta}}, \hat{\boldsymbol{\nu}})}$$

$$q_{\boldsymbol{\theta}} = -2 \ln \lambda(\boldsymbol{\theta})$$

$$\boldsymbol{\theta} = (\theta_1, \ldots, \theta_N)$$

$$\boldsymbol{\nu} = (\nu_1, \ldots, \nu_M)$$

$$Z = \Phi^{-1}(1 - p) = 3.6$$

$$P(n|s, b) = \frac{(s + b)^n}{n!} e^{-(s+b)}$$

$$m = \sqrt{2E_1 E_2 (1 - \cos \theta_{12})}$$

$$P(n|s, b) = \frac{(s + b)^n}{n!} e^{-(s+b)}$$

$$Q = -2 \ln \frac{L_{s+b}}{L_b} = 2s - 2 \sum_{i=1}^{n} \ln \left[1 + \frac{s}{b} \frac{f(\mathbf{x}_i|s)}{f(\mathbf{x}_i|b)} \right]$$

$$\lambda(\mu) = \frac{L(\mu)}{L(\hat{\mu})}$$

$$\hat{\mu} = \underset{\mu}{\operatorname{argmax}} L(\mu)$$

$$p = 1 - \Phi\left(\sqrt{q_0}\right)$$

$$Z = \sqrt{q_0} = \hat{\mu}/\sigma_{\hat{\mu}}$$

$$L(\mu) = \frac{(\mu s + b)^n}{n!} e^{-(\mu s+b)} \prod_{i=1}^{n} f(x_i|\mu)$$

$$f(x|\mu) = \frac{\mu s}{\mu s + b} f(x|\mathrm{s}) + \frac{b}{\mu s + b} f(x|\mathrm{b})$$

$$x_1, \ldots, x_n$$

$$n \sim \mathrm{Poisson}(\mu s + b)$$

$$y(\mathbf{x}) = \frac{p(\mathbf{x}|\mathrm{s})}{p(\mathbf{x}|\mathrm{b})}$$

$$\frac{p(\mathbf{x}|\mathrm{s})}{p(\mathbf{x}|\mathrm{b})} > c$$

$$t_{\mathrm{p}}(x) = \frac{L_{\mathrm{p}}(x|s)}{L_{\mathrm{p}}(x|b)} = \frac{L(x|\hat{\hat{\nu}}(s), s)}{L(x|\hat{\hat{\nu}}(b), b)}$$

$$t_{\mathrm{m}}(x) = \frac{L_{\mathrm{m}}(x|s)}{L_{\mathrm{m}}(x|b)} = \frac{\int L(x|\nu, s)\pi_\nu(\nu)\, d\nu}{\int L(x|\nu, b)\pi_\nu(\nu)\, d\nu}$$

$$P(\theta < \theta_{\mathrm{up}}(x)|\theta) = ?$$

$$P(\theta < \theta_{\mathrm{up}}|x) = \int_{-\infty}^{\theta_{\mathrm{up}}} p(\theta|x)\, d\theta = 95\%$$

$$\pi_\nu(\nu) = \pi(\nu|y) \propto L(y|\nu)\pi_0(\nu)$$

$$p(\theta, \nu|x) \propto L(x|\theta, \nu)\pi_\theta(\theta)\pi_\nu(\nu)$$

$$p(\theta, \nu|x, y) \propto L(x|\theta, \nu)L(y|\nu)\pi_\theta(\theta)\pi_0(\nu)$$

$$p(\theta|x) = \int p(\theta, \nu|x)\, d\nu \propto \int L(x|\theta, \nu)\pi_\nu(\nu)\pi_\theta(\theta)\, d\nu = L_{\mathrm{m}}(x|\theta)\pi_\theta(\theta)$$

$$L_{\mathrm{m}}(x|\theta) = \int L(x|\theta, \nu)\, \pi_\nu(\nu)\, d\nu$$

$$p(\theta|x) = \int p(\theta, \nu|x)\, d\nu$$

$$p(\theta, \nu|x) \propto L(x|\theta, \nu)\pi(\theta, \nu)$$

$$\pi_\theta(\theta)$$

$$\pi_\nu(\nu)$$

$$\pi(\theta, \nu) = \pi_\theta(\theta)\pi_\nu(\nu)$$

$$\pi(\theta, \nu)$$

$$p_\theta = \int_{t_{\theta,\mathrm{obs}}}^{\infty} f(t_\theta|\theta, \nu)\, dt_\theta$$

$$t_\theta = -2 \ln \frac{L(\theta, \hat{\hat{\nu}}(\theta))}{L(\hat{\theta}, \hat{\nu})}$$

$$L_\mathrm{p}(\theta) = L(\theta, \hat{\hat{\nu}}(\theta))$$

$$\hat{\hat{\nu}}(\theta) = \operatorname*{argmax}_{\nu} L(\theta, \nu)$$

$$L(\theta)$$

$$L(x|\theta)$$

$$L_\mathrm{p}(\mu) = L(\mu, \hat{\hat{\theta}}(\mu))$$

$$L_\mathrm{m}(\mu) = \int L(\mu, \theta)\pi_\theta(\theta)\, d\theta$$

$$L(L_1, L_2, T, \tilde{\tau}_0, \tilde{\alpha}_1, \tilde{\alpha}_2|\lambda, \tau, \tau_0, \alpha_1, \alpha_2) =$$

$$\frac{1}{\sqrt{2\pi}\sigma_T} e^{-(T-\tau)^2/2\sigma_T^2} \prod_{i=1}^{2} \frac{1}{\sqrt{2\pi}\sigma_i} e^{-(L_i - \lambda + \alpha_i(\tau - \tau_0))^2/2\sigma_i^2}$$

$$\times \frac{1}{\sqrt{2\pi}\sigma_{\tilde{\tau}_0}} e^{-(\tilde{\tau}_0 - \tau_0)^2/2\sigma_{\tilde{\tau}_0}^2}$$

$$\prod_{i=1}^{2} \frac{1}{\sqrt{2\pi}\sigma_{\tilde{\alpha}_i}} e^{-(\tilde{\alpha}_i - \alpha_i)^2/2\sigma_{\tilde{\alpha}_i}^2}$$

$$\mathrm{cov}[y_1, y_2] = \alpha_1\alpha_2\sigma_T^2$$

$$\mathrm{cov}[y_i, T] = \alpha_i\sigma_T^2$$

$$y_i = L_i + \alpha_i(T - \tau_0)$$

$$L(T, L_1, L_2|\lambda, \tau) = \frac{1}{\sqrt{2\pi}\sigma_T} e^{-(T-\tau)^2/2\sigma_T^2} \prod_{i=1}^{2} \frac{1}{\sqrt{2\pi}\sigma_i} e^{-(L_i - \lambda + \alpha_i(\tau - \tau_0))^2/2\sigma_i^2}$$

$$L_i \sim \mathrm{Gauss}(\lambda - \alpha_i(\tau - \tau_0), \sigma_i)$$

$$T \sim \mathrm{Gauss}(\tau, \sigma_T)$$

$$E[L_i] = \lambda - \alpha_i(T - \tau_0), \qquad i = 1, 2$$

$$\mathrm{cov}[\theta_i, \theta_j] = \int \theta_i \theta_j p(\boldsymbol{\theta}|x)\, d\boldsymbol{\theta} \; - \int \theta_i p(\boldsymbol{\theta}|x)\, d\boldsymbol{\theta} \int \theta_j p(\boldsymbol{\theta}|x)\, d\boldsymbol{\theta}$$

$$\rho_{xy} = \frac{\mathrm{cov}[x, y]}{\sigma_x \sigma_y}$$

$$\mathrm{cov}[x, y] = E[xy] - E[x]E[y]$$

$$t_\theta = -2 \ln \frac{L(\theta)}{L(\hat{\theta})}$$

$$\widehat{V}_{ij}^{-1} \approx -\frac{\partial^2 \ln L}{\partial \theta_i \, \partial \theta_j}\bigg|_{\theta = \hat{\theta}}$$

$$b = E[\hat{\theta}] - \theta$$

$$V_{ij} = \mathrm{cov}[\hat{\theta}_i, \hat{\theta}_j]$$

$$\hat{\theta} = \operatorname*{argmax}_\theta L(x|\theta)$$

$$p(\theta|x) = \frac{L(x|\theta)\pi(\theta)}{\int L(x|\theta)\pi(\theta)\, d\theta}$$

$$Z = \sqrt{2 \left((s + b) \ln \left(1 + \frac{s}{b} \right) - s \right)}$$

$$t_{\mathrm{float}} = -2 \ln \frac{L(0)}{L(\hat{\mu}, \hat{m})} \equiv -2 \ln \lambda(0)$$

$$\chi^2(\boldsymbol{\mu}) + \tau \sum_i \left[(\mu_{i+1} - \mu_i) - (\mu_i - \mu_{i-1})^2 \right]$$

$$\chi^2 = \sum_{i=1}^{N} \frac{(\nu_{0i} - n_i)^2}{\nu_{0i}}$$

$$\nu_{0i} = \sum_{j=1}^{M} R_{ij} \mu_{0j} + \beta_i$$

$$\Phi(\boldsymbol{\mu}) = \alpha\left(\ln L(\boldsymbol{\mu}) + \ln L_\theta(\theta)\right) + S(\boldsymbol{\mu})$$

$$C_i = 0.1$$

$$n_i = 100$$

$$\beta_i = 0$$

$$\hat{\mu}_i = C_i n_i = 10$$

$$\sigma_{\hat{\mu}_i} = C_i\sqrt{n_i} = 1.0$$

$$E[\hat{\boldsymbol{\mu}}] = R^{-1}(E[\mathbf{n}] - \boldsymbol{\beta}) = \boldsymbol{\mu}$$

$$U_{ij} = \mathrm{cov}[\hat{\mu}_i, \hat{\mu}_j] = \sum_{k,l=1}^{N} (R^{-1})_{ik}(R^{-1})_{jl}\,\mathrm{cov}[n_k, n_l]$$

$$= \sum_{k=1}^{N} (R^{-1})_{ik}(R^{-1})_{jk}\,\nu_k$$

$$\frac{\partial^2 \ln L}{\partial \mu^2} = -\frac{n}{\mu^2}$$

$$\ln L(\boldsymbol{\mu}) = \sum_{i=1}^{N} (n_i \ln \nu_i - \nu_i)$$

$$\hat{\boldsymbol{\nu}} = \mathbf{n}$$

$$\hat{\boldsymbol{\mu}} = R^{-1}(\mathbf{n} - \boldsymbol{\beta})$$

$$P(n_i; \nu_i) = \frac{\nu_i^{n_i}}{n_i!} e^{-\nu_i}$$

$$\boldsymbol{\mu} = R^{-1}(\boldsymbol{\nu} - \boldsymbol{\beta})$$

$$\boldsymbol{\mu} = (\mu_1, \ldots, \mu_M), \quad \mu_{\mathrm{tot}} = \sum_{j=1}^{M} \mu_j$$

$$\mathbf{p} = (p_1, \ldots, p_M) = \boldsymbol{\mu}/\mu_{\mathrm{tot}}$$

$$\boldsymbol{\nu} = (\nu_1, \dots, \nu_N)$$

$$\varepsilon_j = \sum_{i=1}^{N} R_{ij}$$

$$\boldsymbol{\beta} = (\beta_1, \dots, \beta_N)$$

$$E[\mathbf{n}] = \boldsymbol{\nu} = R\boldsymbol{\mu} + \boldsymbol{\beta}$$

$$\sum_{i=1}^{N} R_{ij} = \sum_{i=1}^{N} P(\text{observed in bin } i \,|\, \text{true value in bin } j)$$

$$= P(\text{observed anywhere} \,|\, \text{true value in bin } j)$$

$$= \varepsilon_j$$

$$\nu_i = \sum_{j=1}^{M} R_{ij}\mu_j + \beta_i$$

$$\mathbf{n} = (n_1, \dots, n_N)$$

$$\nu_i = E[n_i] = \sum_{j=1}^{M} R_{ij}\mu_j \,, \quad i = 1, \dots, N$$

$$R_{ij} = P(\text{observed in bin } i \,|\, \text{true in bin } j)$$

$$f_{\text{meas}}(x) = \int R(x|y) f_{\text{true}}(y)\, dy$$

$$p_j = \int_{\text{bin}\, j} f(y)\, dy$$

$$\mu_j = \mu_{\text{tot}} p_j$$

$$\pi(b) = \frac{1}{\sqrt{2\pi}\sigma_b} e^{-(b-b_{\text{meas}})^2/2\sigma_b^2}$$

$$p_{\text{float}} \approx p_{\text{fix}} + \langle N(c) \rangle$$

$$c = t_{\text{fix}} = Z_{\text{fix}}^2$$

$$Z_{\rm fix} = \Phi^{-1}(1 - p_{\rm fix})$$

$$\mathcal{N} = \frac{\langle N(c)\rangle}{1 - F_{\chi^2_2}(c)}$$

$$\langle N(c)\rangle \approx \langle N(c_0)\rangle e^{-(c-c_0)/2}$$

$$F_{\rm trials} \equiv \frac{p_{\rm float}}{p_{\rm fix}} \approx \sqrt{\frac{\pi}{2}}\,\frac{\langle N(t_{\rm float,obs})\rangle}{1 - F_{\chi^2_2}(t_{\rm float,obs})} Z_{\rm fix}$$

$$F_{\rm trials} \equiv \frac{p_{\rm float}}{p_{\rm fix}} \approx \sqrt{\frac{\pi}{2}} \mathcal{N} Z_{\rm fix}$$

$$P(t_{\rm float} > t_{\rm float,obs}|H_0) \leq 1 - F_{\chi^2_1}(t_{\rm float,obs}) + \langle N(t_{\rm float,obs})\rangle$$

$$-2\ln\lambda(\mu) = -2\ln(L(\mu)/L(\hat{\mu})) < 1 \text{ i.e., } \ln L(\mu) > \ln L(\hat{\mu}) - \frac{1}{2}$$

$$\frac{P(\mathbf{x}|H_1)}{P(\mathbf{x}|H_0)} \geq k$$

$$x \sim \mathrm{Gauss}(\mu, \sigma)$$

$$\alpha = 1 - \Phi\left(\frac{x_{\rm c} - \mu_0}{\sigma}\right)$$

$$\text{power} = 1 - \beta = P(x > x_{\rm c}|\mu) = 1 - \Phi\left(\frac{\mu_0 - \mu}{\sigma} + \Phi^{-1}(1-\alpha)\right)$$

$$x_{\rm c} = \mu_0 + \sigma\Phi^{-1}(1-\alpha)$$

$$L(x, y|\theta, \nu) = L(x|\theta,\nu)L(y|\nu)$$

$$\pi(\nu|y) \propto L(y|\nu)\pi_0(\nu)$$

$$L_{\rm m}(x|\theta) = \int L(x|\theta,\nu)\pi(\nu)\,d\nu$$

$$\hat{\hat{\nu}}(\theta)$$

$$p_\theta = \int_{q_{\theta,\rm obs}}^{\infty} f(q_\theta|\theta,\nu)\,dq_\theta$$

$$L(x|\theta) \to L(x|\theta, \nu)$$

$$L(x|\theta) = \theta x$$

$$L(x|\theta) = \theta x + \alpha x^2 + \beta x^3 + \cdots$$

$$p_\theta = \int_{q_{\theta,\mathrm{obs}}}^{\infty} f(q_\theta|\theta)\, dq_\theta$$

$$p(s|n) = \int p(s, b|n)\, db$$

$$p(\mathbf{x}|H_0), p(\mathbf{x}|H_1)$$

$$s_{\mathrm{up}} = \tfrac{1}{2} F^{-1}_{\chi^2}(1 - \alpha; 2(n+1)) - b$$

α

$s_{\mathrm{up}} = -\ln \alpha \approx 3.00$

$$\begin{aligned} P(\mathrm{D}) &= 0.001 \\ P(\text{no D}) &= 0.999 \end{aligned}$$

$$\begin{aligned} P(+|\mathrm{D}) &= 0.98 \\ P(-|\mathrm{D}) &= 0.02 \end{aligned}$$

$$\begin{aligned} P(+|\text{no D}) &= 0.03 \\ P(-|\text{no D}) &= 0.97 \end{aligned}$$

$$\begin{aligned} p(\mathrm{D}|+) &= \frac{P(+|\mathrm{D})P(\mathrm{D})}{P(+|\mathrm{D})P(\mathrm{D}) + P(+|\text{no D})P(\text{no D})} \\ &= \frac{0.98 \times 0.001}{0.98 \times 0.001 + 0.03 \times 0.999} \\ &= 0.032 \end{aligned}$$