

Model Building for Light Sterile Neutrinos



Alexander Merle

University of Southampton, UK



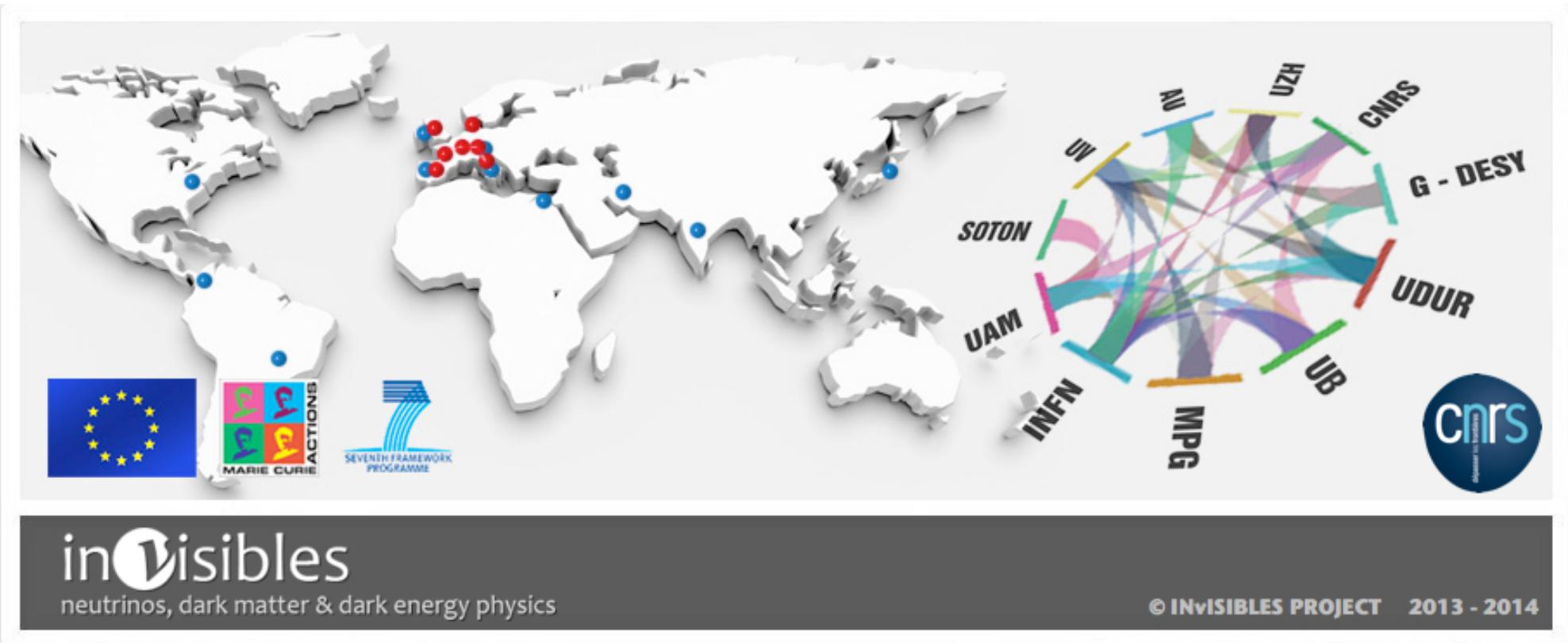
White paper & pedagogical review available:

Abazajian,... AM,...: 1204.5379 [hep-ph]

AM: Int. J. Mod. Phys. D22 (2013) 1330020

Invisibles 14 Workshop, Paris, France, 14-07-2014

**First, I would like to thank the organisers
of this workshop for inviting me!**



THANK YOU!!!

CONGRATULATIONS!!!



To **GERMANY** for Winning the World Cup!!!

At least the participants from that country will enjoy my talk no matter what...

Contents:

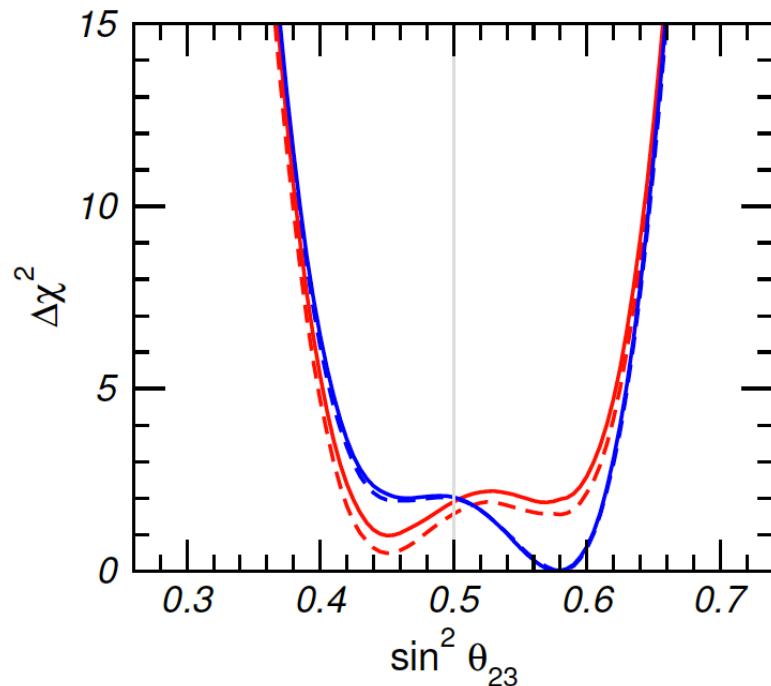
1. Introduction: *Why sterile neutrinos?!*
2. Experiments/Observations/Applications
3. General principles for ν_s models
4. A few example models
5. Conclusions and Outlook

1. Introduction

1. Introduction

Who needs sterile neutrinos?!? Why and what for?!?

- ordinary neutrinos are doing fine, we see to have a clear picture:



$$\begin{aligned}\theta_{12} &\approx 33.48^\circ \\ \theta_{13} &\approx 8.52^\circ \\ \theta_{23} &\approx 49.4^\circ \\ \delta_{CP} &\approx 251^\circ (?) \\ \Delta m^2_{21} &\approx 7.50 \times 10^{-5} \text{ eV}^2 \\ |\Delta m^2_{31}|_{\text{NO}} &\approx 2.458 \times 10^{-3} \text{ eV}^2 \\ |\Delta m^2_{31}|_{\text{IO}} &\approx 2.448 \times 10^{-3} \text{ eV}^2\end{aligned}$$

www.nu-fit.org

$m_\nu < 1 \text{ eV}$ [MAINZ, Planck, KamLAND-Zen, EXO-200, GERDA,...]

Why on Earth would we need sterile neutrinos?!?

1. Introduction

Why on Earth would we need sterile neutrinos???

Simple answer:

Actually, we don't need them at all...

HOWEVER:

We have measured/observed a couple of anomalies/hints/puzzles, all of which could be explained by sterile neutrinos!!!

1. Introduction

Why on Earth would we need sterile neutrinos!!!

Simple answer:

Actually, we don't need them at all...

HOWEVER:

We have measured/observed a couple of anomalies/hints/puzzles, all of which could be explained by sterile neutrinos!!!

It is your choice which position to take, but personally I think we should never discard anything as long as experiments do not tell us to do so.

2. Experiments/Observations/Applications

2. Experiments/Observations/Applications

Reactor anomaly

(eV & $\theta^2 \approx 10^{-1}$)

LSND/MiniBooNE

(eV & $\theta^2 \approx 10^{-3} - 10^{-2}$)

Gallium anomaly

(eV & $\theta^2 \approx 10^{-1}$)

Why are sterile
neutrinos useful?!?

Dark Radiation

(eV & $\theta^2 \approx 10^{-5} - 10^{-2}$)

(Warm) Dark Matter

(keV & $\theta^2 \approx 10^{-8}$ or tinier)

+ interesting phenomenology at various experiments:

Daya Bay, ESS, JUNO, NUCIFER, STEREO,...

2. Experiments/Observations/Applications

Example 1: The reactor anomaly

- re-evaluation of reactor neutrino flux

[Mueller *et al.*: Phys. Rev. **C83** (2011) 054615]

[Huber: Phys. Rev. **C83** (2011) 024617]

[Hayes *et al.*: Phys. Rev. Lett. **112** (2014) 202501]

→ 3.5% **ABOVE** previous predictions

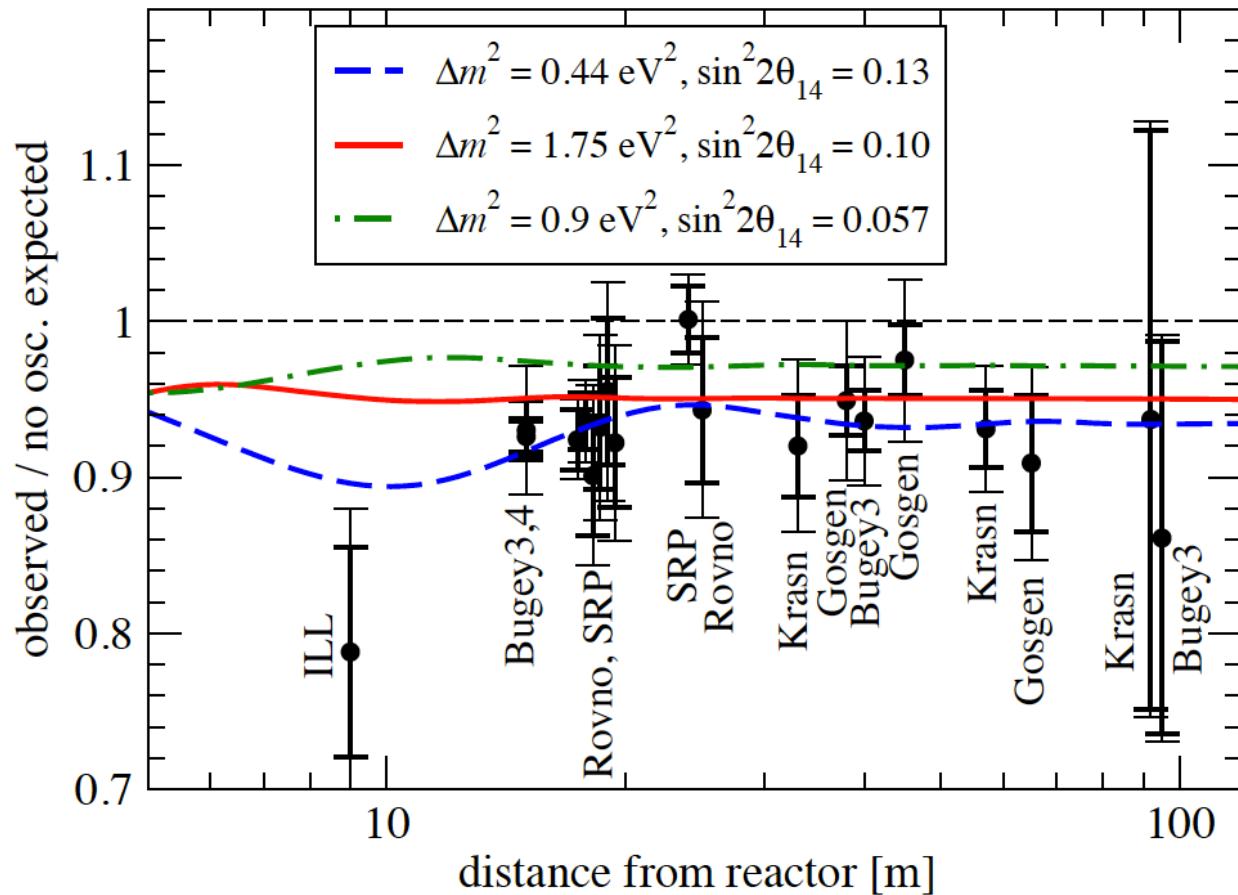
- **BUT**: experiments were okay with old predictions
→ systematics?!? or maybe **NEW PHYSICS**?!?

- **FACT**: this so-called “reactor anomaly” **CAN** be interpreted in terms of oscillations into light sterile neutrinos

2. Experiments/Observations/Applications

Example 1: The reactor anomaly

- possible 3+1 fits to the data:



3+1 scenarios:

- all provide a good fit
- all strongly disfavour the “no oscillation (i.e., no sterile ν) hypothesis”

BUT:

- only fitting
- systematics cannot be excluded

2. Experiments/Observations/Applications

Example 2: (Warm) Dark Matter → Talk by Oleg Ruchayskiy

HOT

- highly relativistic
- light neutrinos
- only DM within SM
(Higgs is unstable) ✓
- ruled out by
structure formation ✗

WARM/COOL

- hardly relativistic
- gravitino, axino,
keV neutrino,...
- exotic ✗
- Dwarf galaxies ✓ (?)
- model building ✓

COLD

- non-relativistic
- WIMP paradigm
- good for SUSY, etc. ✓
- no direct detection
so far (**LUX**) ✗
- Dwarf problem ✗ (?)

EXCLUDED!!!

Still okay.

Sterile neutrinos with keV-ish masses are very good (and typically warm) Dark Matter candidates!!!

2. Experiments/Observations/Applications

Example 2: (Warm) Dark Matter → Talk by Oleg Ruchayskiy

- production mechanisms (ordinary thermal freeze-out does not work):

$v_a - v_s$ oscillations [Dodelson, Widrow: Phys. Rev. Lett. **72** (1994) 17]

Resonant oscillations [Shi, Fuller: Phys. Rev. Lett. **82** (1999) 2832]

Scalar decay [Asaka *et al.*: Phys. Lett. **B638** (2006) 401], [Anisimov *et al.*: Phys. Lett. **B671** (2009) 211], [Bezrukov, Gorbunov: JHEP **1005** (2010) 010], [Kusenko, Petraki: Phys. Rev. **D77** (2008) 065014], [AM, Niro, Schmidt: JCAP **1403** (2013) 028], [AM, Schneider: *in preparation*]

Diluted thermal overproduction [Bezrukov *et al.*: Phys. Rev. **D81** (2010) 085032], [King, AM: JCAP **1208** (2012) 016], [Nemevsek *et al.*: JCAP **1207** (2012) 006]

2. Experiments/Observations/Applications

Example 2: (Warm) Dark Matter → Talk by Oleg Ruchayskiy

- production mechanisms (ordinary thermal freeze-out does not work):

EXCLUDED!!!

~~$v_a - v_s$ oscillations~~ [Dodelson, Widrow: Phys. Rev. Lett. **72** (1994) 17]

Resonant oscillations [Shi, Fuller: Phys. Rev. Lett. **82** (1999) 2832]

Scalar decay [Asaka *et al.*: Phys. Lett. **B638** (2006) 401], [Anisimov *et al.*: Phys. Lett. **B671** (2009) 211], [Bezrukov, Gorbunov: JHEP **1005** (2010) 010], [Kusenko, Petraki: Phys. Rev. **D77** (2008) 065014], [AM, Niro, Schmidt: JCAP **1403** (2013) 028], [AM, Schneider: *in preparation*]

Diluted thermal overproduction [Bezrukov *et al.*: Phys. Rev. **D81** (2010) 085032], [King, AM: JCAP **1208** (2012) 016], [Nemevsek *et al.*: JCAP **1207** (2012) 006]

2. Experiments/Observations/Applications

Example 2: (Warm) Dark Matter → Talk by Oleg Ruchayskiy

- production mechanisms (ordinary thermal freeze-out does not work):

EXCLUDED!!!

~~$v_a - v_s$ oscillations~~ [Dodelson, Widrow: Phys. Rev. Lett. **72** (1994) 17]

Resonant oscillations [Shi, Fuller: Phys. Rev. Lett. **82** (1999) 2832]

Scalar decay [Asaka *et al.*: Phys. Lett. **B638** (2006) 401], [Anisimov *et al.*: Phys. Lett. **B671** (2009) 211], [Bezrukov, Gorbunov: JHEP **1005** (2010) 010], [Kusenko, Petraki: Phys. Rev. **D77** (2008) 065014], [AM, Niro, Schmidt: JCAP **1403** (2013) 028], [AM, Schneider: *in preparation*]

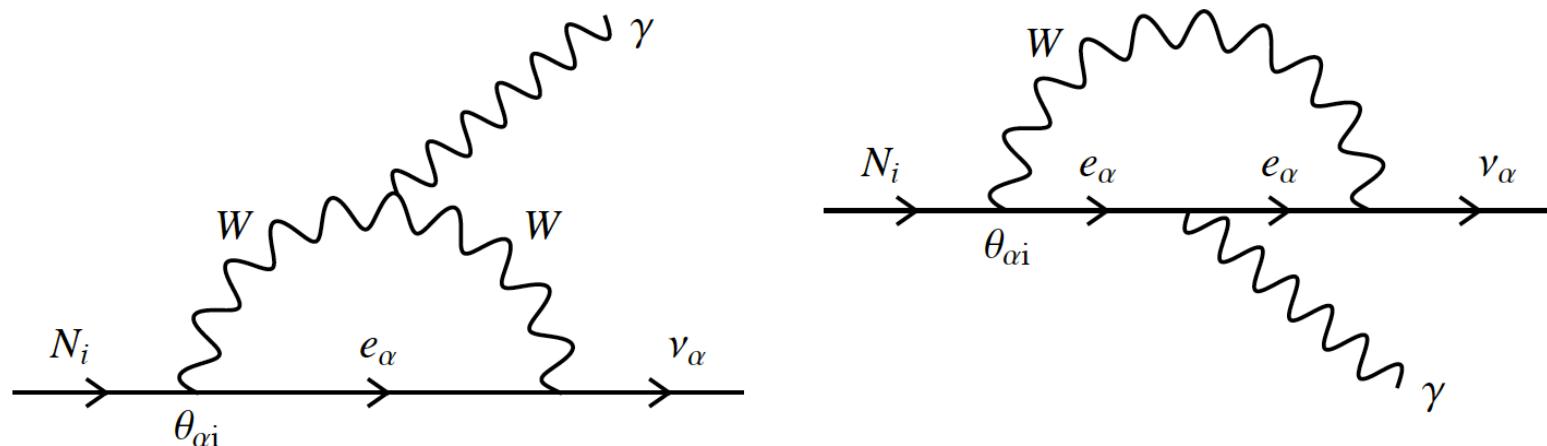
Diluted thermal overproduction [Bezrukov *et al.*: Phys. Rev. **D81** (2010) 085032], [King, AM: JCAP **1208** (2012) 016], [Nemevsek *et al.*: JCAP **1207** (2012) 006]

OKAY!!!

2. Experiments/Observations/Applications

Example 2: (Warm) Dark Matter → Talk by Oleg Ruchayskiy

- a possible hint: the “3.5” keV line (ACTUALLY: 3.6 keV, at least according to the first paper)
 - stacked XMM-Newton spectrum of 73 galaxy clusters + division into 3 subsamples (e.g. Perseus cluster only):
 $m_s \approx 7.1 \text{ keV}$ & $\sin^2(2\theta) \approx 7 \times 10^{-11}$ [Bulbul *et al.*: *Astrophys. J.* **789** (2014) 13]
 - XMM-Newton spectrum of Perseus cluster & Andromeda galaxy:
 $m_s = 7.06 \pm 0.05 \text{ keV}$ & $\sin^2(2\theta) = (2.2-20) \times 10^{-11}$ [Boyarsky *et al.*: 1402.4119]



3. General principles for v_s models

3. General principles for ν_s models

- **MINIMAL TASKS:**

- eV sterile neutrino:
explain eV-scale masses and potentially sizeable active-sterile mixing of $O(0.01 - 0.1)$
- keV sterile neutrino:
explain keV-scale masses and tiny active-sterile mixing of $O(10^{-5} - 10^{-4})$ or even smaller
 - + *more generations*
 - + *active neutrino masses & mixings*
 - (+ *correct Dark Matter abundance*)

3. General principles for v_s models

- **Differences to “ordinary” model building:**

- we need an explanation for the eV or keV scale:
 - not considered to be “fundamental”
 - need some mechanism → two generic schemes:

$$\overline{\overline{M_{2,3}}}=O(M_R)$$

Bottom-up
scheme

$$\overline{\overline{M_{1,2,3}}}=O(M_R)$$

$$M_R \gg \text{keV}$$

AM: Int. J. Mod.
Phys. D22 (2013)
1330020

$$\overline{M_1}=O(\text{keV})$$

$$\overline{M_1} \equiv 0$$

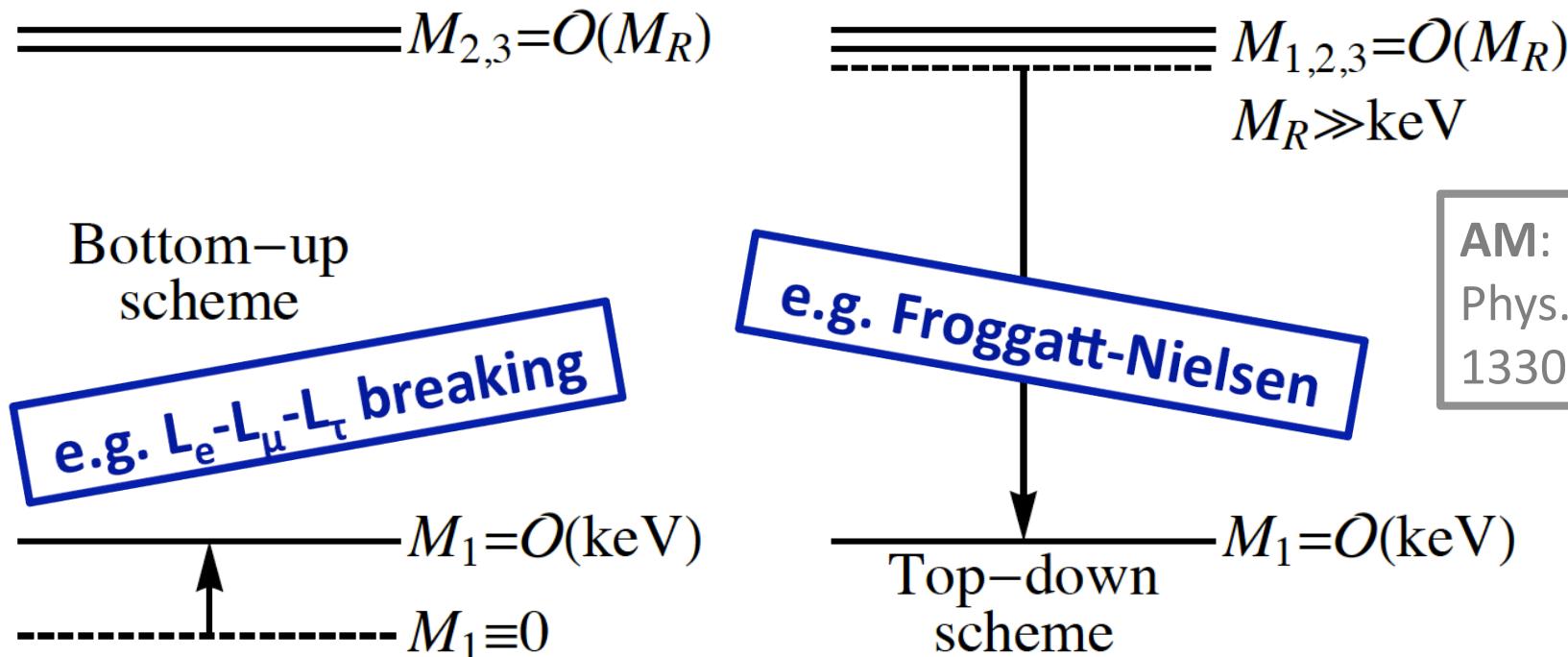
$$\overline{M_1}=O(\text{keV})$$

Top-down
scheme

3. General principles for v_s models

- **Differences to “ordinary” model building:**

- we need an explanation for the eV or keV scale:
 - not considered to be “fundamental”
 - need some mechanism → two generic schemes:

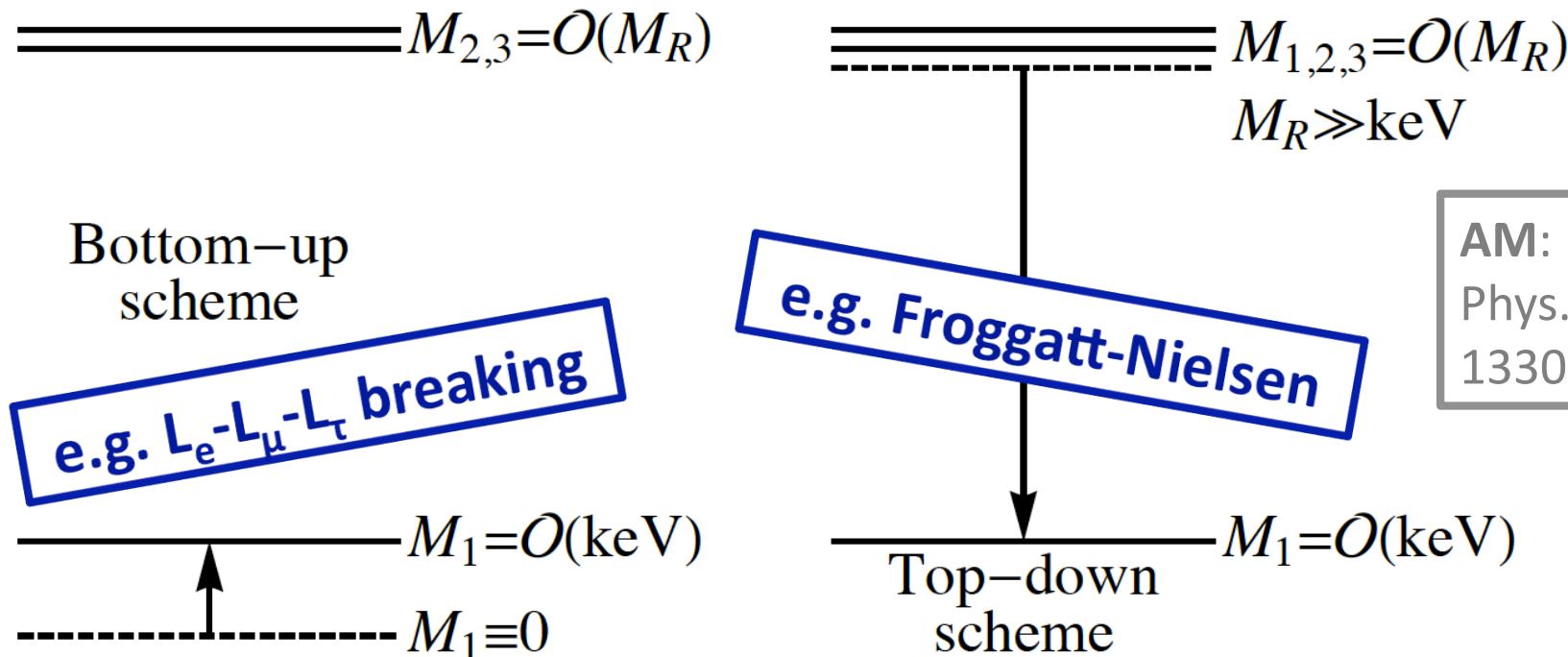


AM: Int. J. Mod.
Phys. D22 (2013)
1330020

3. General principles for v_s models

- **Differences to “ordinary“ model building:**

- we need an explanation for the eV or keV scale:
 - not considered to be “fundamental”
 - need some mechanism → two generic schemes:



AM: Int. J. Mod.
Phys. D22 (2013)
1330020

BUT: One cannot explain absolute scales!!!! Only relative ones.

3. General principles for ν_s models

- **Differences to “ordinary” model building:**

- we need to get the mixing right, i.e. $O(0.1)$ for eV or tiny for keV neutrinos:

- **explain mass and mixing at once:**

- 😊 difficult, quite constrained
 - 😊 predictive

- **use two different mechanisms:**

- 😊 less elegant, needs more ingredients
 - 😊 more freedom, better chance to fit

Both possibilities appear in the literature...

3. General principles for ν_s models

- **Differences to “ordinary” model building:**

- we need to get the mixing right, i.e. $O(0.1)$ for eV or tiny for keV neutrinos:

- **explain mass and mixing at once:**

- ⌚ difficult, quite constrained
 - 😊 predictive

- **use two different mechanisms:**

- ⌚ less elegant, needs more ingredients
 - 😊 more freedom, better chance to fit

Both possibilities appear in the literature...

3. General principles for ν_s models

- use two different mechanisms:

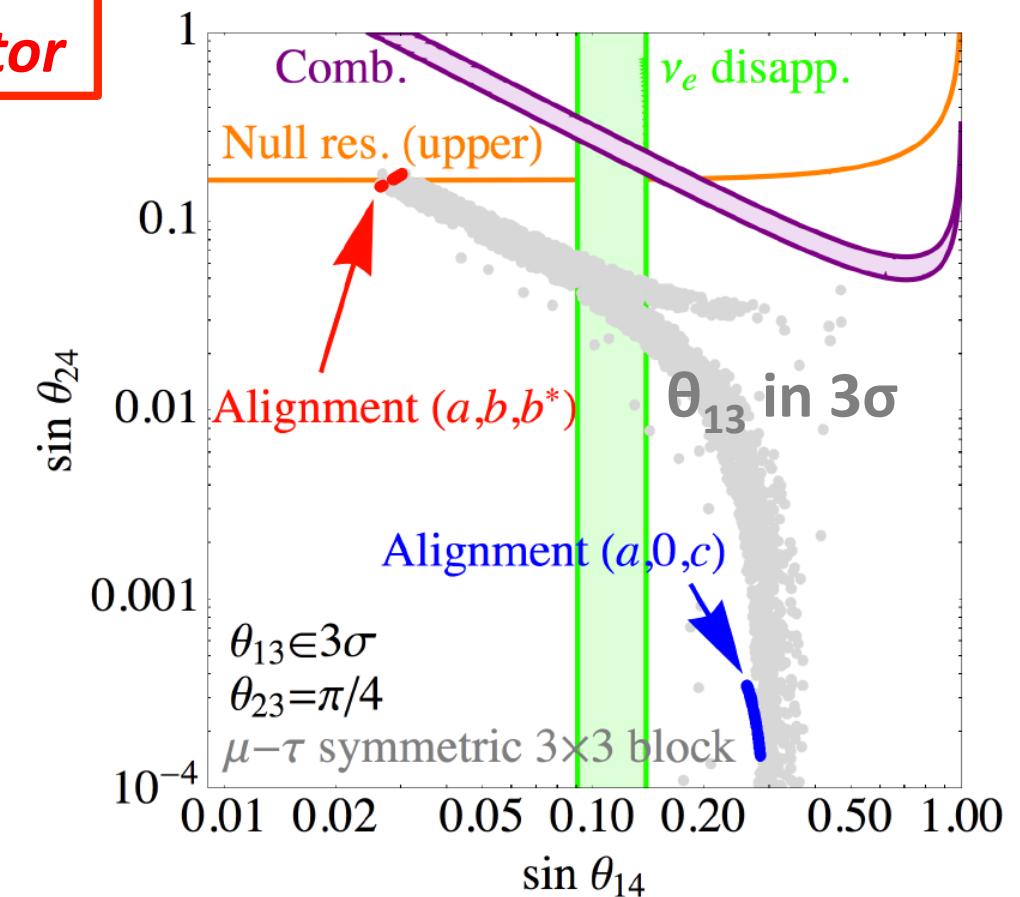
e.g. generating active-sterile mixing of $O(\theta_{13})$

Enforces $\theta_{13}=0$ in the 3x3 subsector

$$M_\nu^{4 \times 4} = \begin{pmatrix} M_{\mu-\tau} & A \\ -A^T & m_s \end{pmatrix}$$

Generates $\theta_{13} \approx \theta_{14}$ for suitable A (i.e., no eigenvector of $M_{\mu-\tau}$)

$A = (a, b, c)^T$



4. Example models

4. Example models

- **CATEGORY 1:** Models based on broken symmetries

4. Example models

- **CATEGORY 1:** Models based on broken symmetries
 - typically, these models apply a bottom-up scheme:

$$===== M_{2,3} = \mathcal{O}(M_R)$$

Bottom-up
scheme

$$\xrightarrow{\hspace{1cm}} M_1 = \mathcal{O}(\text{keV})$$
$$\xrightarrow{\hspace{1cm}} M_1 \equiv 0$$

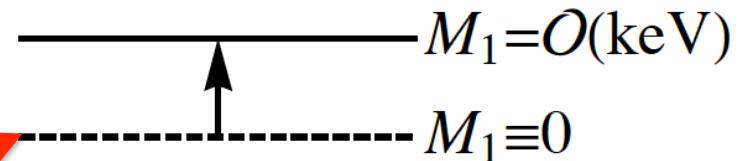
4. Example models

- **CATEGORY 1:** Models based on broken symmetries
 - typically, these models apply a bottom-up scheme:

$$===== M_{2,3} = \mathcal{O}(M_R)$$

Bottom-up
scheme

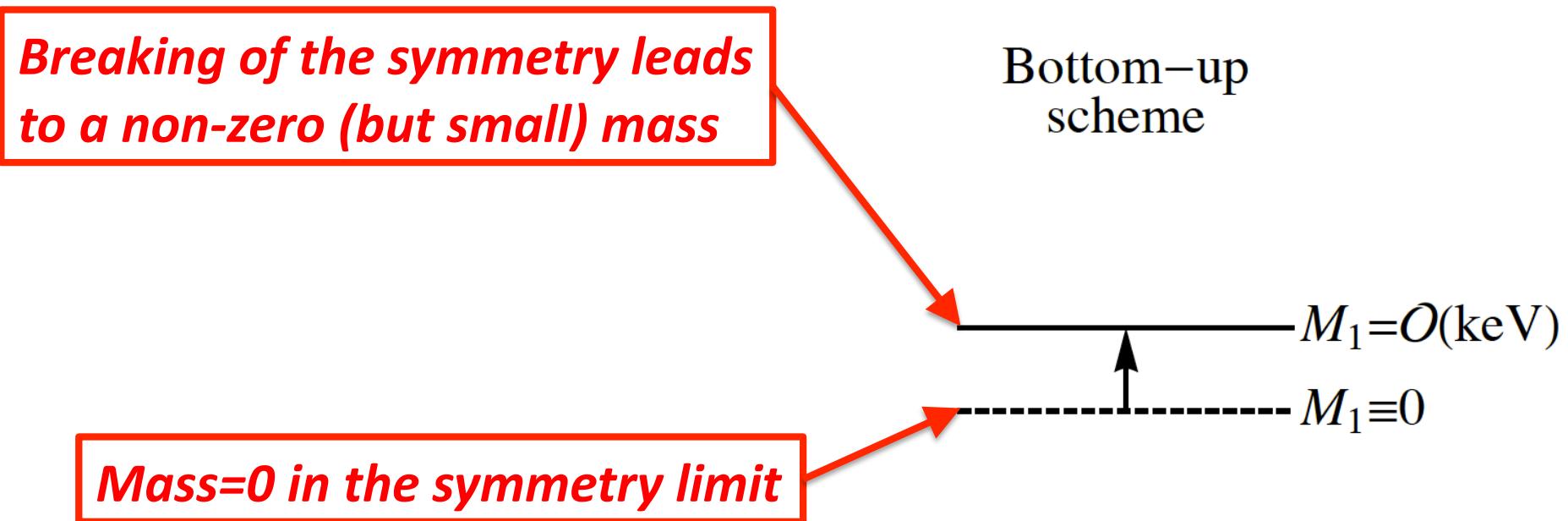
Mass=0 in the symmetry limit



4. Example models

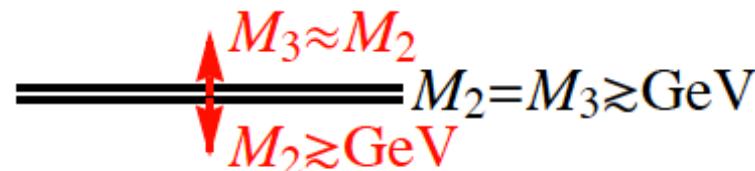
- **CATEGORY 1:** Models based on broken symmetries
 - typically, these models apply a bottom-up scheme:

$$M_{2,3} = \mathcal{O}(M_R)$$



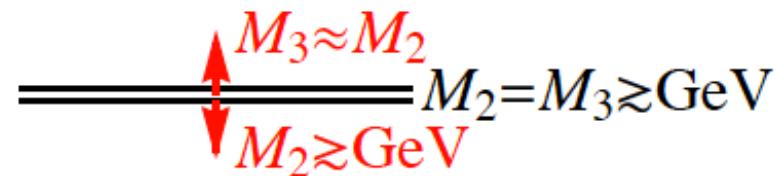
4. Example models

- **CATEGORY 1:** Models based on broken symmetries
 - examples from the literature:



Q_6 symmetry

$O(1/\Lambda^0)$ $\cancel{O(1/\Lambda^2)}$



$L_e - L_\mu - L_\tau \& \mu - \tau$

~~$L_e - L_\mu - L_\tau \& \mu - \tau$~~



[Araki, Li: Phys. Rev. **D85** (2012) 065016]



[Shaposhnikov: Nucl. Phys. **B763** (2007) 49]

[Lindner, AM, Niro: JCAP **1101** (2011) 034]

4. Example models

- model based on Q_6 (=double cover of D_3):

Field	L_1	$L_D \equiv L_{2,3}$	$\overline{e_R}$	$\overline{e_{DR}} = (\overline{\mu_R}, \overline{\tau_R})$	$\overline{N_1}$	$\overline{N_D} \equiv \overline{N_{2,3}}$	H	S_x	S_y	S_z	D
$SU(2)_L$	2	2	1	1	1	1	2	1	1	1	1
Q_6	1	2'	1'	2	1''	2'	1	1''	1'''	1	2
Z_2	+	+	+	+	+	-	+	+	+	-	-
Z_3	1	1	ω	ω^2	1	1	1	ω^2	ω^2	1	1

[Araki, Li: Phys. Rev. D85 (2012) 065016]

$$M_R = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & M_a \\ 0 & M_a & 0 \end{pmatrix} + \frac{1}{\Lambda^2} \begin{pmatrix} M_c s_x s_y & 0 & 0 \\ 0 & M_b d_2^2 & 0 \\ 0 & 0 & M_b d_1^2 \end{pmatrix}$$

4. Example models

- model based on Q_6 (=double cover of D_3):

Field	L_1	$L_D \equiv L_{2,3}$	$\overline{e_R}$	$\overline{e_{DR}} = (\overline{\mu_R}, \overline{\tau_R})$	$\overline{N_1}$	$\overline{N_D} \equiv \overline{N_{2,3}}$	H	S_x	S_y	S_z	D
$SU(2)_L$	2	2	1	1	1	1	2	1	1	1	1
Q_6	1	2'	1'	2	1''	2'	1	1''	1'''	1	2
Z_2	+	+	+	+	+	-	+	+	+	-	-
Z_3	1	1	ω	ω^2	1	1	1	ω^2	ω^2	1	1

[Araki, Li: Phys. Rev. D85 (2012) 065016]

(0, M_a , M_a) pattern

$$M_R = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & M_a \\ 0 & M_a & 0 \end{pmatrix} + \frac{1}{\Lambda^2} \begin{pmatrix} M_c s_x s_y & 0 & 0 \\ 0 & M_b d_2^2 & 0 \\ 0 & 0 & M_b d_1^2 \end{pmatrix}$$

4. Example models

- model based on Q_6 (=double cover of D_3):

Field	L_1	$L_D \equiv L_{2,3}$	$\overline{e_R}$	$\overline{e_{DR}} = (\overline{\mu_R}, \overline{\tau_R})$	$\overline{N_1}$	$\overline{N_D} \equiv \overline{N_{2,3}}$	H	S_x	S_y	S_z	D
$SU(2)_L$	2	2	1	1	1	1	2	1	1	1	1
Q_6	1	2'	1'	2	1''	2'	1	1''	1'''	1	2
Z_2	+	+	+	+	+	-	+	+	+	-	-
Z_3	1	1	ω	ω^2	1	1	1	ω^2	ω^2	1	1

[Araki, Li: Phys. Rev. D85 (2012) 065016]

(0, M_a , M_a) pattern

$$M_R = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & M_a \\ 0 & M_a & 0 \end{pmatrix} + \frac{1}{\Lambda^2} \begin{pmatrix} M_c s_x s_y & 0 & 0 \\ 0 & M_b d_2^2 & 0 \\ 0 & 0 & M_b d_1^2 \end{pmatrix}$$

Flavon VEVs

$$\langle S_i \rangle = s_i$$

$$\langle D \rangle = (d_1, d_2)^T$$

4. Example models

- model based on Q_6 (=double cover of D_3):

Field	L_1	$L_D \equiv L_{2,3}$	$\overline{e_R}$	$\overline{e_{DR}} = (\overline{\mu_R}, \overline{\tau_R})$	$\overline{N_1}$	$\overline{N_D} \equiv \overline{N_{2,3}}$	H	S_x	S_y	S_z	D
$SU(2)_L$	2	2	1	1	1	1	2	1	1	1	1
Q_6	1	2'	1'	2	1''	2'	1	1''	1'''	1	2
Z_2	+	+	+	+	+	-	+	+	+	-	-
Z_3	1	1	ω	ω^2	1	1	1	ω^2	ω^2	1	1

[Araki, Li: Phys. Rev. D85 (2012) 065016]

(0, M_a , M_a) pattern

Flavon VEVs

$$\langle S_i \rangle = s_i$$

$$\langle D \rangle = (d_1, d_2)^T$$

$$M_R = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & M_a \\ 0 & M_a & 0 \end{pmatrix} + \frac{1}{\Lambda^2} \begin{pmatrix} M_c s_x s_y & 0 & 0 \\ 0 & M_b d_2^2 & 0 \\ 0 & 0 & M_b d_1^2 \end{pmatrix}$$

→ $M_1 = \frac{s_x s_y}{\Lambda^2} M_c \ll M_{2,3} \simeq M_a \mp \frac{M_b}{2\Lambda^2} (d_1^2 + d_2^2)$

4. Example models

- predictions of the Q_6 model for light neutrinos:

- *one light neutrino is massless:*

$$\underline{m_1 = 0} \quad , \quad m_2 = \frac{|\beta' + \gamma'|^2 + 2|\alpha'|^2}{M_a} v^2 \quad , \quad m_3 = -\frac{(\beta' - \gamma')^2}{M_a} v^2$$

- *correlation between mixing angles:*

$$\sin \theta_{12} \simeq \sin \theta$$

$$\sin \theta_{13} \simeq \left| \frac{\sqrt{2}\alpha'(2\beta' - \gamma')}{(\beta' - \gamma')^2} \epsilon_d \right|$$
$$\tan \theta_{23} \simeq 1 - \frac{2\sqrt{2}\beta'\gamma'}{|\alpha'(2\beta' - \gamma')|} \sin \theta_{13}$$
$$\tan \theta_{23} \simeq \left| 1 - \frac{4\beta'\gamma'}{(\beta' - \gamma')^2} \epsilon_d \right|$$

4. Example models

- predictions of the Q_6 model for light neutrinos:

- *one light neutrino is massless:*

$$\underline{m_1 = 0} \quad , \quad m_2 = \frac{|\beta' + \gamma'|^2 + 2|\alpha'|^2}{M_a} v^2 \quad , \quad m_3 = -\frac{(\beta' - \gamma')^2}{M_a} v^2$$

- *correlation between mixing angles:*

$$\sin \theta_{12} \simeq \sin \theta$$

$$\sin \theta_{13} \simeq \left| \frac{\sqrt{2}\alpha'(2\beta' - \gamma')}{(\beta' - \gamma')^2} \epsilon_d \right|$$

$$\tan \theta_{23} \simeq \left| 1 - \frac{4\beta'\gamma'}{(\beta' - \gamma')^2} \epsilon_d \right|$$

$$\tan \theta_{23} \simeq 1 - \frac{2\sqrt{2}\beta'\gamma'}{|\alpha'(2\beta' - \gamma')|} \sin \theta_{13}$$

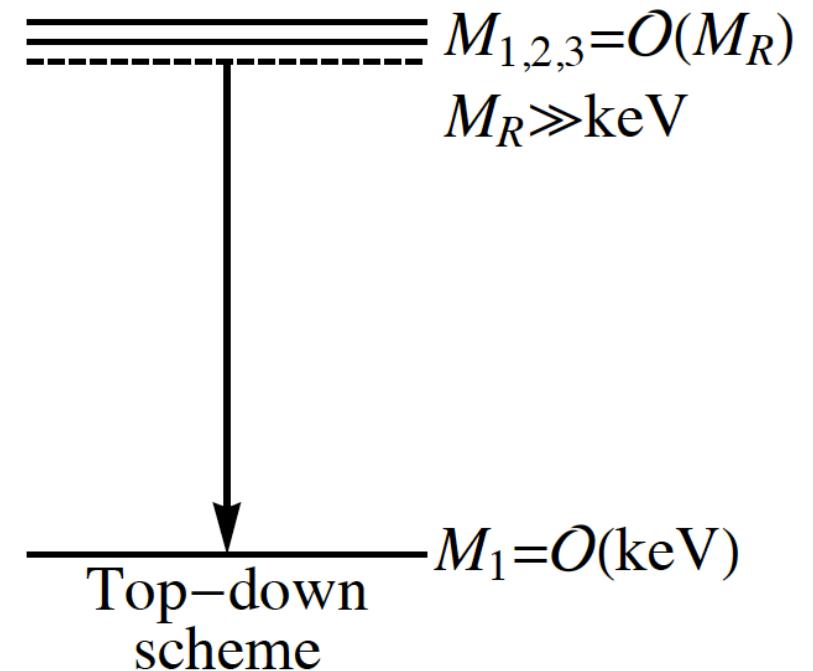
→ TESTABLE PREDICTIONS

4. Example models

- **CATEGORY 2:** Models with suppression mechanisms

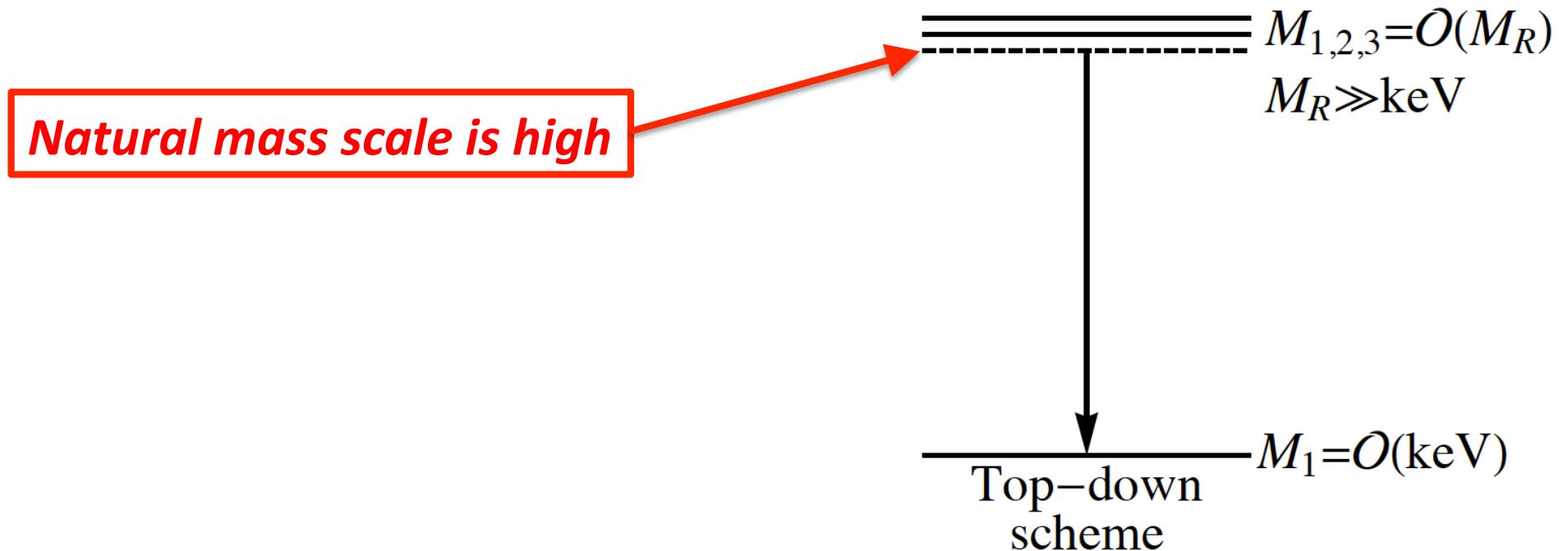
4. Example models

- **CATEGORY 2:** Models with suppression mechanisms
 - typically, these models apply a top-down scheme:



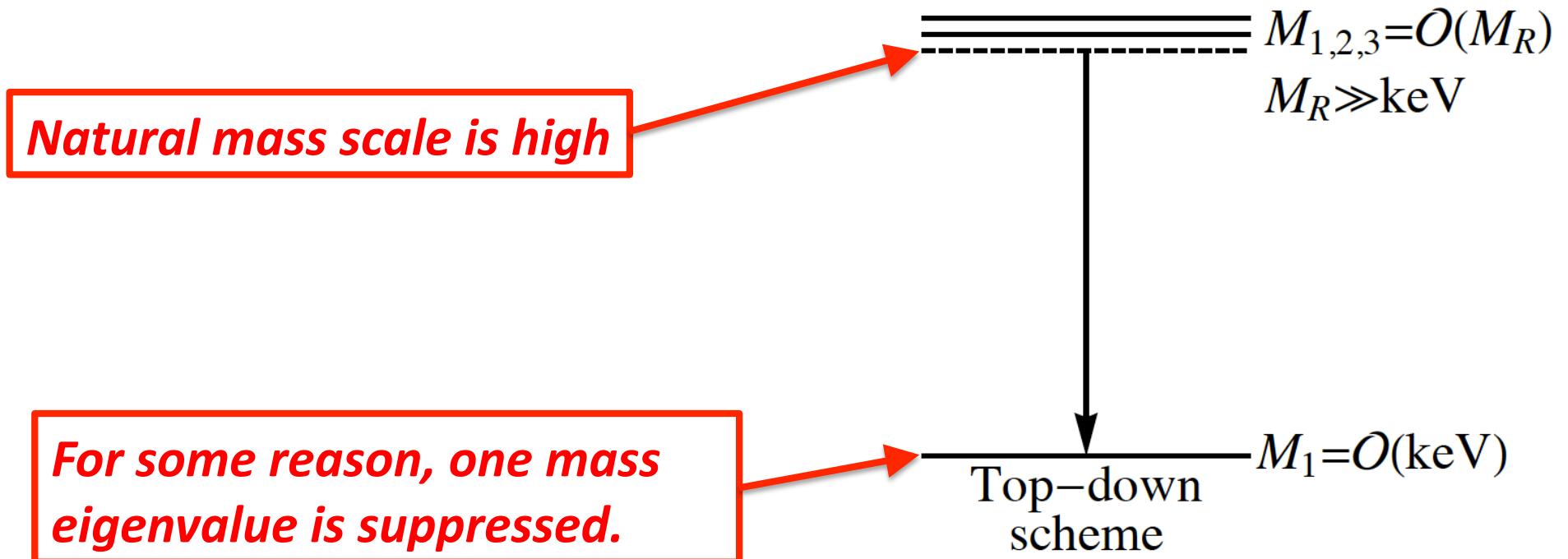
4. Example models

- **CATEGORY 2:** Models with suppression mechanisms
 - typically, these models apply a top-down scheme:



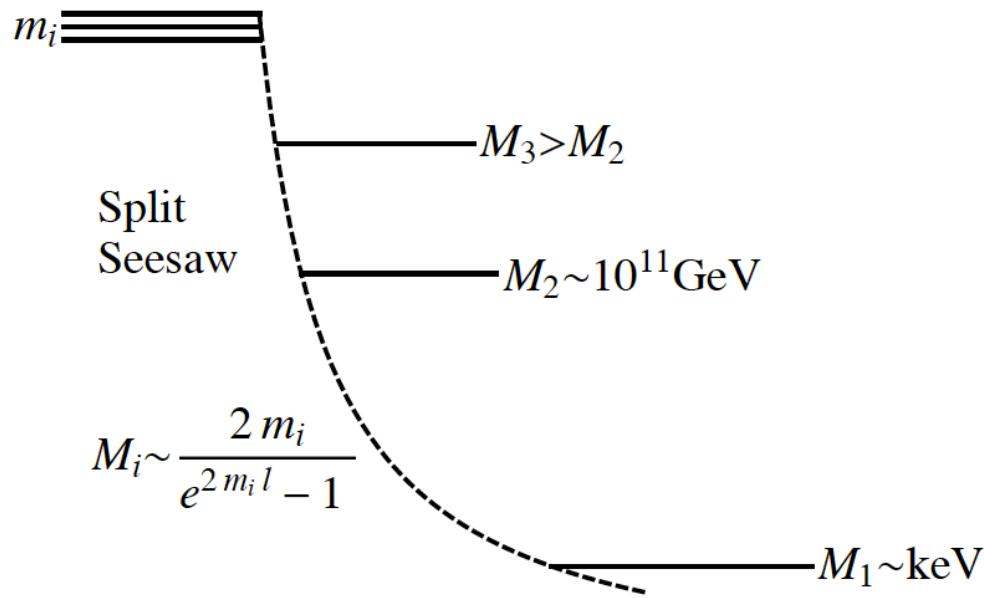
4. Example models

- **CATEGORY 2:** Models with suppression mechanisms
 - typically, these models apply a top-down scheme:



4. Example models

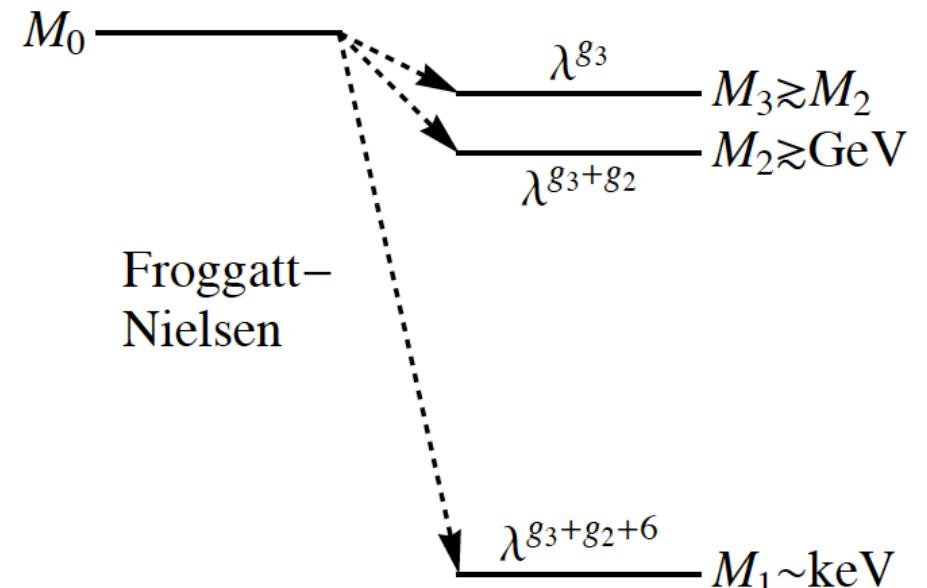
- **CATEGORY 2:** Models with suppression mechanisms
 - examples from the literature:



[Kusenko *et al.*: Phys. Lett. **D693** (2010) 144]

[Adulpravitchai, R. Takahashi: JHEP **1109** (2011) 127]

[R. Takahashi: PTEP **6** (2013) 063B04]



[AM, Niro: JCAP **1107** (2011) 023]

[Barry *et al.*: JHEP **1107** (2011) 091]

[Barry *et al.*: JCAP **1201** (2012) 052]

4. Example models

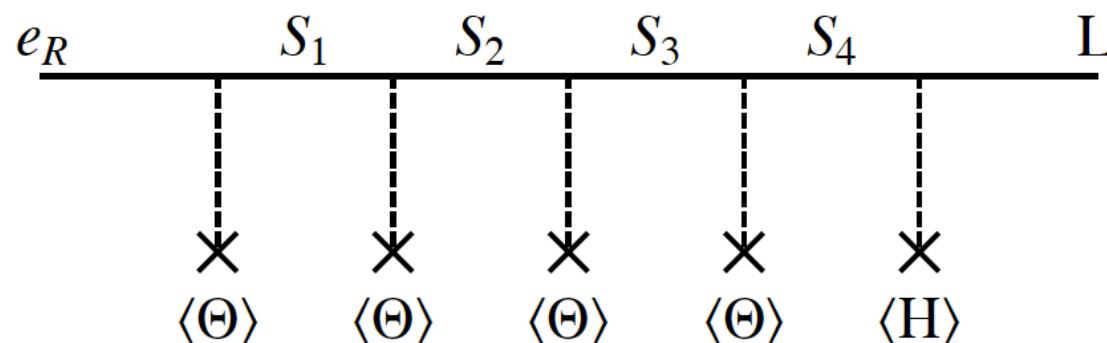
- Froggatt-Nielsen models:

- **Froggatt-Nielsen mechanism:**

high energy sector suppresses low-energy
masses like “multiple seesaws”

[Froggatt, Nielsen: Nucl. Phys. B147 (1979) 277]

- heavy fermions S_i and “flavons” Θ are charged
suitably under a global $U(1)_F$ symmetry:



4. Example models

- **Froggatt-Nielsen models:**
 - this leads to generation-dependent Yukawa couplings, which are suppressed:

$$Y_{ij} = Y_{ij}^{\text{nat}} \lambda^{a_i + b_j}$$

4. Example models

- Froggatt-Nielsen models:
 - this leads to generation-dependent Yukawa couplings, which are suppressed:

$$Y_{ij} = Y_{ij}^{\text{nat}} \lambda^{a_i + b_j}$$

“natural” Yukawa coupling: $O(1)$

4. Example models

- Froggatt-Nielsen models:

- this leads to generation-dependent Yukawa couplings, which are suppressed:

$$Y_{ij} = Y_{ij}^{\text{nat}} \lambda^{a_i + b_j}$$

“natural” Yukawa coupling: $O(1)$

$\lambda = \langle \Theta \rangle / \Lambda$: suppression factor

$\langle \Theta \rangle$: Flavon VEV
 Λ : High scale

4. Example models

- Froggatt-Nielsen models:

- this leads to generation-dependent Yukawa couplings, which are suppressed:

$$Y_{ij} = Y_{ij}^{\text{nat}} \lambda^{a_i + b_j}$$

“natural” Yukawa coupling: $O(1)$

$\lambda = \langle \Theta \rangle / \Lambda$: suppression factor

Generation-dependent $U(1)_{\text{FN}}$ charges

$\langle \Theta \rangle$: Flavon VEV
 Λ : High scale

4. Example models

- Froggatt-Nielsen models:

- this leads to generation-dependent Yukawa couplings, which are suppressed:

$$Y_{ij} = Y_{ij}^{\text{nat}} \lambda^{a_i + b_j}$$

Generation-dependent
 $U(1)_{\text{FN}}$ charges

“natural” Yukawa coupling: $O(1)$

$\lambda = \langle \Theta \rangle / \Lambda$: suppression factor

$\langle \Theta \rangle$: Flavon VEV
 Λ : High scale

→ *Physical Yukawa couplings can be small!!*

4. Example models

- **Froggatt-Nielsen models:**

- for light sterile neutrinos, one generation of N_R needs a large FN-charge:

$$\Theta_{1,2} : (\theta_1, \theta_2; +, -)$$

$$L_{1,2,3} : (f_1, f_2, f_3; +, +, -)$$

$$\overline{e_{1,2,3}} : (k_1, k_2, k_3; +, +, -)$$

$$\overline{N_{1,2,3}} : (g_1, g_2, g_3; +, +, -)$$

$$A(3,0,0) : M_1 = M_0 \lambda^6 2R_0^2 \sqrt{1 + R_0^4 + 2R_0^2 \cos(2\alpha_0)}$$

$$M_2 = M_0$$

$$M_3 = M_0 (1 + \lambda^6 [1 + R_0^2 (3 \cos(2\alpha_0) + 3R_0^2 \cos(4\alpha_0) + R_0^4 \cos(6\alpha_0)])]$$

4. Example models

- **Froggatt-Nielsen models:**

- for light sterile neutrinos, one generation of N_R needs a large FN-charge:

$$\Theta_{1,2} : (\theta_1, \theta_2; +, -)$$

$$L_{1,2,3} : (f_1, f_2, f_3; +, +, -)$$

$$\overline{e_{1,2,3}} : (k_1, k_2, k_3; +, +, -)$$

$$\overline{N_{1,2,3}} : (\underline{g_1, g_2, g_3}; +, +, -)$$

$$A(3,0,0) : M_1 = M_0 \lambda^6 2R_0^2 \sqrt{1 + R_0^4 + 2R_0^2 \cos(2\alpha_0)}$$

$$M_2 = M_0$$

$$M_3 = M_0 (1 + \lambda^6 [1 + R_0^2 (3 \cos(2\alpha_0) + 3R_0^2 \cos(4\alpha_0) + R_0^4 \cos(6\alpha_0)])]$$

4. Example models

- **Froggatt-Nielsen models:**

- for light sterile neutrinos, one generation of N_R needs a large FN-charge:

$$\Theta_{1,2} : (\theta_1, \theta_2; +, -)$$

$$L_{1,2,3} : (f_1, f_2, f_3; +, +, -)$$

$$\overline{e_{1,2,3}} : (k_1, k_2, k_3; +, +, -)$$

$$\overline{N_{1,2,3}} : (\underline{g_1, g_2, g_3}; +, +, -)$$

$$A(3,0,0) : M_1 = M_0 \lambda^6 2R_0^2 \sqrt{1 + R_0^4 + 2R_0^2 \cos(2\alpha_0)}$$

$$M_2 = M_0$$

$$M_3 = M_0 (1 + \lambda^6 [1 + R_0^2 (3 \cos(2\alpha_0) + 3R_0^2 \cos(4\alpha_0) + R_0^4 \cos(6\alpha_0)])]$$

4. Example models

- **Froggatt-Nielsen models:**

- for light sterile neutrinos, one generation of N_R needs a large FN-charge:

$$\Theta_{1,2} : (\theta_1, \theta_2; +, -)$$

$$L_{1,2,3} : (f_1, f_2, f_3; +, +, -)$$

$$\overline{e_{1,2,3}} : (k_1, k_2, k_3; +, +, -)$$

$$\overline{N_{1,2,3}} : (g_1, g_2, g_3; +, +, -)$$

One eigenvalue suppressed

$$A(3,0,0) : M_1 = M_0 \lambda^6 2R_0^2 \sqrt{1 + R_0^4 + 2R_0^2 \cos(2\alpha_0)}$$

$$M_2 = M_0$$

$$M_3 = M_0 (1 + \lambda^6 [1 + R_0^2 (3 \cos(2\alpha_0) + 3R_0^2 \cos(4\alpha_0) + R_0^4 \cos(6\alpha_0)])]$$

4. Example models

- Froggatt-Nielsen models:
 - the good, the bad, and the ugly:
- :-) strong suppression, seesaw guaranteed to work, additional flavour symmetry possible
- :-(not predictive per se, enhances active-sterile mixing (bad for keV neutrinos due to X-ray)
- ?-/ unknown high energy sector swept under the carpet... only shift of problems...

4. Example models

- **CATEGORY 3:** Other kinds of models

4. Example models

- **CATEGORY 3:** Other kinds of models
 - some models do not follow any of the schemes,
e.g. because they rely on *exotic new physics* or
because they incorporate *more than one new scale*

4. Example models

- **CATEGORY 3:** Other kinds of models
 - some models do not follow any of the schemes,
e.g. because they rely on *exotic new physics* or
because they incorporate *more than one new scale*
 - **EXAMPLE: Composite Sterile Neutrinos**
[Grossmann, Robinson: JHEP 1101 (2011) 132]
[Robinson, Tsai: JHEP 1208 (2012) 161]
 - *idea*: sterile neutrinos are not fundamental but
consist of other fermions (“preons”)
 - if the preons are charged under a new hidden
gauge group (“v-colour”), they could form
QCD-like bound states → **NEW MASS SCALE**

4. Example models

- **EXAMPLE: Composite Sterile Neutrinos**

- condensation of the preons q :

$$\mathcal{L}_{\text{Yukawa}} = -\frac{\lambda}{M^{3(n-1)/2}} \bar{L} \tilde{H} q^n \rightarrow -\lambda \epsilon^{3(n-1)/2} \bar{L} \tilde{H} N_R + h.c.$$

4. Example models

- **EXAMPLE: Composite Sterile Neutrinos**

- condensation of the preons q :

$$\mathcal{L}_{\text{Yukawa}} = -\underbrace{\frac{\lambda}{M^{3(n-1)/2}} \overline{L} \tilde{H} q^n}_{\text{Effective Interaction}} \rightarrow -\lambda \epsilon^{3(n-1)/2} \overline{L} \tilde{H} N_R + h.c.$$

**Effective Interaction
(below high scale M)**

4. Example models

- **EXAMPLE: Composite Sterile Neutrinos**

- condensation of the preons q :

$$\mathcal{L}_{\text{Yukawa}} = -\frac{\lambda}{M^{3(n-1)/2}} \bar{L} \tilde{H} q^n \rightarrow -\lambda \epsilon^{3(n-1)/2} \bar{L} \tilde{H} N_R + h.c.$$

**Effective Interaction
(below high scale M)**

Condensate $q^n \rightarrow N_R \Lambda^{3(n-1)/2}$

The diagram illustrates the process of condensation. A red bracket underlines the interaction term $\lambda / M^{3(n-1)/2}$ in the Yukawa Lagrangian. An arrow points from this bracket to the text "Effective Interaction (below high scale M)". Another arrow points from the bracket to the condensate equation $q^n \rightarrow N_R \Lambda^{3(n-1)/2}$.

4. Example models

- **EXAMPLE: Composite Sterile Neutrinos**

- condensation of the preons q :

$$\mathcal{L}_{\text{Yukawa}} = -\frac{\lambda}{M^{3(n-1)/2}} \bar{L} \tilde{H} q^n \rightarrow -\lambda \epsilon^{3(n-1)/2} \bar{L} \tilde{H} N_R + h.c.$$

**Effective Interaction
(below high scale M)**

Condensate
$$q^n \rightarrow N_R \Lambda^{3(n-1)/2}$$

**Suppression due to
small $\epsilon = \Lambda/M$**

The diagram illustrates the derivation of the effective interaction term. It starts with the Yukawa Lagrangian $\mathcal{L}_{\text{Yukawa}}$, which contains a term proportional to $\lambda / M^{3(n-1)/2}$. This term is highlighted with a red bracket and labeled "Effective Interaction (below high scale M)". Below this, a purple box contains the condensate equation $q^n \rightarrow N_R \Lambda^{3(n-1)/2}$. An arrow points from this condensate equation upwards to the red bracketed term. To the right of the red bracket, another red circle highlights the term $\epsilon^{3(n-1)/2}$, which is also labeled "Suppression due to small $\epsilon = \Lambda/M$ ".

4. Example models

○ EXAMPLE: Composite Sterile Neutrinos

- condensation of the preons q :

$$\mathcal{L}_{\text{Yukawa}} = -\frac{\lambda}{M^{3(n-1)/2}} \bar{L} \tilde{H} q^n \rightarrow -\lambda \epsilon^{3(n-1)/2} \bar{L} \tilde{H} N_R + h.c.$$

**Effective Interaction
(below high scale M)**

Condensate
$$q^n \rightarrow N_R \Lambda^{3(n-1)/2}$$

**Suppression due to
small $\epsilon = \Lambda/M$**

- this generates DIRAC sterile neutrinos:

$$\mathcal{L}_{LH'} = -\Lambda'_N \epsilon^{3(n'-1)/2} \bar{N}_L N_R + h.c.$$

4. Example models

- ***EXAMPLE: Composite Sterile Neutrinos***
 - corresponding mass shifting scheme:

4. Example models

- ***EXAMPLE: Composite Sterile Neutrinos***

- corresponding mass shifting scheme:



4. Example models

- ***EXAMPLE: Composite Sterile Neutrinos***

- corresponding mass shifting scheme:

$$\longrightarrow \text{---} N_R$$

(because an entirely new QCD-like scale is introduced)

4. Example models

- **EXAMPLE: Composite Sterile Neutrinos**

- corresponding mass shifting scheme:



(because an entirely new QCD-like scale is introduced)

Don't take this slide
too seriously...

4. Example models

- other possibilities (hopefully I didn't miss too many...):

- **extended seesaw** [Zhang: Phys. Lett. **B714** (2012) 262; Heeck, Zhang: JHEP **1305** (2013) 164]
- **type II seesaw in 331-models** [Dias, Peres, Silva: Phys. Lett. **B628** (2005) 85; Cogollo, Diniz, Peres: Phys. Lett. **B677** (2009) 338]
- **$U(1)$ -symmetries close to M_p** [Allison, JHEP **1305** (2013) 009]
- **Dark GUTs** [Babu, Seidl: Phys. Rev. **D70** (2004) 113014]
- **many EDs** [Ioannision, Valle: Phys. Rev. **D63** (2001) 073002]
- **MRISM** [Dev, Pilaftsis: Phys. Rev. **D87** (2013) 053007]
- **Exotic Loops** [Ma: Phys. Rev. **D80** (2009) 013013; Zhang *et al.*: JHEP **1310** (2013) 104; Borah, Adhikari: Phys. Lett. **B729** (2014) 143]
- **global symmetries** [Sayre, Wiesenfeldt, Willenbrock: Phys. Rev. **D72** (2005) 015001]
- **RPV SUSY** [Frank, Selbuz: Phys. Rev. **D88** (2013) 055003]
- **gravitational torsions** [Mavromatos, Pilaftsis: Phys. Rev. **D86** (2012) 124038]

There is a lot goin' on....

5. Conclusions and Outlook

5. Conclusions and Outlook

- let us look back for a second:

5. Conclusions and Outlook

- let us look back for a second:

Reactor anomaly

(eV & $\theta^2 \approx 10^{-1}$)

LSND/MiniBooNE

(eV & $\theta^2 \approx 10^{-3} - 10^{-2}$)

Gallium anomaly

(eV & $\theta^2 \approx 10^{-1}$)

Why are sterile
neutrinos useful?!

Dark Radiation

(eV & $\theta^2 \approx 10^{-5} - 10^{-2}$)

(Warm) Dark Matter

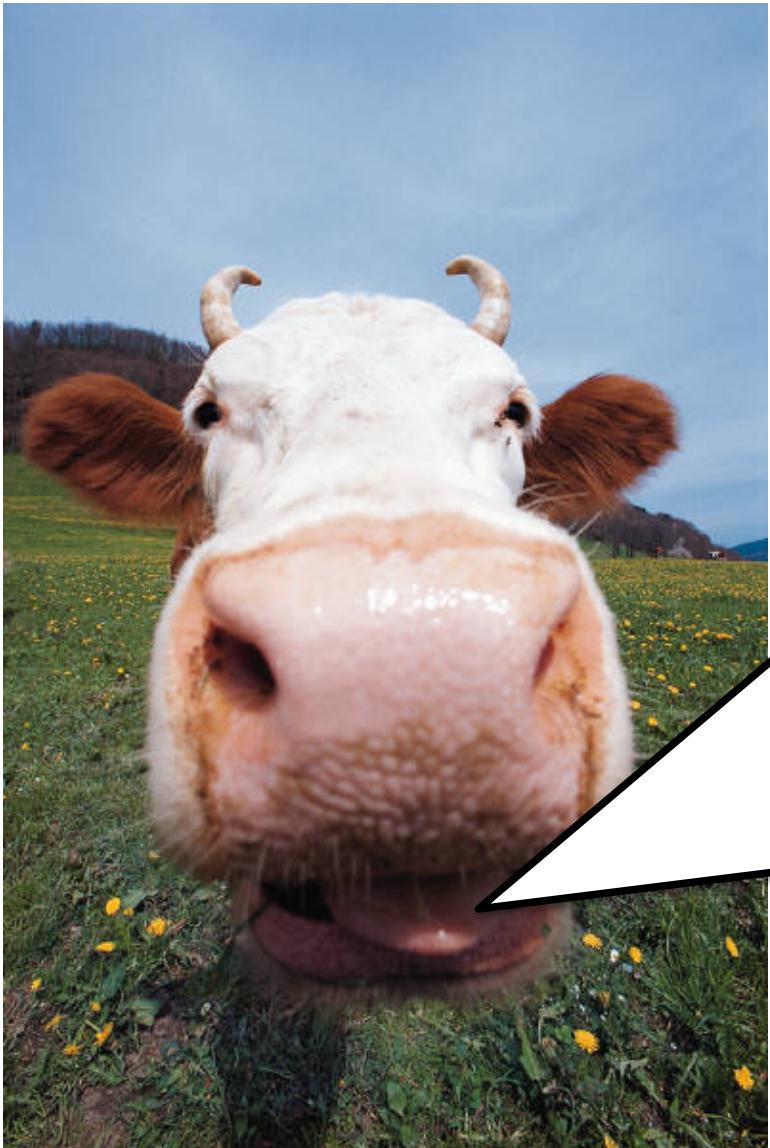
(keV & $\theta^2 \approx 10^{-8}$ or tinier)

5. Conclusions and Outlook

- let us look back for a second:
 - *All wrong?!? Probably not...*
→ ***probably some anomalies will survive***
 - *on top of that: no experiment/observation/calculation done solely to probe sterile neutrinos (except for MiniBooNE)*
→ ***human factor considerably reduced***

5. Conclusions and Outlook

- **Past Evidences:** STAY TUNED!!!
- **Model Building:** other requirements compared to light neutrino sector, but several proposals are out there
- **Dark Matter:** we should follow what happens to the line signal
- **Experiments:** NUCIFER, STEREO... *On the way!!!*



ADVERTISEMENT

International Journal of Modern Physics D
© World Scientific Publishing Company

KEV NEUTRINO MODEL BUILDING

ALEXANDER MERLE

*Physics and Astronomy, University of Southampton,
Highfield, Southampton, SO17 1BJ. United Kingdom*

A.Merle@soton.ac.uk

~~Received~~

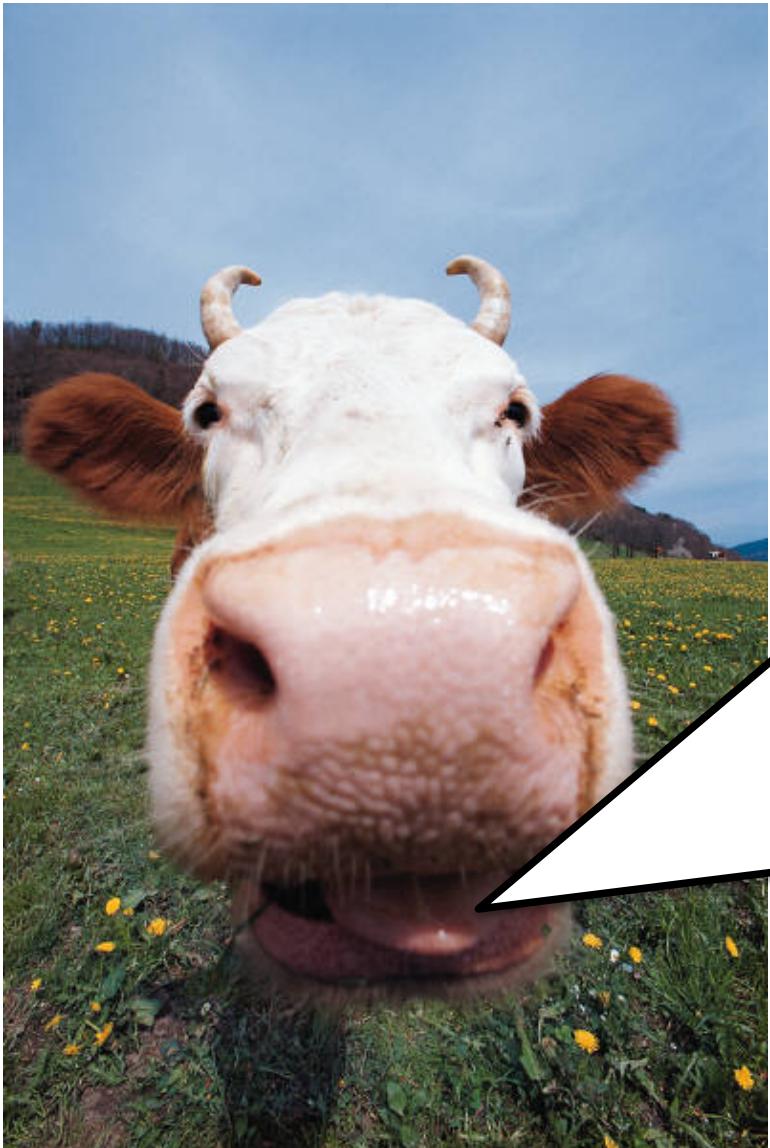
Please have a look if you got interested!

In latter candidate aspects, we explain the

got interested generic methods used. Model building are discussed, being pointing out some finer details where necessary and should enable a grad student or an interested colleague from astrophysics with some prior experience to start working on the field.

Keywords: Neutrinos; Dark Matter; Model Building.

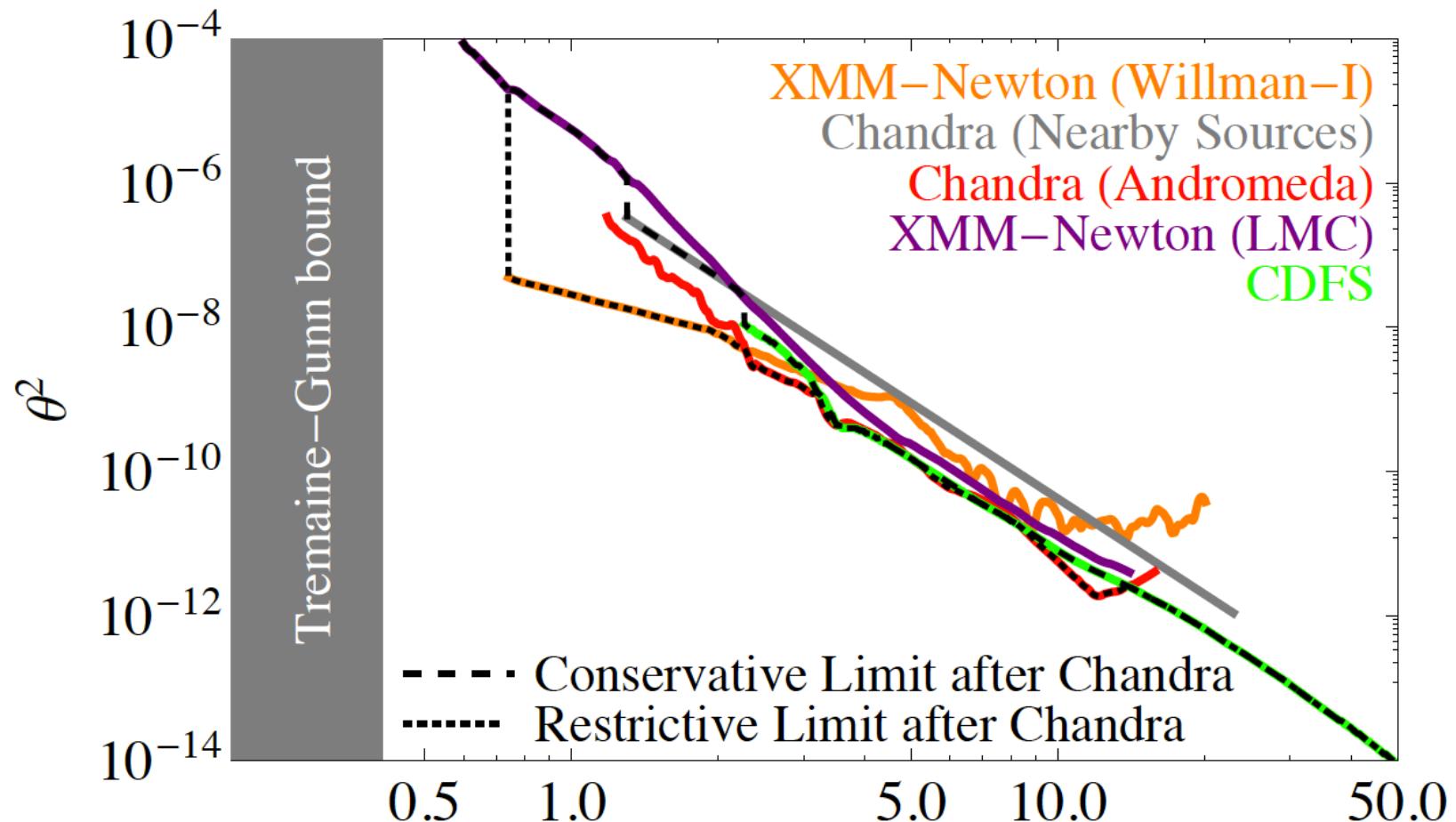
PACS numbers: 14.60.Pq; 14.60.St; 12.90.+b; 95.35.+d



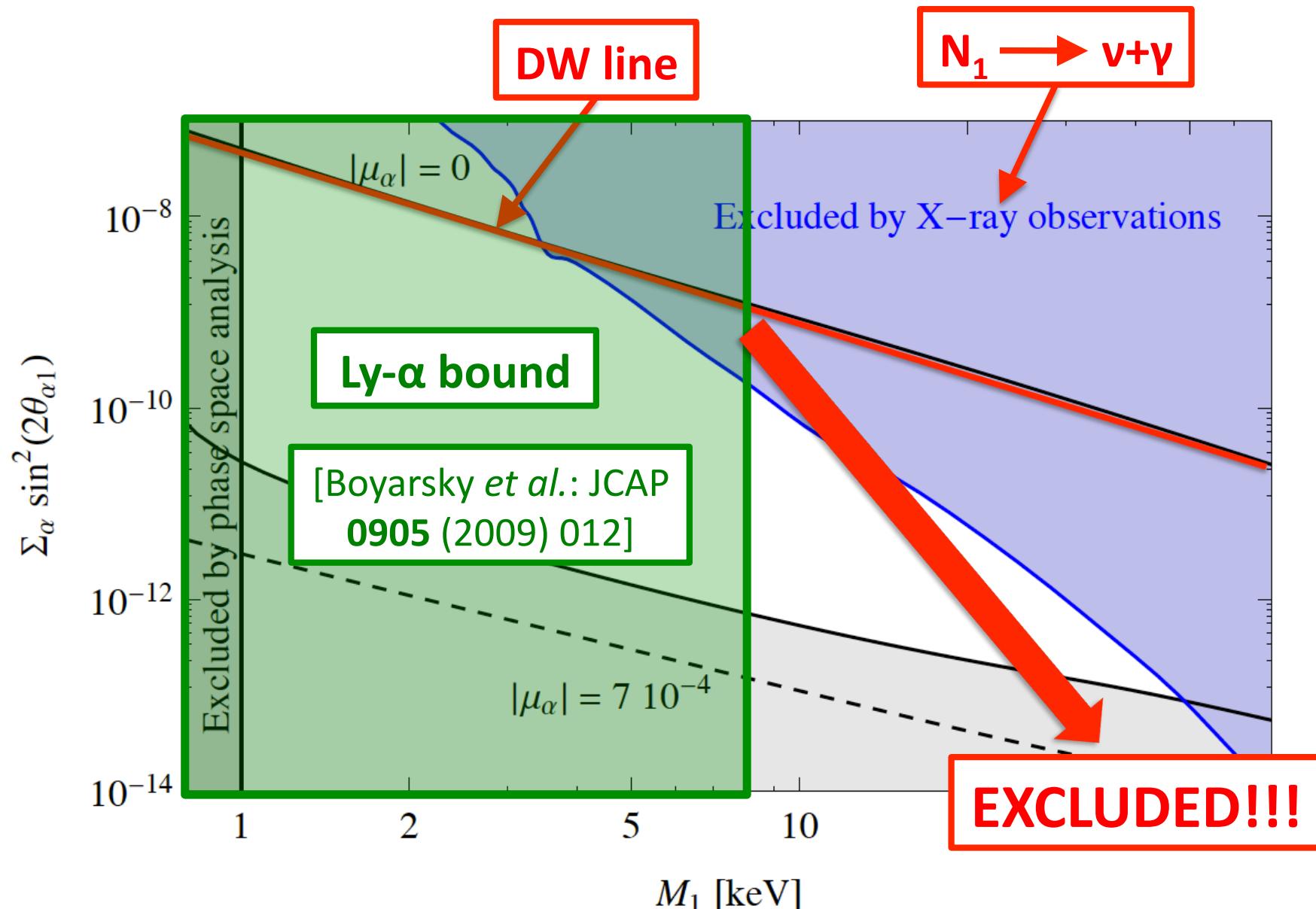
**BACK-UP
SLIDES**

Model building for keV neutrinos

- Differences to “ordinary” model building:
 - we need to respect the X-ray bound: $N_1 \rightarrow \nu\gamma$



Dodelson-Widrow exclusion



[Canetti *et al.*: Phys. Rev. D87 (2013) 093006]

Production by singlet scalar decay

- EXAMPLE: SCALAR “S” ONLY FEEBLY INTERACTING

[AM,Niro,Schmidt: JCAP 1403 (2014) 028]

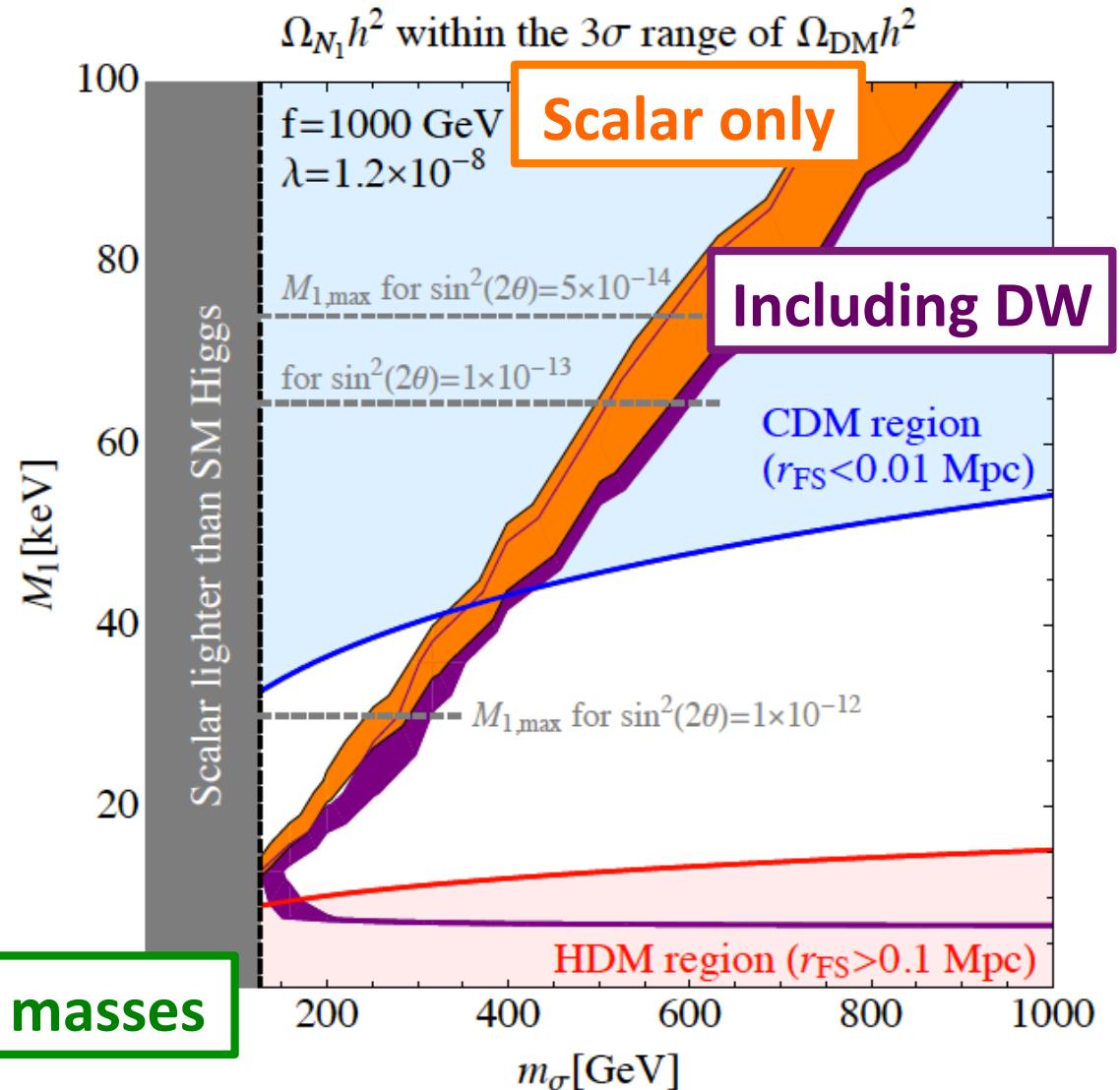
Free-streaming horizon

$$r_{\text{FS}} = \int_{t_{\text{in}}}^{t_0} \frac{\langle v(t) \rangle}{a(t)} dt$$



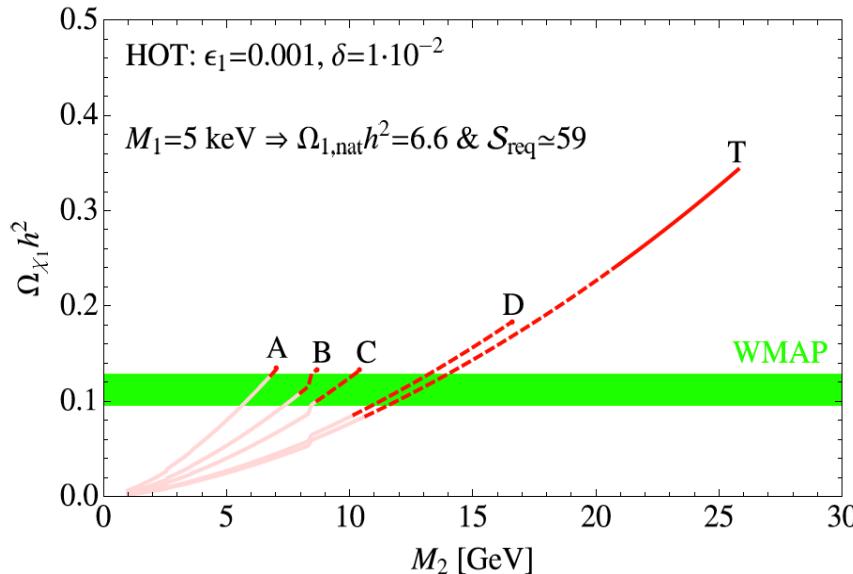
Decides about whether the keV sterile neutrinos are **HOT**, **WARM**, or **COLD**

→ all possible depending on masses



The generalisation: *keVins*

- the generalisation: keV inert fermions
 - setting: χ_1 at $O(\text{keV})$ to be Dark Matter, χ_2 at $O(\text{GeV})$
 - proposed mechanism: thermal overproduction of χ_1 plus subsequent dilution by entropy production
 - can produce the correct abundance:



- other production mechanisms and more or less model-independent studies possible

Model building for keV neutrinos

- Differences to “ordinary” model building:
 - seesaw mechanism for keV neutrinos:
 - guaranteed to work for models based on the split seesaw or Froggatt-Nielsen mechanisms
[Kusenko, Takahashi, Yanagida: Phys. Lett. **B693** (2010) 144]
[AM, Niro: JCAP **1107** (2011) 023]
 - all models that respect the X-ray bound have no problems with the seesaw mechanism
[AM: Phys. Rev. **D86** (2012) 121701(R)]

➔ *Actually okay in most of the cases!*

4. Example models

- $\mathcal{F}=L_e-L_\mu-L_\tau$ symmetry:

- original: [Petcov: Phys. Lett. **B110** (1982) 245]
 - 2 RH neutrinos: [Grimus,Lavoura: JHEP **0009** (2000) 007]
 - 3 RH neutrinos:

[Barbieri,Hall,Tucker-Smith,Strumia,Weiner: JHEP **9812** (1998) 017]

[Mohapatra: Phys. Rev. **D64** (2001) 091301]

- *application to keV sterile neutrinos:*

[Shaposhnikov: Nucl. Phys. **B763** (2007) 49]

[Lindner,**AM**,Niro: JCAP **1101** (2011) 034]

- general features:

- *symmetry* → patterns: (0,m,m) & (0,M,M)
 - *broken* → small mass, degeneracy lifted

4. Example models

- $\mathcal{F} = L_e - L_\mu - L_\tau$ symmetry:

- charge assignment under global $U(1)$ [or: Z_4]:

	L_{eL}	$L_{\mu L}$	$L_{\tau L}$	e_R	μ_R	τ_R	N_{1R}	N_{2R}	N_{3R}	ϕ	Δ
\mathcal{F}	1	-1	-1	1	-1	-1	1	-1	-1	0	0

- then, only symmetry preserving terms are allowed:

$$\mathcal{L}_{\text{mass}} = -\frac{1}{2} \overline{\Psi^C} \mathcal{M}_\nu \Psi + h.c.$$

with: $\Psi \equiv ((\nu_{eL})^C, (\nu_{\mu L})^C, (\nu_{\tau L})^C, N_{1R}, N_{2R}, N_{3R})^T$

→ mass matrix:

$$\mathcal{M}_\nu = \left(\begin{array}{ccc|ccc} 0 & m_L^{e\mu} & m_L^{e\tau} & m_D^{e1} & 0 & 0 \\ m_L^{e\mu} & 0 & 0 & 0 & m_D^{\mu 2} & m_D^{\mu 3} \\ m_L^{e\tau} & 0 & 0 & 0 & m_D^{\tau 2} & m_D^{\tau 3} \\ \hline m_D^{e1} & 0 & 0 & 0 & M_R^{12} & M_R^{13} \\ 0 & m_D^{\mu 2} & m_D^{\tau 2} & M_R^{12} & 0 & 0 \\ 0 & m_D^{\mu 3} & m_D^{\tau 3} & M_R^{13} & 0 & 0 \end{array} \right)$$

4. Example models

- **$\mathcal{F}=L_e-L_\mu-L_\tau$ symmetry:**
 - eigenvalues of \mathcal{M}_ν (with μ - τ symmetry):
 - light neutrinos: $(\lambda_+, \lambda_-, 0)$
 - heavy neutrinos: $(\Lambda_+, \Lambda_-, 0)$
 - with: $\lambda_{\pm} = \pm \sqrt{2} \left[m_L - \frac{m_D^2}{M_R} \right]$ $\Lambda_{\pm} = \pm \sqrt{2} M_R$
 - mass patterns:
 - light ν 's: $(0, \lambda_+, \lambda_-) \rightarrow$ okay up to degeneracy
 - heavy N's: $(0, \Lambda_+, \Lambda_-) \rightarrow 0 \ll M$, but still $0 \neq \text{keV}$
 - WAY OUT: broken symmetry
 - \rightarrow will remedy the above issues
 - \rightarrow important: no matter how the breaking is achieved, the results will always look similar

4. Example models

- $\mathcal{F} = L_e - L_\mu - L_\tau$ symmetry:
 - pragmatic: *soft breaking* [Lindner,AM,Niro: JCAP **1101** (2011) 034]
 - we assumed *small* breaking terms and worked out their consequences:

→ new mass matrix:

$$\left(\begin{array}{ccc|ccc} s_L^{ee} & m_L^{e\mu} & m_L^{e\tau} & m_D^{e1} & 0 & 0 \\ m_L^{e\mu} & s_L^{\mu\mu} & 0 & 0 & m_D^{\mu 2} & m_D^{\mu 3} \\ m_L^{e\tau} & 0 & s_L^{\tau\tau} & 0 & m_D^{\tau 2} & m_D^{\tau 3} \\ \hline m_D^{e1} & 0 & 0 & S_R^{11} & M_R^{12} & M_R^{13} \\ 0 & m_D^{\mu 2} & m_D^{\tau 2} & M_R^{12} & S_R^{22} & 0 \\ 0 & m_D^{\mu 3} & m_D^{\tau 3} & M_R^{13} & 0 & S_R^{33} \end{array} \right)$$

→ new eigenvalues: $\Lambda_s = S$, $\Lambda'_\pm = S \pm \sqrt{2}M_R$

$$\lambda_s = s \quad \lambda'_\pm = s \pm \sqrt{2} \left[m_L - \frac{m_D^2}{M_R} \right] + \frac{5m_D^2 S}{4M_R^2}$$

4. Example models

- $\mathcal{F}=L_e-L_\mu-L_\tau$ symmetry:

- pragmatic: *soft breaking* [Lindner,AM,Niro: JCAP **1101** (2011) 034]
- we assumed **small** breaking terms and worked out their consequences

natural assumption: like p-n isospin symmetry

→ new mass matrix:

$$\begin{pmatrix} s_L^e & m_L^\mu & m_L^\tau & m_D^\tau & 0 & 0 \\ m_L^{e\mu} & s_L^{\mu\mu} & 0 & 0 & m_D^{\mu 2} & m_D^{\mu 3} \\ m_L^{e\tau} & 0 & s_L^{\tau\tau} & 0 & m_D^{\tau 2} & m_D^{\tau 3} \\ \hline m_D^{e1} & 0 & 0 & S_R^{11} & M_R^{12} & M_R^{13} \\ 0 & m_D^{\mu 2} & m_D^{\tau 2} & M_R^{12} & S_R^{22} & 0 \\ 0 & m_D^{\mu 3} & m_D^{\tau 3} & M_R^{13} & 0 & S_R^{33} \end{pmatrix}$$

→ new eigenvalues: $\Lambda_s = S$, $\Lambda'_\pm = S \pm \sqrt{2}M_R$

$$\lambda_s = s \quad \lambda'_\pm = s \pm \sqrt{2} \left[m_L - \frac{m_D^2}{M_R} \right] + \frac{5m_D^2 S}{4M_R^2}$$

4. Example models

- $\mathcal{F}=L_e-L_\mu-L_\tau$ symmetry:
 - pragmatic: *soft breaking* [Lindner,AM,Niro: JCAP **1101** (2011) 034]
 - we assumed **small** breaking terms and worked out their consequences

natural assumption: like p-n isospin symmetry

→ new mass matrix:

s_L^e	m_L^μ	m_L^τ	m_D^τ	0	0
$m_L^{e\mu}$	$s_L^{\mu\mu}$	0	0	$m_D^{\mu 2}$	$m_D^{\mu 3}$
$m_L^{e\tau}$	0	$s_L^{\tau\tau}$	0	$m_D^{\tau 2}$	$m_D^{\tau 3}$
<hr/>	<hr/>	<hr/>	<hr/>	<hr/>	<hr/>
m_D^{e1}	0	0	S_R^{11}	M_R^{12}	M_R^{13}
0	$m_D^{\mu 2}$	$m_D^{\tau 2}$	M_R^{12}	S_R^{22}	0
0	$m_D^{\mu 3}$	$m_D^{\tau 3}$	M_R^{13}	0	S_R^{33}

keV neutrino

→ new eigenvalues: $\Lambda_s = S$ $\Lambda'_\pm = S \pm \sqrt{2}M_R$

$$\lambda_s = s \quad \lambda'_\pm = s \pm \sqrt{2} \left[m_L - \frac{m_D^2}{M_R} \right] + \frac{5m_D^2 S}{4M_R^2}$$

4. Example models

- **\mathcal{F} - L_e - L_μ - L_τ symmetry:**

- mixings also require soft breaking:

$$\mathcal{M}_l \mathcal{M}_l^\dagger \simeq \begin{pmatrix} m_e^2 + m_\mu^2 \lambda^2 & m_\mu^2 \lambda & 0 \\ m_\mu^2 \lambda & m_\mu^2 & 0 \\ 0 & 0 & m_\tau^2 \end{pmatrix}$$

$$\tan^2 \theta_{12} \simeq 1 - 2\sqrt{2}\lambda + 4\lambda^4 - 2\sqrt{2}\lambda^3 \rightarrow \theta_{12} \simeq 33.4^\circ$$

$$\lambda = \theta_{12} - \pi/4$$

$$|U_{e3}| \simeq \frac{\lambda}{\sqrt{2}} \rightarrow \theta_{13} \simeq 8^\circ,$$

$$\sin^2 2\theta_{23} \simeq 1 - 4\lambda^4 \rightarrow \theta_{23} \simeq 45^\circ.$$

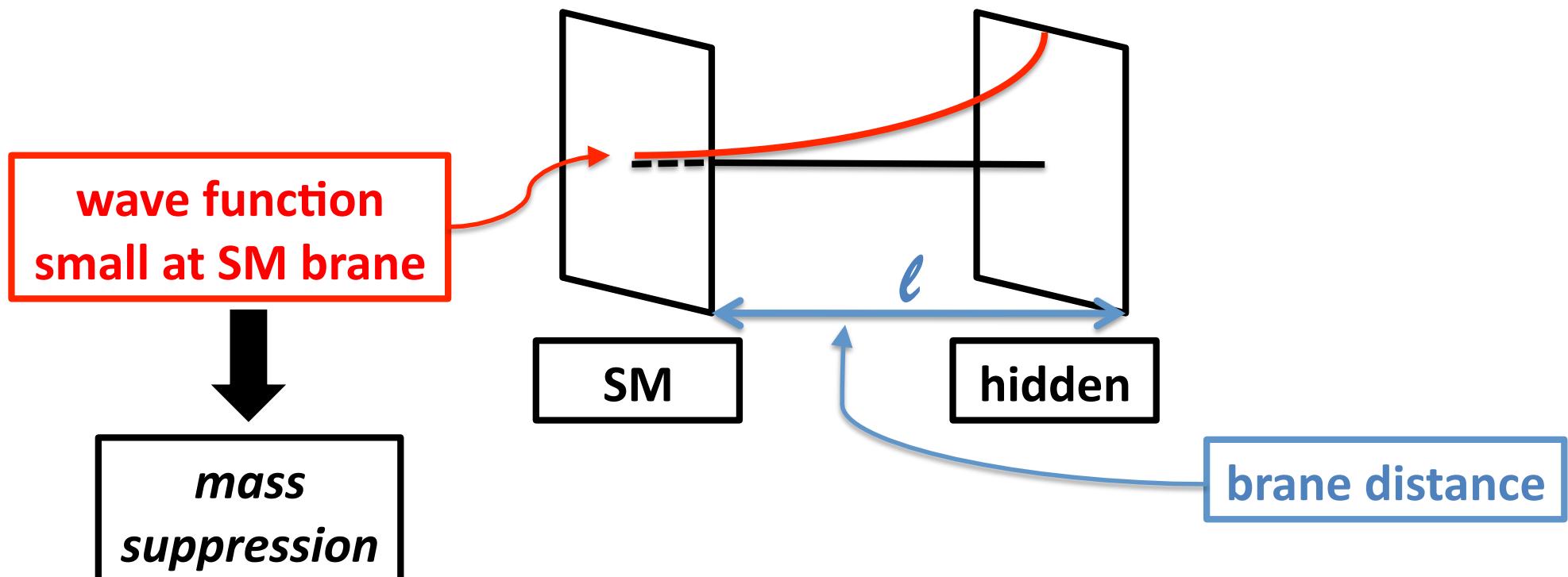
- prediction for the masses (under assumptions):

$$|m_1| = 0.0486 \text{ eV}, |m_2| = 0.0494 \text{ eV}, \text{ and } |m_3| = 0.0004 \text{ eV}$$

4. Example models

- Split seesaw mechanism:

- *idea*: brane (*NOT* “brain”!!!) splitting in extra dimensions → **mass suppression**



[Kusenko *et al.*: Phys. Lett. D693 (2010) 144]

4. Example models

- **Split seesaw mechanism:**

- starting point: 5D action

$$S = \int d^4x \int_0^l dy M_0 (i\bar{\Psi}\Gamma^A \partial_A \Psi - m\bar{\Psi}\Psi)$$

- Fourier expansion of the field:

$$\Psi_{L,R}(x^\mu, y) = \sum_n \psi_{L,R}^{(n)}(x^\mu) f_{L,R}^{(n)}(y)$$

→ equation of motion in the Extra Dimension:

$$(\pm \partial_y - m) f_{L,R}^{(n)}(y) = m_n f_{L,R}^{(n)}(y)$$

- solution (“**bulk profile**”) for the zero mode: $m_n=0$

$$f_{L,R}^{(0)}(y) = C e^{\mp my}$$

$$C = \sqrt{\frac{2m}{e^{2ml} - 1}} \frac{1}{\sqrt{M_0}}$$

4. Example models

- **Split seesaw mechanism:**

- for a more complicated action, this leads to strong mass suppression and hierarchy enhancement:

$$S = \int d^4x \int dy \left[M_0 \left(\overline{\Psi_{iR}^{(0)}} i\Gamma^A \partial_A \Psi_{iR}^{(0)} - m_i \overline{\Psi_{iR}^{(0)}} \Psi_{iR}^{(0)} \right) \right.$$

$$\left. - \delta(y) \left(\frac{\kappa_i}{2} v_{B-L} \overline{(\Psi_{iR}^{(0)})^c} \Psi_{iR}^{(0)} + \tilde{\lambda}_{i\alpha} \overline{\Psi_{iR}^{(0)}} L_\alpha H \right) \right]$$

- the bulk profile leads to suppressions of masses AND Yukawa couplings:

$$M_i = \kappa_i \frac{v_{B-L}}{M_0} \frac{2m_i}{e^{2m_il} - 1}$$

$$\lambda_{i\alpha} = \frac{\tilde{\lambda}_{i\alpha}}{\sqrt{M_0}} \sqrt{\frac{2m_i}{e^{2m_il} - 1}}$$

4. Example models

- **Split seesaw mechanism:**

- for a more complicated action, this leads to strong mass suppression and hierarchy enhancement:

$$S = \int d^4x \int dy \left[M_0 \left(\overline{\Psi_{iR}^{(0)}} i\Gamma^A \partial_A \Psi_{iR}^{(0)} - m_i \overline{\Psi_{iR}^{(0)}} \Psi_{iR}^{(0)} \right) \right.$$

5D mass of the sterile N_i's

$$\left. - \delta(y) \left(\frac{v_{B-L}}{2} (\overline{\Psi_{iR}^{(0)}})^c \Psi_{iR}^{(0)} + \tilde{\lambda}_{i\alpha} \overline{\Psi_{iR}^{(0)}} L_\alpha H \right) \right]$$

- the bulk profile leads to suppressions of masses AND Yukawa couplings:

$$M_i = \kappa_i \frac{v_{B-L}}{M_0} \frac{2m_i}{e^{2m_il} - 1}$$

$$\lambda_{i\alpha} = \frac{\tilde{\lambda}_{i\alpha}}{\sqrt{M_0}} \sqrt{\frac{2m_i}{e^{2m_il} - 1}}$$

4. Example models

- **Split seesaw mechanism:**

- for a more complicated action, this leads to strong mass suppression and hierarchy enhancement:

$$S = \int d^4x \int dy \left[M_0 \left(\overline{\Psi}_{iR}^{(0)} i\Gamma^A \partial_A \Psi_{iR}^{(0)} - m_i \overline{\Psi}_{iR}^{(0)} \Psi_{iR}^{(0)} \right) \right.$$

$e^{-2m_il} << 1 \text{ for } m_i l >> 1$

STRONG SUPPRESSION!!!

$$\left. - g(y) \left(\frac{v_{B-L}}{2} \overline{\Psi}_{iR}^{(0)} \Gamma^A \Psi_{iR}^{(0)} + \lambda_{i\alpha} \Psi_{iR}^{(0)} \Gamma^\alpha \Pi \right) \right]$$

- the bulk profile leads to suppressions of masses AND Yukawa couplings:

$$M_i = \kappa_i \frac{v_{B-L}}{M_0} \frac{2m_i}{e^{2m_il} - 1}$$

$$\lambda_{i\alpha} = \frac{\tilde{\lambda}_{i\alpha}}{\sqrt{M_0}} \sqrt{\frac{2m_i}{e^{2m_il} - 1}}$$

4. Example models

- **Split seesaw mechanism:**
 - in particular: exponential enhances hierarchies

$$M_i = \kappa_i \frac{v_{B-L}}{M_0} \frac{2m_i}{e^{2m_i l} - 1}$$

$m_3 < m_2 < m_1 \rightarrow M_3 \gg M_2 \gg M_1 !!!$

\rightarrow this mechanism is very well suited to generate strong mass hierachies!

- additional enhancement: $v_{B-L} \ll M_0$
(M_0 : fundamental Planck scale in 5D)
- bonus: seesaw guaranteed to work, due to conspiracy between the suppressions

4. Example models

- **Split seesaw mechanism:**

- **issue #1:** slight enhancement of active-sterile mixing

$$\theta_1 \propto M_1^{-1/2} \text{ instead of } \theta_1 \sim \frac{m_D}{M_R} \propto M_1^{-1}$$

→ not a very big problem

- **issue #2:** we do not have an explanation for having $m_1 > m_2 > m_3$ in the first place

→ can be cured by A_4 extension:

$$m_1 > m_2 = m_3 \rightarrow M_1 \ll M_2 = M_3$$

[Adulpravitchai,Takahashi: JHEP **1109** (2011) 127]

BUT: $\theta_{13}=0$, $\theta_{23}=\pi/4$, excluded by X-ray bound!

→ needs seesaw type II situation to work

4. Example models

- **Froggatt-Nielsen models:**

- rewrite:

$$\lambda_1^a \lambda_2^b \equiv \left(\frac{\langle \Theta_1 \rangle}{\Lambda} \right)^a \left(\frac{\langle \Theta_2 \rangle}{\Lambda} \right)^b = \lambda^{a+b} R^b$$

→ 3 real parameters: $\lambda = \frac{\langle \Theta_1 \rangle}{\Lambda}$, $R = \frac{\langle \Theta_1 \rangle}{\langle \Theta_2 \rangle} = R_0 e^{i\alpha_0}$

- two example scenarios: A(3,0,0) & B(4,1,0)

A(3, 0, 0) : $M_1 = M_0 \lambda^6 / 2R_0^2 \sqrt{1 + R_0^4 + 2R_0^2 \cos(2\alpha_0)}$

$M_2 = M_0$

$M_3 = M_0 (1 + \lambda^6 [1 + R_0^2 (3 \cos(2\alpha_0) + 3R_0^2 \cos(4\alpha_0) + R_0^4 \cos(6\alpha_0))])$

Quasi-Degeneracy

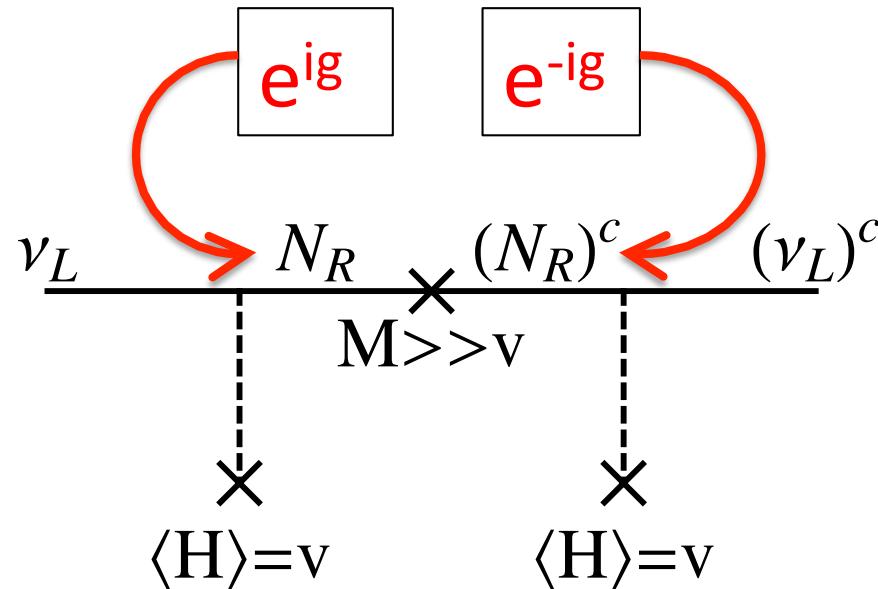
B(4, 1, 0) : $M_1 = M_0 \lambda^8 / 2R_0^4 \sqrt{1 + R_0^4 + 2R_0^2 \cos(2\alpha_0)}$

$M_2 = M_0 \lambda^2$

$M_3 = M_0 (1 + R_0^2 \lambda^2 \cos(2\alpha_0))$

4. Example models

- **Froggatt-Nielsen models:**
 - important point: seesaw guaranteed to work



- $U(1)_{FN}$ is a global $U(1)$, just like lepton number, which gets broken by the seesaw-diagram!
- the charges (g_1, g_2, g_3) drop out of the light neutrino mass matrix ✓

4. Example models

- **Froggatt-Nielsen models:**
 - **interesting to note:** FN not as arbitrary as it looks!
 - does not work with Left-Right symmetry
 - disfavours one production mechanism
 - favours SU(5) compared to SO(10)
 - reason: g_i unconstrained for singlet N_i
 - BUT: associated problems with p^+ decay...
 - Renormalization Group Running negligible
 - excludes bimaximal *neutrino mixing* (for the diagonalization of the light neutrino mass matrix)
 - disfavours democratic Yukawa couplings
 - no anomalies within SU(5)