Dark Energy to Modified Gravity

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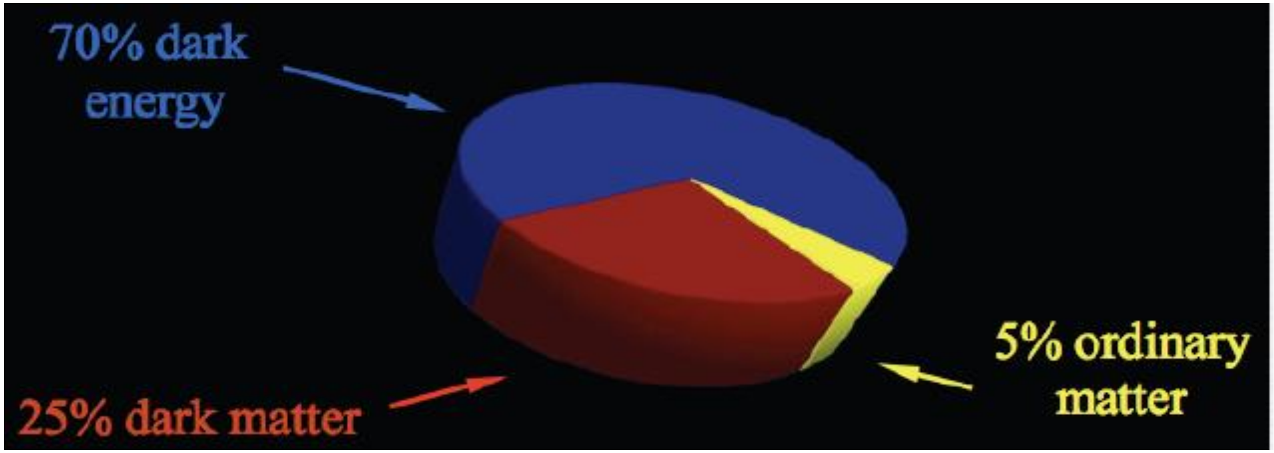
The Big Puzzle



Acceleration of the expansion

Dark Energy ?

Modified gravity on large enough scales?

The acceleration of the expansion of the Universe could have (at least) two dynamical explanations:

Dark Energy: a new form of matter

$$S = \frac{1}{16\pi G_N}\int d^4x\sqrt{-g}(R - \mathcal{L}_{DE})$$

Einstein-Hilbert action describing General Relativity. Einstein's equations are simply obtained using the least action principle

Lagrangian density of dark energy

Modified law of gravity on large scales

$$S = \frac{1}{16\pi G_N}\int d^4x\sqrt{-g}h(R, R_{\mu\nu}, R_{\mu\nu\rho\sigma})$$

Arbitrary corrections to the Einstein-Hilbert Lagrangian involving the Riemann and Ricci tensors.

Modifying Gravity

Mass term for a graviton

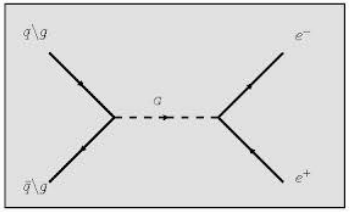
The simplest modification is massive gravity (Pauli-Fierz):

$$\delta\mathcal{L} = \frac{m_G^2}{4}(h^{\mu\nu}h_{\mu\nu} - h^2)$$

Pauli-Fierz gravity is ghost free (negative kinetic energy terms) . Unfortunately, a massive graviton carries 5 polarisations when a massless one has only two polarisations. In the presence of matter, the graviton wave function takes the form:

$$h_{\mu\nu} = \frac{8\pi G_N}{p^2}(T_{\mu\nu} - \frac{1}{2}T\eta_{\mu\nu}) \to h_{\mu\nu} = \frac{8\pi G_N}{p^2 + m_G^2}(T_{\mu\nu} - \frac{1}{3}T\eta_{\mu\nu})$$

The massless limit does not gives GR! (van Dam-Velman-Zakharov discontinuity). The extra polarization is lethal.



Pauli- Fierz massive gravity has a ghost in curved backgrounds (cosmology).

An infinite class of modified gravity models can be considered:

$$S_{\mathsf{MG}} = \frac{1}{16\pi G_N}\int d^4x\sqrt{-g}\,f(R, R_{\mu\nu}, R_{\mu\nu\rho\sigma})$$

These Lagrangian field theories fall within the category of higher derivative theories.

Ostrosgradski's theorem states that these theories are *generically* plagued with ghosts. Quantum mechanically, this implies an explosive behaviour with particles popping out of the vacuum continuously. In particular an excess in the gamma ray background.

A large class is ghost-free though, the f(R) models:

$$S_{\mathrm{MG}} = \frac{1}{16\pi G_N} \int d^4x \sqrt{-g} f(R)$$

As promised, f(R) is totally equivalent to an effective field theory with gravity and scalars!

$$\boxed{S = \int d^4x\sqrt{-g}\left(\frac{1}{16\pi G_N}R - \frac{1}{2}(\partial\phi)^2 - V(\phi) + \mathcal{L}_m(\psi_m, e^{2\phi/\sqrt{6}m_{\mathrm{Pl}}}g_{\mu\nu})\right)}$$

The potential V is directly related to f(R)

Crucial coupling between matter and the scalar field

$$V(\phi) = m_{\mathrm{Pl}}^2\frac{Rf' - f}{2f'^2},\ \ f' = e^{-2\phi/\sqrt{6}m_{\mathrm{Pl}}}$$

Acceleration of the expansion

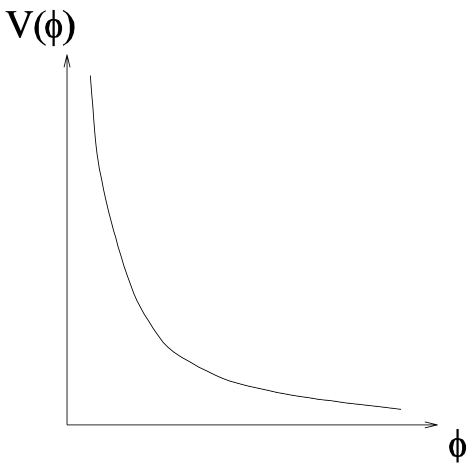
Modified gravity on large enough scales.

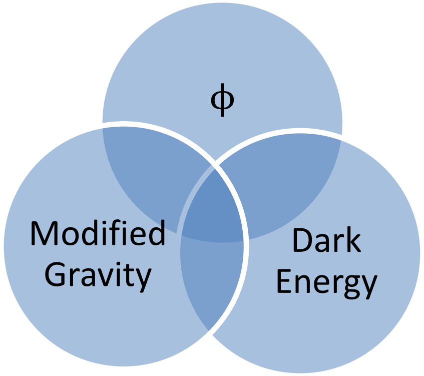
Parameterised by a scalar field: ϕ

Massive graviton always involves a scalar field.

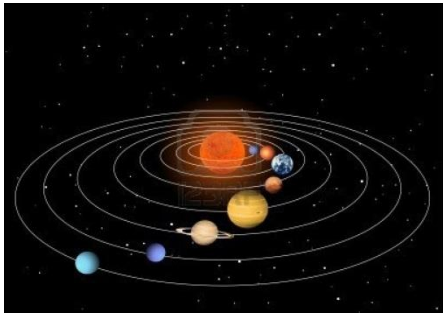
Potential energy leads to dark energy.

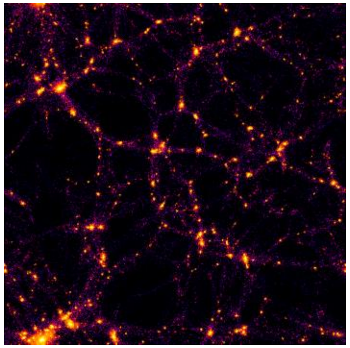
At low energy: generalised Galileon Models.





GRAVITY ACTS ON ALL SCALES!





Nothing guarantees that a modification of gravity on large scales is consistent with the gravity tests in the solar system.

For these ubiquitous scalars: very low masses

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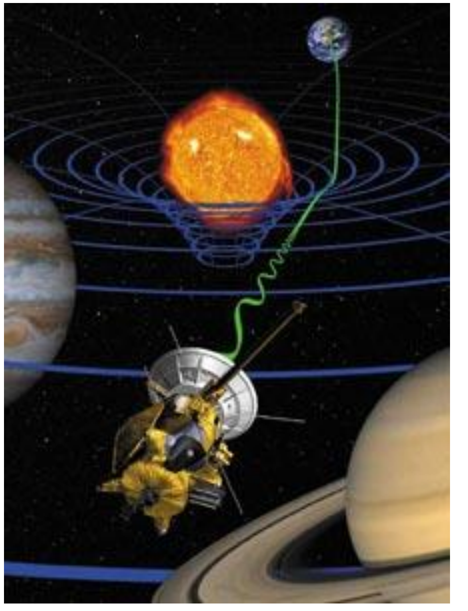
Major gravitational problem!

Deviations from Newton's law are parametrised by:

$$\phi_N = -\frac{G_N}{r}(1 + 2\beta^2 e^{-r/\lambda})$$

For large range forces with large λ, the tightest constraint on the coupling β comes from the Cassini probe measuring the Shapiro effect (time delay):

$$\beta^2 \leq 4 \cdot 10^{-5}$$



Bertotti et al. (2004)

SCREENING

SCREENING

Mechanisms whereby nearly massless scalars evade local gravitational tests

Around a background configuration and in the presence of matter, the Lagrangian can be linearised and the main screening mechanisms can be schematically distinguished :

$$\mathcal{L} \supset \boxed{-\frac{Z(\phi_0)}{2}}(\partial\delta\phi)^2 - \frac{m^2(\phi_0)}{2}\delta\phi^2 + \frac{\beta(\phi_0)}{M_P}\delta\phi\delta T \ ,$$

The Vainshtein mechanism reduces the coupling by increasing Z

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The Damour-Polyakov mechanism reduces the coupling β

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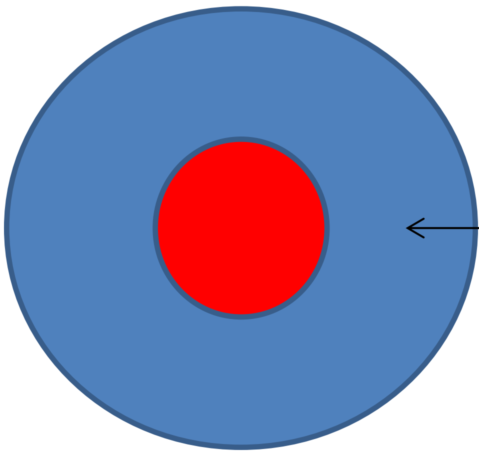
The chameleon mechanism increases the mass.

A simple example: the CUBIC GALILEON

$$\mathcal{L}_\phi = -\frac{1}{2}(\partial\phi)^2 - \frac{1}{2\Lambda^3}(\partial\phi)^2\partial^2\phi + \frac{\beta\phi}{M_P}T \, .$$

$\Lambda^3 = m^2 m_{\mathrm{Pl}}$ m graviton mass



$$\frac{F_\phi}{F_N} = 2\beta^2\left(\frac{r}{R_V}\right)^{3/2} \, .$$

Well inside the Vainshtein radius, Newtonian gravity is restored. Well outside gravity is modified.

What is specific about this model?

The Vainshtein radius is very large for stars, typically 0.1 kpc for the sun, and a mass for the graviton of the order of the Hubble rate .

The Vainshtein and K-mouflage mechanisms can be nicely understood:

Effective Newtonian potential:

$$\Psi = (1 + \frac{2\beta^2(\phi)}{Z(\phi)})\Phi_N$$

For theories with second order eom:

$$Z(\phi) = 1 + a(\phi) L^2 \frac{D^\mu D_\mu \phi}{m_{\mathrm{Pl}}} + b(\phi)\frac{(\partial\phi)^2}{M^4} + \dots$$

Vainshtein

K-mouflage

$$M^4 \sim 3H_0^2 m_{\mathrm{Pl}}^2, \quad L \sim H_0^{-1}$$

Vainshtein

Newtonian gravity retrieved when the curvature is large enough:

$$\nabla^2 \Phi_N \geq \frac{1}{2\beta L^2}$$

On large cosmological scales, this tells us that overdensities such as galaxy clusters are screened :

$$\delta \geq \frac{1}{3\Omega_{m0}\beta}$$

On small scales (solar system, galaxies) screening only occurs within the Vainshtein radius:

$$R_V = (\frac{3\beta m L^2}{4\pi m_{\rm Pl}^2})^{1/3}$$

K-mouflage

Newtonian gravity retrieved when the gravitational acceleration is large enough:

$$|\nabla\Phi_N| \geq \frac{M^2}{2\beta m_{\rm Pl}}$$

On large cosmological scales, this tells us that overdensities such as galaxy clusters are not screened :

$$\frac{k}{H_0} \leq \beta\delta$$

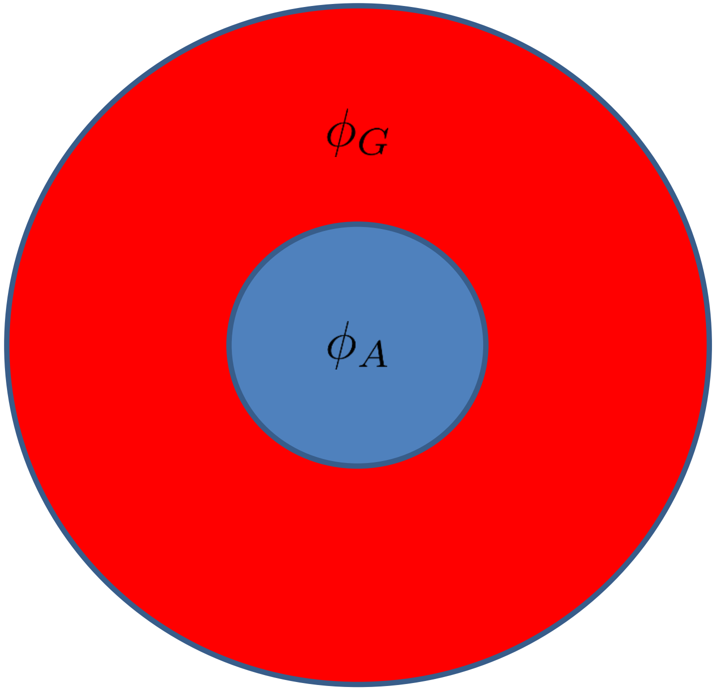
On small scales (solar system, galaxies) screening only occurs within the K-mouflage radius:

$$R_K = \left(\frac{\beta m}{4\pi m_{\rm Pl} M^2}\right)^{1/2}$$

Dwarf galaxies are not screened.

Chameleons:

The screening criterion for an object BLUE embedded in a larger region RED expresses the fact that the Newtonian potential of an object must be larger than the variation of the field:



Scalar charge: $$Q_A = \frac{|\phi_G - \phi_A|}{2 m_{\rm Pl} \Phi_A}$$

$$Q_A \leq \beta_G$$

Self screening: large Newton potential

Blanket screening: due to the environment G

Φ_A Newton's potential at the surface

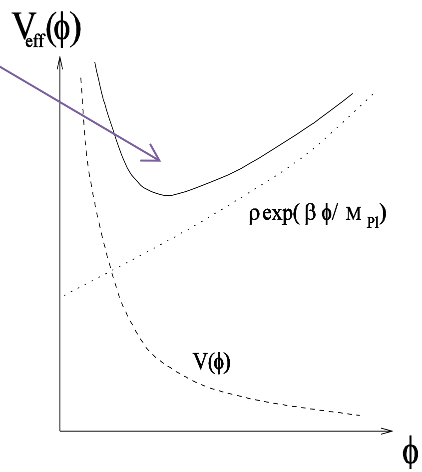
The effect of the environment

When coupled to matter, scalar fields have a matter dependent effective potential

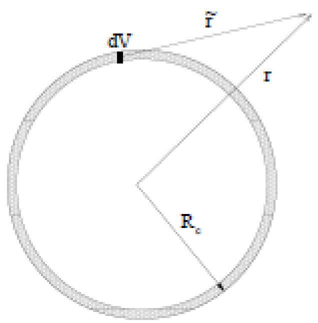
$$V_{eff}(\phi) = V(\phi) + \rho_m(A(\phi) - 1)$$

Environment dependent minimum

$$\beta(\phi) = m_{\rm Pl}\frac{d\ln A(\phi)}{d\phi}$$

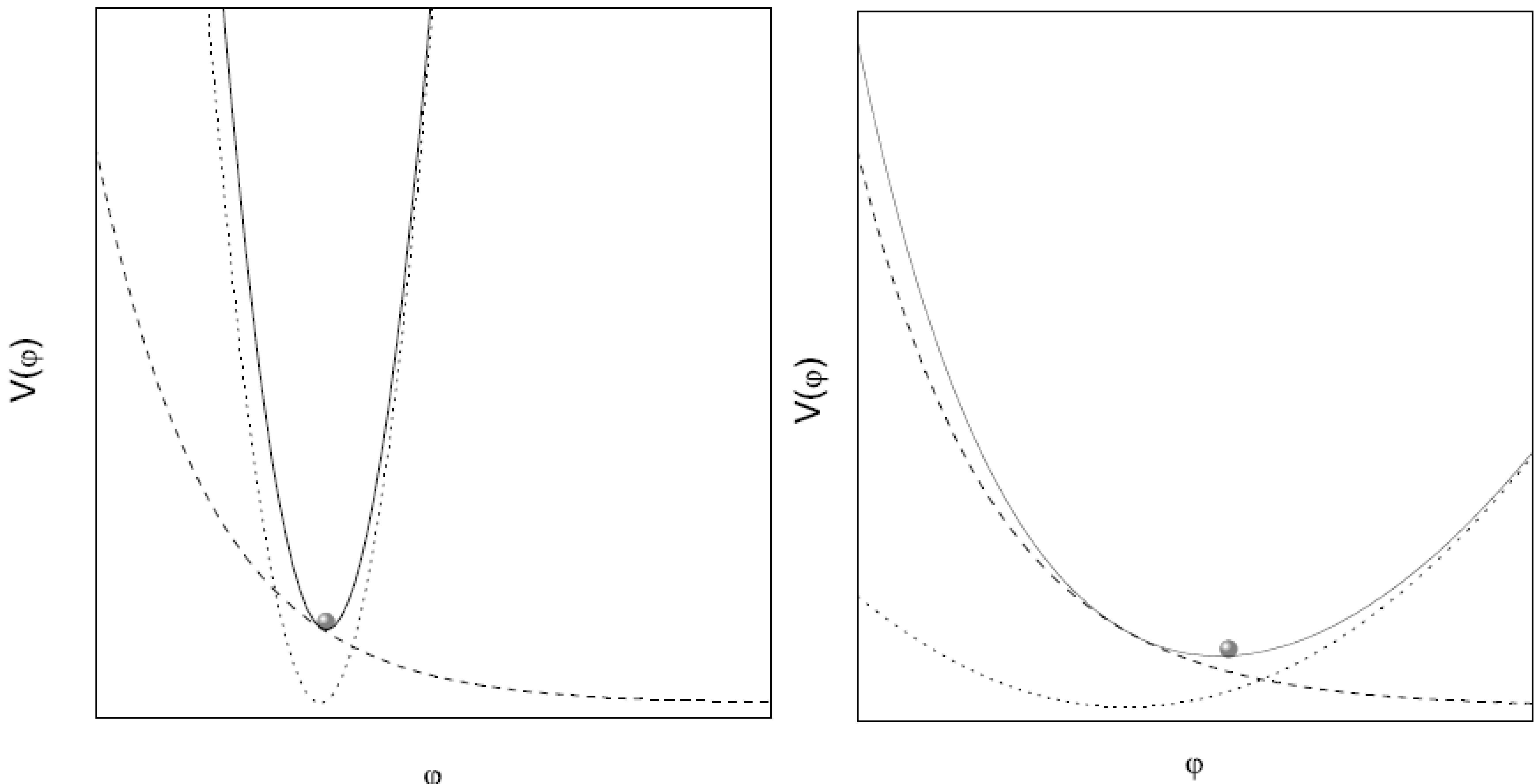


Chameleon



The field generated from deep inside is Yukawa suppressed. Only a thin shell radiates outside the body. Hence suppressed scalar contribution to the fifth force.

The string dilaton:



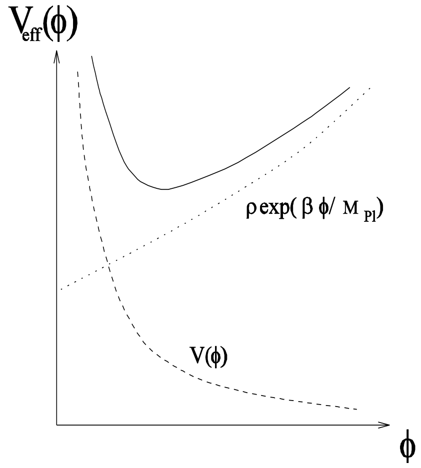
Dilaton

$$V(\phi) = V_0 e^{-\phi/m_{\rm Pl}}, \quad A(\phi) = 1 + \frac{A_2}{2m_{\rm Pl}^2}(\phi - \phi_\star)^2$$

All these models can be entirely characterised by 2 time dependent functions. The non-linear potential and coupling of the model can be reconstructed using:

Works for chamelons, dilatons, symmetrons etc…

$$\phi(a) = \phi_i + \frac{3}{m_{\rm Pl}} \int_{a_i}^{a} da \frac{\beta(a)\rho(a)}{a m^2(a)}$$
$$V(a) = V_i - \frac{3}{m_{\rm Pl}} \int_{a_i}^{a} da \frac{\beta^2(a)\rho^2(a)}{a m^2(a)}$$



tomography

$m(a)$
$\beta(a)$

$\longleftrightarrow$

$m(\rho)$
$\beta(\rho)$

$$\rho(a) = \frac{\rho_0}{a^3}$$

For chameleons, dilatons and large curvature models:

$$\beta(a) = \beta_0 a^{-s}, \quad m(a) = m_0 a^{-r}$$

3/2<r<3, s=0 : chameleons

r>3, s=0 : large curvature

r=3/2, s=-3: dilaton

Self-screening of the Milky Way:

$$\frac{m_0^2}{H_0^2} \geq \frac{9\Omega_{m0}}{2(2r-3-s)} 10^6$$

This bounds the range of the scalar interaction to be less than a few Mpc's on cosmological scales

Big Bang Nucleosynthesis tells us that particle masses should not vary more than 10% between BBN and now.

This is realised provided that:

The field follows the minimum of the effective potential since BBN.

The mass is always much larger than the Hubble rate m>>H

The equation of states varies very little from the concordance model:

$$1 + w = \mathcal{O}\left(\frac{H^2}{m^2}\right) \leq 10^{-6}$$

At the background level, these models are indistinguishable from Λ-CDM.

At the linear level, CDM perturbations grow differently from GR in chameleon, dilaton, symmetron models:

$$\delta_c'' + \mathcal{H}\delta_c' - \frac{3}{2}\Omega_m \mathcal{H}^2 \left(1 + \frac{2\beta(a)^2}{1 + \frac{m(a)^2 a^2}{k^2}}\right)\delta_c = 0$$

Inside the Compton wavelength k<<m(a)a, anomalous growth depending on the coupling to matter β(a).

$$\lambda_c = \frac{1}{ma}$$

Outside the Compton wavelength, growth is not modified: $\delta \sim a$

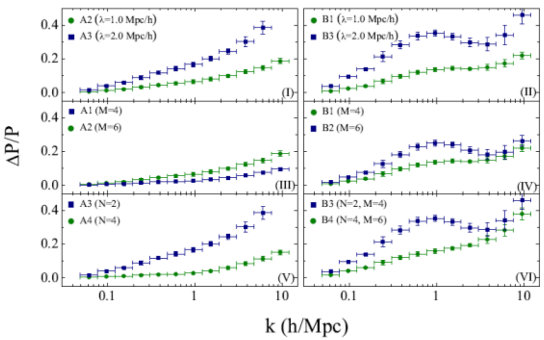
Inside the Compton wavelength, more growth:

$$\delta \sim a^{\frac{\nu}{2}}, \quad \nu = \frac{-1 + \sqrt{1 + 24(1 + 2\beta_c^2)}}{2}$$

$$\frac{m_0}{H_0} \geq 10^3$$

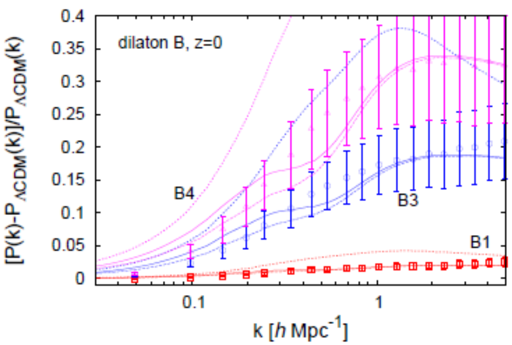
Modification of gravity on quasi-linear to non-linear scales

N-body simulations



ECOSMOG simulations using a modification of the RAMSES code.

Semi-analytical methods

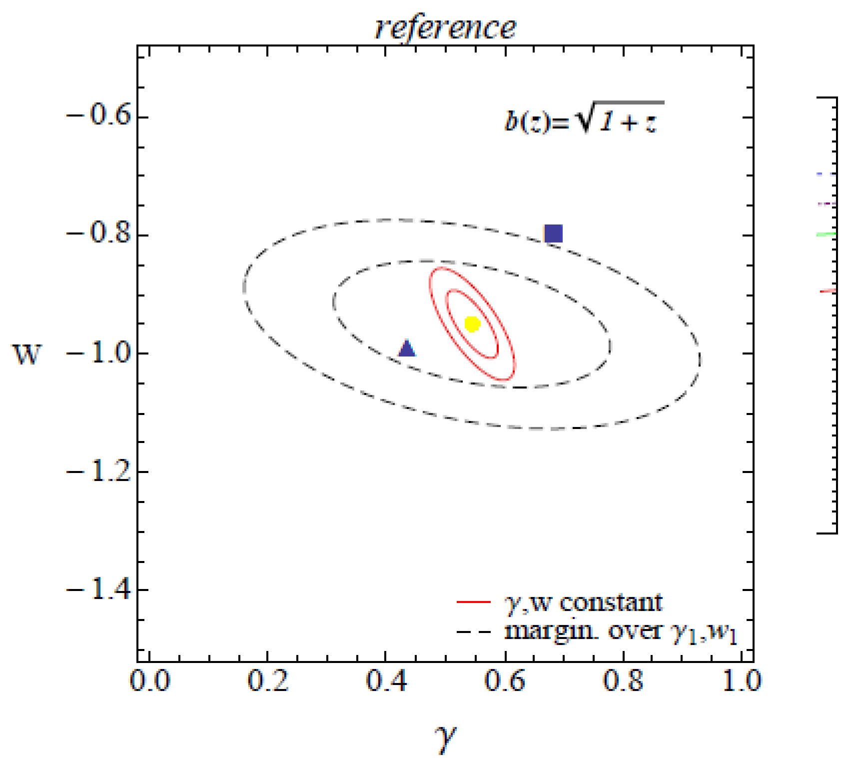


Perturbation theory and halo model.

EUCLID Forecast

$$f_g(z) = \frac{d\ln\delta}{d\ln a}$$

Equation of state



$$f_g(z) = \Omega_m^{\gamma}$$

Growth index

Model independent parameterisation valid on linear scales only.

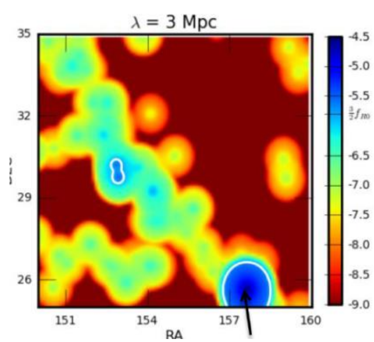
Summary

Dark energy must be nearly massless on large scales: gravitational problem.

Must be embedded in screened models of modified gravity can be classified: chameleons, Damour-Polyakov and Vainshtein.

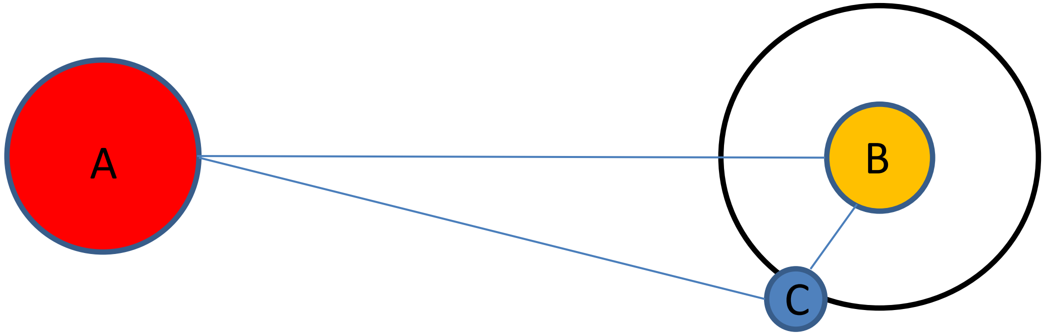
Local test imply bounds on the range of the scalar interaction.

Consequences for large scale structure.



Due to the scalar interaction, within the Compton wavelength of the scalar field, the inertial and gravitational masses differ for screened objects:

VIOLATION OF THE STRONG EQUIVALENCE PRINCIPLE



$$\eta_{BC} \equiv \left|\frac{a_C - a_B}{a_C + a_B}\right| = Q_A |Q_C - Q_B|$$

$$\eta_{\mathrm{moon-earth}} \approx Q_\oplus^2$$

$$\eta_{\mathrm{moon-earth}} \leq 10^{-13}$$

$$Q_\oplus \leq 10^{-7}$$

The Lunar Ranging constraint becomes:

$$9\Omega_{m0}H_0^2\int_0^{a_G} da\,\frac{\beta(a)}{a^4m^2(a)} \le 10^{-7}\Phi_\oplus$$

$$\Phi_\oplus \sim 10^{-9}$$

This leads to a tight bound on the range:

$$\frac{m_0^2}{H_0^2} \ge 9\,\frac{\Omega_{m0}\beta_0}{2r-3-s}\,a_G^{2r-3-s}10^{16}$$

For (2r-s)>8, this is a weaker condition than the screening of the Milky Way.

Large curvature f(R): for n>1 the Milky Way condition is the strongest.

$$|f_{R_0}| \le 10^{-6}$$

Chameleons: the cosmological range is always less than 10 kpc.

Dilaton: the cosmological range is less than 10 kpc.