

# Theory Perspective on Flavour

Ferruccio Feruglio  
Universita' di Padova

## Invisibles Workshop 2014

14-18 July 2014  
Institut de Cordeliers, Paris

# Approaches to the flavour puzzle

1  $\gamma$  should be deduced from first principles

most striking fact: nothing approaching a standard theory of  $\gamma$ , despite decades of experimental progress and theoretical efforts

2  $\gamma$  are due to chance

many variants

bottom-up: anarchy, FN models, fermions in ED, partial compositeness

top-down: fundamental theory with a landscape of ground states

observed  $\gamma$  are environmental and cannot be fully predicted



relative sizes of solar planetary orbits

assumptions

knowledge of statistical distribution of  $\gamma$  in the fundamental theory

the observed  $\gamma$  are typical

[any anthropic selection?]

relevant questions

how typical are the  $\gamma$  we observe?

which is the statistical distribution of  $\gamma$  in the fundamental theory?

fundamental theory



[symmetry and/or dynamical principle]



$\gamma$

# any empirical evidence for a symmetry from the quark sector?

$$G_f = U(1)_{FN}$$

[Froggatt, Nielsen 1979]

mass ratios and mixing angles are small, hierarchical parameters

$$\frac{m_u}{m_t} \ll \frac{m_c}{m_t} \ll 1 \quad \frac{m_d}{m_b} \ll \frac{m_s}{m_b} \ll 1 \quad |V_{ub}| \ll |V_{cb}| \ll |V_{us}| \equiv \lambda < 1$$

easily reproduced by  $G_f = U(1)_{FN}$

mass ratios and mixing angles are powers of a small SB parameter  $\lambda$

$U(1)_{FN}$  broken by

$$\lambda = \frac{\langle \varphi \rangle}{\Lambda_f} \approx 0.2$$

<i>flavon</i>	$Q_{FN}$
$\varphi$	-1

$$y_u = F_{U^c} Y_u F_Q$$

$$y_d = F_{D^c} Y_d F_Q$$

$$F_X = \begin{pmatrix} \lambda^{FN(X_1)} & 0 & 0 \\ 0 & \lambda^{FN(X_2)} & 0 \\ 0 & 0 & \lambda^{FN(X_3)} \end{pmatrix}$$

$$Y_{u,d} \approx O(1)$$

undetermined by  $U(1)_{FN}$

$FN(X_i)$  are  $U(1)_{FN}$  charges

( $X = Q, U^c, D^c$ )

# not a mere book-keeping

take  $\text{FN}(Q_1) > \text{FN}(Q_2) > \text{FN}(Q_3) \geq 0$

$$(V_{u,d})_{ij} \approx \frac{F_{Q_i}}{F_{Q_j}} < 1 \quad (i < j) \quad V_{CKM} = V_u^+ V_d$$

$$V_{ud} \approx V_{cs} \approx V_{tb} \approx O(1)$$

$$V_{ub} \approx V_{td} \approx V_{us} \times V_{cb}$$

[O.K. within a factor of 2]

independently from the specific charge choice

correct orders of magnitude of  $V_{ij}$   
reproduced by e.g.

$$\text{FN}(Q) = (3, 2, 0)$$

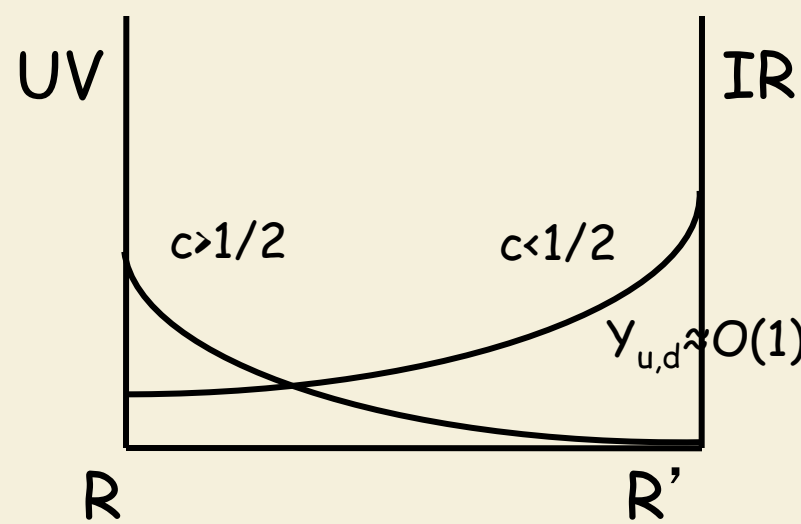
correct orders of magnitude of  
quark/charged lepton mass ratios  
[up to a couple of moderate tunings]  
reproduced by e.g.

$$\begin{aligned} \text{FN}(U^c) &= \text{FN}(E^c) = \text{FN}(Q) = (3, 2, 0) \\ \text{FN}(D^c) &= \text{FN}(L) = (2, 0, 0) \end{aligned}$$

is a symmetry really needed ?

$$y_u = F_{U^c} Y_u F_Q$$

$$y_d = F_{D^c} Y_d F_Q$$



split fermions in an Extra Dimension

$$F_{X_i} = \sqrt{\frac{2\mu_i}{1 - e^{-2\mu_i\rho}}}$$

ED	$\mu_i$	$\rho$
Flat $[0, \pi R]$	$M_i / \Lambda$	$\Lambda \pi R$
Warped $[R, R']$	$1/2 - M_i R$	$\log R'/R$

no symmetry:  
hierarchy produced by geometry

$M_i$  = bulk mass of fermion  $X_i$   
 $Y_{u,d} = O(1)$  Yukawa couplings between bulk fermions and a Higgs localized at one brane

partial compositeness

$$F_{X_i} = \Delta_i M_i^{-1}$$

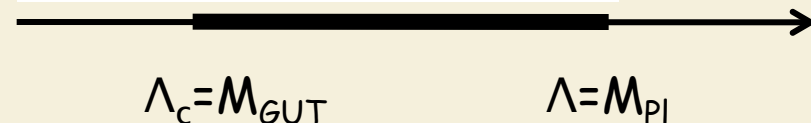
$M_i$  = masses of composite fermions  
 $\Delta_i$  = elementary-composite mixing  
 $Y_{u,d} = O(1)$  Yukawa couplings in composite sector

chiral multiplets  $X_i$  of the MSSM coupled to a superconformal sector

[Nelson-Strassler 0006251]

$$F_{X_i} = \left(\frac{\Lambda_c}{\Lambda}\right)^{\frac{\gamma_i}{2}} < 1$$

$\gamma_i$  anomalous dimension of  $X_i$



# can be extended to the lepton sector

no evidence for big hierarchies in neutrino mixing angles  
clear hierarchy only in the charged lepton masses



$$F_{E_1^c} \ll F_{E_2^c} \ll F_{E_3^c}$$
$$F_{L_1} \approx F_{L_2} \approx F_{L_3}$$

[viable both for Majorana or Dirac neutrinos, here focus on Majorana]

## 1 an extreme possibility

$$F_{L_1} = F_{L_2} = F_{L_3}$$

**Anarchy**

[Hall, Murayama, Weiner 1999  
De Gouvea, Murayama 1204.1249]

$$m_\nu \propto \begin{pmatrix} O(1) & O(1) & O(1) \\ O(1) & O(1) & O(1) \\ O(1) & O(1) & O(1) \end{pmatrix}$$

mixing angles  
and mass ratios  
from random  $O(1)$   
quantities

$$|U_{PMNS}| \approx \begin{pmatrix} 0.8 & 0.5 & 0.2 \\ 0.4 & 0.6 & 0.6 \\ 0.4 & 0.6 & 0.8 \end{pmatrix}$$

consistent with data

$\vartheta_{13} \approx 0.15$  rad and the hint for non maximal  $\vartheta_{23}$  (from global fits)  
have strengthened the case for anarchy

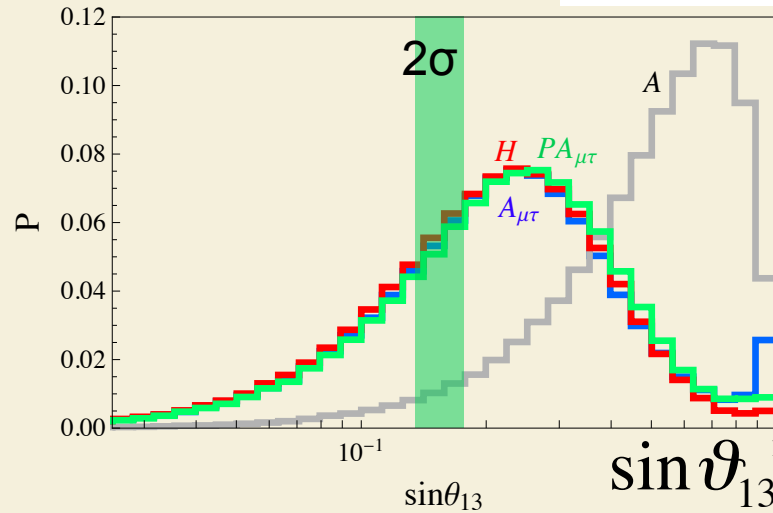
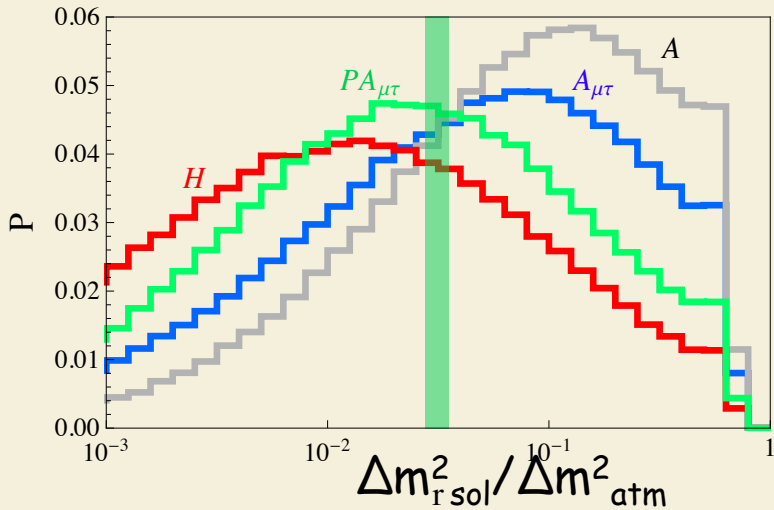
## 2 with many variants...

- variants of Anarchy e.g. in  $U(1)_{FN}$  models, quarks and leptons treated on equal foot
- compatible with  $SU(5)$  unification

[Buchmuller, Domcke, Schmitz, 1111.387;  
Altarelli, F, Masina, Merlo 1207.0587;  
Bergstrom, Meloni, Merlo, 1403.4528]

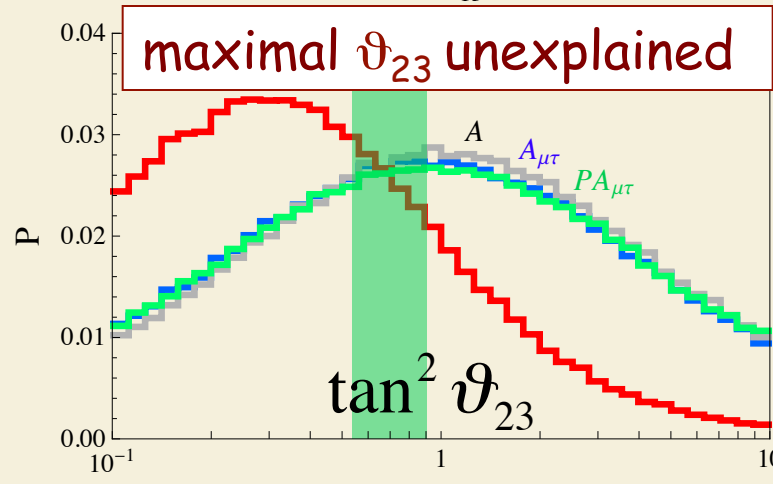
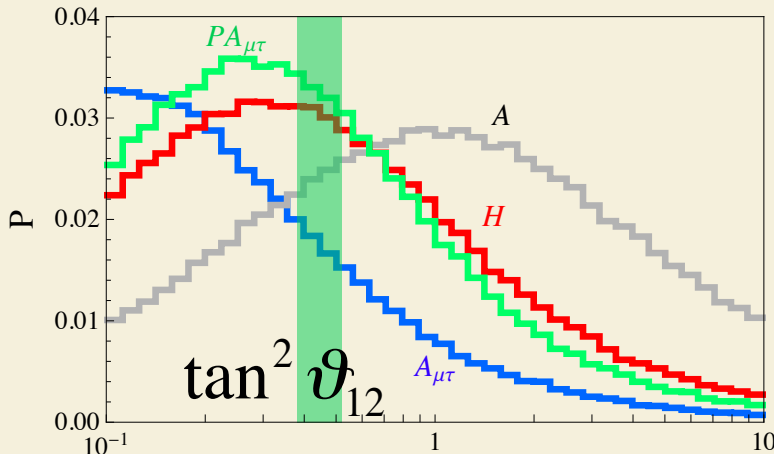
$$F(L_i) = \lambda^{FN(L_i)}$$

	$FN(L)$	$\lambda$
$A$	(0,0,0)	
$A_{\mu\tau}$	(1,0,0)	0.25
$PA_{\mu\tau}$	(2,0,0)	0.35
$H$	(2,1,0)	0.45



$$\sin^2 \vartheta_{13} \approx \frac{\Delta m_{12}^2}{\Delta m_{13}^2}$$

NH favoured



difficult to go beyond order-of-magnitude predictions

# flavor puzzle made simpler in $SU(5)$ ?

$$\bar{5} = (l, d^c) \quad 10 = (q, u^c, e^c) \quad 1 = \nu^c$$

$$5_H = (D, T) \quad \text{Higgs}$$

$$y_d = y_e^T$$

not a bad first-order approximations  
only corrections of  $O(1)$  needed  
remedies well-known

$$L_Y = -y_u^{ij} 10_i 10_j 5_H - y_d^{ij} \bar{5}_i 10_j \bar{5}_H \\ - y_\nu^{ij} 1_i \bar{5}_j 5_H - \frac{1}{2} M_{ij} 1_i 1_j + h.c.$$

only three matrices  $F_{10}, F_{\bar{5}}, F_1$

$$y_\nu = F_1 Y_\nu F_{\bar{5}} \quad M = F_1 \mu F_1$$

$$y_u = F_{10} Y_u F_{10} \quad y_d = F_{\bar{5}} Y_d F_{10} \quad y_e = F_{10} Y_e^T F_{\bar{5}}$$

$$m_\nu \propto F_{\bar{5}} Y_\nu^T \mu^{-1} Y_\nu F_{\bar{5}}$$

adopt anarchy ansatz

$F_1$  dependence  
cancels in  $m_\nu$

$$F_{\bar{5}} \approx \text{diag}(1, 1, 1)$$

large  $l$  mixing corresponds  
to a large  $d^c$  mixing:  
unobservable in weak int.  
of quarks

hierarchy mostly due to  $F_{10}$

$$m_u : m_c : m_t \approx m_d^2 : m_s^2 : m_b^2 \approx m_e^2 : m_\mu^2 : m_\tau^2$$

$$F_{10} \approx \begin{pmatrix} \lambda^{3+4} & 0 & 0 \\ 0 & \lambda^2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

realistic picture  
after correcting  
 $y_d = y_e^T$



# and $SO(10)$ ?

previous picture seems incompatible with  $SO(10)$  at first sight

a whole SM generation in a 16

$$L_Y = -y_{10}^{ij} 16_i 16_j 10_H + h.c. \quad \text{only one matrix}$$

$$F_{16}$$

$$y_{10} = F_{16} Y_{10} F_{16}$$

$O(1)$

affects all members of the 16 in the same way: no distinction between quarks and leptons

adding more Yukawa couplings does not help, if  $Y$  are all  $O(1)$

$$L_Y = -16_i \left[ y_{10}^{ij} 10_H + y_{120}^{ij} 120_H + y_{126}^{ij} 126_H \right] 16_j + h.c.$$

$$\frac{m_{d_i}}{m_{d_j}} \approx \frac{m_{u_i}}{m_{u_j}} \approx \frac{m_{e_i}}{m_{e_j}}$$

# extension to $SO(10)$ made possible by Kitano-Li

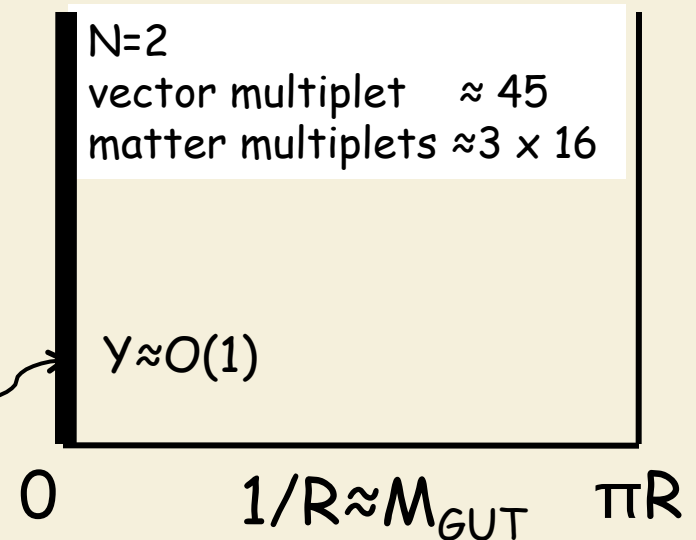
[Phys.Rev. D67 (2003) 116004]

SUSY  $SO(10)$  model in (flat) 5D

5D N=1 SUSY is  
4D N=2 SUSY

5D =  $S^1/Z_2$   
breaks N=2 SUSY  
down to N=1

Higgs and Yukawa  
interactions on  $y=0$   
brane



$O^{\text{th}}$  order approximation

$$F_{16_i} = \sqrt{\frac{2\mu_i}{1 - e^{-2\mu_i\rho}}}$$

$$\mu_i = \frac{M_i}{\Lambda}$$

$$\rho = \Lambda\pi R$$

bulk mass of  $16_i$  in units of  $\Lambda$

length of ED in units  $1/\Lambda$

zero-mode wave-function of  $16_i$  evaluated at  $y=0$

no distinction between quarks and leptons within the same family

# 1<sup>st</sup> key ingredient

bulk gauge interaction between  $16_i$  and vector multiplet = universal Yukawa interaction

$$16_i^c \left[ M_i - \sqrt{2} g_5 45_\Phi \right] 16_i$$



N=1 chiral multiplet being part of N=2 vector multiplet

$$SO(10) \rightarrow SU(5) \times U(1)_X \rightarrow SU(5) \rightarrow G_{SM}$$

$$\langle 45_\Phi \rangle = v_\Phi^{3/2}$$

$$\mu_i \rightarrow \mu_i^r = \mu_i - Q_X^r k$$

$r$	10	$\bar{5}$	1
$Q_X^r$	-1	3	-5

$$k = \sqrt{2} g_5 \frac{v_\Phi^{3/2}}{\Lambda}$$

we are back to the SU(5) case

$$y_u = F_{10} Y_u F_{10}$$

$$y_d = F_{\bar{5}} Y_d F_{10}$$

$$y_e = F_{10} Y_e F_{\bar{5}}$$

$$m_\nu \propto F_{\bar{5}} Y_\nu^T \mu^{-1} Y_\nu F_{\bar{5}}$$

$$r = (10, \bar{5}, 1) \quad F_{r_i} = \sqrt{\frac{2\mu_i^r}{1 - e^{-2\mu_i^r \rho}}}$$

profiles F controlled by only 4 free parameters:  $\mu_i$  and k

## 2<sup>nd</sup> key ingredient

multiple contributions to  $O(1)$  Yukawa couplings at  $y=0$  such that  $Y_u, Y_d, Y_e, Y_\nu$  can be treated as independent [otherwise incorrect mass relations]

$$\frac{\delta(y)}{\Lambda} \left[ Y_{ij} 16_i 16_j 10_H + Y'_{ij} 16_i 16_j 10_H \frac{45_H}{\Lambda} + \dots \right]$$

more details  
in poster session  
by Denise Vicino

we have reconsidered the KL model [F, Patel, Vicino 1407.2913]

- 1 modified Yukawa couplings at  $y=0$  such that  $Y_u, Y_d, Y_e, Y_\nu$  arise from operators of the same dimensionality

$$\frac{\delta(y)}{\Lambda} \left[ Y_{10}^{ij} 16_i 16_j 10_H + Y_{120}^{ij} 16_i 16_j 120_H + Y_{126}^{ij} 16_i 16_j 126_H + \dots \right]$$

cut-off scale  $\Lambda$   
can be  $> M_{GUT}$

27  $O(1)$  free parameters  $\longleftrightarrow$  chance

- 2 explicit solution to the doublet-triplet splitting problem through the missing partner mechanism

light sector:  $10_H$  and  $120_H$  [3 pairs of D and 3 pairs of T]

heavy sector:  $126_H, 126_H, 45_H, 120_H$  [2 pairs of D and 3 pairs of T]

8  $O(1)$  free parameters define the light Higgs combinations

### 3 fit to an idealized set of 17 data

Observable	Normal ordering		Inverted ordering	
	Fitted value	Pull	Fitted value	Pull
$y_t$	0.51	0	0.54	1.00
$y_b$	0.37	0	0.37	0
$y_\tau$	0.51	0	0.47	-1.00
$m_u/m_c$	0.0027	0	0.0031	0.67
$m_d/m_s$	0.051	0	0.045	-0.86
$m_e/m_\mu$	0.0048	0	0.0048	0
$m_c/m_t$	0.0023	0	0.0023	0
$m_s/m_b$	0.016	0	0.015	-0.50
$m_\mu/m_\tau$	0.050	0	0.049	-0.50
$ V_{us} $	0.227	0	0.227	0
$ V_{cb} $	0.037	0	0.038	1.00
$ V_{ub} $	0.0033	0	0.0030	-0.50
$J_{CP}$	0.000023	0	0.000021	-0.51
$\Delta_S/\Delta_A$	0.0309	0	0.0320	0.73
$\sin^2 \theta_{12}$	0.308	0	0.309	0.06
$\sin^2 \theta_{23}$	0.425	0	0.435	-0.07
$\sin^2 \theta_{13}$	0.0234	0	0.0237	-0.10
$\chi^2_{\min}$	$\approx 0$		$\approx 5.75$	
	Predicted value		Predicted value	
$m_{\nu_{\text{lightest}}}$ [meV]	0.08		2.15	
$ m_{\beta\beta} $ [meV]	1.63		30.4	
$\sin \delta_{CP}^l$	0.265		0.510	
$M_{N_1}$ [GeV]	$3.85 \times 10^6$		$1.13 \times 10^4$	
$M_{N_2}$ [GeV]	$9.31 \times 10^7$		$3.06 \times 10^6$	
$M_{N_3}$ [GeV]	$2.19 \times 10^{14}$		$2.02 \times 10^{13}$	
$v_R$ [GeV]	$0.05 \times 10^{16}$		$0.18 \times 10^{16}$	

TABLE II. Results from numerical fit corresponding to minimized  $\chi^2$  for normal (NO) and inverted ordering (IO) in neutrino masses. The fit is carried out for the GUT scale extrapolated data given in Table I for  $\tan\beta = 50$ . The input parameters are collected in Appendix.

35  $O(1)$  + 4 profile parameters

agreement not completely trivial: indeed only large  $\tan\beta$  is allowed [here  $\tan\beta=50$ ]

both NO and IO neutrino spectrum lead to a decent fit

# are the 27 $O(1)$ Yukawa coupling fine-tuned?

we reiterate the fit by first generating a random sample of Yukawas  
 for every such fit 27 parameters are now fixed:  $8+4=12$  free parameters  
 $\nu=(17-12)=5$  dof

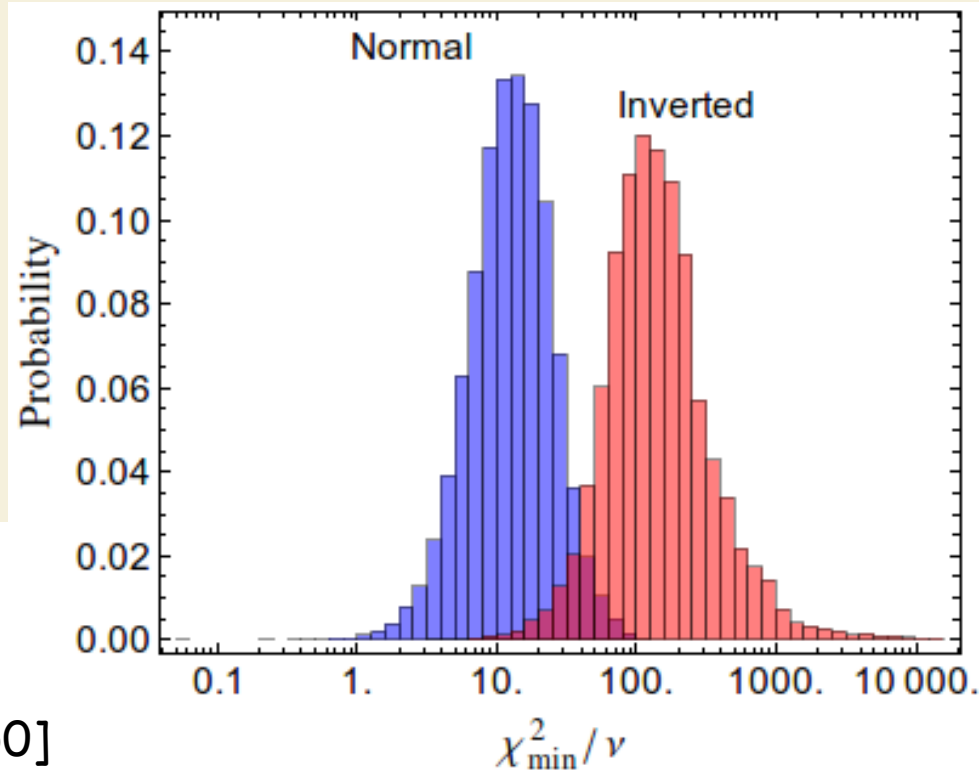
flat distributions

$$|Y_{ij}| \in [0.5, 1.5]$$

$$\arg(Y_{ij}) \in [0, 2\pi]$$

or

$$Y_{ij} \in (1+i, -1+i, -1-i, 1-i)$$



IO is fine-tuned:  
no anarchy

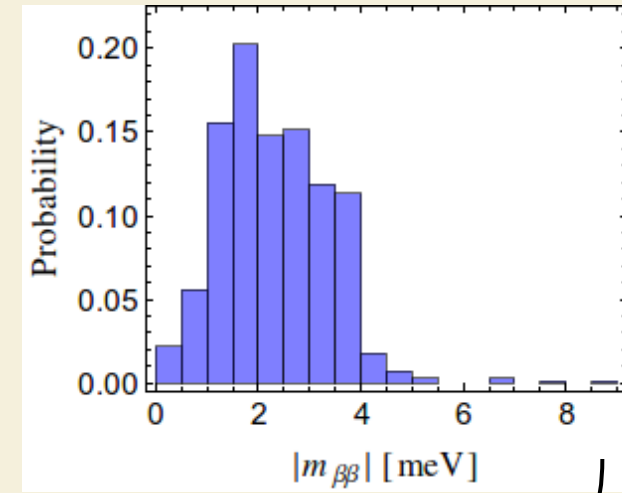
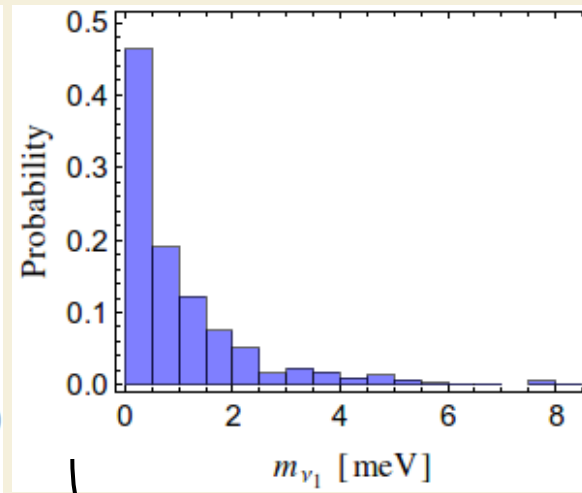
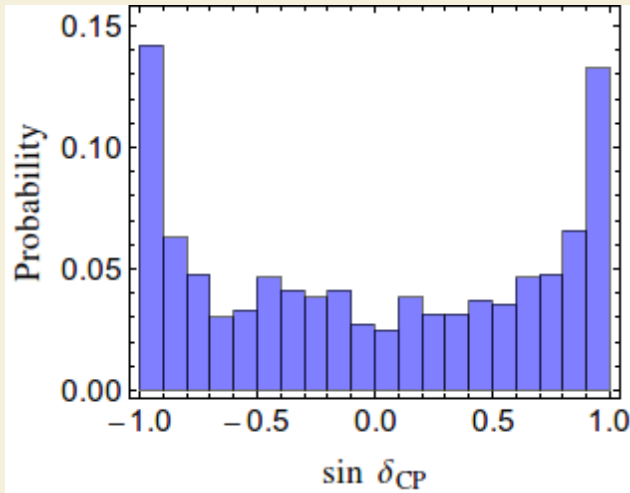
for NO fine-tuning  
acceptable

[ $\tan\beta=50$ ]

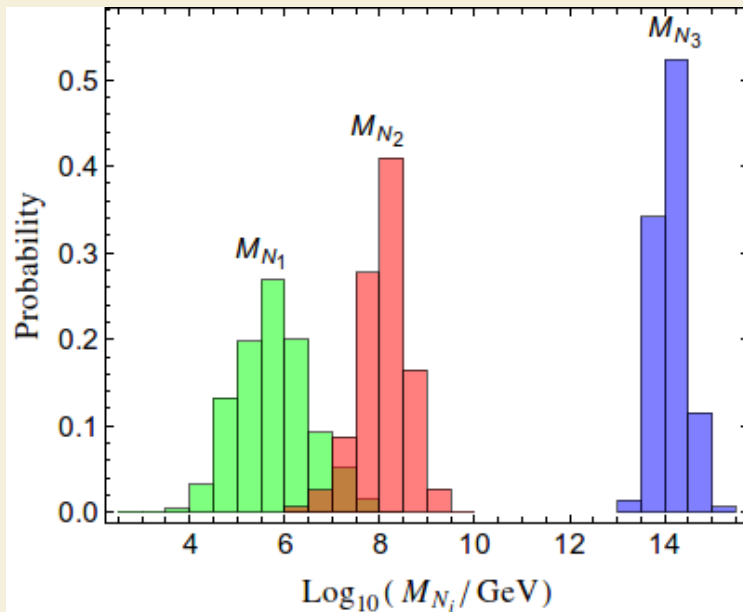
$p$ -value	0.50	0.10	<b>0.05</b>	0.001
$\chi_{\min}^2/\nu$ (for $\nu = 5$ )	$\leq 0.87$	$\leq 1.85$	$\leq$ <b>2.21</b>	$\leq 4.10$
successful cases (NO)	0.1%	0.7%	<b>1.2%</b>	5.6%
successful cases (IO)	$< 10^{-3}\%$	$< 10^{-3}\%$	<b><math>10^{-3}\%</math></b>	0.01%

TABLE III. The fraction of successful events obtained for different  $p$ -values from random samples of  $O(1)$  Yukawa couplings in case of normal and inverted ordering in the neutrino masses.

# our predictions for NO [ $\tan\beta=50$ ]



beyond the reach of current experiments:  
the model can be easily falsified



RH neutrinos too-light:  
leptogenesis from Lightest or NTL  
neutrino decay does not work

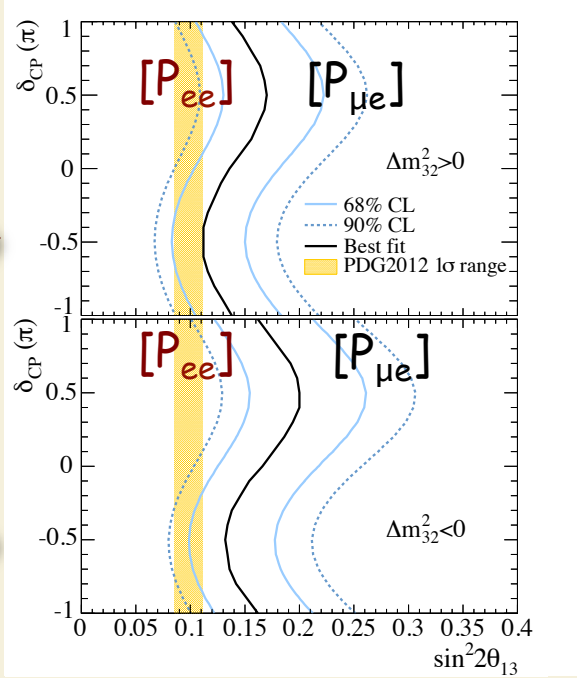
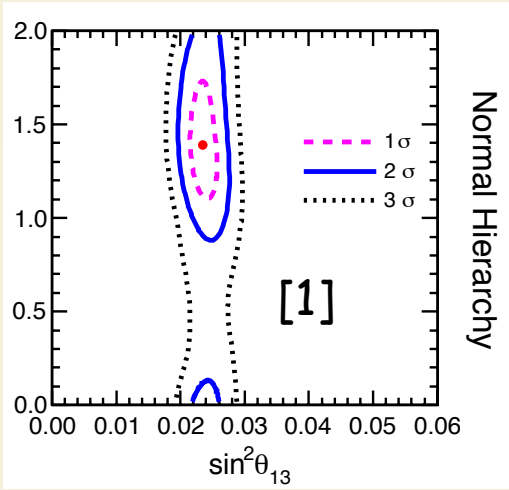
special features of data should be considered accidental in previous picture

1 today most precise single determination of  $\vartheta_{23}$  is from T2K ( $P_{\mu\mu}$ ) [1403.1532]

$$\sin^2 \vartheta_{23} = \begin{cases} 0.514^{+0.055}_{-0.056} & \text{(NH)} \\ 0.511^{+0.055}_{-0.055} & \text{(IH)} \end{cases}$$

well compatible with  $\vartheta_{23}$  maximal

2  $\delta_{CP} = -\pi/2$  ?



[T2K: 1311.4750 and 1311.4114]

global fit:  
*G.-Garcia, Maltoni, Salvado, Schwetz* 1209.3023  
 [1] *Capozzi, Fogli, Lisi, Marrone, Montanino, Palazzo* 1312.2878  
*Forero, Tortola, Valle* 1405.7540

both  $\vartheta_{23}$  maximal and  $\delta_{CP} = -\pi/2$  can be explained by flavour symmetries [-> next talks]



# Conclusions

flavour symmetries are a useful tool in our quest of the origin of  $\mathcal{Y}$  but no compelling and unique picture have emerged so far.  
Present data can be described within widely different frameworks [despite the constant, impressive progress on the experimental side]

simple schemes with a minimal amount of structure can well reproduce the main features of  $\mathcal{Y}$  in both quark and lepton sectors also in a GUT framework

main drawbacks: -- no precise questions/no precision tests allowed  
[e.g. maximal  $\vartheta_{23}$  unexplained]

-- more structure needed to suppress FCNC and CPV if there is new physics at the TeV scale

some special features [ $\vartheta_{23}$  maximal,  $\delta_{CP} = -\pi/2$ ,  $U_{PMNS} \approx TB, BM, \dots$ ]  
can survive experimental refinements and guide us in the search of first principles

back up slides

Observables	$\tan \beta = 10$	$\tan \beta = 50$
$y_t$	$0.48 \pm 0.02$	$0.51 \pm 0.03$
$y_b$	$0.051 \pm 0.002$	$0.37 \pm 0.02$
$y_\tau$	$0.070 \pm 0.003$	$0.51 \pm 0.04$
$m_u/m_c$	$0.0027 \pm 0.0006$	$0.0027 \pm 0.0006$
$m_d/m_s$	$0.051 \pm 0.007$	$0.051 \pm 0.007$
$m_e/m_\mu$	$0.0048 \pm 0.0002$	$0.0048 \pm 0.0002$
$m_c/m_t$	$0.0025 \pm 0.0002$	$0.0023 \pm 0.0002$
$m_s/m_b$	$0.019 \pm 0.002$	$0.016 \pm 0.002$
$m_\mu/m_\tau$	$0.059 \pm 0.002$	$0.050 \pm 0.002$
$ V_{us} $	$0.227 \pm 0.001$	
$ V_{cb} $	$0.037 \pm 0.001$	
$ V_{ub} $	$0.0033 \pm 0.0006$	
$J_{CP}$	$0.000023 \pm 0.000004$	
$\Delta_S/10^{-5} \text{ eV}^2$	$7.54 \pm 0.26$ (NO or IO)	
$\Delta_A/10^{-3} \text{ eV}^2$	$2.44 \pm 0.08$ (NO)	$2.40 \pm 0.07$ (IO)
$\sin^2 \theta_{12}$	$0.308 \pm 0.017$ (NO or IO)	
$\sin^2 \theta_{23}$	$0.425 \pm 0.029$ (NO)	$0.437 \pm 0.029$ (IO)
$\sin^2 \theta_{13}$	$0.0234 \pm 0.0022$ (NO)	$0.0239 \pm 0.0021$ (IO)

TABLE I. The GUT scale values of the charged fermion masses and quark mixing parameters from [30] that we use in our analysis. The lepton mixing angles and solar and atmospheric mass differences are taken from a global fit analysis [34] ignoring the running effects. NO (IO) stands for the normal (inverted) ordering in the neutrino masses.

$$F_{16_i} = \sqrt{\frac{2\mu_i}{1 - e^{-2\mu_i\rho}}} = \begin{cases} \sqrt{2\mu_i} & (\mu_i > 0 \quad \mu_i\rho \geq 1) \\ \sqrt{2|\mu_i|} e^{-|\mu_i|\rho} & (\mu_i < 0 \quad |\mu_i|\rho \geq 1) \\ 1/\sqrt{\rho} & (|\mu_i|\rho < 1) \end{cases}$$

# constraints from lepton flavour violation

take the limit  $m_\nu = 0$   
 if MFV applied, we would expect no LFV [ $y_e$  diagonal]



in our setup, in general  $F_{E^c}, F_L, Y_e$  do not commute  
 [not even when  $F_L$  is universal]  
 LFV expected at some level

dominant LFV dipole operator

$$L_{dip} = \frac{e}{\Lambda_{NP}^2} E^c (\sigma_{\mu\nu} F^{\mu\nu}) \underbrace{(F_{E^c} Y_e Y_e^+ Y_e F_L)}_{\substack{\text{not diagonal} \\ \text{when } y_e = F_{E^c} Y_e F_L \text{ diagonal}}} (H^+ L)$$

## Explicit computation in RS

[Agashe, Blechman, Petriello 0606021  
 Csaki, Grossman, Tanedo, Tsai 1004.2037]

$$BR(\mu \rightarrow e\gamma) < 5.7 \times 10^{-13}$$

$$M_{KK} > O(10) \text{ TeV}$$

comparable bounds from  $e$  EDM

[Keren-Zur, Lodone, Nardecchia, Pappadopulo, Rattazzi, Vecchi, 1205.5803]



$F_L$  universality is not enough

a sufficient condition for the absence of LFV:

$$F_{E^c}, Y_e, F_L$$

diagonal in the same basis

for instance:

$$F_L \propto 1$$

$$F_{E^c} \propto Y_e Y_e^+$$

[M.C. Chen and Yu, 08042503  
 Perez, Randall 0805.4652]

# anything special from data, requiring a symmetry?

1  $\vartheta_{23}$  maximal ?

2  $\delta_{CP} = -\pi/2$  ?

3  $U_{PMNS}$  close to TB (BM,...) ?

3 examples from a longer list...

1 today most precise single determination of  $\vartheta_{23}$  is from T2K ( $P_{\mu\mu}$ )

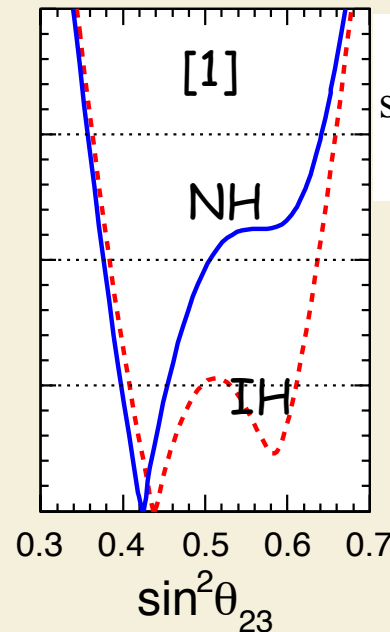
$$[1403.1532] \quad \sin^2 \vartheta_{23} = \begin{cases} 0.514^{+0.055}_{-0.056} & \text{(NH)} \\ 0.511^{+0.055}_{-0.055} & \text{(IH)} \end{cases}$$

well compatible with  $\vartheta_{23}$  maximal

global fits hint at  $\vartheta_{23}$  non-maximal  
main effect: interplay between  
SBL reactor experiments ( $P_{ee}$ ) and  
LBL experiments searching ( $P_{\mu e}$ )

$$P_{ee} = 1 - \sin^2 2\vartheta_{13} \sin^2 \frac{\Delta m_{32}^2 L}{4E} + \dots$$

$$P_{\mu e} = \sin^2 \vartheta_{23} \sin^2 2\vartheta_{13} \sin^2 \frac{\Delta m_{32}^2 L}{4E} + \dots$$



$$[2] \quad \sin^2 \vartheta_{23} = \begin{cases} 0.567^{+0.032}_{-0.128} & \text{(NH)} \\ 0.573^{+0.025}_{-0.043} & \text{(IH)} \end{cases}$$

global fit:  
[1] Capozzi, Fogli, Lisi, Marrone,  
Montanino, Palazzo 1312.2878  
[2] Forero, Tortola, Valle  
1405.7540

a small change of  $P_{ee}$  and/or  $P_{\mu e}$  within about  $1\sigma$  can bring back  $\vartheta_{23}$  to maximal

difficult to improve  $\vartheta_{23}$  from  $P_{\mu\mu}$

$$\delta\vartheta_{23} \approx \sqrt{\delta P_{\mu\mu}} / 2$$

$$\delta P_{\mu\mu} \approx 0.01$$



$$\delta\vartheta_{23} \approx 0.05 \text{ rad } (2.9^\circ)$$

$\vartheta_{23}$  nearly maximal would be a crucial piece of information

$\vartheta_{23}$  cannot be made maximal by RGE evolution  
[barring tuning of b.c. and/or threshold corrections]

when a flavour symmetry is present,  $\vartheta_{23}$  is determined entirely by breaking effects [no maximal  $\vartheta_{23}$  from an exact symmetry]

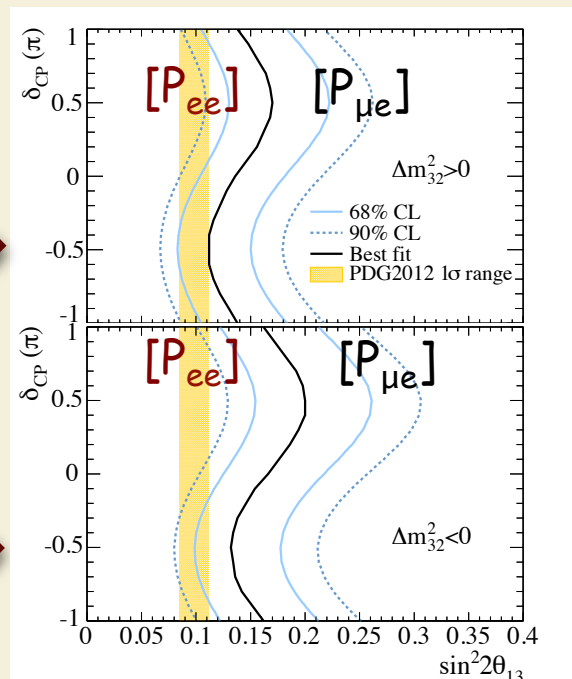
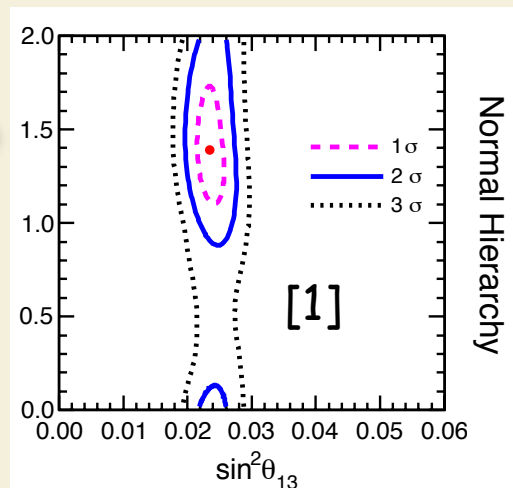
broken abelian symmetries do not work  
[not a theorem but no counterexamples]



we are left with broken non-abelian symmetries

2

$$\delta_{CP} = -\pi/2 ?$$



[T2K: 1311.4750 and 1311.4114]

3

 $U_{PMNS}$  close to TB (BM,...) ?

discrete flavor symmetries showed very efficient to reproduce  $U_{TB}$ ,  $U_{BM}, \dots$

indirect: symmetries of  $m_\nu$  and  $(m_e + m_e)$  have no direct relation to  $G_f$

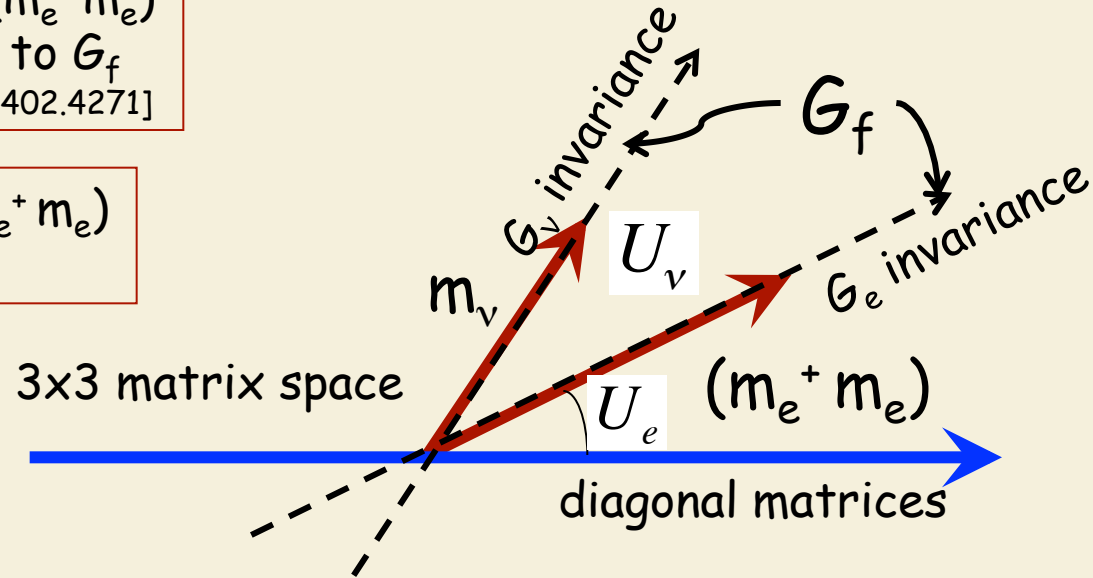
[see King, Merle, Morisi, Shimizu and Tanimoto 1402.4271]

direct: symmetries of  $m_\nu$  and  $(m_e + m_e)$  are subgroups of  $G_f$

4 predictions

$$\vartheta_{12}^0 \quad \vartheta_{23}^0 \quad \vartheta_{13}^0 \quad \delta^0 \pmod{\pi}$$

$$U_{PMNS} = U_{PMNS}^0 + \text{corrections}$$



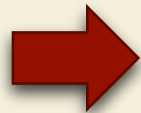
Majorana neutrinos imply  $G_\nu \leq Z_2 \times Z_2$

smallest group leading to TB:  $S_4 \approx (A_4 + \text{accidental symmetry})$

neutrino masses fitted, not predicted.

expectation for  $U_{PMNS}^0 = U_{TB}$

$$\begin{cases} \vartheta_{13}^0 = 0 \\ \vartheta_{23}^0 = \frac{\pi}{4} \end{cases}$$



$$\begin{cases} \vartheta_{13} = O(\text{few degrees}) \\ \vartheta_{23} = \text{close to } \frac{\pi}{4} \end{cases}$$

not to spoil the agreement with  $\vartheta_{12}$

wrong!



# 1 add large corrections $O(\vartheta_{13}) \approx 0.2$

- predictability is lost since in general correction terms are many
- new dangerous sources of FC/CPV if NP is at the TeV scale

# 2 relax symmetry requirements

[Hernandez, Smirnov 1204.0445]

$G_e$  as before

$$G_\nu = Z_2$$

2 predictions:

2 combinations of

$$\vartheta_{12}^0 \quad \vartheta_{23}^0 \quad \vartheta_{13}^0 \quad \delta_{CP}$$

two deformations of TB, called Trimaximal [TM] mixing

TM<sub>1</sub>

$$U^0 = U_{TB} \times \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha & e^{i\delta} \sin \alpha \\ 0 & -e^{-i\delta} \sin \alpha & \cos \alpha \end{pmatrix}$$

TM<sub>2</sub>

$$U^0 = U_{TB} \times \begin{pmatrix} \cos \alpha & 0 & e^{i\delta} \sin \alpha \\ 0 & 1 & 0 \\ -e^{-i\delta} \sin \alpha & 0 & \cos \alpha \end{pmatrix}$$

leads to testable sum rules

$$\sin^2 \vartheta_{12} = \frac{1}{3} - \frac{2}{3} \sin^2 \vartheta_{13} + O(\sin^4 \vartheta_{13})$$

$$\sin^2 \vartheta_{12} = \frac{1}{3} + \frac{1}{3} \sin^2 \vartheta_{13} + O(\sin^4 \vartheta_{13})$$

$$\sin^2 \vartheta_{23} = \frac{1}{2} - \sqrt{2} \sin \vartheta_{13} \cos \delta_{CP} + O(\sin^2 \vartheta_{13})$$

$$\sin^2 \vartheta_{23} = \frac{1}{2} + \frac{1}{\sqrt{2}} \sin \vartheta_{13} \cos \delta_{CP} + O(\sin^2 \vartheta_{13})$$

deviation from TB is linear in  $\alpha$  for  $\sin^2\theta_{23}$ , whereas is quadratic for  $\sin^2\theta_{12}$ , the best measured angle

sum rules can be tested by measuring  $\delta_{CP}$  and improving on  $\sin^2\theta_{23}$

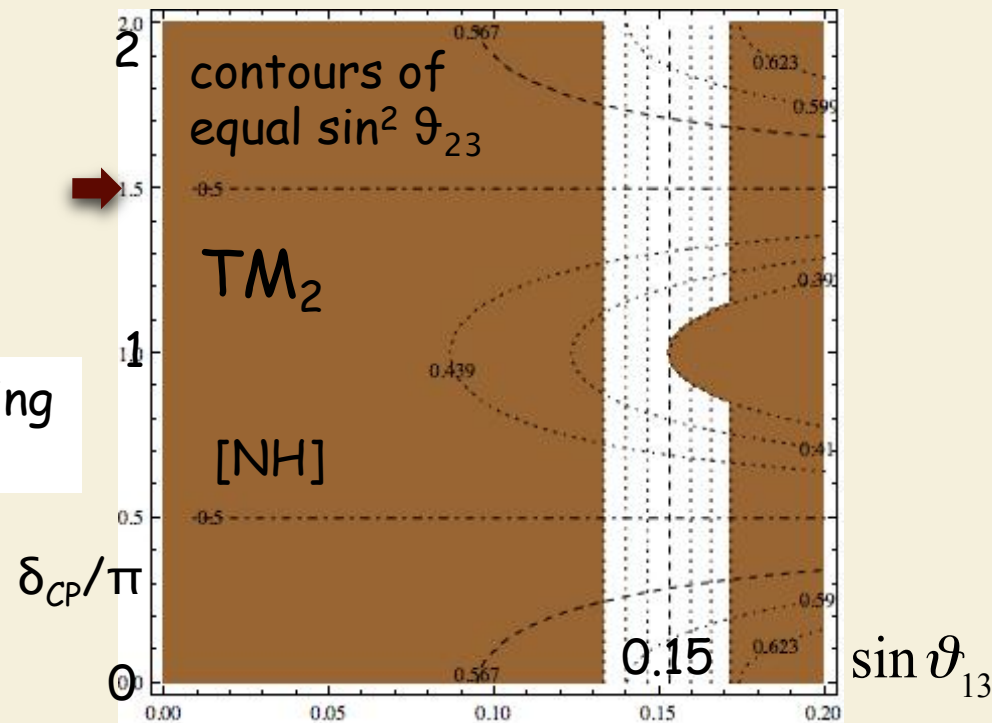
### 3 change discrete group $G_f$

- solutions exist  
special forms of  $TM_2$

$G_f$	$\Delta(96)$	$\Delta(384)$	$\Delta(600)$
$\alpha$	$\pm\pi/12$	$\pm\pi/24$	$\pm\pi/15$
$\sin^2\vartheta_{13}^0$	0.045	0.011	0.029

$\delta^0 = 0, \pi$  (no CP violation) and  $\alpha$  "quantized" by group theory

complete classification of  $|U_{PMNS}|$  from **any** finite group available now!



$$U^0 = U_{TB} \times \begin{pmatrix} \cos\alpha & 0 & e^{i\delta} \sin\alpha \\ 0 & 1 & 0 \\ -e^{-i\delta} \sin\alpha & 0 & \cos\alpha \end{pmatrix}$$

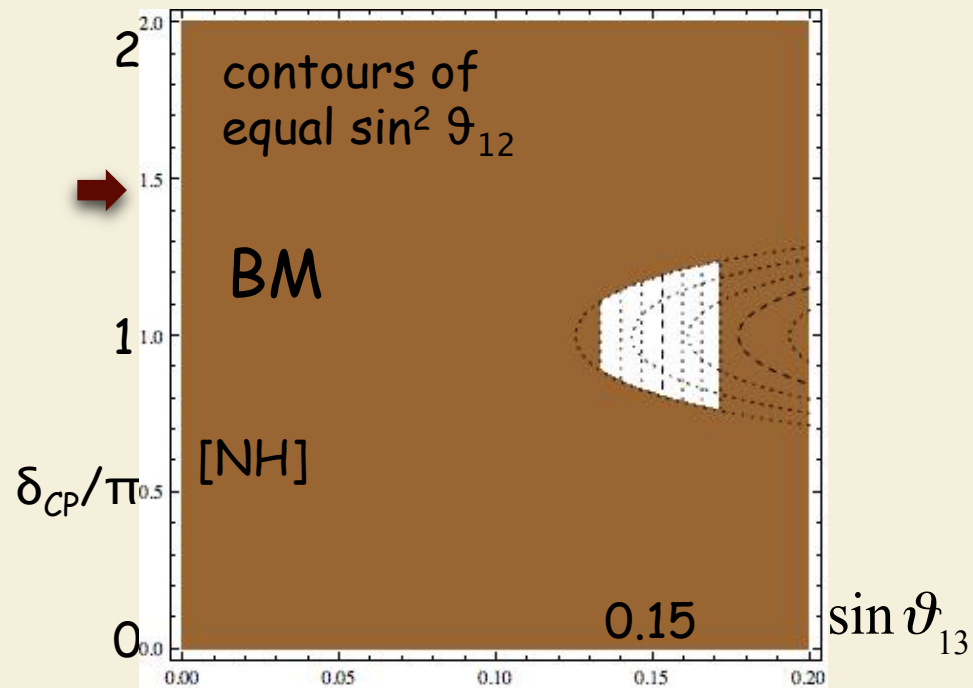
F.F., C. Hagedorn, R. de A. Torroop  
 hep-ph/1107.3486 and hep-ph/1112.1340  
 Lam 1208.5527 and 1301.1736  
 Holthausen1, Lim and Lindner 1212.2411  
 Neder, King, Stuart 1305.3200  
 Hagedorn, Meroni, Vitale 1307.5308  
 [Fonseca, Grimus 1405.3678]

#### 4 change LO pattern

$$U_{PMNS}^0 = U_{BM}$$

corrected by  $U_{e_{12}}$

$$\sin^2 \vartheta_{12} = \frac{1}{2} + \sin \vartheta_{13} \cos \delta_{CP} + O(\sin^2 \vartheta_{13})$$



#### 5 include CP in the SB pattern

$$G_{CP} = G_f \rtimes CP$$

[F. F. C. Hagedorn and R. Ziegler 1211.5560, 1303.7178  
Ding, King, Luhn, Stuart 1303.6180  
Ding, King, Stuart 1307.4212]

$$G_e$$

$$G_\nu = Z_2 \times CP$$

mixing angles and CP violating phases

$$(\vartheta_{12}^0, \vartheta_{23}^0, \vartheta_{13}^0, \delta^0, \alpha^0, \beta^0)$$

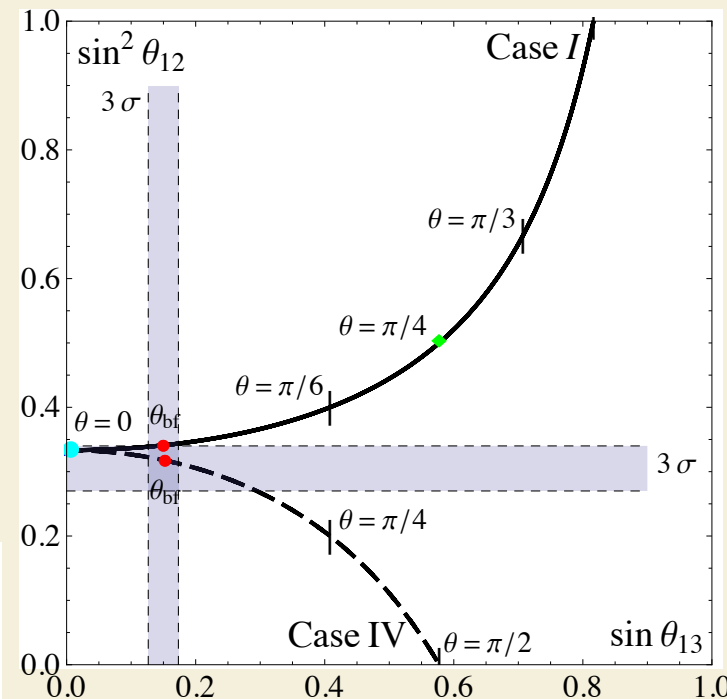
predicted in terms of a single real parameter  $0 \leq \vartheta \leq \pi$

2 examples with  $G_f = S_4$   $G_e = Z_3$

$$\sin^2 \vartheta_{23}^0 = \frac{1}{2} \quad |\sin \delta^0| = 1$$

$$\sin \alpha^0 = 0$$

$$\sin \beta^0 = 0$$



back up slides

# $\theta_{23}$ maximal from some flavour symmetries ?

a no-go theorem

[F. 2004]

$\vartheta_{23} = \pi/4$  can never arise in the limit of an **exact realistic symmetry**

charged lepton mass matrix:

$$m_l = m_l^0 + \delta m_l^0$$

symmetric limit

symmetry breaking effects:  
vanishing when flavour symmetry F is **exact**

realistic symmetry:

(1)  $|\delta m_l^0| < |m_l^0|$

(2)  $m_l^0$  has rank  $\leq 1$



$$m_l^0 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & m_\tau \end{pmatrix}$$

$\vartheta_{12}^e$  undetermined

$$U_{PMNS} = U_e^+ U_\nu$$

[omitting phases]

$$\tan \vartheta_{23}^0 = \tan \vartheta_{23}^\nu \cos \vartheta_{12}^e + \left( \frac{\tan \vartheta_{13}^\nu}{\cos \vartheta_{23}^\nu} \right) \sin \vartheta_{12}^e$$

undetermined

$$\vartheta_{23} = \frac{\pi}{4}$$

determined entirely by breaking effects  
(different, in general, for  $\nu$  and  $e$  sectors)

# 2011/2012 breakthrough

- LBL experiments searching for  $\nu_\mu \rightarrow \nu_e$  conversion
- SBL reactor experiments searching for anti- $\nu_e$  disappearance

[see Fogli's talk]

	Lisi [NeuTel 2013]	[1209.3023] [G-Garcia, Maltoni, Salvado, Schwetz]
$\sin^2 \vartheta_{13}$	$0.0241^{+0.0025}_{-0.0025}$ (NO) $0.0244^{+0.0023}_{-0.0025}$ (IO)	$0.0227^{+0.0023}_{-0.0024}$
$\sin^2 \vartheta_{23}$	$0.386^{+0.024}_{-0.021}$ (NO) $0.392^{+0.039}_{-0.022}$ (IO)	$0.413^{+0.037}_{-0.025} \oplus 0.594^{+0.021}_{-0.022}$



10 $\sigma$  away from 0

impact on flavor symmetry (part 3)

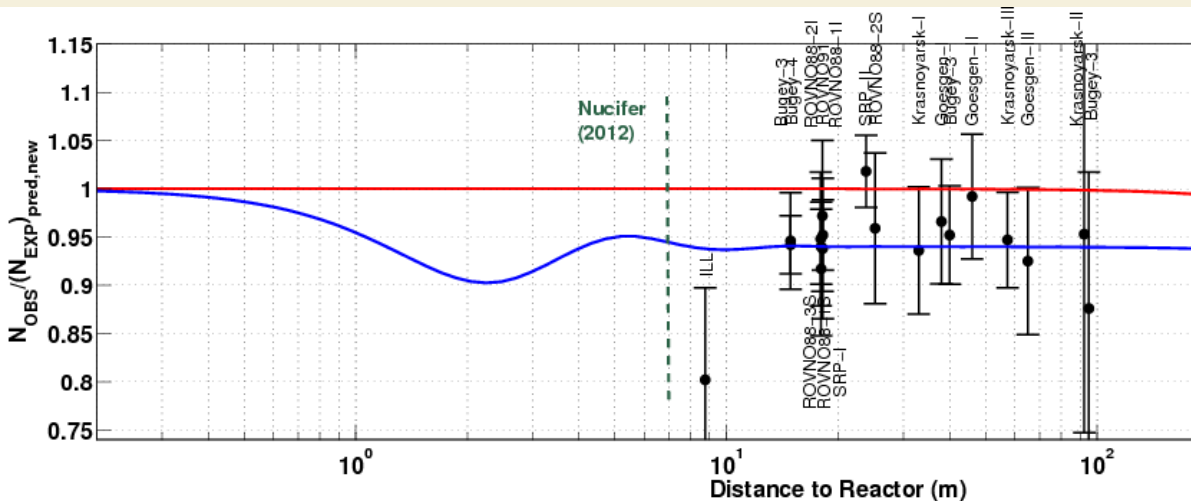


hint for non maximal  $\vartheta_{23}$

## sterile neutrinos coming back

### 1 reactor anomaly (anti- $\nu_e$ disappearance)

re-evaluation of reactor anti- $\nu_e$  flux: new estimate 3.5% higher than old one



$$(\Phi_{\text{exp}} - \Phi_{\text{th}}) / \Phi_{\text{th}} \approx -6\%$$

[th. uncertainty?]

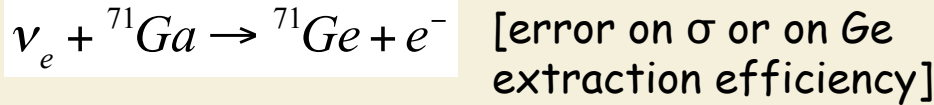
very SBL  $L \leq 100$  m

$$\vartheta_{es} \approx 0.2$$

$$\Delta m^2 \approx m_s^2 \geq 1 \text{ eV}^2$$

supported by the **Gallium anomaly**

$\nu_e$  flux measured from high intensity radioactive sources in Gallex, Sage exp



## most recent cosmological limits

[depending on assumed cosmological model, data set included,...]

relativistic degrees of freedom at recombination epoch

$$N_{\text{eff}} = 3.30 \pm 0.27$$

[Planck, WMAP, BAO, high multiple CMB data]

fully thermalized non relativistic  $\nu$

$$N_{\text{eff}} < 3.80 \quad (95\% \text{ CL})$$

$$m_s < 0.42 \text{ eV} \quad (95\% \text{ CL})$$

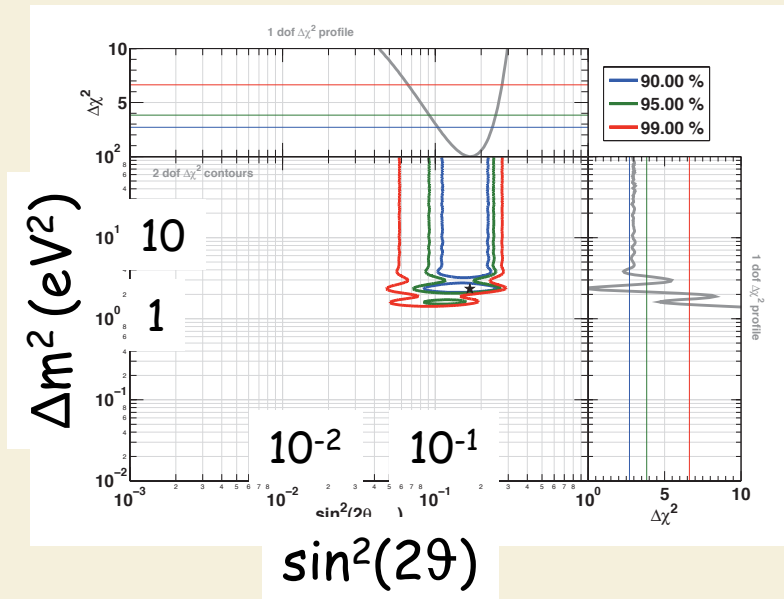
## 2 long-standing claim

evidence for  $\nu_\mu \rightarrow \nu_e$  appearance in accelerator experiments

exp		$E(\text{MeV})$	$L(\text{m})$
<i>LSND</i>	$\bar{\nu}_\mu \rightarrow \bar{\nu}_e$	$10 \div 50$	30
<i>MiniBoone</i>	$\nu_\mu \rightarrow \nu_e$	$300 \div 3000$	541
	$\bar{\nu}_\mu \rightarrow \bar{\nu}_e$		

$3.8\sigma$

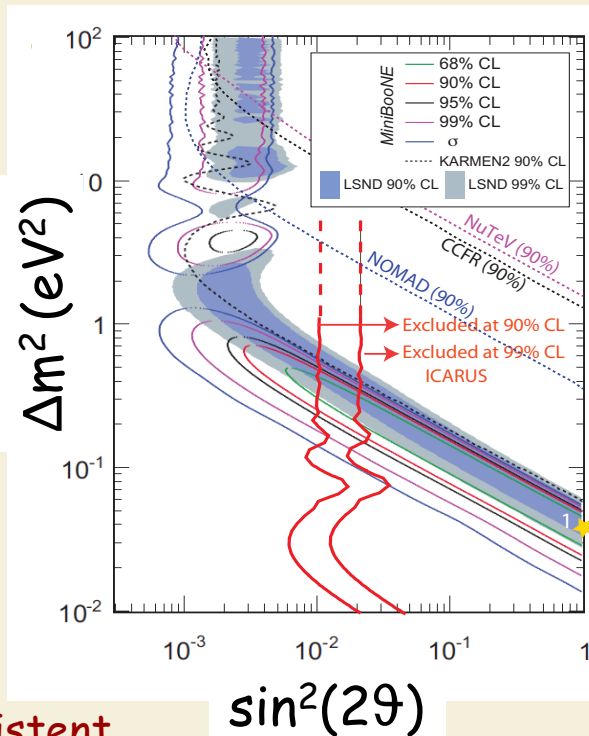
$3.8\sigma$  [signal from low-energy region]



parameter space limited by negative results from Karmen and ICARUS

$$\vartheta_{e\mu} \approx 0.035$$

$$\Delta m^2 \approx 0.5 \text{ eV}^2$$

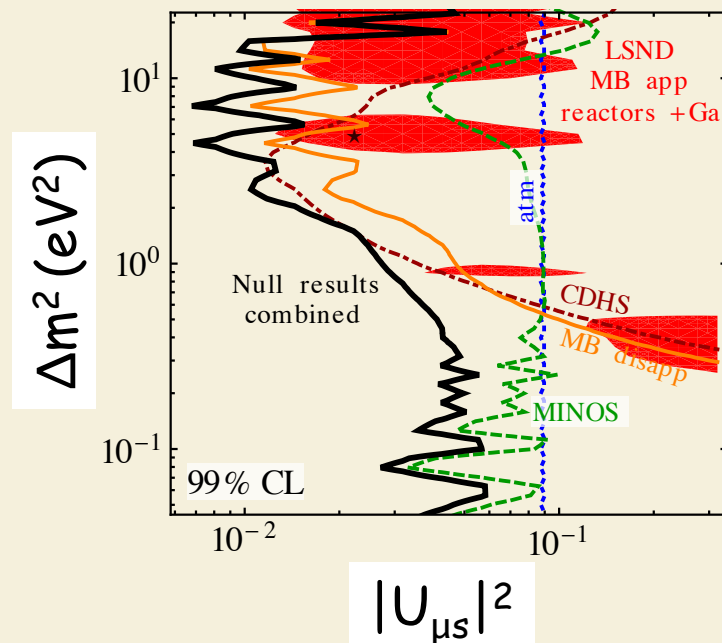


3 interpretation in 3+1 scheme: **inconsistent** (more than 1s disfavored by cosmology)

$$\underbrace{\vartheta_{e\mu}}_{0.035} \approx \underbrace{\vartheta_{es}}_{0.2} \times \vartheta_{\mu s} \quad \Rightarrow \quad \vartheta_{\mu s} \approx 0.2$$

predicted suppression in  $\nu_\mu$  disappearance experiments: **undetected**

by ignoring LSND/Miniboone data the reactor anomaly can be accommodated by  $m_s \geq 1 \text{ eV}$  and  $\vartheta_{es} \approx 0.2$   
[not suitable for WDM, more on this later]



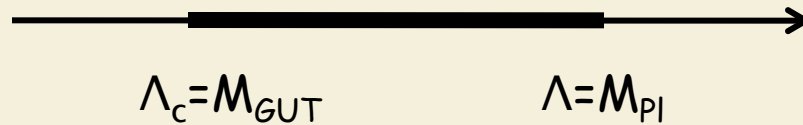


# other realizations of Anarchy (II)

Nelson-Strassler [0006251 “Suppressing Flavor Anarchy”]

Anarchy can arise when matter chiral supermultiplets  $X_i$  of the MSSM are coupled to a superconformal sector in some finite energy range

e.g.



large positive anomalous dimensions for  $X_i$ :

$$\frac{\gamma_i}{2} \equiv d(X_i) - 1 > 0$$

$$K = \sum_i Z_i X_i^+ X_i + \dots \quad Z_i(\Lambda_c) = \underbrace{Z_i(\Lambda)}_1 \left( \frac{\Lambda_c}{\Lambda} \right)^{-\gamma_i}$$

Anarchy through wave function renormalization:  $X_i \rightarrow F_{X_i} X_i$

$$w = Y_{ij} X_i X_j H + \dots \rightarrow (F_{X_i} Y_{ij} F_{X_j}) X_i X_j H + \dots$$

$$F_{X_i} = \left( \frac{\Lambda_c}{\Lambda} \right)^{\frac{\gamma_i}{2}} < 1$$

[as in FN with a single flavon and positive FN charges]

**no underlying flavour symmetry**

[an anomaly free R symmetry is generated dynamically at the IR stable fixed point:  $\dim(X_i) = 2/3$   $R(X_i)$ ]

**anomalous dimensions  $\gamma_i$  calculable** when gauge group and field content are known

[Polland, Simmons-Duffin 0910.4585]